The following question given below consist of an "Assertion" (A) and "Reason" (R) Type questions. Use the following Key to choose the appropriate answer.

(A) If both (A) and (R) are true, and (R) is the correct explanation of (A).

(B) If both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) If (A) is true but (R) is false.

(D) If (A) is false but (R) is true.

Q.1 **Assertion :** The coefficient of viscosity of a liquid is the viscous force acting on unit area of a liquid layers having unit velocity gradient perpendicular to the direction of flow.

Reason : Viscous force is directly proportional to area of liquid layer and velocity gradient.

Sol. **[A]**

$$F_v = -\eta A \frac{dv}{dy}$$

Q.2 **Assertion** : Buoyancy force and viscous force of the air reduce the descending speed of parachute.

Reason : Viscous force acts downward whereas buoyancy force acts upward on a body falling downward. **[C]**

Q.3 **Assertion** : Bigger rain drops have smaller terminal speeds.

Reason : Terminal speed of a body depends on its size and density. **[D]**

Q.4 **Assertion :** The coefficient of viscosity of a liquid is the viscous force acting on unit area of a liquid layers having unit velocity gradient perpendicular to the direction of flow.

Reason : Viscous force is directly proportional to area of liquid layer and velocity gradient.

Sol. **[A]**

$$F_v = -\eta A \frac{dv}{dy}$$

Q.1 A solid sphere moves at a terminal velocity of 20 m/s in air at a place where g = 9.8 m/s^2. The sphere is taken in a gravity free hall having air at the same pressure and pushed down at a speed of 20 m/s -

(A) Its initial acceleration will be 9.8 m/s^2 downward

(B) Its initial acceleration will be 9.8 m/s^2 upward

(C) The magnitude of acceleration will decrease as the time passes

(D) It will eventually stop **[B,C,D]**

Q.2 The viscous force acting on a solid ball of surface area A moving with terminal velocity v is proportional to :

(A) A (B) $A^{1/2}$

(C) v (D) $v^{1/2}$ **[B, C]**

Sol. $F = 6\pi\eta rv$

$\therefore F \propto v$ and $F \propto r$ or $F \propto A^{1/2}$

Q.3 An oil drop falls through air with a terminal velocity of 5×10^{-4} m/s. Viscosity of oil is 1.8×10^{-5} N-s/m^2 and density of oil is 900 kg/m^3. Neglecting density of air as compared to that of the oil :

(A) radius of the drop is 6.20×10^{-2} m

(B) radius of the drop is 2.14×10^{-6} m

(C) terminal velocity of the drop at half of this radius is 1.25×10^{-4} m/s

(D) terminal velocity of the drop at half of this radius is 2.5×10^{-4} m/s **[B, C]**

Sol. Viscous force = weight

$$6\pi\eta rv = \frac{4}{3}\pi r^3 \rho g \qquad \ldots (1)$$

Substituting the values we can find r. Further, $v \propto r^2$

It r is halved, terminal velocity will remain $\frac{1}{4}$th.

PHYSICS

Q.1 What is the velocity v of a metallic ball of radius r falling in a tank of liquid at the instant when its acceleration is one half that of a freely falling body ? (The densities of metal and of liquid are ρ and σ respectively and the viscosity coefficient of the liquid is η) -

(A) $\frac{r^2 g}{9\eta}(\rho - 2\sigma)$ (B) $\frac{r^2 g}{9\eta}(2\rho - \sigma)$

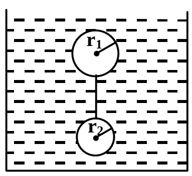
(C) $\frac{r^2 g}{9\eta}(\rho - \sigma)$ (D) $\frac{2r^2 g}{9\eta}(\rho - \sigma)$

[C]

Sol. $\eta = \frac{2r^2}{9}\left(\frac{\rho-\sigma}{V}\left(\frac{g}{2}\right)\right)$

So $V = \frac{r^2 g(\rho-\sigma)}{9\eta}$

Q.2 Two solid spherical balls of radius r_1 & r_2 ($r_2 < r_1$), of density σ are tied up with a string and released in a viscous liquid of lesser density ρ and coefficient of viscosity η, with the string just taut as shown. The terminal velocity of spheres is-

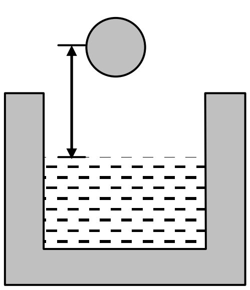


(A) $\frac{2}{9}\frac{r_2^2 g}{\eta}(\sigma-\rho)$ (B) $\frac{2}{9}\frac{r_1^2 g}{\eta}(\sigma-\rho)$

(C) $\frac{2}{9}\frac{(r_1^3+r_2^3)}{r_1+r_2}\frac{(\sigma-\rho)g}{\eta}$ (D) $\frac{2}{9}\frac{(r_1^3-r_2^3)}{r_1-r_2}\frac{(\sigma-\rho)g}{\eta}$

[C]

Q.3 A ball of radius r and density ρ falls freely under gravity through a distance h before entering water. Velocity of ball does not change even on entering water. If viscosity of water is η, the value of h is given by -

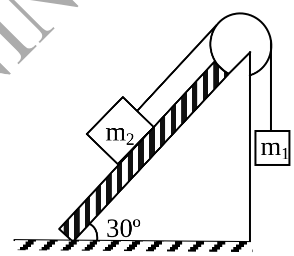


(A) $\frac{2}{9}r^2\left(\frac{1-\rho}{\eta}\right)g$ (B) $\frac{2}{81}r^2\left(\frac{\rho-1}{\eta}\right)g$

(C) $\frac{2}{81}r^4\left(\frac{\rho-1}{\eta}\right)^2 g$ (D) $\frac{2}{9}r^4\left(\frac{\rho-1}{\eta}\right)^2 g$

[C]

Q.4 The mass of block m_1 = 4 kg and m_2 = 20 kg, m_2 slides on the incline on a film of oil 7μm thick. Assume linear velocity profile. Block m_2 is cube of length 10cm. viscosity of oil is 7×10^{-3} Pa-s : Terminal velocity of blocks is –



(A) 2 m/s (B) 3 m/s

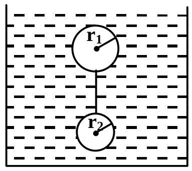
(C) 4 m/s (D) 5 m/s **[B]**

Q.5 The velocity of a small ball of mass M and density d_1, when dropped in a container filled with glycerine becomes constant after some time. If the density of glycerine is d_2, the viscous force acting on the ball will be

(A) $\frac{M d_1 g}{d_2}$ (B) $M g\left(1-\frac{d_2}{d_1}\right)$

(C) $\frac{M(d_1+d_2)}{g}$ (D) $M d_1 d_2$ **[B]**

Q.6 Two solid spherical balls of radius r_1 & r_2 ($r_2 < r_1$), of density σ are tied up with a string and released in a viscous liquid of lesser density ρ and coefficient of viscosity η, with the string just taut as shown. The terminal velocity of spheres is-



(A) $\frac{2}{9}\frac{r_2^2 g}{\eta}(\sigma-\rho)$

(B) $\frac{2}{9}\frac{r_1^2 g}{\eta}(\sigma-\rho)$

(C) $\frac{2}{9}\frac{(r_1^3+r_2^3)}{r_1+r_2}\frac{(\sigma-\rho)g}{\eta}$

(D) $\frac{2}{9}\frac{(r_1^3-r_2^3)}{r_1-r_2}\frac{(\sigma-\rho)g}{\eta}$

Sol. **[C]** at terminal velocity net force is zero.

$6\pi\eta(r_1+r_2)\,V_T+\frac{4}{3}\pi\,(r_1^3+r_2^3)\rho g=\frac{4}{3}\pi\,(r_1^3+r_2^3)\,\sigma g$

Q. 7 A wide jar is filled with glycerin having specific gravity 1.26, in this jar a steel ball of radius 0.25 cm has been dropped. After some time it has observed that ball is taking equal interval of time 1.8 sec to cover equal successive distances of 20 cm. The viscosity of glycerin in N-s/m^2 would be [ρ_{steal} = 7.8 × 10^3 kg/m^3, g = 9.8 m/s^2]-

(A) 0.802 (B) 1.67

(C) 0.76 (D) 0.963 **[A]**

Q.8 A solid ball of density ρ_1 and radius r falls vertically through a liquid of density ρ_2. Assume that the viscous force acting on the ball is F = krν, where k is a constant and ν its velocity. What is the terminal velocity of the ball ?

(A) $\frac{4\pi r^2(\rho_1-\rho_2)}{3k}$ (B) $\frac{2\pi r(\rho_1-\rho_2)}{3gk}$

(C) $\frac{2\pi g(\rho_1+\rho_2)}{3gr^2k}$ (D) none of these

[A]

Sol. Net force on the ball = 0
(when terminal velocity is attained).
Hence,
Weight = upthrust + viscous force

$\therefore\ \frac{4}{3}\pi r^3\rho_1 g = \frac{4}{3}\pi r^3\rho_2 g + kr\nu_T$

$\therefore\ \nu_T = \frac{4\pi g r^2}{3k}(\rho_1-\rho_2)$

Q.9 A newtonion fluid fills the clearance between a shaft and a sleeve. When a force of 800 N is applied to the shaft, parallel to the sleeve, the shaft attains a speed of 2 cm/s. If a force of 2.4 kN is applied instead, the shaft would move with a speed of -

(A) 2 cm/s (B) 15 cm/s

(C) 6 cm/s (D) None of these **[C]**

Sol. $F = \eta A\,\frac{v}{d}$

$\frac{F_1}{F_2}=\frac{v_1}{v_2}$

$\frac{800}{2400}=\frac{2}{v_2}$

v_2 = 6 cm/s

Q.10 A small drop of steel falls from rest through a long height h in coaltar, the final velocity will be proportional to h^n, then n is -

(A) 1/2 (B) 1

(C) – 1 (D) 0 **[D]**

Sol. $v \propto h^0$ so n is equal to zero.

Q.11 The velocity of a small ball of mass M and density d_1, when dropped in a container filled with glycerine becomes constant after some time. If the density of glycerine is d_2, the viscous force acting on the ball will be –

(A) $\frac{Md_1 g}{d_2}$ (B) $\frac{M(d_1+d_2)}{g}$

(C) $Mg\left(1-\frac{d_2}{d_1}\right)$ (D) M d_1 d_2 **[C]**

Q.12 A small lead ball is falling freely in a viscous liquid. The velocity of the ball –

(A) goes on increasing

(B) goes on decreasing

(C) remains constant

(D) first increases and then becomes constant

[D]

Q.13 The terminal velocity of a spherical ball of radius r falling through a viscous liquid is proportional to –

(A) r (B) r^2

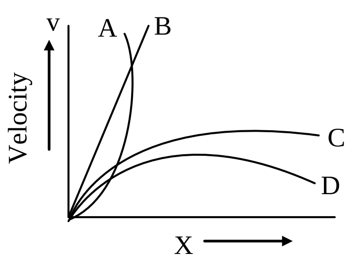
(C) r^3 (D) r^{-1} **[B]**

Q.14 The viscous force acting on a solid ball moving in air with terminal velocity ν is directly proportional to -

(A) $\sqrt{\nu}$ (B) ν

(C) $\frac{1}{\sqrt{\nu}}$ (D) ν^2 **[B]**

Q.15 A small spherical solid ball is dropped in a viscous liquid. Its journey in the liquid is best described in the figure by –



(A) Curve A (B) Curve B

(C) Curve C (D) Curve D **[C]**

Q.16 The relative velocity between two parallel layers of water is 8 cm/s and the perpendicular distance between them is 0.1 cm. Calculate the velocity- gradient -

(A) 90/s (B) 80.5 /s

(C) 80 /s D) None of these

[C]

Q.17 There is a 1 mm thick layer of oil between a flat plate of area 10^{-2} m^2 and a big plate. How much force is required to move the plate with a velocity of 1.5 cm/s^2 ? The coefficient of viscosity of oil is 1 poise -

(A) 1.5×10^{-3} N (B) 1.3×10^{-5} N

(C) 1.5×10^{-2} N (D) 1.5×10^{2} N **[C]**

Q.18 A steel shot of diameter 2 mm is dropped in a viscous liquid filled in a drum. Find the terminal speed of the shot. Density of the material of the shot = 8.0×10^3 kg/m^3, density of liquid = 1.0×10^3 kg/ m^3. Coefficient of viscosity of liquid = 1.0 kg/(m-s), g = 10 m/s^2

(A) 1.55 cm/s (B) 1.455 cm/s

(C) 5.1 cm/s (D) None of these

[A]

Q.19 An air bubble (radius 0.4 mm) rises up in water. If the coefficient of viscosity of water be 1×10^{-3} kg/(m–s), then determine the terminal speed of the bubble density of air is negligible-

(A) 0.843 m/s (B) 3.048 m/s

(C) 0.483 m/s (D) 0.348 m/s **[D]**

Q.20 If an oil drop of density 0.95×10^3 kg/ m^3 and radius 10^{-4} cm is falling in air whose density is 1.3 km/m^3 and coefficient of viscosity is 18×10^{-6} kg/(m–s). Calculate the terminal speed of the drop.

(A) 0.00015 cm/s (B) 0.0005 cm/s

(C) 0.0115 cm/s (D) None of these

[C]

Q.21 The terminal velocity of a ball in air is v, where acceleration due to gravity is g. Now the same ball is taken in a gravity free space where all other conditions are same. The ball is now pushed at a speed v, then –

(A) The terminal velocity of the ball will be v/2
(B) The ball will move with a constant velocity
(C) The initial acceleration of the ball is 2g in opposite direction of the ball's velocity
(D) The ball will finally stop (Given that density of the ball ρ = 2 times the density of air σ)

[D]

Q.22 A tank is filled up to a height 2H with a liquid and is placed on a platform of height H from the ground. The distance x from the ground where a small hole is punched to get the maximum range R is -

(A) H (B) 1.25 H

(C) 1.5 H (D) 2H **[A]**

Q.23 Which one of the following represents the correct dimensions of the coefficient of viscosity ? **[AIEEE-2004]**

(A) $ML^{-1}T^{-1}$ (B) MLT^{-1}

(C) $ML^{-1}T^{-2}$ (D) $ML^{-2}T^{-2}$ **[A]**

Q.24 Spherical balls of radius 'R' are falling in a viscous fluid of viscosity 'η' with a velocity 'v'. The retarding viscous force acting on the spherical ball is – **[AIEEE-2004]**

(A) inversely proportional to both radius 'R' and velocity 'v'

(B) directly proportional to both radius 'R' and velocity 'v'

(C) directly proportional to 'R' but inversely proportional to 'v'

(D) inversely proportional to 'R' but directly proportional to velocity 'v' **[B]**

Q.25 If the terminal speed of a sphere of gold (density = 19.5 kg/m^3) is 0.2 m/s in a viscous liquid (density = 1.5 kg/m^3), find the terminal speed of a sphere of silver (density = 10.5 kg/m^3) of the same size in the same liquid– **[AIEEE-2006]**

(A) 0.1 m/s (B) 0.2 m/s

(C) 0.4 m/s (D) 0.133 m/s **[A]**

Q.26 A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density ρ_2 ($\rho_2 < \rho_1$). Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed υ, i.e., $F_{viscous} = -k\upsilon^2$ (k > 0). The terminal speed of the ball is - **[AIEEE-2008]**

(A) $\frac{Vg\rho_1}{k}$ (B) $\frac{Vg\rho_1}{k}$

(C) $\frac{Vg(\rho_1-\rho_2)}{k}$ (D) $\sqrt{\frac{Vg(\rho_1-\rho_2)}{k}}$

[D]

Q.27 Two rain drops reach the earth with their terminal velocities in the ratio 4 : 9. The ratio of their radii is -

(A) 4 : 9 (B) 2 : 3 (C) 3 : 2 (D) 9 : 4

[B]

Sol. $v \propto r^2$; $\frac{r_1}{r_2} = \sqrt{\frac{v_1}{v_2}}$

Q.28 Blood is flowing at the rate of 100 cm^3/s in a capillary of cross-sectional area 0.25 m^2. The velocity of flow is -

(A) .1 mm/s (B) 0.2 mm/s

(C) 0.3 mm/s (D) 0.4 mm/s **[D]**

Sol. $Q = Av \Rightarrow v = \frac{Q}{A} = \frac{100\times10^{-6}}{0.25}$

$v = 400\times10^{-3}$ mm/s = 0.4 mm/s

Q.29 The mass of a lead ball is M. It falls down in a viscous liquid with terminal velocity V. The terminal velocity of another lead ball of mass 8M in the same liquid will be -

(A) 64 V (B) 4 V (C) 8 V (D) V

Sol. **[B]**

mass = $\frac{4}{3}\pi r^3 \times \rho$. so when mass become 8M so radius will become 2r & terminal velocity $V_t \propto r^2$ so it becomes 4 times of its previous value.

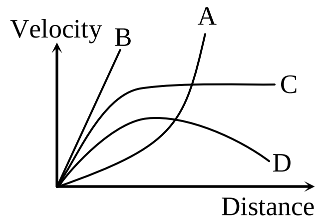
Q.30 When body falls in liquid with terminal velocity, the ratio of resistive force of liquid to its weight is-

(A) $\frac{2r^2\rho_s g}{9\eta^2}$ (B) $\frac{2r^2(\rho_s-\rho_m)g}{9\eta}$

(C) $\frac{2r^2\rho_m g}{9\eta}$ (D) 1

Sol. **[D]** $F_v + F_u = Mg$; so $\frac{F_v+F_u}{Mg} = 1$

Q.31 A small spherical solid ball is dropped in a viscous liquid. Its journey in the liquid is best described in figure drawn by -



(A) curve A (B) curve B
(C) curve C (D) curve D

Sol. **[C]**

Q.32 A liquid whose coefficient of viscosity is η flows on a horizontal surface. Let dx represent the vertical distance between two layers of liquid and dv represent the difference in the velocities of the two layers. Then the quantity $\eta(dv/dx)$ has the same dimensions as -

(A) acceleration (B) force
(C) momentum (D) pressure

Sol. **[D]** $F = \eta A \dfrac{dv}{dx} \Rightarrow \dfrac{F}{A} = \eta \dfrac{dv}{dx}$ = Dimension of pressure

Q.33 Viscosity is closely related to -

(A) friction
(B) adhesive molecular force
(C) cohesive molecular force
(D) Barnoulli's theorem

Sol. [A] Viscous force is a opposing force.

Q.34 Water sticks to glass because of -

(A) force of cohesion
(B) force of adhesion
(C) vander wall force
(D) None

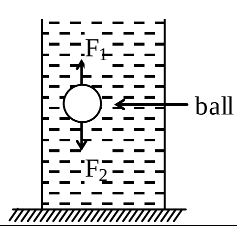
Sol. [B] Adhesive force of attraction act between molecules of different substances.

Q.35 With the increase in temperature viscosity of a liquid -

(A) increases (B) decreases
(C) remain same (D) None

Sol.[B] With increase of temperature, free flow of liquid increase, hence viscosity decrease.

Q.36 A sphere falls from top and travels through some distance in liquid when it attains terminal velocity. Then -



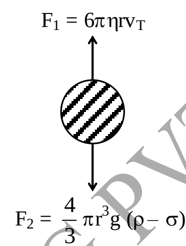
(A) F_2 is total weight of the sphere

(B) if F_2 is total weight of sphere then F_1 is upward force of viscosity

(C) If F_2 is net weight of body then F_1 is upward thrust

(D) None

Sol.[B]



Q.37 Two solid spheres of radii r_1 and r_2 have surfaces of same nature and are of same material. Both spheres are at same high temperature 'T'. They are allowed to cool under same conditions. Initial rate of heat loss is -

(A) $\left(\dfrac{r_2}{r_1}\right)^2$ (B) $\left(\dfrac{r_1}{r_2}\right)^2$ (C) $\sqrt{\dfrac{r_1}{r_2}}$ (D) $\sqrt{\dfrac{r_2}{r_1}}$

Sol.[B] Since ratio of rate of loss of heat is equal to ratio between rate of emission, we have

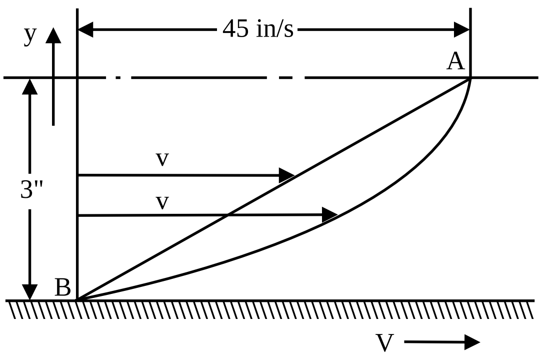
$$\frac{Q_1}{Q_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{eA_1\sigma\theta^4}{eA_2\sigma\theta^4} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

Q.38 Two spheres A and B fall through the same viscous fluid. A and B have the same density and B has the larger radius -

(A) A has the larger terminal velocity
(B) A and B have the same terminal velocity
(C) B has the larger terminal velocity
(D) Insufficient information is given to reach a conclusion.

Sol. **[C]**

Q.1 The fluid flowing in Fig. has an absolute viscosity (μ) og 0.0010 lb.s/ft^2 and specific gravity of 0.913. Calculate the velocity gradient and intensity of shear stress at the boundary and at points 1 in, 2 in and 3 in from the boundary, assuming **(a)** a straight-line velocity distribution and **(b)** a parabolic velocity distribution. The parabola in the sketch has its vertex at A and origin at B.



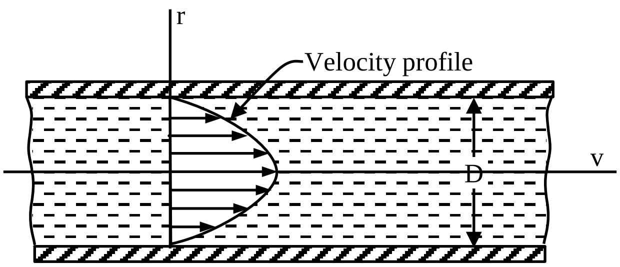
Sol. **(a)** For the straight-line assumption, the relation between velocity v and distance y is v = 15y, dv = 15dy. The velocity gradient = dv/dy = 15. Since $\mu = \tau$ (dv/dy), $\tau = \mu$ (dv/dy). For y = 0 (i.e., at the boundary), v = 0 and dv/dy = 15 s^{-1}; τ = (0.0010)(15) = 0.015 lb/ft^2. For y = 1 in, 2 in, and 3 in, dv/dy and τ are also 15 s^{-1} and 0.015 lb/ft^2, respectively. **(b)** For the parabolic assumption, the parabola pases through the points v = 0 when y = 0 and v = 45 when y = 3. The equation of this parabola is v = 45 – 5(3 – y)^2, dv/dy = 10(3 – y), τ = 0.0010 (dv/dy). For y = 0 in, v = 0 in/s, v = 0 in/s, dv/dy = 30 s^{-1}, and τ = 0.030 lb/ft^2. For y = 1 in, v = 25 in/s, dv/dy = 20 s^{-1}, and τ = 0.020 lb/ft^2. For y = 2 in, v = 40 in/s, dv/dy = 10 s^{-1}, and τ = 0.010 lb/ft^2. For y = 3 in, v = 45 in/s, dv/dy = 0 s^{-1}, and τ = 0 lb/ft^2.

Q.2 A cylinder of 0.040-ft radius rotates concentrically inside a fixed cylinder of 0.42-ft radius. Both cylinders are 1.00 ft long. Determine the viscosity of the liquid that fills the space between the cylinders if a torque of 0.650 lb. ft is required to maintain an angular velocity of 60 rpm.

Sol. The torque is transmitted through the field layers to the outer cylinder. Since the gap between the cylinders is small, the calculations may be made without integration. The tangential velocity v, of the inner cylinder = rω, where r = 0.40 ft and $\omega = 2\pi$ rad/s. Hence, v_t = (0.40)(2π) = 2.51 ft/s. For the small space between cylinders, the velocity gradient may be assumed to be a straight line and the mean radius can be used. Then,

dv/dy = (2.51 – 0)/(0.42 – 0.40) = 125.5 s^{-1}. Since applied torque equals resisting torque, applied torque = (τ)(area)(arm), 0.650 = τ[(1.00)(2π)(0.40 + 0.42)/2][(0.40 + 0.42)/2], τ = 0.615 lb/ft^2 = μ(dv/dy), 0.615 = (μ)(125.5), μ = 0.00490 lb.s/ft^2.

Q.3 Water is moving through a pipe. The velocity profile at some section is shown in Fig. and is given mathematically as v = ($\beta/4\mu$)(d^2/4 – r^2), where v = velocity of water at any position r, β = a constant, μ = viscosity of water, d = pipe diameter, and r = radial distance from centerline. What is the shear stress at the wall of the pipe due to the water? What is the shear stress at a position r = d/4? If the given profile persists a distance L along the pipe, what drag is induced on the pipe by the water in the direction of flow over this distance?



Sol. $v = (\beta/4\mu)(d^2/4 - r^2)$ $\quad dv/dr = (\beta/4\mu)(-r^2) = -2\beta r/4\mu$

$\tau = \mu\,(dv/dr) = \mu(-2\beta r/4\mu) = -2\beta r/4$

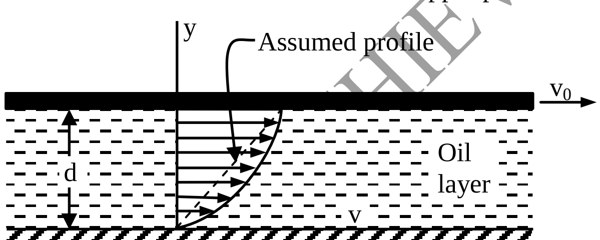
At the wall, r = d/2. Hence,

$\tau_{wall} = \dfrac{-2\beta(d/2)}{4} = -\dfrac{\beta d}{4} \qquad \tau_{r=d/4}$

$= \dfrac{-2\beta(d/4)}{4} = \dfrac{\beta d}{8}$

Drag = (τ_{wall})(area) = $(\tau_{wall})(\pi dL) = (\beta d/4)(\pi dL) = \beta d^2\pi L/4$.

Q.4 A large plate moves with speed v_0 over a stationary plate on a layer of oil **(see Fig.)**. If the velocity profile is that of a parabola, with the oil at the plates having the same velocity as the plates, what is the shear stress on the moving plate from the oil? If a linear profile is assumed, what is the shear stress on the upper plate?



Sol. For a parabolic profile, $v^2 = ay$. When y = d, $v = v_0$. Hence, $v_0^2 = ad$, $a = v_0^2/d$. Therefore,

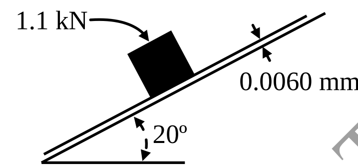
$v^2 = (v_0^2/d)(y) = (v_0^2)(y/d) \qquad v = v_0\sqrt{y/d}$

$dv/dy = [(v_0)(1/\sqrt{d})(½)(y^{-1/2})]$

$\tau = \mu(dv/dy) = \mu[(v_0)(1/\sqrt{d})(½)(y^{-1/2})]$

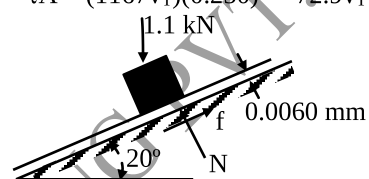
For y = d, $\tau = \mu[(v_0)(1/\sqrt{d})(½)(d^{-1/2})] = \mu v_0/(2d)$. For a linear profile, $dv/dy = v_0/d$, $\tau = \mu(v_0/d)$.

Q.5 A square block weighing 1.1 kN and 250 mm on an edge slides down an incline on a film of oil 6.0 μm thich **(see Fig.)**. Assuming a linear velocity profile in the oil, what is the terminal speed of the block? The viscosity of the oil is 7 mPa ⋅s.



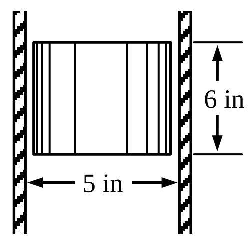
Sol. $\tau = \mu\,(dv/dy) = (7 \times 10^{-3})[v_T/(6.0 \times 10^{-6})] = 1167\,v_T$

$F_f = \tau A = (1167v_T)(0.250)^2 = 72.9v_T$



At the terminal condition, equilibrium occurs. Hence, 1100 sin 20º = $72.9v_T$, v_T = 5.16 m/s.

Q.6 A piston of weight 21 lb slides in a lubricated pipe, as shown in Fig. The clearance between piston and pipe is 0.001 in. If the piston decelerates at 2.1 ft/s² when the speed is 21 ft/s, what is the viscosity of the oil?

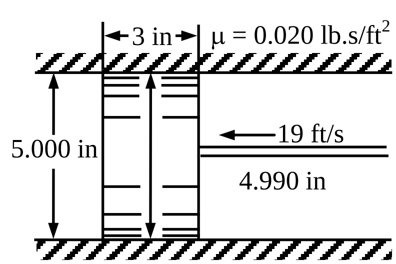


Sol. $\tau = \mu(dv/dy) = \mu[v/(0.001/12)] = 12000\,\mu v$

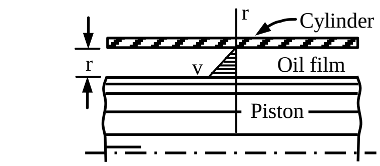
$F_f = \tau A = 12000\,\mu v[(\pi)\left(\frac{6}{12}\right)\left(\frac{5}{12}\right)] = 7854\,\mu v$

$\Sigma F = ma \quad 21 - (7854)(\mu)(21) = (21/32.2)(-2.1)$ $\quad \mu = 1.36 \times 10^{-4}$ lb.s/ft².

Q.7 A piston is moving through a cylinder at a speed of 19 ft/s, as shown in Fig. The film of oil separating the piston from the cylinder has a viscosity of 0.020 lb.s/ft². What is the force required to maintain this motion?



Sol. Assume a cylindrically symmetric, linear velocity profile for the flow of oil in the film.

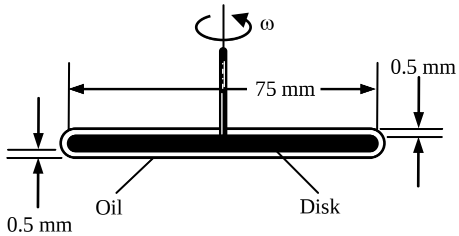


To find the frictional resistance, compute the shear stress at the piston surface.

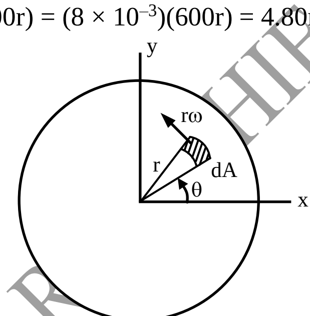
$$\tau = \mu\frac{dv}{dr} = 0.020\left[\frac{19}{(5.000-4.990)/2}\right](12) = 912 \text{ lb/ft}^2$$

$$F_f = \tau A = 912\left[\pi\left(\frac{4.990}{12}\right)\left(\frac{3}{12}\right)\right] = 298 \text{ lb.}$$

Q.8 To damp oscillations, the pointer of a galvanometer is fixed to a circular disk which turns in a container of oil (see Fig.). What is the damping torque for $\omega = 0.3$ rad/s if the oil has a viscosity of 8×10^{-3} Pa . s? Neglect edge effects.



Sol. Assume at any point that the velocity profile of the oil is linear $dv/dn = r\omega/(0.5/1000) = (r)(0.3)/(0.5/1000) = 600r$; $\tau = \mu(dv/dn) = \mu(600r) = (8 \times 10^{-3})(600r) = 4.80r$.

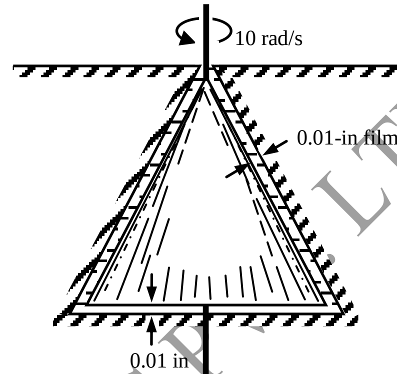


The force dF_f on dA on the upper face of the disc is then $dF_f = \tau dA = (4.80r)(r\, d\theta\, dr) = 4.80r^2\, d\theta\, dr$. The torque dT for dA on the upper face is then $dT = r\, dF_f = r(4.80r^2\, d\theta\, dr) = 4.80r^3\, d\theta\, dr$. The total resisting torque on both faces is

$$T = 2\left[\int_0^{0.075/2}\int_0^{2\pi} 4.80r^3 d\theta\, dr\right]$$

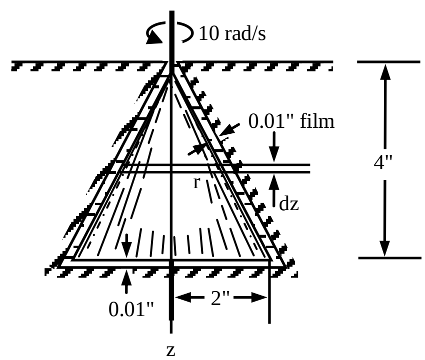
$$= (9.60)(2\pi)\left[\frac{r^4}{4}\right]_0^{0.075/2} = 2.98 \times 10^{-5} \text{ N.m}$$

Q.9 A conical body turns in a container, as shown in Fig., at constant speed 11 rad/s. A uniform 0.01-in film of oil with viscosity 3.125×10^{-7} lb.s/in^2 separates the cone from the container. What torque is required to maintain this motion, if the cone has a 2-in radius at its base and is 4 in tall?



Sol. Consider the conical surface first ($r/2 = z/4$, $r = z/2$). The stress on this element is $\tau = \mu(dv/dx) = \mu(r\omega/0.01) = (3.125 \times 10^{-7})[(z/2)(11)/0.01] = 1.719 \times 10^{-4}z$. The area of the strip shown is $dA = 2\pi r\, ds = (2\pi z/2)[dz/(4/\sqrt{20})] = 3.512z\, dz$. The torque on the strip is $dT = \tau(dA)(r) = (1.719 \times 10^{-4}z)(3.512z\, dz)(z/2) = 3.019 \times 10^{-4}z^3\, dz$.

$$T_1 = \int_0^4 3.019\times10^{-4}z^3\, dz = 3.019 \times 10^{-4}\left[\frac{z^4}{4}\right]_0^4 = 0.01932 \text{ in.lb}$$



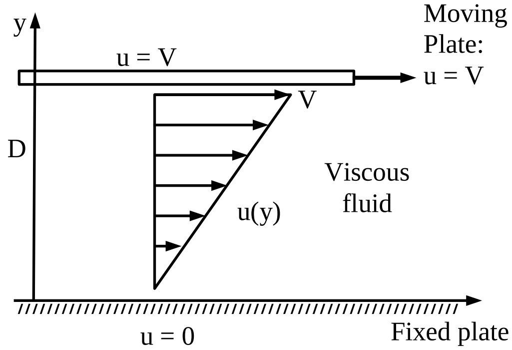
Next consider the base:

$dF_f = \tau dA$, $\tau = \mu(r\omega/0.01) = (3.125 \times 10^{-7})[(r)(11)/0.01] = 3.438 \times 10^{-4}r$, $dF_f = (3.438 \times 10^{-4}r)(r\, d\theta\, dr) = 3.438 \times 10^{-4}\, r^2\, d\theta\, dr$, $dT_2 = (3.438 \times 10^{-4}\, r^2\, d\theta\, dr)(r) = 3.438 \times 10^{-4}\, r^3\, d\theta\, dr$.

$$T_2 = \int_0^2\int_0^{2\pi} 3.438\times10^{-4}r^3\, d\theta\, dr = (3.438 \times 10^{-4})(2\pi)\left[\frac{r^4}{4}\right]_0^2 = 0.00864 \text{ in . lb}$$

$T_{tot} = 0.01932 + 0.00864 = 0.280$ in . lb

Q.10 In Fig., if the fluid is oil of viscosity 0.440 $Nm^{-2}S$ at 20ºC and D = 7 mm, what shear stress is required to move the upper plate at 3.5 m/s? Compute the Reynolds number based on D.



Sol. $\tau = \mu\,(dv/dh) = (0.440) = (0.440)\,[3.5/(7/1000)]$ = 220 Pa

$N_R = \rho Dv/\mu = (888)(7/1000)(3.5)/0.440 = 49.4$

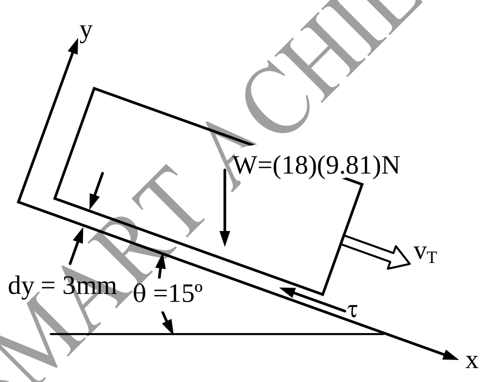
Q.11 An 18-kg slab slides down a 15º inclined plane on a 3-mm-thick film of oil of viscosity 8.14×10^{-2} $Nm^{-2}S$ at 20ºC; the contact area is 0.3 m^2. Find the terminal velocity of the slab.

Sol. See Fig.

$\Sigma F_s = 0 \quad W \sin\theta - \tau A_{bottom} = 0$

$\tau = \mu\,(dv/dy) = (8.14 \times 10^{-2})(v_T/0.003) = 27.1v_T$

$[(18)(9.81)](\sin 15º) - (27.1v_T)(0.3) = 0 \qquad v_T$ = 5.62 m/s



Q.12 A disk of radius r_0 rotates at angular velocity $\boldsymbol{\omega}$ inside an oil bath of viscosity $\boldsymbol{\mu}$, as shown in Fig. Assuming a linear velocity profile and neglecting shear on the outer disk edges, derive an expression for the viscous torque on the disk.

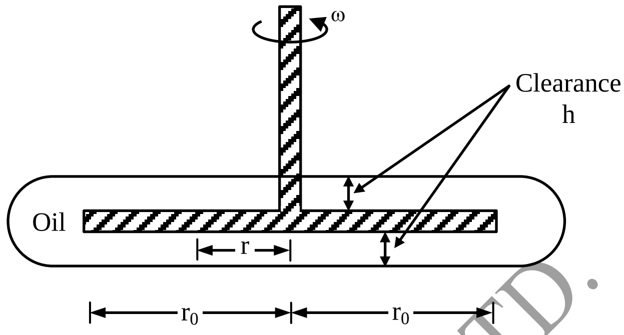


Fig.

Sol. $\tau = \mu\,(dv/dy) = \mu\,(r\omega/h)$ (on both sides)

$dT = (2)(r\tau dA) = (2)\,\{(r)[\mu(r\omega/h)](2\pi r dr)\} = (4\mu\omega\pi/h)(r^3 dr)$

$$T = \int_0^{r_0} \frac{4\mu\omega\pi}{h}\,(r^3 dr) = \frac{4\mu\omega\pi}{h}\left[\frac{r^4}{4}\right]_0^{r_0} = \frac{\pi\mu\omega\, r_0^4}{h}$$

Q.13 A Newtonian fluid fills the gap between a shaft and a concentric sleeve. When a force of 788 N is applied to the sleeve parallel to the shaft, the sleeve attains a speed of 2 m/s. If a 1400-N force is applied, what speed will the sleeve attain? The temperature of the sleeve remains constant.

Sol. $\tau = F/A = \mu\,(dv/dx)$; $F/dv = \mu A/dx$ = constant. Therefore, $F_1/dv_1 = F_2/dv_2$, $788/2 = 1400/dv_2$, $dv_2 = 3.55$ m/s.

Q.14 A 10.00-cm shaft rides in an 10.03-cm sleeve 12 cm long, the clearance space (assumed to be uniform) being filled with lubricating oil at 40ºC (μ = 0.11 Pa.s). Calculate the rate at which heat is generated when the shaft turns at 100 rpm.

Sol. $dv = \omega$ (circumference) $= 100/60[\pi(0.10)]$ = 0.5236 m/s

$dx = (0.1003 - 0.1000)/2 = 0.00015$ m

$\tau = \mu\,(dv/dx) = (0.11)(0.5236)/0.00015) = 384.0$ N/m^2

$F_f = \tau A = 384.0\,[\pi(0.12)(0.10)] = 14.48$ N

Rate of energy loss $= F_f v = (14.48)(0.5236)$ = 7.852 N . m/s = 7.582 W

Q.15 A body weighing 100 lb with a flat surface area of 3 ft^2 slides down a lubricated inclined plane making a 35º angle with the horizontal. For viscosity of 0.002089 lb . s/ft^2 and a body speed of 3.5 ft/s, determine the lubricant film thickness.

Sol. F = weight of body along inclined plane = 100 sin 35º = 57.4 lb

$\tau = F/A = \mu \ (dv/dx)$ 57.4/3 = (0.002089)(3.5/dx) dx = 0.0003821 ft or 0.00459 in.