Vectors

Multiple Correct Answer Type

1. Let $\bar{a}$ and $\bar{b}$ be two non collinear unit vectors. If $\bar{u} = \bar{a} - (\bar{a}.\bar{b})\bar{b}$ and $\bar{v} = \bar{a} \times \bar{b}$ then $|\bar{v}| =$

a) $|\bar{u}|$ b) $|\bar{u}| + |\bar{u}.\bar{a}|$

c) $|\bar{u}| + |\bar{u}.\bar{b}|$ d) $|\bar{u}| + \bar{u}.(\bar{a} + \bar{b})$

Key. A,C

Sol. Given $\bar{v} = \bar{a} \times \bar{b} \Rightarrow |\bar{v}| = |\bar{a}||\bar{b}|\sin\theta = \sin\theta$

$\bar{u} = \bar{a} - (\bar{a}.\bar{b})\bar{b}$ = $\bar{a} - \bar{b}\cos\theta$

$\Rightarrow |\bar{u}|^2 = (\bar{a} - \bar{b}\cos\theta)^2 = |\bar{a}|^2 + |\bar{b}|^2\cos^2\theta - 2\bar{a}.\bar{b}\cos\theta$

= $1 + \cos^2\theta - 2\cos^2\theta$

= $1 - \cos^2\theta$

= $\sin^2\theta$

$\Rightarrow |\bar{u}| = |\bar{v}|$

Again $\bar{u}.\bar{b} = \bar{a}.\bar{b} - (\bar{a}.\bar{b})(\bar{b}.\bar{b}) = 0$

$\Rightarrow |\bar{u}.\bar{b}| = 0$

2. Three vectors $\bar{a}\left(|\bar{a}| \neq 0\right)$, $\bar{b}$ and $\bar{c}$ are such that $\bar{a} \times \bar{b} = 3\bar{a} \times \bar{c}$. Also $|\bar{a}| = |\bar{b}| = 1$ and

$|\bar{c}| = \frac{1}{3}$. If the angle between $\bar{b}$ and $\bar{c}$ is 60^0, then.

a) $\bar{b} = 3\bar{c} + \bar{a}$ b) $\bar{b} = 3\bar{c} - \bar{a}$ c) $\bar{a} = 6\bar{c} + 2\bar{b}$ d) $\bar{a} = 6\bar{c} - 2\bar{b}$

Key. A,B

Sol. $\bar{a} \times (\bar{b} - 3\bar{c}) = \bar{0}$

$\Rightarrow \bar{b} - 3\bar{c} = \lambda\bar{a}$

$\Rightarrow |\bar{b} - 3\bar{c}| = |\lambda\bar{a}|$

$\Rightarrow 1 + 1 - 6.1.\frac{1}{3}.\frac{1}{2} = |\lambda| \Rightarrow \lambda = \pm 1$

$\therefore \bar{b} - 3\bar{c} = \pm\bar{a}$

3. If $\vec{a}$ and $\vec{b}$ are two unit vectors perpendicular to each other and $\vec{c} = \lambda_1\vec{a} + \lambda_2\vec{b} + \lambda_3(\vec{a} \times \vec{b})$,

then the following is (are) true

a) $\lambda_1 = \vec{a}.\vec{c}$ b) $\lambda_2 = \left|\vec{b}\times\vec{a}\right|$

c) $\lambda_3 = \left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right|$ d) $\lambda_1+\lambda_2+\lambda_3 = \left(\vec{a}+\vec{b}+\vec{a}\times\vec{b}\right).\vec{c}.$

Key. A,D

Sol. (a) is proved if we take dot product of both sides with $\vec{a}$.

(b) If we take dot produuct with $\vec{b}$, we get

$$\lambda_2 = \vec{b}.\vec{c}$$

$\Rightarrow$ Choice (b) is not true.

(c) If we take dot product of both sides with $\vec{a}\times\vec{b}$, we get $[\vec{c}\,\vec{b}\,\vec{a}] = \lambda_3[\vec{a}\times\vec{b}]^2$

$\Rightarrow \lambda_3 = [\vec{a}\,\vec{b}\,\vec{c}]$ OR $\vec{c}.(\vec{a}\times\vec{b})$

$\Rightarrow$ Choice (c) is wrong.

(d) is correct since $\lambda_1+\lambda_2+\lambda_3 = \vec{c}.\vec{a}+\vec{b}.\vec{c}+[\vec{a}\,\vec{b}\,\vec{c}]$.

4. $\vec{a} = (\cos q)\vec{i} - (\sin q)\vec{j}$, $\vec{b} = (\sin q)\vec{i} + (\cos q)\vec{j}$, $\vec{c} = \vec{k}$, $\vec{r} = 7\vec{i} + \vec{j} + 10\vec{k}$

if $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c}$, then

a) min. of x + y + z = 0 b) min. of x + y + z = 5

c) max. of x + y + z = 15 d) max. of x + y + z = 20

Key. A,D

Sol. $x = 7\cos q - \sin q,\ y = 7\sin q + \cos q,\ z = 10$

$x + y + z = 8\cos q + 6\sin q + 10$

min value = $10 - \sqrt{8^2 + 6^2} = 0$, max value = 10 + 10 = 20

5. If a vector $\vec{r}$ satisfies the equation $\vec{r}\times\left(\hat{i}+2\hat{j}+\hat{k}\right) = \hat{i}-\hat{k}$, then $\vec{r}$ is equal to

(A) $\hat{i}+3\hat{j}+\hat{k}$

(B) $3\hat{i}+7\hat{j}+3\hat{k}$

(C) $\hat{j}+t\left(\hat{i}+2\hat{j}+\hat{k}\right)$ where t is any scalar

(D) $\hat{i}+(t+3)\hat{j}+\hat{k}$ where t is any scalar

Key. A,B,C

Sol. $\vec{r}\times(\hat{i}+2\hat{j}+\hat{k})=\hat{i}-\hat{k}$

Let $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$\therefore \quad (x\hat{i}+y\hat{j}+z\hat{k})\times(\hat{i}+2\hat{j}+\hat{k})=\hat{i}-\hat{k}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 2 & 1 \end{vmatrix}=\hat{i}-\hat{k}$$

Put values from options and check.

6. If $\bar{a}$ *and* $\bar{b}$ are unit vectors and $\bar{c}$ is a vector such that $\bar{c}=\bar{a}\times\bar{c}+\bar{b}$ then

(A) $\left[\bar{a}\ \bar{b}\ \bar{c}\right]=\bar{b}.\bar{c}-(\bar{a}.\bar{b})^2$ (B) $\left[\bar{a}\ \bar{b}\ \bar{c}\right]=0$

(C) Maximum value of $\left[\bar{a}\ \bar{b}\ \bar{c}\right]=\frac{1}{2}$ (D) Minimum value of $\left[\bar{a}\ \bar{b}\ \bar{c}\right]$ is $\frac{1}{2}$

Key. A,C

Sol. $\bar{c}.\bar{a}=((\bar{a}\times\bar{c})+\bar{b}).\bar{a}=\bar{b}.\bar{a}$

$\bar{b}\times\bar{c}=(\bar{b}.\bar{c})+\bar{a}-(\bar{a}-\bar{b}).\bar{c}$

$\therefore \left[\bar{a}\bar{b}\bar{c}\right]=\bar{b}.\bar{c}-(\bar{a}-\bar{b}).(\bar{a}.\bar{c})$

Also $\bar{c}.\bar{b}=1-\left[\bar{a}\bar{b}\bar{c}\right]$

$\therefore 2\ \left[\bar{a}\bar{b}\bar{c}\right]=1-(\bar{a}.\bar{b})^2\le 1$

$\therefore \left[\bar{a}\bar{b}\bar{c}\right]\le\frac{1}{2}$

7. If a vector $\vec{r}$ satisfies the equation $\vec{r}\times(\hat{i}+2\hat{j}+\hat{k})=\hat{i}-\hat{k}$, then $\vec{r}$ is equal to

(A) $\hat{i}+3\hat{j}+\hat{k}$

(B) $3\hat{i}+7\hat{j}+3\hat{k}$

(C) $\hat{j}+t(\hat{i}+2\hat{j}+\hat{k})$ where t is any scalar

(D) $\hat{i}+(t+3)\hat{j}+\hat{k}$ where t is any scalar

Key. A,B,C

Sol. $\vec{r}\times(\hat{i}+2\hat{j}+\hat{k})=\hat{i}-\hat{k}$

Let $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$\therefore \quad (x\hat{i}+y\hat{j}+z\hat{k})\times(\hat{i}+2\hat{j}+\hat{k})=\hat{i}-\hat{k}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 2 & 1 \end{vmatrix}=\hat{i}-\hat{k}$$

Put values from options and check.

8. In a four-dimensional space where unit vectors along axes are $\hat{i}, \hat{j}, \hat{k}$ and $\hat{\ell}$ and $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ are four non zero vectors such that no vector can be expressed as linear combination of others and $(\lambda-1)(\vec{a}_1-\vec{a}_2)+\mu(\vec{a}_2+\vec{a}_3)+\gamma(\vec{a}_3+\vec{a}_4-2\vec{a}_2)+\vec{a}_3+\delta\vec{a}_4=\vec{o}$ then

(A) $\lambda=1$ (B) $\mu=-\frac{2}{3}$ (C) $\lambda=\frac{2}{3}$ (D) $\delta=\frac{1}{3}$

Key. A,B,D

Sol. (a, b, d)

$$(\lambda-1)(\vec{a}_1-\vec{a}_2)+\mu(\vec{a}_2+\vec{a}_3)+\gamma(\vec{a}_3+\vec{a}_4-2\vec{a}_2)+\vec{a}_3+\delta\vec{a}_4=\vec{o}$$

i.e $(\lambda-1)\vec{a}_1+(1-\lambda+\mu-2\gamma)\vec{a}_2+(\mu+\gamma+1)\vec{a}_3+(\gamma+\delta)\vec{a}_4=0$

since $\vec{a}_1,\vec{a}_2,\vec{a}_3,\vec{a}_4$ are linearly independent

$\therefore$ $\lambda-1=0, 1-\lambda+\mu-2\gamma=0, \mu+\lambda+1=0$ $\gamma+\delta=0$

i.e. $\lambda=1, \mu=2\gamma, \mu+\gamma+1=0, \gamma+\delta=0$

i.e. $\lambda=1, \mu=-\frac{2}{3}, \gamma=-\frac{1}{3}, \delta=\frac{1}{3}$

9. A vector $(\vec{d})$ is equally inclined to three vectors $\vec{a}=\hat{i}-\hat{j}+\hat{k}, \vec{b}=2\hat{i}+\hat{j}$ and $\vec{c}=3\hat{j}-2\hat{k}$. Let $\vec{x},\vec{y},\vec{z}$ be three vector in the plane of $\vec{a},\vec{b};\vec{b},\vec{c};\vec{c},\vec{a}$ respectively then

(A) $\vec{x}.\vec{d}=14$ (B) $\vec{y}.\vec{d}=3$

(C) $\vec{z}.\vec{d}=0$ (D) $\vec{r}.\vec{d}=0$ where $\vec{r}=\lambda\vec{x}+\mu\vec{y}+\delta\vec{z}$

Key. C,B

Sol. (c, d)

since $[\vec{a},\vec{b},\vec{c}]=0$

$\therefore$ $\vec{a},\vec{b}$ and $\vec{c}$ are complanar vectors

Further since $\vec{d}$ is equally inclined to $\vec{a},\vec{b}$ and $\vec{c}$

$\therefore$ $\vec{d}.\vec{a}=\vec{d}.\vec{b}=\vec{d}.\vec{c}=0$

$\therefore$ $\vec{d}.\vec{r}=0$

10. Identify the statement(s) which is/are incorrect ?

(A) $\vec{a}\times[\vec{a}\times(\vec{a}\times\vec{b})]=(\vec{a}\times\vec{b})(\vec{a}^2)$

(B) If $\vec{a},\vec{b},\vec{c}$ are non-zero, non coplanar vector and $\vec{v}.\vec{a}=\vec{v}.\vec{b}=\vec{v}.\vec{c}=0$ then $\vec{v}$ must be a null vector.

(C) If $\vec{a}$ and $\vec{b}$ lie in a plane normal to the plane contaning the vectors $\vec{c}$ and $\vec{d}$ then $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{o}$

(D) If $\vec{a},\vec{b},\vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vectors then $\vec{a}.\vec{b}'+\vec{b}.\vec{c}'+\vec{c}.\vec{a}'=3$

Key. A,C,D

Sol. (a, c, d)

(A) $\vec{a}\times[\vec{a}\times(\vec{a}\times\vec{b})]=\vec{a}\times[(\vec{a}\,.\,\vec{b})\vec{a}-(\vec{a}.\vec{a})\,b]=-(\vec{a}.\vec{a})(\vec{a}\times\vec{b})$

(A) is not correct

(B) Let $\vec{a},\vec{b},\vec{c}$ ne no coplanar vector

then $\vec{v}=\alpha\vec{a}+\beta\vec{b}+\gamma\vec{c}$

now $\vec{v}.\vec{a}=0$

$\Rightarrow$ $\alpha(\vec{a}.\vec{a})+\beta(\vec{b}.\vec{a})+\gamma(\vec{c}.\vec{a})=0$ ......(1)

and similarly

$\alpha(\vec{a}.\vec{b})+\beta(\vec{b}.\vec{b})+\gamma(\vec{c}.\vec{b})=0$ ....(2)

$\alpha(\vec{a}.\vec{c})+\beta(\vec{b}.\vec{c})+\gamma(\vec{c}.\vec{c})=0$ ....(3)

here $\begin{vmatrix} \vec{a}.\vec{a} & \vec{b}.\vec{a} & \vec{c}.\vec{a} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} & \vec{c}.\vec{b} \\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c} \end{vmatrix}=[\vec{a}\,\vec{b}\,\vec{c}]\neq 0$

Equation (1) (2) and (3) will have only one solution i.e. $\alpha=\beta=\gamma=0$

$\therefore$ (B) is true

(C) Let $\vec{a}.\vec{b}$ lie in the plane P_1

$\therefore$ $\vec{a}\times\vec{b}\perp P_1$

Let $\vec{c},\vec{d}$ lie in the plane P_2

$\therefore$ $\vec{c}\times\vec{d}\perp P_2$

as P_1 & P_2 are $\perp\ \perp$ to each other.

$\therefore$ $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})=0\,\&\,(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})\neq 0$

(D) $\vec{a}.\vec{b}'+\vec{b}.\vec{c}'+\vec{c}.\vec{a}=0$ (property of reciprocal system)

11. The equation of a plane is 2x – y – 3z = 5 and A(1, 1, 1), B(2, 1, –3), C(1, –2, –2) and D(–3, 1, 2) are four points. Which of the following line segments are intersected by the plane?

(A) AD (B) AB (C) AC (D) BC

Key. B,C

Sol. For A(1, 2, 3), 2x – y – 3z – 5 = 2 – 1 – 3 – 5 < 0

For B(2, 1, –3), 2x – y – 3z – 5 = 4 – 1 + 9 – 5 > 0

For C(1, –2, –2), 2x – y – 3z – 5 = 2 + 2 + 6 – 5 > 0

For D(–3, 1, 2), 2x – y – 3z – 5 = – 6 – 1 – 6 – 5 < 0

$\therefore$ AD are on one side of the plane and B, C are on the other side

$\therefore$ the line segments AB, AC, BD, CD intersect the plane.

12. If $\vec{a}$, $\vec{b}$, $\vec{c}$ be three non zero vectors satisfying the condition $\vec{a} \times \vec{b} = \vec{c}$ & $\vec{b} \times \vec{c} = \vec{a}$ then

(A) $\vec{a}$, $\vec{b}$, $\vec{c}$ are orthogonal in pairs (B) $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = |\vec{a}|^2$

(C) $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = |\vec{c}|^2$ (D) $|\vec{b}| = |\vec{c}|$

Key. B,C

Sol. Clearly $\vec{a} \cdot \vec{c} = 0$ & $\vec{b} \cdot \vec{c} = 0$ Also $\vec{a} \cdot \vec{b} = 0 \Rightarrow$ A

Again $\left.\begin{matrix} |\vec{a}|\,|\vec{b}| = |\vec{c}| \\ |\vec{b}|\,|\vec{c}| = |\vec{a}| \end{matrix}\right] \Rightarrow \dfrac{|\vec{a}|}{|\vec{c}|} = \dfrac{|\vec{c}|}{|\vec{a}|} \Rightarrow |\vec{a}| = |\vec{c}|$ & $|\vec{b}| = 1$

$\Rightarrow \vec{a} \times \vec{b} \cdot \vec{c} = |\vec{a}|\,|\vec{b}|\,|\vec{c}| = |\vec{a}|^2 = |\vec{c}|^2$]

13. If $\vec{r} = \hat{i} + \hat{j} + \lambda\left(2\hat{i} + \hat{j} + 4\hat{k}\right)$ and $\vec{r} \cdot \left(\hat{i} + 2\hat{j} - \hat{k}\right) = 3$ are the equations of a line and a plane respectively then which of the following is incorrect?

(A) line is perpendicular to the plane

(B) line lies in the plane

(C) line is parallel to the plane but does not lie in the plane

(D) line cuts the plane obliquely

Key. A,C,D

Sol. Since $(2\hat{i} + \hat{j} + 4\hat{k}).(\hat{i} + 2\hat{j} - \hat{k}) = 0$ and, $(\hat{i} + \hat{j}).(\hat{i} + 2\hat{j} - \hat{k})$ 1 + 2 = 3 $\Rightarrow$ line lies in the plane

14. If $\bar{r}$ is a vector satisfying $\bar{r} \times \left(\hat{i} + j + 2k\right) = \hat{i} - j$ then $|\bar{r}|$ can be

A) π B) e C) $\dfrac{1}{3}$ D) $\dfrac{1}{\sqrt{5}}$

Key. A,B

Sol. Solving the equation we get $\bar{r} = \hat{i} + \hat{j} + \hat{k} + \lambda\left(\hat{i} + \hat{j} + 2\hat{k}\right),\ \lambda \in R$

15. If each of $\bar{a}, \bar{b}, \bar{c}$ is orthogonal to the sum of the other two vectors and $|\bar{a}| = 3,\ |\bar{b}| = 4,\ |\bar{c}| = 5$ then which of the following statement(s) is/are true

a) if $\vec{a}$ makes angles of equal measures with x,y,z axes, then tangent of this angle is $\pm\sqrt{2}$

b) range of $|\bar{a} - \bar{b}|$ is $[1,\ 7]$ c) range of $|\bar{b} - \bar{c}|$ is $[1,\ 9]$ d) $|\bar{a} + \bar{b} + \bar{c}| = 2\sqrt{5}$

Sol : ans: a

a)according to the given condition

$a_1 = a_2 = a_3$ $\qquad a_1 = \pm\dfrac{1}{\sqrt{3}}$

$\cos\alpha = \pm\dfrac{1}{\sqrt{3}} \Rightarrow \tan\alpha = \pm\sqrt{2}$

b) $|\bar{a}-\bar{b}|^2 = 1 \text{ or } 49$ c) $|\bar{b}-\bar{c}|^2 = b^2+c^2-2\bar{b}.\bar{c} = 1 \text{ or } 81$

d) $|\bar{a}+\bar{b}+\bar{c}|^2 = 50+0 \Rightarrow |\bar{a}+\bar{b}+\bar{c}| = 5\sqrt{2}$

16. The position vector of a point P is $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$, when x, y, z $\in$ N and $\vec{a} = \hat{i}+\hat{j}+\hat{k}$. If $\vec{r}.\vec{a} = 10$, the number of possible position of P is

(A) 36 (B) 72

(C) 66 (D) ${}^9C_2$

Key : A, D

Sol : $\because \vec{r}.\vec{a} = 10$

$\therefore$ x + y + z = 10; x $\geq$ 1, y $\geq$ 1, z $\geq$ 1

The required number of positions

$= {}^{10-1}C_{3-1} = {}^9C_2 = 36$

17. Let $\bar{a}$ and $\bar{b}$ be two non collinear unit vectors. If $\bar{u} = \bar{a} - (\bar{a}.\bar{b})\bar{b}$ and $\bar{v} = \bar{a}\times\bar{b}$ then $|\bar{v}|$ =

a) $|\bar{u}|$ b) $|\bar{u}| + |\bar{u}.\bar{a}|$ c) $|\bar{u}| + |\bar{u}.\bar{b}|$ d) $|\bar{u}| + \bar{u}.(\bar{a}+\bar{b})$

Key. A,C

Sol. Given $\bar{v} = \bar{a}\times\bar{b} \Rightarrow |\bar{v}| = |\bar{a}||\bar{b}|\sin\theta = \sin\theta$

$\bar{u} = \bar{a} - (\bar{a}.\bar{b})\bar{b} = \bar{a} - \bar{b}\cos\theta$

$\Rightarrow |\bar{u}|^2 = (\bar{a} - \bar{b}\cos\theta)^2 = |\bar{a}|^2 + |\bar{b}|^2\cos^2\theta - 2\bar{a}.\bar{b}\cos\theta$

$= 1+\cos^2\theta - 2\cos^2\theta$

$= 1-\cos^2\theta$

$= \sin^2\theta$

$\Rightarrow |\bar{u}| = |\bar{v}|$

Again $\bar{u}.\bar{b} = \bar{a}.\bar{b} - (\bar{a}.\bar{b})(\bar{b}.\bar{b}) = 0$

$\Rightarrow |\bar{u}.\bar{b}| = 0$

18. If the unit vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle 2θ such that $|\vec{a}-\vec{b}| < 1$ and $0 \leq \theta \leq \pi$, then θ lies in the interval

(A) [0, π/6) (B) (5π/6, π]

(C) (π/6, π/2] (D) [π/2, 5π/6)

Key. A,B

Sol. Since, $\vec{a}$ and $\vec{b}$ are unit vectors, we have

$$|\vec{a}-\vec{b}| = \sqrt{(\vec{a}-\vec{b})^2}$$

$$\therefore \sqrt{(\vec{a})^2+(\vec{b})^2-2\vec{a}.\vec{b}} = \sqrt{1+1-2\cos 2\theta} = 2|\sin\theta|$$

Therefore, $|\vec{a}-\vec{b}|<1$

$\Rightarrow \quad 2|\sin\theta|<1$

$|\sin\theta|<\dfrac{1}{2}$

$\Rightarrow \quad \theta \in \left[0, \dfrac{\pi}{6}\right)$

or $\left(\dfrac{5\pi}{6}, \pi\right]$

19. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-zero, non-collinear vectors such that a vector $\vec{p}$ = a b cos $\left(2\pi - \left(\vec{a} \wedge \vec{b}\right)\right)$ $\vec{c}$ and a vector $\vec{q}$ = a c cos $\left(\pi - \left(\vec{a} \wedge \vec{c}\right)\right)$ $\vec{b}$ then $\vec{p}$ + $\vec{q}$ is

(A) parallel to $\vec{a}$ (B) perpendicular to $\vec{a}$

(C) coplanar with $\vec{b}$ & $\vec{c}$ (D) coplanar with $\vec{a}$ and $\vec{c}$

Key. B,C

Sol. $\vec{p} = a\,b\cos(2\pi-\theta)\vec{c}$ where θ is the angle between $\vec{a}$ and $\vec{b}$ and

$\vec{q} = a\,c\cos(\pi-\varphi)\vec{b}$ where ϕ is the angle between $\vec{a}$ and $\vec{c}$

Now $\vec{p}+\vec{q}$ = $(a\,b\cos\theta)\vec{c} - a\,c\,\cos\varphi\vec{b}$ = $(\vec{a}.\vec{b})\vec{c}-(\vec{a}.\vec{c})\vec{b}$ = $\vec{a}\times(\vec{c}\times\vec{b})$ $\Rightarrow$ B and C

20. Given three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ such that they are non – zero, non – coplanar vectors, then which of the following are coplanar.

(A) $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$ (B) $\vec{a}-\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$

(C) $\vec{a}+\vec{b}$, $\vec{b}-\vec{c}$, $\vec{c}+\vec{a}$ (D) $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}-\vec{a}$

Key. B,C,D

Sol. Verify $\vec{v}_1+\vec{v}_2=\vec{v}_3$ in order to quickly answer

21. Let OABC be a tetrahedron whose four faces are equilateral triangles of unit side. Let $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{OB}=\vec{b}$ and $\overrightarrow{OC}=\vec{c}$, then

(A) $\vec{c}=\dfrac{1}{3}\left(\vec{a}+\vec{b}\pm2\sqrt{2}\,\vec{a}\times\vec{b}\right)$ (B) $\vec{c}=\dfrac{1}{2}\left(\vec{a}+\vec{b}\pm2\sqrt{3}\,\vec{a}\times\vec{b}\right)$

(C) volume of the tetrahedron is $\dfrac{1}{2\sqrt{3}}$ (D) $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right| = \dfrac{1}{\sqrt{2}}$

Key. A,D

Sol. Let $\vec{C}=x\vec{a}+y\vec{b}+z(\vec{a}\times\vec{b})$. Taking succecive dots with $\vec{a},\vec{b},\vec{c}$ and $\vec{a}\times\vec{b}$ we get x = y = $\dfrac{1}{3}$ and z = ± $\dfrac{2\sqrt{2}}{3}$.

22. If $\vec{a},\vec{b},\vec{c},\vec{d}$ are unit vectors such that $\left(\vec{a}\times\vec{b}\right).\left(\vec{c}\times\vec{d}\right)=1$ and $\vec{a}.\vec{c}=\frac{1}{2}$ then

(A) $\vec{a},\vec{b},\vec{c}$ are non coplanar
(B) $\vec{b},\vec{d}$ are non parallel
(C) $\vec{b},\vec{c},\vec{d}$ are coplanar
(D) $\vec{a},\vec{d}$ are parallel and $\vec{b},\vec{c}$ are parallel

Key. B,C

Sol. $\left(\vec{a}\times\vec{b}\right).\left(\vec{c}\times\vec{d}\right)=1 \Rightarrow \sin\alpha\ \sin\beta\ ((\hat{n}_1.\hat{n}_2)=1 \Rightarrow \sin\alpha\ \sin\beta\ \cos\theta = 1$

$\Rightarrow \sin\alpha = 1,\ \sin\beta = 1$ and $\cos\theta = 1 \Rightarrow \alpha = \beta = \pi/2,\ \theta = 0$ i.e., $\hat{n}_1 \| \hat{n}_2$

So, $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar. Again $\vec{a}.\vec{c}=\frac{1}{2} \Rightarrow \cos\gamma = \frac{1}{2} \Rightarrow \gamma = \pi/3$

So, no two of vectors $\vec{a},\vec{b},\vec{c},\vec{d}$ are parallel.

23. *ABCDEFGH* is a regular octagon. If $\overline{AB}=\overline{a},\overline{BC}=\overline{b},\overline{CD}=l\overline{a}+m\overline{b}$ and $\overline{DE}=p\overline{a}+q\overline{b}$, then

A) *m, p* are irrational
B) l,q are rational
C) $m+p=0$
D) $l-q=0$

Key. A,B,C

Sol. $\overline{CD}=-\overline{a}+\sqrt{2}\overline{b}$

$\overline{DE}=-\sqrt{2}\overline{a}+\overline{b}$.

24. In a triangle ABC, the point D divides BC in the ratio 3:4 and the point E divides BA in the ratio 4:3.If AD and CE intersects at F, then

a) AF:FD =21 :16
b) AF:FD = 2:1
c) CF:FE= 28:9
d) CF:FE=9:28

Key. A,C

Sol. Using Menelau's theorem or by vectors

$$\frac{AF}{DF}=\frac{21}{16},\frac{CF}{FE}=\frac{28}{9}$$

25. If $A_1 B_1 C_1$ and $A_2B_2C_2$ are two coplanar triangles such that perpendicular from A_1, B_1, C_1 to the sides B_2C_2, C_2A_2, A_2B_2 of the triangles $A_2B_2C_2$ are concurrent, then

(A) $\Sigma\ \vec{a}_1\ (\vec{c}_2-\vec{b}_2)=0$
(B) $\Sigma\ \vec{a}_1\,\vec{b}_2\vec{c}_2=0$
(C) $\Sigma\ \vec{a}_1\ (\vec{c}_2+\vec{b}_2)=0$
(D) $\Sigma\ \vec{a}_2\ (\vec{c}_1-\vec{b}_1)=0$

Key. A,D

Sol. Let H be the point of concurrency

$A_1H \perp B_2C_2 \Rightarrow (\vec{h}-\vec{a}_1)\ (\vec{c}_2-\vec{b}_2) = 0$

$B_1H \perp C_2A_2 \Rightarrow (\vec{h}-\vec{b}_1)\ (\vec{a}_1-\vec{c}_1) =$

$C_1H \perp A_1B_1 \Rightarrow (\vec{h}-\vec{c}_1)\ (\vec{b}_2-\vec{a}_2)=0$

$\Rightarrow \Sigma\ \vec{a}_1\ (\vec{c}_2-\vec{b}_2)=0$

26. $\overline{a},\overline{b},\overline{c}$ are unit vectors which are linearly dependent. $\overline{d}$ is a unit vector perpendicular to the plane containing $\overline{a},\overline{b},\overline{c}$. If $(\overline{a}\times\overline{b})\times(\overline{c}\times\overline{d})=\frac{1}{6}(i-2j+2k)$ and the angle between $\overline{a},\overline{b}$ is $\frac{\pi}{6}$ then $\overline{c}$ can be

A) $\dfrac{i-2j+2k}{3}$ B) $\dfrac{2i+j-k}{3}$ C) $\dfrac{-2i-2j+k}{3}$ D) $\dfrac{-i+2j-2k}{3}$

Key. A,D

Sol. Conceptual

27. If $\bar{r}=x\bar{a}\times(\bar{a}\times\bar{b})+y\bar{a}\times\bar{b}$ and $\bar{r}$ satisfies the conditions $\bar{r}.\bar{b}=1;[\bar{r}\ \bar{a}\ \bar{b}]=1$ and also $\bar{a}.\bar{b}\neq 0$ then

A) $\bar{r}.\bar{a}=0$ B) $x=\dfrac{-1}{(\bar{a}\times\bar{b})^2}$ C) $x=\dfrac{\bar{a}.\bar{b}}{(\bar{a}\times\bar{b})^2}$ D) $x+y=0$

Key. A,B,D

Sol. Conceptual

28. $\vec{u}=\hat{i}-\hat{j}+\hat{k},\vec{v}=\alpha\hat{i}+\alpha\hat{j}+(\beta+1)\hat{k}$, $\vec{w}=\beta\hat{i}+\beta\hat{j}+(2\alpha+1)\hat{k}$. If it is possible to construct a parallelo piped using $\vec{u},\vec{v},\vec{w}$ as its 3-coterminus sides for any value of α , then which of the following is/are false.

A) $\dfrac{-1-\sqrt{2}}{2\sqrt{2}}<\beta<\dfrac{\sqrt{2}-1}{2\sqrt{2}}$ B) $\dfrac{-1-\sqrt{2}}{2\sqrt{2}}<\beta<\dfrac{1-\sqrt{2}}{2\sqrt{2}}$

C) $\dfrac{-1+\sqrt{2}}{2\sqrt{2}}<\beta<\dfrac{1+\sqrt{2}}{2\sqrt{2}}$ D) $\dfrac{1-\sqrt{2}}{2\sqrt{2}}<\beta<\dfrac{1+\sqrt{2}}{2\sqrt{2}}$

Key. C,D

Sol. $[\vec{u}\ \ \vec{v}\ \ \vec{w}]\neq 0\Rightarrow 2\alpha^2+\alpha-\beta^2-\beta\neq 0$

$\therefore D<0$

29. Let $\vec{a}\ \&\ \vec{c}$ are unit vectors and $\left|\vec{b}\right|=4$ with $\vec{a}\times\vec{b}=2\vec{a}\times\vec{c}$. The angle between $\vec{a}\ \&\ \vec{c}$ is $\cos^{-1}\left(1/4\right)$. If $\vec{b}-2\vec{c}=\lambda\vec{a}$, then λ equals

A) 1/3 B) 1/4 C) -4 D) 3

Key. C,D

Sol. $\left|\vec{b}\right|=\left|2\vec{c}+\lambda\vec{a}\right|$

30. Unit vectors $\vec{a}$ and $\vec{b}$ are perpendicular and unit vector $\vec{c}$ be inclined at an angle θ to both $\vec{a}$ and $\vec{b}$. If $\vec{c}=\alpha\vec{a}+\beta\vec{b}+\gamma(\vec{a}\times\vec{b})$ then

(A) $\alpha=\beta$ (B) $1-2\alpha^2=\gamma^2$ (C) $\gamma^2=1-2\cos^2\theta$ (D) $\alpha^2-\beta^2=\gamma^2$

Key. A,B,C

Sol. $\vec{c}=\alpha\vec{a}+\beta\vec{b}+\gamma(\vec{a}\times\vec{b})$

$\vec{c}.\vec{a}=\alpha\Rightarrow\cos\theta=\alpha\rightarrow(1)$

$\vec{c}.\vec{b}=\beta\Rightarrow\cos\theta=\beta\rightarrow(2)$

Also $2\cos^2\theta+\cos^2\left(\vec{c},\vec{a}\times\vec{b}\right)=1$

$\Rightarrow\gamma^2=1-2\alpha^2\rightarrow(3)$

From (1) , (2) and (3) it follows

31. If ABCD be a tetrahedron with G as centroid and position vectors of A,B,C,D are $\vec{a},\vec{b},\vec{c},\vec{d}$ respectively then volume of the tetrahedron GABC =

(A) $\dfrac{1}{6}\left|\left[\vec{a}\,\vec{b}\,\vec{c}\right]\right|$ (B) $\dfrac{1}{6}\left|\left[\vec{b}\,\vec{c}\,\vec{d}\right]\right|$ (C) $\dfrac{1}{3}\left|\left[\vec{b}\,\vec{c}\,\vec{d}\right]\right|$ (D) $\dfrac{1}{3}\left|\left[\vec{a}\,\vec{b}\,\vec{c}\right]\right|$

Key. A,B

Sol. Conceptual

32. If $\bar{a}$ and $\bar{c}$ are unit vectors and $|\bar{b}| = 4$ with $\bar{a}\times\bar{b} = 2\bar{a}\times\bar{c}$. The angle between $\bar{a}$ and $\bar{c}$ is $\cos^{-1}\left(\frac{1}{4}\right)$, $\bar{b}-2\bar{c}=\lambda\bar{a}$, then λ =

(a) 3 (b) -3 (c) 4 (d) -4

Key. A,D

Sol. $|\bar{a}| = |\bar{c}| = 1, |\bar{b}| = 4$

$\bar{a}\times\bar{b} = 2(\bar{a}\times\bar{c}); (\bar{a},\bar{c}) = \cos^{-1}\left(\frac{1}{4}\right)$

Now $\bar{a}.\bar{c} = |\bar{a}||\bar{c}|\frac{1}{4} = \frac{1}{4} \Rightarrow \bar{a}.\bar{c} = \frac{1}{4} \rightarrow$ (1)

Given $\bar{b}-2\bar{c}=\lambda\bar{a}$

$|\bar{b}-2\bar{c}|^2 = \lambda^2|\bar{a}|^2$

$\Rightarrow b^2+4c^2-4\bar{b}\cdot\bar{c}=\lambda^2$

$\Rightarrow 4\bar{b}\cdot\bar{c}=20-\lambda^2$

$\Rightarrow \bar{b}\cdot\bar{c}=\frac{20-\lambda^2}{4}$

$\bar{b}\cdot\bar{c}-2\bar{c}\cdot\bar{c}=\lambda\bar{a}\cdot\bar{c}$

$\frac{20-\lambda^2}{4}-2=\frac{\lambda}{4}$

$\Rightarrow 20-\lambda^2-8=\lambda$

$\Rightarrow \lambda^2+\lambda-12=0$

$\Rightarrow (\lambda-3)(\lambda+4)=0$

$\Rightarrow \lambda = 3 \ or \ -4$

33. If (x, y, z) $\neq$ (0, 0, 0) and $\left(\hat{i}+j+3\hat{k}\right)x+\left(3\hat{i}-3j+\hat{k}\right)y+\left(-4\hat{i}+5j\right)z=\lambda\left(x\hat{i}+yj+z\hat{k}\right)$

where $\hat{i},\hat{j},\hat{k}$

are unit vectors along the coordinate axes, then

(a) λ = 0 (b) λ = 2 (c) λ = 1 (d) λ = -1

Key. A,D

Sol. Here $\left(\hat{i}+\hat{j}+3\hat{k}\right)x+\left(3\hat{i}-3\hat{j}+\hat{k}\right)y+\left(-4\hat{i}+5\hat{j}\right)z=\lambda\left(x\hat{i}+y\hat{j}+z\hat{k}\right)$

On equating we obtain

$(1-\lambda)x+3y-4z=0$

$x-(3+\lambda)y+5z=0$

$3x+y-\lambda z=0$

Since the equation have non trivial solutions

Hence $\begin{vmatrix} 1-\lambda & 3 & -4 \\ 1 & -(3+\lambda) & 5 \\ 3 & 1 & -\lambda \end{vmatrix} = 0 \quad \Rightarrow \lambda = 0 \text{ or } -1$

34. If $\bar{a} = \bar{i} + \bar{j} + \bar{k}$, $\bar{b} = 4\bar{i} + 3\bar{j} + 4\bar{k}$ and $\bar{c} = \bar{i} + \alpha\bar{j} + \beta\bar{k}$ are linearly dependent vectors and $|\bar{c}| = \sqrt{3}$ then

(a) $\beta = \pm 1$ (b) $\beta = 1$ (c) $\alpha = 1$ (d) $\alpha = -1$

Key. B,C,D

Sol. If $\bar{a}, \bar{b}, \bar{c}$ are linearly independent vectors, then $\bar{c}$ should be a linear combination of $\bar{a}$ and $\bar{b}$

$\bar{c} = p\bar{a} + q\bar{b}$ for some scalars p and q

i.e., $\bar{i} + \alpha\bar{j} + \beta\bar{k} = p(\hat{i} + \hat{j} + k) + q(4\hat{i} + 3\hat{j} + 4k)$

$\Rightarrow 1 = p + 4q \quad \alpha = p + 3q \quad \beta = p + 4q$

$\Rightarrow \beta = 1$ Now $|\bar{c}| = \sqrt{3} \quad \Rightarrow 1 + \alpha^2 + \beta^2 = 3$

$\Rightarrow 1 + \alpha^2 + 1 = 3$

$\Rightarrow \alpha^2 = 1 \quad \Rightarrow \alpha = \pm 1$

35. Let $\bar{a}, \bar{b}, \bar{c}$ be three non coplanar vectors such that $\bar{r}_1 = \bar{a} - \bar{b} + \bar{c}$, $\bar{r}_2 = \bar{b} + \bar{c} - \bar{a}$, $\bar{r}_3 = \bar{a} + \bar{b} + \bar{c}$, $\bar{r}_4 = 2\bar{a} - 3\bar{b} + 4\bar{c}$. If $\bar{r}_4 = p_1\bar{r}_1 + p_2\bar{r}_2 + p_3\bar{r}_3$ then

(a) $p_1 = 7$ (b) $p_1 + p_3 = 3$

(c) $p_1 + p_2 + p_3 = 4$ (d) $p_2 + p_3 = 0$

Key. B,C

Sol. $\bar{r}_4 = p_1\bar{r}_1 + p_2\bar{r}_2 + p_3\bar{r}_3$

$\Rightarrow 2\bar{a} - 3\bar{b} + 4\bar{c} = (p_1 - p_2 + p_3)\bar{a} + (-p_1 + p_2 + p_3)\bar{b} + (p_1 + p_2 + p_3)\bar{c}$

Since $\bar{a}, \bar{b}, \bar{c}$ are non coplanar

$\Rightarrow p_1 - p_2 + p_3 = 2, \quad -p_1 + p_2 + p_3 = -3, \quad p_1 + p_2 + p_3 = 4$

Solving $\Rightarrow p_1 = \frac{7}{2}, \quad p_2 = 1, \quad p_3 = -\frac{1}{2}$

36. P is the point $\vec{i} + x\vec{j} + 3\vec{k}$ .The vector $\overrightarrow{OP}$ ('O' is the origin) is rotated about the point 'O' through an angle θ.Q is the point $4\vec{i} + (4x-2)\vec{j} + 2\vec{k}$ on the new support of $\overrightarrow{OP}$ such that OQ = 2OP. Then *x* value is

a) 2 b) $\frac{2}{3}$ c) $\frac{1}{3}$ d) $\frac{-2}{3}$

Key. A,D

Sol. $\overrightarrow{OP}=\vec{i}+x\vec{j}+3\vec{k}$

$\overrightarrow{OQ}=\overrightarrow{4i}+(4x-2)\vec{j}+2\vec{k}$, $OQ=2OP \Rightarrow 16+(4x-2)^2+4=4(1+x^2+9)$

$\Rightarrow 12x^2-16x-16=0 \quad \Rightarrow 3x^2-4x-4=0 \qquad \Rightarrow (3x+2)(x-2)=0$

$\Rightarrow x=2,\dfrac{-2}{3}$

37. If $\bar{a}\times\bar{b}=\bar{c}$ and $\bar{b}\times\bar{c}=\bar{a}$ then

a) $\left|\bar{a}\right|=1$ b) $\left|\bar{b}\right|=1$

c) $\left|\bar{a}\right|=\left|\bar{c}\right|$ d) $\left|\bar{b}\right|=\left|\bar{c}\right|$

Key. B,C

Sol. $\bar{a}\times\bar{b}=\bar{c}\Rightarrow\bar{c}$ is perpendicular to $\bar{a}$ and $\bar{b}$.

$\bar{b}\times\bar{c}=\bar{a}\Rightarrow\bar{a}$ is perpendicular to $\bar{b}$ *and* $\bar{c}$

$\Rightarrow$ $\bar{a}, \bar{b}, \bar{c}$ are mutually perpendicular

Again $\bar{a}\times\bar{b}=\bar{c}\Rightarrow\left|\bar{a}\times\bar{b}\right|=\left|\bar{c}\right|\Rightarrow\left|\bar{a}\right|\left|\bar{b}\right|=\left|\bar{c}\right| \quad \rightarrow(1)$

$\bar{b}\times\bar{c}=\bar{a}\Rightarrow\left|\bar{b}\times\bar{c}\right|=\left|\bar{a}\right|\Rightarrow\left|\bar{b}\right|\left|\bar{c}\right|=\left|\bar{a}\right| \quad \rightarrow(2)$

$\therefore$ from (1) & (2) $\left|\bar{c}\right|=\left|\bar{a}\right|\ \&\left|\bar{b}\right|=1$

38. The lines whose vector equations are $\vec{r}=2\hat{i}-3j+7k+\lambda\left(2\vec{i}+p\vec{j}+5\vec{k}\right)$ and $\vec{r}=\hat{i}+2j+3k+\mu\left(3\vec{i}-p\vec{j}+p\vec{k}\right)$ are perpendicular for all values of □ and □, if

a) p = - 6 b) p = - 1 c) p = 1 d) p = 6

Ans. b, d

Sol. Given lines are perpendicular if $2\vec{i}+p\vec{j}+5\vec{k}$ and $3\vec{i}-p\vec{j}+p\vec{k}$ are perpendicular.

$\Rightarrow 2\cdot3+p(-p)+5p=0\Rightarrow p=-1,6$

39. The vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are of the same length and taken pair wise they form equal angles. If $\vec{a}=\hat{i}+j$ and $\vec{b}=j+k$, the coordinates of $\vec{c}$ can be

a) (1, 0, 1) b) (-1, 1, 2) c) $\left(-\dfrac{1}{3},\dfrac{4}{3},-\dfrac{1}{3}\right)$ d) $\left(\dfrac{1}{3},0,\dfrac{2}{3}\right)$

Ans. a,c

Sol. Let $\vec{c}=c_1\hat{i}+c_2j+c_3k$. Then $\left|\vec{c}\right|=\sqrt{2}=\sqrt{c_1^2+c_2^2+c_3^2}$ - (1)

And = $\dfrac{\vec{a}\cdot\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}=\dfrac{1}{2}\Rightarrow\dfrac{1}{2}=\dfrac{\vec{a}\cdot\vec{c}}{\left|\vec{a}\right|\left|\vec{c}\right|}=\dfrac{c_1+c_2}{\sqrt{2}\cdot\sqrt{2}}\Rightarrow c_1+c_2=1$

and $\dfrac{1}{2}=\dfrac{\vec{b}\cdot\vec{c}}{\left|\vec{b}\right|\left|\vec{c}\right|}=\dfrac{c_2+c_3}{2}\Rightarrow c_2+c_3=1$

From (1)

$2=\left(1-c_2\right)^2+c_2^2+\left(1-c_2\right)^2$

$\Rightarrow 3c_2^2-4c_2=0$

$$\Rightarrow c_2 = 0 \ or \ c_2 = \frac{4}{3}$$

Therefore, the points are (1, 0, 1) and $\left(-\frac{1}{3}, \frac{4}{3}, -\frac{1}{3}\right)$