PHYSICS

The following questions consists of two statements each, printed as Assertion and Reason. While answering these questions you are to choose any one of the following four responses.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true but Reason is not correct explanation of the Assertion.

(C) If Assertion is true but the Reason is false.

(D) If Assertion is false but Reason is true.

Q.1 **Assertion :** If a boat moves a distance x downstream in time 't', then turns back and moves a distance y upstream in same time 't'. If $\frac{x}{y}$ = m and velocity of boat is n times greater than velocity of river, where n is greater than unity, then m = $\frac{n+1}{n-1}$.

Reason : In downstream motion net velocity of boat is less than that in upstream motion. **[C]**

Sol. $x = (v_b + v_r)t = (n + 1)\, v_r t$

$y = (v_b - v_r)t = (n - 1)\, v_r t$

$\therefore m = \frac{x}{y} = \frac{n+1}{n-1}$

Q.2 **Statement I :** $\bar{v} = \bar{\omega} \times \bar{r}$ and $\bar{v} \neq \bar{r} \times \bar{\omega}$

Statement II : Cross product is commutative.

[C]

Q.3 **Statement I :** When $\bar{P} + \bar{Q} = \bar{R}$ and P + Q = R, the angle between $\bar{P}$ & $\bar{Q}$ must be 0º.

Statement II : Here θ = 0º

$R = \sqrt{P^2 + Q^2 + 2PQ\cos 0^\circ} = P + Q.$ **[A]**

Q.4 **Statement-I :** If $\vec{a}.\vec{b} = \vec{a}.\vec{c}$, It follows that $\vec{b} = \vec{c}$

Statement-II : If $\vec{a} \times \vec{b} = \vec{a} \times \vec{c} = 0$, $\vec{b}$ must be perpendicular to $\vec{c}$. **[C]**

(A) Statement (I) and (II) are correct, and Statement (II) is the correct explanation of Statement (I).

(B) Statement (I) and (II) are correct but Statement (II) is not the correct explanation of Statement (I).

(C) Statement (I) is correct but Statement (II) is wrong.

(D) Statement (I) is wrong but Statement (II) is correct.

Q.5 **Assertion :** Velocity of boat is n times the river flow velocity then if n = 1 boat cannot cross river without drifting.

Reason : In downstream motion net speed of boat is greater than in the upstream motion. **[B]**

Sol. To cross without drifting boat velocity must be greater than river velocity.

Q.6 **Assertion :** Motion in two or three dimensions, velocity and acceleration vectors may have any angle between 0º and 180º.

Reason : Component of acceleration along the direction of velocity is equal to rate of change of speed. **[A]**

Q.7 **Assertion :** The sum of two vectors can be zero.

Reason : When they are equal and opposite. **[A]**

Q. 8 **Assertion (A) :** If $\vec{R} = \vec{a} + \vec{b}$ and $|\vec{R}|$ = 5 units, $|\vec{a}|$ = 3 units $|\vec{b}|$ = 4 units. Then the angle between vectors $\vec{a}$ and $\vec{b}$ may be 270º.

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Reason (R) : If $\vec{R} = \vec{a} + \vec{b}$ and if $\vec{R}$ makes an angle α with $\vec{b}$ then, cot $\alpha = \frac{b}{a}$ cosecθ + cot θ where θ is angle between $\vec{a}$ and $\vec{b}$. **[D]**

Sol. R = $\sqrt{a^2+b^2+2ab\cos\theta}$ and tan α = $\frac{b\sin\theta}{a+b\cos\theta}$

Q. 9 **Assertion (A) :** The direction of resultant vector of two vectors $\vec{a}$ and $\vec{b}$ must change when magnitude of vectors $\vec{a}$ and $\vec{b}$ are simultaneously changed.

Reason (R) : If $\vec{R} = \vec{a}+\vec{b}$ and $\vec{R}$ makes angles α with $\vec{a}$ then tan α = $\frac{b\sin\theta}{a+b\cos\theta}$, where θ is angle between $\vec{a}$ and $\vec{b}$ **[D]**

Sol. Conceptual

Q.10 **Assertion (A) :** Angle made by vector $\vec{A} = \sqrt{3}\ \hat{i} - \hat{j}$ with x-axis is θ = 330º.

Reason (R) : If angle between x-axis & vector $\vec{A}$ is θ then tan $\theta = -\frac{1}{\sqrt{3}}$. **[B]**

Sol. 'A' & 'R' both correct but 'R' is not correct explanation of 'A'.

Q.11 **Assertion (A) :** 'y' vs 'x' graph of equation $y = 2x^2 + 3$ is a parabola symmetrical to y-axis.

Reason (R) : Graph of a quadratic equation in form $y = ax^2 + bx + c$ is a parabola symmetrical to y-axis if b = 0. **[A]**

Sol. 'A' & 'R' both correct & 'R' is correct explanation of 'A'.

Q.12 **Statement I :** The minimum number of non coplanar vector whose sum can be zero, is four.

Statement II : The resultant of two vectors of unequal magnitude can be zero.

Sol.[C] Statement I is true but Statement II is wrong.

Q.13 **Assertion :** Angle between $\hat{i}+\hat{j}$ and $\hat{i}$ is 45°

Reason : $\hat{i}+\hat{j}$ is equally inclined to both $\hat{i}$ and $\hat{j}$ and the angle between $\hat{i}$ and $\hat{j}$ is 90°

Sol.[A] $\cos\theta = \frac{(\hat{i}+\hat{j}).(\hat{i})}{|\hat{i}+\hat{j}||\hat{i}|} = \frac{1}{\sqrt{2}}$. Hence $\theta = 45°$.

Q.14 **Statement - I :** $\vec{A}\times\vec{B}$ is perpendicular to both $\vec{A}+\vec{B}$ as well as $\vec{A}-\vec{B}$.

Statement - II : $\vec{A}+\vec{B}$ as well as $\vec{A}-\vec{B}$ lie in the plane containing $\vec{A}$ and $\vec{B}$. But $\vec{A}\times\vec{B}$ lies perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

Sol. [A] assertion and reason both are true

Q.15 **Statement - I :** A null vector is a vector whose magnitude is zero and direction is arbitrary.

Statement - II : A null vector does not exist.

Sol. [C] assertion is true reason is false

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Q.1 Two ships A and B are 10 km apart on a line running from south to north. A is towards north of B and moving west with a speed of 20 Km/hr while B is moving towards north with 20 Km/hr. The distance of their closest approach in metres is ℓ and the time in second taken to reach this position is t.

Column-I	**Column-II**
(A) ℓ	(P) North – West
(B) t	(Q) 7071
(C) $\vec{V}_{AB}$	(R) 900
(D) $\vec{V}_{BA}$	(S) South – East

(A) $\to$ Q **(B)** $\to$ R **(C)** $\to$ S **(D)** $\to$ P

Q.2 A boat moves a distance x downstream in time t_1 and turns back, moves same distance x in time t_2. If velocity of boat v_b is greater than the river flow velocity v_r -

Column-I	**Column-II**
(A) $\frac{t_1}{t_2}$	(P) $(v_b - v_r)t_2$
(B) $(v_b + v_r)t_1$	(Q) less than unity
(C) v_b	(R) $\frac{x}{\left(\frac{2t_1t_2}{t_1+t_2}\right)}$
(D) v_r	(S) $\frac{x}{\left(\frac{2t_1t_2}{t_2-t_1}\right)}$

(A) $\to$ Q **(B)** $\to$ P **(C)** $\to$ R **(D)** $\to$ S

Q.3 If $\vec{R} = \vec{a} + \vec{b}$ and $\vec{S} = \vec{a} - \vec{b}$ also θ is angle between $\vec{a}$ and $\vec{b}$.

Column-I	**Column-II**
(A) $R^2 + S^2$	(P) $\vec{R}$ is perpendicular to $\vec{a}$
(B) $R^2 - S^2$	(Q) $2(a^2 + b^2)$
(C) $\frac{R}{S}$	(R) $4\ \vec{a} \cdot \vec{b}$
(D) R < S	(S) $\tan\left(\frac{\theta}{2}\right)$ If $|\vec{a}| = |\vec{b}|$

(A) $\to$ Q **(B)** $\to$ R **(C)** $\to$ S **(D)** $\to$ P

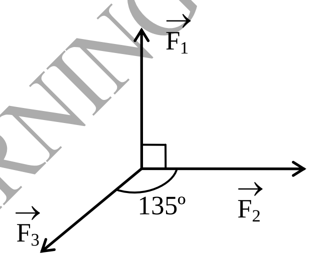
Q.4 Velocity of three bodies A, B and C varies with time as $\vec{V}_A = (2t\hat{i} + 6\hat{j})$m/s, $\vec{V}_B = (3\hat{i} + 4\hat{j})$ m/s and $\vec{V}_C = (6\hat{i} - 4t\hat{j})$m/s. Match the following column for pseudo force.

Column-I	**Column-II**
(A) on A w.r.t B	(P) zero
(B) on B w.r.t C	(Q) along negative x-axis
(C) on A w.r.t C	(R)along positive y-xis
(D) on C w.r.t A	(S)along negative y-axis

Sol. **A $\to$ P ; B $\to$ R ; C $\to$ R ; D $\to$ Q**

$\vec{a}_A = 2\hat{i}$, $\vec{a}_B = 0$, $\vec{a}_C = -4\hat{j}$

Q.5



$|\vec{F}_1| = 10$N

$|\vec{F}_2| = 10$N

$|\vec{F}_3| = 10\sqrt{2}$N

Three coplanar forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ are acting simultaneously on a particle. Column I contains different operation between forces and Column II contains their magnitude. Match them.

Column-I	**Column-II**
(A) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3$	(P) 0
(B) $\vec{F}_1 + \vec{F}_2 - \vec{F}_3$	(Q) 20
(C) $\vec{F}_1 - \vec{F}_2 + \vec{F}_3$	(R) $20\sqrt{2}$
(D) $\vec{F}_3 \cdot (\vec{F}_1 + \vec{F}_2)$	(S) 200

A $\to$ P ; B $\to$ R ; C $\to$ Q ; D $\to$ S

Q.6 If $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} + 2\hat{k}$ then -

Column I	**Column II**
(A) $|\vec{A} \times \vec{B}|$	(P) $\sqrt{11}$
(B) $|\vec{A} - \vec{B}|$	(Q) 6
(C) $\vec{A} \cdot \vec{B}$	(R) $\sqrt{35}$
(D) $|\vec{A} + \vec{B}|$	(S) $\sqrt{90}$

A $\to$ S ; B $\to$ P ; C $\to$ Q ; D $\to$ R

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Q.7 In column-I condition on velocity, force and acceleration of a particle is given. Resultant motion is described in column-II. $\vec{u}$ = instantaneous velocity -

Column-I	Column-II
(A) $\vec{u} \times \vec{F} = 0$ and $\vec{F}$ = constant	(P) path will be circular path
(B) $\vec{u} \cdot \vec{F} = 0$ and $\vec{F}$ = constant	(Q) speed will increase
(C) $\vec{v}.\vec{F} = 0$ all the time and $\lvert\vec{F}\rvert$ = constant and the particle always remains in one plane	(R) path will be straight line
(D) $\vec{u} = 2\hat{i} - 3\hat{j}$ and acceleration at all time $\vec{a} = 6\hat{i} - 9\hat{j}$	(S) path will be parabolic

Sol. **A → R ; B → Q,S ; C → P ; D → Q,R**

(A) $\vec{F}$ = constant and $\vec{u} \times \vec{F} = 0$
Therefore initial velocity is either in direction of constant force or opposite to it. Hence the particle will move in straight line and speed may increase or decrease.

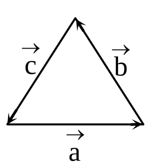
(B) $\vec{u}.\vec{F} = 0$ and $\vec{F}$ = constant
Initial velocity is perpendicular to constant force, hence the path will be parabolic with speed of particle increasing.

(C) $\vec{v}.\vec{F} = 0$ means instantaneous velocity is always perpendicular to force. Hence the speed will remain constant. And also $\lvert\vec{F}\rvert$ = constant. Since the particle moves in one plane, the resulting motion has to be circular.

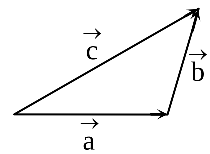
(D) $\vec{u} = 2\hat{i} - 3\hat{j}$ and $\vec{a} = 6\hat{i} - 9\hat{j}$. Hence initial velocity is in same direction of constant acceleration, therefore particle moves in straight line with increasing speed.

Q.8 Column-I contains vector diagram of three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ & Column-II contains vector equation. Match them.

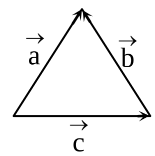
Column-I **Column-II**

(A) 

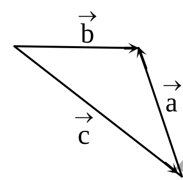
(P) $\vec{a} - (\vec{b} + \vec{c}) = 0$

(B) 

(Q) $\vec{b} - \vec{c} = \vec{a}$

(C) 

(R) $\vec{a} + \vec{b} = -\vec{c}$

(D) 

(S) $\vec{a} + \vec{b} = \vec{c}$

Sol.2 A → R; B → S; C → P; D → Q

A : $\vec{a} + \vec{b} + \vec{c} = 0$ (polygon law)

B : $\vec{a} + \vec{b} = \vec{c}$ (Δ law)

C : $\vec{c} + \vec{b} = \vec{a}$ (Δ law)

D : $\vec{c} + \vec{a} = \vec{b}$

Q.9 In vector algebra match the vector operations with conditions under which operation is possible.

	Column I	Column II
(a)	$\vec{A}+\vec{B} = \vec{A}-\vec{B}$	1. Not possible
(b)	$\lvert\vec{A}+\vec{B}\rvert = \lvert\vec{A}-\vec{B}\rvert$	2. θ = 90°
(c)	$\lvert\vec{A}\times\vec{B}\rvert = \lvert\vec{A}.\vec{C}\rvert$	3. B = 0
(d)	$\vec{A}\times\vec{B} = \vec{A}.\vec{C}$	4. θ = 45°

(A) a-4, b-3, c-2, d-1
(B) a-4, b-2, c-3, d-1
(C) a-3, b-2, c-4, d-1
(D) a-1, b-2, c-3, d-4

Sol. [C] (A) $\vec{A}+\vec{B} = \vec{A} - \vec{B} \Rightarrow B = 0$

(B) $\lvert\vec{A}+\vec{B}\rvert = \lvert\vec{A}-\vec{B}\rvert \Rightarrow \theta = \frac{\pi}{2}$

(C) $\lvert\vec{A}\times\vec{B}\rvert = \lvert\vec{A}.\vec{C}\rvert \Rightarrow \theta = \frac{\pi}{4}$

(D) $\vec{A}\times\vec{B} = \vec{A}.\vec{C}$ (Not possible) because

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$\vec{A} \times \vec{B}$ is a vector and $\vec{A}.\vec{C}$ is a scalar quantity

Q.10 Two vectors of the same physical quantity are unequal if -

(a) they have the same magnitude and the same direction

(b) they have different magnitude but the same direction

(c) they have the same magnitude but different directions

(d) they have different magnitudes and different directions

(A) a, b, c (B) a, c, d

(C) a, b, d (D) b, c, d

Sol. [D] Two vectors are called equal if they have same magnitude and same direction.

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Q.1 Vector $\vec{R}$ is the resultant of the vectors $\vec{A}$ and $\vec{B}$. Ratio of maximum value of $|\vec{R}|$ to the minimum value of $|\vec{R}|$ is $\frac{3}{1}$. The $\frac{|\vec{A}|}{|\vec{B}|}$ may be equal to -

(A) $\frac{2}{1}$ (B) $\frac{1}{2}$

(C) $\frac{4}{1}$ (D) $\frac{3}{1}$ **[A,B]**

Sol. $\frac{|\vec{A}+\vec{B}|_{max}}{|\vec{A}+\vec{B}|_{min}} = \frac{|\vec{A}|+|\vec{B}|}{|\vec{A}|-|\vec{B}|} = \frac{3}{1}$

If $|\vec{A}| > |\vec{B}|$

$\frac{|\vec{A}|+|\vec{B}|}{|\vec{A}|-|\vec{B}|} = \frac{3}{1} \Rightarrow \frac{|\vec{A}|}{|\vec{B}|} = \frac{2}{1}$

If $|\vec{B}| > |\vec{A}|$

$\frac{|\vec{A}|+|\vec{B}|}{|\vec{A}|-|\vec{B}|} = \frac{3}{1} \Rightarrow \frac{|\vec{A}|}{|\vec{B}|} =$

Q.2 A boat can travel at a speed of 3m/s in still water. A boat man wants to cross a river whilst covering the shortest possible distance -

(A) If the speed of water is 2 m/s, then direction in which he should row his boat is $\cos^{-1}$ 2/3 with respect to bank

(B) If speed of water is 4 m/s, then direction in which he should row his boat is $\cos^{-1}3/4$ with respect to bank

(C) The resultant velocity in the case (A) of the boat is 2.24 m/s

(D) The resultant velocity of boat in case (B) is 2.65 m/s **[A,B,C,D]**

Sol. (i) The shortest path is one perpendicular to the bank and the boat goes in this direction if the boatman rows in the direction shown in fig.1

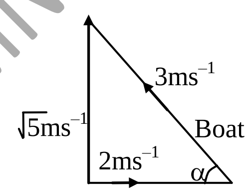

Fig.(1)

The resultant speed of boat (in the direction perpendicular to the bank) is $\sqrt{5}$ ms$^{-1} \approx$ 2.24 ms^{-1}. The boatman has to row upstream at an angle α to the bank, where $\cos\alpha = \frac{2}{3}$; this gives $\alpha \approx 48º$.

(ii) In this case, the current is so strong that the boat will move downstream even if the boatman rows at full speed against the stream. This means, in contrast to the previous case, he cannot choose his direction with respect to the bank and in particular, he cannot travel across in a direction perpendicular to the bank.

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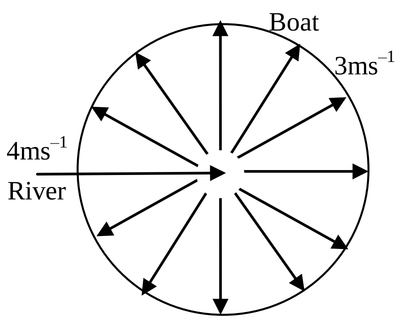
The possible directions he can take may be determined by adding all the possible still-water velocities of the boat to the velocity of the river. Draw the velocity vector of the river and, from the endpoint of this vector, draw velocity vectors in all directions, with a magnitude equal to the speed of the boat in still water. The endpoints of these vectors will form a circle as shown in fig.(2)



The possible resultant velocities of the boat can be obtained by joining the starting point of the velocity of the river to the points on this circle. The resultant corresponding to the shortest path will be the one that makes the greatest angle with the direction of the current, i.e., when the line of action of the resultant velocity vector is a tangent to the circle (see fig.3)

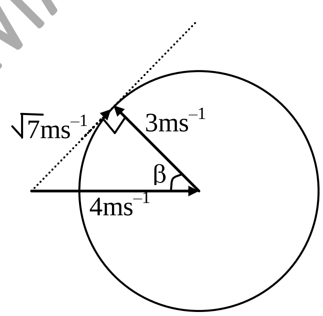


Fig.(3)

Thus, the velocity of the boat with respect to the shore is $\sqrt{7}$ m/s ≈ 2.65 ms^{-1}. Again, the boatman has to row upstream, but this time at an angle β to the bank, where cos β = $\frac{3}{4}$, yielding β ≈ 41º. The figure also shows that in this case the distance traveled by the boat will be $\frac{4}{3}$ times the width of the river.

Q.3 A man is walking toward east with a velocity of 8 km/h. Wind is blowing toward north-east at angle of 45°. To the man wind appears to blow of angle of 60° north of west –

(A) True velocity of wind is $\frac{8\sqrt{6}}{1+\sqrt{3}}$ km/hr

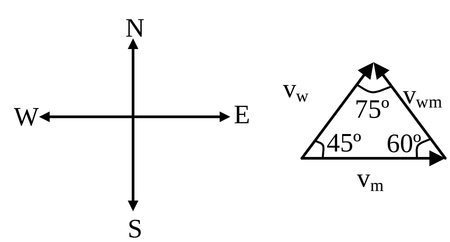
(B) Velocity of wind relative to man is $\frac{16}{1+\sqrt{3}}$ km/h

(C) True velocity of wind is $\frac{\sqrt{6}}{1+\sqrt{3}}$ km/h

(D) Velocity of wind relative to man is $\frac{8\sqrt{3}}{1+\sqrt{3}}$ km/h **[A, B]**

Sol. $\vec{v}_{wm} = \vec{v}_w - \vec{v}_m$

$\vec{v}_w = \vec{v}_{wm} + \vec{v}_m$



From sine rule

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$$\frac{v_w}{\sin 60^o} = \frac{v_{wm}}{\sin 45^o} = \frac{v_m}{\sin 75^o}$$

$v_w = \left(\frac{\sin 60^o}{\sin 75^o}\right) v_m$; $v_w = \frac{\sqrt{3}}{2\frac{(1+\sqrt{3})}{3\sqrt{2}}} v_m$;

$v_m = \frac{8\sqrt{6}}{1+\sqrt{3}}$ km/h

Q.4 Maximum value of resultant of $\vec{A}$ and $\vec{B}$ is 10 and minimum value of resultant of these two vectors is 4. The value of $|\vec{A}|$ may be -

(A) 10 (B) 4

(C) 7 (D) 3 **[C,D]**

Sol. $|\vec{A} + \vec{B}|_{max} = |\vec{A}| + |\vec{B}| = 10$

$|\vec{A} + \vec{B}|_{min} = \big||\vec{A}| - |\vec{B}|\big| = 4$

Q.5 A particle is moving with acceleration $\vec{a} = (\hat{i} + \hat{j})$ m/s^2 and its initial velocity (at t = 0) is $\vec{v}_0 = (\hat{i} - \hat{j})$ m/s. Then select the correct statement –

(A) Magnitude of displacement of particle in first second is $\sqrt{\frac{5}{2}}$ m

(B) Rate of change of speed at t = 0 is zero

(C) Rate of change of speed at t = 0 is $\sqrt{2}$ m/s^2

(D) Speed of particle at t = 2s is $\sqrt{10}$ m/s

[A,B,D]

Sol. $\vec{S} = \vec{u}\,t + \frac{1}{2}\vec{a}\,t^2$

$= (\hat{i} - \hat{j}) \times 1 + \frac{1}{2}(\hat{i} + \hat{j})\, 1^2$

$= (\hat{i} - \hat{j}) + \left(\frac{\hat{i}}{2} + \frac{\hat{j}}{2}\right) = \frac{3\hat{i}}{2} - \frac{\hat{j}}{2}$

$|\vec{S}| = \sqrt{\frac{5}{2}}$ m

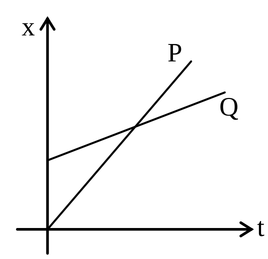
$|\vec{v}| = (1 + t)^2 + (t - 1)^2$

$\frac{d|\vec{v}|}{dt} = 2(1 + t) + 2(t - 1)$

At t = 0

$\frac{d|\vec{v}|}{dt} = 0$

Q.6 Two bodies P and Q are moving along positive x-axis their position-time graph is shown below if $\vec{V}_{PQ}$ is velocity of P w.r.t Q and $\vec{V}_{QP}$ is velocity of Q w.r.t P then –



(A) $|\vec{V}_{PQ}| = |\vec{V}_{QP}|$ = constant

(B) $\vec{V}_{PQ}$ is towards origin

(C) $\vec{V}_{QP}$ is towards origin

(D) $\vec{V}_{PQ}$ and $\vec{V}_{QP}$ both can be towards origin at same time **[A,C]**

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Sol. Use definition of relative velocity $\vec{v}_{PQ} = \vec{v}_P - \vec{v}_Q$

Q.7 If $\vec{a}$ and $\vec{b}$ are two vectors with $|\vec{a}| = |\vec{b}|$ and $|\vec{a}+\vec{b}| + |\vec{a}-\vec{b}| = 2|\vec{a}|$, then angle between $\vec{a}$ and $\vec{b}$ –

(A) 0º (B) 90º (C) 60º (D) 180º

[A,D]

Sol. Conceptual.

Q.8 If $\vec{P} = 5a\hat{i} + 6\hat{j}$ and $\vec{Q} = 3a\hat{i} + 10\hat{j}$. The vectors $\vec{P} + \vec{Q}$ makes an angle α with $\vec{P}$ and β with $\vec{Q}$ then –

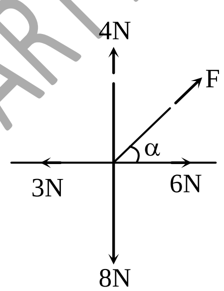
(A) $\alpha = \beta$ if a = 2 (B) $\alpha > \beta$ if a > 2

(C) $\alpha < \beta$ if a > 2 (D) $\alpha > \beta$ if a = 0

[A,C,D]

Sol. $\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta}$ and $\tan\beta = \frac{P\sin\theta}{Q+P\cos\theta}$

Q.9 Five concurrent forces are acting on a body. For the body to remain in equilibrium under these five forces -



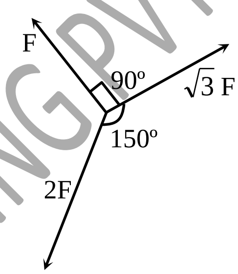
(A) F = 10 N (B) F = 5N

(C) 90º < α < 180º (D) 180º < α < 270º

[B,C]

Sol. Except F, resultant of rest four forces is 5N in fourth quadrant. Therefore F should be equal & opposite to this resultant or it should be 5 N in 2nd quadrant.

Q.10 The arrow shown below represent all the force vectors that are applied to a single point. Select the correct statements -



(A) The point may be moving at a constant velocity

(B) The point may not moving

(C) The point is accelerating at a constant rate

(D) The point is not accelerating

Sol. **[A,B,D]** $\Sigma\vec{F} = 0$

Q.11 A boat can travel at a speed of 3m/s in still water. A boat man wants to cross a river whilst covering the shortest possible distance -

(A) If the speed of water is 2 m/s, then direction in which he should row his boat is $\cos^{-1}$ 2/3 with respect to bank

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(B) If speed of water is 4 m/s, then direction in which he should row his boat is $\cos^{-1}3/4$ with respect to bank

(C) The resultant velocity in the case (A) of the boat is 2.24 m/s

(D) The resultant velocity of boat in case (B) is 2.65 m/s

Sol.[A,B,C,D]

(i) The shortest path is one perpendicular to the bank and the boat goes in this direction if the boatman rows in the direction shown in fig.1

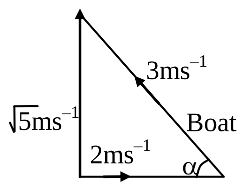
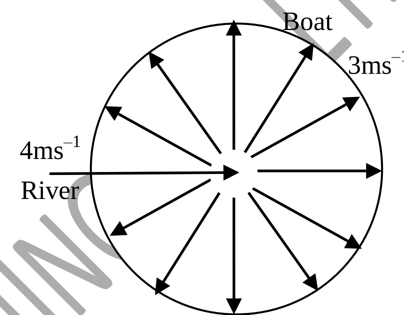


Fig.(1)

The resultant speed of boat (in the direction perpendicular to the bank) is $\sqrt{5}$ ms^{-1} ≈ 2.24 ms^{-1}. The boatman has to row upstream at an angle α to the bank, where $\cos\alpha = \frac{2}{3}$; this gives $\alpha \approx 48º$.

(ii) In this case, the current is so strong that the boat will move downstream even if the boatman rows at full speed against the stream. This means, in contrast to the previous case, he cannot choose his direction with respect to the bank and in particular, he cannot travel across in a direction perpendicular to the bank.

The possible directions he can take may be determined by adding all the possible still-water velocities of the boat to the velocity of the river. Draw the velocity vector of the river and, from the endpoint of this vector, draw velocity vectors in all directions, with a magnitude equal to the speed of the boat in still water. The endpoints of these vectors will form a circle as shown in fig.(2)



The possible resultant velocities of the boat can be obtained by joining the starting point of the velocity of the river to the points on this circle. The resultant corresponding to the shortest path will be the one that makes the greatest angle with the direction of the current, i.e., when the line of action of the resultant velocity vector is a tangent to the circle (see fig.3)

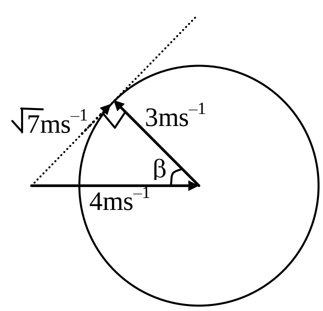


Fig.(3)

Thus, the velocity of the boat with respect to the shore is $\sqrt{7}$ m/s ≈ 2.65 ms^{-1}. Again, the boatman has to row upstream, but this time at an angle β to

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the bank, where cos $\beta = \frac{3}{4}$, yielding $\beta \approx$ 41º. The figure also shows that in this case the distance traveled by the boat will be $\frac{4}{3}$ times the width of the river.

Q.12 A man is walking toward east with a velocity of 8 km/h. Wind is blowing toward north-east at angle of 45°. To the man wind appears to blow of angle of 60° north of west -

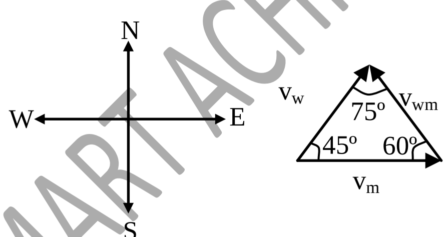
(A) True velocity of wind is $\frac{8\sqrt{6}}{1+\sqrt{3}}$ km/hr

(B) Velocity of wind relative to man is $\frac{16}{1+\sqrt{3}}$ km/h

(C) True velocity of wind is $\frac{\sqrt{6}}{1+\sqrt{3}}$ km/h

(D) Velocity of wind relative to man is $\frac{8\sqrt{3}}{1+\sqrt{3}}$ km/h

Sol.[A, B] $\vec{v}_{wm} = \vec{v}_w - \vec{v}_m$ $\vec{v}_w = \vec{v}_{wm} + \vec{v}_m$



From sine rule

$$\frac{v_w}{\sin 60^\circ} = \frac{v_{wm}}{\sin 45^\circ} = \frac{v_m}{\sin 75^\circ}$$

$v_w = \left(\frac{\sin 60^\circ}{\sin 75^\circ}\right) v_m$; $v_w = \frac{\sqrt{3}}{2\frac{(1+\sqrt{3})}{3\sqrt{2}}} v_m$;

$v_m = \frac{8\sqrt{6}}{1+\sqrt{3}}$ km/h

Q.13 If $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = \hat{i} + \hat{j} + \hat{k}$ are two vectors, then the unit vector-

(A) perpendicular to $\vec{A}$ is $\left(\frac{-\hat{j}+\hat{k}}{\sqrt{2}}\right)$

(B) parallel to $\vec{A}$ is $\frac{(2\hat{i}+\hat{j}+\hat{k})}{\sqrt{6}}$

(C) perpendicular to $\vec{B}$ is $\left(\frac{-\hat{j}+\hat{k}}{\sqrt{2}}\right)$

(D) parallel to $\vec{A}$ is $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

Sol. **[B,D]**

Q.14 If $\vec{a}$ and $\vec{b}$ are two vectors with $|\vec{a}| = |\vec{b}|$ and $|\vec{a}+\vec{b}| + |\vec{a}-\vec{b}| = 2|\vec{a}|$, then angle between $\vec{a}$ and $\vec{b}$ –

(A) 0º (B) 90º (C) 60º (D) 180º **[A,D]**

Q.15 If $\vec{P} = 5a\hat{i} + 6\hat{j}$ and $\vec{Q} = 3a\hat{i} + 10\hat{j}$. The vectors $\vec{P} + \vec{Q}$ makes an angle α with $\vec{P}$ and β with $\vec{Q}$ then –

(A) $\alpha = \beta$ if a = 2 (B) $\alpha > \beta$ if a > 2

(C) $\alpha < \beta$ if a > 2 (D) $\alpha > \beta$ if a = 0

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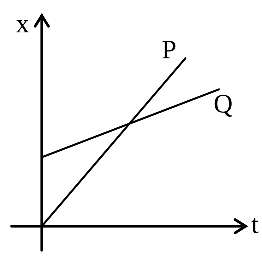
[A,C,D]

Q.16 Two bodies P and Q are moving along positive x-axis their position-time graph is shown below if $\vec{V}_{PQ}$ is velocity of P w.r.t Q and $\vec{V}_{QP}$ is velocity of Q w.r.t P then –



(A) $|\vec{V}_{PQ}| = |\vec{V}_{QP}|$ = constant

(B) $\vec{V}_{PQ}$ is towards origin

(C) $\vec{V}_{QP}$ is towards origin

(D) $\vec{V}_{PQ}$ and $\vec{V}_{QP}$ both can be towards origin at same time **[A,C]**

Q.17 If a boat moves a distance x down stream in time 't', then turns back and moves a distance y in same time 't'. If $\frac{x}{y}$ = m and velocity of boat is n times greater than velocity of river, where n is greater than unity then -

(A) $m \le 1$ (B) $m > 1$

(C) $m = \frac{n+1}{n-1}$ (D) $m = \frac{n-1}{m+1}$

[B,C]

Q.18 At t = 0 a body is at point whose position vector is $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and starts with velocity $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$. If point O is at $\vec{r}$ = 0, then body is always moving towards point O, when –

(A) $\frac{v_x}{x} = \frac{v_y}{y} = \frac{v_z}{z}$

(B) $\left(\frac{v_x}{x}\right).\left(\frac{v_y}{y}\right).\left(\frac{v_z}{z}\right) < 0$

(C) $xv_x + yv_y + zv_z < 0$

(D) $\left(\frac{v_x}{x}\right).\left(\frac{v_y}{y}\right) > 0$ **[A,B,C,D]**

Q.19 Velocity of a boat is n times the river flow velocity then –

(A) If n < 1 boat cannot cross river

(B) If n = 1 boat cannot cross river without drifting

(C) If n > 1 boat can cross river along shortest path

(D) If n > 0 boat can cross river **[B,C,D]**

Q.20 $\vec{A} \times \vec{B}$ is perpendicular to (where $\vec{A}$ and $\vec{B}$ are non zero non-collinear vector) -

(A) $\vec{A}$ (B) $\vec{B}$

(C) $\vec{A}+\vec{B}$ (D) $\vec{A}-\vec{B}$

[A,B,C,D]

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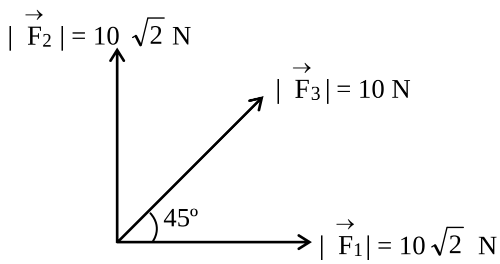
PHYSICS

Q.1 On a horizontal ground, rabbit is at origin and a cat is at position (46m, 28m). When the rabbit starts running with velocity $(7.5\hat{i} + 10\hat{j})$ m/s cat also starts running. If cat can run with maximum speed of 5 m/s. What is the minimum time in which cat can catch the rabbit ? (in seconds) **[0004]**

Q.2 Two forces of magnitude 6 N and 10 N are acting at a point. Resultant of these force is perpendicular to 6N. Then find the magnitude of the resultant force. **[0008]**

Q.3 Three forces are acting at a point as shown in figure. All forces are acting in a plane. Find the magnitude of resultant of these forces.

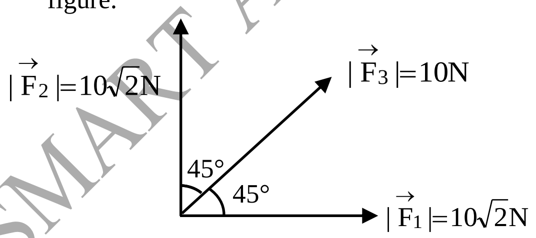


[0030]

Q.4 On an open ground, a motorist follows a truck that runs to his left by an angle 60º after every 500 m. Starting from a given turn specify the displacement (in meter) of motorist at the third turn. **[1000]**

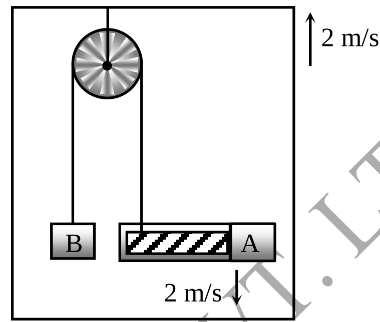
Q.5 Two forces of magnitude 6N and 10N are acting at a point. Resultant of these force is perpendicular to 6 N. Then find the magnitude of the resultant force. **[0008]**

Q.6 Three forces are acting at a point as shown in figure.



All forces are acting in a plane. Find the magnitude of resultant of these forces. **[0030]**

Q.7 In the figure shown, the velocity of lift is 2m/s while string is winding on the motor shaft with velocity 2m/s and block A is moving downwards with a velocity of 2m/s, the velocity of block B in m/s is-



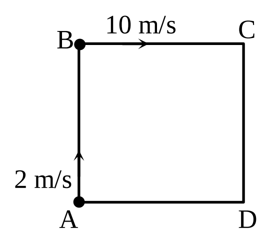
Sol. $\vec{V}_{B\ell} = 4$ m/s ↑

$\vec{V}_{B\ell} = \vec{V}_B - \vec{V}_\ell$

4 m/s = $\vec{V}_B$ – 2m/s

$\vec{V}_B$ = 4 + 2 = 6 m/s.

Q.8 Two men P & Q are standing at corners A & B of square ABCD of side 8 m. They start moving along the tank with constant speed 2m/s and 10 m/s respectively. The time when they will meet for the first time, in second is -



Sol. Relative displacement = relative velocity × time

8 × 3 = (10 – 2)t

⇒ t = 3 sec

Q.9 Two motor vehicles run at constant speeds 5 m/s each along highways intersecting at an angle 60º. In what time after they meet at the intersection will the distance between the vehicles be $10\sqrt{3}$ m.

Sol. u_{12} = Speed of second vehicle with respect to the first one

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$u_{12}^2 = v_1^2 + v_2^2 + 2v_1v_2 \cos \alpha$

$= (5^2 + 5^2 + 2 \times 5 \times 5 \cos 60^o)\ m^2/s^2$

$u_{12} = 5\sqrt{3}$ m/s

$t = \frac{S}{v_{1/2}} = \frac{10\sqrt{3}}{5\sqrt{3}}$ s = 2 s

Q.10 Magnitude of resultant of two vector $\vec{A}$ and $\vec{B}$ is equal to 2. Angle between two vectors is 180º. If $|\vec{A}| = 3$ then find $|\vec{B}|$ ($|\vec{B}|$ must be less then 2)

Sol. [1] (Given |A| = 3)

$|\vec{A}| - |\vec{B}| = 2$ $\qquad$ $|\vec{B}| > |\vec{A}|$

$|\vec{B}| = 1$

Q.11 Magnitude of subtraction of two vector $\vec{A}$ and $\vec{B}$ is equal to 5. Angle between both is 180º. Find the magnitude of resultant of these two vectors. If $|\vec{A}| = 2$. **[1]**

Sol. $|\vec{A}| + |\vec{B}| = 5$

$2 + |\vec{B}| = 5$

$|\vec{B}| = 3$

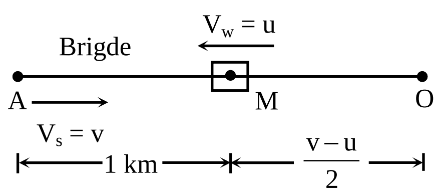
resultant of $|\vec{A}|$ and $|\vec{B}|$

$|\vec{B}| - |\vec{A}| = 3 - 2 = 1$

Q.12 A swimmer jumps from a bridge over a canal and swims 1 km up stream. After that first km, he passes a floating cork. He continues swimming for half an hour and then turns around and swims back to the bridge. The swimmer and the cork reach the bridge at the same time. The swimmer has been swimming at a constant speed. How fast does the water in the canal flow in km/hr.

Sol.[1] Let $V_w = u$ & $U_{sw} = v$

Time taken by swimmer to go from M to O and O to B = time taken by float to reach B from M.



$= \frac{1}{2} + \frac{1 + \frac{v-u}{2}}{v+u} = \frac{1}{u}$

$\Rightarrow \frac{1}{2} + \frac{2+v-u}{2(v+u)} = \frac{1}{u}$

$\Rightarrow \frac{(v+u+2+v-u)}{2(v+u)} = \frac{1}{u}$

$\Rightarrow$ (2v + 2) u = 2(v + u)

$\Rightarrow$ 2vu + 2u = 2v + 2u

u = 1 km/hr

Q.16 Two particles are moving with velocity $\vec{v}_1 = \hat{i} - 2t\,\hat{j}\ m/s$ and $\vec{v}_2 = 4\hat{i} + \hat{j}\ m/s$ respectively. Time at which they are moving perpendicular to each other is.

Sol. [2] $\vec{v}_1 \cdot \vec{v}_2 = 0 \Rightarrow t = 2\,sec$

Q.17 Motion of a particle is defined by the position vector $\vec{r}$ =A (cos t + t sin t) $\hat{i}$ + A (sin t– t cos t) $\hat{j}$, where t is time in sec. Value of 't' for which position vector and acceleration are perpendicular to each other is –

Sol. [1] $\vec{a} = \frac{d^2\vec{r}}{dt^2}$

$\vec{r}$ and $\vec{a}$ are perpendicular if $\vec{r} \cdot \vec{a} = 0$

Q.18 Magnitude of resultant of two vector $\vec{A}$ and $\vec{B}$ is equal to 2. Angle between two vectors is 180º. If $|\vec{A}| = 3$ then find $|\vec{B}|$ ($|\vec{B}|$ must be less then 2)

Sol.[1] (Given |A| = 3)

$|\vec{A}| - |\vec{B}| = 2$ $\qquad$ $|\vec{B}| > |\vec{A}|$

$|\vec{B}| = 1$

Q.19 Magnitude of subtraction of two vector $\vec{A}$ and $\vec{B}$ is equal to 5. Angle between both is 180º. Find the magnitude of resultant of these two vectors. If $|\vec{A}| = 2$.

Sol.[1] $|\vec{A}| + |\vec{B}| = 5$

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$2+|\vec{B}| = 5$

resultant of $|\vec{A}|$ and $|\vec{B}|$

$$|\vec{B}| - |\vec{A}| = 3 - 2 = 1$$

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PHYSICS

Q.1 Force acting on a particle is $(2\hat{i} + 3\hat{j})$ N. Work done by this force is zero, when a particle is moved on the line 3y + kx = 5. Here value of k is -

(A) 2 (B) 4 (C) 6 (D) 8

[A]

Sol. Force is parallel to a line $y = \frac{3}{2}x + c$

The equation of given line can be written as

$y = -\frac{k}{3}x + \frac{5}{3}$

Work done will be zero, when force is perpendicular to the displacement i.e., the above two lines are perpendicular or

$m_1m_2 = -1$

or $\left(\frac{3}{2}\right)\left(-\frac{k}{3}\right) = -1$

or k = 2

Q. 2 boat travels upstream in a river and at t = 0 a wooden cork is thrown over the side with zero initial velocity. After 7.5 minutes the boat turns and starts moving downstream catches the cork when it has drifted 1 km downstream. Then the velocity of water current is -

(A) 2 Km/hr (B) 4 Km/hr

(C) 6 Km/hr (D) 8 Km/h

Sol. **[B]**

Assume observer standing on water of flowing river.

Then, $V_r = \frac{d}{t}$ = 4 km/hr

Q. 3 Two vectors $\vec{a}$ and $\vec{b}$ lie in one plane. Vector $\vec{c}$ lies in different plane, then $\vec{a} + \vec{b} + \vec{c}$

(A) may be zero

(B) must be zero

(C) must not be zero

(D) All of above are possible

Sol. **[C]**

sum of three non coplanar vectors can not be zero

Q.4 A man crosses the river perpendicular to river flow in time t seconds and travels an equal distance down the stream in T seconds. The ratio of man's speed in still water to the speed of river water will be :

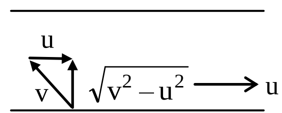
(A) $\frac{t^2 - T^2}{t^2 + T^2}$ (B) $\frac{T^2 - t^2}{T^2 + t^2}$

(C) $\frac{t^2 + T^2}{t^2 - T^2}$ (D) $\frac{T^2 + t^2}{T^2 - t^2}$

Sol. **[C]**

Let velocity of man in still water be v and that of water with respect to ground be u.

Velocity of man perpendicular to river flow with respect to ground = $\sqrt{v^2 - u^2}$



Velocity of man downstream = v + u

As given. $\sqrt{v^2 - u^2}\, t = (v + u)T$

$\Rightarrow (v^2 - u^2)t^2 = (v + u)^2 T^2$

$\Rightarrow (v - u)t^2 = (v + u)T^2$

$\therefore \frac{v}{u} = \frac{t^2 + T^2}{t^2 - T^2}$

Q.5 If $\vec{a}_1$ and $\vec{a}_2$ are two non collinear unit vectors and if $|\vec{a}_1 + \vec{a}_2| = \sqrt{3}$, then the value of $(\vec{a}_1 - \vec{a}_2) \cdot (2\vec{a}_1 + \vec{a}_2)$ is –

(A) 2 (B) $\frac{3}{2}$ (C) $\frac{1}{2}$ (D) 1

Sol. **[C]**

$a_1 = a_2 = 1$ and

$a_1^2 + a_2^2 + 2a_1a_2\cos\theta = (\sqrt{3})^2 = 3$

Or $1 + 1 + 2\cos\theta = 3$ or $\cos\theta = \frac{1}{2}$

Now $(\vec{a}_1 - \vec{a}_2)\ (2\vec{a}_1 + \vec{a}_2) = 2a_1^2 - a_2^2 - a_1a_2$ $\cos\theta$

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$= 2 - 1 - \frac{1}{2} = \frac{1}{2}$

Q.6 If $\vec{a}$ and $\vec{b}$ are two unit vectors and $\vec{R} = \vec{a} + \vec{b}$ and also if $|\vec{R}| = R$, then -

(A) $R < 0$

(B) $R > 2$

(C) $0 \le R \le 2$

(D) R must be 2

Sol. **[C]**

if $|\vec{R}| = |\vec{a} + \vec{b}|$ then, $(a - b) \le R \le (a + b)$

Q.7 Two forces $\vec{F_1}$ and $\vec{F_2}$ are acting on a particle. Magnitude of resultant of these force is $|\vec{F_1}|$. Then :

(i) $|\vec{F_2}|$ may be zero.

(ii) $|\vec{F_2}|$ may not be zero.

Select correct one -

(A) Only (i) is correct

(B) Only (ii) is correct

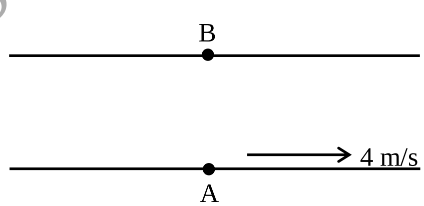
(C) Both (i) and (ii) will be correct

(D) Neither (i) nor (ii) will be correct

[C]

Sol. If $|\vec{F_1}| = |\vec{F_2}|$ and angle between these two forces is 120º then resultant is equal $|\vec{F_1}|$ or $|\vec{F_2}|$.

Q.8 A conveyor belt of width 10 m is moving along x-axis with speed 4 m/s as shown in the figure. Two points A and B are situated on the conveyor belt. A person want to move from A to B in least time. His speed with respect to belt is 2 m/s. The time taken by the person is –

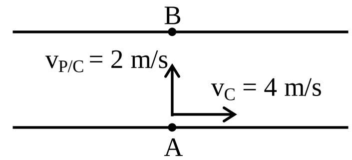


(A) 2.5 s (B) 5 s

(C) 4 s (D) 3 s

[B]

Sol.



$t_{min} = \frac{AB}{v_{P/C}}$

$= \frac{10}{2} = 5$ s

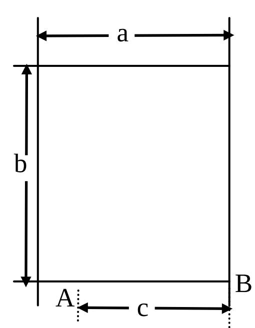
Q.9 There are two vectors $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} - 2\hat{k}$, then vector component of $\vec{A}$ along $\vec{B}$ is -

(A) $\frac{2\hat{i} + 2\hat{j} - 2\hat{k}}{9}$ (B) $\frac{2}{3}(2\hat{i} + \hat{j} + \hat{k})$

(C) $\frac{2}{3}(\hat{i} + 2\hat{j} - 2\hat{k})$ (D) None of these **[A]**

Sol. Component of $\vec{A}$ along $\vec{B} = \frac{\vec{A}.\vec{B}}{|\vec{B}|}.\hat{B}$

Q.10 A billiards-ball is at point A on a billiards-table whose dimensions are given in fig. At what angle should the ball be struck so that it should rebound from two cushions and go into pocket B? Assume that in striking the cushion, the ball's direction of motion changes according to the law of reflection of light from a mirror, i.e., the angle of reflection equals the angle of incidence.



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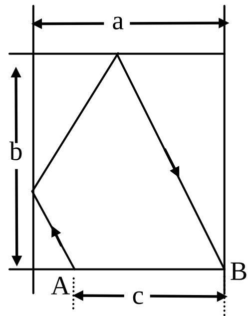
(A) $\frac{a-2c}{b}$ (B) $\frac{2a-c}{b}$

(C) $\frac{2a-c}{2b}$ (D) $\frac{a}{b}-\frac{a}{2c}$

[C]

Sol. Let us resolve the velocity v imparted to the ball into component parallel with the sides of the table and consider the path of a ball as shown, for example, in the diagram (fig.).



We obtain two equations, evident from the diagram :

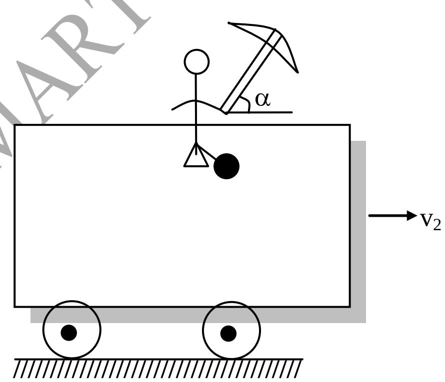
$\frac{2a-c}{t} = v\cos\alpha$, $\frac{2b}{t} = v\sin\alpha$,

From these equations we get :

$\mathbf{\cot\alpha = \frac{2a-c}{2b}}$,

i.e., we find angle α, at which the ball must be struck. The value for the velocity v which is imparted to the ball plays no part at all.

Q.11 A man is moving with constant velocity v_2 = 20m/s in horizontal plane. A what angle to the horizontal should the man hold his umbrella so that the can protect himself from rain falling vertically with velocity 60m/sec.



(A) $\sin^{-1} 3$ (B) $\tan^{-1} 3$

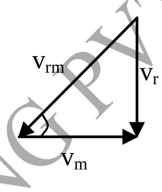
(C) $\cos^{-1} 3$ (D) $\tan^{-1}\frac{1}{\sqrt{3}}$

[B]

Sol. Man should hold the umbrella in direction in which rain appears to come i.e., $\vec{v}_{rm}$

$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$; $v_{rm} \rightarrow$ Velocity of rain with respect to man.

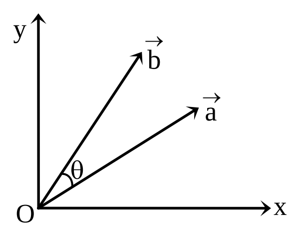
$\vec{v}_r = \vec{v}_{rm} + \vec{v}_m$



$\tan\alpha = \frac{v_r}{v_m} = \frac{60}{20} = 3$

$\mathbf{\alpha = \tan^{-1} 3}$

Q.12 For the vectors $\vec{a}$ and $\vec{b}$ shown in figure, $\vec{a} = \sqrt{3}\,\hat{i} + \hat{j}$ and $|\vec{b}|$ = 10 units while θ = 23º, then the value of R = $|\vec{a} + \vec{b}|$ is nearly –

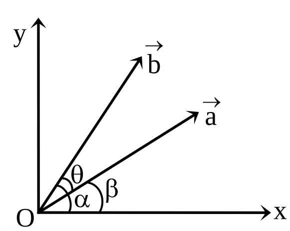


(A) 12 (B) 13

(C) 14 (D) 15

[A]

Sol.



$\tan\beta = \frac{1}{\sqrt{3}}$

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$\Rightarrow \quad \beta = 30^o$

$\therefore \quad \alpha = \theta + \beta = 53^o$

$\therefore \quad \vec{b} = 10\cos 53\,\hat{i} + 10\sin 53\,\hat{j} = 6\hat{i} + 8\hat{j}$

$\therefore \vec{a} + \vec{b} = (6 + \sqrt{3})\hat{i} + 9\hat{j}$

Q.13 The sum, difference and cross product of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are mutually perpendicular if :

(A) $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other and $|\vec{\mathbf{A}}| = |\vec{\mathbf{B}}|$

(B) $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other

(C) $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are perpendicular but their magnitudes are arbitrary

(D) $|\vec{\mathbf{A}}| = |\vec{\mathbf{B}}|$ and their directions are arbitrary

[D]

Sol. Let $\vec{\mathbf{A}} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}}$

Given that $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ is perpendicular to $\vec{\mathbf{A}} - \vec{\mathbf{B}}$

i.e., $(\vec{\mathbf{A}} + \vec{\mathbf{B}}) \cdot (\vec{\mathbf{A}} - \vec{\mathbf{B}}) = 0$

or $(a_1 + b_1)(a_1 - b_1) + (a_2 + b_2)(a_2 - b_2) + (a_3 + b_3)(a_3 - b_3) = 0$

or $a_1^2 + a_2^2 + a_3^2 = b_1^2 + b_2^2 + b_3^2$

or $|\vec{\mathbf{A}}| = |\vec{\mathbf{B}}|$

cross product of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is perpendicular to the plane formed by $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ or $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ and $\vec{\mathbf{A}} - \vec{\mathbf{B}}$.

Q.14 A particle moves in x-y plane. The position vector of particle at any time t is = $\vec{\mathbf{r}}$ $\{(2t)\hat{\mathbf{i}} + (2t^2)\hat{\mathbf{j}}\}$ m. The rate of change of θ at time t = 2 s. (where θ is the angle which its velocity vector makes with positive x-axis) is :

(A) $\frac{2}{17}$ rad/s (B) $\frac{1}{14}$ rad/s

(C) $\frac{4}{7}$ rad/s (D) $\frac{6}{5}$ rad/s

[A]

Sol. $x = 2t \Rightarrow v_x = \frac{dx}{dt} = 2$

$y = 2t^2 \Rightarrow v_y = \frac{dy}{dt} = 4t$

$\therefore \tan\theta = \frac{v_y}{v_x} = \frac{4t}{2} = 2t$

Differentiating with respect to time we get,

$(\sec^2\theta)\frac{d\theta}{dt} = 2$

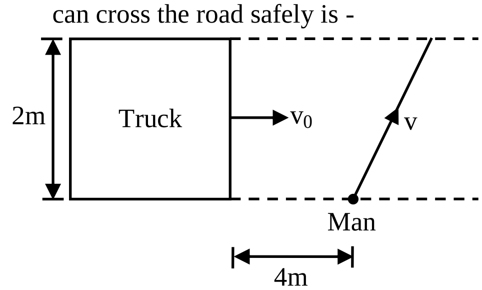
or $(1 + \tan^2\theta)\frac{d\theta}{dt} = 2$

or $(1 + 4t^2)\frac{d\theta}{dt} = 2$

or $\frac{d\theta}{dt} = \frac{2}{1+4t^2}$

$\frac{d\theta}{dt}$ at t = 2 s is $\frac{d\theta}{dt} = \frac{2}{1+4(2)^2} = \frac{2}{17}$ rad/s

Q.15 A 2 m wide truck is moving with a uniform speed v_0 = 8 m/s along a straight horizontal road. A pedestrian starts to cross the road with a uniform speed **v** when the truck is 4 m away from him. The minimum value of **v** so that he can cross the road safely is -



(A) 2.62 m/s (B) 4.6 m/s

(C) 3.57 m/s (D) 1.414 m/s

[C]

Sol. Let the man starts crossing the road at an angle θ as shown in figure. For safe crossing the condition is that the man must cross the road by the time the truck describes the distance 4 + AC or 4 + 2 cot θ.

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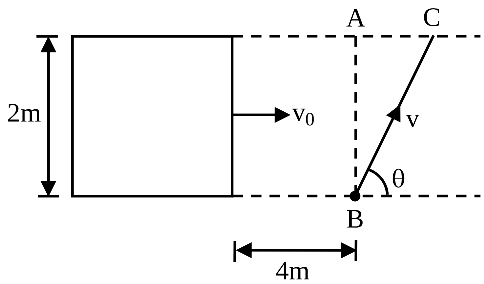


$$\therefore \frac{4+2\cot\theta}{8} = \frac{2/\sin\theta}{v}$$

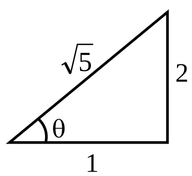
or $v = \dfrac{8}{2\sin\theta+\cos\theta}$ … (1)

For minimum v, $\dfrac{dv}{d\theta} = 0$

or $\dfrac{-8(2\cos\theta-\sin\theta)}{(2\sin\theta+\cos\theta)^2} = 0$

or $2\cos\theta - \sin\theta = 0$

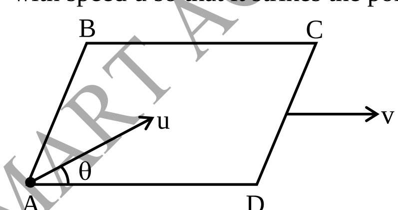
or $\tan\theta = 2$



From Eq. (1) $v_{min} = \dfrac{8}{2\left(\dfrac{2}{\sqrt{5}}\right)+\dfrac{1}{\sqrt{5}}} = \dfrac{8}{\sqrt{5}} =$

3.57 m/s

Q.16 A smooth square platform ABCD is moving towards right with a uniform speed v. At what angle θ must a particle be projected from A with speed u so that it strikes the point B –



(A) $\sin^{-1}\left(\dfrac{u}{v}\right)$ (B) $\cos^{-1}\left(\dfrac{v}{u}\right)$

(C) $\cos^{-1}\left(\dfrac{u}{v}\right)$ (D) $\sin^{-1}\left(\dfrac{v}{u}\right)$

[B]

Sol. Particle will strike the point B if velocity of particle with respect to platform is along AB or component of its relative velocity along AD is zero i.e.,

$u\cos\theta = v$

or $\theta = \cos^{-1}\left(\dfrac{v}{u}\right)$

Q.17 The vectors from origin to the points A and B are $\vec{A} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} + \hat{j} - 2\hat{k}$ respectively. The area of the triangle OAB will be -

(A) $\dfrac{5}{2}\sqrt{17}$ sq. unit (B) $\dfrac{2}{5}\sqrt{17}$ sq. unit

(C) $\dfrac{3}{5}\sqrt{17}$ sq. unit (D) $\dfrac{5}{3}\sqrt{17}$ sq. unit

3.[A] Area of $\Delta = \dfrac{1}{2}\left|\overrightarrow{OA}\times\overrightarrow{OB}\right|$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & 2 \\ 2 & 1 & -2 \end{vmatrix}$$

$$= \frac{1}{2}\left|(10\hat{i}+10\hat{j}+15\hat{k})\right| = \frac{1}{2}\sqrt{425} = \frac{5}{2}\sqrt{17}$$

Q.18 The force $(3\hat{i}-\hat{j}+\hat{k})$ N displaces a body from (1, 2, 0) to (3, 4, 5). Coordinates are in metre. The work done is -

(A) 9 J (B) 18 J

(C) 11 J (D) 29 J

[A]

Sol. $\vec{F} = 3\hat{i} - \hat{j} + \hat{k}$

$\vec{s} = 2\hat{i} + 2\hat{j} + 5\hat{k}$

W = 6 – 2 + 5 = 9 J

Q.19 12 forces, each equal to P, act on a body. If each force makes an angle of 30º with the next one, the resultant of all the forces is -

(A) zero (B) 3 P

(C) 6 P (D) 12 P

[A]

Sol. $N = 12,\ \theta = \dfrac{2\pi}{12} = 30^{\circ}$

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Resultant force = 0

Q.20 A force of 6 kgf and another of 8 kgf can be applied to produce the effect of a single force equal to -

(A) 16 kgf (B) 1 kgf

(C) 10 kgf (D) 0 kgf

[C]

Sol. 10 kgf

Q.30 The resultant of two vectors $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If the magnitude of $\vec{Q}$ is doubled, the new resultant becomes perpendicular to $\vec{P}$, then the magnitude of $\vec{R}$ is -

(A) $\frac{P^2-Q}{2PQ}$ (B) $\frac{P+Q}{P-Q}$

(C) Q (D) $\frac{P}{Q}$

[C]

Sol. $R = \sqrt{P^2+2PQ\cos\theta+Q^2}$ …(1)

$\tan 90^o = \frac{2Q\sin\theta}{P+2Q\cos\theta} = \frac{1}{0}$

P + 2Q cos θ = 0

from (1)

$R = \sqrt{P(P+2Q\cos\theta)+Q^2}$

R = Q

Q.31 Given : $\vec{P} = \vec{A} - \vec{B}$ and P = A + B. The angle between $\vec{A}$ and $\vec{B}$ is -

(A) 0º (B) 90º

(C) 180º (D) 270º

[C]

Sol. $(A + B) = \sqrt{A^2+B^2-2AB\cos\theta}$

$A^2 + B^2 + 2AB = A^2 + B^2 - 2AB\cos\theta$

2AB (1 + cos θ) = 0

cos θ = – 1

θ = 180º

Q.32 If $\vec{A} = \hat{i}+2\hat{j}+2\hat{k}$ and $\vec{B} = 3\hat{i}+6\hat{j}+2\hat{k}$, then the vector in the direction of $\vec{A}$ and having same magnitude as $|\vec{B}|$, is -

(A) $\frac{7}{3}(\hat{i}+2\hat{j}+2\hat{k})$ (B) $7(\hat{i}+2\hat{j}+2\hat{k})$

(C) $\frac{3}{7}(\hat{i}+2\hat{j}+2\hat{k})$ (D) $\frac{7}{9}(\hat{i}+2\hat{j}+2\hat{k})$

[A]

Sol. Vector in the direction of $\vec{A}$ and having same magnitude as B is = $B\hat{A}$

$= B\left(\frac{\vec{A}}{A}\right)$

$= \frac{7}{3}(\hat{i}+2\hat{j}+2\hat{k})$

Q.33 Two forces of magnitudes F and $\sqrt{3}$ F act at right angles to each other. Their resultant makes an angle β with F. The value of β is -

(A) 30º (B) 45º

(C) 60º (D) 135º

[C]

Sol. $\tan\beta = \frac{B\sin\theta}{A+B\cos\theta}$

$= \frac{F\sqrt{3}\sin\theta}{F+F\sqrt{3}\cos\theta}$

$= \frac{\sqrt{3}\sin\theta}{1+\sqrt{3}\cos\theta}$

$\tan\beta = \frac{\sqrt{3}}{1}$ (θ = 90º)

β = 60º

Q.34 A truck travelling due north at 20 m s^{-1} turns west and travels with same speed. What are the changes in velocity?

(A) $20\sqrt{2}$ m s^{-1} south-west

(B) 40 m s^{-1} south-west

(C) $20\sqrt{2}$ m s^{-1} north-west

(D) 40 m s^{-1} north-west

[A]

Sol. $\vec{v}_1$ = 20 m/s due north

$\vec{v}_2$ = 20 m/s due west

$|\vec{v}_2 - \vec{v}_1| = \sqrt{v_1^2+v_2^2-2v_1v_2\cos\theta}$

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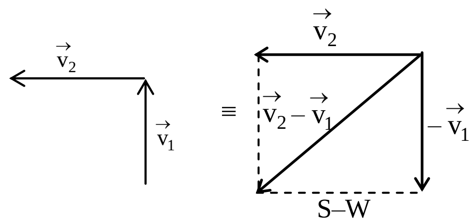
$= \sqrt{20^2 + 20^2 - 0}$ [$\because \theta = 90°$]

$= 20\sqrt{2}$ m/s



Direction of $\vec{v}_2 - \vec{v}_1$ is due South -West

Q.35 Let $\vec{A} = \frac{1}{\sqrt{2}} \cos\theta\,\hat{i} + \frac{1}{\sqrt{2}} \sin\theta\,\hat{j}$ be any vector. What will be the unit vector $\hat{n}$ in the direction of $\vec{A}$?

(A) $\cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

(B) $-\cos\theta\,\hat{i} - \sin\theta\,\hat{j}$

(C) $1/\sqrt{2}\,(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$

(D) $1/\sqrt{2}\,(\cos\theta\,\hat{i} - \sin\theta\,\hat{j})$

[A]

Sol. $\because\ \vec{A} = \frac{1}{\sqrt{2}}\cos\theta\hat{i} + \frac{1}{\sqrt{2}}\sin\theta\hat{j}$

$\therefore\ |\vec{A}| = \frac{1}{\sqrt{2}}\sqrt{\cos^2\theta + \sin^2\theta} = \frac{1}{\sqrt{2}}$

$\Rightarrow \hat{n} = \frac{\vec{A}}{|\vec{A}|} = \cos\theta\hat{i} + \sin\theta\hat{j}$

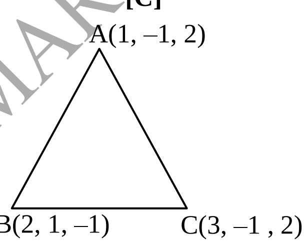
Q.36 The area of the triangle whose vertices are A (1, –1, 2), B(2, 1, –1) and C (3, –1, 2) is -

(A) 26 (B) $7\sqrt{13}$

(C) $\sqrt{13}$ (D) 8

[C]

Sol.



$\overrightarrow{AB} = \hat{i} + 2\hat{j} - 3\hat{k}$

$\overrightarrow{BC} = \hat{i} - 2\hat{j} + 3\hat{k}$

$$\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 1 & -2 & 3 \end{vmatrix} = -6\hat{j} - 4\hat{k}$$

$|\overrightarrow{AB} \times \overrightarrow{BC}| = \sqrt{(-6)^2 + (-4)^2} = \sqrt{52}$ unit

$\therefore$ Area of $\Delta = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{BC}| = \sqrt{13}$ unit

Q.37 An engine exerts a force $\vec{F} = (20\hat{i} - 3\hat{j} + 5\hat{k})$N and moves with velocity $\vec{v} = (6\hat{i} + 20\hat{j} - 3\hat{k})$ m/s. The power of the engine (in watt) is -

(A) 45 (B) 75

(C) 20 (D) 10

[A]

Sol. $\because\ P = \vec{F}.\vec{v} = (20\hat{i} - 3\hat{j} + 5\hat{k}) \cdot (6\hat{i} + 20\hat{j} - 3\hat{k})$

= 120 – 60 – 15 = 45 W

Q.38 The maximum and the minimum magnitudes of the resultant of two given vectors are 17 unit and 7 unit respectively. If these two vectors are at right angles to each other, the magnitude of their resultant is -

(A) 14 (B) 16

(C) 18 (D) 13

[D]

Sol. $\because$ A + B = 17

& A – B = 7 $\Rightarrow$ $\boxed{\begin{matrix} A = 12 \\ B = 5 \end{matrix}}$

$\because \theta = 90°$

$\therefore\ R = \sqrt{A^2 + B^2} = \sqrt{12^2 + 5^2} = 13$ unit

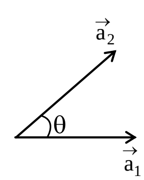
Q.39 A vector of magnitude a is rotated through an angle θ. What is the magnitude of the change in the vector?

(A) $2a \sin\frac{\theta}{2}$ (B) $2a \cos\frac{\theta}{2}$

(C) $2a \sin\theta$ (D) $2a \cos\theta$

[A]

Sol.



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$|\vec{a}_1| = |\vec{a}_2| = a$

$|\Delta\vec{a}| = |\vec{a}_2 - \vec{a}_1|$

$= \sqrt{a_1^2 + a_2^2 - 2a_1a_2\cos\theta}$

$= \sqrt{a^2 + a^2 - 2a^2\cos\theta}$

$= 2a \sin \theta/2$

Q.40 Resultant of two vectors $\vec{F}_1$ and $\vec{F}_2$ is of magnitude P. If $\vec{F}_2$ is reversed, then resultant is of magnitude Q. What is the value of $P^2 + Q^2$?

(A) $F_1^2 + F_2^2$ (B) $F_1^2 - F_2^2$

(C) $2(F_1^2 - F_2^2)$ (D) $2(F_1^2 + F_2^2)$

[D]

Sol. $\because |\vec{F}_1 + \vec{F}_2| = P$

$\Rightarrow F_1^2 + F_2^2 + 2F_1F_2\cos\theta = P^2$ …..(1)

& $|\vec{F}_1 - \vec{F}_2| = Q$

$\Rightarrow F_1^2 + F_2^2 - 2F_1F_2\cos\theta = Q^2$ …..(2)

(1) + (2), $\boxed{2(F_1^2 + F_2^2) = P^2 + Q^2}$

Q.41 An athlete completes one round of a circular track of radius R in 40 second. What will be his displacement at the end of 2 minute 20 second?

(A) Zero (B) 2R

(C) $2\pi R$ (D) $7\pi R$

[B]

Sol. 2 min 20 second = 140 second

$\because$ He complete one round in 40 sec, thus he will complete 3 round in 120 sec. and half round in 20 sec, thus his displacement is 2R.

Q.42 A body covered a distance of 5 m along a semicircular path. The ratio of distance to displacement is -

(A) 11 : 7 (B) 12 : 5

(C) 8 : 3 (D) 7 : 5

[A]

Sol. In semi-circular path, Distance = πr

& Displacement = 2r

$\therefore$

$$\frac{\text{Distance}}{\text{Displacement}} = \frac{\pi r}{2r} = \frac{\pi}{2} = \frac{11}{7}$$

Q.43 For two vectors $\vec{a}$ and $\vec{b}$, if $\vec{R} = \vec{a} + \vec{b}$ and $\vec{S} = \vec{a} - \vec{b}$, if $2|\vec{R}| = |\vec{S}|$, when $\vec{R}$ is perpendicular to $\vec{a}$, then –

(A) $\frac{a}{b}\sqrt{\frac{3}{7}}$ (B) $\frac{a}{b} = \sqrt{\frac{7}{3}}$

(C) $\frac{a}{b} = \sqrt{\frac{1}{5}}$ (D) $\frac{a}{b} = \sqrt{\frac{5}{1}}$

[A]

Sol. As $\vec{R}$ is perpendicular to $\vec{a}$ therefore

$\cos\theta = \frac{-a}{b} \Rightarrow R = \sqrt{b^2 - a^2}$ and $S = \sqrt{3a^2 + b^2}$

As $2|\vec{R}| = |\vec{S}|$

$\Rightarrow 4b^2 - 4a^2 = 3a^2 + b^2 \Rightarrow 3b^2 = 7a^2$

Q.44 If $\vec{r} = bt^2\,\hat{i} + ct^3\,\hat{j}$, where b and c are positive constant, the time at which velocity vector makes an angle $\theta = 60°$ with positive y-axis is -

(A) $\frac{c}{b}$ (B) $\frac{2b}{3\sqrt{3}c}$

(C) $\frac{2c}{\sqrt{3}b}$ (D) $\frac{2b}{\sqrt{3}c}$

[B]

Sol. $\frac{d\vec{r}}{dt} = 2bt\,\hat{i} + 3ct^2\,\hat{j}$

$\tan 60° = \frac{2bt}{3ct^2}$

$\Rightarrow t = \frac{2b}{3\sqrt{3}\,c}$

Q.45 Two vectors $\vec{A}$ and $\vec{B}$ are given in the figure :

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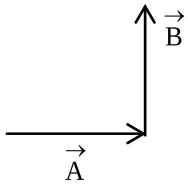
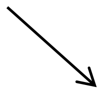
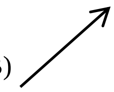
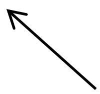


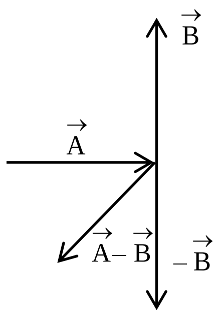
Then $\vec{A} - \vec{B}$ is given by -

(A)  (B) 

(C)  (D) None of these

[D]

Sol.



Q.46 Resultant of two vectors $\vec{A}$ and $\vec{B}$ is $\vec{R}$. Now magnitude of vector is doubled keeping direction same, then magnitude of the resultant becomes $|\vec{B}|$. Angle between vector $\vec{A}$ and $\vec{B}$ is 120º. Then magnitude of $\vec{A}$ is equal to -

(A) $|\vec{B}|$ (B) $2|\vec{B}|$

(C) $\frac{|\vec{B}|}{2}$ (D) $4|\vec{B}|$

[C]

Sol. $2|\vec{A}| = |\vec{B}|$

$|\vec{A}| = \frac{|\vec{B}|}{2}$

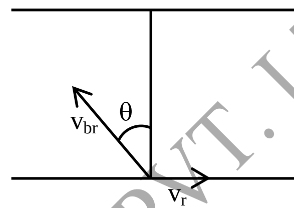
Q.47 A man can row a boat with speed v_{br} in still water, speed of river flow is v_r and v_r is two times of v_{br} . Man reach one bank to other bank and follow a path so that the path traveled by boat is shortest. Then angle between velocity vector of boat in still water and current flow of water in river is -

(A) 30º (B) 60º

(C) 90º (D) 120º

[D]

Sol. For shortest path when $v_{br} < v_r$.



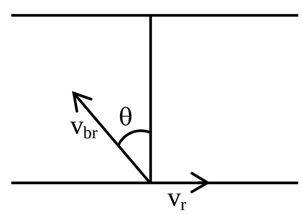
$\sin\theta = \frac{v_{br}}{v_r}$

Q.48 A man can row a boat with speed v_{br} in still water, speed of river flow is v_r and v_r is two times of v_{br} . Man reach one bank to other bank and follow a path so that the path traveled by boat is shortest. Then angle between velocity vector of boat in still water and current flow of water in river is -

(A) 30º (B) 60º

(C) 90º (D) 120º **[D]**

Sol. For shortest path when $v_{br} < v_r$.



$\sin\theta = \frac{v_{br}}{v_r}$

Q.48 Vector $\vec{R}$ is the resultant of the vectors $\vec{A}$ and $\vec{B}$. Ratio of minimum value of $|\vec{R}|$ and maximum value of $|\vec{R}|$ is $\frac{1}{4}$. Then $\frac{|\vec{A}|}{|\vec{B}|}$ may be –

(A) $\frac{4}{1}$ (B) $\frac{2}{1}$

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(C) $\frac{3}{5}$ (D) $\frac{1}{4}$

[C]

Sol. $\frac{|\vec{R}|_{min}}{|R|_{max}} = \frac{1}{4} = \frac{||\vec{A}|-|\vec{B}||}{||\vec{A}|+|\vec{B}||}$

$|\vec{A}|+|\vec{B}| = 4\,||\vec{A}|-|\vec{B}||$

If $|\vec{A}| > |\vec{B}|$

$|\vec{A}| + |\vec{B}| = 4\,(|\vec{A}| - |\vec{B}|)$

$3|\vec{A}| = 5|\vec{B}| \Rightarrow \frac{|\vec{A}|}{|\vec{B}|} = \frac{5}{3}$

If $|\vec{B}| > |\vec{A}|$

$|\vec{A}|+|\vec{B}| = 4\,(|\vec{B}|-|\vec{A}|)$

$\frac{|\vec{A}|}{|\vec{B}|} = \frac{3}{5}$

Q.49 A launch takes 3 hours to go downstream from point A to point B and 6 hours to come back. Time taken by this launch to cover the distance AB downstream when its engine cut-off is –

(A) 3 hr (B) 6 hr

(C) 9 hr (D) 12 hr

[D]

Sol. $T = \frac{2t_1t_2}{t_2 - t_1} = 12$ hr

Q.50 There are two vectors $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = \hat{i} - \hat{j} + \hat{k}$, then component of $\vec{A}$ along $\vec{B}$ –

(A) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$ (B) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

(C) $\sqrt{3}(\hat{i} + \hat{j} + \hat{k})$ (D) $(\hat{i} - \hat{j} + \hat{k})$

[B]

Sol. Component of $\vec{A}$ along $\vec{B} = \left(\frac{\vec{A}.\vec{B}}{|\vec{B}|}\right)\hat{B}$

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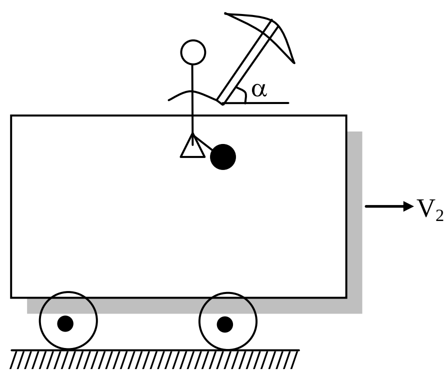
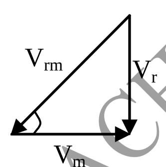
Q.1 A man is moving with constant velocity v_2 = 20m/s in horizontal plane. A what angle to the horizontal should the man hold his umbrella so that the can protect himself from rain falling vertically with velocity 60m/sec.



Sol. Man should hold the umbrella in direction in which rain appears to come i.e., $\vec{V}_{rm}$

$\vec{V}_{rm} = \vec{V}_r - \vec{V}_m$; $V_{rm} \rightarrow$ Velocity of rain with respect to man.

$\vec{V}_r = \vec{V}_{rm} + \vec{V}_m$



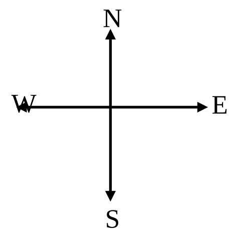
$\tan\alpha = \frac{V_r}{V_m} = \frac{60}{20} = 3$

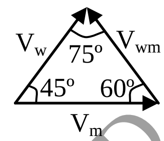
$\boldsymbol{\alpha = \tan^{-1}3}$

Q.2 A man is walking toward east with a velocity of 8 km/h. Wind is blowing toward north-east at angle of 45º. To man wind appears to blow at angle of 60º north of west. Find the
(i) true velocity of wind.
(ii) Velocity of wind relative to man.

Sol. $\vec{V}_{wm} = \vec{V}_w - \vec{V}_m$

$\vec{V}_w = \vec{V}_{wm} + \vec{V}_m$





From since rule

$\frac{V_w}{\sin 60^o} = \frac{V_{wm}}{\sin 45^o} = \frac{V_m}{\sin 75^o}$

$V_w = \left(\frac{\sin 60^o}{\sin 75^o}\right) V_m$; $V_w = \frac{\sqrt{3}}{2\frac{(1+\sqrt{3})}{3\sqrt{2}}} V_m$;

$V_m = \frac{8\sqrt{6}}{1+\sqrt{3}}$ km/h **Ans**

Q.3 A boat can travel at a speed of $3ms^{-1}$ on still water. A boat man wants to cross a river whilst covering the shortest possible distance. In what direction he row with respect to the bank if the speed of the water is
(i) 2 ms^{-1} (ii) 4 ms^{-1}
Assume that speed of water is the same every where.

Sol. (i) The shortest path is one perpendicular to the bank and the boat goes in this direction if the boatman rows in the direction shown in fig.1

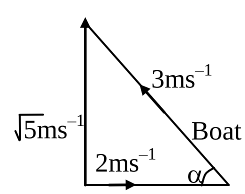


Fig.(1)

The resultant speed of boat (in the direction perpendicular to the bank) is $\sqrt{5}$ $ms^{-1} \approx 2.24$ ms^{-1}. The boatman has to row

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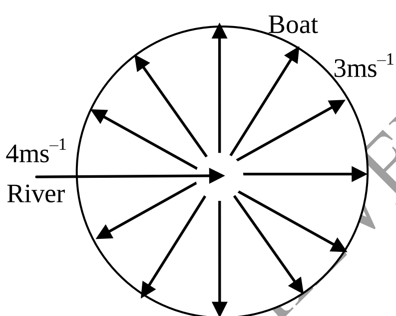
upstream at an angle α to the bank, where cos α = $\frac{2}{3}$; this gives α ≈ 48°.

(ii) In this case, the current is so strong that the boat will move downstream even if the boatman rows at full speed against the stream. This means, in contrast to the previous case, he cannot choose his direction with respect to the bank and in particular, he cannot travel across in a direction perpendicular to the bank.

The possible directions he can take may be determined by adding all the possible still-water velocities of the boat to the velocity of the river. Draw the velocity vector of the river and, from the endpoint of this vector, draw velocity vectors in all directions, with a magnitude equal to the speed of the boat in still water. The endpoints of these vectors will form a circle as shown in fig.(2)



The possible resultant velocities of the boat can be obtained by joining the starting point of the velocity of the river to the points on this circle. The resultant corresponding to the shortest path will be the one that makes the greatest angle with the direction of the current, i.e., when the line of action of the resultant velocity vector is a tangent to the circle (see fig.3)

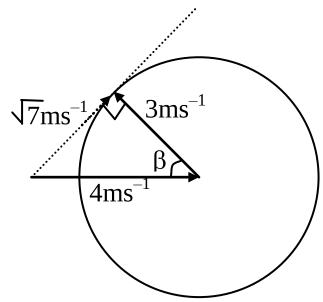


Fig.(3)

Thus, the velocity of the boat with respect to the shore is $\sqrt{7}$ m/s ≈ 2.65 ms^{-1}. Again, the boatman has to row upstream, but this time at an angle β to the bank, where cos β = $\frac{3}{4}$, yielding β ≈ 41°. The figure also shows that in this case the distance traveled by the boat will be $\frac{4}{3}$ times the width of the river.

Q.4 Two men decide to fight a duel with revolvers in unusual circumstances. They are to fire while standing on a round about of radius R, which is turning with angular velocity ω. The first duellist stands at the center O of the round about, the second at its edge. How should they each aim so as to hit his opponent ? Which is in the more favourable position ? Assume that the first duellist's bullet is fired from O at a velocity v.

Sol. The first duellist must take into account that during the flight of his bullet, his opponent will have moved to another position. The time of flight of the first duellist's bullet does not depend on the roundabout's rotation: t = R/v. During time t, the roundabout will turn and point A will move to position B (fig.) i.e., through an arc of length s=ωR.t;so he must

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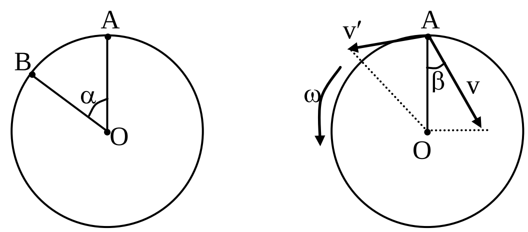


(a) (b)

Fire in the direction OB, not in the direction OA. The angle α can be found from the equation $\alpha = \omega Rt/vt$. The duellist standing on the circumference moves with a velocity of v′ =

ωR, so the velocity of his bullet is compounded of two velocities v and v ′. If he is to hit the centre of the roundabout, he too must not fire along AO, but at an angle which may be found from the formula

$$\sin\beta = \frac{\omega R}{v}$$

while v′ < v

$\beta \approx \omega R/v$, i.e., both duellists should aim at the same angle towards left of their respective opponents (if the rotation of the roundabout is in the direction given in the diagram). But as ωR increase β must increase, and the resultant velocity of the bullet will increase and therefore the danger of being hit will decrease for first duellist.

When ωR = v , sin β must equal l, i.e., the second duellist should aim in the direction opposite to v′. But in this case resultant velocity of the bullet would be zero.

The time of flight of the second duellist's bullet t′ depends on the roundabout's speed of rotation

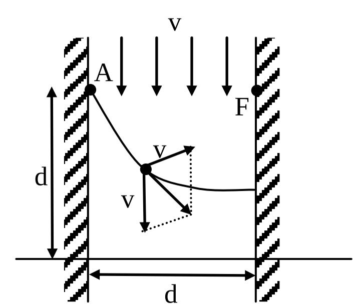
$$t' = \frac{R}{v\cos\beta} = \frac{R}{v}\cdot\frac{1}{\sqrt{1-\frac{\omega^2R^2}{v^2}}} = \frac{1}{\sqrt{\frac{v^2}{R^2}-\omega^2}}$$

When ωR = v, t′ = ∞, i.e. the bullet of the second due 'hangs' in the air at A and he travels up to his own bullet which case, as we have seen above, the resultant velocity bullet equals zero).

When ωR > v the resultant velocity cannot anyhow directed towards O, i.e., the first duellist cannot be hit by second duellist's bullets. But the first can hit the second, if selects angle α aright.

Q.5 A boatman sets off from one bank of a straight, uniform canal for a mark directly opposite the starting point. The speed of the water flowing in the canal is **v** everywhere. The boatman rows steadily at such a rate that, were there no current, the boat's speed would also be **v**. He always set the boat's course in the direction of the mark, but the water carries him downstream. Fortunately he never tires! How far downstream does the water carry the boat ? What trajectory does it follow with respect to the bank ?

Sol. **Method 1 :** Denote the width of the canal by d and draw a straight line perpendicular to its banks a distance d downstream from the boat's starting point A (see figure).



The boat is initially at distance d both from the mark F on the opposite bank and from this straight line. As both the speed of the water and that of the boat with respect to the water are v, the water takes the boat downstream by the same distance as is covered by the boat in the direction of F.

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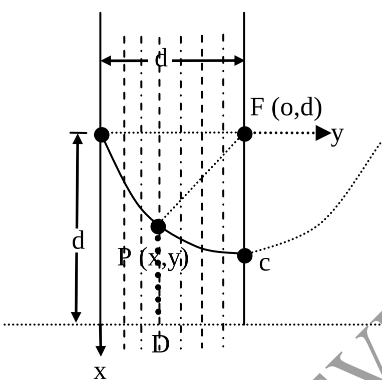
This means that the boat is always equally far from point F and the straight line. The path of the boat is therefore a parabola, with F as its focus and the straight line as its directrix . After a very long time, the boat approaches the opposite bank at a point d/2 from F. Because the speed of the current equals that of the boat, the boatman cannot land closer than this.

Method 2 :

For calculating the distance of boat from point F i.e., FC



PF = FD is the trajectory is parabola

$\sqrt{x^2+(d-y)^2} = (d-x)$

$x^2 + (d-y)^2 = (d-x)^2$

$x^2 + d^2 + y^2 - 2dy = d^2 + x^2 - 2dx$

$y^2 - 2dy = 2dx$

$(d-y)^2 = d^2 - 2dx$

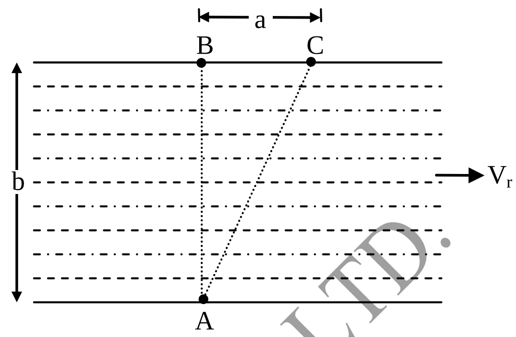
$(d-y)^2 = d\,[d - 2x]$

$(d-y)^2 = 2d\,[x - d/2]$

When y = d then x = d/2

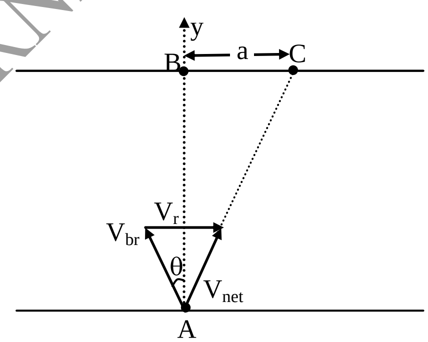
i.e., FC = d/2.

Q.6



What should be the minimum velocity of boat with respect to the river so that starting from point A the boat reaches point C on the opposite bank. Velocity of river flow is V_r.

Sol. **Method 1 :**



$\overrightarrow{V_{net}} = \overrightarrow{V_b} = \overrightarrow{V_{br}} + \overrightarrow{V_r}$

$(V_{net})_x = V_r - V_{br}\sin\theta$

$(V_{net})_y = V_{br}\cos\theta$

Time taken to cross the river

$$T = \frac{b}{(V_{net})_y} = \frac{b}{V_{br}\cos\theta}$$

Drift BC = $(V_{net})_x t$

$$a = BC = (V_r - V_{br}\sin\theta)\,\frac{b}{V_{br}\cos\theta}$$

$$V_{br} = \frac{bV_r}{a\cos\theta + b\sin\theta}$$

………..(i)

For V_{br} to minimum

$$\frac{d}{d\theta}(V_{br}) = 0$$

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i.e., $-\dfrac{bv_r[-a\sin\theta + b\cos\theta]}{(a\cos\theta + b\sin\theta)^2} = 0$

$\tan\theta = \tan\theta_0 = \dfrac{b}{a}$ ………….(ii)

Putting (ii) in (i)

$$(V_{br})_{min} = \frac{bV_r}{\dfrac{a^2}{\sqrt{a^2+b^2}} + \dfrac{b^2}{\sqrt{a^2+b^2}}}$$

$$(V_{br})_{min} = \frac{bV_r}{\sqrt{a^2+b^2}}$$

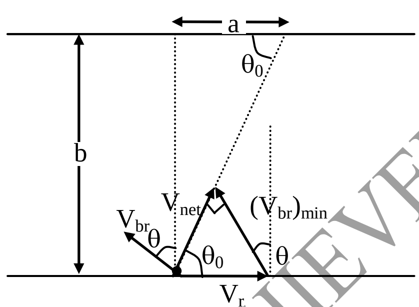
Method 2 :

From $\overrightarrow{V_{net}} = \overrightarrow{V_{br}} + \overrightarrow{V_r}$

Draw a vector triangle. Draw a perpendicular from tip of V_r on line of V_{net}. That will give minimum V_{br}, because minimum distance is perpendicular distance.

From Vector triangle

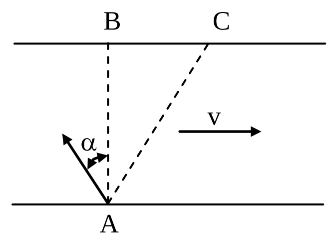
$(V_{br})_{min} = V_r\sin\theta_0$



$(V_{br})_{min} = V_r \dfrac{b}{\sqrt{b^2+a^2}}$

Since from figure $\tan\theta_0 = \dfrac{b}{a}$.

Q.7 A man in a boat crosses a river from point A (fig.) after he rows perpendicular to the banks then, 10 minutes S = 120 m downstream from point B. If the man heads at a certain angle α to the straight line AB (AB is perpendicular to the banks) against the current he will reach point B after 12.5 minutes.



Find the width of the river ℓ, the velocity of the boat u relative to the water, the speed of the current v and the angle α. Assume the velocity of the boat relative to the water to be constant and of the same magnitude in both cases.

Sol. **ℓ = 200 m ; u = 20 m/min; v = 12 m/min; α = 36º50′.**

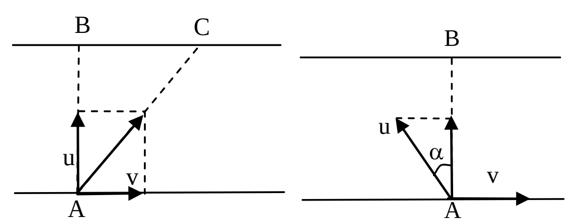
In both cases the motion of the boat is composed of its motion relative to the water and its motion together with the water relative to the bank.

First case (fig.) The boat moves along the river with a velocity v and covers during the crossing a distance downstream of

$\ell_1 = vt_1$ ………(i)

The boat moves across the river with a velocity u and traverses a distance of

$\ell_2 = ut_1$ ………(ii)



Second case (fig.). The velocity of the boat along the river is zero, i.e.,

$u \sin \alpha = v$ ……...(iii)

The velocity across the river is equal to u cos α and the distance ℓ_2 covered during the distance ℓ_2 covered during the crossing will be

$\ell_2 = u \cos \alpha t_2$ …….(iv)

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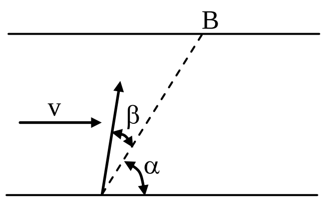
Solving the system of the four equations (i), (ii), (iii), (iv) we obtain

$$\ell = \frac{t_2 S}{\sqrt{t_2^2 - t_1^2}},\ u = n\frac{\ell}{t_1},\ v = \frac{S}{t_1},\ \alpha = \text{arc sin}\frac{v}{u}$$

Q.8 A launch plies between two points A and B on the opposite banks of a river (Fig.), always following the line AB. The distance S between points A and B is 1,200 m. The velocity of the river current v = 1.9m/s is constant over the entire width of the river. The line AB makes an angle $\alpha = 60^\circ$ with the direction of the current. With what velocity **u** and at what angle **β** to the line AB should be launch move to cover the distance AB and back in a time t = 5 min ? The angle β remains the same during the passage from A to B and from B to A.

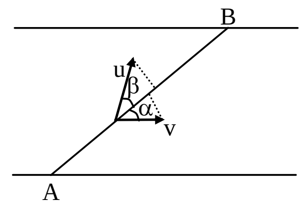


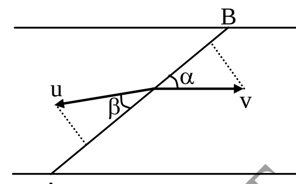
Sol. **u = 8 m/s; β ≈ 120º**

As in the previous problem the velocities of the current and of the launch should be resolved into components along the line AB and perpendicularly to it (fig.). In order that the moving launch is always on the straight line AB the components of the velocity of the current and of the launch in the direction perpendicular to AB should be equal, i.e.,

$$u \sin\beta = v \sin\alpha$$

When the launch moves from A to B its velocity to the banks





will be u cos β + v cos α and the time of motion will be determined from the equation

$S = (u \cos\beta + v \cos\alpha)\, t_1$ …………(ii)

The time of motion form B to A (fig.) can be found from the equation

$S = (u \cos\beta - v \cos\alpha)\, t_2$ …………(iii)

From the known conditions,

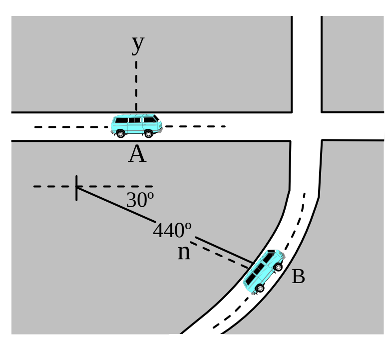
$t_1 + t_2 = t$ …………(iv)

Solving the system of the four equations we find

$$\beta = \text{arc cot}\frac{S + \sqrt{S^2 + v^2 t^2 \cos^2\alpha}}{vt\sin\alpha} \approx 12^\circ$$

$$u = v\frac{\sin\alpha}{\sin\beta} \approx 8 \text{ m/s}$$

Q.9 Car A is accelerating in the direction of its motion at the rate of 3 ft/sec.^{2}. Car B is rounding a curve of 440ft radius at a constant speed of 30 mi/hr. Determine the velocity and acceleration which car B appears to have to an observer in car A if car A has reached a speed of 45 mi/hr for the positions represented.



Sol. We choose nonrotating reference axes attached to car A since the motion of B with respect to A is desire.

Velocity : The relative velocity equation is

$\mathbf{v_B = v_A + v_{B/A}}$

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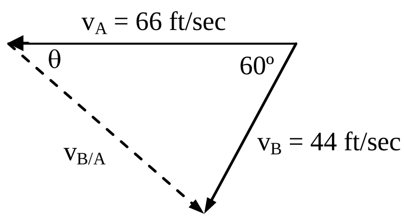


And the velocities of A and B for the position considered have the magnitude

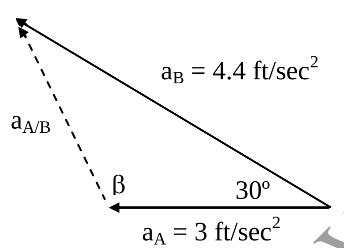
$v_A = 45\frac{5280}{60^2} = 45\frac{44}{30} = 66$ ft/sec $v_B = 30\frac{44}{30} = 44$ ft/sec

The triangle of velocity vectors is drawn in the sequence required by the equation, and application of the law of cosines and the law of sines gives

(1) $v_{B/A} = 58.2$ ft/sec $\theta = 40.9^o$ Ans.

Acceleration. The relative-acceleration equation is

$\mathbf{a_B = a_B + a_{B/A}}$



The acceleration of A is given, and the acceleration of B is normal to the curve in the n-direction and has the magnitude

$[a_n = v^2/\rho]$ $a_B = (44)^2/440 = 4.4$ ft/sec^2

The triangle of acceleration vectors is drawn in the sequence required by the equation as illusrated. Solving for the x-and y-components of $a_{B/A}$ gives us

$(a_{B/A})_x = 4.4 \cos 30^o - 3 = 0.810$ ft/sec^2

$(a_{B/A})_y = 4.4 \sin 30^o = 2.2$ ft/sec^2

form which $a_{B/A} = \sqrt{(0.810)^2 + (2.2)^2}$

= **2.34 ft/sec^2** Ans.

The direction of $a_{B/A}$ may be specified by the angle β which, by the law of sines, becomes

(2) $\frac{4.4}{\sin\beta} = \frac{2.34}{\sin 30^o}$ $\beta = \sin^{-1}\left(\frac{4.4}{2.34}0.5\right) =$ **110.2º** Ans.

Helpful Hints

(1) Alternatively, we could use either a graphical or a vector algebraic solution.

(2) Be careful to choose between the two values 69.8º and 180 – 69.8 = 110.2º.

Q.10 Two vectors $\vec{a}$ & $\vec{b}$ are varying with time as $a = 3t\hat{i} + 4t^2\hat{j}$ & $\vec{b} = (6t+3)\hat{i} + (7\sin t)\hat{j}$. Find the magnitude of the rate of change of $\vec{a}\cdot\vec{b}$ at t = π/2 sec.

Sol. 46π + 9

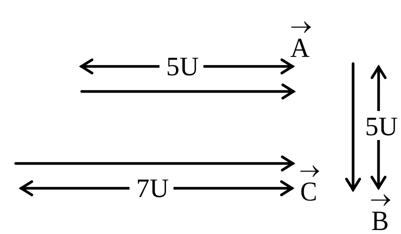
Q.11 Resolve a vector $\vec{A}$ into two perpendicular components so that :

(a) the components are of equal magnitude.

(b) the magnitude of one component is twice that of the other.

Sol. (a) . $\frac{A}{\sqrt{2}}, \frac{A}{\sqrt{2}}$ (b). $\frac{A}{\sqrt{5}}, \frac{2A}{\sqrt{5}}$

Q.12 It can be easily understood that $\vec{A}+\vec{B}+\vec{C} = (\vec{A}+\vec{B})+\vec{C}$ using triangle law represent $\vec{A}+\vec{B}+\vec{C}$ and $\vec{A}-\vec{B}+\vec{C}$ and find their magnitude –



Sol. 13 unit

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Q.13 A charged particle is moving in magnetic field with velocity $2\hat{i} + 4\hat{j} + 4\hat{k}$. If acceleration of particle is $4\hat{i} - x\hat{j} + 2\hat{k}$ m/s^2, find value of x and hence magnitude of acceleration.

Sol. 4, 6 m/s^2

Q.14 A mosquito net over a 7ft x 4ft bed is 3ft high. The net has a hole at one corner of the bed through which a mosquito enters the net it files and sits at the diagonally opposite upper corner of the net.

(a) Find the magnitude of the displacement of the mosquito.

(b) Taking the hole as the origin, the length of the bed as the x-axis, its width as the y-axis and vertically up as the z-axis, write the components of the displacement vector.

Ans. (a) $\sqrt{74}$ ft (b) 7ft; 4 ft, 3 ft

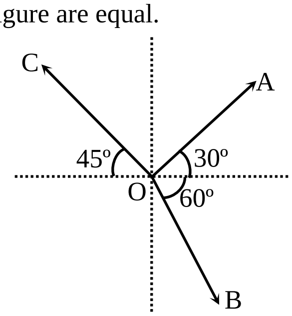
Q.15 A mosquito net over a 7ft x 4ft bed is 3ft high. The net has a hole at one corner of the bed through which a mosquito enters the net it files and sits at the diagonally opposite upper corner of the net.

(a) Find the magnitude of the displacement of the mosquito.

(b) Taking the hole as the origin, the length of the bed as the x-axis, its width as the y-axis and vertically up as the z-axis, write the components of the displacement vector.

Ans. (a) $\sqrt{74}$ ft (b) 7ft; 4 ft, 3 ft

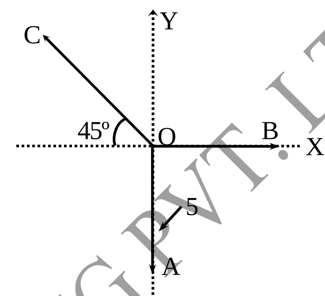
Q.16 The magnitudes of vectors **OA**, **OB** and **OC** in figure are equal.



Find the direction of **OA** + **OB** – **OC**.

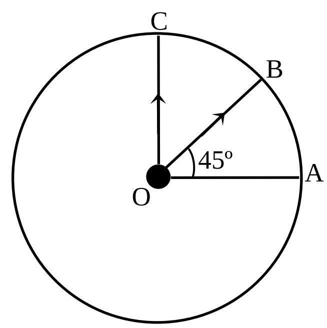
Ans. $\tan\alpha = \left[\dfrac{1-\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}+1}\right]$

Q.17 The sum of three vectors shown in figure is zero. Find the magnitude of the vectors $\overrightarrow{OB}$ and $\overrightarrow{OC}$.



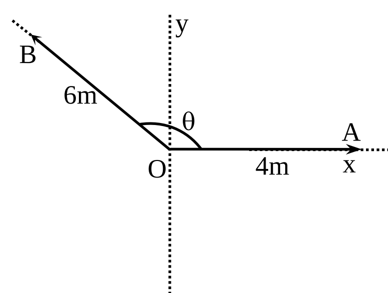
Ans. $|OB| = 5$, $|OC| = 5\sqrt{5}$

Q.18 Find the resultant of three vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ as shown in figure. Radius of the circle is R.



Ans. $R(1+\sqrt{2})$

Q.19 The resultant of vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is perpendicular to $\overrightarrow{OA}$. Find the angle ∠AOB.



Ans. $\theta = \cos^{-1}(-2/3)$

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Q.20 Two vectors have magnitudes 2m and 3m. The angle between them is 60º. Find (a) the scalar product of the two vectors, (b) the magnitude of their vector product.

Ans. (a) 3 (b) $3\sqrt{3}$

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