Trigonometry
Single Correct Answer Type

1. $\sec h^{-1}(\sin\theta) =$

   1) $\log\tan\dfrac{\theta}{2}$ 2) $\log\sin\dfrac{\theta}{2}$ 3) $\log\cos\dfrac{\theta}{2}$ 4) $\log\cot\dfrac{\theta}{2}$

Key. 4

Sol. $\log_e\left[\dfrac{1+\sqrt{\cos^2\theta}}{\sin\theta}\right]$

$= \log_e \cot\theta/2$

2. The value of the expression $\sec h^2\left(\operatorname{Tanh}^{-1}(1/2)\right) + \operatorname{cosech}^2\left(\cot h^{-1}3\right)$ is

   A) $\dfrac{35}{9}$ B) $\dfrac{43}{4}$ C) $\dfrac{35}{4}$ D) $\dfrac{43}{9}$

Key. 3
Sol. Conceptual

3. If $x = \log\left[\cot\left(\dfrac{\pi}{4}+\theta\right)\right]$ *then* $\sinh x =$

   1) $\tan 2\theta$ 2) $\cot 2\theta$ 3) $-\tan 2\theta$ 4) $-\cot 2\theta$

Key. 3

Sol. $x = \log\left[\cot\left(\pi/4+\theta\right)\right]$

$= \log\left[\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right] \Rightarrow e^x = \dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$

$\sinh x = \dfrac{e^x-e^{-x}}{2} = \dfrac{1}{2}\left[\dfrac{(\cos\theta-\sin\theta)^2-(\cos\theta+\sin\theta)^2}{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}\right]$

$= \dfrac{1}{2}\left[\dfrac{-4\cos\theta\sin\theta}{\cos^2\theta-\sin^2\theta}\right] = \dfrac{-\sin 2\theta}{\cos 2\theta} = -\tan 2\theta$

4. If $Sinh^{-1}2x = 2\,Cosh^{-1}y, then$

   1) $x^2 + y^2 = x^4$ 2) $x^2 + y^2 = 4$

   3) $x^2 + y^2 = y^4$ 4) $x^2 = y^2$

Key. 3

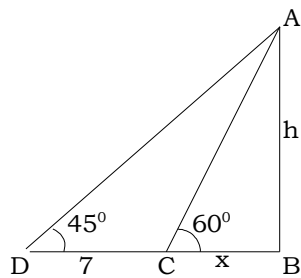Sol. $\sinh^{-1}2x = 2\cosh^{-1}y$

$2x = \sinh\left(2\cosh^{-1}y\right) = 2\sinh\left(\cosh^{-1}y\right)\cosh\left(\cosh^{-1}y\right)$

$= 2\sinh\left(\sinh^{-1}\left(\sqrt{y^2-1}\times y\right)\right)$

$2x = 2y\sqrt{y^2-1}$

$\Rightarrow x^2 + y^2 = y^4$

5.   AB is a vertical pole with B at the ground level and A at the top. A man finds
      that the angle of elevation of the point A from a certain point C on the
      ground is 60^0. He moves away from the pole along the line BC to a point D
      such that CD =7 m. From D the angle of elevation of the point A is 45^0. Then
      the height of the pole is

   1) $\dfrac{7\sqrt{3}}{2}\left(\sqrt{3}+1\right)m$ 2) $\dfrac{7\sqrt{3}}{2}\left(\sqrt{3}-1\right)m$ 3) $\dfrac{7\sqrt{3}}{2}\dfrac{1}{\sqrt{3}+1}m$ 4) $\dfrac{7\sqrt{3}}{2}\dfrac{1}{\sqrt{3}-1}m$

Key.   1

Sol. $x = h\cot 60^0 = h/\sqrt{3}$
        $x+7 = h\cot 45^0 \Rightarrow h = h - h/\sqrt{3} = 7$
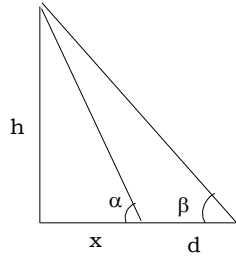 $\Rightarrow h = \dfrac{7\sqrt{3}}{\sqrt{3}-1}$

6.   The angle of elevation of an object from a point P on the level ground is α.
      Moving d metres on the ground towards the object, the angle of elevation is
      found to be β. Then the height (in metres) of the object is

   1) d tan α 2) d cot β

   3) $\dfrac{d}{\cot \alpha + \cot \beta}$ 4) $\dfrac{d}{\cot \alpha - \cot \beta}$

Key.   4

Sol. $\text{Tan}\alpha = \dfrac{h}{x+d}$
        $\Rightarrow x+d = h\cot\alpha$
        $\text{Tan}\beta = \dfrac{h}{x} \Rightarrow x = h\cot\beta$
        $x+d-x = h[\cot\alpha - \cot\beta]$
        $h = \dfrac{d}{\cot\alpha - \cot\beta}$

7. The angle of elevation of a cloud from a point h mt above the surface of a lake is θ and the angle of depression of its reflection in the lake is φ. The height of the cloud is

   1) $\dfrac{h\sin(\varphi+\theta)}{\sin(\varphi-\theta)}$
   2) $\dfrac{h\sin(\varphi-\theta)}{\sin(\varphi+\theta)}$
   3) $\dfrac{h\sin(\theta+\varphi)}{\sin(\theta-\varphi)}$
   4) $\dfrac{h\sin(\theta-\varphi)}{\sin(\theta+\varphi)}$
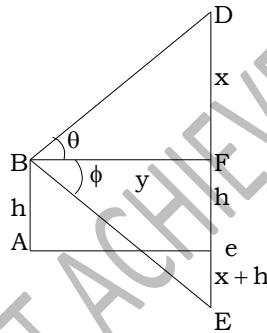
Key. 1

Sol. $\text{Tan}\theta = \dfrac{x}{y}$

$\text{Tan}\phi = \dfrac{2h+x}{y}$

$\Rightarrow x = \dfrac{2h}{\cot\theta.\tan\phi - 1}$

$CD = h + x = \dfrac{h\sin(\phi+\theta)}{\sin(\phi-\theta)}$

8. If $\tan\left(\dfrac{\pi}{4}+\dfrac{y}{2}\right) = \tan^3\left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)$, then $\sin x\left(\dfrac{3+\sin^2 x}{1+3\sin^2 x}\right)$ equals

   (A) cos y   (B) sin y
   (C) sin 2y   (D) 0

Key. B

Sol. $\dfrac{1+\tan\dfrac{y}{2}}{1-\tan\dfrac{y}{2}} = \left(\dfrac{1+\tan\dfrac{x}{2}}{1-\tan\dfrac{x}{2}}\right)^3$

Square both sides, we get

$\dfrac{1+\sin y}{1-\sin y} = \dfrac{(1+\sin x)^3}{(1-\sin x)^3}$

Using componendo and dividendo

$$\frac{2\sin y}{2} = \frac{(3+\sin^2 x)}{1+3\sin^2 x}\sin x$$

9. If $x = \sin\frac{2\pi}{7} + \sin\frac{4\pi}{7} + \sin\frac{8\pi}{7}$ and $y = \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{8\pi}{7}$ then $x^2 + y^2 =$

A. 1   B. 2   C. 3   D. 4

KEY.   B

SOL. $x^2 + y^2 = 3 + 2\left(\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}\right) = 2$

10. If $0 < A < B < \pi, \sin A - \sin B = \frac{1}{\sqrt{2}}, \cos A - \cos B = \sqrt{\frac{3}{2}}$ then A+B=

A. $\frac{2\pi}{3}$ B. $\frac{5\pi}{6}$ C. π D. $\frac{4\pi}{3}$

KEY.   D

SOL. $(\sin A - \sin B)^2 + (\cos A - \cos B)^2 = 2 \Rightarrow B = A + \frac{\pi}{2}$ and $A = \frac{5\pi}{12}$

11. $\cot\frac{7\pi}{6} + 2\cot\frac{3\pi}{8} + \cot\frac{15\pi}{16} =$

A. -4   B. 4   C. 1   D. 0

KEY.   A

SOL. $\tan\frac{\pi}{16} - \cot\frac{\pi}{16} + 2\cot\left(\frac{3\pi}{8}\right) = -2\cot\frac{\pi}{8} + 2\tan\frac{\pi}{6} = -4$

12. $Tan\frac{4\pi}{5} - Tan\frac{2\pi}{15} + \sqrt{3}Tan\frac{4\pi}{5}Tan\frac{2\pi}{15} =$

A. $\sqrt{3}$ B. $\frac{1}{\sqrt{3}}$ C. $-\sqrt{3}$ D. $-\frac{1}{\sqrt{3}}$

KEY.   C

SOL. $TanA - TanB - TanATanBTan(A-B) = Tan(A-B)$

13. If $x_1, x_2, x_3, \ldots x_n$ are in A.P. Whose common difference is α, then the value of

$\sin \alpha [\sec x_1 \sec x_2 + \sec x_2 \sec x_3 + ... + \sec x_{n-1} \sec x_n] =$

A. $\dfrac{\sin n\alpha}{\cos x_n \cos x_1}$ B. $\dfrac{\sin(n-1)\alpha}{\cos x_n \cos x_1}$ C. $\dfrac{\sin(n+1)\alpha}{\cos x_n \cos x_1}$ D. $\dfrac{\cos(n-1)\alpha}{\cos x_n \cos x_1}$

KEY.   B

SOL. $= \dfrac{\sin(x_2 - x_1)}{\cos x_1 \cos x_2} + \dfrac{\sin(x_3 - x_2)}{\cos x_2 \cos x_3} + ... + \dfrac{\sin(x_n - x_{n-1})}{\cos x_{n-1} \cos x_n}$

$= \tan x_2 - tax_1 + \tan x_3 - \tan x_2 + ... + \tan x_n - \tan x_{n-1}$

$= \tan x_n - \tan x_1 = \dfrac{\sin(x_n - x_1)}{\cos x_n \cos x_1} = \dfrac{\sin(n-1)\alpha}{\cos x_n \cos x_1}$

14.

If $a \sin^2 x + b \cos^2 x = c, b \sin^2 y + a \cos^2 y = d$ and $a\tan x = b\tan y$ then $\dfrac{a^2}{b^2} =$

A. $\dfrac{(a-d)(c-a)}{(b-c)(d-b)}$ B. $\dfrac{(a+d)(c+a)}{(b+c)(d+b)}$ C. $\dfrac{(a-d)(b-a)}{(a-c)(c-b)}$ D. $\dfrac{(d-a)(c-a)}{(b-c)(d-b)}$

KEY.   A

SOL. $a \tan^2 x + b = c(1 + \tan^2 x)$

$$\Rightarrow \tan^2 x = \left(\dfrac{c-b}{a-c} \right), \tan^2 y = \left(\dfrac{d-a}{b-d} \right)$$

$$\dfrac{a^2}{b^2} = \dfrac{\tan^2 y}{\tan^2 x} = \dfrac{(a-d)(c-a)}{(b-c)(d-b)}$$

15.

If $\cos^3 x \sin 2x = \displaystyle\sum_{r=0}^{n} a_r \sin(rx), \forall x \in R$ then

A. $n = 5, a_1 = \dfrac{1}{2}$ B. $n = 5, a_1 = \dfrac{1}{4}$ C. $n = 5, a_2 = \dfrac{1}{8}$ D. $n = 5, a_2 = \dfrac{1}{4}$

KEY.   B

SOL. $\cos^3 x \sin 2x = \cos^2 x . \cos x \sin 2x$

$$= \left(\dfrac{1 - \cos 2x}{2} \right)\left(\dfrac{2 \sin 2x \cos x}{2} \right) = \dfrac{1}{4}(1 - \cos 2x)(\sin 3x + \sin x)$$

$$= \dfrac{1}{4}[\sin 3x + \sin x - \dfrac{1}{2}(2 \sin 3x \cos 2x) - \dfrac{1}{2}(2 \cos 2x \sin x)]$$

$$= \dfrac{1}{4}[\sin 3x + \sin x - \dfrac{1}{2}(\sin 5x + \sin x) - \dfrac{1}{2}(\sin 3x - \sin x)] = \dfrac{1}{4}[\sin x + \dfrac{1}{2}\sin 3x - \dfrac{1}{2}\sin 5x]$$

$$a_1 = \frac{1}{4}; a_3 = \frac{1}{8}; n = 5$$

16. If , $\cos\theta = \dfrac{a\cos\phi + b}{a + b\cos\phi}$ then $\tan\theta/2$ is equal to

A. $\sqrt{\left(\dfrac{a-b}{a+b}\right)}\tan(\phi/2)$ B. $\sqrt{\left(\dfrac{a+b}{a-b}\right)}\cos(\phi/2)$ C. $\sqrt{\left(\dfrac{a-b}{a+b}\right)}\sin(\phi/2)$ D. none of these

Key. A

Sol. $\tan\theta/2 = \sqrt{\left(\dfrac{1-\cos\theta}{1+\cos\theta}\right)}$

$$= \sqrt{\dfrac{1 - \left(\dfrac{a\cos\phi + b}{a + b\cos\phi}\right)}{1 + \left(\dfrac{a\cos\phi + b}{a + b\cos\phi}\right)}}$$

$$= \sqrt{\dfrac{(a-b)(1-\cos\phi)}{(a+b)(1+\cos\phi)}}$$

$$= \sqrt{\dfrac{(a-b)}{(a+b)}}\tan(\phi/2)$$

17. If in a triangle ABC, $\cos 3A + \cos 3B + \cos 3C = 1,$ then one angle must be exactly equal to

A. $\dfrac{\pi}{3}$ B. $\dfrac{2\pi}{3}$ C. π D. $\dfrac{\pi}{6}$

Key. B

Sol. $\therefore \cos 3A + \cos 3B + \cos 3C = 1$

$\Rightarrow \cos 3A + \cos 3B + \cos 3C - 1 = 0$

$\Rightarrow \cos 3A + \cos 3B + \cos 3C + \cos 3\pi = 0$

$\Rightarrow 2\cos\left(\dfrac{3A+3B}{2}\right)\cos\left(\dfrac{3A-3B}{2}\right) + 2\cos\left(\dfrac{3\pi+3C}{2}\right)\cos\left(\dfrac{3\pi-3C}{2}\right) = 0$

$\Rightarrow 2\cos\left(\dfrac{3\pi-3C}{2}\right)\left\{\cos\left(\dfrac{3A-3B}{2}\right) + \cos\left(\dfrac{3\pi+3C}{2}\right)\right\} = 0$

$\Rightarrow 2\cos\left(\dfrac{3\pi}{2} - \dfrac{3C}{2}\right) 2\cos\left(\dfrac{3\pi+3C+3A-3B}{4}\right).\cos\left(\dfrac{3\pi+3C-3A+3B}{4}\right) = 0$

$\Rightarrow 2\cos\left(\dfrac{3\pi}{2} - \dfrac{3C}{2}\right) 2\cos\left(\dfrac{3\pi}{2} - \dfrac{3B}{2}\right).\cos\left(\dfrac{3\pi}{2} - \dfrac{3A}{2}\right) = 0$

$$\Rightarrow \quad -4\sin\left(\frac{3A}{2}\right)\sin\left(\frac{3B}{2}\right)\sin\left(\frac{3C}{2}\right)=0$$

$$\Rightarrow \quad \sin\left(\frac{3A}{2}\right)\sin\left(\frac{3B}{2}\right)\sin\left(\frac{3C}{2}\right)=0$$

$$\therefore \frac{3A}{2}=\pi \text{ or } \frac{3B}{2}=\pi \text{ or } \frac{3C}{2}=\pi$$

$$\therefore A=\frac{2\pi}{3} \text{ or } B=\frac{2\pi}{3} \text{ or } C=\frac{2\pi}{3}$$

18. The value of $\sum_{r=0}^{10}\cos^3\frac{\pi r}{3}$ is equal to

(A) $\frac{-9}{2}$ (B) $\frac{-7}{2}$

(C) $\frac{-9}{8}$ (D) $\frac{-1}{8}$

Key. D

Sol.
$$I=\sum_{r=0}^{10}\frac{1}{4}\left(\cos 3\frac{\pi r}{3}+3\cos\frac{\pi r}{3}\right)$$
$$=\sum_{r=0}^{10}\frac{1}{4}\left(\cos\pi r+3\cos\frac{\pi r}{3}\right)$$
$$=\frac{1}{4}(I_1+I_2)$$

$$\therefore I_1=\sum_{r=0}^{10}\cos\pi r=1-1+1-1+.....-1+1=1$$

$$I_2=3\sum_{r=0}^{10}\cos\frac{\pi r}{3}=\frac{3\cos\left(\frac{10}{2}\frac{\pi}{3}\right)\sin\frac{11\pi}{3}}{\sin\frac{\pi}{6}}=-\frac{1\times 3}{2}=-\frac{3}{2}$$

$$\Rightarrow I=\frac{1}{4}\left(1-\frac{3}{2}\right)=-\frac{1}{8}$$

19. The number of distinct real roots of the equation $\tan x=mx, m>1$ in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ is

A) 1   B) 2   C) 3   D) 0

Key. C
Sol. Conceptual

20. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points in the XY-Plane whose co-ordinates satisfy the

equation $\cot^2(x+y) + \tan^2(x+y) + y^2 + 2y - 1 = 0$. The minimum distance between P and

Q is

A) $\pi/4$ B) $\pi/2$ C) $3\pi/4$ D) π

Key.   B

Sol. $\left[\cot(x+y) - \tan(x+y)\right]^2 + (y+1)^2 = 0$

     $\therefore \tan^2(x+y) = 1$ and $y = -1$

21. If α is the angle which each side of a regular polygon of n sides subtends at its centre then
$1 + \cos\alpha + \cos 2\alpha + \cos 3\alpha + ... + \cos(n-1)\alpha$ is equal to

    (a) n   (b) 0   (c) 1   (d) n − 1

Key.   B

Sol. $\cos\alpha + \cos(\alpha + \beta) + ... + \cos(\alpha + (n-1)\beta) = \dfrac{\sin\dfrac{nB}{2}}{\sin\dfrac{B}{2}} \sin\left(\alpha + \dfrac{(n-1)\beta}{2}\right)$

22.   If $\lfloor C = 90^0$ in ΔABC, then $\tan^{-1}\left(\dfrac{a}{b+c}\right) + \tan^{-1}\left(\dfrac{b}{c+a}\right)$ is equal to

     a) $\dfrac{\pi}{2}$ b) $\dfrac{\pi}{4}$ c) $\dfrac{\pi}{3}$ d) π

Ans.   b

Sol. $\tan^{-1}\left(\dfrac{\dfrac{a}{b+c} + \dfrac{b}{c+a}}{1 - \dfrac{a}{b+c} \cdot \dfrac{b}{c+a}}\right)$ as $\dfrac{ab}{(b+c)(c+a)} < 1$

     But in right angled ΔABC
     c² = a² + b²

     $\therefore \tan^{-1}(1) = \dfrac{\pi}{4}$

23.   In a ΔABC, $\dfrac{a^2 + b^2 + c^2}{\Delta}$ is always

     a) $\geq 6\sqrt{3}$ b) $\geq 4\sqrt{3}$ c) $\geq 8\sqrt{3}$ d) $\geq 12\sqrt{3}$

Ans.   b

Sol. $\dfrac{a^2 + b^2 + c^2}{\Delta} \geq 4\sqrt{3}$: use the fact that $\Delta \leq \dfrac{(a+b+c)^2}{12\sqrt{3}}$

24.   In triangle ABC, the value of the expression $\displaystyle\sum_{r=0}^{n} {}^nC_r \, a^r b^{n-r} \cos(rB - (n-r)A)$ is equal

     to
     a) C^n b) Zero   c) a^n d) b^n

Ans.   a

Sol.   It is the expansion of $(a\cos B + b\cos A)^n = C^n$

25.   Total number of solution of $2^{\cos x} = |\sin x|$ in $[-2\pi, 5\pi]$ is equal to

a) 12   b) 14   c) 16   d) 15

Ans.   b

Sol.   Drawn the graphs of both. Total intersection points are 14.

26.   If $2\sec 2\alpha = \tan\beta + \cot\beta$, then one positive value of $\alpha + \beta$ is

a) $\dfrac{\pi}{2}$ b) $\dfrac{\pi}{4}$ c) $\dfrac{\pi}{3}$ d) 0

Ans.   b

Sol. $2\sec 2\alpha = \left(\dfrac{1}{\sin\beta\cos\beta}\right)$

$\Rightarrow 2\alpha = \dfrac{\pi}{2} - 2\beta \Rightarrow \alpha + \beta = \dfrac{\pi}{4}$

27.   If in a triangle $\dfrac{s-a}{11} = \dfrac{s-b}{12} = \dfrac{s-c}{13}$ and $\lambda\tan^2(A/2) = 455$, then λ must be

a) 1155   b) 1551   c) 5511   d) 1515

Ans.   a

Sol. $\dfrac{s-a}{11} = \dfrac{s-b}{12} = \dfrac{s-c}{13} = \dfrac{s}{36}$ calculate $\tan^2(A/2) = \dfrac{13}{33}$

$\lambda = 1155$

28.   The value of $\sin^3 10^0 + \sin^3 50^0 - \sin^3 70^0$ is equal to

a) $-\dfrac{3}{2}$ b) $\dfrac{3}{4}$ c) $-\dfrac{3}{4}$ d) $-\dfrac{3}{8}$

Ans.   d

Sol.   We have $\sin^3 10^0 + \sin^3 50^0 - \sin^3 70^0$

$= \dfrac{1}{4}\left[(3\sin 10^0 - \sin 30^0) + (3\sin 50^0 - \sin 150^0) - (3\sin 70^0 - \sin 120^0)\right]$

$= \dfrac{1}{4}\left[3(\sin 10^0 + \sin 50^0 - \sin 70^0) - \dfrac{3}{2}\right]$

$= \dfrac{1}{4}\left[3(\sin 10^0 - 2\cos 60^0 \cdot \sin 10^0) - \dfrac{3}{2}\right] = -\dfrac{3}{8}$

29.   If $\tan(\alpha - \beta) = \dfrac{\sin 2\beta}{3 - \cos 2\beta}$, then

a) $\tan\alpha = 2\tan\beta$ b) $\tan\beta = 2\tan\alpha$ c) $2\tan\alpha = 3\tan\beta$ d) $3\tan\alpha = 2\tan\beta$

Ans.   a

Sol.   We have $\dfrac{\sin 2\beta}{3 - \cos 2\beta} = \dfrac{2\sin\beta\cdot\cos\beta}{2 - 2\cos 2\beta + 1 + \cos 2\beta}$

$= \dfrac{2\sin\beta\cdot\cos\beta}{4\sin^2\beta + 2\cos^2\beta} = \dfrac{\tan\beta}{1 + 2\tan^2\beta} = \dfrac{2\tan\beta - \tan\beta}{1 + 2\tan^2\beta}$

$$= \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \cdot \tan\beta} = \frac{2\tan\beta - \tan\beta}{1 + 2\tan^2\beta}$$

$$\therefore \tan\alpha = 2\tan\beta$$

30. In a triangle ABC, if angle C is obtuse and angles A and B are given by roots of the equation $\tan^2 x + p\tan x + q = 0$, then the value of q is
    a) greater than 1   b) less than 1   c) equal to 1   d) 0

Ans. b

Sol. We have $A + B = \pi - C$

$$= \tan(A + B) = -\tan C$$

$$= \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} > 0 \qquad [\because \tan A > 0, \tan B > 0, \tan C < 0]$$

$$= \tan A \cdot \tan B < 1 \Rightarrow q < 1$$

31. If $2\sin x - \cos 2x = 1$, then $\cos^2 x + \cos^4 x$ is equal to
    a) 1   b) -1 c) $-\sqrt{5}$ d) $\sqrt{5}$

Ans. a

Sol. Given $2\sin x + 2\sin^2 x - 1 = 1$

Or, $\sin^2 x + \sin x - 1 = 0$

$$\therefore \sin x = \frac{-1 \pm \sqrt{1 + 4}}{2} = \frac{-1 + \sqrt{5}}{2}$$

$$\Rightarrow \sin^2 x = \frac{3 - \sqrt{5}}{2} \Rightarrow \cos^2 x = \frac{\sqrt{5} - 1}{2}$$

$$\therefore \cos^2 x (1 + \cos^2 x) = \frac{\sqrt{5} - 1}{2} \times \frac{\sqrt{5} + 1}{2} = 1$$

32. If ABCD is a cyclic quadrilateral such that $13\cos A + 12 = 0$ and $3\tan B - 4 = 0$, then the quadratic equation whose roots are tanC and cosD is
    a) $15x^2 + 60x - 11 = 0$ b) $60x^2 + 11x - 15 = 0$
    c) $11x^2 + 60x - 15 = 0$ d) none of these

Ans. b

Sol. In a cyclic quadrilateral, no angle is greater than 180^0

Here $\cos A = -\frac{12}{13} \Rightarrow \frac{\pi}{2} < A < \pi$ and $0 < C < \pi/2$ (since A + C = 180^0)

$$\therefore \tan A = -\frac{5}{12} \Rightarrow \tan C = \frac{5}{12}$$

Also $\tan B = \frac{4}{3} \Rightarrow 0 < B < \frac{\pi}{2}$ and $\frac{\pi}{2} < D < \pi$ (since B + D = 180^0)

$$\therefore \cos B = \frac{3}{5} \Rightarrow \cos D = -\frac{3}{5}$$

Now, the required equation is

$$x^2 - \left(\frac{5}{12} - \frac{3}{5}\right)x + \left(\frac{5}{12}\right)\left(-\frac{3}{5}\right) = 0$$

$$\Rightarrow 60x^2 + 11x - 15 = 0$$

33. If A, B, C are the angles of a triangle such that $\cot\dfrac{A}{2}=3\tan\dfrac{C}{2}$, then sinA, sinB, sinC are in

    a) A.P   b) G.P   c) H.P   d) none of these

Ans. a

Sol. Given $\cot\dfrac{A}{2}\cdot\cot\dfrac{C}{2}=3$

$$\Rightarrow \frac{\cos\dfrac{A}{2}\cdot\cos\dfrac{C}{2}}{\sin\dfrac{A}{2}\cdot\sin\dfrac{C}{2}}=3 \Rightarrow \frac{\cos\dfrac{A-C}{2}}{\cos\dfrac{A+C}{2}}=2 \qquad \text{(using componendo and dividendo)}$$

$$\Rightarrow \frac{2\sin\dfrac{A+C}{2}\cos\dfrac{A-C}{2}}{2\sin\dfrac{A+C}{2}\cdot\cos\dfrac{A+C}{2}}=2$$

= 2sinB = sinA + sinC

34. If $\dfrac{2\tan\alpha}{1+\sec\alpha+\tan\alpha}=\lambda,$ then $\dfrac{2\tan\alpha/2}{1+\tan\alpha/2}$ is equal to

    a) $\dfrac{1}{\lambda}$ b) λ c) $1-\lambda$ d) $1+\lambda$

Ans. b

Sol. We have $\dfrac{2\tan\alpha}{1+\sec\alpha+\tan\alpha}=\dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}$

$$=2\frac{2\tan\alpha/2}{\left(1+\tan^2\alpha/2\right)+\left(1-\tan^2\alpha/2\right)+2\tan\alpha/2}=\frac{2\tan\alpha/2}{1+\tan\alpha/2}$$

35. In $\triangle$ABC, if $b^2 + c^2 = 2a^2$, then the value of $\dfrac{\cot A}{\cot B+\cot C}$ is

    a) 1/2   b) 3/2   c) 5/2   d) 5/3

Ans. a

Sol. $\dfrac{\cot A}{\cot B+\cot C}=\dfrac{\dfrac{R\left(b^2+c^2-a^2\right)}{abc}}{\dfrac{R\left(a^2+c^2-b^2\right)}{abc}+\dfrac{R\left(a^2+b^2-c^2\right)}{abc}}=1/2$

36. If $0\le A,B,C\le\pi$ and A + B + C = π, then the minimum value of

    sin 3A + sin 3B + sin 3C is

    a) -2   b) $-\dfrac{3\sqrt{3}}{2}$ c) 0   d) none of these

Ans. a

Sol. Since A + B + C = π

$\Rightarrow$ all of sin3A, sin3B, sin3C can't be negative

Let us take sin3A = - 1 $\Rightarrow$ A = π/2

$\Rightarrow$ sin3A = - 1, sin3B = - 1 and sin 3C = 0

So minimum value is – 2 1.   Let $\theta\in\left(0,\pi/4\right)$ and $t_1=\left(\tan\theta\right)^{\tan\theta}$,

$t_2=\left(\tan\theta\right)^{\cot\theta}$, $t_3=\left(\cot\theta\right)^{\tan\theta}$, $t_4=\left(\cot\theta\right)^{\cot\theta}$ then

a) $t_1 > t_2 > t_3 > t_4$ b) $t_4 > t_3 > t_1 > t_2$

c) $t_3 > t_1 > t_2 > t_4$ d) $t_2 > t_3 > t_1 > t_4$

Key.   B

Sol. $\theta \in \left(0, \dfrac{\pi}{4}\right)$

therefore, $\tan\theta < \cot\theta$

since $\tan\theta < 1 \,\&\, \cot\theta > 1$

therefore, $(\tan\theta)^{\cot\theta} < 1$ and $(\cot\theta)^{\tan\theta} > 1$

therefore, $t_4 > t_1$

2.   If $\theta = \dfrac{2\pi}{7}$ then the value of $\tan\theta\tan2\theta + \tan2\theta\tan4\theta + \tan4\theta\tan\theta$ is

a) -1 b) 0   c) $\dfrac{1}{8}$ d) -7

Key.   D

Sol. $7\theta = 2\pi$

$\theta + 2\theta + 4\theta = 2\pi$

$\cos(\theta + 2\theta + 4\theta) = 1$

Expanding and dividing with $\cos\theta\cos2\theta\cos4\theta$ we have

$$\tan\theta\tan2\theta + \tan2\theta\tan4\theta + \tan4\theta\tan\theta = 1 - \dfrac{1}{\cos\theta\cos2\theta\cos4\theta} = 1 - \dfrac{1}{\left(\dfrac{1}{8}\right)} = -7$$

$$\left(\because \cos\theta\cos2\theta\cos4\theta = \dfrac{\sin8\theta}{8\sin\theta} = \dfrac{1}{8}\right)$$

3.   If $k_1 = \tan27\theta - \tan\theta$ and $k_2 = \dfrac{\sin\theta}{\cos3\theta} + \dfrac{\sin3\theta}{\cos9\theta} + \dfrac{\sin9\theta}{\cos27\theta}$ then

a) $k_1 = 2k_2$ b) $k_1 = k_2$ c) $k_1 = -k_2$ d) $2k_1 = k_2$

Key.   A

Sol. $\tan3\theta - \tan\theta = \dfrac{\sin2\theta}{\cos3\theta\cos\theta} = \dfrac{2\sin\theta}{\cos3\theta}$ (1)

$\tan9\theta - \tan3\theta = \dfrac{2\sin3\theta}{\cos9\theta}$ (2)

$\tan27\theta - \tan9\theta = \dfrac{2\sin9\theta}{\cos27\theta}$ (3)

Adding (1), (2), (3) $k_1 = 2k_2$

4.   If $\dfrac{\cos x}{a} = \dfrac{\cos(x+\theta)}{b} = \dfrac{\cos(x+2\theta)}{c} = \dfrac{\cos(x+3\theta)}{d}$ then $\dfrac{a+c}{b+d}$ is equal to

a) $\dfrac{a}{d}$ b) $\dfrac{c}{b}$

c) $\dfrac{b}{c}$ d) $\dfrac{d}{a}$

Key.   C

Sol.   For each of the ratio be k

$$\frac{a+c}{b+d} = \frac{k\cos x + k\cos(x+2\theta)}{k\cos(x+\theta) + k\cos(x+3\theta)} = \frac{2\cos(x+\theta)\cos\theta}{2\cos(x+2\theta)\cos\theta}$$

$$= \frac{\cos(x+\theta)}{\cos(x+2\theta)} = \frac{k\cos(x+\theta)}{k\cos(x+2\theta)} = \frac{b}{c}$$

$\Rightarrow$ (c) is correct.

5.   If $\cos\alpha = \dfrac{2\cos\beta - 1}{2 - \cos\beta}\,(0 < \alpha, \beta < \pi)$, then $\dfrac{\tan\dfrac{\alpha}{2}}{\tan\dfrac{\beta}{2}}$ is equal to

         a) 1   b) $\sqrt{2}$

         c) $\sqrt{3}$ d) $\dfrac{1}{\sqrt{3}}$

Key.   C

Sol.   Take $\beta = 120^\circ$, then

$$\cos\alpha = \frac{2\left(-\dfrac{1}{2}\right) - 1}{2 - \left(-\dfrac{1}{2}\right)} = \frac{2}{5/2} = \frac{4}{5}$$

$$\cos\alpha = -\frac{4}{5} \Rightarrow \tan\alpha = \frac{3}{4}$$

If $\tan\alpha = -\dfrac{3}{4}$, then we get $\dfrac{\tan\alpha/2}{\tan\beta/2} = \sqrt{3}$ and $\tan\dfrac{\alpha}{2} = 3$

$$\Rightarrow \frac{3}{4} = \frac{2\tan\dfrac{\alpha}{2}}{1 - \tan^2\dfrac{\alpha}{2}} \;\Rightarrow\; \tan\frac{\alpha}{2} = 3 \;\Rightarrow\; \frac{\tan\dfrac{\alpha}{2}}{\tan\dfrac{\beta}{2}} = \frac{\dfrac{1}{3}}{\dfrac{1}{\sqrt{2}}} = \sqrt{3}$$

6.   Cot 16^0 Cot 44^0 + Cot44^0 Cot 76^0 – Cot 76^0 Cot 16^0 =

         a) 1   b) 2   c) 3   d) 4

Key.   C

Sol.

$$\frac{\cos 16\cos 44}{\sin 16.\sin 44} - 1 + \frac{\cos 44.\cos 76}{\sin 44\sin 76} - 1 - \frac{\cos 76\cos 16}{\sin 76.\sin 16} - 1 + 3$$

$$= \frac{\cos 60}{\sin 16.\sin 44} + \frac{\cos 120}{\sin 44\sin 76} - \frac{\cos 60}{\sin 76.\sin 16} + 3 = \frac{1}{2}\left(\frac{\sin 76 - \sin 16}{\sin 16.\sin 44.\sin 76}\right) - \frac{1}{2\sin 76.\sin 16} + 3 = 3.$$

37.   The value of x which satisfies equation $2\tan^{-1} 2x = \sin^{-1}\dfrac{4x}{1 + 4x^2}$ is

         a) $\left[\dfrac{1}{2}, \infty\right)$ b) $\left(-\infty, -\dfrac{1}{2}\right]$ c) $[-1, 1]$ d) $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$

Ans.   d

Sol. $-\dfrac{\pi}{2} \le 2\tan^{-1} 2x \le \dfrac{\pi}{2}$

$\Rightarrow -\dfrac{1}{2} \le x \le \dfrac{1}{2}$

38.   Number of integral solutions of the equation $3\tan^{-1} x + \cos^{-1}\left(\dfrac{1-3x^2}{\left(1+x^2\right)^{3/2}}\right) = 0$ is

a) 1   b) 2   c) 0   d) infinite

Ans.   b

Sol.   Let $\tan^{-1} x = \theta, \theta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

$3\theta + \cos^{-1}\left(\cos 3\theta\right) = 0$

$\cos^{-1}\left(\cos 3\theta\right) = -3\theta \Rightarrow -\pi \le 3\theta \le 0$

$\Rightarrow -\dfrac{\pi}{3} \le \theta \le 0$

$\Rightarrow x \in \left[-\sqrt{3}, 0\right]$, so number of integral solutions is 2.

39.   In a triangle ABC, with $A = \dfrac{\pi}{7}, B = \dfrac{2\pi}{7}, C = \dfrac{4\pi}{7},$ then $a^2 + b^2 + c^2$ is (R = circumradius of

ΔABC)

a) $4R^2$ b) $6R^2$ c) $7R^2$ d) $8R^2$

Ans.   c

Sol. $a^2 + b^2 + c^2 = 4R^2 \left(\sin^2 A + \sin^2 B + \sin^2 C\right)$

$= 2R^2 \left(1 - \cos 2A + 1 - \cos 2B + 1 - \cos 2C\right) = 2R^2 \left[3 - \left(\cos \dfrac{2\pi}{7} + \cos \dfrac{4\pi}{7} + \cos \dfrac{8\pi}{7}\right)\right]$

$= 2R^2 \left[3 - \left(\cos \dfrac{2\pi}{7} + \cos \dfrac{4\pi}{7} + \cos \dfrac{6\pi}{7}\right)\right]$

$= 2R^2 \left[3 - \left(2\sin \dfrac{\pi}{7} \cdot \cos \dfrac{2\pi}{7} + 2\sin \dfrac{\pi}{7} \cdot \cos \dfrac{4\pi}{7} + 2\sin \dfrac{\pi}{7} \cdot \cos \dfrac{6\pi}{7}\right) \dfrac{1}{2\sin \dfrac{\pi}{7}}\right]$

$= 2R^2 \left[3 - \dfrac{1}{2\sin \dfrac{\pi}{7}} \left(\sin \dfrac{3\pi}{7} - \sin \dfrac{\pi}{7} + \sin \dfrac{5\pi}{7} - \sin \dfrac{3\pi}{7} + \sin \dfrac{7\pi}{7} - \sin \dfrac{5\pi}{7}\right)\right]$

$= 2R^2 \left[3 + \dfrac{1}{2}\right] = 7R^2$

40.   For which value of x, sin(cot⁻¹(x + 1)) = cos(tan⁻¹ x)

a) $\dfrac{1}{2}$ b) 0   c) 1   d) $-\dfrac{1}{2}$

Ans.   d

Sol. $\sin\left(\cot^{-1}(x+1)\right) = \sin\left(\sin^{-1}\left(\dfrac{1}{\sqrt{x^2+2x+2}}\right)\right)$

$\Rightarrow \sin\left(\cot^{-1}(x+1)\right) = \dfrac{1}{\sqrt{x^2+2x+2}}$

$\cos\left(\tan^{-1}x\right) = \cos\left(\cos^{-1}\left(\dfrac{1}{\sqrt{1+x^2}}\right)\right) = \dfrac{1}{\sqrt{1+x^2}}$

$\Rightarrow \dfrac{1}{x^2+2x+2} = \dfrac{1}{1+x^2}$

41.   If the equation $x^2 + 12 + 3\sin(a+bx) + 6x = 0$ has atleast one real solution where $a, b \in [0, 2\pi]$, then value of $\cos\theta$ where θ is least positive value of a + bx is

    a) π b) 2π c) 0   d) $\dfrac{\pi}{2}$

Ans.   c

Sol. $(x+3)^2 + 3 + 3\sin(a+bx) = 0$

x = - 3, sin(a + bx) = - 1

$\Rightarrow \sin(a-3b) = -1$

$a - 3b = (4n-1)\dfrac{\pi}{2}, \; n \in 1$

n = 1

a − 3b = 3π/2

cos(a − 3b) = 0

42.   In any ΔABC, which is not right angled, $\sum \cos A \cos ec B \cos ec C$ is

    a) constant   b) less than 1   c) greater than 2   d) none of these

Ans.   a

Sol. $\sum \dfrac{\cos A}{\sin B \sin C} = \dfrac{-\sum \cos(B+C)}{\sin B \sin C} = \sum(1 - \cot B \cot C) = 3 - \sum \cot A \cot B = 2$

1.   Range of $f(x) = \sin^6 x + \cos^6 x$ is

    (A) $[0, 1]$ (B) $[0, \sqrt{2}]$

    (C) $\left[\dfrac{1}{\sqrt{2}}, \dfrac{3}{4}\right]$ (D) $\left[\dfrac{1}{4}, 1\right]$

    Key.   D

    Sol. $f(x) = (\sin^2 x + \cos^2 x)^3 - 3\sin^2 x \cos^2 x(\sin^2 x + \cos^2 x) \; = 1 - \dfrac{3}{4}\sin^2 2x$

    Range of $\sin^2 2x$ is $[0,1]$

    Range of $f(x)$ is $\left[\dfrac{1}{4}, 1\right]$.

Note: Certain questions are better done by avoiding derivatives. Derivatives is one of the tools to determine extrema.

20.   The maximum value of $4\sin^2 x + 3\cos^2 x + \sin\dfrac{x}{2} + \cos\dfrac{x}{2}$ is

a) $4 + \sqrt{2}$ b) $3 + \sqrt{2}$ c) 9   d) 4

Key.   A

Sol.   Maximum value of $4\sin^2 x + 3\cos^2 x$ i.e., $\sin^2 x + 3$ is 4 and that of $\sin\dfrac{x}{2} + \cos\dfrac{x}{2}$ is

$\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}} = \sqrt{2}$, both attained at $x = \pi/2$. Hence the given function has maximum value of

$4 + \sqrt{2}$

21.   If $\sin\theta + \sin 2\theta + \sin 3\theta = \sin\alpha$ and $\cos\theta + \cos 2\theta + \cos 3\theta = \cos\alpha$, then θ is equal to

a) $\dfrac{\alpha}{2}$ b) α c) 2α d) $\dfrac{\alpha}{6}$

Key.   A

Sol.   sinθ+sin3θ+sin2θ=sinα

$\Rightarrow \qquad 2\sin 2\theta \cos\theta + \sin 2\theta = \sin\alpha$

$\Rightarrow \qquad \sin 2\theta (2\cos\theta + 1) = \sin\alpha$ ....(1)

Now $\qquad \cos\theta + \cos 3\theta + \cos 2\theta = \cos\alpha$

$\cos 2\theta (2\cos\theta + 1) = \cos\alpha$ ....(2)

From (1) and (2),

$\tan 2\theta = \tan\alpha \Rightarrow 2\theta = \alpha \Rightarrow \theta = \dfrac{\alpha}{2}$

22.   If $\pi < 2\theta < \dfrac{3\pi}{2}$, then $\sqrt{2 + \sqrt{2 + 2\cos 4\theta}}$ equals to

a) $-2\cos\theta$ b) $-2\sin\theta$ c) $2\cos\theta$ d) $2\sin\theta$

Key.   D

Sol. $\sqrt{2 + 2(1 + \cos 4\theta)} = \sqrt{2 + 2|\cos 2\theta|}$

$= \sqrt{2(1 - \cos 2\theta)}$

$= 2|\sin\theta| = 2\sin\theta$ as $\dfrac{\pi}{2} < \theta < \dfrac{3\pi}{4}$

23. $\cos^2\alpha + \cos^2(\alpha + 120^0) + \cos^2(\alpha - 120^0)$ is equal to

a) $\dfrac{3}{2}$ b) 1   c) $\dfrac{1}{2}$ d) 0

Key.   A

Sol. $\cos^2\alpha + \cos^2(\alpha + 120^0) + \cos^2(\alpha - 120^0)$

$= \cos^2\alpha + \{\cos(\alpha + 120^0) + \cos(\alpha - 120^0)\}^2 - 2\cos(\alpha + 120^0)\cos(\alpha - 120^0)$

$= \cos^2\alpha + \{2\cos\alpha\cos 120^0\}^2 - 2\{\cos^2\alpha - \sin^2 120^0\}$

$= \cos^2\alpha + \cos^2\alpha - 2\cos^2\alpha + 2\sin^2 120^0$

$= 2\sin^2 120^0 = 2 \times \dfrac{3}{4} = \dfrac{3}{2}$