PHYSICS

The following question given below consist of an "Assertion" (A) and "Reason" (R) Type questions. Use the following Key to choose the appropriate answer.

(A) If both (A) and (R) are true, and (R) is the correct explanation of (A).

(B) If both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) If (A) is true but (R) is false.

(D) If (A) is false but (R) is true.

Q. 1 **Assertion :** From a large metal sheet a small circular piece of radius R is removed leaving a hole in the sheet. Now the sheet with hole and the piece are heated to the same temperature. The piece can still exactly fit into the hole

Reason : The coefficient of surface expansion for both the piece and the hole is same.

[D]

Q.2 **Assertion :** Coefficient of volume expansion at constant pressure for diatomic gas is more than that for mono-atomic gas.

Reason : Coefficient of volume expansion of gas at constant pressure is independent of the degrees of freedom of gas.

Sol. **[D]**

$\delta = \frac{1}{V}\frac{dV}{dT}$: coefficient of volume expansion

$PV = nRT$

$V = \frac{nR}{P}T$

$\frac{dV}{dT} = \frac{nR}{P}$

$\delta = \frac{1}{V}\frac{dV}{dT} = \frac{nR}{PV} = \frac{nR}{nRT} = \frac{1}{T}$

Q.3 **Assertion** : When a solid sphere is heated, increase in its surface area is maximum.

Reason : Surface area involves expansion in two dimensions. **[D]**

Q.4 **Assertion** : When a liquid with coefficient of cubical expansion γ is heated in a vessel of coefficient of linear expansion $\gamma/3$, the level of liquid in the vessel remains unchanged.

Reason : $\gamma_a = \gamma_r - \gamma_g = \gamma - 3\left(\frac{\gamma}{3}\right) = 0.$ **[A]**

Q.5 **Assertion** : A beaker is completely filled with water at 4ºC. It will overflow, both when heated or cooled.

Reason : Overflowing is on account of expansion of water. **[B]**

Q.6 **Assertion :** From a large metal sheet a small circular piece of radius R is removed leaving a hole in the sheet. Now the sheet with hole and the piece are heated to same temperature. The piece can still exactly fit into the hole.

Reason : The coefficient of surface expansion for both the piece and the hole is same. **[A]**

Q.7 **Statement - 1 :** Cooking is faster in pressure cooker.

Statement - 2 : Aluminium is good conductor of heat.

Sol. **[B]**

Q.8 **Statement - 1 :** Two bodies at different temperatures, if brought in contact do not necessary settle to the mean temperature.

Statement - 2 : The two bodies may have different thermal capacities.

Sol. **[A]**

Q.9 **Statement - 1 :** A body is in equilibrium in an inertial frame but not be equilibrium in an non-inertial frame.

Statement - 2 : The body can be in thermal equilibrium in both the frames.

Sol. **[B]**

Q.10 **Statement - 1 :** Work and heat are two equivalent forms of energy.

Statement - 2 : Work is the transfer of mechanical energy irrespective of temperature difference, whereas heat is the transfer of thermal energy because of temperature difference only.

Sol. **[A]**

Q.10 **Statement - 1 :** In isothermal process whole of the heat energy supplied to the body is converted into internal energy.

Statement - 2 : According to the first law of thermodynamics $\Delta Q = \Delta U + W$.

Sol. **[D]**

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Q.1 **Column I** **Column II**

(A) When temperature increases then time period of pendulum [rod is of metal] (P) Decrease

(B) When temperature decreases then time period of pendulum [rod is of metal] (Q) Increase

(C) A cavity is inside of metal sphere then on increasing the temperature (R) Same

(D) A hole in a circular plate (S) Can't say anything

Ans. **A → P ; B → Q ; C → Q ; D → Q**

Q.2 **Column - I** **Column- II**

(A) Specific heat capacity S (P) $l_1 - l_2 =$ constant for $l_1\,\alpha_1 = l_2\,\alpha_2$

(B) Two metals (l_1, α_1) and (l_2, α_2) are heated uniformly (Q) Y is same

(C) Thermal stress (R) S = ∞ for $\Delta T = 0$

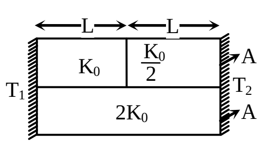
(D) Four wires of same Material (S) $Y \propto \Delta t$

Ans. **(A) → (R), (B) → (P), (C) → (S), (D) → (Q)**

Q.3 **Match the column**

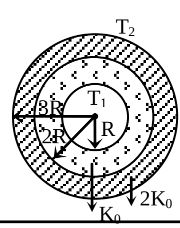
Column -I **Column-II**

(A) $K_{eq} > K_0$ (P)



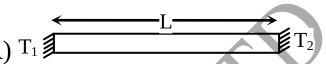
Lateral surfaces are insulated

(B) $K_{eq} < 3K_0$ (Q)



Cross-section of sphere with layers of conductivity as shown. Inner surface at T_1 and outer surface at T_2

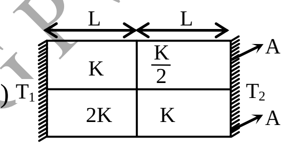
(C) $K_{eq} < 2K_0$ (R)



Rod of length (L) with K varies as

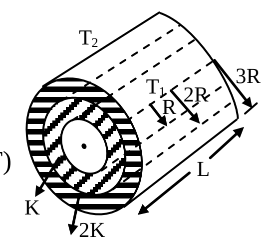
$$K = \frac{K_0(L + x)}{L}$$

(D) $K_{eq} > 2K_0$ (S)



Lateral surfaces are insulated

(T)



Inner surface T_1

Outer surface T_2

Sol. A → P,Q,R,S,T ; B → P,Q,R,S,T; C→Q,R,S ; D → P,T

K_{eq} is defined for same geometrical and physical dimensions.

Q.4 Match the column :

Whenever a liquid is heated in a container, expansion in liquid as well a container takes place. If γ is the volume expansion coefficient of liquid and α is coefficient of linear expansion. Match the entries of column I and column II.

Column- I **Column-II**

(A) Liquid level rises with respect to container (P) $\gamma = 2\alpha$

(B) Liquid level remains (Q) $\gamma < 3\alpha$

same with respect to container

(C) Liquid level drops with respect to container — (R) $\gamma = 3\alpha$

(D) Liquid level remains same with respect to ground — (S) $\gamma > 3\alpha$

Sol. (A) $\rightarrow$ (S) ; (B) $\rightarrow$ (R) ; (C) $\rightarrow$ (Q) ; (D) $\rightarrow$ (P)

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Q. 1 The temperature of an isotropic cubical solid of length L, density ρ, and coefficient of linear expansion α per kelvin is raised by 10ºC. Then at this temperature to a good approximation.

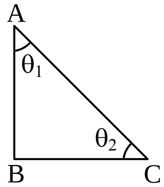
(A) Length is $L(1 + 10\alpha)$

(B) total surface area is $L^2(1 + 20\alpha)$

(C) density is $\rho(1 + 30\alpha)$

(D) density is $\frac{\rho}{1+30\alpha}$ **[A,D]**

Q.2 Three identical rods of same material are joined to form a triangular shape ABC as shown. Angles at edge A and C are respectively θ_1 and θ_2 as shown. When this triangular shape is heated then -



(A) θ_1 decreases and θ_2 increases

(B) θ_1 increases and θ_2 decreases

(C) θ_1 increases

(D) θ_2 increases **[C,D]**

Sol. In thermal expansion all dimensions will increases.

Q.3 Which of the following statements is correct ?

(A) Bimetal is used in metal thermometers

(B) Bimetal is used in thermostats for regulating the heating or cooling of rooms

(C) Bimetal relays are used to open or close electric circuits.

(D) Bimetals are used to generate electricity

[A,B,C]

Q.4 If α, β and γ are coefficients of linear, superficial and volume expansion respectively, then–

(A) $\frac{\beta}{\alpha} = \frac{1}{2}$ (B) $\frac{\beta}{\gamma} = \frac{2}{3}$

(C) $\frac{\gamma}{\alpha} = \frac{3}{1}$ (D) $\frac{\beta}{\alpha} = \frac{\gamma}{\beta}$

[B,C]

Q.5 A bimetallic strip is formed out of two identical strips, one of copper and the other of brass. The coefficient of linear expansion of the two metals are α_C and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then R is –

[IIT–99]

(A) Proportional to ΔT

(B) Inversely proportional to ΔT

(C) Proportional to $|\alpha_B - \alpha_C|$

(D) Inversely proportional to $|\alpha_B - \alpha_C|$

[B,D]

Q.6 When the temperature of a copper coin is raised by 80ºC, it's diameter increase by 0.2% -

(A) Percentage rise in the area of a face is 0.4%

(B) Percentage rise in the thickness is 0.4%

(C) Percentage rise in the volume is 0.6%

(D) Coefficient of linear expansion of copper is 0.25×10^{-4} /ºC **[A,C,D]**

Sol. $\frac{\Delta A}{A} \times 100 = 2\left[\frac{\Delta \ell}{A}\right] \times 100$

$\Rightarrow$ % increase in Area = $2 \times 0.2 = 0.4$

$\frac{\Delta V}{V} \times 100 = 3 \times 0.2 = 0.6\%$

Since $\Delta\ell = \ell\,\alpha\,\Delta T$

$\frac{\Delta \ell}{\ell} \times 100 = \alpha\,\Delta T \times 100 = 0.2$

$\Rightarrow \alpha = 0.25 \times 10^{-4}$/ºC

Q.7 A driver of an automobile gets his gasoline tank made of steel of volume 75L, filled with 75 L of gasoline at 10°C. As the temperature in the day becomes 30°C -

$[\alpha_{steel} = 12 \times 10^{-6}/°C,\ \gamma_{gasoline} = 9.5 \times 10^{-4}/°C]$

(A) Change in volume of gasoline is 190 L

(B) Change in volume of steel tank is 0.054 L

(C) Gasoline overflow is 1.37 L

(D) Gasoline will not overflow **[B,C]**

Sol. $\Delta V = V_0\gamma\Delta T = V_0(3\,\alpha\,\Delta T)$

If change in volume for gasoline is more than that of tank it will overflow.

Q.8 A vessel is partly filled with liquid. When the vessel is cooled to a lower temperature, the space in the vessel, unoccupied by the liquid remains constant. Then the volume of the liquid (V_L), volume of the vessel (V_v), the coefficients of cubical expansion of the material of the vessel (γ_V) and of the liquid (γ_L) are related as -

(A) $\gamma_L > \gamma_V$ (B) $\gamma_L < \gamma_V$

(C) $\gamma_V/\gamma_L = V_V/V_L$ (D) $\gamma_V/\gamma_L = V_L/V_V$

[A,D]

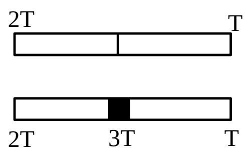
Sol. $\Delta V_L = \Delta V_V$

$\Rightarrow \gamma_L V_L = \gamma_V V_V$ or $\dfrac{\gamma_L}{\gamma_V} = \dfrac{V_V}{V_L}$

best $V_V > V_L \Rightarrow \gamma_L > \gamma_V$

Q.9 The diagram shows two identical rods connected in series. The rods are conducting heat in steady state. The ends of the rod are maintained at temperatures 2T and T. Now, a small block at temperature 3T is placed between the rods.

Now choose the incorrect statements :



(A) The temperature of the block will decrease.

(B) The block will release heat towards both the sides.

(C) The final temperature of the block will be $\dfrac{3T}{2}$.

(D) The temperature of block will first increase then it will decrease.

Sol. [D] Rod will be in unsteady state.

Q.10 Select the correct statements :

(A) The empty space in a beaker containing a liquid, partially filling it, on heating both beaker and liquid through the same temperature rise, may not change in volume, only if coefficient of cubical expansion of the two are different

(B) Two copper spheres (one solid and other hollow) have same initial radius and temperature. They are heated to same higher temperature. At the end of the heating, the radius of the hollow one is greater than that of solid.

(C) Two spheres of the same material have radii 1 m and 4 m and temperature 4000 K and 2000 K respectively. The energy radiated per second by the second sphere is greater than that by the first

(D) When you touch two bodies, the body felt warmer on the touch must be of higher temperature than the other

Sol. **[A,C]**

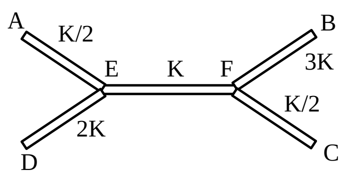
Q.1 A bar measured with a Vernier Caliper is found to be 1800 mm long. The temperature during the measurement is 10ºC.

The measurement error if the scale of the Vernier Caliper has been graduated at a temperature of 20ºC is found to $x \times 10^{-2}$ mm. Find x.

Sol.[2] $x = \ell_0 \alpha t$

$x = 2$

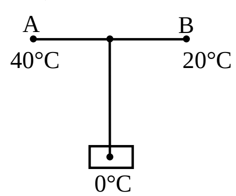
Q.2 Five rods with identical geometries are arranged as shown. Their thermal conductivity are shown. Only A and C are maintained at 100°C and 0°C respectively. If temperature difference between ends B and C can be writted as 10x °C where x is an integer. Then find x .



Sol. [2] $\frac{d\theta}{dt} = \frac{100-0}{R_{eq}}$; $T_B = 40°C$, $T_D = 60°C$

Q.2 In the figure shown AB is a rod of length L and thermal resistance $R_H = 10$ SI unit and end A and B are maintained by 20°C and 40°C. At mid point of rod another rod of thermal resistance $R'_H = R_H/4$ SI unit connected and other end of rod is inserted into ice.

After steady state reach, we start counting of time, and find the amount of ice (in kg) melt in 5.6×10^4 s. ($L_f = 3.36 \times 10^5$ J/kg)



Sol. [1] $\frac{40-T}{R_H/2} = \frac{T-20}{R_H/2} + \frac{T-0}{R_H/4}$

$T = 15°C$

$\frac{T-0}{R_H/4} = i_H \Rightarrow i_H = 6$ J/s

Heat supplied $= 6 \times 5.6 \times 10^4 = 3.36 \times 10^5$ J In 5.6×10^4s. amount of ice $mL_f = 3.36 \times 10^5$

Q. 1 A liquid of cubical expansivity Y is heated in a vessel having linear expansivity $\frac{Y}{3}$.Then level of liquid -

(A) Increase (B) decrease
(C) remain same (D) all-possible **[C]**

Sol. $y_s = 3\,\alpha_s$

$\alpha_s = \frac{y}{3}$ given

So $y_s = 3 \times \frac{y}{3} = y$

Q. 2 The diameters of steel rods A and B having the same length are 2 cm and 4 cm respectively. They are heated through 100°C. What is the ratio of increase of length of A to that of B.

(A) 1 : 2 (B) 2 : 1
(C) 1 : 1 (D) 4 : 1 **[C]**

Sol. $\Delta L = L_I \propto \Delta t$

Q.3 There is a metallic rod of length L (at room temperature) and coefficient of linear expansion (α/ºC). The length of rod at 1ºC above room temperature is 102 cm and at 2ºC above room temperature is 104 cm then the values of L and α are –

(A) 101 cm, 0.02/ºC (B) 100 cm, 0.01/ºC
(C) 100 cm, 0.02/ºC (D) 99 cm, 0.02/ºC **[C]**

Sol. $\ell = L\,(1 + \alpha\Delta T)$

$\therefore\ 102 = L\,(1 + \alpha)$ and $104 = L\,(1 + 2\alpha)$

Q. 4 A metallic bar is heated from 0ºC to 100ºC. The coefficient of linear expansion is $10^{-5}\ K^{-1}$. What will be the percentage increase in length ?

(A) 0.01 % (B) 0.1 %
(C) 1 % (D) 10 % **[B]**

Sol. $\frac{\Delta\ell}{\ell} = \alpha\Delta T = 10^{-5} \times 100 = 10^{-3}$

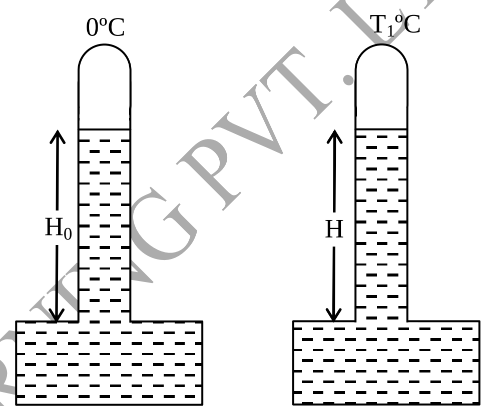
$\frac{\Delta\ell}{\ell} \times 100\% = 10^{-3} \times 100 = 10^{-1} = 0.1\%$

Q.5 The height of mercury column measured with brass scale at temperature T_0 is H_0. What height H' will the mercury column have at T = 0ºC. Coefficient of volume expansion of mercury is γ. Coefficient of linear expansion of brass is α -

(A) $H_0(1 + \alpha T_0)$ (B) $\frac{H_0(1+3\alpha T_0)}{1+\gamma T_0}$

(C) $\frac{H_0(1+3\alpha T_0)}{(1+\gamma/3)T_0}$ (D) $\frac{H_0(1+\alpha T_0)}{1+\gamma T_0}$

Sol. **[D]**



$P_{atm} = \rho_0 gH'$; $P_{atm} = \frac{\rho_0\, gH_0}{1+\gamma T_0}$

H → true reading at T_0ºC

Let H_0 be observed reading at T_0ºC

$\therefore\ H_0 = H[1 - \alpha T_1]$

$\rho_0 gH' = \frac{\rho_0 gH_0[1+\alpha T_0]}{1+\gamma T_0}$

$\Rightarrow H' = \frac{H_0[1+\alpha T_1]}{1+\gamma T_1}$

Q.6 Robin wants to shove rubber pipe up a plastic tap. The problem is that the pipe's diameter is smaller than that required to secure a tight fit. So Robin switches on a half-dryer on and pointed it towards the pipe. He found that now the pipe fitted in easily. Why is this so ?

(A) The pipe expanded due to the hot air, and then contracted back again.
(B) the tap got deformed because of the hot air.
(C) The hot air caused adhesion between rubber and plastic.
(D) None of the above.

Sol. **[A]**

Plastic tube expanded because of heat and then when one stopped applying heat, it contracted by cooling.

Q.7 A parallel plate capacitor of plate area A and separation d is provided with thin insulating spacers to keep its plates aligned in an environment of fluctuating temperature. If the coefficient of thermal expansion of material of plate is α then the coefficient of thermal expansion (α_S) of the spacers in order that the capacitance does not vary with temperature (ignore effect of spacers on capacitance)

(A) $\alpha_S = \frac{\alpha}{2}$ (B) $\alpha_S = 3\alpha$

(C) $\alpha_S = 2\alpha$ (D) $\alpha_S = \alpha$ **[C]**

Sol. $C = \frac{\varepsilon_0 A}{x}$, where x is separation between plates

$$\frac{1}{C}\frac{dC}{dT} = \frac{1}{A}\frac{dA}{dT} - \frac{1}{x}\frac{dx}{dT}$$

for $\frac{dC}{dT} = 0$, $\frac{1}{x}\frac{dx}{dT} = \frac{1}{A}\frac{dA}{dT} \Rightarrow \alpha_S = 2\alpha$

Q. 8 A gas is at pressure P and temperature T. Coefficient of volume expansion of one mole of gas at constant pressure is –

(A) $\frac{1}{T}$ (B) T

(C) $\frac{1}{T^2}$ (D) T^2 **[A]**

Sol. $PV = RT \Rightarrow PdV = RdT$

$\therefore$ Coefficient of volume expansion

$$= \frac{1}{V}\frac{dV}{dT} = \frac{R}{PV} = \frac{1}{T}$$

Q.9 If two rods of length L and 2 L having coefficients of linear expansion α and 2α respectively are connected so that total length becomes 3L, the average coefficient of linear expansion of the composite rod equals –

(A) $\frac{3}{2}\alpha$ (B) $\frac{5}{2}\alpha$

(C) $\frac{5}{3}\alpha$ (D) None of these

[C]

Sol. $(3L)\alpha_{eff}\Delta\theta = L\alpha\,\Delta\theta + 2L(2\alpha)(\Delta\theta)$

$\therefore \alpha_{eff} = \frac{5}{3}\alpha$

Q.10 A metal ball immersed in water weighs w_1 at 0ºC and w_2 at 50ºC. The coefficient of cubical expansion of metal is less than that of water. Then –

(A) $w_1 > w_2$ (B) $w_1 < w_2$

(C) $w_1 = w_2$ (D) data is insufficient

[B]

Sol. Apparent weight (w_a)

= actual weight (w) – upthrust (F)

Here, $F = V\rho_w g$ (ρ_w = density of water)

i.e., $F_{0^oC} = V_{0^oC}(\rho_w)_{0^oC}g$

and $F_{50^oC} = V_{50^oC}(\rho_w)_{50^oC}g$

$$\therefore \frac{F_{50^oC}}{F_{0^oC}} = \frac{V_{50^oC}}{V_{0^oC}} \cdot \frac{(\rho_w)_{50^oC}}{(\rho_w)_{0^oC}} = \frac{(1+\gamma_m\Delta\theta)}{(1+\gamma_w\Delta\theta)}$$

($\Delta\theta = 50^o$ C)

Given that $\gamma_m < \gamma_w$ $\therefore F_{50^oC} < F_{0^oC}$

or apparent weight at 50º C will be more.

Q.11 The design of some physical apparatus requires that there be a constant difference in length at any temperature between iron and copper cylinder laid side by side. What should be the length of cylinders at 0ºC for difference in length to be 10 cm at all temperatures (Given: Iron $\alpha = 1.1 \times 10^{-5}$ ºC^{-1}, Copper $\alpha = 1.7 \times 10^{-5}$ ºC^{-1}) –

(A) L_{Iron} = 18.33 cm, L_{Copper} = 28.33 cm

(B) L_{Iron} = 15 cm, L_{Coper} = 25 cm

(C) L_{Iron} = 28.33 cm, L_{Copper} = 18.33 cm

(D) L_{Iron} = 25 cm, L_{Copper} = 15 cm **[C]**

Sol. For Iron

$L_t - L_0 = L_0 \times \alpha \times \Delta t$

$L_t - L_0 = (L_0)_{Iron} \times 1.1 \times 10^{-5}\,\Delta t$

For Copper

$L_t - L_0 = (L_0)_{Copper} \times 1.7 \times 10^{-5}\,\Delta t$

Since $L_t - L_0$ is equal for both metals at all temperatures.

$(L_0)_{Iron} \times 1.1 \times 10^{-5}\,\Delta t = (L_0)_{Copper} \times 1.7 \times 10^{-5}\,\Delta t$

$(L_0)_{Iron} \times 11 = (L_0)_{Copper} \times 17$

But $(L_0)_{Iron} - (L_0)_{Copper} = 10$ cm

or $(L_0)_{Iron} = (L_0)_{Copper} + 10$ cm

$((L_0)_{Copper} + 10) \times 11 = (L_0)_{Copper} \times 17$

or $6(L_0)_{copper} = 110$ cm

or $(L_0)_{copper} = 110/6$ cm $= 18.33$ cm.

$(L_0)_{copper} = 18.33$ cm.

$(L_0)_{Iron} = 28.33$ cm.

Q.12 An anisotropic material has coefficient of linear expansion α, 2α and 3α along the three co-ordinate axis. Coefficient of cubical expansion of material will be equal to –

(A) 2α (B) $\sqrt[3]{6}\,\alpha$

(C) 6α (D) None of these **[C]**

Sol. For anisotropic material

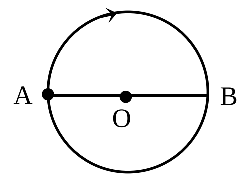
$\gamma = \alpha + 2\alpha + 3\alpha = 6\alpha$

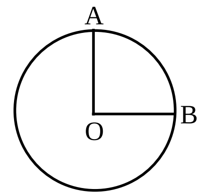
Q.13 A and B are two points on a uniform metal ring whose centre is O. The angle AOB = θ. A and B are maintained at two different constant temperatures. When $\theta = 180°$, the rate of total heat flow from A to B is 1.2 W. When $\theta = 90°$, this rate will be -

(A) 0.6 watt (B) 0.9 watt

(C) 1.6 watt (D) 1.8 watt **[C]**

Sol.





$$1.2 = \frac{KA(T_1 - T_2)}{R/4}$$

$$\tau = \frac{KA(T_1 - T_2)}{\frac{R}{4} + \frac{3R}{4}}$$

$\tau = 1.6$ watt.

Q.14 A parallel-sided slab is made of two different materials. The upper half of the slab is made of material X, of thermal conductivity λ; the lower half is made of material Y, of thermal conductivity 2λ. In the steady state, the left hand face of the composite slab is at a higher, uniform temperature than the right-hand face, What fraction of the total heat flow through the slab passes through material X ?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$

(C) $\frac{1}{2}$ (D) $\frac{2}{3}$ **[B]**

Sol. $$\tau = \frac{(T_1 - T_2)\;3\lambda A}{L}$$

$$\tau_1 = \frac{(T_1 - T_2)\;\lambda A}{L}$$

$$\frac{\tau}{\tau_1} = 3 \Rightarrow \tau_1 = \tau/3$$

Q.15 The thermal conductivity of two materials are in the ratio 1 : 2. What will be the ratio of thermal resistances of rods of these materials having length in the ratio 1 : 2 and area of cross-section in the ratio 1 : 2:

(A) 1 : 2 (B) 1 : 4

(C) 1 : 8 (D) 1 : 16 **[A]**

Sol. $$\frac{K_1A(T_h - T_j)}{L} = \frac{K_2A(T_j - T_c)}{L}$$

Q.16 The co-efficient of thermal expansion of a rod is temperature dependent and is given by the formula α = aT, where a is a positive constant and T in ºC. If the length of the rod is ℓ at temperature 0ºC, then the temperature at which the length will be 2ℓ is -

(A) $\sqrt{\dfrac{\ell n2}{\alpha}}$ (B) $\sqrt{\dfrac{\ell n4}{a}}$

(C) $\dfrac{1}{\alpha}$ (D) $\dfrac{2}{\alpha}$ **[B]**

Sol. As; $d\ell = \alpha\, \ell dT$ $\quad \therefore \int_0^{2\ell} \dfrac{d\ell}{\ell} = a\int_0^T TdT$

$\ell n2 = a\ \dfrac{T^2}{2}$ $\quad \therefore\ T = \left[\dfrac{\ell n4}{a}\right]^{1/2}$

Q.17 Three rods A, B and C have the same dimensions. Their thermal conductivities are k_A, k_B, and k_C respectively. A and B are placed end to end, with their free ends kept at certain temperature difference. C is placed separately with its ends kept at same temperature difference. The two arrangements conduct heat at the same rate. k_C must be equal to -

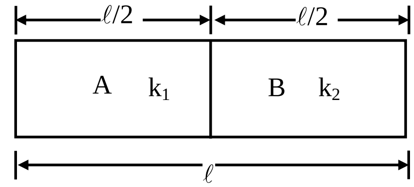
(A) $k_A + k_B$ (B) $\dfrac{k_A k_B}{k_A + k_B}$

(C) $\dfrac{1}{2}(k_A + k_B)$ (D) $\dfrac{2k_A k_B}{k_A + k_B}$ **[B]**

Sol. $\dfrac{T_1 - T_2}{\dfrac{L}{k_A A} + \dfrac{L}{k_B A}} = \dfrac{T_1 - T_2}{\dfrac{L}{k_C A}}$

$\dfrac{k_A k_B}{k_A + k_B} = k_C$

Q.18 A composite slab consists of two slabs A and B of different materials but of the same thickness placed one on top of the other. The thermal conductivities of A and B are k_1 and k_2 respectively. A steady temperature difference of 12ºC is maintained across the composite slab. If $k_1 = k_2/2$, the temperature difference across slab A will be :

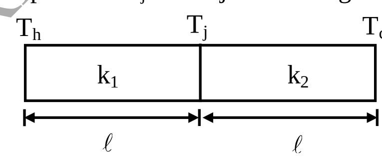


(A) 4ºC (B) 8ºC (C) 12º (D) 16º **[B]**

Sol. $R = \dfrac{L}{KA}$

$\dfrac{R_1}{R_2} = \dfrac{L_1}{L_2} \times \dfrac{K_2}{K_1} \times \dfrac{A_2}{A_1}$

Q.19 Two bars of equal length and the same cross-sectional area but of different thermal conductivities, k_1 and k_2, are joined end to end as shown in figure. One end of the composite bar is maintained at temperature T_h where as the opposite end is held at T_c . If there are no heat losses from the sides of the bars, the temperature T_j of the junction is given by -



(A) $\dfrac{k_2}{k_1}\ \dfrac{(T_h + T_c)}{2}$

(B $\dfrac{k_2}{k_1 + k_2}\ (T_h + T_c)$

(C) $\dfrac{k_1 + k_2}{k_2}\ \dfrac{(T_h + T_c)}{2}$

(D) $\dfrac{1}{k_1 + k_2}(k_1 T_h + k_2 T_c)$ **[D]**

Sol. $\tau \propto \dfrac{Kr^2}{\ell}$

$\therefore \quad \tau_1 = \tau_2$

$\dfrac{K_1 r_1^2}{\ell_1} = \dfrac{K_2 r_2^2}{\ell_2}$

Q.20 A rectangular block is heated from 0ºC to 100ºC. The percentage increase in its length is 0.10%. What will be the percentage increase in its volume ?

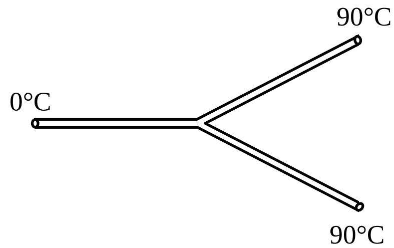
(A) 0.03 % (B) 0.10 %

(C) 0.30 % (D) None of these **[C]**

Sol. $\alpha = 10^{-5}$/ºC $\quad \dfrac{\Delta\ell}{\ell}$ = 0.10%, ΔT = 100ºC

$\therefore \frac{\Delta\ell}{\ell} = \alpha\,\Delta\theta$ and $\frac{\Delta V}{V} = \gamma.\,\Delta\theta = 3\alpha.\Delta\theta$

Q.21 Three rods made of the same material and having the same cross-section have been joined as shown in the figure. Each rod is of the same length.The left and right ends are kept at 0°C and 90°C respectively. The temperature of the junction of the three rods will be –



(A) 45°C (B) 60°C
(C) 30°C (D) 20°C **[B]**

Sol. $\tau = \tau_1 + \tau_2 =$

$$\frac{KA(T-0)}{L} = \frac{KA(90-T)}{L} + \frac{KA(90-T)}{L}$$

Q.22 Two metallic rods are connected in series. Both are of same material of same length and same area of cross-section. If the conductivity of each rod be k, then what will be the conductivity of the combination ?

(A) 4 k (B) 2 k (C) k (D) k/2 **[C]**

Sol. $\frac{2L}{k_{eq.}A} = \frac{L}{kA} + \frac{L}{kA}$

$\frac{2}{k_{eq.}} = \frac{2}{k} \Rightarrow k_{eq.} = k$

For parallel

$k_{eq.} = \frac{k_1A_1 + k_2A_2}{A_1 + A_2} = k$

Q.23 The ratio of thermal conductivities of two rods of different material is 5 : 4. The two rods of same area of cross-section and same thermal resistance will have the lengths in the ratio -

(A) 4 : 5 (B) 9 : 1
(C) 1 : 9 (D) 5 : 4 **[D]**

Sol. $\frac{L_1}{K_1A} = \frac{L_2}{K_2A}$

$\frac{K_1}{K_2} = \frac{L_1}{L_2}$

Q.24 Two vessels A and B of different materials are similar in shape and size. The same quantity of ice filled in them melts in times t_1 and t_2 respectively. The ratio of the thermal conductivities of A and B is -

(A) $t_1 : t_2$ (B) $t_2 : t_1$
(C) $t_1^2 : t_2^2$ (D) $t_2^2 ; t_1^2$ **[B]**

Sol. $\frac{mL}{t_1} = \frac{K_1A(T_1-T_2)}{L}$

$\frac{mL}{t_2} = \frac{K_2A(T_1-T_2)}{L}$

$\frac{K_1}{K_2} = \frac{t_2}{t_1}$

Q.25 One end of a conducting rod is maintained at temperature 50°C and at the other end ice is melting at 0°C. The rate of melting of ice is doubled if -

(A) The temperature is made 200°C and the area of cross-section of rod is doubled

(B) The temperature is made 100°C and length of the rod is made of four times

(C) Area of cross section of rod is halved and length is doubled

(D) The temperature is made 100°C and area of cross-section of rod and length both are doubled. **[D]**

Sol. $\frac{\Delta Q}{\Delta t} \propto \frac{A(\Delta T)}{\ell}$

If A is double the rate of flow of heat is double.

Q.26 Wires A and B have identical lengths and have circular cross-sections. The radius of A is twice the radius of B i.e. $R_A = 2R_B$. For a given temperature difference between the two ends, both wires conduct heat at the same rate. The relation between the thermal conductivities is given by -

(A) $K_A = 4\,K_B$ (B) $K_A = 2\,K_B$
(C) $K_A = K_B/2$ (D) $K_A = K_B/4$ **[D]**

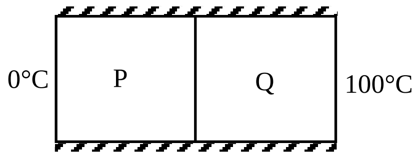
Sol. $\tau_1 = \frac{K_A\pi(2R_B)^2(T_1-T_2)}{L}$

$\tau_2 = \frac{K_B\pi R_B^2(T_1-T_2)}{L}$

$\therefore \tau_1 = \tau_2$

$K_B = 4\,K_A$

Q.27 The diagram below shows rods of the same size of two different materials P and Q placed end to end in thermal contact and heavily lagged at their sides. The outer ends of P and Q are kept at 0°C and 100°C, respectively. The thermal conductivity of P is four times that of Q. What is the steady-state temperature of the interface ?

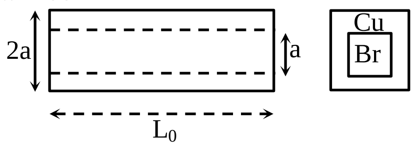


(A) 20°C (B) 75°

(C) 25°C (D) 80°C **[A]**

Sol. $\dfrac{4K(T-0)}{L} = \dfrac{K(100-T)}{L}$

Q.28 There is a copper rod of square cross-section, the dimension of each side being 2a, but having a square hole through out the rod, dimension of each side of hole being a. A brass rod exactly fitting inside the hole is inserted in the hole. Now, both sides of the rod are welded together. The temperature is elevated by θ. Neglecting any lateral expansion or stresses developed, the composite length of the system at temperature θ will be -



(A) $L_0\left[1+\dfrac{\alpha_{Br}Y_{Br}+\alpha_{Cu}Y_{Cu}}{Y_{Br}+Y_{Cu}}(\theta)\right]$

(B) $L_0\left[1+\dfrac{\alpha_{Br}Y_{Br}-\alpha_{Cu}Y_{Cu}}{Y_{Br}-Y_{Cu}}(\theta)\right]$

(C) $L_0\left[1+\dfrac{3\alpha_{Br}Y_{Br}+\alpha_{Cu}Y_{Cu}}{3Y_{Br}+Y_{Cu}}(\theta)\right]$

(D) $L_0\left[1+\dfrac{\alpha_{Br}Y_{Br}+3\alpha_{Cu}Y_{Cu}}{Y_{Br}+3Y_{Cu}}(\theta)\right]$

Sol. **[D]**

$F_1 = F_2$

$\dfrac{Y_{Cu}(3a^2)(x-L_0\alpha_{Cu}\theta)}{L_0}$

$= \dfrac{Y_{Br}(a^2)(L_0\alpha_{Br}\theta - x)}{L_0}$

Composite length = $L_0 + x$

$= L_0\left[1+\dfrac{\alpha_{Br}Y_{Br}+3Y_{Cu}\alpha_{Cu}}{3Y_{Cu}+Y_{Br}}(\theta)\right]$

Q.29 Two rods made of same material having same length and diameter are joined in series. The thermal power dissipated through then is 2W. If they are joined in parallel, the thermal power dissipated under the same conditions on the two ends of the rods, will be -

(A) 16 W (B) 8 W

(C) 4 W (D) 2 W **[B]**

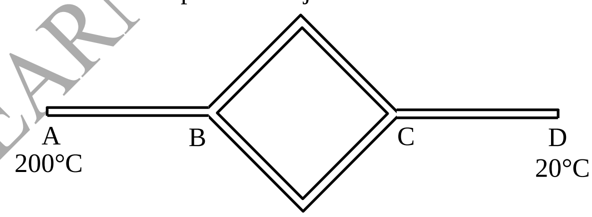
Sol. $2W = \dfrac{T_1 - T_2}{2R}$

$\tau = \dfrac{T_1 - T_2}{R/2}$

$\dfrac{2\text{ watt}}{\tau} = \dfrac{T_1 - T_2}{2R} \times \dfrac{R}{2(T_1 - T_2)}$

$\tau = 8$ watt

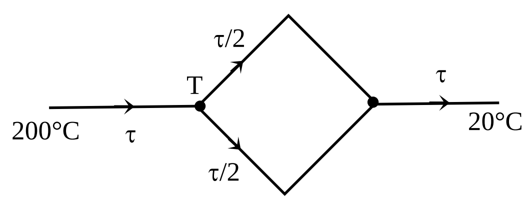
Q.30 Six identical conducting rods are joined as shown in figure. Points A and D are maintained at temperatures 200°C and 20°C respectively. The temperature of junction B will be –



(A) 120°C (B) 100°C

(C) 140° C (D) 80°C **[C]**

Sol.



$\dfrac{200 - T_B}{R} = \dfrac{T_C - 20}{R}$

$200 - T_B = T_C - 20$

$220 = T_B + T_C$

$T_B + T_C = 220$ …….(i)

$\dfrac{\tau}{2} = \dfrac{T_B - T_C}{2R}$

$\dfrac{200 - T_B}{R} = \dfrac{T_B - T_C}{R}$

$200 - T_B = T_B - T_C$

$200 - T_B = T_B - (220 - T_B)$

$420 = 3\,T_B$

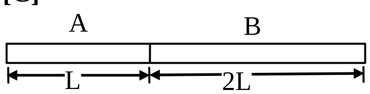
$T_B = 140\ °C$

Q.31 If two rods of length L and 2L having coefficient of linear expansion α and 2α respectively are connected so that total length becomes 3L, the average coefficient of linear expansion of the composition rods equals -

(A) $\frac{3}{2}\alpha$ (B) $\frac{5}{2}\alpha$

(C) $\frac{5}{3}\alpha$ (D) None of these

Sol. **[C]**



$\therefore \alpha = \frac{\Delta\ell}{L\Delta T}$

for A : $\alpha = \frac{\Delta\ell_1}{L\Delta T} \Rightarrow \Delta\ell_1 = L\,\alpha\,\Delta T$

for B : $2\alpha = \frac{\Delta\ell_2}{2L\Delta T} \Rightarrow \Delta\ell_2 = 4L\,\alpha\,\Delta T$

For composition

$\alpha_{avg.} = \frac{\Delta\ell_1 + \Delta\ell_2}{3L\Delta T} = \frac{\alpha L\Delta T + 4L\alpha\Delta T}{3L\Delta T} = \frac{5}{3}\alpha$

Q.32 A thin copper wire of length L increase in length by 1% when heated from temp T_1 to T_2. What is the percentage change in area when a thin copper plate having dimensions $2L \times L$ is heated from T_1 to T_2 ?

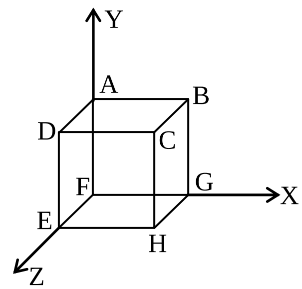
(A) 1% (B) 2%

(C) 3% (D) 4%

Sol. **[B]**

$\frac{\%\text{change in length}}{\%\text{change in Area}} = \frac{\alpha}{\beta}$

Q.33 A cuboid ABCDEFGH is anisotropic with $\alpha_x = 1 \times 10^{-5}/°C$, $\alpha_y = 2 \times 10^{-5}/°C$, $\alpha_z = 3 \times 10^{-5}/°C$. Coefficient of superficial expansion (β) of faces can be (Take approximation)



(A) $\beta_{ABCD} = 5 \times 10^{-5}/°C$ (B) $\beta_{BCGH} = 4 \times 10^{-5}/°C$

(C) $\beta_{CDEH} = 3 \times 10^{-5}/°C$ (D) $\beta_{EFGH} = 2 \times 10^{-5}/°C$

Sol. **[C]**

$\ell_x = \ell_{x_0}(1 + \alpha_x T)$, $\ell_y = \ell_{y_0}(1 + \alpha_y T)$

$\ell_x\ell_y = \ell_{x_0}\ell_{y_0}[1 + (\alpha_x + \alpha_y + \alpha_x\alpha_y)T]$

$A_x = A_{x_0}[1 + (\alpha_x + \alpha_y + \alpha_x\alpha_y)T]$

$A_x = A_{x_0}[1 + (1 \times 10^{-5} + 2 \times 10^{-5} + 1 \times 2 \times 10^{-10})T]$

$A_x = A_{x_0}[1 + 3 \times 10^{-5}T]$, $\beta_{CDEH} = 3 \times 10^{-5}/°C$

Q.34 A clock which keeps correct time at 25ºC has a pendulum made of a metal. The temperature falls to 0ºC. If the coefficient of linear expansion of the metal is 1.9×10^{-5} per ºC, then number of second the clock gains per day is -

(A) 10.25 s (B) 20.52 s

(C) 30.75 s (D) 41 s

Sol. **[B]**

$\frac{\Delta T}{T} = \frac{1}{2}\alpha(\theta_2 - \theta_1)$

$\Delta T = \frac{1}{2} \times 1.9 \times 10^{-5}(25 - 0) \times 24 \times 3600$

Q.35 Equation of a gas is given by $(T^7/P^2)^{1/5}$ = constant. Coefficient of volume expansion of that gas in isobaric process is -

(A) $-\frac{1}{T}$ (B) $-\frac{1.5}{T}$

(C) $-\frac{2.5}{T}$ (D) $-\frac{3.5}{T}$

Sol. **[C]**

$$T^{\frac{7}{5}} P^{-\frac{2}{5}} = C \; ; \; T^{\frac{7}{5}} \left(\frac{nRT}{V}\right)^{-\frac{2}{5}} = C$$

$$T^{\frac{7}{5}-\frac{2}{5}} V^{\frac{2}{5}} = C \; ; \; TV^{\frac{2}{5}} = C$$

$$\ln T + \frac{2}{5} \ln V = \ln C$$

$$\frac{\Delta T}{T} + \frac{2}{5}\frac{\Delta V}{V} = 0 \; ; \; -\frac{5}{2T} = \frac{\Delta V}{V\Delta T} = \gamma$$

$$\gamma = -\frac{2.5}{T}$$

Q.36 The temperature of a thin uniform circular disc, of diameter 1m is increased by 10ºC. The percentage increase in its moment of inertia about an axis passing through its centre and perpendicular to its surface is - ($\alpha = 11 \times 10^{-6}$ / ºC)

(A) 0.0055% (B) 0.011%

(C) 0.022% (D) 0.044%

Sol. **[C]**

$I = \frac{1}{2} mr^2$

$\ln I = \ln \frac{1}{2} m + \ln r^2$

$\ln I = 2\ln r + \ln \frac{1}{2} m$

$\frac{\Delta I}{I} = 2 \frac{\Delta r}{r} = 2\, \alpha\, \Delta\theta$

$= 2 \times 11 \times 10^{-6} \times 10$

$= 22 \times 10^{-5} = 22 \times 10^{-3}\,\% = 0.022\%$

Q.37 A glass flask of volume one litre at 0ºC is filled, level full of mercury at this temperature. The flask and mercury are now heated at 100ºC. How much mercury will spill out, if coefficient of volume expansion of mercury is 1.82×10^{-4}/ºC and linear expansion of glass is 0.1×10^{-4}/ºC respectively -

(A) 21.2 cc (B) 15.2 cc

(C) 1.52 cc (D) 2.12 cc

Sol.[B] $\Delta V_A = \Delta V_R - \Delta V_s$

$= V (r_A - r_s)\, \Delta\theta$

$= 10^3$ c.c. $(1.82 \times 10^{-4} - 0.3 \times 10^{-4}) \times 100$

$= 1.52 \times 10$ cc $= 15.2$ cc

Q.38 A crystal has a coefficient of expansion 13×10^{-7} in one direction and 231×10^{-7} in every other direction at right angles to it. Then the cubical coefficient of expansion is -

(A) 462×10^{-7} (B) 244×10^{-7}

(C) 475×10^{-7} (D) 257×10^{-7}

Sol.[C] $\gamma = \alpha_1 + 2\alpha_2 = (13 + 2 \times 231) \times 10^{-7} = 475 \times 10^{-7}$

Q.39 An iron tyre of diameter 2 m is to be fitted on to a wooden wheel of diameter 2.01 m. The temperature to which the tyre must be heated, if $\alpha = 11 \times 10^{-6}$/ºC and room temperature is 20ºC, will be -

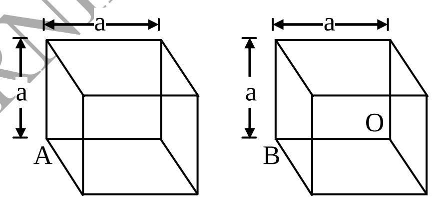
(A) 474.5ºC (B) 490.5ºC

(C) 440.5ºC (D) 460.5ºC

Sol.[A] $\Delta\theta = \frac{\Delta\ell}{\ell\alpha} = \frac{0.01}{2 \times 11 \times 10^{-6}} = \frac{10^4}{22} = 454.5$

$\theta = 454.5 + 20 = 474.5$ºC

Q.40 A and B are made up of an isotropic medium. Both A and B are of equal volume. Body B has cavity as shown in Fig.(b). Which of the following statements is true?



(a) (b)

(A) Expansion in volume of A > expansion in B

(B) Expansion in volume of B > expansion in A

(C) Expansion in A = expansion in B

(D) None of these

Sol. **[C]** Thermal expansion in isotropic bodies is independent of shape size & availability of cavity.

Q. 41 A piece of metal floats on Hg. The coefficient of expansion of metal and Hg are γ_1 and γ_2 respectively. If the temperature of both Hg and metal are increased by an amount ΔT, by what factor the fraction of the volume of metal submerged in mercury changes?

(A) $(\gamma_2 - \gamma_1)\, \Delta T$ (B) $\left(\frac{\gamma_2 + \gamma_1}{2}\right) \Delta T$

(C) $\frac{2\gamma_1\gamma_2}{\gamma_1 + \gamma_2}\, \Delta T$ (D) $\frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2}\, \Delta T$

Sol. **[A]**

$f_{in} = \frac{V_{in}}{V} = \frac{\rho}{\sigma}$ $\rho \rightarrow$ density of metal, $\sigma \rightarrow$ density of Hg

$$\frac{\Delta f}{f} = \frac{f'_{in} - f_{in}}{f_{in}} = \frac{f'_{in}}{f_{in}} - 1$$

$$= \frac{\frac{\rho}{\sigma}\left(\frac{1+\gamma_2 \Delta T}{1+\gamma_1 \Delta T}\right) - 1}{\rho/\sigma}$$

$= (\gamma_2 - \gamma_1)\, \Delta T$ (using Binomial theorem)

Q.42 A Cu rod and a steel rod maintain a difference in their lengths constant = 10 cm at all temperatures. If their coefficients of expansion are $1.6 \times 10^{-5}\ K^{-1}$ and $1.2 \times 10^{-5}\ K^{-1}$, then the length of the Cu rod is -

(A) 40 cm (B) 30 cm
(C) 32 cm (D) 24 cm

Sol. **[B]**

$\Delta\ell_1 = \Delta\ell_2 \Rightarrow \ell_1\alpha_1\,\Delta T = \ell_2\alpha_2\Delta T$

$\Rightarrow \ell_1\alpha_1 = \ell_2\alpha_2 \Rightarrow \frac{\ell_2}{\ell_1} = \frac{\alpha_1}{\alpha_2} \Rightarrow \frac{\ell_2}{\ell_1} - 1 = \frac{\alpha_1}{\alpha_2} - 1$

$\frac{\ell_2 - \ell_1}{\ell_1} = \frac{\alpha_1 - \alpha_2}{\alpha_1} \Rightarrow \ell_1 = \frac{\alpha_1(\ell_2 - \ell_1)}{(\alpha_1 - \alpha_2)}$

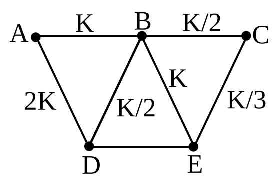
Q.43 A clock which keeps correct time at 20ºC has a pendulum rod made of brass. How many seconds will it gain or lose per day when temperature falls to 0ºC ($\alpha = 18 \times 10^{-6}$/ºC) ?

(A) 155.5 s (B) 15.55 s
(C) 25.55 s (D) 18.55 s

Sol.[B] $\Delta T = \frac{1}{2} \propto T\Delta\theta = \frac{1}{2} \times 18 \times 10^{-6} \times 86400\ s \times 20$

$= 1.8 \times 8.64\ s = 15.55\ s$

Q.44 Five rods of identical geometries are arranged as shown in figure. Temperatures are maintained at points A, C, D and E as 100ºC, 0ºC, 40ºC and 60ºC respectively. Thermal conductivities of rods are shown in the figure. Heat current in the branch BE is - [Take length of each rod ℓ = 1m and A = 1m^2, K = 0.5 W/m-K]



(A) 10 W (B) 20 W
(C) zero (D) None of these

Sol.[C] Net heat current at B must be zero so T_B = 60ºC

Q.45 A clock with a metal pendulum beating seconds keeps correct time at 0ºC. If it loses 12.5 seconds a day at 25ºC, the coefficient of linear expansion of metal of pendulum is -

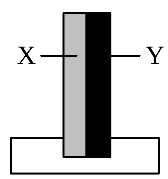
(A) $\frac{1}{86400}$/ºC (B) $\frac{1}{43200}$/ºC

(C) $\frac{1}{14400}$/ºC (D) $\frac{1}{28800}$/ºC

Sol. [A] $\Delta t = \frac{1}{2}\alpha t\,\Delta\theta$

$\alpha = \frac{2\Delta t}{t\Delta\theta} = \frac{2 \times 12.5}{86400 \times 25} = \frac{1}{86400}$/ºC

Q.46 A bimetallic strip consists of metals X and Y. It is mounted rigidly at the base as shown in the figure. The metal X has a higher coefficient of expansion as compared to that for metal Y. When the bimetallic strip is placed in a cold bath ?



(A) It will bend towards the right
(B) It will bend towards the left
(C) It will not bend but shrink
(D) It will neither bend nor shrink

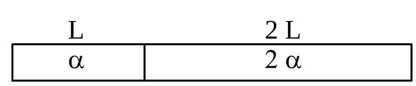
Sol.[B] As, $\alpha_x > \alpha_y$ then on colling X contracts more than Y. So, the strip bends towards X, i.e., towards left.

Q.47 If two rods of length L and 2L having coefficients of linear expansion α and 2α respectively are connected so that total length becomes 3L, the average coefficient of linear expansion of the composition rod equals -

(A) $\frac{3}{2}\alpha$ (B) $\frac{5}{2}\alpha$

(C) $\frac{5}{3}\alpha$ (D) none of these

Sol.[C]



$\Delta\ell = \Delta\ell_1 + \Delta\ell_2$

$\alpha_{eq} 3L\Delta T = \alpha L\Delta T + 2\alpha \times 2\, L\Delta T$

$3\alpha_{eq} = 5\alpha \Rightarrow \alpha_{eq} = \frac{5}{3}\alpha$

Q.48 At 4ºC, 0.98 of the volume of a body is immersed in water. The temperature at which the entire body gets immersed in water ($\gamma_w = 3.3 \times 10^{-4} K^{-1}$) is (neglect the expansion of the body)–

(A) 40.8ºC (B) 65.8ºC

(C) 60.6ºC (D) 58.8ºC **[B]**

Q.49 A solid ball of metal has a spherical cavity inside it. If the ball is heated, the volume of the cavity will–

(A) Increase

(B) Decrease

(C) Remains unchanged

(D) Have its shape changed **[A]**

Q.50 A beaker is completely filled with water at 4°C. It must overflow -

(A) when heated but not when cooled

(B) when cooled but not when heated

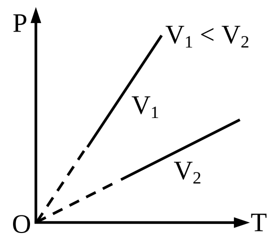
(C) both when heated or cooled

(D) neither when heated nor when cooled **[C]**

Q.1 A certain mass of gas is heated first in a small vessel and then in a large one. During heating the volumes of the vessels remain constant.

How will the pressure-temperature graphs differ in the first and the second case?

Sol. See Fig.

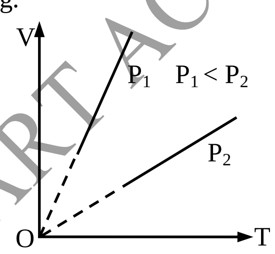


At any temperature a given mass of gas will create a pressure which will increase, as the volume of vessel containing the gas decreases. When the gas is heated in the small vessel the pressure will increase faster than during heating in the vessel. The constant-volume line corresponding to the small volume will always form a larger angle with the X-axis on the plot (P, T) than the constant-volume line that corresponds to the larger volume **(Fig.)**.

Q.2 The gas in a cylinder is enclosed by a freely moving piston. Plot the dependence of volume on temperature: **(a)** when the gas is heated with a small load on the piston; and **(b)** with a large load.

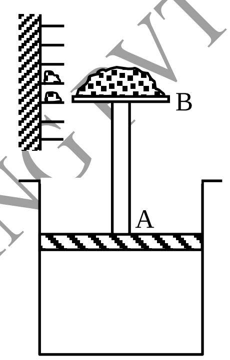
How will the position of the volume versus temperature curve change at constant (internal) pressure when the external pressure is altered?

Sol. See Fig.

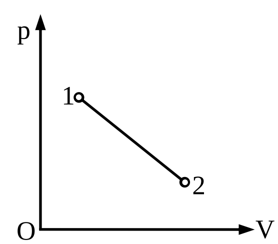


At a given temperature and a high pressure the gas will occupy a smaller volume than at a small pressure although at the same temperature. The higher the pressure at which the constant-pressure process occurs, the smaller the angle formed by the constant-pressure line with the X-axis on the plot (V, T) (Fig.).

Q.3 The gas in a cylinder is enclosed by a piston A (Fig.). The piston end has an area B supporting a certain amount of sand that exerts the necessary pressure on the piston. If some sand is pushed in small portions onto the shelves near the support the pressure exerted on the piston will gradually change. It is also possible to change the temperature of the gas by placing the cylinder on heaters or coolers.

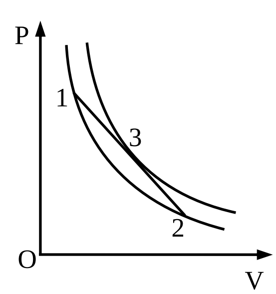


A plot of pressure versus volume for a gas expanding in such a cylinder made from direct measurements is illustrated in **Fig.**

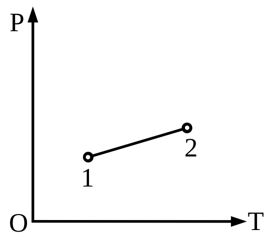


How can this plot be used to determine the nature of change in the temperature of the gas?

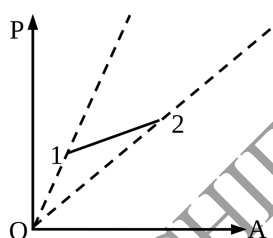
Sol. To determine the temperature of the gas at the initial point 1, at the final point 2 and at a certain point 3, draw constant-temperature lines **(Fig.)** through these points and determine the ratio of the temperature lines. The gas is heated in the section 1-3 and cooled in the section 1-2.



Q.4 A curve showing the dependence of pressure on absolute temperature was obtained for a certain gas **(Fig.)**. Does compression or expansion take place when the gas is being heated?

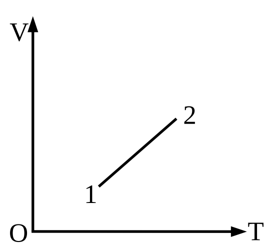


Sol. In order to determine the nature of change in the gas volume during heating, draw constant-volume liens on the drawing passing through the initial and final points 1 and 2 (Fig.). Point 2 lies on the constant-volume line which is more sloping with respect to the X-axis than the constant-volume line passing through point 1 and therefore (see the solution to Q 1) the gas occupies a larger volume at point 2 than at point 1. Heating was conducted for an increasing volume of gas.

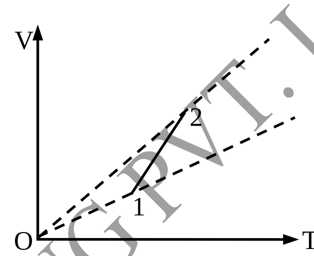


The gas expands during heating. **Ans.**

Q.5 Use the volume-temperature curve **(Fig.)** to find graphically the nature of change in the pressure of a gas during heating.



Sol. In order to solve the problem, draw lines of constant pressure on which points 1 and 2 lie **(Fig.)**. Point 1 lies on the constant-pressure line which forms a smaller angle with the X-axis than the constant-pressure line passing through point 2 and therefore (see the solution to Problem **Q. 2**) the gas is present at point 1 at a larger pressure than at point 2. Heating was conducted with a diminishing pressure of the gas.

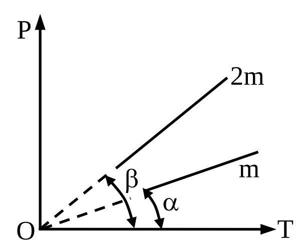


The pressure constantly diminishes. **Ans.**

Q.6 A constant volume vessel is used to heat first **m** grams of a certain gas and then **2m** grams of the same gas.

Draw the curves showing the dependence of pressure on temperature in each case. Indicate the difference in the positions of the curves.

Sol. See Fig.



At any given temperature, **2m** grams of gas will produce twice as great a pressure than **m** grams enclosed within the same volume.

The constant-volume line for **2m** grams of gas will be at a larger angle with respect to the X-axis than the constant-volume line for m grams, and

tan β = 2 tan α **Ans.**

Q.7 A movable piston is inserted in a cylinder closed on both ends. One end of the cylinder contains **m** grams of a certain gas and the other **2m** grams of the same gas.

What fraction of the cylinder by volume will be occupied by 2m grams of the gas when the piston is in equilibrium?

Sol. If P_1, V_1 and T_1 are the pressure, volume and temperature of m grams of gas and P_2, V_2 and T_2 are the pressure, volume and temperature of 2m grams of gas the following ratio will always hold:

$$\frac{P_2V_2}{T_2} = 2\frac{P_1V_1}{T_1}$$

From the given conditions, when the piston is in equilibrium, $T_1 = T_2$ and $P_1 = P_2$. Therefore, the piston should take such a position when

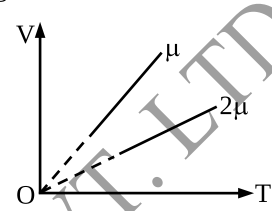
$V_2 = 2V_1$.

2/3 of the cylinder volume. **Ans.**

Q.8 A gas of molecular weight **μ** is heated in a cylinder enclosed by a freely moving piston. A gas of molecular weight **2μ** is then heated in the same cylinder. The masses piston are the same in each case.

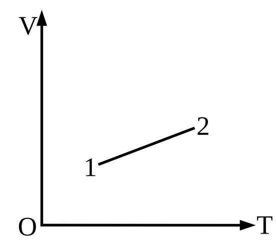
Will the plots of volume versus temperature be the same in each case?

Sol. With the same masses, the gas with a molecular weight **2μ** will contain half as many molecules as the gas with a molecular weight **μ** If the temperatures of the gases are the same, the mean stores of energy in the molecules of both gases will also be the same. For the gases in these conditions to create the same pressure the number of impacts of the molecules per each **cm²** of the surface of the vessels should be the same too. This is only possible when the heavier gas occupies a smaller volume. Hence, if the pressures and temperatures are the same, **m** grams of the heavy gas always occupy a smaller volume than m grams of the light gas. The constant-pressure line of the heavy gas on the plot (V, T) will always be more sloping than that of the light gas.



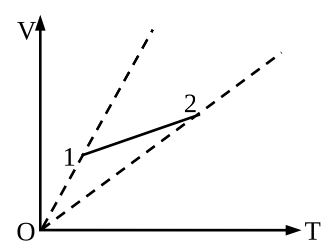
The plots will be different. **Ans.**

Q.9 The piston in a gas-filled cylinder is loosely fitted against the wall of the cylinder and can slowly let the gas go past. The volume-temperature curve for the gas at constant pressure has the form shown in **Fig.**



Use this curve to determine whether the amount of gas in the cylinder has increased or decreased.

Sol. In order to solve the problem, draw lines of constant pressure corresponding to the constant masses of the gas through points **1** and **2 (Fig.)**.



The constant-pressure line on which point 1 lies is steeper than the line on which point 2 lies. Therefore (see the solution to Q. 6) the mass of the gas in state 1 is larger than in state 2.

The amount of gas has decreased. **Ans.**

Q.10 On the thermometric scale of the Russian Academician I.N. Delil the boiling point of water corresponds to zero and the melting point of ice to –150º.

What value of the temperature coefficient of gas expansion at constant pressure should be taken when temperature is measured on Delil's scale?

Sol. The coefficient of gas expansion $\boldsymbol{\alpha}$ shows that the volume of gas is increased by 1º during heating with respect to the volume it occupied at 0ºC on the temperature scale.

But 0ºC on Delil's sacle corresponds to the boiling point of water. For this reason, utilizing the usual definition of the coefficient of gas expansion all the changes in volume should be referred to the volume occupied by the gas at the boiling point of water.

Since 1 deg by Delil $= \frac{2}{3}$ by centigrade sacle

and $\alpha V_0 = \frac{1}{273} \times V_0 = \frac{1}{373} V_{100} = \alpha_1 V_{100}$, the coefficient of gas expansion on Delil's cale will be

$$\alpha_D = \frac{2}{3} a_1 \approx \frac{1}{560}$$

and the equation of Gay-Lussac's law may be written as

$$V = V_0' (1 + \alpha_D t) \qquad (1)$$

Where V_0' is the volume occupied by the gas at 0º on Delil's scale and **t** is the temperature in Delil's degrees.

If the change in the volume of gas is referred, as before, to the volume occupied by the gas at the melting point of ice (i.e., as it is commonly done when measuring temperature on the centigrade scale) it will be necessary to:

(a) determine the coefficient of gas expansion as the number showing the increase in the volume of the gas when it is heated by 1º on Delil's scale with respect to the volume occupied by the gas at the temperature –150º Delil';

(b) assume the coefficient α_D' as equal to

$$\alpha_D' = \frac{2}{3} \alpha = \frac{2}{3} \frac{1}{273} \approx \frac{1}{410}$$

In this case the equation of Gay-Lussac's law will take the form

$$V = V_{-150} [1 + \alpha_D' (t + 150)] \qquad (2)$$

It is easy to see that equations (1) and (2) always produce the same results when the volume of gas is calculated.

Two answers are possible: 1/610 or 1/360. **Ans.**

Q.11 An open glass tube is immersed in mercury so that an end of length $\boldsymbol{l_1}$ = 8 cm projects above the mercury. The tube is then closed and raised 44 cm.

What fraction of the tube will be occupied by the air after it has been raised? The atmospheric pressure is P = 76 cm Hg.

Sol. **Note:** In this case Boyle's law equation will be

$l_1 P = h_2 (P + h_2 – l)$.

where $\boldsymbol{l}$ is the total length of the part of the tube projecting from the mercury.

h_2 = 15.8 cm. **Ans.**

Q.12 At a temperature t_0 the pendulum of a clock has a length of $\boldsymbol{l_0}$ and the clock then goes accurately. The coefficient of linear expansion of the pendulum material is $\boldsymbol{\alpha} = 1.85 \times 10^{-5}$.

How much will the clock gain or lose in twenty-four hours if the ambient temperature is 10ºC higher than t_0? In deriving the formula allow for a small value of the coefficient of linear expansion of the pendulum.

Sol. A definite number N of oscillations of the pendulum will correspond to one full revolution of the hour-hand. If the clock is accurate, these

N° oscillations are performed in twenty-four hours. From the given conditions

$$N = \frac{24 \times 60 \times 60}{2\pi\sqrt{\frac{l_0}{g}}}$$

when the temperature changes by **t** degrees the length of the pendulum will be $\boldsymbol{l} = l_0(1 + \alpha t)$ and the period of oscillations of the pendulum will change by

$$T - T_0 = 2\pi\left(\sqrt{\frac{l}{g}} - \sqrt{\frac{l_0}{g}}\right) = \frac{2\pi}{g}\,\frac{l - l_0}{\sqrt{\frac{l}{g}} + \sqrt{\frac{l_0}{g}}} \approx \frac{\pi}{g}\,\frac{l - l_0}{\sqrt{\frac{l_0}{g}}} = \frac{\pi}{g}\,\frac{\alpha l_0 t}{\sqrt{\frac{l_0}{g}}}$$

The clock will gain or lose

$$\tau = N(T - T_0) = \frac{24 \times 60 \times 60}{2\pi\sqrt{\frac{l_0}{g}}}\,\frac{2\pi}{g}\,\frac{\alpha l_0 t}{\sqrt{\frac{l_0}{g}}} = 12 \times 60$$

$\times\, 60\alpha t$ sec (In twenty-four hours.)

The clock will lose $\tau = 8$ s **Ans.**

Q.13 A steel rod with a cross-sectional area S = 10 cm^2 is set lengthwise between two rigidly secured massive steel plates.

Sol. If the rod were free, heating it by t° would expand it by a length

$l - l_0 = \alpha l_0 t$

Since, from the given conditions the distance between the steel plates remains constant, the value $l - l_0$ will determine the compressive deformation of the rod caused by the heating.

According to Hooke's law, the force of pressure of the rod will be

$$F = \frac{SE}{l_0}(l - l_0) = SE\alpha t.$$

$F = SE\alpha t = 3{,}465$ kgf. **Ans.**

Q.14 When making a certain physical instrument it was found necessary to ensure that the difference between the lengths of an iron and a copper cylinder remained the same whatever the temperature change.

How long should these cylinders be at 0°C so that the difference between them is 10 cm whatever the temperature change? The coefficient of linear expansion of iron is $\boldsymbol{\alpha_1} = 1.1 \times 10^{-5}$ and of copper $\boldsymbol{\alpha_2} = 1.7 \times 10^{-5}$.

Sol. At any temperature the lengths of the iron and copper cylinders will be

$l_1 = l_{01}(1 + \alpha_1 t)$, $l_2 = l_{02}(1 + \alpha_2 t)$ ….. (1)

From the given conditions

$l_1 - l_2 = 10$ and $l_{01} - l_{02} = 10$ ….. (2)

If follows from (1) and (2) that

$$\frac{l_{02}}{l_{01}} = \frac{\alpha_1}{\alpha_2} \quad \text{….. (3)}$$

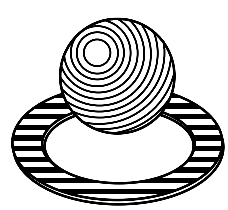
The initial lengths of the cylinders should be inversely proportional to the coefficients of linear expansion.

It follows from (3) and (2) that

$$l_{01} = \frac{10\alpha_2}{\alpha_2 - \alpha_1}, \quad l_{02} = \frac{10\alpha_1}{\alpha_2 - \alpha_1}.$$

Iron cylinder 28.3 cm and copper cylinder 18.3 cm. **Ans.**

Q.15 The experiment demonstrating the expansion of metal under heat is well known: a metal ball which passes through a metal ring gets stuck when it is heated (Fig.). What will happen if the ring, instead of the ball, is heated?



Sol. The ball passes through the ring at any temperature which is the same for both ball and ring. Heating the ring is equivalent to cooling the ball and consequently the ball will pass freely through the ring when the latter is heated.

Q.16 Three are two layers of water in a calorimeter, the lower one colder, the upper one hotter. Will the overall volume of the water be altered if the temperatures are evened out?

Sol. Let t be the final total temperature; let $\mathbf{m_1}$, $\mathbf{v_1}$, $\mathbf{t_1}$ be the mass, volume and temperature respectively of the colder water and let $\mathbf{v_1}'$ be the volume of this water at temperature **t**; also let $\mathbf{m_2}$, $\mathbf{v_2}$, $\mathbf{t_2}$ be the mass, volume and temperature respectively of the warmer water and let $\mathbf{v_2}'$ be the volume of this water at temperature **t**. The equation of thermal balance gives

$cm_2(t_2 - t) = cm_2(t - t_1)$ …..(1)

(c, the thermal capacity, can be cancelled).

On the other hand, the change of the volumes with the change of temperature will be expressed thus:

$$v_1 = \frac{m_1}{\rho_1} = \frac{m_1(1+\alpha t_1)}{\rho_0},$$

where $\boldsymbol{\rho_1}$ is the density of the water at temperature $\mathbf{t_1}$, $\boldsymbol{\rho_0}$ is its density at a temperature of 0ºC, and α is the coefficient of cubic expansion (taking it as constant). Similarly

$$v_1' = \frac{m_1(1+\alpha t)}{\rho_0}; \quad v_2 = \frac{m_2(1+\alpha t)}{\rho_0}; \quad v_2' = \frac{m_2(1+\alpha t)}{\rho_0},$$

Hence we shall find the alteration in volume:

$$v_2 - v_2' = \frac{m_2\alpha(t_2 - t)}{\rho_0}, \quad \text{….. (2)}$$

$$v'_1 - v_1 = \frac{m_1\alpha(t - t_1)}{\rho_0}, \quad \text{….. (2')}$$

Substituting (1) in (2) and (2'), we find:

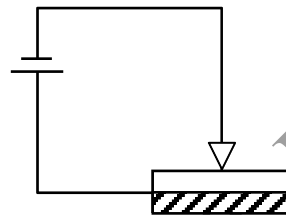
$v_2 - v_2' = v_1' - v_1$

or

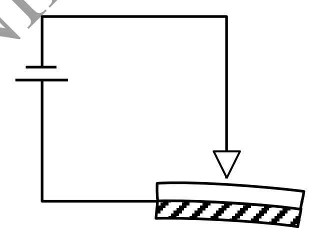
$v_2 + v_2 = v_2' + v_1'$,

i.e. the total volume of liquid will not be altered.

Q.17 A plate composed of welded sheets of copper and iron is connected to an electrical circuit as shown in Fig. Describe what will happen if a fairly strong current be passed through the circuit.



Sol. The coefficient of thermal expansion of copper is greater than that of iron: therefore when it is heated, the welded plate will bend, as shown in Fig., and this will cause the electric circuit to be broken.



As soon as the circuit breaks, heating will cease, the plate will cool and straighten itself out, returning to its previous position and recompleting the circuit; heating will begin again and so on. Such bimetallic plates can consequently be used as interrupters. Sometimes they are used for automatic shutdown of a section of an electric grid under overloading.

Q.18 It is well known that to take a man's temperature with a thermometer 5-10 min are needed; but to shake the mercury down when the thermometer is taken out sometimes requires only a few seconds. Why is this?

Sol. When a man's temperature is taken with a clinical thermometer, this is what happens. At first the difference in the temperatures of man and thermometer is considerable and the mercury expands with rapid heating. When the thermometer's temperature is near to that of the man's body, the heating of the thermometer takes place slowly and the mercury also expands slowly. Therefore a considerable time is required for the thermometer to be heated to the temperature of the man. When the thermometer is taken out there is a greater difference between the temperatures of the thermometer and the surrounding air, the volume of the mercury contracts rapidly and it is enough to shake the thermometer for the column of mercury to occupy the space which is empty in the bulb.

Q

Q.19 What properties of red copper make it an exceptionally suitable substance for soldering irons? Is there any other substance which has such valuable properties for the purpose?

Sol. Copper has a high specific thermal capacity and it is thanks to this that when a copper soldering iron is heated, a great amount of heat is transmitted to it. Further, copper has high thermal conductivity, so that a copper soldering iron quickly gives up a large amount of heat to tin or any other material, which must be fused. The other substance which has the same high qualities is silver, but it is too expensive.

Q.20 The brass scale of a mercury barometer has been checked at 0ºC. At 18ºC the barometer shows a pressure of 760 mm.
Reduce the reading of the barometer to 0ºC. The coefficient of linear expansion of brass is α = 1.9×10^{-5} and the coefficient of volume expansion of mercury $\beta = 1.8 \times 10^{-4}$.

Sol. Since, from the given conditions, the scale has been checked at 0ºC, then l_1 = 760 graduations on the scale will correspond to the length of the mercury column
$l_2 = l_1 (1 + \alpha t)$
The column of mercury of height l_2 will set up a pressure $P = \gamma l_2$, where γ is the specific gravity of mercury at a temperature t = 18ºC. At 0ºC the same pressure will be built up by the mercury column of height l_0, such that $P = \gamma_0 l_0$.

Since $\gamma = \dfrac{\gamma_0}{1+\beta t}$ the actual pressure expressed in millimeters of the mercury column will at 0ºC be equal to

$l_0 = \dfrac{\gamma}{\gamma_0} l_2 = l_1 \dfrac{1+\alpha t}{1+\beta t}$ = **757.3 m Hg.** **Ans.**