PHYSICS

The following questions consists of two statements each, printed as Assertion and Reason. While answering these questions you are to choose any one of the following four responses.

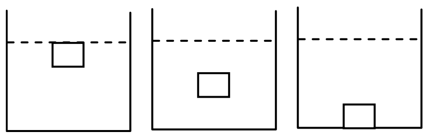
(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true but Reason is not correct explanation of the Assertion.

(C) If Assertion is true but the Reason is false.

(D) If Assertion is false but Reason is true.

Q.1 **Assertion :** Water in a container is to be cooled by putting an ice cube in it. Water will get cooled fastest in case 'A'.



Reason : In 'A' water is cooled through convection only, in 'B' partly through convection and partly through conduction and in 'C' through conduction only. **[A]**

Q.2 **Assertion** : The SI unit of thermal conductivity is watt $m^{-1}K^{-1}$.

Reason : Thermal conductivity is a measure of ability of the material to allow the passage of heat through it. **[B]**

Q.3 **Assertion** : When temperature difference across the two sides of a wall is increased, its thermal conductivity increases.

Reason : Thermal conductivity depends on nature of material of the wall. **[D]**

Q. 4 **Assertion :** In lake or ocean water does not freeze completely.

Reason : Water has minimum volume at 4ºC.

[A]

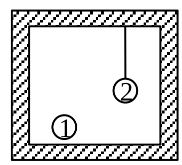
Q.5 **Assertion :** In steady state temperature at each point of rod remain constant with time.

Reason : In steady state heat current through rod does not flow. **[C]**

Q.6 **Assertion :** A body is emitting red light. As the temperature of body is increased it starts emitting light of yellow colour.

Reason : Rate of radiation emitted by a body increases as the temperature increases. **[D]**

Q. 7 Two rigid identical spheres of same material, are in a closed chamber. The walls, floor and ceiling are thermally non-conducting. The thread with which sphere 2 is hanging is also non-conducting.



Assertion : Sphere 1 will absorb more heat than sphere 2 for the same temperature rise from 5ºC to 100ºC

Reason : Heat supplied to a system is used to raise the internal energy and do work against the external forces. **[A]**

Q.8 **Assertion (A) :** Two bodies at different temperatures, it brought in thermal contact do not necessary settle to the mean temperature.

Reason (R) : The two bodies may have different thermal capacities.

[A]

PHYSICS

Q.1

Column-I	**Column-II**
(A) Ice formation in a lake	(P) Molecular transfer leads to uniform heat content
(B) The mode not associated with solids	(Q) heat transfer by convection
(C) Water heated in a round Bottomed flask	(R) Time taken is inversely proportional to ambient temperature
(D) Land breeze	(S) process of heat transfer by conduction

Sol. (A) $\rightarrow$ (R) & (S), (B) $\rightarrow$ (Q), (C) $\rightarrow$ (Q) & (P), (D) $\rightarrow$ (Q)

Time in which a thickness 'x' of ice is formed in an ambience ($-\theta$) in a lake is given by

$t = \frac{\rho L}{2K\theta}(x^2)$ where K is thermal conductivity of ice, ρ is density of water and L is latent heat energy.

So, (A) $\rightarrow$ (R). Ice as it is formed start conducting heat from water underneath to the ambience.

So, (A) $\rightarrow$ (S)

Solids always transfer heat by radiation and/or conduction.

So, (B) $\rightarrow$ (Q).

Water when heated, will raise its molecules upward and start transferring heat. Since actual molecules are involved, it is convection current and is responsible for uniform heating.

So, (C) $\rightarrow$ (P) and (Q)

Land and sea breeze are caused due to convection currents,

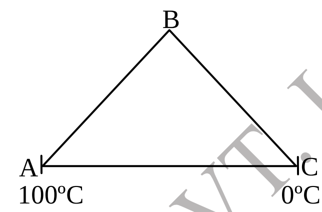
So, (D) $\rightarrow$ (Q).

Q.2 Match the following :

Column-1	**Column-2**
(A) Specific heat	(P) $[MLT^{-3}\theta^{-1}]$
(B) Coefficient of thermal conductivity	(Q) $[MT^{-3}\theta^{-4}]$
(C) Boltzmann constant	(R) $[L^2T^{-2}\theta^{-1}]$
(D) Stefan's constant	(S) $[ML^2T^{-2}\theta^{-1}]$

Ans. **A $\rightarrow$ R ; B $\rightarrow$ P ; C $\rightarrow$ S ; D $\rightarrow$ Q**

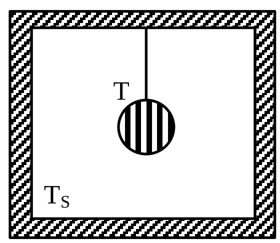
Q.3 Three rods of equal length of same material are joined to form an equilateral triangle ABC as shown in figure. Area of cross-section of rod AB is S, of rod BC is 2S and that of AC is S. Then match the following :



Column-1	**Column-2**
(A) Temperature of junction B	(P) Greater than 50ºC
(B) Heat current in AB	(Q) Less than 50ºC
(C) Heat current in BC	(R) Is equal to heat current in BC
	(S) is 2/3 times heat current in AC
	(T) None

Ans. **A $\rightarrow$ P ; B $\rightarrow$ R ; C $\rightarrow$ T**

Q.4



A body at initial temperature T_0 is kept in a uniform temperature enclosure at temperature T_S. ($T_S > T_0$)

Column I	**Column II**
(A) Net Rate of energy loss initially of radiation energy by enclosme depends on	(P) $T_0^4 - T_S^4$
(B) Net Rate of energy loss by body initially depends on	(Q) $T_S^4 - T_0^4$
(C) In thermal equilibrium Rate of energy emitted by body depends on	(R) T_S^4
(D) In thermal equilibrium Rate of energy absorbed by enclosure depends on.	(S) T_0^4

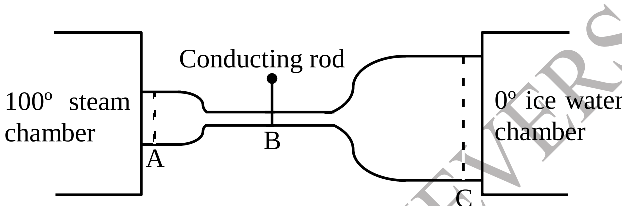
(A) $\rightarrow$ Q **(B)** $\rightarrow$ P **(C)** $\rightarrow$ R **(D)** $\rightarrow$ R

Q.5

Column I	**Column II**
(A) Steady state	(P) A blackened platinum wire when gradually heated appear first red and then blue
(B) Wein's displacement Law	(Q) Radiated power is proportional to fourth power of absolute temperature of body
(C) Stefan's Law	(R) Energy absorbed is equal to energy emitted
(D) Black Body	(S) Absorptive power of body is unity

(A) $\rightarrow$ R ; **(B)** $\rightarrow$ P ; **(C)** $\rightarrow$ Q ; **(D)** $\rightarrow$ P,S

Q.6 A copper rod (initially at room temperature 20ºC) of non-uniform cross section is placed between a steam chamber at 100ºC and ice-water chamber at 0ºC.



Column-I	**Column-II**
(A) Initially rate of heat flow $\left[\frac{dQ}{dt}\right]$ will be	(P) maximum at section A
(B) At steady state rate of heat flow $\left[\frac{dQ}{dt}\right]$ will be	(Q) maximum at section B
(C) At steady state temperature gradient $\left\|\frac{dT}{dx}\right\|$ will be	(R) minimum at section C
(D) At steady state rate of change of temperature $\left[\frac{dT}{dx}\right]$ at a certain point will be	(S) same for all section

Sol. A $\rightarrow$ P,R ; B $\rightarrow$ Q,R ; C $\rightarrow$ S; D $\rightarrow$ S

(A) Initially more heat will enter through section A but the metal will absorb same heat and less heat will leave from C.

(B) At steady state heat accumulation =

$\Rightarrow \frac{dQ}{dt}$ is same for all sections

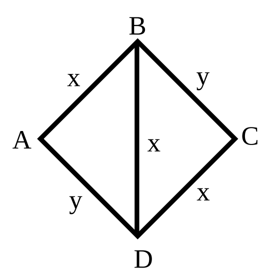
(C) At steady state $\frac{dQ}{dt} = kA\left|\frac{dT}{dx}\right|$ or $\left|\frac{dT}{dx}\right|$

$= \frac{1}{kA}\left[\frac{dQ}{dt}\right] \Rightarrow \left|\frac{dT}{dx}\right|$ is inversely proportional to area of cross section. Hence it is maximum at B and minimum at C.

(D) At steady state heat accumulation = 0

So $\frac{dT}{dt} = 0$ for any section.

Q.7 Three rods of material x and two rods of material y are connected as shown in figure.



All the rods are of identical length and cross-sectional area. The end A is maintained at 100ºC and the junction C at 0ºC. It is given that resistance of rod of material x is R. Further, $K_x = 2K_y$. Match the entries of Column I and II

Column I	Column II
(A) Temperature of junction B	(P) 3R/5

(B) Temperature of junction D (Q) 7R/5

(C) Thermal resistance between (R) 400/7

ºC

B and D

(D) Thermal resistance between (S) 300/7 ºC

A and C

Sol. (A) $\rightarrow$ (R) ; (B) $\rightarrow$ (S) ; (C) $\rightarrow$ (P) ; (D) $\rightarrow$ (Q)

Q.1 A spherical shell has inner and outer radius R_1 and R_2 respectively. Inner surface is maintained at temperature $5T_0$ while outer surface is maintained at temperature T_0. Thermal conductivity is K, then –

(A) Temperature gradient decrease in radially outward direction

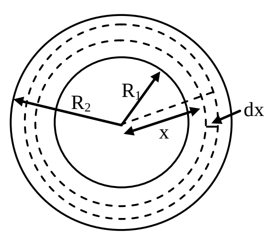
(B) Isothermal surface will be spherical

(C) Temperature of mid-point of shell $\dfrac{5R_2 + R_1}{R_1 + R_2}T_0$

(D) Heat flown in time Δt is $\dfrac{4\pi(R_2 - R_1)KT_0}{R_1R_2} . \Delta t$

[A,B,C,D]

Sol.



$$dR = \frac{1}{K}\frac{dx}{4\pi x^2} \qquad \text{... (i)}$$

$\therefore$ Thermal resistance of shell of inner radius 'R_1' and outer radius 'x' is

$$R = \frac{1}{4\pi K}\left\{\frac{1}{R_1} - \frac{1}{x}\right\} \qquad \text{... (ii)}$$

From (i) $\dfrac{dR}{dx} \propto \dfrac{1}{x^2}$

$\therefore$ Temperature gradient decrease with increase in x

$$\Rightarrow \frac{dT}{dx} \propto \frac{1}{x^2}$$

From (ii) resistance of shell from inner to the mid-point

$$R_{inner} = \frac{1}{4\pi K}\left(\frac{R_2 - R_1}{R_1(R_1 + R_2)}\right)$$

and that of shell from mid-point to outer

$$R_{outer} = \frac{1}{4\pi K}\left(\frac{R_2 - R_1}{R_1(R_1 + R_2)}\right)$$

Temperature of mid-point is given by,

$$\frac{5T_0 - T}{R_{inner}} = \frac{T - T_0}{R_{outer}} \quad \Rightarrow T = \frac{5R_2 + R_1}{R_1 + R_2}.T_0$$

Thermal resistivity of shell is

$$R = \frac{1}{4\pi K}\left\{\frac{R_1R_2}{R_2 - R_1}\right\}$$

$\therefore$ Heat flown in time Δt

$$\Delta Q = \frac{5T_0 - T_0}{R}.\Delta t$$

$$= \frac{4\pi(R_2 - R_1)KT_0}{R_1R_2}.\Delta t$$

Q.2 For transmission of heat from one place to the other, medium is required in–

(A) conduction (B) convection

(C) radiation (D) all **[A,B]**

Q.3 In steady state–

(A) Temperature does not change with time

(B) There is no absorption of heat

(C) There is no flow of heat

(D) All parts of the body are at same temp.

[A,B]

Q. 4

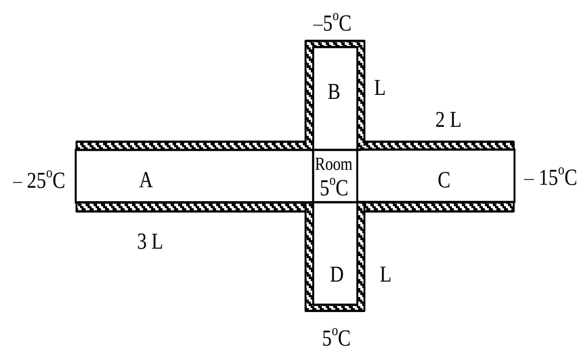


Figure shows a horizontal cross section (top view) of a square room surrounded on four sides by thick walls. The walls are all made of the same material and all have the same face area. They have thickness of either L, 2L or 3L as shown, and they are maintained at 5°C, and the conduction of energy outward through the walls is steady. H be heat current through walls. $\left(\dfrac{\Delta T}{\Delta x}\right)$ be the temperature gradient across the wall, then

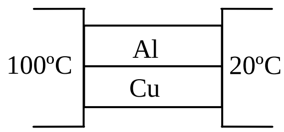
(A) $\left(\dfrac{\Delta T}{\Delta x}\right)_A = \left(\dfrac{\Delta T}{\Delta x}\right)_B = \left(\dfrac{\Delta T}{\Delta x}\right)_C > \left(\dfrac{\Delta T}{\Delta x}\right)_D$

(B) $H_A = H_B = H_C > H_D$

(C) $\left(\frac{\Delta T}{\Delta x}\right)_A < \left(\frac{\Delta T}{\Delta x}\right)_B < \left(\frac{\Delta T}{\Delta x}\right)_C = \left(\frac{\Delta T}{\Delta x}\right)_D$

(D) $H_A = H_B < H_C > H_D$ **[A,B]**

Q. 5 Two metal cubes with 3 cm edges of copper and aluminium are arranged as shown. Thermal conductivities of copper and aluminium are 401 W/mK and 237 W/mK.



(A) Thermal resistance of aluminium is 0.05 K/W

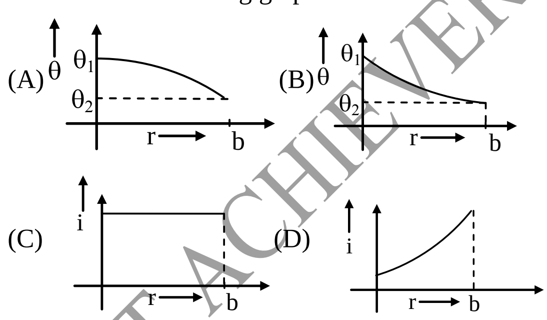
(B) Equivalent thermal resistance is 0.05 K/W

(C) Heat current is 1.6×10^3 W

(D) Thermal resistance of copper is 0.08 K/W

[A,C,D]

Q.6 Radius of rod changes linearly from a to b (a > b). Temperature of the two ends are maintained at θ_1 and θ_2 ($\theta_1 > \theta_2$) respectively. Let 'i' be the heat passing per unit cross sectional area of rod and 'θ' be temperature at a distance 'r' from one end (having cross section radius 'a'), then which of the following graphs is/are correct -



[A,C]

Sol. Temperature gradient at distance 'r'

$$\frac{d\theta}{dr} = \frac{1}{K} \times \frac{1}{\pi x^2}$$

[Where x = cross-sectional radius at distance 'r']

$\Rightarrow$ Temperature increases with increase in 'r'.

Q.7 When two bodies at different temperature are kept in contact. Net heat flow takes place between them till -

(A) Thermal equilibrium has reached

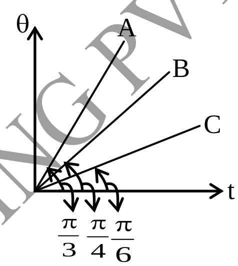
(B) Temperature of both the bodies become same

(C) Internal energies of both the bodies become equal

(D) None of these **[A, B]**

Sol. conceptual

Q.8 Three bodies A, B, C of masses m, m and $\sqrt{3}$ m respectively are supplied heat at a constant rate. The change in temperature (θ) versus time graph for A, B, C is as shown. If there specific heats are s_A, s_B, and s_C, then which of the following can not be correct ? (Initial temperature of bodies are 0ºC)



(A) $s_A > s_B > s_C$ (B) $s_B = s_C < s_A$

(C) $s_A = s_B = s_C$ (D) $s_B = s_C > s_A$

[A,B,C]

Sol. $\frac{dH}{dt} = ms\frac{dT}{dt}$ = constant

slope of graph $\left(\frac{dT}{dt}\right) = \frac{\text{constant}}{\text{mass} \times \text{specific heat}}$

Q.9 The ends of a rod of uniform thermal conductivity are maintained at different (constant) temperatures. After the steady state is achieved -

(A) heat flows in the rod from high temperature to low temperature even if the rod has non uniform cross sectional area

(B) temperature gradient along length is same even if the rod has non uniform cross sectional area

(C) heat current is same even if the rod has non uniform cross sectional area

(D) if the rod has uniform cross sectional area the temperature is same at all points of the end.

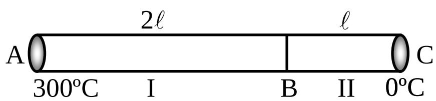
[A,C]

Sol. Heat obviously flows from higher temperature to lower temperature in steady state. $\Rightarrow$ A is true.

Temperature gradient $\propto \frac{1}{\text{cross section area}}$ in steady state $\Rightarrow$ B is false.

Thermal current through each cross section area is same ⇒ C is true. Temperature decreases along the length of the rod from higher temperature end to lower temperature end ⇒ D is false.

Q.10 Two conducting rods of the same cross-section are connected end to end, while the temperature at A and C are maintained at θ_A = 300ºC and θ_C = 0ºC, respectively. There is no loss of heat from the sides of the rods. Let θ_B be the temperature of function B -



(A) if θ_B = 200ºC in the steady state, the conductivities of the rods are equal

(B) if θ_B < 50ºC in the steady state, $K_I < K_{II}$

(C) if θ_B > 200ºC in the steady state, $K_I > K_{II}$

(D) if θ_B = 100ºC in the steady state, $K_I < K_{II}$

Here K_I, K_{II} denote the thermal conductivities of the rods I and II respectively.

Sol. **[B,C]**

The temperature at B is such that

$$K_I \frac{(300-\theta_B)}{2\ell} = \frac{K_{II}(\theta_B - 0)}{\ell} \; ;$$

wherefrom, the ratio $\frac{K_I}{K_{II}}$ can be found.

Q.11 The two ends of a uniform rod of thermal conductivity k are maintained at different but constant temperature. The temperature gradient at any point on the rod is $\frac{d\theta}{d\ell}$. The heat flow per unit time per unit cross-section of the rod is I.

(A) $\frac{d\theta}{d\ell}$ is the same for all points on the rod

(B) I will decrease as we move from higher to lower temperature

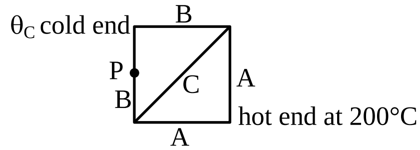
(C) $I = k.\frac{d\theta}{d\ell}$

(D) All the above options are incorrect

Sol. **[A,C]**

Use law of conduction $dq = \frac{kA(\Delta\theta)\,dt}{\ell}$

Q.12 A, B & C are three types of thermally conducting rods making a square frame with a diagonal as shown. All rods have identical cross-section but their thermal conductivies are in the ratio A : B : C = 1 : 2 : 4. The hot end and cold end are maintained at 200°C & θ_C respectively. Temperature at mid point of rod C is 160°C. Point P is the mid point of rod B as shown -



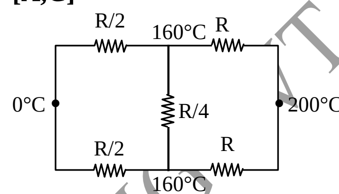
(A) θ_C in °C is 140°C

(B) θ_C in °C is 120°C

(C) temp. in centigrade at point P is 150°C

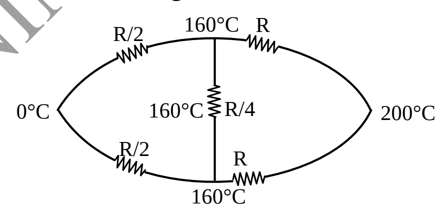
(D) temp. in centigrade at point P is 120°C

Sol. **[A,C]**



$$R = \frac{L}{KA}$$

using wheat stone bridge



$$\frac{200-160}{R} = \frac{160-\theta_C}{R/2}$$

$$\frac{40}{2} = 160 - \theta_C$$

θ_C = 160 – 20 = 140°C

$$\theta_P = \frac{\theta_C + 160}{2} = \frac{140+160}{2} = 150°C$$

Q.13 The heat capacity of a body depends upon -

(A) Heat given

(B) Temperature raised

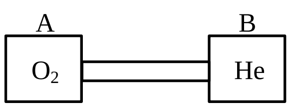
(C) Mass of the body

(D) Material of the body

Sol.[C, D]

PHYSICS

Q.1 Two containers A and B are connected by a conducting solid cylindrical rod of length $\frac{242}{7}$ cm and radius $\sqrt{8.3}$ cm. Thermal conductivity of the rod is 693 watt/mole-K. The container A contains two mole of oxygen gas and the container B contains four mole of helium gas. At time t = 0 temperature difference of the containers is 50ºC, after what time (in seconds) temperature difference between them will be 25ºC. Transfer of heat takes place through the rod only. Neglect radiation loss. Take R = 8.3 J/mole-K and $\pi = \frac{22}{7}$.



Sol. **[3]**

$R = \frac{\ell}{KA}$

When heat is transferred from first vessel to second, temperature of first vessel decreases while that of second vessel increases. Due to both there reasons, difference between temperature of vessels decreases.

Let at an instant t, the temperature difference between two vessels be θ.

$$H = \frac{\theta}{R} = \frac{KA\theta}{\ell}$$

$$dQ = Hdt = \frac{KA\theta}{\ell}dt \qquad \text{….(i)}$$

Since gases are contained in two vessels, therefore, processes on gases in two vessels are isochoric.

Hence, decrease in temperature of gas in first vessel,

$$\Delta\theta_1 = \frac{dQ}{nC_v} = \frac{dQ}{2\times\frac{5R}{2}} = \frac{dQ}{5R}$$

Increase in temperature of gas in second vessel is

$$\Delta\theta_2 = \frac{dQ}{4\times\frac{3R}{2}} = \frac{dQ}{6R}$$

$\therefore$ Decrease in temperature difference

$(-d\theta) = \Delta\theta_1 + \Delta\theta_2$

$$-d\theta = \frac{dQ}{R}\times\frac{11}{30}$$

$$\text{or } -\int_{50}^{25}\frac{d\theta}{\theta} = \frac{KA\times 11}{30\ell R}\int_0^t dt$$

t = 3 seconds.

2. An electric heater is used in a room of total wall area 137 m^2 to maintain a temperature of +20ºC inside it, when the outside temperature is –10ºC. The walls have three different layers materials. The innermost layer is of wood of thickness 2.5 cm, the middle layer is of cement of thickness 1.0 cm and the outermost layer is brick 25.0 cm. Find the power of the electric heater. Assume that there is no heat loss through the floor and the ceiling. The thermal conductivities of wood, cement and brick are 0.125, 1.5 and 1.0 watt/m/ºC respectively.

Sol.[9000] $$K = \frac{d_1 + d_2 + d_3}{\frac{d_1}{K_1} + \frac{d_2}{K_2} + \frac{d_3}{K_3}}$$

Rate of flow of heat is

$$\frac{Q}{t} = KA\frac{\theta_1 - \theta_2}{(d_1 + d_2 + d_3)} = 9000\text{ W}$$

Q.3 For a temperature difference ΔT = 20ºC one slab of material conducts 10.0 w/m^2 another of the same shape conducts 20.0 w/m^2. What is the rate of heat flow per m^2 of surface area when the slabs are placed side by side wth ΔT_{tot} = 20.0ºC. Answer should be given in single digit after rounding off.

Sol.[7] 7 W/m^2

Q.1

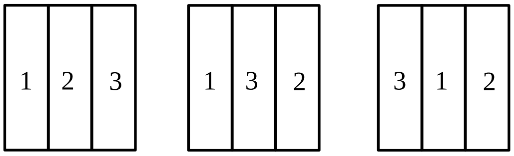


Fig.(a) Fig.(b) Fig.(c)

Figure (a), (b), (c) shows three different arrangements of materials 1, 2 and 3 to form a wall. Thermal conductivities are $K_1 > K_2 > K_3$. The left side of the wall is 20ºC higher than the right side. Temperature difference ΔT across the material 1 has following relation, in three cases:

(A) $\Delta T_a > \Delta T_b > \Delta T_c$

(B) $\Delta T_a = \Delta T_b = \Delta T_c$

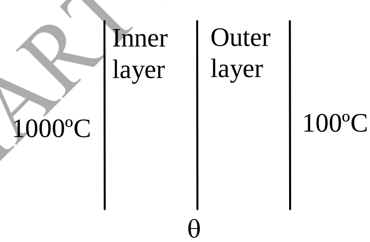
(C) $\Delta T_a = \Delta T_b > \Delta T_c$

(D) $\Delta T_a = \Delta T_b < \Delta T_c$

Sol. **[B]**

All are in series therefore current remains same. Hence temperature difference = (current × thermal resistance) are equal for every case.

Q.2 The temperature drop through a two layer furnace wall is 900ºC. Each layer is of equal area of cross-section. Which of the following actions will result in lowering the temperature θ of the interface ?



(A) By increasing the thermal conductivity of outer layer

(B) By increasing the thermal conductivity of inner layer

(C) By increasing thickness of outer layer

(D) By decreasing thickness of inner layer

Sol. **[A]**

H = rate of heat flow

$$= \frac{900}{\frac{\ell_i}{K_iA} + \frac{\ell_0}{K_0A}}$$

Now $100 - \theta = \frac{H\ell_i}{K_iA}$

or $\theta = 1000 - \left[\frac{900}{\frac{\ell_i}{K_iA} + \frac{\ell_0}{K_0A}}\right]\frac{\ell_i}{K_iA}$

$$= 100 - \frac{900}{1 + \frac{\ell_0}{K_0}\frac{K_i}{\ell_i}}$$

Now, we can see that θ can be decreased by increasing thermal conductivity of outer layer (K_0) and thickness of inner layer (ℓ_i).

Q.3 A students performs cooling experiment with a solid sphere and hollow sphere of same material and size which are heated to the same temperature. If the temperature difference between each sphere and surroundings is around 30°C , then -

(A) The hollow sphere will cool at a faster rate

(B) The solid sphere will cool at a faster rate

(C) Both spheres will cool at the same rate

(D) Both spheres will cool at the same rate if temp. difference more than 30°C

[A]

Sol. Rate of cooling $\frac{d\theta}{dt} \propto \frac{dQ}{ms}$

dQ is same so $\frac{d\theta}{dt} \propto \frac{1}{m}$

$m_{solid} > m_{hollow}$

hence hollow sphere will cool fast.

Q.4 The ends of the two rods of different materials with their lengths, diameters of cross-section and thermal conductivities all in the ratio 1:2 are maintained at the same temperature difference. The rate of flow of heat in the shorter rod is 1 cal s^{-1}. What is the rate of flow of heat in the larger rod ?

(A) 1 cal s^{-1} (B) 4 cal s^{-1}

(C) 8 cal s^{-1} (D) 16 cal s^{-1} **[B]**

Sol. $\left(\frac{Q}{t}\right)_1 = \frac{KA(\theta_1-\theta_2)}{d} = 1 \text{ cal s}^{-1}$

$\left(\frac{Q}{t}\right)_2 = \frac{2k(4A)(\theta_1-\theta_2)}{2d}$

$= \frac{4kA(\theta_1-\theta_2)}{d}$

$= 4 \text{ cal s}^{-1}$

Q.5 Two metallic spheres P and Q of the same surface area are taken. The weight of P is twice that of Q. Both the spheres are heated to the same temperature and left in a room to cool by radiation. The ratio of the rate of cooling of Q to P is :

(A) $\sqrt{2} : 1$ (B) 2 : 1

(C) 1 : 2 (D) $1 : (2)^{1/3}$

Sol. **[D]**

P and Q have same surface finish and same temperature difference. Hence rate of radiation will depend only on the surface area and mass. That is, rate of radiation will be proportional to mass per unit area $m/4\pi r^2$, i.e., proportional to radius r. Since the mass of P is twice that of Q, $r_P^3 = 2r_Q^3$.

$r_P = (2)^{1/3} r_Q$. where r_P and r_Q are the radii of spheres of P and Q respectively. Hence ratio of rates of cooling is $1 : 2^{1/3}$

Q.6 In preparation for a landing on the bright side of the moon, surface temperature of moon has to be estimated. Assume Lunar surface material is a good insulator. It is given that solar constant is 1353 watts/m^2.

Assume absorptivity and emissivity of moon is same. Approximate surface temperature of moon is

(stefan's constant = 5.67×10^{-8} Wm^{-2}K^{-4}) -

(A) 120ºC (B) 300ºC

(C) 500ºC (D) 720ºC

Sol. **[A]**

At equilibrium

Rate of emission = Rete absorption

$e\sigma T^4 = aI_0$

…(i)

where e = emissivity of moon

a = absorptivity of moon

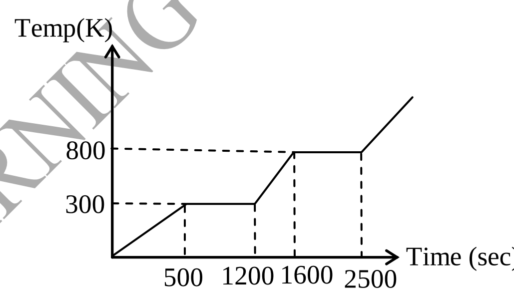
I_0 = solar constant.

given that e = a

$\sigma T^4 = I_0$

$T = \left(\frac{I_0}{\sigma}\right)^{1/4} = \left(\frac{1353}{5.67\times10^{-8}}\right)^{1/4}$ = 393 K = 120º C

Q.7 Temperature variation with time is plotted for an object as shown in figure. The mass of the object is 200 g. Heat is supplied to the object at constant rate of 1 KW. Specific heat of object in liquid phase is -



(A) 3000 J/kg-K (B) 1000 J/kg-K

(C) 4000 J/kg-K (D) 2000 J/kg-K

Sol. **[C]**

$H = \frac{dQ}{dt} = mcdT/dt$

$\frac{H}{mdT/dt} = C = \frac{1\times10^3}{0.2\times\frac{500}{400}} = \frac{10^4\times2}{5}$

$C = 0.4 \times 10^4$

$C = 4000 \frac{J}{Kg\,K}$.

Q.8 Consider a black sphere of radius R at temperature T which radiates to distant black surroundings at T = 0 K. The sphere is surrounded by nearby heat shield in the form of black shell whose temperature is determined by radiative equilibrium -

(A) The temperature of the shell is $\frac{T}{\sqrt{2}}$

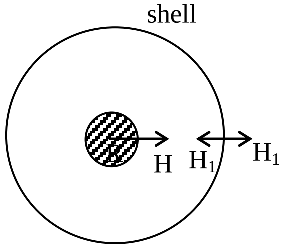
(B) The temperature shell is $\frac{T}{(2)^{1/4}}$

(C) Total power radiated to the surroundings remains the same

(D) Total power radiated to the surroundings is reduces to one fourth of the initial value

[B]

Sol.



Rate of energy absorbed by shell = Rate of energy radiated by shell

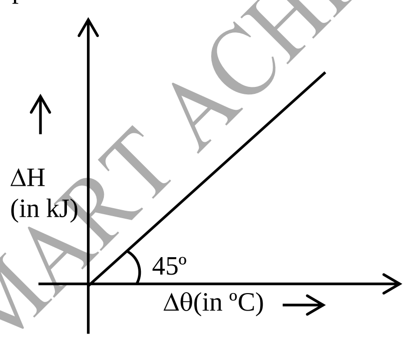
$H - H_1 = H_1$

$H_1 = \frac{H}{2}$; power radiated to the surroundings

$T_1^4 = \frac{T^4}{2}$

$T_1 = \frac{T}{(2)^{1/4}}$

Q.9 A solid of mass 2kg is heated and ΔH (Heat given) vs Δθ (change in temperature) is plotted. Specific heat of solid is –



(A) 1 J/kg/°C (B) 0.5 J/kg/°C

(C) 2 kJ/kg/°C (D) 0.5 kJ/kg/°C **[D]**

Sol. Slope of ΔH vs Δθ graphs give 'heat capacity'.

$\therefore$ Specific heat capacity = $\frac{\text{heat capacity}}{\text{mass}}$

$= \frac{\tan 45^o}{2}$ kJ/kg/°C

= 0.5 kJ/kg/°C

Q.10 Mechanism of heat-transfer involved in freezing of lakes in colder region -

(A) Conduction only (B) Convection only

(C) Radiation only (D) None of these

[D]

Sol. Water of lake gets cooled by conduction till 4°C after that it is cooled by conduction through ice.

Q.11 A cylinder of radius R made of material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius 3R made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperature. What is the effective thermal conductivity of the system ?

(A) $K_1 + K_2$ (B) $\frac{K_1 + 8K_2}{9}$

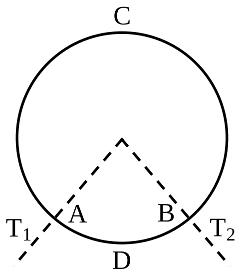
(C) $\frac{K_1K_2}{K_1 + K_2}$ (D) $\frac{8K_1 + K_2}{9}$ **[B]**

Sol. $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

$$\frac{K_{eq}(9\pi R^2)}{L} = \frac{K_1\pi R^2}{L} + \frac{K_2 8\pi R^2}{L}$$

$$K_{eq} = \frac{K_1 + 8K_2}{9}$$

Q.12 A ring consisting of two parts ADB and ACB of same conductivity K carries an amount of heat H. The ADB part is now replaced with another metal keeping the temperatures T_1 and T_2 constant. The heat carried increases to 2H. What should be the conductivity of the new ADB part ? (Given $\frac{ACB}{ADB} = 3$)



(A) $\frac{7}{3}$ K (B) 2 K

(C) $\frac{5}{2}$ K (D) 3 K **[A]**

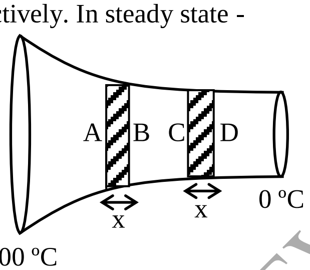
Sol. $H_1 + H_2 = \frac{KA(T_1 - T_2)}{3l} + \frac{KA(T_1 - T_2)}{l} = \frac{4}{3l}$ KA $(T_1 - T_2)$

In later case

$H_2 = 2H - H_1 = \frac{7KA}{3l}(T_1 - T_2) = \frac{K'A}{l}(T_1 - T_2)$

$\Rightarrow K' = \frac{7}{3}$ K

Q.13 Two ends of a conducting rod of varying cross-sections are maintained at 200 ºC and 0 ºC respectively. In steady state -



(A) Temperature difference across AB and CD are equal

(B) Temperature difference across AB is greater than that of across CD

(C) Temperature difference across AB is less than that of across CD

(D) Temperature difference may be equal or different depending on the thermal conductivity of the rod

Sol. **[C]**

Sol. Rate of flow of heat $\frac{dQ}{dt}$ or H is equal throughout the rod.

Temperature difference = (H) (thermal resistance)

or Temperature difference $\propto$ thermal resistance (R)

where $R = \frac{l}{KA}$ or $R \propto \frac{1}{A}$

Area across CD is less. Therefore, temperature difference across CD will be more.

Q.14 A cylindrical tube of diameter 3 m and length 20 m is lined with 3 cm of insulating material of conductivity 10^{-4} cal cm^{-1} s^{-10} C^{-1}. It is maintained at 20^oC, although the outer temperature is -30^oC. What is the rate of heating required to maintain the inner temperature –

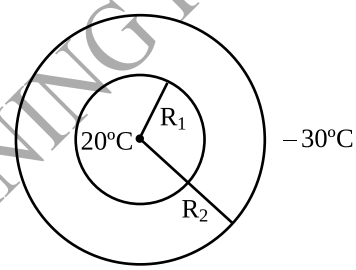


Fig.

(A) 1298 J/s (B) 12980 J/s

(C) 12098 J/s (D) 3100 J/s **[B]**

Sol. R = 1.5 m, length = 20 m.

Consider a cylinder of radius = R and thickness = dR.

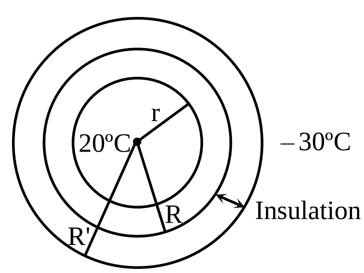


Fig.

Rate of flow of heat

$Q = KA\frac{dt}{dR}$

dt = temp difference in thickness dR.

$A = 2\pi R\ell$, $Q = K(2\pi R)\frac{\ell dt}{dR}$

The rate of flow of heat kept constant to maintain temperature inside.

or $\frac{dR}{R} = \frac{k\,2\pi\ell}{Q}\,dt$

$$\int_R^{R'} \frac{dR}{R} = \frac{k\,2\pi\ell}{Q}\int_{t_1}^{t_2} dt$$

$$[\log R]_R^{R'} = \frac{k\,2\pi\ell}{Q}(t_2 - t_1)$$

$$Q = \frac{k\,2\pi\ell(20-(-30))}{\log\frac{1.53}{1.50}}$$

Q = 3100 cal/sec.

Q = 12980 J/s.

Q.15 A rod of length ℓ and cross section area A has a variable thermal conductivity given by K = α T, where α is a positive constant and T is temperature in Kelvin. Two ends of the rod are maintained at temperatures T_1 and T_2 ($T_1 > T_2$). Heat current flowing through the rod will be -

(A) $\frac{A\alpha(T_1^2 - T_2^2)}{3\ell}$ (B) $\frac{A\alpha(T_1^2 + T_2^2)}{\ell}$

(C) $\frac{A\alpha(T_1^2 + T_2^2)}{3\ell}$ (D) $\frac{A\alpha(T_1^2 - T_2^2)}{2\ell}$ **[D]**

Sol. Heat current $i = -KA\frac{dT}{dX}$

$$idX = -KA\,dT$$

$$i\int_0^{\ell} dX = -A\alpha\int_{T_1}^{T_2} T\,dT$$

$$\Rightarrow \quad i\ell = -A\,\alpha\frac{(T_2^2 - T_1^2)}{2}$$

$$\Rightarrow \quad i = A\,\alpha\,\frac{(T_1^2 - T_2^2)}{2\ell}$$

Q.16 A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius 2R made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is –

(A) $K_1 + K_2$ (B) $(K_1 + 3K_2)/4$

(C) $K_1K_2/(K_1 + K_2)$ (D) $(3K_1 + K_2)/4$ **[B]**

Sol. Parallel combination of cross-section area

πR^2 and $\pi[(2R)^2 - R^2] = 3\pi R^2$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \text{ with } R = \frac{L}{KA}$$

$$\frac{K_{eq}.4\pi R^2}{L} = \frac{K_1\pi R^2}{L} + \frac{K_2(3\pi R^2)}{L}$$

$$\Rightarrow K_{eq} = \frac{K_1 + 3K_2}{4}$$

Q.17 A steel ball of mass m_1 = 1 kg moving with velocity 50 m/sec collides with another steel ball of mass m_2 = 200 gm lying on the ground and both come to rest. During the collision their internal energies changes equally and T_1 and T_2 are the rise in temperature of masses m_1 and m_2 respectively. If s_{steel} = 0.105 cal/gm ºC and J = 4.18, then -

(A) T_1 = 7.1ºC, T_2 = 1.47ºC

(B) T_1 = 1.42ºC, T_2 = 7.1ºC

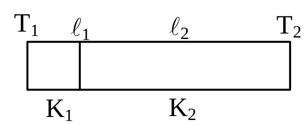
(C) T_1 = 3.4 K, T_2 = 17.0 K

(D) None of these **[B]**

Sol. Half of KE is shared by each ball

$$\frac{1}{2}\text{KE} = m_1s_1T_1 = m_2s_2T_2$$

Q.18 One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed of two sections of length ℓ_1 and ℓ_2 and thermal conductivities K_1 and K_2 respectively. The temperature at the interface of the two section is –



(A) $\frac{(K_1\ell_1T_1 + K_2\ell_2T_2)}{(K_1\ell_1 + K_2\ell_2)}$

(B) $\frac{(K_2\ell_2T_1 + K_1\ell_1T_2)}{(K_1\ell_1 + K_2\ell_2)}$

(C) $\frac{(K_2\ell_1T_1 + K_1\ell_2T_2)}{(K_2\ell_1 + K_1\ell_2)}$

(D) $\frac{(K_1\ell_2T_1 + K_2\ell_1T_2)}{(K_1\ell_2 + K_2\ell_1)}$ **[D]**

Sol. $H_1 = H_2 \therefore \dfrac{k_1A[T_1-T]}{\ell_1} = \dfrac{k_2A[T-T_1]}{\ell_2}$

Q.19 The diameter of a rod is given by $d = d_0(1 + ax)$ where 'a' is a constant and x is distance from one end. If thermal conductivity of material is K. Then the thermal resistance of the rod if its length is ℓ is -

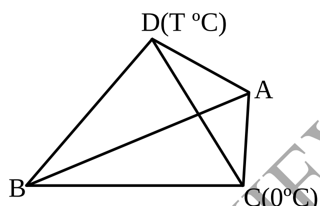
(A) $\dfrac{1}{K\pi d_0^2}$ (B) $\dfrac{4\ell}{K\pi d_0^2(a\ell+1)}$

(C) $\dfrac{2\ell}{K\pi d_0^2(a\ell+1)^2}$ (D) $\dfrac{2\ell}{K\pi d_0^2(a\ell+1)}$ **[B]**

Sol. $dR = \dfrac{dx}{K\dfrac{\pi d^2}{4}} = \dfrac{4dx}{K\pi d_0^2(1+ax)^2}$

$\therefore R = \int dR$

Q.20 Six similar bars each of thermal resistance R are joined to form a regular tetrahedron as shown in the figure. Point D is maintained at a constant temperature T ºC and point C at 0ºC. Temperature of junction A is -



(A) $\dfrac{T}{4}$ (B) $\dfrac{T}{3}$ (C) $\dfrac{T}{2}$ (D) $\dfrac{T}{5}$

Sol. **[C]**

Temperature of junction A is T_A

$\dfrac{KA(T-T_A)}{\ell} = \dfrac{KA(T_A-0)}{\ell}$

$\Rightarrow T - T_A = T_A$

$\Rightarrow T_A = \dfrac{T}{2}$

Q.21 Two rods P and Q of same length and same diameter having thermal conductivity ratio 2 : 3 joined end to end. If temperature at one end of P is 100ºC and at one end of Q 0ºC, then the temperature of the interface is -

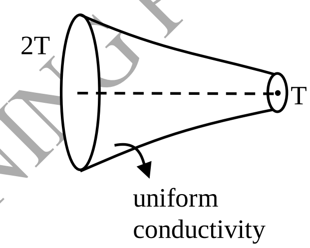
(A) 40ºC (B) 50ºC

(C) 60ºC (D) 70ºC **[A]**

Sol. $\dfrac{K_pA[100-\theta]}{\ell} = \dfrac{K_Q.A[\theta]}{\ell}$

$\therefore \dfrac{K_p}{K_Q} = \dfrac{2}{3} = \dfrac{\theta}{100-\theta}$ OR (θ = 40ºC)

Q.22 In the given conical distorted shape of rod heat is being conducted under steady state. The two ends are maintained at different temperature. Now choose the correct alternative –



(A) The rate of heat flow will not be constant through the rod

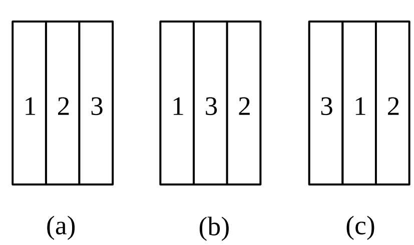
(B) The magnitude of temperature gradient increase from left to right

(C) The temperature at mid point will be $\dfrac{3T}{2}$

(D) The temperature at mid point will be less than $\dfrac{3T}{2}$ **[B]**

Sol. $\dfrac{d\theta}{dt} = KA\left(\dfrac{-dT}{dx}\right)$ so $\left(-\dfrac{dT}{dx}\right) \propto \dfrac{1}{A}$

Q.23 Following figure shows three different arrangements of materials 1, 2 and 3 to form a wall, thermal conductivities are $K_1 > K_2 > K_3$. The left side of wall is 20ºC higher than the right side. Temperature difference ΔT across the material 1 has following relation, in three cases–



(A) $\Delta T_a > \Delta T_b > \Delta T_c$ (B) $\Delta T_a = \Delta T_b = \Delta T_c$
(C) $\Delta T_a = \Delta T_b > \Delta T_c$ (D) $\Delta T_a = \Delta T_b < \Delta T_c$ **[B]**

Sol. Heat current is same in all cases and resistance of material remain same in all cases.

Q.24 Two rods of same length and transfer a given amount of heat 12 second, when they are joined as shown in figure (i). But when they are joined as shown in figure (ii), then they will transfer same heat in same conditions in

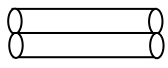


Figure (i)

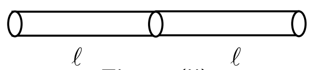
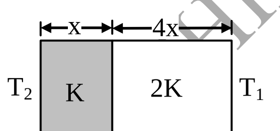


Figure (ii)

(A) 24 s (B) 13 s
(C) 15 s (D) 48 s **[D]**

Sol. $t \propto \dfrac{\ell}{A}$, $t' \propto \dfrac{2\ell}{A/2}$

Q.25 The temperature of the two outer surface of a composite slab consisting of two materials having coefficient of thermal conductivity K and 2K and thickness x and 4x respectively are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab, in steady state is $\left(\dfrac{A(T_2 - T_1)K}{x}\right).f$ with f equal to –



(A) 1 (B) 1/2
(C) 2/3 (D) 1/3 **[D]**

Sol.

Series Req = $R_1 + R_2 = \dfrac{x}{KA} + \dfrac{4x}{2KA} = \dfrac{3x}{KA}$

Rate of heat $\dfrac{dQ}{dt} = \dfrac{T_2 - T_1}{R_{eq}} = \dfrac{T_2 - T_1}{3x}$ KA

given $\dfrac{dQ}{dt} = \dfrac{A[T_2 - T_1]K}{x} f = \dfrac{T_2 - T_1}{3x}$ KA

$f = \dfrac{1}{3}$

Q.26 A slab consists of two parallel layers of copper and brass of the same thickness and having thermal conductivities in the ratio 1 : 4. If the free face of brass is at 100ºC and that of copper at 0ºC, the temperature of interface is -
(A) 80ºC (B) 20ºC
(C) 60ºC (D) 40ºC **[A]**

Sol. $4K(100 - \theta) = K(\theta - 0)$
$\Rightarrow 400 - 4\theta = \theta$
$\Rightarrow \theta = 80$ºC

Q.27 Two solid spheres, of radii R_1 and R_2 are made of the same material and have similar surfaces. The spheres are raised to the same temperature and then allowed to cool under identical conditions. Assuming spheres to be perfect conductors of heat, their initial rates of loss of heat are -

(A) R_1^2/R_2^2 (B) R_1/R_2
(C) R_2/R_1 (D) R_2^2/R_1^2 **[A]**

Sol. Rate of hoss of heat

$= \dfrac{\sigma\pi^4 \times A_1 \times e_1}{\sigma\pi^4 \times A_2 \times e_2} = \dfrac{R_1^2}{R_2^2}$ [$\therefore e_1 = e_2$]

Q.28 Two identical rods of copper and iron are coated with wax uniformly. When one end of each is kept at temperature of boiling water, the length upto which wax melts are 8.4 cm and 4.2 cm, respectively. If thermal conductivity of copper is 0.92, then thermal conductivity of iron is -
(A) 0.23 (B) 0.46
(C) 0.115 (D) 0.69 **[A]**

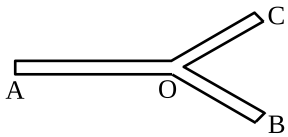
Sol. $\dfrac{K_1}{K_2} = \dfrac{\ell_1^2}{\ell_2^2}$

Q.29 Two vessels of different materials are similar in size in every respect. The same quantity of ice filled in them gets melted in 20 min and 35 min, respectively. The ratio of coefficients of thermal conduction of the metals is -
(A) 4 : 7 (B) 7 : 4
(C) 25 : 16 (D) 16 : 25 **[B]**

Sol. $Q_1 = k_1 A \dfrac{\Delta t}{L} . t_1 = K_2 A \dfrac{\Delta t}{L} . t_2$

$\therefore \dfrac{k_1}{k_2} = \dfrac{t_2}{t_1} = \dfrac{7}{4}$

Q.30 Three rods of same material & thickness with $BO = CO = \frac{AO}{2}$. End A & B are maintained at 10°C & 100°C and temperature of C is varied from 65°C to 75°C extremely slowly. Then the correct options is -



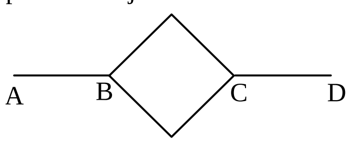
(A) heat always flows from O to C

(B) heat always flows from C to O

(C) heat first flows from O to C & then from C to O (D) heat first flows from C to O & then from O to C **[C]**

Sol. At 70°C there will be equilibrium.

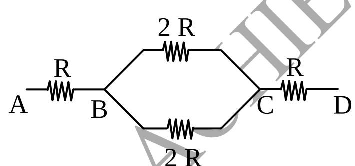
Q.31 Six identical conducting rods are joined as shown in figure. Points A and D are maintained at 200ºC and 20ºC respectively. The temperature of junction B will be –



(A) 120ºC (B) 100ºC

(C) 140ºC (D) 80ºC **[C]**

Sol. Electrical circuit



different between A & D = 200 – 20 = 180ºC.

Resistances for three parts equal $\therefore \frac{180}{3}$ = 60ºC

Temperature B = 200 – 60 = 140ºC

Q.32 The emissive power of a black body at T = 300 K is 100 watt/m^2. Consider a body B of area A = 10 m^2 coefficient of reflectivity r = 0.3 and coefficient of transmission t = 0.5. Its temperature is 300 K. Then which of the following is correct -

(A) The emissive power of B is 20 watt/m^2

(B) The emissive power of B is 200 watt /m^2

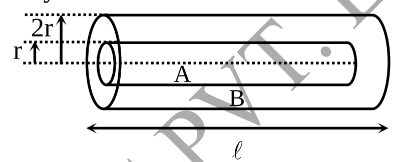
(C) The power emitted by B is 200 watts

(D) The emissivity of B is = 0.2 **[A,C,D]**

Sol. Since, e = a = 0.2 (since a = (1 – r – t) = 0.2 for the body B) E = (100) (0.2) = 20 watt/m^2

Power emitted = e.A = 20 × 10 = 200 watt.

Q.33 A composite cylinder is made of two different materials A and B of thermal conductivities K_A and K_B. The dimensions of the cylinder are as shown in the figure. The thermal resistance of the cylinder between the two end faces is -



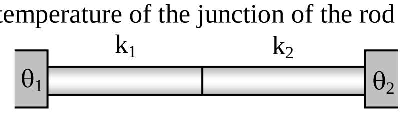
(A) $\frac{\ell}{\pi r^2(K_A + 3K_B)}$ (B) $\frac{\ell}{\pi r^2}\left(\frac{1}{K_A} + \frac{3}{K_B}\right)$

(C) $\frac{\ell}{\pi r^2}\left(\frac{1}{K_A} + \frac{4}{K_B}\right)$ (D) $\frac{\ell}{\pi r^2(K_A + 4K_B)}$

[A]

Sol. $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

Q.34 Two identical rods are made of different materials whose thermal conductivities are k_1 and k_2. They are placed end to end between two heat reservoirs at temperature θ_1 and θ_2. The temperature of the junction of the rod is –



(A) $\frac{\theta_1 + \theta_2}{2}$ (B) $\frac{k_1\theta_1 + k_2\theta_2}{2}$

(C) $\frac{k_1\theta_2 + k_2\theta_1}{k_1 + k_2}$ (D) $\frac{k_1\theta_1 + k_2\theta_2}{k_1 + k_2}$

Sol. **[D]**

In series the rate of heat flow is same

$$\frac{\theta_1 - \theta}{\frac{\ell}{k_1 A}} = \frac{\theta - \theta_2}{\frac{\ell}{k_2 A}} \Rightarrow k_1\theta_1 - k_1\theta = k_2\theta - k_2\theta_2$$

$$\theta = \frac{k_1\theta_1 + k_2\theta_2}{k_1 + k_2}$$

Q.35 The ratio of thermal capacities of two spheres A and B, if their diameters are in the ratio 1 : 2, densities in the ratio 2 : 1, and the specific heat in the ratio of 1 : 3, will be -

(A) 1 : 6 (B) 1 : 12

(C) 1 : 3 (D) 1 : 4

Sol. **[B]**

$$\frac{H_1}{H_2} = \frac{m_1 s_1}{m_2 s_2} = \frac{\rho_1 V_1 s_1}{\rho_2 V_2 s_2}$$

Q.36 In a steady state of thermal conduction, temperature of the ends A and B of a 20 cm long rod are 100ºC and 0ºC respectively. What will be the temperature of the rod at a point at a distance of 6 cm from the end A of the rod ?

(A) –30ºC (B) 70ºC

(C) 5ºC (D) None of these

Sol. **[B]**

$$\frac{100 - T}{6} = \frac{T - 0}{14} \Rightarrow 1400 - 14\,T = 6T$$

$$\Rightarrow 1400 = 20\,T \quad \Rightarrow \quad T = 70\ ºC$$

Q.37 A cylindrical rod with one end in a steam chamber and the other end in ice results in melting of 0.1 g of ice per second. If the rod is replaced by another rod with half the length and double the radius of the first and if the thermal conductivity of material of the second rod is 0.25 times that of first, the rate at which ice melts in g s^{-1} will be -

(A) 0.1 (B) 0.2

(C) 1.6 (D) 3.2

Sol. **[B]**

For first rod $\frac{\Delta Q}{\Delta t} = \frac{\Delta m}{\Delta t} L = \frac{KA(T_1 - T_2)}{L}$

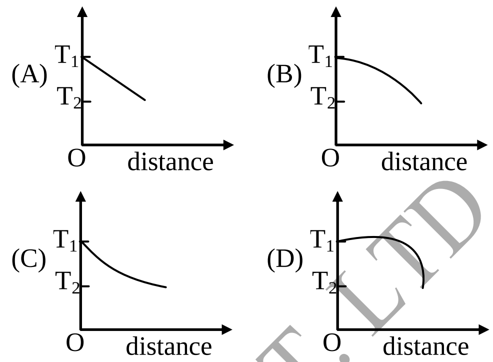
$$0.1 \times L = \frac{KA(T_1 - T_2)}{L} \quad \text{….(i)}$$

For second rod

$$\frac{\Delta m}{\Delta t} L = \frac{0.25K \times 4A(T_1 - T_2)}{L/2} \quad \text{….(ii)}$$

$$\therefore \ \frac{\Delta m}{\Delta t} = 0.2 \text{ g/s}$$

Q.38 The ends of a metal bar of constant cross-sectional area are maintained at temperatures T_1 and T_2 which are both higher than the temperature of the surroundings. If the bar is unlagged, which one of the following sketches best represents the variation of temperature with distance along the bar ?



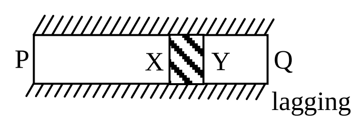
Sol. **[A]**

$$\frac{T_1 - T}{x} = \frac{T - T_2}{L - x}$$

$\Rightarrow T_1L - T_1x + Tx - TL = Tx - T_2x$

$TL = (T_2 - T_1)x + T_1L$

Q.39 PQ is fully-lagged metal bar, containing a section of XY of a material of lower thermal conductivity. The thermal conductivities of the two materials are independent of temperature. Ends P and Q are maintained at different temperatures.



In the steady state, the temperature difference across XY would be independent of–

(A) the temperature difference between P and Q

(B) the metal of which the bar is made

(C) the thickness of the section XY

(D) the distance of the section XY from the end P

[D]

Q.40 Curved surface of a uniform rod is isolated from surrounding. Ends of the rod are maintained at temperatures T_1 and T_2 ($T_1 > T_2$) for a long time. At an instant, temperature T_1 starts to decrease at a constant and slow rate. If thermal capacity of material of the rod is considered, then which of the following statements is/are correct –

(A) At an instant, rate of heat flow near the hotter end is equal to that near the other end.

(B) Rate of heat flow through the rod starts to decrease near the hotter end and remains constant near the other end.

(C) Rate of heat flow is maximum at mid section of the rod

(D) None of these **[D]**

Q.41 A and B are two points on a uniform metal ring whose centre is C. The angle ACB = θ. A and B are maintained at two different constant temperatures. When θ = 180º, the rate of total heat flow from A to B is 1.2 W. When θ = 90º, this rate will be -

(A) 0.6 W (B) 0.9 W

(C) 1.6 W (D) 1.8 W

Sol. **[C]**

R_{total} = R for θ = 180º

Two sections of resistance R/2 each are in parallel R_{eq} = R/4

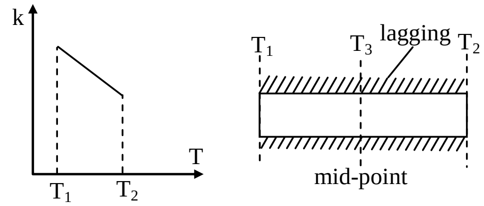
$\therefore$ Rate of heat flow $I_1 = 1.2 = \Delta T/R/4$

$$\Rightarrow 0.3 = \frac{\Delta T}{R}$$

θ = 90º two sections of resistance R/4 & 3R/4 in parallel $R_{eq} = \frac{R/4.3R/4}{R/4+3R/4} = \frac{3R}{16}$

$\therefore \quad I_2 = \frac{\Delta T}{3R/16} = \frac{16}{3}$ (0.3) = 1.6 watt

Q.42 Over a certain temperature range, the thermal conductivity k of a metal is not constant but varies as indicated in figure. A lagged rod of the metal has its ends maintained at temperatures T_1 and T_2(T_2> T_1) as shown in figure–



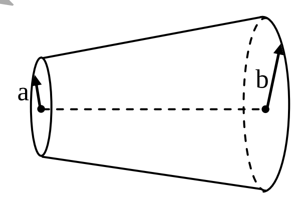
Which one of the following correctly describes how T_3, the temperature at the mid-point of the rod, compares with T_1 and T_2 ?

(A) $T_3 = (T_1 + T_2)/2$ (B) $T_3 = (T_1 - T_2)/2$

(C) $T_3 > (T_1 + T_2)/2$ (D) $T_3 < (T_1 + T_2)/2$

[C]

Q.43 Figure shows a rod of variable cross-section area. Two ends of rod is maintained at different temperature. Let radius of cross-section, where temperature is arithmetic mean of temperature at two ends be r. Then 'r' –



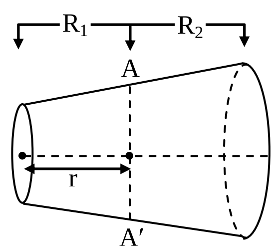
(A) Is equal to arithmetic mean of a and b

(B) Is equal to geometric mean of a and b

(C) Is equal to harmonic mean of a and b

(D) Depends upon length of rod, a and b **[C]**

Sol. Let the cross-section be AA′



$\therefore R_1 = R_2$

[R_1 = Thermal resistance of rod left of cross-section AA′]

$$\Rightarrow \frac{1}{a} - \frac{1}{r} = \frac{1}{r} - \frac{1}{b}$$

$$\Rightarrow r = \frac{2ab}{a+b}$$

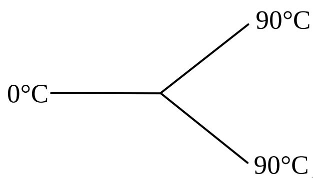
Q.44 Three rods of identical cross-sectional area and made from the same metal form the sides of an isosceles triangle ABC, right-angled at B. The points A and B are maintained at temperature T and $(\sqrt{2})$ T respectively. In the steady state, the temperature of the point C is T_C. Assuming that only heat conduction takes place, Tc/T is-

[IIT – 1995]

(A) $\frac{1}{2(\sqrt{2}-1)}$ (B) $\frac{3}{\sqrt{2}+1}$

(C) $\frac{1}{\sqrt{3}(\sqrt{2}-1)}$ (D) $\frac{1}{\sqrt{2}+1}$ **[B]**

Q.45 Three rods made of the same material and having the same cross section have been joined as shown in figure. The left and right ends are kept at 0°C and 90°C respectively. The temperature of the junction of the three rods will be –

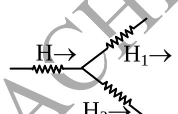


(A) 45°C (B) 60°C

(C) 30°C (D) 20°C

Sol. **[B]**

$H = H_1 + H_2$



Q.46 An ice cube is floating in water kept in glass. The water in glass will get cooled majorly by –

(A) Convection

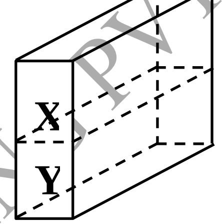
(B) Conduction

(C) Radiation

(D) Conduction and convection both **[A]**

Sol. Cooled water being denser will go down displacing hotter water and thereby forming convection cycle.

Q.47 A parallel-sided slab is made of two different materials. The upper half of the slab is made of material X, of thermal conductivity λ ; the lower half is made of material Y, of thermal conductivity 2λ. In the steady state, the left hand face of the composite slab is at a higher, uniform temperature than the right-hand face, and the flow of heat through the slab is parallel to its shortest sides. What fraction of the total heat flow through the slab passes through material X ?



(A) 1/4 (B) 1/3 (C) 1/2 (D) 2/3

Sol. **[B]**

$$\frac{\Delta Q}{\Delta t} = -\lambda A\frac{dT}{dx} \Rightarrow \lambda_y = 2\,\lambda_x = 2\lambda$$

$$\Rightarrow -A\frac{dT}{dx} = \text{constant} = a,\ \frac{\Delta Q_x}{\Delta t} = a\lambda$$

$$\Rightarrow \frac{\Delta Q_y}{\Delta t} = 2a\lambda \Rightarrow \frac{\Delta Q}{\Delta t} = \frac{\Delta Q_x}{\Delta t} + \frac{\Delta Q_y}{\Delta t}$$

$$= a\lambda + 2a\lambda = 3a\lambda$$

$$\therefore \quad \text{Fraction} = \frac{\Delta Q_x/\Delta t}{Q/\Delta t} = \frac{1}{3}$$

Q.48 The ratio of thermal conductivity of two rods of different materials is 5 : 4. The two rods of same area of cross-section and same thermal resistance will have the lengths in the ratio -

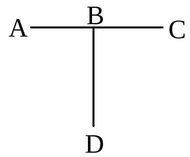
(A) 5 : 4 (B) 1 : 9

(C) 9 : 1 (D) 4 : 5

Sol. **[A]** $\frac{K_1}{K_2} = \frac{5}{4}$ $\because$ R = $\frac{L}{KA}$ $\therefore L \propto K$

$$\frac{L_1}{L_2} = \frac{K_1}{K_2} = \frac{5}{4}$$

Q.49 Three conducting rods of same material and cross-sectional area are joined as shown. Temperatures at points A, D and C are maintained at 20°C, 90°C and 0°C respectively. The ratio of lengths BD to BC if there is no heat flow in AB is-



(A) $\frac{2}{7}$ (B) $\frac{7}{2}$

(C) $\frac{9}{2}$ (D) $\frac{2}{9}$ **[B]**

Sol. As no heat flows in rod AB

Therefore $T_A = T_B = 20°C$

$\therefore$ Rate of heat flow through CB and BD are same.

Hence, $\frac{90-20}{\frac{BC}{KA}} = \frac{20-0}{\frac{BD}{KA}}$

$$\frac{7}{BC} = \frac{2}{BD}$$

$$\frac{BD}{BC} = \frac{7}{2}$$

Q.50 One end of a conducting rod is maintained at temperature 50°C and at the other end ice is maintained at 0°C. The rate of melting of ice is doubled if -

(A) the temperature is made 200°C and the area of cross-section of the rod is doubled

(B) the temperature is made 100°C and length of the rod is made four times

(C) area of cross-section of the rod is halved and length is doubled

(D) the temperature is made 100°C and area of cross-section of rod and length both are doubled **[D]**

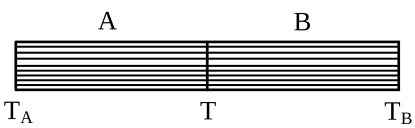
Sol. Rate of melting of ice $\propto$ rate of heat transfer $\left(\frac{dQ}{dt}\right)$

further, $\frac{dQ}{dt} = \frac{\text{temperature difference}}{(\ell / KA)}$

or $\frac{dQ}{dt} \propto \frac{\text{tempdiff}}{\ell} \times A$

If temp. diff, A and ℓ are all doubled, then dQ and hence rate of melting of ice are doubled

Q.1 Two cylinders of identical dimensions but of different materials are welded together at their butt-ends (Fig.). The thermal capacity of cylinder A is twice as great as that of cylinder B, but its thermal conductivity is only half that of B. One of the free ends is heated and the other is cooled, so that constant temperature is maintained at each end. Will the overall quantity of heat flowing through the cylinders depend on whether it is A's end that is heated and B's cooled, or vice versa? (Do not take into account the loss of heat through the curved sides of the cylinders.)



Sol. Consider the state that obtains when the temperature T of the joint is constant. Let $\mathbf{c_A}$ and $\mathbf{c_B}$ be the thermal capacities of the cylinders, let $\mathbf{k_A}$ and $\mathbf{k_B}$ be their coefficients of thermal conductivity, and T_A and T_B be the temperatures of their ends. Since the cylinders are homogeneous and heat is not given up through their sides, the temperature in the state that is set up changes along each of the cylinders according to a linear equation and the temperature T of the join can be found from the equality of the flow of heat through any two cross-sections of the two cylinders

$K_A(T_A - T) = k_B(T - T_B)$,

$$T = \frac{k_A T_A + k_B T_B}{k_A + k_B} \qquad \text{.....(1)}$$

The amount of heat which flows through each cylinder can be found by replacing the variable temperature along the whole cylinder by the average temperature (since the distribution of temperature along the cylinder changes according to a linear equation). The total amount of heat which flows through the cylinders

$$Q = \frac{c_A(T_A + T)}{2} + \frac{c_B(T + T_B)}{2}$$

or

$$Q = \frac{c_A T_A}{2} + \frac{c_B T_B}{2} + \frac{(c_A + c_B)}{2} T. \quad \text{.....(2)}$$

Substituting the value for T from expression (1), we shall get:

$$Q = \frac{c_A T_A}{2} + \frac{c_B T_B}{2} + \frac{(c_A + c_B)(k_A T_A + k_B T_B)}{2(k_A + k_B)}.$$

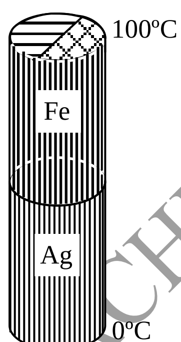
Bringing the right-hand side over a common denominator and collecting terms containing T_A and T_B, we finally find that:

$$Q = \frac{2k_A c_A + k_A c_B + k_B c_A}{2(k_A + k_B)} T_A + \frac{2k_B c_B + k_A c_B + k_B c_A}{2(k_A + k_B)} T_B.$$

And this expression gives us the answer to the problem posed. Since the coefficients are, in general, different at temperatures T_A and T_B, Q will have a different value if we replace $\mathbf{T_A}$ by $\mathbf{T_B}$, i.e. the sum total of heat flowing through the cylinders will in general depend on which of the ends is heated and which is cooled. And since the coefficients at temperatures $\mathbf{T_A}$ and $\mathbf{T_B}$ differ only in the values of the terms $\mathbf{k_A c_A}$ and $\mathbf{k_B c_B}$, the amount of heat will be greater if we heat the end of that cylinder for which the product of thermal capacity and thermal conductivity is greater. This is quite clear physically: the role of thermal capacity is obvious; and the greater the thermal conductivity, the less will be the fall of temperature along the cylinders and the higher will be the temperature of all points.

Only in the special case when $\mathbf{k_A c_A = k_B c_B}$, as is laid down by our problem, will the amount of heat flowing through the cylinders be independent of which end is heated and which is cooled.

Q.2 Two cylinders of the same dimensions, one of iron, the other of silver, stand one upon the other **(Fig.)**. The lower end of the silver one is kept at a temperature of 0ºC, and the upper end of the iron one is kept at a temperature of 100ºC. The thermal conductivity of silver is eleven times greater than that of iron. What is the temperature of the ends which are in contact with each other, if we assume that no heat escapes into the surrounding medium through the side surfaces of the cylinders?



Sol. Since there are no losses of heat through the sides of the cylinders, the amount of heat which flows through any section of our system in unit time will be the same. If the difference in temperature between any two cross-sections of a homogeneous medium, distance l apart, is $T_1 - T_2$, the amount of heat which flows from the first to the second in unit time through any section between them will be expressed as

$$Q = kS\,\frac{T_1 - T_2}{l},$$

Where **k** is the coefficient of thermal conductivity and S is the sectional area.

Let $\mathbf{T_1}$ be the temperature of the upper end of the iron cylinder, $\mathbf{T_2}$ be temperature of the ends in contact, $\mathbf{T_3}$ be the amount of heat which flows through any section of the iron cylinder equals

$$Q_{Fe} = k_{Fe}S\,\frac{T_1 - T_2}{l}.$$

Similarly for any section of the silver cylinder we shall have

$$Q_{Ag} = k_{Ag}S\,\frac{T_2 - T_3}{l}$$

Comparing these expressions, we shall get:

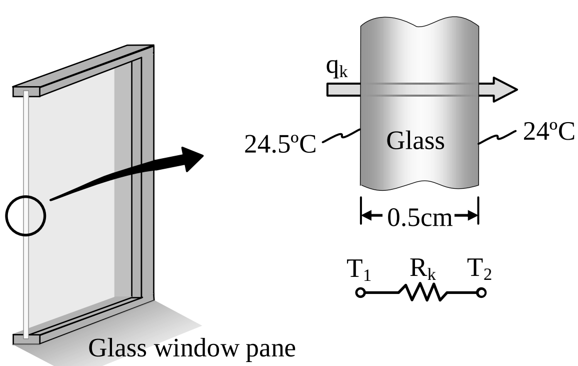
$K_{Fe}(T_1 - T_2) = k_{Ag}(T_2 - T_3)$.

Substituting the values of $T_1 = 100ºC$, $T_2 = T_2 = 0ºC$ and $k_{Ag} = 11k_{Fe}$ in the above equation we find that the temperature of the ends in contact is

$$T_2 = \frac{100}{12} = 8.3ºC.$$

Q.3 Calculate the thermal resistance and the rate of heat transfer through a pane of window glass (k = 0.81 W/mK) 1 m high, 0.5 m wide, and 0.5 cm thick, if the outer-surface temperature is 24ºC and the inner-surface temperature is 24.5ºC.

Sol.



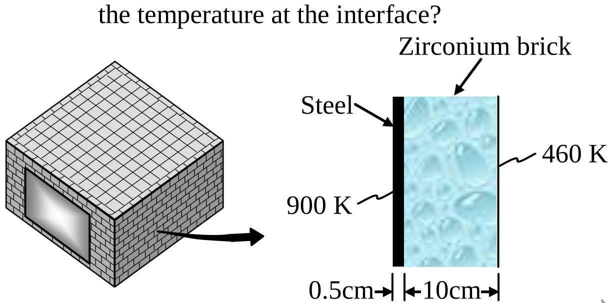
A schematic diagram of the system is shown in **Fig**. Assume that steady state exists and that the temperature is uniform over the inner and outer surfaces. The thermal resistance to conduction R_k is from

$$R_k = \frac{L}{kA} = \frac{0.005\,\text{m}}{0.81\,\text{W/m K} \times 1\,\text{m} \times 0.5\,\text{m}} = 0.0123 \text{ K/W}$$

The rate of heat loss from the interior to the exterior surface is obtained from:

$$q_k = \frac{T_1 - T_2}{R_k} = \frac{(24.5 - 24.0)^\circ\text{C}}{0.0123\ \text{K/W}} = 40\text{ W}$$

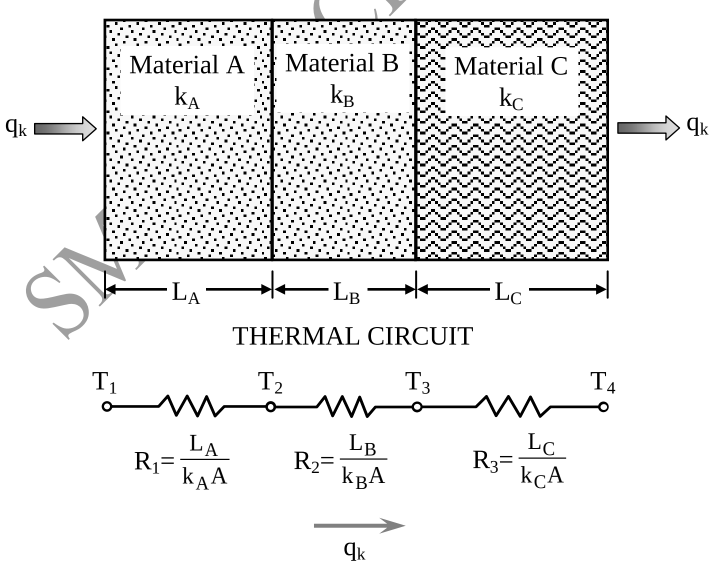
Q.4 Calculate the rate of heat loss from a furnace wall per unit area. The wall is constructed from an inner layer of 0.5-cm-thick steel (k = 40 W/m K) and an outer layer of 10-cm zirconium brick (k = 2.5 W/m K) as shown in Fig. The inner-surface temperature is 900 K and the outside surface temperature is 460 K. What is the temperature at the interface?



Wall cross section

Sol. Assume that steady state exists, neglect effects at the corners and edges of the wall, and assume that the surface temperatures are uniform. The physical system and the corresponding thermal circuit are similar to those in Fig., but only two layers or walls are present.

PHYSICAL SYSTEM



The rate of heat loss per unit area can be calculated from:

$$\frac{q_k}{A} = \frac{(900 - 460)\text{K}}{(0.005\,\text{m})/(40\,\text{W/m K}) + (0.1\,\text{m})(2.5\,\text{W/m K})}$$

$$= \frac{440\text{K}}{(0.000125 + 0.04)(\text{m}^2\,\text{K/W})} = 10.965\text{ W/m}^2$$

The interface temperature T_2 is obtained from

$$\frac{q_k}{A} = \frac{T_1 - T_2}{R_1}$$

Solving for T_2 gives

$$T_2 = T_1 - \frac{q_k}{A_1}\frac{L_1}{k_1} = 900\text{ K} - \left(10{,}965\frac{\text{W}}{\text{m}^2}\right)\left(0.000125\,\frac{\text{m}^2\,\text{K}}{\text{W}}\right)$$

= 898.6 K

Note that the temperature drop across the steel interior wall is only 1.4 K because the thermal resistance of the wall is small compared to the resistance of the brick, across which the temperature drop is many times larger.

Q.5 The steady temperatures at the ends of a copper rod of length 25 cm and area of cross-section 1.0 cm^2 are 125ºC and 0ºC respectively. Calculate the temperature-gradient, rate of heat-flow, and the temperature at a distance 10 cm from the hot end. K for copper = 9.2×10^{-2} kilo-cal/(sec-meter-ºC).

Sol. The rate of linear, steady heat-flow is given by

$$\frac{\Delta Q}{\Delta t} = KA\,\frac{\theta_1 - \theta_2}{d},$$

Where the symbols have their usual meanings.

Here, temperature-gradient, $\frac{\theta_1 - \theta_2}{d} = \frac{125}{25} = 5$ ºC/cm.

K = 9.2×10^{-2} kilo-cal/(see-meter-ºC)

= 9.2×10^{-1} cal/(see-cm-ºC),

and A = 1.0 cm^2.

$\therefore \ \frac{\Delta Q}{\Delta t} = (9.2 \times 10^{-1}) \times 1.0 \times 5 = 4.6$ cal/sec.

Now, in the steady state, the rate of heat-flow is same through all the sections of the bar. If the temperature at a distance of 10 cm from the hot end (i.e. d = 10 cm) be θ; then from eq. (i), we have

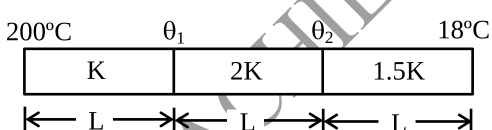
$$\frac{\Delta Q}{\Delta t} = 4.6 = 9.2 \times 10^{-1} \times 1.0 \times \frac{125-\theta}{10}$$

or $(125 - \theta) = \frac{4.6\times10}{9.2\times10^{-1}} = 50$

$\therefore \ \theta = 125 - 50 = 75^\circ$ C.

Q.6 Three bars each of area of cross-section A and length L are connected in series. The thermal conductivities of their materials are K, 2K and 1.5 K. If the temperatures of the external ends of the first and the last bar are 200ºC and 18ºC, then calculate the temperatures of both the junctions. The loss of heat due to radiation is negligible.

Sol.



Let the temperatures of the junctions (see Fig.) be θ_1 and θ_2 respectively. The loss of heat due to radiation is negligible. Therefore, in the steady state, the rate of flow of heat in the whole system will be same. Therefore

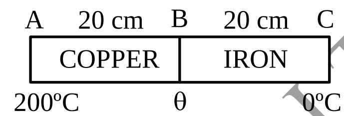
$$\frac{\Delta Q}{\Delta t} = KA\frac{(200-\theta_1)}{L} = (2\,K)\,A\frac{(\theta_1-\theta_2)}{L} = (1.5\,K)\,A\frac{(\theta_2-18)}{L}$$

or $200 - \theta_1 = 2\,\theta_1 - 2\,\theta_2 = 1.5\,\theta_2 - 27$.

Solving : $\theta_1 = 116$ ºC, $\theta_2 = 74$ºC.

Q.7 A bar of length 40 cm and uniform cross-section 5 cm^2 consists of two halves, AB of copper and CB of iron welded at B. The end A is maintained at 200ºC and C at 0ºC. The sides are thermally insulated. Find the rate of heat-flow along the bar in the steady state. K for Cu and Fe are 0.9 and 0.1 cal/(sec-cm-ºC) respectively.

Sol.



Let θ be the temperature of the junction. In the steady state, the rate of heat-flow will be same throughout the bar. Therefore

$$\frac{\Delta Q}{\Delta t} = 0.9 \times 5 \times \frac{200-\theta}{20} = 0.1 \times 5 \times \frac{\theta-0}{20}.$$

Solving for θ, we get

$\theta = 180^\circ$.

Now, putting the value of θ in the last eq., we get

$$\frac{\Delta Q}{\Delta t} = 0.1 \times 5 \times \frac{180-0}{20}$$

= 4.5 cal/sec.

Q.8 An electric hot plate of 100 cm^2 area has temperature of 820ºC on the heater side. A kettle of a perfect conductor with 250 cm^2 of water at a temperature of 20ºC is kept over it. Find how much time will be required for the water to just boil if the water equivalent of the kettle is 20 gm and thickness of the hot plate is 0.9 cm and thermal conductivity is 0.9 C.G.S. units.

Sol. The mass of the water is 250 gm and the water equivalent of the kettle is 20 gm. The specific heat of water is 1 C.G.S. unit. Hence the heat required for the water to reach from 20ºC to 100ºC (boiling) is giving by

Q = (250 × 1 + 20) (100 – 20) = 21600 cal.

Let t be the time required for the heat Q to flow through the hot plate. We know that, in usual notations,

$$Q = KA\frac{\theta_1-\theta_2}{d}\,t.$$

Here Q = 21600 cal, K = 0.9 C.G.S. units, A = 100 cm^2, $(\theta_1 - \theta_2) = \left\{820 - \frac{20+100}{2}\right\} = 760$ºC and d = 0.9 cm.

$$\therefore \ 21600 = \frac{0.9\times100\times760\times t}{0.9}$$

or $t = \frac{21600\times0.9}{0.9\times100\times760} = \frac{27}{95}$ sec.

Q.9 An insulating wall of area of 4 m^2 is made from cork, 16.5 cm thick, protected externally by 12.5 cm brick and lined on the inside with 10.0 cm wood. If the exposed surface of wood is maintained at – 5.0ºC and the outer surface of the brick at 20.0ºC, calculate **(i)** the rate of energy transfer through the wall and **(ii)** the interface temperatures. **Given**: thermal conductivity for brick = 2.5×10^{-3}, for cork = 1.1×10^{-4} and for wood = 4.0×10^{-4} cal-sec^{-1} cm^{-1}-ºC^{-1}.

Sol. Let θ_1 be the temperature of the brick-cork interface and θ_2 that of the cork-wood interface. The temperature of the outer surface of brick is 20.0ºC and that of the exposed surface of wood is – 5ºC. In the steady state, the heat flowing per second is same throughout. Therefore, equating the heat flowing per second through brick, cork and wood, we have

$$2.5 \times 10^{-3} \times A \frac{20-\theta_1}{12.5} = 1.1 \times 10^{-4}. \times A \frac{\theta_1-\theta_2}{16.5} = 4.0 \times 10^{-4} \times A \frac{\theta_2-(-5)}{10}.$$

From this, we get two equations

$$25 \times \frac{20-\theta_1}{12.5} = 1.1 \times \frac{\theta_1-\theta_2}{16.5}.$$

and $$25 \times \frac{20-\theta_1}{12.5} = 4 \times \frac{\theta_1-5}{10}.$$

Solving we get: θ_1 = 19.3ºC and θ_2 = – 1.53ºC.

The rate of heat-flow through brick, say, is

$$\frac{\Delta Q}{\Delta t} = 2.5 \times 10^{-3} \times A \times \frac{20-\theta_1}{12.5}$$

$$= 2.5 \times 10^{-3} \times (400 \times 400) \times \frac{20-19.3}{12.5} = 22.4$$

cal/sec.

Q.10 In Lee's experiment for finding the thermal conductivity of cardboard, the following readings were taken:

Steady temperature of copper disc = 99.5ºC. Steady temperature of lower disc = 83.5ºC. Time taken for the lower disc to cool from 86ºC to 81ºC = 4 min. Thickness of cardboard disc = 4.8 mm. Thickness of lower disc = 1 cm. The discs are of copper whose density is 9 gm/cm^3 and sp. heat is 0.1 cal/(gm-ºC). Find the conductivity of cardboard.

Sol. The heat conducted per second from the upper to the lower disc through the cardboard disc is

$$Q = K(\pi r^2)\frac{\theta_1-\theta_2}{d},$$

where **r** is the radius and **d** the thickness of the cardboard disc. In the steady state, this heat is emitted away per second from the lower disc. Thus, if **m** is the mass, **s** the specific heat and $d\theta/dt$ the rate of cooling of the lower disc, then

$$Q = ms\frac{d\theta}{dt} = (\pi r^2 d' \rho)s\frac{d\theta}{dt},$$

where **d'** is the thickness of the lower disc and **ρ** and density of copper. Thus

$$K(\pi r^2)\frac{\theta_1-\theta_2}{d} = (\pi r^2 d' \rho)s\frac{d\theta}{dt}$$

or $$K = \frac{d'\rho s(d\theta/dt)d}{\theta_1-\theta_2}.$$

Here d' = 1 cm, ρ = 9 gm/cm^3, s = 0.1 cal/(gm-ºC), $\frac{d\theta}{dt} = \frac{86-81}{4\times 60} = \frac{1}{48}$ ºC/sec,

d = 0.48 cm and $\theta_1 - \theta_2$ = 99.5 – 83.5 = 16ºC.

$$\therefore \quad K = \frac{1\times 9\times 0.1\times(1/48)\times 0.48}{16}$$

= 5.6×10^{-4} cal-cm^{-1}-sec^{-1}-ºC^{-1}.

Q.11 A rubber tube of length 20 cm, through which steam at 100ºC is passing, is immersed in a calorimeter whose water equivalent is 15 gm and which contains 300 gm of water at 16ºC. The temperature of water rises at the rate of 2ºC per minute. If the outer and inner diameters of the tube are 1.0 cm and 0.6 cm respectively, calculate the thermal conductivity of rubber.

Sol. The radial rate of heat-flow through a tube of length ***l*** cm and inner and outer radii $\mathbf{r_1}$ and $\mathbf{r_2}$ cm is given by

$$\frac{dQ}{dt} = 2\pi K\, l \frac{\theta_1 - \theta_2}{\log_e(r_2/r_1)} \text{ cal/sec,}$$

where θ_1 and θ_2 are temperatures inside and outside the tube. If this heat raises the temperature of **m** gm of water (specific heat 1) contained in a calorimeter of water equivalent w gm by $\Delta\theta$, then

$$\frac{dQ}{dt} = (m + w)\,\Delta\theta \text{ cal/sec.}$$

Equating the above two equations, we get

$$2\pi K\, l \frac{\theta_1 - \theta_2}{\log_e(r_2/r_1)} = (m + w)\,\Delta\theta$$

$$\therefore\ K = \frac{(m+w)\Delta\theta \log_e(r_2/r_1)}{2\pi\, l(\theta_1 - \theta_2)}.$$

Here m + w = 300 + 15 = 315 gm, $\Delta\theta$ = 2ºC per minute $= \left(\frac{1}{30}\right)$ºC/sec, $\log_e(r_2/r_1) = 2.3 \log_{10}$ (1.0/0.6) = 2.3 × 0.2219, l = 20 cm, $(\theta_1 - \theta_2)$ = 100 – 16 = 84ºC.

$$\therefore\ K = \frac{315 \times (1/30) \times (2.3 \times 0.2219)}{2 \times 3.14 \times 20 \times 84}$$

$= 5 \times 10^{-4}$ cal-cm^{-1}-sec^{-1}-ºC^{-1}.

Q.12 A heating-wire of 0.0005 meter diameter is embedded along the axis of a cylinder of 0.12 meter diameter. When a current is passed through the wire it gives out a power of 3 kilowatt per meter of its length. If the temperature of the wire be 1500ºC and that of the outer surface of the cylinder be 20ºC, compute the thermal conductivity of the material of the cylinder.

Sol. Let l meter be the length of the wire. Then the power being generated

= 3 l kilowatt = 3000 l watt

= 3000 l joule/sec (1 watt = 1 joule/sec)

$= \frac{3000}{4.18}\, l$ cal/sec. (1 cal = 4.18 joule)

The rate of radial heat-flow through a cylinder of length l and internal and external radii r_1 and r_2 is given by

$$\frac{dQ}{dt} = 2\pi K\, l \frac{\theta_1 - \theta_2}{2.3026 \log_{10}(r_2/r_1)}$$

Here $\frac{dQ}{dt} = \frac{3000}{4.18}\, l$ cal/sec, θ_1 = 15000ºC, θ_2 = 20ºC, r_1 = 0.0005 meter and r_2 = 0.12 meter, so that $\frac{r_2}{r_1} = \frac{0.12}{0.0005}$ = 240. Therefore

$$\frac{3000}{4.18}\, l = 2 \times 3.14 \times K \times l \times \frac{1500 - 20}{2.3026 \log_{10} 240}$$

or $\frac{3000}{4.18} = 2 \times 3.14 \times K \times \frac{1480}{2.3026 \times 2.3802}$

$\therefore$ K $= \frac{3000 \times 2.3026 \times 2.3802}{4.18 \times 2 \times 3.14 \times 1480} =$ 0.423 cal/(meter-sec-ºC).

Q.13 A copper plate of radius 12 cm and thickness 5 cm conducts when the temperature difference between the circular end faces is 20ºC. A spherical shell of aluminium of radii 9 cm and 4 cm conducts heat radially under the same temperature difference. Compare the amounts of heat transferred per second in the two cases. Thermal conductivities of copper and aluminium are 0.9 and 0.5 C.G.S. units.

Sol. The linear rate of the heat-flow through the circular copper plate of area A (= π r^2) and thickness d is

$$\left\{\frac{dQ}{dt}\right\}_{plate} = KA \frac{\theta_1 - \theta_2}{d}$$

$$= \frac{0.9 \times \pi \times (12)^2 \times 20}{5}. \quad \text{..... (i)}$$

The radial rate of heat-flow through a spherical shell of radii r_1 and r_2 is

$$\left\{\frac{dQ}{dt}\right\}_{plate} = \frac{4\pi K r_1 r_2 (\theta_1 - \theta_2)}{r_2 - r_1}$$

$$= \frac{4 \times \pi \times 0.5 \times 4 \times 9 \times 20}{5} \quad \text{….. (ii)}$$

Comparing the above two expression:

$$\frac{(dQ/dt)_{plate}}{(dQ/dt)_{shell}} = \frac{0.9 \times (12)^2}{4 \times 0.5 \times 4 \times 9}$$

= 1.8.

Q.14 Two thin concentric spherical shells of radii 5 cm and 10 cm respectively have their annular cavity filled with charcoal powder. When energy is supplied at the rate of 10.5 watt to a heater at the centre, a temperature difference of 60ºC is set up between the shells. Find the thermal conductivity of charcoal. (J = 4.2 joule/cal.)

Sol. The thermal conductivity of the charcoal will be given by

$$K = \frac{(dQ/dt)(r_2 - r_1)}{4\pi r_1 r_2(\theta_1 - \theta_2)}, \qquad \text{[see problem 5]}$$

where dQ/dt is the rate of heat supply.

Here radius of one shell r_1 = 5cm,

Radius of other shell r_2 = 10 cm,

Temperature of difference $(\theta_1 - \theta_2)$ = 60ºC,

And heat supplied per sec, dQ/dt = energy dissipated in heater per sec,

= 10.5 wall = 10.5 joule/sec

$= \frac{10.5}{4.2}$ cal/sec.

Hence from the above expression, we get

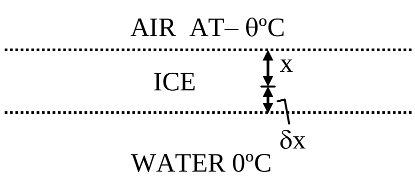
$$K = \frac{\frac{10.5}{4.2}\times(10-5)}{4\times3.14\times10\times5\times60}$$

$$= \frac{10.5\times5}{4.2\times4\times3.14\times10\times5\times60}$$

$= 3.3 \times 10^{-4}$ cal – cm^{-1}-cm^{-1}-sec^{-1}-$ºC^{-1}$.

Q.15 The thickness of ice in a lake is 5 cm and the temperature of air is –10ºC. Calculate the time required for the thickness of ice to be doubled. Constants for ice are : conductivity = 0.004 cal-cm^{-1}-sec^{-1}-$ºC^{-1}$, density = 0.92 gm/cm^3, latent heat = 80 cal/gm.

Sol. The cold air (below 0ºC) above the water in a lake takes in heat from the water. The water therefore begins to freeze into ice layer. Let us determine the rate of growth of this layer.



Let us consider a layer of ice x cm thick already formed on a lake at 0ºC, the temperature of air above it being – θºC. Let A be the area of the ice layer, L the latent heat of fusion of ice and **ρ** its density. Then the heat given up when the ice layer increases in thickness by an amount **dx**

= mass × latent heat

= A dx ρ × L calorie.

If this quantity of heat is conducted upwards through the ice layer in dt second, then we have

$$A\,dx\,\rho \times L = KA\,\frac{0-(-\theta)}{x}\,dt,$$

where K is thermal conductivity of ice. This gives

$$\frac{dx}{dt} = \frac{K\theta}{\rho L x}.$$

dx/dt is the rate at which the thickness of ice layer increases. Rearranging the above expression, we get

$$dt = \frac{\rho L x}{K\theta}\,dx.$$

Therefore, the time in which thickness of ice will increase from x_1 to x_2 will be given by

$$t = \frac{\rho L}{K\theta}\int_{x_1}^{x_2} x\,dx$$

$$= \frac{\rho L}{K\theta}\left[\frac{x^2}{2}\right]_{x_1}^{x_2} = \frac{\rho L}{2K\theta}\,(x_2^2 - x_1^2).$$

In the given problem, we have K = 0.004 cal-cm^{-1}-sec^{-1}-$ºC^{-1}$, ρ = 0.92 gm/cm^3, L = 80 cal/gm, – θ = – 10ºC, x_1 = 5 cm, x_2 = 10 cm (doubled).

$$\therefore\ t = \frac{0.92\times80}{2\times0.004\times10}\,(10^2 - 5^2)$$

= 69000 seconds = 19.1 hours.

Q.16 How much time will it take for a layer of ice of thickness 20 cm to increase by 10 cm on the surface of a pond when the temperature of the surroundings is – 15ºC. Given : K = 0.005 cgs units, L = 80 cal/gm, ρ = 0.90 gm/cm^3.

Sol. As in the last problem:

$$t = \frac{\rho L}{2K\theta}(x_2^2 - x_1^2)$$

$$= \frac{0.92\times80}{2\times0.005\times15}(30^2 - 20^2)$$

$$= \frac{0.90\times80\times500}{2\times0.005\times15} = 240000 \text{ sec} = 66.67 \text{ hours.}$$

Q.17 A lake is covered with ice 4 cm thick and the temperature of air above the ice is – 12ºC. At what rate, expressed in cm per hour, will the ice thicken? Conductivity of ice = 0.0052 cgs units, density of ice = 0.92 gm/cm^3 and latent heat of ice = 80 cal/gm.

Sol. See problem 5.

$$\frac{dx}{dt} = \frac{K\theta}{\rho Lx}$$

$$= \frac{0.0052 \text{cal}/(\text{cm}-\text{sec}-^\circ\text{C})\times(12^\circ\text{C})}{0.92\text{gm}/\text{cm}^3\times80\text{cal}/\text{gm}\times4\text{cm}}$$

= 2.12×10^{-4} cm/sec

= $(2.12 \times 10^{-4}) \times 3600 = 0.763$ cm/hour.

Q.18 The air above a large lake is at –2ºC, whilst the water of the lake is at 0ºC. Assuming that only thermal conduction is important, and using relevant data selected from that given below, estimate how long it would take for a layer of ice 10 cm thick to form on the lake's surface.

Data:

Thermal conductivity of water, λ_w = 0.56 W m^{-1}K^{-1}

Thermal conductivity of ice, λ_i = 2.3 W m^{-1}K^{-1}

Specific latent heat of fusion of ice, L_i = 3.3 × 105 J kg^{-1}

Density of water, ρ_w = 1000 kg m^{-3}

Density of ice, ρ_i = 920 kg m^{-3}

Sol. The heat conducted away in a given time must equal (minus) the latent heat of fusion of the additional ice formed in that time. Thus for an area A of the lake

$$\lambda_1 A\frac{\Delta T}{x} = L_i\frac{d}{dt}(\rho_i Ax).$$

Simple integration gives that $t = \frac{1}{2}x^2B$, where $B = (\rho_i L_i)/(\lambda_i\Delta T)$. Inserting the relevant numerical values gives the time as about 90 hours.

Q.19 If it takes two days to defrost a frozen 5-kg turkey, estimate how long it would take to defrost an 8-tonne Siberian mammoth.

Sol. According to the law of heat conduction the heat transferred is directly proportional to the thermal gradient (temperature difference divided by distance), the area and the time. So

$$\text{Time} \propto \frac{\text{Heat capacity}}{\text{Area}\times\text{Temperature gradient}} \propto \frac{L^3}{L^2L^{-1}}$$

$= L^2$.

Since $M \propto L^3$, the time is proportional to $M^{2/3}$ and a mammoth should take $2(800/5)^{2/3}$ days, i.e. about nine months, assuming that the turkey and the mammoth both start to thaw from the same temperature and are defrosting in similar environments. (In fact, the Siberian summer is too short to defrost a mammoth.)

Q.20 A 0.6-kg block of ice at –10ºC is placed into a closed empty 1 m^3 container, also at a temperature of –10ºC. The temperature of the container is then increased to 100ºC. How much greater is the heat required than that necessary to raise the empty container alone to that temperature?

Sol. Consider first the state of the ice and the pressure in the container at 100ºC. Since the density of saturated water vapour is 0.5977($\approx$ 0.6) kg m^{-3} and its pressure is 1 atm, the 0.6 kg of ice is completely transformed into vapour at 100ºC, indeed into saturated vapour at a pressure of 1 atm.

How does ice, at a temperature of – 10ºC, turn into saturated water vapour at 100ºC? If the temperature is increased very slowly, the system passes through a number of equilibrium states. First, the ice sublimes and the ice phase is in equilibrium with the vapour phase. This lasts until the temperature and pressure of the triple point (0.01ºC, 610 Pa) is reached. At the triple point, a liquid state appears alongside the ice and water vapour. Further heating makes the solid phase disappear, and only water and saturated water vapour remain in the container. It is interesting to note that subsequently the water boils steadily at 100ºC, until all the water has been transformed into vapour.

From the point of view of heat absorption, only the initial and the final states are important. The heat Q absorbed by the system, as it passes with increasing internal energy through the 'ice-water-vapour' states, is independent of the intermediate states. For calculational purposes, the heating of 0.6 kg of ice should be divided into four stages (warming the ice, melting the ice, warming the water and boiling the water) to give:

$Q = c_i m\Delta T_1 + L_f m + c_w m\Delta T_2 + L_v m$,

Where c_i = 2.1 kJ kg^{-1} $ºC^{-1}$ is the specific heat of ice, ΔT_1 = 10ºC, L_f = 334 kJ kg^{-1} is the heat of fusion of ice, c_w = 4.2 kJ kg^{-1} $ºC^{-1}$ is the specific heat of water, ΔT_2 = 100ºC and L_v is the specific heat of vaporization of water.

The specific heats of ice and water are slightly dependent on temperature and pressure, and the heat of fusion of the ice also depends a little on pressure. But these variations are small and can be neglected. However, the situation is different for the heat of vaporization of water. It is true that the heat of vaporization only varies a little with temperature, but its dependence on pressure is significant.

The value usually given in tables, L_v = 2256 kJ kg^{-1}, covers not only the higher internal energy of the vapour, but also the work done against atmospheric pressure. In the present problem, this work was done when the container was evacuated, i.e. the heat to be transferred to the system is smaller by this amount, which is $-p\Delta V$ = –101.3 kJ. As this figure is that for 0.6 kg of water,

$$L_v = \left(2256 - \frac{101.3}{0.6}\right) = 2087 \approx 2090 \text{ kJ kg}^{-1}$$

is the more accurate value to be used for the present calculation. After substitution of the data, the heat transfer is found to be Q = 1720 kJ.

Neglecting the work done against atmospheric pressure would have introduced an error of nearly 6 per cent, while ignoring the slight dependence of the other coefficients on temperature and pressure causes an inaccuracy of approximately 1 per cent. The main reason for this is that the normal change in the volume in the course of a water-vapour transition is very significant (a factor of about 1600). The volumes of the water and ice are negligible compared with that of the vapour, and they are tacitly neglected in the solution.