Straight lines

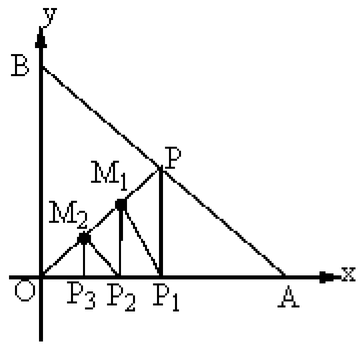
Single Correct Answer Type

1. The line x + y = 1 meets x-axis at A and y-axis at B. P is the mid-point of AB. P_1 is the foot of the perpendicular from P to OA; M_1 is that from P_1 to OP; P_2 is that from M_1 to OA; M_2 is that from P_2 to OP; P_3 is that from M_2 to OA and so on. If P_n denotes the nth foot of the perpendicular on OA from M_{n-1}, then $OP_n =$

A. $\frac{1}{2}$ B. $\frac{1}{2^n}$ C. $\frac{1}{2^{n/2}}$ D. $\frac{1}{\sqrt{2}}$

Key. B

Sol. x + y = 1 meets x-axis at A(1, 0) and y-axis at B(0, 1).



The coordinates of P are (1/2, 1/2) and PP_1 is perpendicular to OA.

$\Rightarrow OP_1 = P_1P = 1/2$

Equation of line OP is y = x

We have $(OM_{n-1})^2 = (OP_n)^2 + (P_nM_{n-1})^2 = 2(OP_n)^2 = 2P_n^2$ (say)

Also, $(OP_{n-1})^2 = (OM_{n-1})^2 + (P_{n-1}M_{n-1})^2$

$$(OP_{n-1})^2 = (OM_{n-1})^2 + (P_{n-1}M_{n-1})^2 = 2p_n^2 + \frac{1}{2}p_{n-1}^2$$

$$\Rightarrow p_n^2 = \frac{1}{4}p_{n-1}^2 \Rightarrow p_n = \frac{1}{2}p_{n-1}$$

$$\therefore OP_n = p_n = \frac{1}{2}p_{n-1} = \frac{1}{2^2}p_{n-2} = ... = \frac{1}{2^{n-1}}p_1 = \frac{1}{2^n}$$

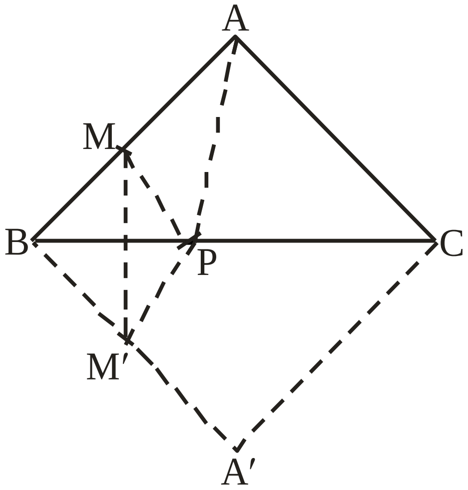
2. M is the mid point of side AB of equilateral triangle ABC. P is a point on BC such that AP + PM is minimum. If AB = 20 then AP + PM is

(A) $10\sqrt{7}$ (B) $10\sqrt{3}$

(C) $10\sqrt{5}$ (D) 10

Key. A

Sol. Take the reflection of ΔABC in BC.



PM = PM′

PA + PM = PA + PM′ it is minimum when M′ PA lies in a line

Now apply cosine rule in triangle ABM′

We will get $AM' = 10\sqrt{7}$

3. The algebraic sum of distances of the line ax + by + 2 = 0 from (1, 2), (2, 1) and (3, 5) is zero and the lines bx – ay + 4 = 0 and 3x + 4y + 5 = 0 cut the co-ordinate axes at concyclic points then

(a) $a + b = -\frac{2}{7}$

(b) area of the triangle formed by the line ax + by + 2 = 0 with coordinate axes is $\frac{14}{5}$.

(c) line ax + by + 3 = 0 always passes through the point $(-1, 1)$

(d) max {a, b} = $\frac{5}{7}$

Key. C

Sol. Line always passes through the point $\left(2, \frac{8}{3}\right)$ hence 6a + 8b + 6 = 0 $\Rightarrow$ 3a + 4b + 3 = 0

bx – ay + 4 = 0 and 3x + 4y + 5 = 0 are concyclic.

So, $m_1 m_2 = 1$

$\frac{b}{a} \cdot -\frac{3}{4} = 1 \Rightarrow 4a + 3b = 0$

Solving a = 9/7, b = –12/7

4. The algebraic sum of distances of the line ax + by + 2 = 0 from (1, 2), (2, 1) and (3, 5) is zero and the lines bx – ay + 4 = 0 and 3x + 4y + 5 = 0 cut the co-ordinate axes at concyclic points then

(a) $a+b=-\frac{2}{7}$

(b) area of the triangle formed by the line ax + by + 2 = 0 with coordinate axes is $\frac{14}{5}$.

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So, $m_1m_2 = 1$

$\frac{b}{a}.-\frac{3}{4}=1 \Rightarrow$ 4a + 3b = 0

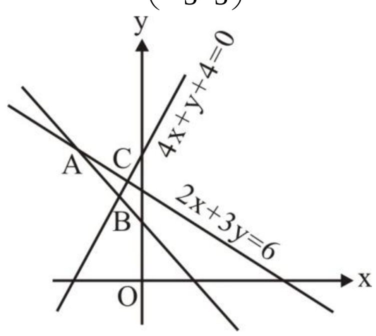
Solving a = 9/7, b = –12/7

5. The orthocentre of the triangle formed by the lines x + y = 1, 2x + 3y = 6 and 4x – y + 4 = 0 lies in

(A) I quadrant (B) II quadrant (C) III quadrant (D) IV quadrant

Key. A

Sol. Coordinates of A and B are (–3, 4) and $\left(-\frac{3}{5},\frac{8}{5}\right)$ if orthocentre p (h, k)



Then, (slope of PA) × (slope of BC) = –1

$\frac{k-4}{h+3}\times 4=-1$

$\Rightarrow$ 4k – 16 = –h –3

$\Rightarrow$ h + 4k = 13 ... (i)

and slope of PB × slope of AC = – 1

$$\Rightarrow \quad \frac{k-\frac{8}{5}}{h+\frac{3}{5}} \times -\frac{2}{3} = -1$$

$$\Rightarrow \quad \frac{5k-8}{5h+3} \times \frac{2}{3} = 1$$

$\Rightarrow$ 10k – 16 = 15th + 9

15th – 10k + 25 = 0

3h – 2k + 5 = 0 ... (ii)

Solivng Eqs. (i) and (ii), we get $h = \frac{3}{7}, k = \frac{22}{7}$

Hence, orthocentre lies in I quadrant.

6. A,B,C are three points on the curve $xy - x - y - 3 = 0$ which are not collinear. D,E,F are foot of perpendiculars from vertices A,B,C to the sides BC,CA and AB of ΔABC respectively. If (α,α) is incentre of ΔDEF then $'\alpha'$ can be

A) 1 B) 2 C) 3 D) 4

Key. C

Sol. Incentre of ΔDEF is ortho-centre of ΔABC. But in a rectangular hyperbola & ortho-centre lies on hyperbola $\Rightarrow \alpha^2 - 2\alpha - 3 = 0 \Rightarrow (\alpha-3)(\alpha+1) = 0 \Rightarrow \alpha = 3$

7. The reflection of the curve $xy = 1$ in the line $y = 2x$ is the curve $12x^2 + r\,xy + s\,y^2 + t = 0$ then the value of $'r'$ is

A) – 7 B) 25 C) – 175 D) 90

Key : A

HINT : The reflection of (α,β) in the line $y = 2x$ is

$$(\alpha_1,\beta_1) = \left(\frac{4\beta - 3\alpha}{5}, \frac{4\alpha + 3\beta}{5}\right) = \alpha_1\beta_1 = 1$$

$$\Rightarrow 12\alpha^2 - 7\alpha\beta - 12\beta^2 + 25 = 0$$

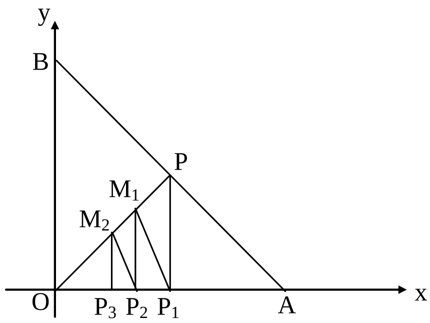
8. The line x + y = 1 meets x-axis at A and y-axis at B. P is the mid-point of AB. P_1 is the foot of the perpendicular from P to OA; M_1 is that from P_1 to OP; P_2 is that from M_1 to OA and so on. If P_n denotes the nth foot of the perpendicular on OA from M_{n-1}, then OP_n =

(a) 1/2 (b) $1/2^n$ (c) $1/2^{n/2}$ (d) $1/\sqrt{2}$

Key: b

Hint:

x + y = 1 meets x-axis at A(1, 0) and y-axis at B(0, 1).
The ordinates of P are (1/2, 1/2) and PP_1 is perpendicular to OA.
$\Rightarrow$ $OP_1 = P_1P = 1/2$
Equation of the line OP is y = x.
We have
$(OM_{n-1})^2 = (OP_n)^2 + (P_nM_{n-1})^2$
$= 2(OP_n)^2 = 2p_n^2$ (say)



Also, $(OP_{n-1})^2 = (OM_{n-1})^2 + (P_{n-1}M_{n-1})^2 = 2p_n^2 + 2p_n^2$

$\Rightarrow p_n^2 = \frac{1}{4}p_{n-1}^2 \Rightarrow p_n = \frac{1}{2}p_{n-1}$

$\therefore$ $OP_n = p_n = \frac{1}{2}p_{n-1} = \frac{1}{2^2}p_{n-2} = ... = \frac{1}{2^{n-1}}p_1 = \frac{1}{2^n}$

9. A line passes through (2, 0). The slope of the line, for which its intercept between $y = x - 1$ and $y = -x + 1$ subtends a right angle at the origin, is/are

(A) $\sqrt{3}$ (B) $-\sqrt{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) $-\frac{1}{\sqrt{3}}$

Key. C,D

Sol. The joined equation of straight line $y = x - 1$ and $y = -x + 1$ is

$$(x - y - 1)(x + y - 1) = 0$$

$\Rightarrow$ $x^2 - y^2 - 2x + 1 = 0$ (1)

Let equation of line passes through (2, 0) is

$y = m(x - 2)$ (2)

By homogenizing equation (1) with help of line (2) is

$$x^2 - y^2 - 2x\left(\frac{mx - y}{2m}\right) + \left(\frac{mx - y}{2m}\right)^2 = 0$$

Q coefficient of x^2 + coefficient of $y^2 = 0$

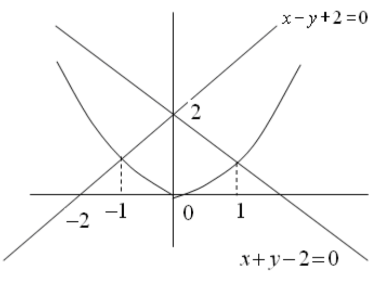
$\Rightarrow$ $m = \pm\frac{1}{\sqrt{3}}$

10. The complete set of values of $'a'$ for which the point (a, a^2), $a \in R$ lies inside the triangle formed by the lines $x - y + 2 = 0$, $x + y = 2$ and $x - axis$ is

(A) $(-2, 2)$ (B) $(-1, 1)$ (C) $(0, 2)$ (D) $(-2, 0)$

KEY : B

HINT :



(a, a^2) *lies of* $y = x^2$

$a - a^2 - 2 = 0 \quad a = -1, 2$

$a + a^2 - 2 = 0 \quad a = 1, -2$

11. The values of k for which lines $kx + 2y + 2 = 0,\ 2x + ky + 3 = 0,\ 3x + 3y + k = 0$ are concurrent

a) $\{2, 3, 5\}$ b) $\{2, 3, -5\}$ c) $\{3, -5\}$ d) $\{-5\}$

Key: C

Hint: Three non parallel lines are concurrent if $\Delta = 0$

$\begin{vmatrix} k & 2 & 2 \\ 2 & k & 3 \\ 3 & 3 & k \end{vmatrix} = 0 \Rightarrow k = 2, 3, -5$ But for k= 2, first two lines are parallel.

12. A straight line passes through the point of intersection $x - 2y - 2 = 0$ and $2x - by - 6 = 0$ and the origin then the complete set of values of b for which the acute angle between this line and y = 0 is less than 45^o

(A) $(-\infty, 4) \cup (7, \infty)$ (B) $(-\infty, 5) \cup (7, \infty)$

(C) $(-\infty, 4) \cup (5, 7) \cup (7, \infty)$ (D) $(-\infty, 4) \cup (4, 5) \cup (7, \infty)$

Key: D

Hint: As line passes through the point of intersection of $x - 2y - 2 = 0$ and $2x - by - 6 = 0$

It can be represented as $\lambda(x - 2y - 2) + (2x - by - 6) = 0$

As it passes through the origin

$-2\lambda - 6 = 0$

$\lambda = -3$

$\therefore$ equation of the line is $-x + (6 - b)y = 0$

Its slope is $\dfrac{1}{6 - b}$

As its angle with y = 0 is less than $\dfrac{\pi}{4}$

$\therefore -1 < \dfrac{1}{6 - b} < 1$

$\Rightarrow 6 - b > 1 \text{ or } < -1 \Rightarrow b < 5 \text{ or } b > 7$

But $b \neq 4$ (as the lines intersect)

$\therefore b \in (-\infty, 4) \cup (4, 5) \cup (7, \infty)$

13. Equation of angle bisector of the lines 3x - 4y + 1 = 0 and 12x + 5y - 3 = 0 containing the point (1, 2) is

(A) 3x + 11y - 4 = 0 (B) 99x - 27y – 2 = 0

(C) 3x + 11y + 4 = 0 (D) 99x + 27y - 2 = 0

Key: B

Hint: Since 3 × 1 - 4 × 2 + 1 and 12 × 1 + 5 × 2 - 3 are of the opposite sign, so required angle bisector is given by

$$\frac{3x-4y+1}{5}=-\left(\frac{12+5y-3}{13}\right)$$

14. Let S be the set of all values of α such that the points $(\alpha,6)$, (-5, 0) and (5, 0) form an isosceles triangle. Then the value of $\sum_{\alpha\in S}\alpha^2$ is

(A) 356 (B) 18 C) 178 (D) 338

Key: A

Hint α can take 5 values :0,3,-3,13.-13

15. If the orthocenter and circumcentre of a triangle are $(0,0)$ and $(3,6)$ respectively then the centroid of the triangle is

(A) $(1,2)$ (B) $(2,4)$ (C) $\left(\frac{2}{3},\frac{4}{3}\right)$ (D) $\left(\frac{1}{3},\frac{2}{3}\right)$

Key: B

Hint In any triangle centroid divides the line joining orthocenter and circumcentre internally in the ratio 2 : 1.

So, centroid is $(2,4)$.

16. The line $L_1\equiv 4x+3y-12=0$ intersects the $x-$axis and $y-$axis at A and B respectively. A variable line perpendicular to L_1 intersect the x and $y-$axes at P and Q respectively. The locus of the circumcentre of triangle ABQ is

A) $6x-8y+7=0$ B) $6x+8y-25=0$ C) $8x-6y+7=0$ D) $14x-12y+3=0$

Key. A

Sol. clearly circumcentre of triangle ABQ will lie on perpendicular bisector of line AB, which is $6x-8y+7=0$

17. If the area of the rhombus enclosed by the lines $lx\pm my\pm n=0$ be 2square units, then

A) $l,2m,n$ are in G.P B) l,n,m are in G.P

C) $lm=n$ D) $ln=m$

Key. B

Sol. By solving the sides of the rhombus, the vertices are

$$\left(0,\frac{-n}{m}\right),\left(\frac{-n}{l},0\right)\left(0,\frac{n}{m}\right),\left(\frac{n}{l},0\right)$$

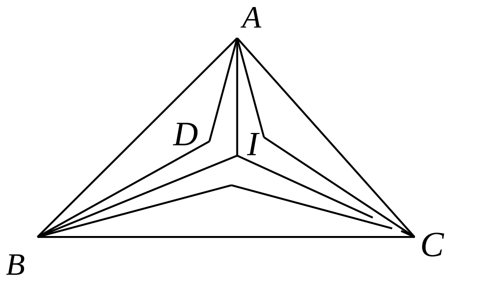
$\therefore$ The area = $\frac{1}{2}\left(\frac{2n}{m}\right)\left(\frac{2n}{l}\right)=2\Rightarrow n^2=lm$

18. If P is a point which moves inside an equilateral triangle of side length 'a' such that it is nearer to any angular bisector of the triangle than to any of its sides, then the area of the region in which P lies is __________ sq units

A) $a^2\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$ B) $\frac{\sqrt{3}a^2}{2}\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$ C) $\sqrt{3}a^2\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$ D) a^2

Key. B

Sol. Shaded area is the region traced by P, its area = $\Delta ABC - 3\Delta ABD$



$$=\frac{\sqrt{3}}{4}a^2-\frac{3}{2}a\times\frac{a}{2}\tan 15^0$$

$$= =\frac{\sqrt{3}}{2}a^2\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$$

19. In ΔABC orthocentre is (6,10) circumcentre is (2,3) and equation of side $\overrightarrow{BC}$ is 2x+y=17.Then the radius of the circumcircle of ΔABC is

a) 4 b) 5 c)2 d) 3

Key: B

Hint Image of orthocenter of ΔABC w.r.t. $\overrightarrow{BC}$ lies on the circle.

20. The area of the triangle formed by the line x + y = 3 and the angular bisectors of pair of straight lines $x^2-y^2+2y=1$ is

A. 8 sq.units B. 6 sq.units C. 4 sq.units D. 2 sq.units

Key. D

Sol. $x^2-(y-1)^2=0$ is given pair of lines

Vertices are (0,1),(0,3),(2,1) ,

Angular bisector is $x(y-1)=0$

Area = 2 sq.units

21. Let $O(0,0),P(3,4),Q(6,0)$ be the verticals of triangle OPQ .The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The point S is such that OS = PS = QS.Then RS =

A. $\frac{13}{16}$ B. $\frac{11}{12}$ C. $\frac{13}{24}$ D. $\frac{11}{24}$

Key. D

Sol. R is centroid .S is circumcentre . $R=\left(3,\frac{4}{3}\right), S=\left(3,\frac{7}{8}\right)$

RS= $\frac{11}{24}$

22. An equilateral triangle has its centroid at origin and one side is $x+y=1$. The equations of the others sides are

A. $y+1=(2\pm\sqrt{3})(x+1)$

B. $y+1=(2\pm\sqrt{3})x, y+1=(3\pm\sqrt{3})x$

C. $y+1=(3\pm\sqrt{3})(x-1), y+1=\sqrt{3}x$

D. $y\pm1=(3\pm\sqrt{3})(x-1), y+1=\frac{\sqrt{3}-1}{\sqrt{3}+1}(x+1)$

Key. A

Sol. Third vertex 'A' lies on x – y =0 and in III quadrent

Perpendicular distance from (0,0) to x + y =1 is $\frac{1}{\sqrt{2}}$

$\therefore AO=\sqrt{2} \Rightarrow A(-1,-1)$

If m is the slope of other side,

$\tan 60^0=\left|\frac{m+1}{1-m}\right|$

$\Rightarrow m=2\pm\sqrt{3}$

23. Triangle is formed by the lines x + y = 0, x – y = 0 and $lx+my=1$. If l and m vary subject to the condition $l^2+m^2=1$, then the locus of its circumcentre is

(A) $(x^2-y^2)^2=x^2+y^2$

(B) $(x^2+y^2)^2=(x^2-y^2)$

(C) $(x^2+y^2)^2=4x^2y^2$

(D) $(x^2-y^2)^2=(x^2+y^2)^2$

Key. A

Sol. Circumcentre of the triangle formed by the given lines is given by

$\left(\frac{l}{l^2-m^2}, \frac{m}{l^2-m^2}\right)$

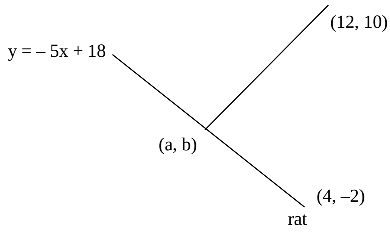
Hence the locus of this point is

$(x^2-y^2)^2 = x^2+y^2$

24. A piece of cheese is located at (12, 10) in a coordinate plane. A mouse is at (4, –2) and is running up the line y = –5x + 18. At the point (a, b), the mouse starts getting farther from the cheese rather than closer to it. The value of (a + b) is

(A) 6 (B) 10 (C) 18 (D) 14

Key. B



Sol.

a = 2, b = 8
a + b = 10

25. $A(3x_1, 3y_1)$, $B(3x_2, 3y_2)$, $C(3x_3, 3y_3)$ are vertices of a triangle with orthocentre H at $(x_1 + x_2 + x_3, y_1 + y_2 + y_3)$ then the $\angle ABC$

A. $\frac{\pi}{2}$ B. $\frac{\pi}{3}$ C. $\frac{\pi}{6}$ D. $\frac{\pi}{4}$

KEY. B

SOL. Centroid $G = \left(\frac{3x_1 + 3x_2 + 3x_3}{3}, \frac{3y_1 + 3y_2 + 3y_3}{3}\right) = (x_1 + x_2 + x_3, y_1 + y_2 + y_3) = H$

$\therefore \angle ABC = \pi / 3$

26. The area of the triangle with vertices $(a,b),(x_1, y_1),(x_2, y_2)$ where a, x_1, x_2 are in G.P. with common ratio 'r' and b, y_1, y_2 are in G.P with common ratio 's' is

A. $|ab(r-1)(s-1)(s-r)|$ B. $\frac{1}{2}|ab(r+1)(s+1)(s-r)|$

C. $\frac{1}{2}|ab(s-1)(r-1)(s-r)|$ D. $\frac{1}{2}abrs$

KEY. C

SOL. a, x_1, x_2 are in GP with C.R is 'r', b, y_1, y_2 are in G.P with C.R is s, $x_1 = ar, x_2 = r^2$,
$y_1 = bs, x_2 = bs^2$

27. If h denote the A.M, k denote G.M of the intercepts made on axes by the lines passing through (1, 1) then (h, k) lies on

A. $y^2 = 2x$ B. $y^2 = 4x$ C. $y = 2x$ D. $x + y = 2xy$

KEY. A

SOL. $a = x -$ intercept, b = y - intercept

2h = a + b, $k^2 = ab$

$\frac{x}{a} + \frac{y}{b} = 1$, substitute (1, 1)

$\frac{1}{a} + \frac{1}{b} = 1$

$a + b = ab$

$2h = k^2 \Rightarrow y^2 = 2x$

28. A straight rod of length 3*l* units slides with its ends A, B always on the x and y axes respectively then the locus of centroid of ΔOAB is

A. $x^2+y^2=3l^2$ B. $x^2+y^2=l^2$ C. $x^2+y^2=4l^2$ D. $x^2+y^2=2l^2$

KEY. B

SOL. Let OA= a, OB = b, AB = 3l

A = (a, 0), b = (0, b)

Let G(x, y) = $\left(\frac{a}{3},\frac{b}{3}\right)$, a = 3x, b = 3y

$a^2+b^2=9l^2 \Rightarrow x^2+y^2=l^2$

29. By translation of axes the equation $xy-x+2y-6=0$ changed as XY =c then c =

A. 4 B. 5 C. 6 D. 7

KEY. A

SOL. New origin $(x_1,y_1)=\left(\frac{-f}{h},\frac{-g}{h}\right)=(-2,1)$

Transformed equation of $xy-x+2y+6=0$ is xy = 4

= C = 4

30. A line has intercepts a, b on axes when the axes are rotated through an angle α, the line makes equal intercepts on axes then $\tan\alpha$ =

A. $\frac{a+b}{a-b}$ B. $\frac{a-b}{a+b}$ C. $\frac{a}{b}$ D. $\frac{b}{a}$

KEY. B

SOL. Equation of the lime $\frac{x}{a}+\frac{y}{b}=1$

Transformed eqution is $\frac{1}{a}(x\cos\alpha-y\sin\alpha)+\frac{1}{b}(x\sin\alpha+y\cos\alpha)=1$

Intercepts are equal

x - coefficient $\equiv$ y - coefficient

$\therefore \tan\alpha=\frac{a-b}{a+b}$

31. In a ΔABC, the coordinates of B are (0,0) AB=2, $\angle ABC=\frac{\pi}{3}$ and the mid point of BC is (2,0). The centroid of triangle is

1) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ 2) $\left(\frac{5}{3}, \frac{1}{\sqrt{3}}\right)$ 3) $\left(\frac{4+\sqrt{3}}{3}, \frac{1}{\sqrt{3}}\right)$ 4) $\left(\frac{4-\sqrt{3}}{3}, \frac{1}{\sqrt{3}}\right)$

Key. 2

Sol. Let $A(h,k)$ then $\cos 60^0 = \frac{h}{2} \Rightarrow h = 1$

$$\sin 60^0 = \frac{k}{2} \Rightarrow k = \sqrt{3}$$

$\therefore A(1, \sqrt{3})$

$\therefore$ centroid = $\left(\frac{5}{3}, \frac{\sqrt{3}}{3}\right)$

32. A point moves in the XY- plane such that the sum of its distances form two mutually perpendicular lines is always equal to 3. The area enclosed by the locus of the point is .

1) 18 Sq. Units 2) 9/2 Sq. Units 3) 9 Sq. Units 4) 27 Sq. Units

Key. 1

Sol. Given $|x| + |y| = 3$

Required area = $\frac{2c^2}{|ab|} = 9 \times 2$ =18 S.U

33. If the point (a, a) falls between the lines $|x+y| = 2$, then

1) $|a| = 2$ 2) $|a| = 1$ 3) $|a| < 1$ 4) $|a| < \frac{1}{2}$

Key. 3

Sol. Origin and (a, a) lies on the same side of the given lines $\therefore |a| < 1$

34. A ray travelling along the line $3x - 4y = 5$ after being reflected from a line $'l'$ travels along the line $5x + 12y = 13$. Then the equation of the line $'l'$ is

1) $x + 8y = 0$ 2) $x - 8y = 0$ 3) $32x + 4y + 65 = 0$ 4) $32x - 4y + 65 = 0$

Key. 2

Sol. The line 'l' can be any one of the bisectors of the angles between the lines $3x - 4y = 5$ and $5x + 12y = 13$

$\therefore$ Angular bisectors, $\frac{3x-4y-5}{5} = \pm\left[\frac{5x+12y-13}{13}\right]$

$\Rightarrow x - 8y = 0, 32x + 4y - 65 = 0$

35. The values of m for which the system of equations $3x + my = m$ and $2x - 5y = 20$ has a solution satisfy the conditions $x > 0, y > 0$ are given by the set

1) $\left\{m : m < \frac{-13}{2}\right\}$ 2) $\left\{m : m > \frac{17}{2}\right\}$

3) $\left\{m : m < \frac{-13}{2} \text{ or } m > \frac{17}{2}\right\}$ 4) $\left\{m : m > 30 \text{ or } m < \frac{-15}{2}\right\}$

Key. 4

Sol. Solve the equations $x = \frac{25m}{2m+15}, y = \frac{2m-60}{2m+15}$

But $x > o, y > 0 \Leftrightarrow 25m > 0, 2m+15 > 0, 2m-60 > 0$

$\Leftrightarrow m > 30$ or $m < \frac{-15}{2}$

36. $A_1, A_2 .....A_n$ are points on the line y=x lying in the positive quadrant such that $OA_n = nOA_{n-1}$ O being the origin. If $OA_1 = 1$ and the coordinates of A_n are $\left(2520\sqrt{2}, 2520\sqrt{2}\right)$, then n=

1) 5 2) 6 3) 7 4) 8

Key. 3

Sol. We have, $OA_n = n.OA_{n-1} = n(n-1).OA_{n-2} = ---$

$\therefore OA_n = \frac{n!}{\sqrt{2}}$

$\Rightarrow \sqrt{2}(2520\sqrt{2}) = n! \Rightarrow n! = 5040$

$\Rightarrow n = 7$

37. M is the mid point of side AB of an equilateral triangle ABC. P is a point on BC such that AP + PM is minimum. If AB = 20 then AP + PM is

(A) $10\sqrt{7}$ (B) $10\sqrt{3}$

(C) $10\sqrt{5}$ (D) 10

Key. A

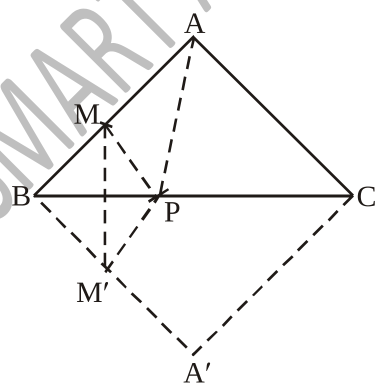
Sol. Take the reflection of ΔABC in BC.

PM = PM′

PA + PM = PA + PM′ it is minimum when M′ PA lies in a line

Now apply cosine rule in triangle ABM′

We will get $AM' = 10\sqrt{7}$



38. All points inside the triangle formed by A(1, 3), B(5, 6), C(–1, 2) will satisfy

(A) 2x + 2y ≤ 0 (B) 2x + y + 1 ≥ 0

(C) 2x + 3y – 12 ≥ 0 (D) –2x + 11 ≤ 0

Key. B

Sol. $L_1 \equiv 2x + 2y = 0$

$L_1(1, 3) > 0$ so a is wrong

$L_2 \equiv 2x + y + 1 = 0$

$L_2(1, 3) > 0$

$L_2(5, 6) > 0 \quad \Rightarrow \quad$ b is ture

$L_3(-1, 2) > 0$

39. Let P(1, 1), Q (2, 4), R(α, β) be the vertices of the triangle PQR. The point S(2, 2) inside the triangle PQR is such that

Area (ΔPQS) = Area (ΔPSR) = Area (Δ RSQ), then (α, β) =

(A) (2, 3) (B) (2, 5/2)

(C) (3, 1) (D) (5/2, 2)

Key. C

Sol. Here S must be cenroid of ΔPQR

$\Rightarrow \frac{1+2+\alpha}{3} = 2 \ \& \ \frac{1+4+\beta}{3} = 2$

$\Rightarrow \alpha = 3 \ \& \ \beta = 1$.

40. A system of line is given as $y = m_i x + c_i$, where m_i can take any value out of 0, 1, –1 and when m_i is positive then c_i can be 1 or –1 when m_i equal 0, c_i can be 0 or 1 and when m_i equals – 1, c_i can take 0 or 2. Then the area enclosed by all these straight line is

(A) $\frac{3}{\sqrt{2}}(\sqrt{2}-1)$ sq. units (B) $\frac{3}{\sqrt{2}}$ sq. units

(C) $\frac{3}{2}$ sq. untis (D) $\frac{3}{4}$ sq. units

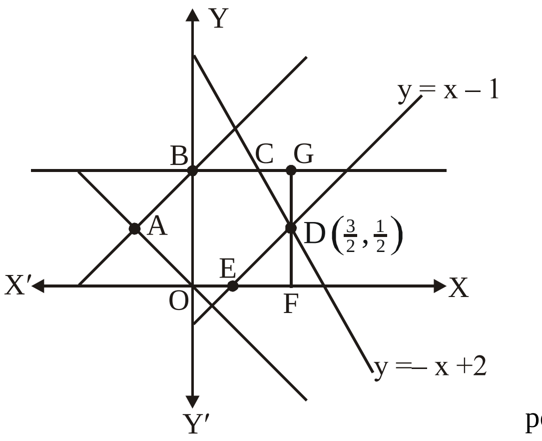
Key. C

Sol. Lines are y = 1, y = 0, y = –x, y = –x + 2, y = x +1, y = x – 1

Area of OABCDE

= area of OBGF

$= \frac{3}{2} \times 1 = \frac{3}{2}$ units.



41. Point A lies on y = x and point B on y = mx so that length AB = 4 units then value of m for which locus of mid point of AB represents a circle is

(A) m = 0 (B) m = –1
(C) m = 2 (D) m = –2

Key. B

SOL. LET CO-ORDINATES OF $A(X_1, X_1)$ AND $B(X_2, MX_2)$.
CLEARLY $(X_1 – X_2)^2 + (X_1 – MX_2)^2 = 16$
LET MID POINT OF P(H, K)
$\Rightarrow$ $X_1 + X_2 = 2H$ AND $X_1 + MX_2 = 2K$
$\Rightarrow$ $(X_1 – X_2)^2 + 4X_1X_2 = 4H^2$ AND $(X_1 – MX_2)^2 + 4MX_1X_2 = 4K^2$
$(X_1 – X_2)^2 + (X_1 – MX_2)^2 = 4H^2 + 4K^2 = 16$
when m = –1

42. The joint equation of two altitudes of an equilateral triangle is $\left(\sqrt{3}x - y + 8 - 4\sqrt{3}\right)\left(\sqrt{3}x + y - 12 - 4\sqrt{3}\right) = 0$. The equation of the third altitude is

a) $\sqrt{3}x = 2 - 4\sqrt{3}$ b) $y - 10 = 0$ c) $\sqrt{3}x = 2 + 4\sqrt{3}$ d) $y + 10 = 0$

Key. B

Sol. The required altitude will be the bisector of obtuse angle between the 2 given altitudes

43. A line $x + 2y = 4$ is translated by 3 units, away from origin and then rotated by 30^0 in clock wise sense about the point where shifted line cuts x-axis . If m is the slope of line in new position then [m] where [.] denotes GIF, is

a) -1 b) -2 c) -3 d) -4

Key. A

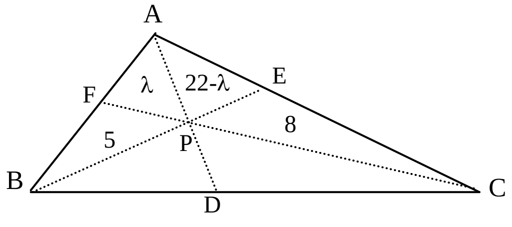
Sol. The required line is at a distance of 3 units from given line and parallel to it . Hence it is $x + 2y - 4 - 3\sqrt{5} = 0,$ cuts x-axis at $C(4 + 3\sqrt{5}, 0)$ with slope $\tan\theta = \frac{-1}{2}$ .After rotation about C by 30^0, slope becomes $m = \tan(\theta - 30^0) = \frac{-(2+\sqrt{3})}{2\sqrt{3}-1} = \frac{-(4+3\sqrt{3})}{11} \Rightarrow [m] = -1$

44. In a triangle ABC, E and F are points on AC and AB respectively. The lines BE and CF intersect at P. If area (BPF) = 5. area (PFAE) = 22, and area (CPE) = 8, then area (BPC) is

(A) 22 (B) 16
(C) 10 (D) not uniquely decidable

Key. C

Sol.



Let area of ΔPBC = x

$\Rightarrow$ $\frac{x}{5+\lambda} = \frac{8}{22-\lambda}$ and $\frac{x}{30-\lambda} = \frac{5}{\lambda}$

$\Rightarrow \frac{\lambda+5}{30-\lambda} = \frac{(22-\lambda)5}{8\lambda}$

$\Rightarrow 8\lambda^2 + 40\lambda = 5(\lambda^2 - 52\lambda + 660)$

$\Rightarrow \lambda^2 + 100\lambda - 1100 = 0$

$\Rightarrow (\lambda + 110)(\lambda - 10) = 0 \Rightarrow \lambda = 10$

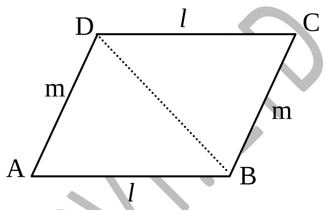
$\Rightarrow x = \frac{(30-\lambda)5}{\lambda} = \frac{(30-10)\times 5}{10}$ = 10 square units.

Ans. (C) 10 square units.

45. The perimeter of a parallelogram is 40. All the sides of the parallelogram are natural numbers, and at least one of its diagonals is a natural number. The number of noncongruent parallelograms so formed is
(A) 10 (B) 30
(C) 60 (D) 100

Key. D

Sol.



Let BD be integer and $l \geq m$

$2(l + m) = 40$

$\Rightarrow l + m = 20$

Possible values of m = 1, 2, 3, ..., 10

Note in any triplet of l, m, BD if atleast one is different parallelogram will be noncongruent

Now $l - m < BD < l + m$ (triangle inequality)

$\Rightarrow 20 - 2m < BD < 20$

$\Rightarrow$ No. of possible values of BD for a given 'm' is $20 - (20 - 2m) - 1 = 2m - 1$

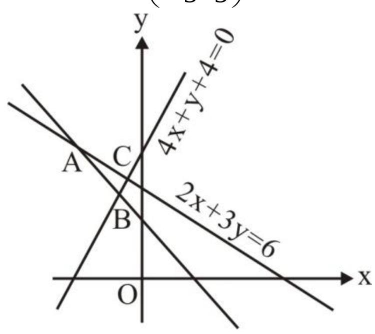
$\Rightarrow$ Total no. of noncongruent parallelogram $= \sum_{m=1}^{10}(2m-1) = 10^2 = 100$

Ans. (D) 100

46. The orthocentre of the triangle formed by the lines x + y = 1, 2x + 3y = 6 and 4x – y + 4 = 0 lies in
(A) I quadrant (B) II quadrant (C) III quadrant (D) IV quadrant

Key. A

Sol. Coordinates of A and B are (–3, 4) and $\left(-\frac{3}{5}, \frac{8}{5}\right)$ if orthocentre p (h, k)



Then, (slope of PA) × (slope of BC) = –1

$\frac{k-4}{h+3} \times 4 = -1$

$\Rightarrow \quad 4k - 16 = -h - 3$

$\Rightarrow \quad h + 4k = 13 \quad \quad ...(i)$

and slope of PB × slope of AC = – 1

$\Rightarrow \quad \dfrac{k-\frac{8}{5}}{h+\frac{3}{5}} \times -\dfrac{2}{3} = -1$

$\Rightarrow \quad \dfrac{5k-8}{5h+3} \times \dfrac{2}{3} = 1$

$\Rightarrow$ 10k – 16 = 15th + 9

15th – 10k + 25 = 0

3h – 2k + 5 = 0 ... (ii)

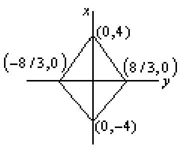
Solivng Eqs. (i) and (ii), we get $h = \dfrac{3}{7}, k = \dfrac{22}{7}$

Hence, orthocentre lies in I quadrant.

47. If $f(x+y) = f(x)f(y) \ \forall x, y \in R$ and $f(1) = 2$, then area enclosed by $3|x| + 2|y| \le 8$ is (in sq.units)

A) $f(4)$ B) $\dfrac{1}{2} f(6)$ C) $\dfrac{1}{3} f(6)$ D) $\dfrac{1}{3} f(5)$

Key. C

Sol. 

Area = $= 4 \times \dfrac{1}{2} \times \dfrac{8}{3} \times 4 = \dfrac{64}{3} = \dfrac{2^6}{3}$

$f(x) = 2^x$

48. $9x^2 + 2hxy + 4y^2 + 6x + 2fy - 3 = 0$ represents two parallel lines then

a) h = 6, f = 2 b) h = -6, f = 2 c) h = 6, f = -2 d) none

Key. A

Sol. Since the given equation represents a pair of parallel lines, we have $h^2 = ab \Rightarrow h = \pm 6$

Condition for pair of lines $\begin{vmatrix} 9 & h & 3 \\ h & 4 & f \\ 3 & f & -3 \end{vmatrix} = 0$

$\Rightarrow 108 \pm 36f - 9f^2 - 144 = 0$

$\Rightarrow$ f = 2 & h = 6

$\Rightarrow$ f = -2, h = -6