Statistics

Single Correct Answer Type

1. The arithmetic mean of the data given by

Variate (x)	0	1	2	3	...	n
Frequency (f)	${}^nC_0$	${}^nC_1$	${}^nC_2$	${}^nC_3$		${}^nC_n$

1) $\frac{1}{2}(n+1)$ 2) $\frac{1}{2}n$ 3) $\frac{2^n}{n}$ 4) $\frac{2^{n-1}}{n}$

Key. 2

Sol. $\overline{x} = \frac{0.{}^nC_0 + 1.{}^nC_1 + 2.{}^nC_2 + ... + n.{}^nC_n}{{}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n} \Rightarrow \overline{x} = \frac{\sum_{r=0}^{n} r.{}^nC_r}{\sum_{r=0}^{n} {}^nC_r} \Rightarrow \overline{x} = \frac{1}{2^n}\sum_{r=1}^{n} r.\frac{n}{r}\,{}^{n-1}C_{r-1}$

$\Rightarrow \overline{x} = \frac{n}{2^n}\sum_{2^n}^{n} {}^{n-1}C_{r-1} = \frac{n}{2^n}2^{n-1} = \frac{n}{2}.$

2. The mean of 10 observations is 16.3 by an error one observation is registered as 32 instead of 23. Then the correct mean is

1) 15.6 2) 15.4 3) 15.7 4) 15.8

Key. 2

Sol. Correct mean $= \frac{10\times16.3 + 23 - 32}{10} = \frac{154}{10} = 15.4$.

3. The mean of *n* items is $\overline{x}$. If the first item is increased by 1, second by 2 and so on, then the new mean is

1) $\overline{x} + n$ 2) $\overline{x} + \frac{n}{2}$ 3) $\overline{x} + \frac{n+1}{2}$ 4) $\overline{x} - \frac{n}{2}$

Key. 3

Sol. Let $x_1, x_2, ..., x_n$ be *n* items. Then $\overline{x} = \frac{1}{n}\Sigma x_i$.

Let $y_1 = x_1 + 1, y_2 = x_2 + 2, y_3 = x_3 + 3, ..., y_n = x_n + n$.

$\therefore$ The mean of the new series is

$\overline{y} = \frac{1}{n}\Sigma y_\iota = \frac{1}{n}\Sigma(x_i + i) \Rightarrow \overline{y} = \frac{1}{n}\Sigma x_i + \frac{1}{n}(1 + 2 + 3 + ... + n)$

$\Rightarrow \overline{y} = \overline{x} + \frac{1}{n}.\frac{n(n+1)}{2} = \overline{x} + \frac{n+1}{2}$.

4. 10 is the mean of a set of 7 observations and 5 is the mean of a set of 3 observations. The mean of the combined set is given by

1) 15 2) 10 3) 8.5 4) 7.5

Key. 3

Sol. Let $n_1 = 7.\bar{x}_1 = 10, n_2 = 3, \bar{x}_2 = 5$.

So, combined mean $= \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{85}{10} = 8.5$.

5. The average marks of boys in a class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is

1) 40 2) 20 3) 80 4) 60

Key. 3

Sol. Take $n_1 + n_2 = 100 \Rightarrow n_2 = 100 - n_1$

$\bar{x}_1$ =Average mark of boys =52, $\bar{x}_2$ = Average mark of girls = 42

$\bar{x} =$ Average mark of boys and girls = 50.

Combined mean $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} \Rightarrow 50 = \frac{n_1.52 + (100 - n_1)42}{100} \Rightarrow n_1 = 80$.

6. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set

1) is decreased by 2 2) is two time the original median

3) remains the same as that of the original set 4) is increased by 2

Key. 3

Sol. The median is the value of 5^{th} item when items are arranged in increasing or decreasing order $\Rightarrow$ median is unchanged.

7. If in a frequency distribution, the mean and median are 21 and 22 respectively, then its mode is approximately

1) 22.0 2) 20.5 3) 25.5 4) 24.0

Key. 4

Sol. Mode = 3 median – 2 mean = $3(22) - 2(21) = 24$.

8. If $\sum_{i=1}^{18}(x_i - 8) = 9$ and $\sum_{i=1}^{18}(x_i - 8)^2 = 45$ then the standard deviation of $x_1, x_2.....x_{18}$ is

1) 4/9 2) 9/4 3) 3/2 4) $\frac{2}{3}$

Key. 3

Sol. Let $d_i = x_i - 8$ but $\sigma_x^2 = \sigma_d^2 = \frac{1}{18}\Sigma d_i^2 - \left(\frac{1}{18}\Sigma di\right)^2 = \frac{1}{18} \times 45 - \left(\frac{9}{18}\right)^2 = \frac{5}{2} - \frac{1}{4} = \frac{9}{4}$.

$\therefore \sigma_x = \frac{3}{2}$.

9. If the standard deviation of $x_1, x_2, ...x_n$ is 3.5, then the standard deviation of

$-2x_1 - 3, -2x_2 - 3, ..., -2x_n - 3$ is

1) -7 2) -4 3) 7 4) 1.75

Key. 3

Sol. We know that if $d_i = \frac{x_i - A}{h}$ then $\sigma_x = |h| \sigma_d$.

In this case $-2x_i - 3 = \frac{x_i - 3/2}{-1/2}$.

So $h = -\frac{1}{2}$.

Thus $\sigma_d = \frac{1}{|h|}\sigma_x = 2 \times 3.5 = 7$.

10. Let $x_1, x_2, ..., x_n$ be *n* observations such that $\Sigma x_i^2 = 400$ and $\Sigma x_i = 80$. Then a possible value of *n* among the following is

1) 15 2) 18 3) 9 4) 12

Key. 2

Sol. $\frac{\Sigma x_i^2}{n} \geq \left(\frac{\Sigma x_i}{n}\right)^2 \Rightarrow n \geq 16$.

11. Marks scored by 100 students in a 25 marks unit test of Mathematics is given below. Their median is

Marks	0-5	5-10	10-15	15-20	20-25
Students	10	18	42	23	7

1) 12 2) 12.62 3) 12.3 4) 12.7

Key. 2

Sol. $l = 10, f = 42, m = 28, n = 100, c = 5$.

$\therefore Median = l + \frac{N/2 - m}{f} \times c = 10 + \frac{100/2 - 28}{42} \times 5 = 10 + \frac{22 \times 5}{42} = 10 + 2.62 = 12.62$.

12. The starting value of the model class of a distribution is 20. The frequency of the model class is 18. The frequencies of the classes preceeding and succeeding are 8,10 and the width of the model class is 5, then mode =

1) 18.5 2) 20.5 3) 21.4 4) 22.78

Key. 4

Sol. Mode $= l + \frac{f - f_1}{2f - f_1 - f_2} \times c = 20 + \frac{18 - 8}{36 - 8 - 10} \times 5 = 20 + \frac{50}{18} = 20 + 2.78 = 22.78$.

13. In the series of *2n* observations, half of them each equal to *a* and remaining half each equal to $-a$. If the standard deviation of the observations is 2, then $|a|$ equals to:

1) $\frac{1}{n}$ 2) $\sqrt{2}$ 3) 2 4) $\frac{\sqrt{2}}{n}$

Key. 3

Sol. Standard.

14. If a variable *x* takes values 0,1,2,….., *n* with frequencies proportional to be binomial coefficients ${}^{n}C_0, {}^{n}C_1, {}^{n}C_2, \ldots\ldots, {}^{n}C_n$ then the variance of *x* is

1) $\frac{n^2-1}{12}$ 2) $\frac{n}{2}$ 3) $\frac{n}{4}$ 4) $\frac{n^2+1}{12}$

Key. 3

Sol. Conceptual

15. The range of a random variable *X* is $\{0,1,2\}$ and $P(X=0)=3K^3, P(X=1)=4K-10K^2$, $P(X=2)=5K-1$. Then we have

1) $P(X=0)<P(X=2)<P(X=1)$ 2) $P(X=0)<P(X=1)<P(X=2)$

3) $P(X=1)+P(X=0)=P(X=2)$ 4) $P(X=1)>P(X=0)+P(X=2)$

Key. 2

Sol. $\Sigma P(X=x_i)=1.$

16. Consider any set of observations $x_1, x_2, x_3, \ldots.., x_{101};$ it being given that $x_1<x_2<x_3<\ldots.<x_{100}<x_{101};$ then the mean deviation of this set of observations about a point *k* is minimum when *k* equals

1) x_1 2) x_{51} 3) $\frac{x_1+x_2+\ldots..+x_{101}}{101}$ 4) x_{50}

Key. 2

Sol. Mean deviation is minimum when it is considered about he item, equidistant from the beginning and the end i.e., the median. In this case median is $\frac{101+1}{2}$ th i.e., 51st item i.e., x_{51}.

17. Mean of the numbers 1,2,3,…..,n with respective weights $1^2+1, 2^2+2, 3^2+3, \ldots\ldots, n^2+n$ is

1) $\frac{3n(n+1)}{2(2n+1)}$ 2) $\frac{2n+1}{3}$ 3) $\frac{3n+1}{4}$ 4) $\frac{3n+1}{2}$

Key. 3

Sol. Here for each $x_i = i$

Weight $w_i = i^2 + i$

Hence, the required mean $= \dfrac{\sum w_i x_i}{\sum w_i} = \dfrac{\sum\limits_{i=1}^{n} i\left(i^2+i\right)}{\sum\limits_{i=1}^{n}\left(i^2+i\right)}$

$$= \frac{\sum\limits_{i=1}^{n} i^3 + \sum\limits_{i=1}^{n} i^2}{\sum\limits_{i=1}^{n} i^2 \sum\limits_{i=1}^{n} i} = \frac{\dfrac{n^2(n+1)^2}{4} + \dfrac{n(n+1)(2n+1)}{6}}{\dfrac{n(n+1)(2n+1)}{6} + \dfrac{n(n+1)}{2}}$$

$$= \frac{\dfrac{n(n+1)}{2}\left\{\dfrac{n(n+1)}{2} + \dfrac{2n+1}{3}\right\}}{\dfrac{n(n+1)}{2}\left\{\dfrac{2n+1}{3} + 1\right\}}$$

$$= \frac{3n^2+7n+2}{2(2n+4)} = \frac{(3n+1)(n+2)}{4(n+2)} = \frac{3n+1}{4}.$$

18. The first and the third quartiles of the data given below:

Marks	*No. of the Students*
0-10	4
10-20	8
20-30	11
30-40	15
40-50	12
50-60	6
60-70	3

are respectively

1) 41.5, 43.8 2) 46.26, 49.69 3) 44.25, 45.2 4) 22.5, 45.2

Key. 4

Sol. Here, we construct the cumulative frequency table.

Class	Frequency	Cumulative frequency
0-10	4	4
10-20	8	12
20-30	11	23
30-40	15	38
40-50	12	50
50-60	6	56
60-70	3	59
Total	59	

For Q_1, Here $n = 59 \Rightarrow \dfrac{n}{4} = \dfrac{59}{4} = 14.75$.

$\therefore$ Class of first quartile is 20 – 30

$\Rightarrow Q_1 = 20 + \dfrac{14.75-12}{11} \times 10 = 20 + \dfrac{27.5}{11} = 22.5$.

For Q_3, Here $\dfrac{3n}{4} = \dfrac{3 \times 59}{4} = 44.25$.

$\therefore$ Class of third quartile is 40 – 50

$\Rightarrow Q_3 = 40 + \frac{44.25-38}{12} \times 10 = 40 + \frac{62.5}{12} = 45.2$.

19. For two data sets, each size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

1) $\frac{5}{2}$ 2) $\frac{11}{2}$ 3) $\sqrt{\frac{11}{2}}$ 4) $\frac{13}{2}$

Key. 2

Sol. $\sigma_1^2 = 4, n_1 = 5, \bar{x}_1 = 2$

$\sigma_2^2 = 5, n_2 = 5, \bar{x}_2 = 4$

$\bar{x}_{12} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{5\times 2 + 5\times 4}{10}$

$d_1 = \left(\bar{x}_1 = \bar{x}_{12}\right) = -1, d_2 = \left(\bar{x}_2 - \bar{x}_{12}\right) - 1$

$\sigma = \sqrt{\left[\frac{n_1\sigma_1^2 + n_2\sigma_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}\right]} = \sqrt{\left[\frac{5.4+5.5+5.1+5.1}{10}\right]} = \sqrt{\left[\frac{55}{10}\right]} = \sqrt{\left[\frac{11}{2}\right]}$.

$\therefore \sigma^2 = \frac{11}{2}$.

20. In a business venture a man can make a profit of Rs.2000/- with probability of 0.4 or have a loss of Rs.1000/- with probability 0.6. His expected profit is

1) Rs. 800/- 2) Rs. 600/- 3) Rs. 200/- 4) Rs. 400/-

Key. 3

Sol. $\mu = \Sigma x_i P\left(X = x_i\right)$.

21. A random variable *X* takes the values $-2, -1, 1$ and 2 with probabilities $\frac{1-a}{4}, \frac{1+2a}{4}, \frac{1-2a}{4}$ and $\frac{1+a}{4}$ respectively then

1) *a* can have any real value 2) $-\frac{1}{2} \le a \le \frac{1}{2}$

3) $-1 \le a \le 1$ 4) $\frac{1}{4} \le a \le \frac{1}{3}$

Key. 2

Sol. $0 \le P\left(A\right) \le 1$, *A* is any event.

22. A discrete random variable *X*, can take all possible integer values from 1 to K, each with a probability $\frac{1}{K}$. Its variance is

1) $\frac{K^2}{4}$ 2) $\frac{(K+1)^2}{4}$ 3) $\frac{K^2-1}{12}$ 4) $\frac{K^2-1}{6}$

Key. 3

Sol. $\Sigma x_i^2 P(X=x_i)-\mu^2$.

23. A player tosses two fair coins. He wins Rs.5/- if two heads occur, Rs.2/- if one head occurs and Rs.1/- if no head occurs. Then his expected gain is

1) $Rs.\frac{8}{3}$ 2) $Rs.\frac{7}{3}$ 3) Rs. 2.5 4) Rs. 1.5

Key. 3

Sol. Mean.

24. The range of random variable *X* is $\{1,2,3,4,....\}$ and the probabilities are $P(X=K)=\frac{3^{CK}}{\angle K}$; $K=1,2,3,4,..........$, then the value of *C* is

1) $\log_e 3$ 2) $\log_e 2$ 3) $\log_3(\log_e 2)$ 4) $\log_2(\log_e 3)$

Key. 3

Sol. $\Sigma P(X=x_i)=1$.

25. A person who tosses an unbiased coin gains two points for turning up a head and loses one point for a tail. If three coins are tossed and the total score *X* is observed, then the range of *X* is

1) $\{0,3,6\}$ 2) $\{-3,0,3\}$ 3) $\{-3,0,3,6\}$ 4) $\{-3,3,6\}$

Key. 3

Sol. HHT, HHT, HHH, TTT

$(2,2,-1)$ $(2,-1,-1)$ $(2,2,2)$ $(-1,-1,-1)$.

26. If the range of a random variable *X* is $\{0,1,2,3,...........\}$ with $P(X=k)=\frac{(k+1)a}{3^k}$ for $k\geq 0$, then $a=$

1) 2/3 2) 4/9 3) 8/27 4) 16/81

Key. 2

Sol. $\Sigma x_i P\left(X = x_o\right) = 1$.

27. If the variance of the random variable *X* is 4, then the variance of the random variable $5X + 10$ is

1) 100 2) 10 3) 50 4) 25

Key. 1

Sol. $V\left(ax \pm b\right) = a^2 V\left(X\right)$.

28. If $f\left(x\right)$ is the cumulative distributive function of a random variable *X* whose range is from $-\alpha$ to $+\alpha$, then $P\left(X < -\alpha\right) =$

1) 1 2) $\frac{1}{2}$ 3) 0 4) $\frac{1}{3}$

Key. 3

Sol. Conceptual.

Statistics

Assertion Reasoning Type

1. Statement-1: The variance of first *n* even natural numbers is $\frac{n^2-1}{4}$.

Statement-2: The sum of first *n* natural numbers is $\frac{n(n+1)}{2}$ and the sum of square of first *n* natural numbers is $\frac{n(n+1)(2n+1)}{6}$.

1) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1

2) Statement-1 is true, Statement-2 is false

3) Statement-1 is false, Statement-2 is true

4) Statement-1 is true, Statement-2 is true. Statement-2 is a correct explanation for Statement-1

Key. 3

Sol. Mean of first *n* even natural numbers $\frac{2+4+6.....+2n}{n}=\frac{2(1+2+....+n)}{n}=n+1$.

Varience of first *n* even natural numbers

$$=\frac{1}{n}\sum_{i=1}^{n}x_i^2-(\overline{x})^2=\frac{2^2+4^2+6^2+....+(2n)^2}{n}-(n+1)^2$$

$$=\frac{4n(n+1)(2n+1)}{n\quad 6}-(n+1)^2=(n+1)\left[\frac{2(2n+1)}{3}-(n+1)\right]=\frac{n^2-1}{3}.$$

2. Let $x_1,x_2,......x_n$ be *n* observations, and let $\overline{x}$ be their arithmetic mean and σ^2 be their variance.

Statement-1: Variance of $2x_1,2x_2,......,2x_n$ is $4\sigma^2$.

Statement-2: Arithmetic mean of $2x_1,2x_2,.......,2x_n$ is $4\overline{x}$.

1) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1

2) Statement-1 is true, Statement-2 is false

3) Statement-1 is false, Statement-2 is true

4) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1

Key. 2

Sol. $E(aX)=aE(X), V(aX)=a^2V(X) \Rightarrow$ Statement-1 is true, Statement-2 is false.