PHYSICS

The following questions consists of two statements each, printed as Assertion and Reason. While answering these questions you are to choose any one of the following four responses.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true but Reason is not correct explanation of the Assertion.

(C) If Assertion is true but the Reason is false.

(D) If Assertion is false but Reason is true.

Q. 1 **Assertion :** When a sphere and a solid cylinder are allowed to roll down an inclined plane, the sphere will reach the ground first even if the mass and radius of the two bodies are different.

Reason : The acceleration of the body rolling down the inclined plane is directly proportional to the radius of the rolling body. **[C]**

Q.2 **Assertion :** When a wheel is in pure rolling with constant acceleration then acceleration of topmost point on the circumference of wheel is not constant.

Reason : Wheel is said to be in pure rolling when the contact point does not slip on the surface.

Sol. **[D]** conceptual

Q.3 **Assertion :** The axis of rotation of a purely rotating body must pass through the centre of mass of body.

Reason : When the body is in pure rotation the axis of rotation remain at rest.

Sol. **[D]**

Conceptual

Q.4 **Assertion :** The centre of mass of a rod must not lie on its centre if linear mass density of rod is variable.

Reason : Centre of mass is point where the whole mass of body is assumed to be concentrated.

Sol. **[D]**

If linear mass density increases up to centre and then decreases in same manner then centre of mass may be at centre.

Q.5 **Assertion :** If there is no external force, on a body, then the angular momentum of the system is conserved.

Reason : If there is no external force torque on the body may or may not be zero. **[C]**

Q.6 **Assertion** : When ice on polar caps of earth melts, duration of the day increases.

Reason : $L = I\omega = I \cdot \frac{2\pi}{T}$ = constant **[A]**

Q.7 **Assertion** : If bodies slide down an inclined plane without rolling then all bodies reach the bottom simultaneously.

Reason : Acceleration of all bodies are equal and independent of the shape. **[A]**

Q.8 **Assertion** : A wheel moving down a perfectly frictionless inclined plane shall undergo slipping (not rolling)

Reason : For rolling, torque is required, which is provided by tangential frictional force. **[A]**

Q.9 **Assertion** : In pure rolling motion, net work done by friction is zero.

Reason : Sum of translational work done by friction and rotational work done by friction is zero. **[D]**

Q.10 **Assertion** : The centre of mass of a circular disc lies always at the centre of the disc.

Reason : Circular disc is a symmetrical body. **[A]**

Q.11 **Assertion :** If there is no external torque on a body about its centre of mass, then the velocity of the center of mass remains constant.

Reason : The linear momentum of an isolated system remains constant. **[IIT – 2007]**

[D]

Q.12 **Assertion :** Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

Reason : By the principle of conversation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline. **[IIT – 2008]**

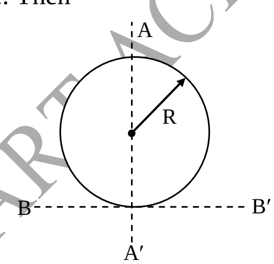
[D]

Q.13 **Assertion :** A rigid disc rolls without slipping on a fixed rough horizontal surface with uniform angular velocity. Then the acceleration of lowest point on the disc is zero.

Reason : For a rigid disc rolling without slipping on a fixed rough horizontal surface, the velocity of the lowest point on the disc is always zero.c **[D]**

Sol. For a disc rolling without slipping on a horizontal rough surface with uniform angular velocity, the acceleration of lowest point of disc is directed vertically upwards and is not zero. (Due to translation part of rolling, acceleration of lowest point is zero. Due to rotational part of rolling, the tangential acceleration of lowest point is zero and centripetal acceleration is non-zero and upwards) Hence Assertion is false.

Q.14 Figure shows a sphere of mass M and Radius 'R'. Let AA′ and BB′ be two axis as shown in figure. Then -



Assertion : Parallel axis theorem is not applicable between axis AA′ and BB′

Reason : $I_{BB'} = I_{AA'} + MR^2$

(A) Both of statements I and II are correct

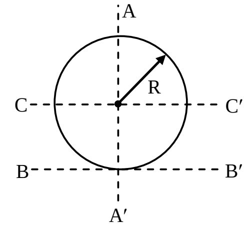
(B) Statement 'I' is correct but 'II' is false

(C) Statement 'I' is false but 'II' is correct

(D) Both of statements I and II are false

[A]

Sol. Parallel axis theorem is applicable only between two parallel axis.



By symmetry $I_{CC'} = I_{AA'}$

Parallel axis theorem

$\Rightarrow I_{BB'} = I_{CC'} + MR^2 = I_{AA'} + MR^2$

Q.15 **Assertion :** Torque acting on a rigid body depends upon the location of the origin of the co-ordinate system.

Reason : Moment of couple is different for different points in its plane. **[C]**

Q.16 **Assertion :** We can write in a equation : Torque + Energy.

Reason : Dimensions of torque and energy are same. **[D]**

Q.17 **Assertion :** When a sphere is rolling without sliding, it is possible that no point on it is at rest.

Reason : For rolling without sliding $v_{CM} = \omega R$, where v_{CM} is velocity of centre of mass with respect to ground. **[C]**

Sol. R is false if surface on which rolling is taking place is moving with respect to ground.

Q.18 **Statement-I:** If the earth expands in size without any change in mass he length of the day would increase.

Statement-II: The rotation of earth about its axis follows the law of conservation of angular momentum. **[A]**

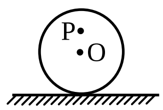
Q.19 **Statement-I:** A body moving in a straight line parallel to Y-axis can have angular momentum.

Statement-II: We can employ the concept of angular momentum only in rotatory motion **[C]**

Q.20 **Statement-I:** When a planet is at maximum distance from the sun, its speed is minimum.

Statement-II: The motion of planet around the sun does not follow the law of conservation of angular momentum. **[C]**

Q.1 A uniform disc rolls without slipping on a rough horizontal surface with uniform angular velocity. Point O is the centre of disc and P is a point on disc as shown. In each situation of column I a statement is given and the corresponding results are given in column II. Match the statements in column-I with the results in column-II.



Column I	**Column II**
(A) The velocity of point P on disc	(P) Change in magnitude with time
(B) The acceleration of point P on disc	(Q) Is always directed from that point (the point on disc given in column-I) towards centre of disc
(C) The tangential acceleration of point P on disc	(R) is always zero
(D) The acceleration of point on disc which is in contact with rough horizontal surface.	(S) is non-zero and remains constant in magnitude

Sol **(A → P)** **(B → Q, S)**
(C → P) **(D → Q, S)**

(A) speed of point P changes with time

(B) Acceleration of point P is equal to $\omega^2 \times$ (ω = angular speed of disc and X = OP). The acceleration is directed from P towards O.]

(C) The angle between acceleration of P (constant in magnitude) and velocity of P changes with time. Therefore , tangential acceleration of P changes with time.

(D) The acceleration of lowest point is directed towards centre of disc and remains constant with time.

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Q.2 A ring of mass m and radius R is placed on a rough inclined plane so that it rolls without slipping then Match the following :

Column-I	**Column-II**
(A) Linear acceleration of centre of mass	(P) is directly proportional to m
(B) Angular acceleration	(Q) is inversely proportional to m
(C) Rotatioal kinetic energy at any instant	(R) is inversely proportional to R
(D) Translational kinetic energy at any instant	(S) None

Sol. **A → S ; B → R ; C → P ; D → P**

$a = \frac{g\sin\theta}{\beta}$ and β is independent of m and R

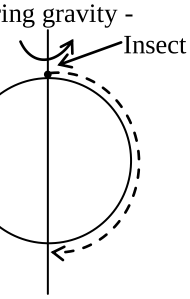
(B) The acceleration of (Q) Is always directed

$\alpha = \frac{a}{R}$

$K_R = \frac{1}{2} I\omega^2 = \frac{1}{2} \times mR^2 \times \left(\frac{at}{R}\right)^2 = \frac{1}{2} ma^2t^2$

$K_T = \frac{1}{2} mv^2 = \frac{1}{2} \times m \times (at)^2 = \frac{1}{2} ma^2t^2$

Q.3 A solid sphere is rotating freely about an axis as shown. An insect moves and follows the dotted path on the circumfercenec of sphere. Match the following ignoring gravity -



Column-I	**Column-II**
(A) Moment of inertia of system	(P) Will remain constant
(B) Angular velocity	(Q) will first increase then decrease
(C) Angular momentum	(R) will first decrease then increase
(D) Rotational kinetic energy	(S) will continuously decrease

Sol. **A → Q ; B → R ; C → P ; D → R**

Angular momentum is conserved and moment of inertia first increases and then decreases

$K_R = \frac{(I\omega)^2}{2I}$

Q.4 Two particles A and B of mass m each, are joined by a rigid massless rod of length ℓ. A particle P of mass m, moving with speed u normal to AB, strikes A and stick to it. The centre of mass of the 'A + B + P' system after collision is C. If u = 4m/s, m = 3 kg, ℓ = 3 m, then (All quantities in column are in SI units)

	Column I		**Column II**
(A)	AC is equal to	**(P)**	12
(B)	Angular momentum of system about C is	**(Q)**	$\frac{2}{3}$
(C)	Moment of inertia of system about C is	**(R)**	18
(D)	Angular velocity of system abut C is	**(S)**	1

Sol. **A → S ; B → P ; C → P ; D → Q**

(A) AC = $\frac{2m\times 0+m\times l}{3m}$

(B) Angular momentum about C = $mu\frac{\ell}{3}$

(C) moment of inertia about C

$= 2m\left(\frac{\ell}{3}\right)^2 + m\left(\frac{2\ell}{3}\right)^2$

(D) Angular velocity = $\frac{\text{Angular momentum}}{\text{moment of inertia}}$

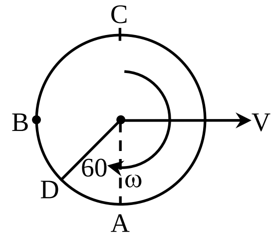
Q.5 A ring, a disc, a solid sphere and a hollow sphere are placed on a rough inclined plane of inclination θ = 30º. Match the following regarding the linear acceleration of:

Column - I	**Column - II**
(A) Ring	(P) $\frac{3g}{10}$
(B) Disc	(Q) $\frac{5g}{14}$
(C) Solid sphere	(R) $\frac{g}{4}$
(D) Hollow sphere	(S) $\frac{g}{3}$

Sol. **(A) → R; (B) → S; (C) → Q; (D) → P**

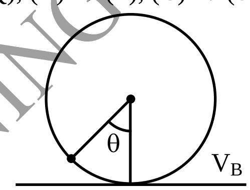
$a = \frac{g\sin\theta}{1+I/MR^2}$

Q.6 A rigid body is rolling without slipping on the horizontal surface



Column-I	**Column-II**
(A) Velocity at point A i.e. V_A	(P) $V\sqrt{2}$
(B) Velocity at point B i.e. V_B	(Q) Zero
(C) Velocity at point C i.e. V_C	(R) V
(D) Velocity at point D i.e. V_D	(S) 2V

Sol. (A) → (Q), (B) → (P), (C) → (S), (D) → (R)



$v_p = 2v\sin\theta/2$

Case-I - At point A i.e. θ = 0

$v_A = 2v\sin\theta/2 = 0$

At point B i.e. θ = 90

$v_B = 2v\sin 45 = v\sqrt{2}$

At point C i.e. θ = 180

$v_C = 2V\sin 90 = 2v$

At point D i.e.

θ = 60

$v_D = 2v\sin 30 = v$

Q.7

Column-I	**Column-II**
(A) Angular displacement	(P) Axial vector
(B) Angular momentum	(Q) It is directed along the axis of Rotation and normal to the plane
(C) Torque is zero	(R) Moment of linear momentum
(D) In pure rolling. K.E. of sphere at any point is	(S) $\frac{7}{10}mv^2$
	(T) Angular momentum is constant

Sol. **(A) → (P) & (Q), (B) → (P),(Q) & (R)**
(C) → (T), (D) → (S)

All axial vectors are directed along the axis of rotation and normal to plane

For (B) angular momentum

$\vec{L} = \vec{r} \times \vec{p}$

It is an axial vector

For (C) $\vec{\tau} = \frac{d\vec{L}}{dt}$

when $\vec{L}$ = constants

$\tau = 0$

For (D) and (S) in pure rolling consist translatory and rotatory motion both.

(K.E.) total $(K.E.)_{total} = (K.E.)_r + (K.E.)_t$

$= \frac{1}{2} I \omega^2 + \frac{1}{2} mv^2$

$= \frac{1}{2} \cdot \frac{2}{5} mR^2\omega^2 + \frac{1}{2} mv^2 \quad (\because \omega^2 = \frac{v^2}{R^2})$

$= \frac{7}{10} mv^2$

Q.8 A solid sphere is rotating about an axis as shown in figure. An insect follows the dotted path on the circumference of sphere as shown. Match the following :

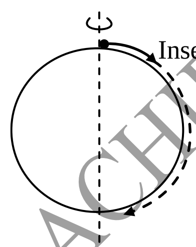


Table-1	**Table-2**
(A) Moment of inertia	(P) will remain constant
(B) Angular velocity	(Q) will first increase then decrease
(C) Angular momentum	(R) will first decrease then increase
(D) Rotational kinetic energy	(S) will continuously decrease

Ans. **A → Q ; B → R ; C → P ; D → R**

Q.9 A disc rolls on ground without slipping. Velocity of centre of mass is v. There is a point P on circumference of disc at angle θ. Suppose v_p is the speed of this point. Then, match the following table :

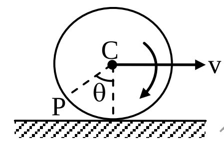


Table-1	**Table-2**
(A) If θ = 60º	(P) $v_p = \sqrt{2}\,v$
(B) If θ = 90º	(Q) $v_p = v$
(C) If θ = 120º	(R) $v_p = 2v$
(D) If θ = 180º	(S) $v_p = \sqrt{3}\,v$

Ans. A → Q ; B → P ; C → S ; D → R

Q.10 A semi-circular ring has mass m and radius R as shown figure. Let I_1, I_2, I_3 and I_4 be the moments of inertias of the four axes as shown. Axis 1 passes through centre and is perpendicular to plane of ring. Then, match the following :

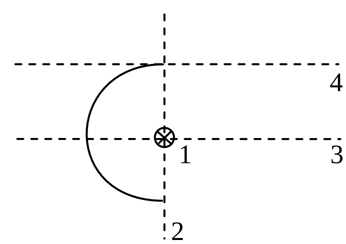
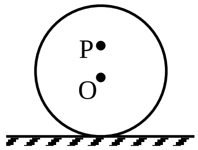


Table-1	**Table-2**
(A) I_1	(P) $\frac{mR^2}{2}$
(B) I_2	(Q) $\frac{3}{2} mR^2$
(C) I_3	(R) mR^2
(D) I_4	(S) Data is insufficient

Ans. **A → R ; B → P ; C → P ; D → Q**

Q.11 A uniform disc rolls without slipping on a rough horizontal surface with uniform velocity. Point O is centre of disc and P is a point on disc as shown. Then match the column I with column II.

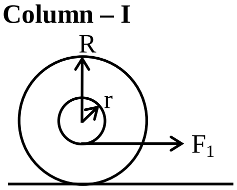


Column-I	**Column-II**
(A) The velocity of point P on disc	(P) Change in magnitude with time
(B) The acceleration of point P on disc	(Q) is always directed towards the centre of disc
(C) The tangential acceleration of point P on disc	(R) is always zero
(D) The acceleration of point on disc which is in contact with rough horizontal surface	(S) is non zero and constant in magnitude
	(T) is constant

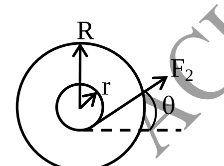
Ans. **A → P ; B → Q,S ; C → P ; D → Q,S,T**

Q.12 A yo-yo is resting on a perfectly rough horizontal table. Forces F_1, F_2, F_3 and F_4 are applied separately as shown.

Column – I **Column – II**

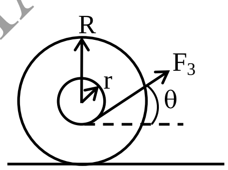
(A) 

(P) Centre of mass accelerates towards left

(B) 

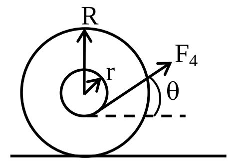
$\cos\theta < \left(\frac{r}{R}\right)$

(Q) Centre of mass accelerates towards right

(C) 

$\cos\theta > \left(\frac{r}{R}\right)$

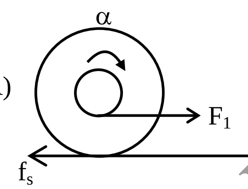
(R) Friction acts towards left

(D) 

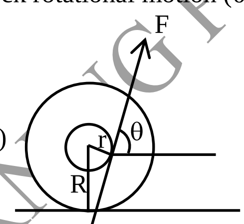
$\cos\theta = \left(\frac{r}{R}\right)$

(S) Friction acts towards right

Sol. **A → Q,R ; B → P,R ; C → Q,R ; D → R**

(A) 

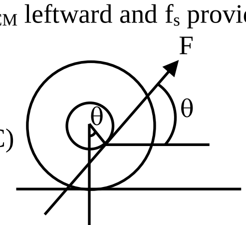
a_{CM} towards right and for clockwise α, f_s leftwards check rotational motion (α) about centre

(B) 

$\cos\theta < \frac{r}{R}$

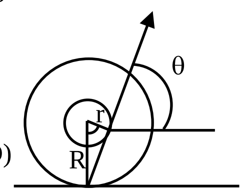
For (B), taking ΣT about instantaneous centre of rotation (ICOR), α is anticlockwise hence for $a_{CM} = \alpha R$

a_{CM} leftward and f_s provides leftward a_{CM}.

(C) 

$\cos\theta > \frac{r}{R}$

For (C) taking ΣT about instantaneous centre of rotation (ICOR), α is clockwise hence a_{CM} rightward and f_s leftward to create clockwise α.

(D) 

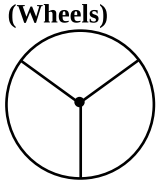
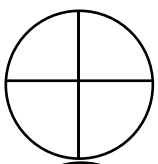
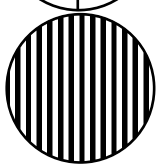
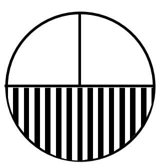
$\cos\theta = \frac{r}{R}$

For (D) ΣT = 0 about instantaneous centre of rotation (ICOR) ⇒ α = 0, a_{cm} = 0.

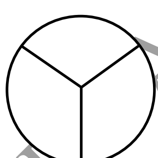
No motion hence f_s leftward to balance horizontal component of F.

Q.13 Column I and column II contains four entries each. Entry of column I are to be uniquely matched with only one entry of column II.

Each of the four wheels in column I has an outer ring having radius R and mass m. Other than the outer ring wheels comprise of same uniform rods (each of mass m and length R) or some lamina (having the same mass m). column II gives radius of gyration about axis passing through centre of ring perpendicular to its plane.

Column – I (Wheels)	**Column – II (Radius of gyration)**
(A) 	**(P)** $R\sqrt{\frac{3}{4}}$
(B) 	**(Q)** $R\sqrt{\frac{11}{18}}$
(C)  One disc + ring	**(R)** $\frac{R}{\sqrt{2}}$
(D)  Outer ring + half disc + one rod	**(S)** $R\sqrt{\frac{7}{15}}$

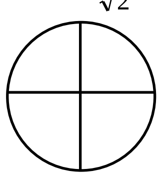
Sol. **A → R ; B → S ; C → P ; D → Q**

(A) 

$$I = mR^2 + 3\left(\frac{mR^2}{3}\right) = 2mR^2$$

$$\therefore 2mR^2 = 4mK^2$$

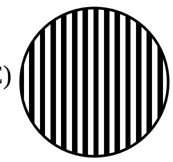
$$K = \frac{R}{\sqrt{2}}$$

(B) 

$$I = mR^2 + 4\left(\frac{mR^2}{3}\right) = \frac{7mR^2}{3}$$

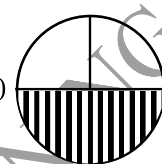
$$\therefore 5mK^2 = \frac{7mR^2}{3}$$

$$K = R\sqrt{\frac{7}{15}}$$

(C) 

$$I = \frac{3}{2}mR^2 = 2mK^2$$

$$K = \frac{\sqrt{3}}{2}R$$

(D) 

$$I = \frac{mR^2}{3} + \frac{mR^2}{2} + mR^2 = \frac{11mR^2}{6}$$

$$\therefore \frac{11mR^2}{6} = 3mK^2$$

$$K = R\sqrt{\frac{11}{18}}$$

Q.14 A particle of mass 1 kg is projected with velocity m/s at 45º with ground. When, the particle is at highest point: ($g = 10\ m/s^2$)

Column-I	**Column-II**
(A) Net torque on the particle about point of projection	(P) 200 SI unit
(B) Angular momentum of the particle about point of projection	(Q) 400 SI unit
(C) Angular velcoity of particle about point of projection	(R) 1.0 SI unit
	(S) None

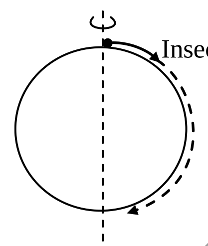
Ans. A → Q; B → Q; C → S

Q.15 Match the following :

Column-I	**Column-II**
(A) In pure rolling work done by friction	(P) is always zero
(B) In forward slipping done by friction	(Q) may be zero
(C) In backward slipping work done by friction	(R) is negative
	(S)is positive
	(T) may be negative
	(U) may be positive

Ans. A $\rightarrow$ Q,T,U; B $\rightarrow$ Q,T,U; C $\rightarrow$ Q,T,U

Q.16 A solid sphere is rotating about an axis as shown in figure. An insect follows the dotted path on the circumference of sphere as shown. Match the following:



Column-I	**Column-II**
(A) Moment of inertia	(P) will remain cons.
(B) Angular velocity	(Q) will first increase then decrease
(C) Angular momentum	(R) will first decrease then increase
(D) Rotational kinetic energy	(S) will continuously decrease
	(T) will continuously increase
	(U) data is insufficient

Ans. A $\rightarrow$ Q; B $\rightarrow$ R; C $\rightarrow$ P; D $\rightarrow$ R

Q.17 A horizontal plane support a vertical cylinder of radius 20 cm and a disk of mass 2 kg is attached to the cylinder by a horizontal thread of length $\pi/5$ m can move frictionlessly on the table. An initial velocity 1 m/s is imparted to the disk. Consider a situation when $\frac{\pi}{20}$ m length of string is wrapped on cylinder.

Column-I	**Column-II**
(A) Angular velocity of disk (in rad/sec)	(P) $\frac{\pi^2}{10}$
(B) Time taken (in sec) (in wrapping $\pi/20$ meter)	(Q) $\frac{40}{3\pi}$
(C) Tension in string (in N)	(R) $\frac{20}{3\pi}$
(D) Time taken (in sec) after which disk will hit cylinder	(S) $\frac{7\pi^2}{160}$

Sol. **A $\rightarrow$ R B $\rightarrow$ S C $\rightarrow$ Q D $\rightarrow$ P**

$\omega = \frac{v}{\ell - R\theta}$ and $t = \frac{\ell\theta - R\frac{\theta^2}{2}}{v}$

Tension in string $T = \frac{mv^2}{(\ell - R\theta)}$

where v = velocity of disk

θ = Angle rotated by disk

ℓ = length of string

Q.18 A cylinder, hollow sphere , solid sphere and a ring all having mass 1 kg are released from rest on a inclined plane having angle of inclination 37º (tan 37º = 3/4). Co-efficient of friction between bodies and plane is 'μ' then match the following column –

Column-I	**Column-II**
(A) If $\mu < 0.25$, body which must not undergo pure rolling motion	(P) Ring
(B) If $\mu \le 0.3$, work done	(Q) Hollow sphere

by friction must be negative for

(C) If $\mu = 0.4$, total mechanical energy will be conserved for — (R) Solid sphere

(D) If $\mu = 0.25$, friction force will be 2N for — (S) Cylinder

Sol. A → P,Q,S ; B → P ; C → P,Q,R,S; D → Q,R

For pure rolling $\mu \geq \dfrac{\tan\theta}{\left(1+\dfrac{mr^2}{I}\right)}$ and

$$f = \frac{mg\sin\theta}{(1+mr^2/I)}$$

Body	condition on μ	Friction force
Ring	$\mu \geq \frac{3}{8}$	3N
Cylinder	$\mu \geq \frac{1}{4}$	2N
Hollow sphere	$\mu \geq \frac{3}{10}$	$\frac{12}{5}$ N
Solid sphere	$\mu \geq \frac{3}{14}$	$\frac{12}{7}$ N

Q.19 A particle moves according to the law given below where 'x', 'v' and 'a' are displacement, velocity and acceleration respectively and ω, A are +ve constants.

Column-I (Motion law of a particle)	**Column-II** (Nature of motion of particle)
(A) $a = -\omega^2 x^3$	(P) Motion is periodic
(B) $a = -\omega^2 x^2$	(Q) Motion is oscillatory
(C) $a = -\omega^2 A \sin\left(\dfrac{\pi x}{12}\right)$	(R) Motion is not a S.H.M.
(D) $v = \dfrac{\omega}{A}\sqrt{A^4 - x^4}$	(S) Mechanical energy is conserved

Sol. A → P,Q,R,S ; B → R,S ; C → P,Q,R,S ;
D → P,Q,R,S

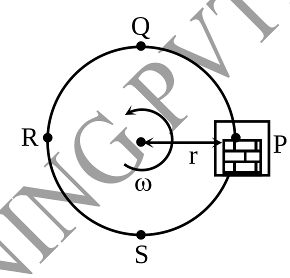
Motion of a particle will be S.H.M. if it satisfies friction

$a = -\omega^2 x$ x = displacement from mean position

ω^2 = constant

All oscillating motion are not S.H.M. but reverse is true.

Q.20 A small block of mass 'm' is placed in the cabin. Cabin is now rotated in the circular path in vertical plane about axis fixed in ground frame with constant angular speed 'ω'. Block rotates in circular path of radius 'r' and remains at rest with respect to chain.



Column-I	**Column-II**
(A) Maximum frictional force on block is at	(P) Position P
(B) Maximum toque about axis through C.M. of block and perpendicular to plane of motion due to normal force on block is at	(Q) Position Q
(C) Maximum contact force on block is at	(R) Position R
(D) Maximum normal contact force on block is at	(S) Position S

Sol. A → P,R ; B → P,R ; C → S ; D → S

Q.1 A uniform disc of mass 2kg and radius 1m is mounted on an axle supported on fixed frictionless bearings. A light chord is wrapped around the rim of the disc and mass of 1kg is tied to the free end. If it is released from rest-

(A) the tension in the chord is 5N

(B) in first four seconds the angular displacement of the disc is 40 rad

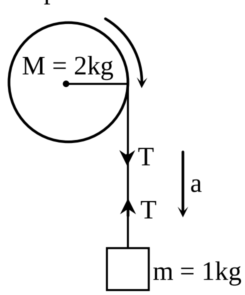
(C) the work done by the torque on the disc in first four seconds is 200J

(D) the increase in the kinetic energy of the disc in the first four seconds in 200J

[A,B,C,D]

Sol.

By FBD of particle



$mg - T = ma$

$10 - T = a$ ... (i)

By FBD of disc

$TR = I\propto = L\frac{a}{R} \Rightarrow T = \frac{MR^2}{2}\frac{a}{R^2}$

$T = Ma/2 = a$ …. (ii)

By eq. (i) and (ii)

(A) $a = 5\ m/s^2$ and $T = 5N$ and $\propto = a/R = 5\ rad/s^2$

(B) For angular displacement of disc :

$\theta = \omega t + 1/2 \propto t^2$

(C) Work done by torque

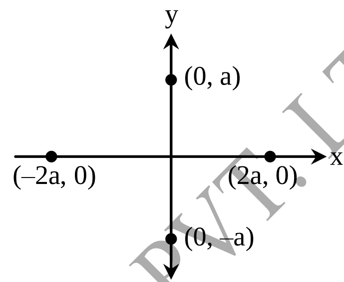
=

$\int \tau d\theta = \tau \int d\theta = 5 \times 40 = 200\ J$

(A) $\Delta K.E. = \Delta w = 200\ J$

$K_2 - K_1 = 200\ J$

Q.2 Four identical particles each of mass m are placed on x-y plane as shown. If I_x, I_y and I_z are the moment of inertia of system about x-axis, y-axis and z-axis respectively then –



(A) $I_x = 2ma^2$

(B) $I_y = 8\ ma^2$

(C) $I_z = 10\ ma^2$

(D) The total moment of inertia of system is 20 ma^2

Sol. **[A,B,C]**

$I_x = ma^2 + ma^2 = 2ma^2$

$I_y = m(2a)^2 + m\,(2a)^2 = 8ma^2$

$I_z = I_x + I_y = 10\ ma^2$

Q.3 Two uniform solid spheres are placed at some distance. The centre of mass of system –

(A) may lie outside spheres

(B) may lie inside one sphere

(C) cannot lie inside any sphere

(D) must lie on the line joining centres of spheres

Sol. **[A,B,D]**

Conceptual.

Q.4 A disc of mass m and radius r lies in x-y plane with its centre at a distance 'a' from origin at (a, 0) then -

(A) its moment of inertia about x-axis is $\frac{mr^2}{4}$

(B) its moments of inertia about y-axis is $\frac{mr^2}{4} + ma^2$

(C) its moments of inertia about z-axis $\frac{mr^2}{2} + ma^2$

(D) None of these

Sol. **[A,B,C]**

As x-axis is along diameter $\therefore I_x = \frac{1}{4} mr^2$ and

$I_y = \frac{mr^2}{4} + ma^2$ using parallel axis theorem

$I_z = \frac{mr^2}{2} + ma^2$

Q.5 A ring rolls without slipping on ground. Its centre c moves with a constant speed u. P is any point on the ring. the speed of P with respect to ground is v then -

(A) $0 \le v \le 2u$

(B) V = u, if cP is horizontal

(C) V = $\sqrt{2}$ u, if cP is horizontal

(D) None of these

Sol. **[A,C]**

Velocity of contact point is minimum is equal to zero and velocity of topmost point is maximum is equal to 2u also when cP is horizontal

$V = \sqrt{u^2 + (r\omega)^2}$ and $u = r\omega$

$\therefore V = \sqrt{2}\, u$

Q.6 A solid sphere is resting over a rough horizontal floor. A sharp impulse is applied on it along its horizontal diameter. Point out false statements :

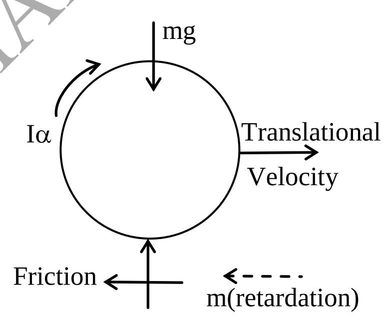
(A) The kinetic energy of the sphere remains constant throughout the motion

(B) The kinetic energy of translation is shared by rotation, but the total kinetic energy decreases initially and finally attains a constant value

(C) The total kinetic energy of the sphere decreases continuously on doing work against friction and finally reduces to zero

(D) A constant frictional force, opposite to the translational motion acts on the sphere throughout its motion.

Sol. **[A,C,D]**



Since, the impulse is applied along a horizontal diameter, therefore, due to that impulse the sphere starts to move translationally without any rotational motion. Since the floor is rough, therefore, friction comes into existence and that opposes forward sliding of the sphere as shown in figure. Hence, friction acts along backward direction which not only provides a retarding force but produces an accelerating moment also. Due to that moment the sphere experiences an angular acceleration. Hence, initially translational velocity of the sphere decreases but angular velocity increases Therefore, KE of the sphere initially decreases. Rotational motion is accelerated till angular velocity (ω) becomes equal to $\frac{v}{r}$ and then the friction disappears. Hence, then total energy of sphere remains constant. Therefore, option (B) alone is correct and rest options are incorrect.

Q.7 Select the wrong statement –

Statement-I : A body may be in pure rotation under the action of single external force.

Statement-II : If a body is in pure rotation under action of single external force then it must not be rigid.

(A) Statement-I

(B) Both Statements are correct.

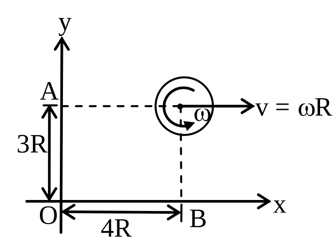
(C) Statement-II

(D) cannot be said.

Sol. **[A,C]**

A single force cannot produce pure rotation.

Q.8 A disc of mass M and radius R moves in the x-y plane as shown in the figure. The angular momentum of the disc at the instant shown is –



(A) $\frac{5}{2}$ mR^2 ω about O

(B) $\frac{7}{2}$ mR^2 ω about O

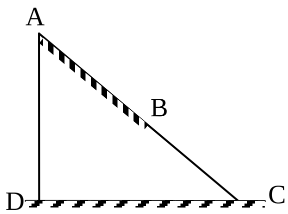
(C) $\frac{1}{2}$ mR^2 ω about A

(D) 4 mR^2 ω about A **[B,C]**

Sol. $L_O = mv \times 3R + \frac{1}{2} mR^2\omega$

$L_A = mv \times 0 + \frac{1}{2} mR^2\omega$

Q.9 Portion AB of the wedge shown in figure is rough and BC is smooth. A solid cylinder rolls without slipping from A to B. If AB = BC, then let m and n is the ratio of translational kinetic energy to rotational kinetic energy when cylinder reaches point B and C respectively, then –



(A) m = 2 (B) $m = \frac{7}{5}$

(C) n = 1 (D) n = 5 **[A,D]**

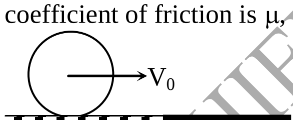
Sol. $K = \beta K_T$ or $K_T + K_R = \beta K_T \Rightarrow K_T = 2K_T$

At point B : $K_T + K_R = mgh \Rightarrow K_R = \frac{mgh}{3}$

At point C : $K_T + \frac{mgh}{3} = mg \times 2h$

$\Rightarrow K_T = \frac{5mgh}{3}$

Q.10 A ring of mass M and radius R sliding with a velocity v_0 suddenly enters into rough surface where coefficient of friction is μ, as shown -



rough(μ)

(A) The ring starts rolling motion when centre of mass becomes stationary

(B) The ring starts rolling motion when the point of contact becomes stationary

(C) The time after which the ring starts rolling is $\frac{v_0}{2\mu g}$

(D) The rolling velocity is $\frac{v_0}{2}$ **[B,C,D]**

Sol. Let rolling velocity is v and angular velocity is ω then,

$v = v_0 - \mu gt$ .... (1)

and $\omega = \frac{\mu g}{r} t$ .... (2)

Also, $v = r\omega$

$\therefore \mu gt = v_0 - \mu gt \Rightarrow t = \frac{v_0}{2\mu g}$ and $v = \frac{v_0}{2}$

Q.11 Which of the following statement(s) is/are correct for a spherical body rolling without slipping on a rough horizontal surface at rest -

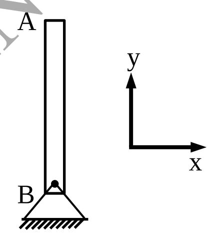
(A) the acceleration of the point of contact with the ground is zero

(B) the speed of some of the point (s) is/are zero

(C) frictional force may or may not be zero

(D) work done by friction may or may not be zero **[B,C]**

Q.12 A uniform rod of mass 1 kg & length L = 1m stands vertically. Rod is free to rotate about hinge at 'B' in x-y plane only. A force $\vec{F} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ starts acting on rod at point 'A'. Then immediately after force starts acting on rod -



(A) Angular acceleration. of rod will be $9(-\hat{k})$ rad/s^2

(B) Angular acceleration. of rod will be $6(-\hat{k})$ rad/s^2

(C) Reaction torque of hinge on rod about B will be zero

(D) Reaction torque of hinge on rod about B will be $-5\hat{i}$ Nm

Sol. **[A, D]**

$\vec{\tau}_F = 3(-\hat{k}) + 5\hat{i}$

$\therefore \alpha = \frac{3}{I} = 9$ rad/s^2

Q.13 Work done by a force on an rigid object having no rotational motion will be zero, if -

(A) the force is always perpendicular to acceleration of object.

(B) the object is at rest relative to ground but the point of application of force moves on the object.

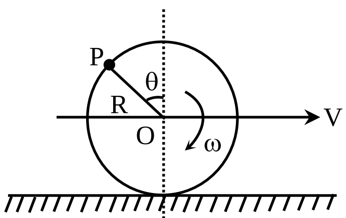
(C) the force is always perpendicular to velocity of object.

(D) the point of application of force is fixed relative to ground by the object moves.

Sol. **[B,C]**

(A) If velocity and acceleration are not in same direction. Work done by force perpendicular to acceleration will not be zero.
(B) If the object is at rest no force can do work.
(C) If force is perpendicular to velocity work done will be zero.
(D) If the point on the body has velocity component in direction of application of force work done will be non-zero.

Q.14 A disc of radius R rolls on a horizontal surface with linear velocity V and angular velocity ω. There is a point P on circumference of disc at angle θ, which has a vertical velocity. Here θ is equal to –



(A) $\pi + \sin^{-1}\frac{V}{R\omega}$ (B) $\frac{\pi}{2} - \sin^{-1}\frac{V}{R\omega}$
(C) $\pi - \cos^{-1}\frac{V}{R\omega}$ (D) $\pi + \cos^{-1}\frac{V}{R\omega}$

[C,D]

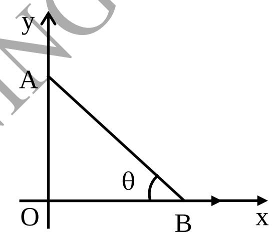
Q.15 A uniform rod kept on the ground falls from its vertical position. Its foot does not slip on the ground –
(A) No part of the rod can have acceleration greater than g in any position
(B) At any one position of rod, different points on it have different acceleration
(C) Any one particular point on the rod has different acceleration at different positions of the rod
(D) The maximum acceleration of any point on the rod, at any position is 1.5 g. **[B,C,D]**

Q.16 A particle moves in a circle of radius r with angular velocity $\vec{\omega}$. At some instant its velocity is $\vec{v}$ and radius vector with respect to centre of the circle is $\vec{r}$. At this particular instant centripetal acceleration $\vec{a}_c$ of the particle would be -
(A) $\vec{\omega}\times\vec{v}$ (B) $\vec{v}\times\vec{\omega}$
(C) $\vec{\omega}\times(\vec{\omega}\times\vec{r})$ (D) $\vec{v}\times(\vec{r}\times\vec{\omega})$ **[A,C]**

Q.17 A particle of mass m is travelling with a constant velocity $\vec{v} = v_0\hat{i}$ along the line y = b, z = 0. Let dA be the area swept out by the position vector from origin to the particle in time dt and L the magnitude of angular momentum of particle about origin at any time t. Then –
(A) L = constant (B) L ≠ constant
(C) $\frac{dA}{dt} = \frac{2L}{m}$ (D) $\frac{dA}{dt} = \frac{L}{2m}$

[A,D]

Q.18 The end B of the rod AB which makes angle θ with the floor is being pulled with a constant velocity v_0 as shown. The length of the rod is ℓ. At the instant when θ = 37º



(A) velocity of end A is $\frac{4}{3}v_0$ downwards
(B) angular velocity of rod is $\frac{5}{3}\frac{v_0}{\ell}$
(C) angular velocity of rod is constant
(D) velocity of end A is constant **[A,B]**

Q.19 A thin uniform rod of mass m and length ℓ is free to rotate about its upper end. When it is at rest, it receives an impulse J at its lowest point, normal to its length. Immediately, after impact -
(A) the angular momentum of rod is $J\ell$
(B) the angular velocity of rod is $\frac{3J}{m\ell}$
(C) the kinetic energy of rod is $\frac{3J^2}{2m}$
(D) the linear velocity of mid point of rod is $\frac{3J}{2m}$ **[All]**

Q.20 A rod of length L is composed of a uniform length L/2 of wood whose mass is M_w and a uniform length L/2 of brass whose mass is M_b.

(A) The moment of inertia of rod about an axis passing through the centre and perpendicular to its is $\frac{1}{12}M_wL^2$

(B) The moment of inertia of rod about an axis as specified in (A) is $\frac{1}{12}(M_w + M_b)L^2$

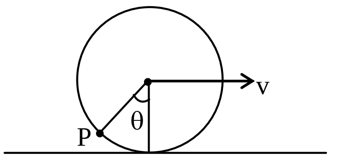
(C) The moment of inertia of rod about an axis passing through the wood end and perpendicular to the rod is $\frac{1}{3}(M_w + M_b)L^2$

(D) The moment of inertia of rod about an axis as specified in (C) is $\frac{1}{12}(M_w + 7M_b)L^2$

[B,D]

Q. 1 A point P is located on the rim of wheel of radius r = 0.5 m which rolls without slipping along a horizontal surface then the total distance traversed by the point P in meters between two successive moments it touches the surface. **[0004]**

Sol.



$V_P = \sqrt{V^2 + V^2 + 2V^2\cos(\pi - \theta)}$

$\therefore V_P = 2\,V\sin\left(\frac{\theta}{2}\right)$

Now $V_P = \frac{ds}{dt} = \frac{ds}{dv}\cdot\frac{dv}{dt}$

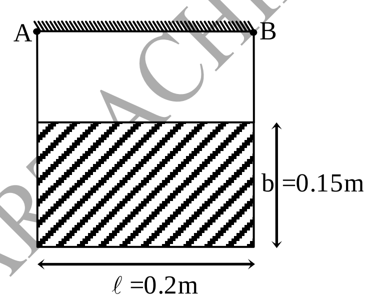
$\therefore V_P = \omega\,\frac{ds}{d\theta} = \frac{V}{R}\,\frac{ds}{d\theta}$

$\therefore \frac{V}{R}\,\frac{ds}{d\theta} = 2V\sin(\theta/2)$

$\Rightarrow ds = 2R\sin(\theta/2)\,d\theta$

$\therefore S = 2R\int_0^{2\pi}\sin\left(\frac{\theta}{2}\right)d\theta = 8R = 4m$

Q.2 A rectangular plate of mass 20 kg is suspended from points A and B as shown. If the pin B is suddenly removed then the angular acceleration in rad/sec^2 of the plate is : (g = 10 m/s^2).



Sol. Moment of inertia about the axis rotation axis is I then,

$I = I_C + md^2$, where $d^2 = \frac{b^2}{4} + \frac{\ell^2}{4} = \frac{0.0625}{4}$

$\therefore I = \frac{20}{12}[(0.2)^2 + (0.15)^2] + \frac{20 \times 0.0625}{4}$

$= 0.416$ kg-m^2

Now $I\alpha = mg\frac{\ell}{2} \Rightarrow \alpha = \frac{mg\ell}{2I} \approx 48$ rad/sec^2

Q.3 A solid sphere rolls on a rough horizontal surface with a linear speed 7 m/s collides elastically with a fixed smooth vertical wall. Then the speed of the sphere when it has started pure rolling in the backward direction in m/s is. **[0003]**

Sol. Let after time t

Sphere starts pure rolling then,

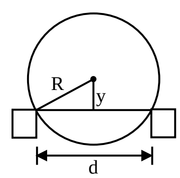
$v = 7 - \mu gt$ ….(1)

and $\frac{v}{r} = \frac{-7}{r} + \frac{5\mu gt}{2r}$ ….(2)

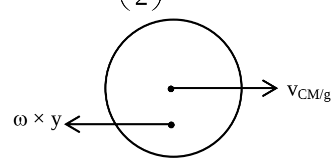
From (1) and (2) v = 3 m/s

Q.4 A uniform ball of radius R = 10 cm rolls without slipping between two rails such that the horizontal distance is d = 16 cm between two contact points of the rail to the ball. If the angular velocity is 5 rad/s, then find the velocity of centre of mass of the ball in cm/s. **[0030]**

Sol.



$y^2 = R^2 - \left(\frac{d}{2}\right)^2$



$v_{CM/g} = \omega \times y$

$= 30$ cm/s

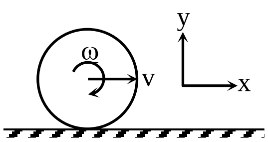
Q.5 A wheel rotating at same angular speed undergoes constant angular retardation. After revolution angular velocity reduces to half its initial value. How many more revolution it will make before stopping ?

5.[3] $\frac{\omega_0^2 - (\omega_{0/2})^2}{{\omega_0}^2} = \frac{9}{\theta_2} \Rightarrow \frac{3}{4} = \frac{9}{\theta_2}$

$\Rightarrow \theta_2 = 12$

$\therefore$ Required No. of revolution = 12 – 9 = 3

Q.6 A disc of radius '5cm' rolls on a horizontal surface with linear velocity $v = 1\hat{i}$ m/s and angular velocity 50 rad/sec. Height of particle from ground on rim of disc which has velocity in vertical direction is (in cm) -

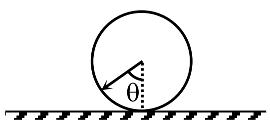


Sol **.[3]**

$v = R\omega \cos\theta$

$\Rightarrow \cos\theta = \dfrac{v}{R\omega} = \dfrac{2}{5}$ cm

$\therefore h = R(1 - \cos\theta) = 3$ cm.



Q.7 A cubical block of mass 6 kg and side 16.1 cm is placed on frictionless horizontal surface. It is hit by a cue at the top as to impart impulse in horizontal direction. Minimum impulse imparted to topple the block must be greater than –

Sol. **[4]**

Let a = side of cube

p = impulse imparted

$\therefore$ After hitting,

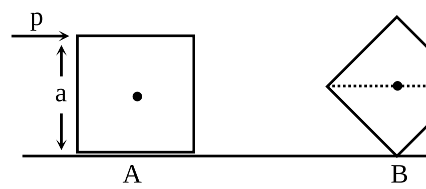
$v_0 = \dfrac{p}{m}$ and $\omega_0 = \dfrac{pa}{2I}$

I : moment of inertial about axis passing through C.M.

For just toppling

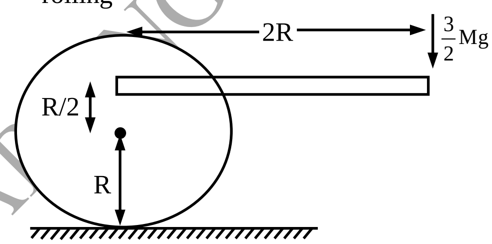
$\dfrac{1}{2} I \omega_0^2 = mg\, a\left(\dfrac{1}{\sqrt{2}} - \dfrac{1}{2}\right)$

(Applying energy conservation between situation A and B)



$\Rightarrow p = \dfrac{2I\omega_0}{a} = 4$ kg m/s.

Q.8 A disc of mass M & radius R is placed a rough horizontal surface with its axis horizontal. A light rod of length '2R' is fixed to the disc at point 'A' as shown in figure and a force $\dfrac{3}{2}$ Mg is applied at the other end. If disc starts to roll without slipping find the value of "$10\times\mu_{min}$" where μ_{min} is minimum coefficient of friction b/w disc & horizontal surface required for pure rolling -



Sol. **[8]**

F = Ma ……….(i)

$\dfrac{3}{2}$ Mg. 2R – F.R = $\dfrac{MR^2}{2} \cdot \dfrac{a}{R}$ ……...(ii)

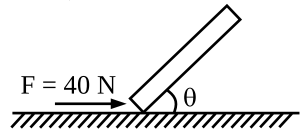
a = 2g

$\therefore F = 2Mg \le \mu N$

Q.9 A homogeneous rod of mass 3 kg is pushed along smooth horizontal surface by a horizontal force

F = 40 N. The angle 'θ' (in degree) for which rod has pure translation motion minus 30 degree is

(g = 10m/s^2) -

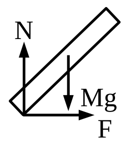


Sol. **[7]**

N = mg

$F\dfrac{L}{2}\sin\theta = \dfrac{N^2}{2}\cos\theta$

$\Rightarrow \tan\theta = \dfrac{3}{4}$



Q.10 Two identical discs are positioned on a vertical axis. The bottom disc is rotating with angular velocity ω_0. The top disc is initially at rest. It is allowed to fall and sticks to the lower disc. Ratio of K.E. before & after collision .

Sol. **[2]** $I\omega_0 = 2I\omega$

(I : M.I. of one disc)

$\therefore K = \dfrac{1}{2}\ 2I\omega^2$

$= \dfrac{1}{2}\left(\dfrac{1}{2}I\omega_0{}^2\right)$

Q.10 A disc is rotating freely about its axis. Percentage change in angular velocity of disc if temperature decreases by 20 °C is (coefficient of linear expansion of material of disc is 5×10^{-4}/°C)

Sol. **[2]**

$I\omega = \text{const.} \Rightarrow \omega \propto \dfrac{1}{I}$

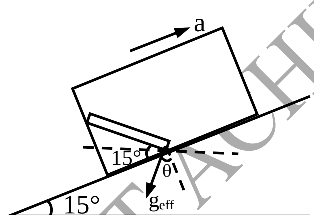
$\therefore$ Percentage change in ω

$= -$ (% change in I)

$= -(2 \times \Delta\theta \times 100)$

$= -(2 \times 5 \times 10^{-4} \times -20 \times 100)$

$= 2\ \%$



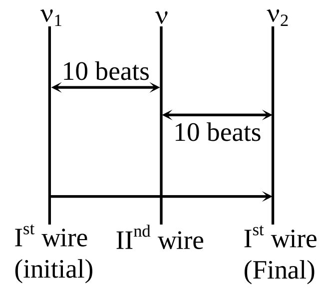
Q.11 Two wires are vibrating together to produce 10 beats/sec. Frequency of one wire is 200Hz. When tension in this wire is increased beat frequency remains unchanged. Frequency (in Hz) of other wire minus 206 Hz is equal to.

Sol. **[4]**

Frequency of first wire is less than that of second wire as upon increasing tension beats frequency remains unchanged.

$\therefore \nu = \nu_1 + 10 = 210$ Hz

$\therefore \nu - 206 = 4$

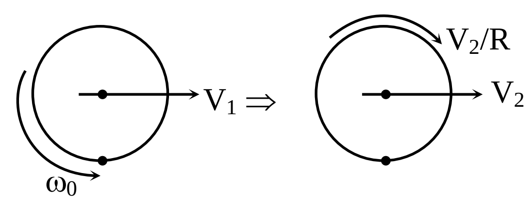


Q.12 A ball of radius R = 20 cm has a mass m = 0.75 kg and moment of inertia about its diameter $I = 0.0125$ kg m^2. The ball rolls without slipping over a rough horizontal floor with velocity $v_0 = 10$ m/s towards a smooth vertical wall. If coefficient of restitution between the wall and ball is e = 0.7 then the velocity of the ball in m/s after long time after collision is (g = 10 m/s^2)

Sol. **[2]**

Let V_1 just after collision and V_2 after long time [ω will not change]

$e = \dfrac{V_1}{V} \Rightarrow V_1 = 0.7 \times 10 = 7$m/s



angular momentum conservation about O

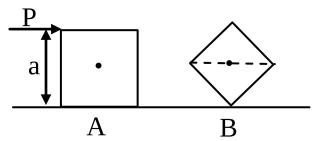
$mV_1R - I\omega_0 = mV_2R + I\ \dfrac{V_2}{R}$

$V_2 = 2$m/s

Q.13 A cubical block of mass 6 kg and side 16.1 cm is placed on frictionless horizontal surface. It is hit by a cue at the top as to impart-impulse in horizontal direction. Minimum impulse imparted to topple the block must be greater than.

Sol.[4] Let, a = side of cube

P = Impulse imparted



∴ After hitting,

$v_0 = \frac{P}{m}$ and $\omega_0 = \frac{Pa}{2I}$

[I = Moment of inertial about axis passing through centre of mass]

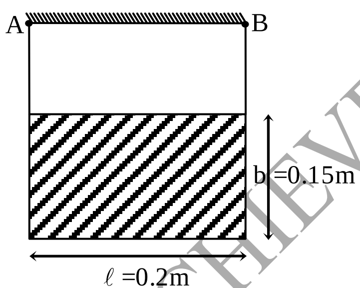
For just toppling

$$\frac{1}{2} I\omega_0^2 = mga\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)$$

(Applying energy conservation between situation A and B)

$$\Rightarrow \quad P = \frac{2I\omega_0}{a} = 4 \text{ kg m/s}$$

Q.14 A rectangular plate of mass 20 kg is suspended from points A and B as shown. If the pin B is suddenly removed then the angular acceleration in rad/sec^2 of the plate divided by 16 is equal to (g = 10 m/s^2)



Sol.[3] Moment of inertia about the axis rotation axis is I then,

$$I = I_C + md^2, \text{ where } d^2 = \frac{b^2}{4} + \frac{\ell^2}{4} = \frac{0.0625}{4}$$

$$\therefore I = \frac{20}{12}[(0.2)^2 + (0.15)^2] + \frac{20 \times 0.0625}{4}$$

$$= 0.416 \text{ kg-m}^2$$

$$\text{Now } I\alpha = mg\frac{\ell}{2} \Rightarrow \alpha = \frac{mg\ell}{2I} \approx 48 \text{ rad/sec}^2$$

Q. 1 A spherical ball rolls on a table without slipping. Then the fraction of its total energy associated with rotation is -

(A) 2/5 (B) 2/7 (C) 3/5 (D) 3/7 **[B]**

Sol. Total energy

$K = K_R + K_T = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$

$= \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2 + \frac{1}{2} mr^2\omega^2$

$= \frac{1}{5}mr^2\omega^2 + \frac{1}{2} mr^2\omega^2 = \frac{7}{10} mr^2\omega^2$

Now, rotational kinetic energy

$K_R = \frac{1}{2} I\omega^2 = \frac{1}{5} mr^2\omega^2$

$\therefore \frac{K_R}{K} = \frac{\frac{1}{5}mr^2\omega^2}{\frac{7}{10}mr^2\omega^2} = \frac{2}{7}$

Q.2 The moment of inertia of a body about a given axis is 1.2 kg × m^2. Initially, the body is at rest. In order to produce a rotational KE of 1500 joule, an angular acceleration of 25 rad/sec^2 must be applied about that axis for a duration of -

(A) 4 s (B) 2 s

(C) 8 s (D) 10 s **[B]**

Sol. $K_R = \frac{1}{2} I\omega^2 = \frac{1}{2} I(\alpha t)^2 = \frac{1}{2} I\alpha^2t^2$

$1500 = \frac{1}{2} \times 1.2 \times (25)^2t^2$

or $t^2 = 4$ or $t = 2s$

Q.3 A body of radius R and mass m is rolling horizontally without slipping with speed v. It then rolls up a hill to a maximum height $h = \frac{3v^2}{4g}$. The body might be a -

(A) solid sphere (B) hollow sphere

(C) disc (D) ring **[C]**

Sol. **[C]**

Let I bt the moment of inertia of the body. Then

total KE = $\frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$

or KE = $\frac{1}{2} mv^2 + \frac{1}{2} I \frac{v^2}{R^2}$ $\left(\omega = \frac{v}{R}\right)$

According to energy conservation loss in KE = gain in PE.

or $\frac{1}{2}\left(m + \frac{I}{R^2}\right)v^2 = mgh = mg\left(\frac{3v^2}{4g}\right)$

Solving this, we get $I = \frac{1}{2} mR^2$

i.e., the solid body is a disc

Q.4 When a wheel of radius R moves a distance smaller than $2\pi R$ making one rotation then -

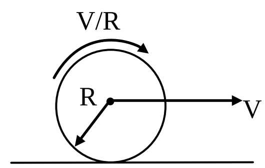
(A) $V_{cm} < R\omega$ (B) $V_{cm} > R\omega$

(C) $V_{cm} \le R\omega$ (D) $V_{cm} \ge R\omega$

Sol. **[A]**

conceptual.

Q.5 A disc is performing pure rolling on a smooth stationary surface with constant angular velocity as shown in figure. At any instant, for the lower most point of the disc.



(A) Velocity is v, acceleration is zero

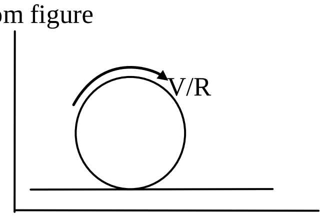
(B) Velocity is zero, acceleration is zero

(C) Velocty is v, acceleration is $\frac{v^2}{R}$

(D) Velocity is zero, acceleration is nonzero

Sol. **[D]**

From figure



V=Rω

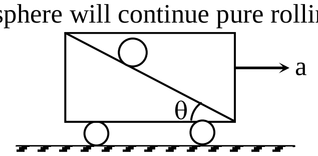
V_{net} (for lowest point) $= v - R\omega = v - v = 0$

and Acceleration $= \frac{v^2}{R} + O = \frac{v^2}{R}$

(Since linear speed is constant)

Hence (D)

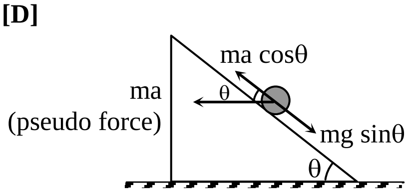
Q.6 Figure shows a smooth inclined plane of inclination θ fixed in a car. A sphere is set in pure rolling on the incline. For what value of 'a' (the acceleration of car in horizontal direction) the sphere will continue pure rolling ?



(A) $g \cos \theta$ (B) $g \sin \theta$

(C) $g \cot \theta$ (D) $g \tan \theta$

Sol. **[D]**

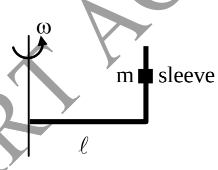


The sphere will continue pure rolling if

$mg \cos \theta = mg \sin \theta$

or $a = g \tan \theta$

Q.7 A L shaped rod whose one rod is horizontal and other is vertical is rotating about a vertical axis as shown with angular speed ω. The sleeve shown in figure has mass m and friction coefficient between rod and sleeve is μ. The minimum angular speed ω for which sleeve cannot sleep on rod is –



(A) $\omega = \sqrt{\frac{g}{\mu \ell}}$

(B) $\omega = \sqrt{\frac{\mu g}{\ell}}$

(C) $\omega = \sqrt{\frac{\ell}{\mu g}}$

(D) None of these

Sol. **[A]**

as $f = \mu N = mg$

or, $\mu m \ell \omega^2 = mg \Rightarrow \omega = \sqrt{\frac{g}{\mu \ell}}$

Q.8 At any instant, a rolling body may be considered to be in pure rotation about an axis through the point of contact but this axis is translating forward with a speed -

(A) zero

(B) equal to centre of mass

(C) twice of centre of mass

(D) None of these

Sol. **[B]**

Conceptual

Q. 9 The speed of wave traveling on the uniform circular ring, which is rotating about an axis passing through its center and perpendicular to its plane with tangential speed v in gravity free space is -

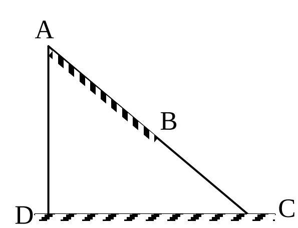
(A) v (B) $\frac{v}{2}$

(C) $\frac{v}{\sqrt{2}}$ (D) $\sqrt{2}v$

Sol. **[A]**

Tension in rotating ring is $T = \mu v^2$

Q.10 Portion AB of the wedge shown in figure is rough and BC is smooth. A solid cylinder rolls without slipping from A to B. If AB = BC, then ratio of translational kinetic energy to rotational kinetic energy, when the cylinder reaches point C is –



(A) 3/5 (B) 5

(C) 7/5 (D) 8/3

Sol. **[B]**

$K = \beta K_T$

or $K_T + K_R = \beta K_T$

$K_R = (\beta - 1)K_T \Rightarrow K_R = \frac{1}{2} K_T$

At point B : $K_T + K_R = mg \times h$

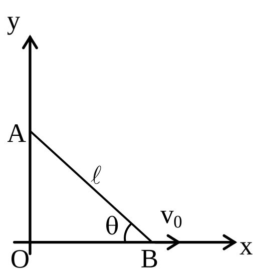
$\therefore K_R = \frac{mgh}{3}$

At point C : $K_T + \frac{mgh}{3} = mg \times 2h$

$K_T = \frac{5mgh}{3}$

$\therefore \frac{K_T}{K_R} = 5$

Q.11 In the figure given below, the end B of the rod AB which makes angle θ with the floor is pulled with a constant velocity v_0 as shown. The length of rod is ℓ. At an instant when $\theta = 37^o$



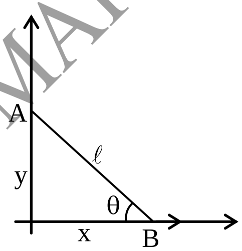
(A) Velocity of end A is $\frac{4v_0}{3}$

(B) angular velocity of rod is $\frac{5v_0}{6\ell}$

(C) angular velocity of rod is constant

(D) velocity of end A is constant

Sol. **[A]**



$x^2 + y^2 = \ell^2$

$\Rightarrow \frac{dy}{dt} = -\left(\frac{x}{y}\right)\frac{dx}{dt}$

$\therefore v_A = -\frac{4}{3} v_0$

Now, $x = \ell \cos\theta$

$\frac{dx}{dt} = -\ell \sin\theta \frac{d\theta}{dt} \Rightarrow \omega = -\frac{5}{3}\left(\frac{v_0}{\ell}\right)$

Q.12 For particle of a purely rotating body, $v = r\omega$, so correct relation will be -

(A) $\omega \propto \frac{1}{r}$ (B) $\omega \propto v$

(C) $v \propto \frac{1}{r}$ (D) ω is independent of r

Sol. **[D]**

Conceptual

Q.13 A ring of mass 100 kg and diameter 2m is rotating at the rate of $\left(\frac{300}{\pi}\right)$ rpm. Then-

(A) moment of inertia is 100 kg – m^2

(B) kinetic energy is 5 kJ

(C) if a retarding torque of 200 N-m starts acting then it will come at rest after 5 sec.

(D) all of these

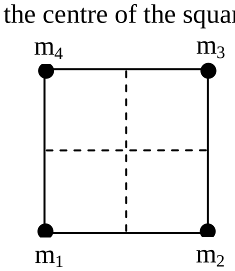
Sol. **[D]**

Moment of inertia = MR^2

k.E of rotation = $\frac{1}{2} I\omega^2$

Torque = $I \propto$ where $\alpha = \frac{\omega_0}{t}$

Q.14 Four particles of mass $m_1 = 2m$, $m_2 = 4m$, $m_3 = m$ and m_4 are placed at four corners of a square. What should be the value of m_4 so that the centre of mass of all the four particles are exactly at the centre of the square ?



(A) 2 m (B) 8 m

(C) 6 m (D) none of these **[D]**

Q.15 Two rings of same radius (r) and mass (m) are placed such that their centres are at a common point and their planes are perpendicular to each other. The moment of inertia of the system about an axis passing through the centre and perpendicular to plane of one of the ring is -

(A) $\frac{1}{2}mr^2$ (B) mr^2

(C) $\frac{3}{2}mr^2$ (D) $2mr^2$ **[C]**

Q.16 A disc is rotating with an angular velocity ω_0. A constant retarding torque is applied on it to stop the disc. The angular velocity becomes $\frac{\omega_0}{2}$ after n rotations. How many more rotations will it make before coming to rest ?

(A) n (B) 2n

(C) $\frac{n}{2}$ (D) $\frac{n}{3}$ **[D]**

Q.17 A particle of mass 1 kg is moving along the line y = x + 2 (here x and y are in metres) with speed 2 m/s. The magnitude of angular momentum of particle about origin is -

(A) 4 kg - m^2/s (B) $2\sqrt{2}$ kg - m^2/s

(C) $4\sqrt{2}$ kg - m^2/s (D) 2 kg - m^2/s **[B]**

Q.18 A circular platform is mounted on a vertical frictionless axle. Its radius is r = 2m and its moment of inertia is I = 200 kg - m^2. It is initially at rest. A 70 kg man stands on the edge of the platform and begins to walk along the edge at speed v_0 = 1.0 m/s relative to the ground. The angular velocity of the platform is -

(A) 1.2 rad/s (B) 0.4 rad/s

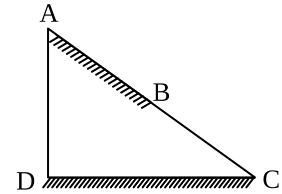
(C) 2.0 rad/s (D) 0.7 rad/s **[D]**

Q.19 The linear velocity of a particle moving with angular velocity $\vec{\omega} = 2\hat{k}$ at position vector $\vec{r} = 2\hat{i} + 2\hat{j}$ is -

(A) $4(\hat{i} - \hat{j})$ (B) $4(\hat{j} - \hat{i})$

(C) $4\hat{i}$ (D) $-4\hat{i}$ **[B]**

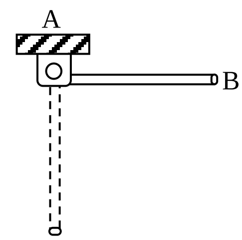
Q.20 Portion AB of the wedge shown in figure is rough and BC is smooth. A solid cylinder rolls without slipping from A to B. If AB = BC, then ratio of translational kinetic energy to rotational kinetic energy, when the cylinder reaches point C is -



(A) 3/5 (B) 5

(C) 7/5 (D) 8/3 **[B]**

Q.21 One end of a uniform rod of length *l* and mass m is hinged at A. It is released from rest from horizontal position AB as shown in figure. The force exerted by the rod on the hinge when it becomes vertical is -



(A) $\frac{3}{2}$ mg (B) $\frac{5}{2}$ mg

(C) 3 mg (D) 5 mg **[B]**

Q.22 A sphere of radius 'R' is rolling over a horizontal surface. All measurement are made with respect to surface over which sphere is rolling. Which of the following strictly confirms pure rolling motion of sphere over horizontal surface ?

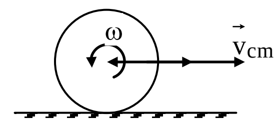
(A) $x_{cm} = R\theta$: x_{cm} & R in meter & 'θ' is in radian

(B) $v_{cm} = R\omega$: R in meter, v_{cm} in m/s, 'ω' in rad/sec

(C) $a_{cm} = R\alpha$: a_{cm} in cm/s^2, R in cm, α in rad/s^2

(D) All of the above **[D]**

Sol. Consider situation shown in figure



Q.23 A square plate is kept in yz-plane. Then according to perpendicular axis theorem -

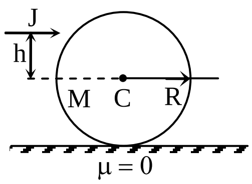
(A) $I_z = I_x + I_y$ (B) $I_x = I_y + I_z$

(C) $I_y = I_x + I_z$ (D) All **[B]**

Sol. For a mass distribution in y-z plane

$$I_x = I_y + I_z$$

Q.24 A solid sphere of mass M and radius R is placed on a smooth horizontal surface. It is given a horizontal impulse J at a height h above the centre of mass and sphere starts rolling then, the value of h and speed of centre of mass are –



(A) $h = \frac{2}{5}R$ and $v = \frac{J}{M}$

(B) $h = \frac{2}{5}R$ and $v = \frac{2}{5}\frac{J}{M}$

(C) $h = \frac{7}{5}R$ and $v = \frac{7}{5}\frac{J}{M}$

(D) $h = \frac{7}{5}R$ and $v = \frac{J}{M}$ **[A]**

Sol. Let the force producing impulse J is F then

$$F \times h = \frac{2}{5} mR^2 \times \alpha$$

and F = ma (where a = Rα)

$$\therefore \ mah = \frac{2}{5} mRa \ \Rightarrow h = \frac{2}{5}R$$

Also impulse = change in momentum

or J = Mv

Q.25 What must be the relation between length 'L' and radius 'R' of the cylinder if its moment of inertia about its axis is equal to that about the equatorial axis ?

(A) L = R (B) L = 2R

(C) L = 3R (D) $L = \sqrt{3}R$ **[D]**

Sol. $$\frac{mR^2}{2} = M\left(\frac{L^2}{12} + \frac{R^2}{4}\right)$$

or $$\frac{R^2}{2} = \frac{L^2}{12} + \frac{R^2}{4}$$

or $L = \sqrt{3}R$

Q.26 A particle performs uniform circular motion with angular momentum 'L'. If the frequency of particles motion is halved and its KE is doubled then the angular momentum becomes –

(A) $\frac{L}{4}$ (B) 4L

(C) 2L (D) L/2 **[B]**

Sol $$K.E. = \frac{1}{2} I\omega^2 = \frac{1}{2}(I\omega)(\omega)$$

or $K.E. = \frac{1}{2} L\omega$

or $L = \frac{2K.E.}{\omega}$

Now $L' = \frac{2(2K.E.)}{(\omega/2)} = 4L$

Q.27 The angular speed of rotating rigid body is increased from 4 ω to 5 ω. The percentage increase in its K.E. is –

(A) 20 % (B) 25 %

(C) 125 % (D) 56 % **[D]**

Sol. $K.E. = \frac{1}{2} I\omega^2 \ \Rightarrow \ K.E. \propto \omega^2$

$$\% \text{ increase K.E.} = \frac{KE_f - KE_i}{KE_i} \times 100$$

$$= \frac{5^2 - 4^2}{4^2} \times 100$$

$$= \frac{9}{16} \times 100 = 56\%$$

Q.28 Two loops P and Q are made from a uniform wire. The radii of P and Q are r_1 and r_2 respectively, and their moments of inertia are I_1 and I_2 respectively. If $I_2 = 4I_1$, then $\frac{r_2}{r_1}$ equals –

(A) $4^{2/3}$ (B) $4^{1/3}$

(C) $4^{-2/3}$ (D) $4^{-1/3}$ **[B]**

Sol. $I = MR^2 = (2\pi RAd)R^2$

or $I \propto R^3$

or $R \propto I^{1/3}$

or $$\frac{R_2}{R_1} = \left(\frac{I_2}{I_1}\right)^{1/3} = \left(\frac{4}{1}\right)^{1/3}$$

Q.29 A loop of radius 3 meter and weighs 150 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 15 cm/sec. How much work has to be done to stop it –

(A) 3.375 J (B) 7.375 J
(C) 5.375 J (D) 9.375 J **[A]**

Sol Required work = Total K.E.

$= \frac{1}{2}\ mv^2 \left(1 + \frac{k^2}{R^2}\right)$

$= \frac{1}{2} Mv^2 \left[1 + \frac{k^2}{R^2}\right]$

$= \frac{1}{2} \times 150 \times (0.15)^2\ (1 + 1) = 3.375$ J

Q.30 A rod of mass **m** and length ***l*** is hinged at one of its end A as shown in figure. A force F is applied at a distance x from A. The acceleration of centre of mass (*a*) varies with x as –

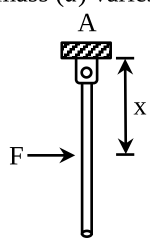
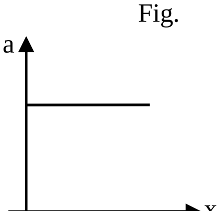
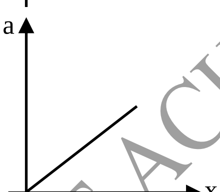
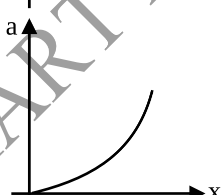
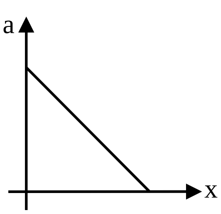


Fig.

(A)


(B)


(C)


(D)


[B]

Sol. The rod will rotate about point A with angular acceleration:

$$\alpha = \frac{\tau}{I} = \frac{Fx}{\frac{ml^2}{3}} = \frac{3Fx}{ml^2}$$

$\therefore\ a = \frac{l}{2}\ \alpha = \frac{3}{2}\ \frac{Fx}{ml}$

or $a \propto x$

i.e., *a*-x graph is a straight line passing through origin.

Q.31 The moment of inertia of a body is I and its coefficient of linear expansion is α if temperature of body rises by a small amount ΔT. Then change in moment of inertia about the same axis –

(A) $\alpha\ I\ \Delta T$ (B) $2\ \alpha\ I\ \Delta T$
(C) $4\ \alpha\ I\ \Delta T$ (D) $\frac{\alpha I \Delta T}{2}$ **[B]**

Sol. Let $I = mr^2$

$\frac{\Delta I}{I} = \frac{2\Delta r}{r} = 2\ \alpha\ \Delta T$

or $\boxed{\Delta I = 2\alpha\, I\, \Delta T}$

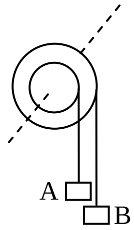
Q.32 A wheel starts rotating from rest and attains an angular velocity of 60 rad/sec in 5 seconds. The total angular displacement in radians will be-

(A) 60 (B) 80
(C) 100 (D) 150 **[D]**

Q.33 A body rotates at 300 rotations per minute. The value in radian of the angle described in 1 sec is-

(A) 5 (B) 5π
(C) 10 (D) 10π **[D]**

Q.34 Figure shows a small wheel fixed coaxially on a bigger one of double the radius. The system rotates about the common axis. The strings supporting A and B do not slip on the wheels. If x and y be the distances travelled by A and B in the same time interval, then-



(A) x = 2y (B) x = y

(C) y = 2x (D) None of these **[C]**

Q.35 A particle is moving with a constant angular velocity about an exterior axis. Its linear velocity will depend upon -

(A) perpendicular distance of the particle form the axis

(B) the mass of particle

(C) angular acceleration of the particle

(D) the linear acceleration of particle **[A]**

Q.36 On account of the earth rotating about its axis-

(A) the linear velocity of objects at equator is greater than at other places

(B) the angular velocity of objects at equator is more than that of objects at poles

(C) the linear velocity of objects at all places at the earth is equal, but angular velocity is different

(D) at all places the angular velocity and linear velocity are uniform **[A]**

Q.37 A chain couples and rotates two wheels in a bicycle. The radii of bigger and smaller wheels in a bicycle. The radii of bigger and smaller wheels are 0.5m and 0.1m respectively. The bigger wheel rotates at the rate of 200 rotations per minute, then the rate of rotation of smaller wheel will be -

(A) 1000 rpm (B) 50/3 rpm

(C) 200 rmp (D) 40 rpm **[A]**

Q.38 If the position vector of a particle is $\hat{r} = (3\hat{i} + 4\hat{j})$ metre and its angular velocity is $\vec{\omega} = (\hat{j} + 2\hat{k})$ rad/sec then its linear velocity is (in m/s)-

(A) $-(8\hat{i} - 6\hat{j} + 3\hat{k})$ (B) $(3\hat{i} + 6\hat{j} + 8\hat{k})$

(C) $-(3\hat{i} + 6\hat{j} + 6\hat{k})$ (D) $(6\hat{i} + 8\hat{j} + 3\hat{k})$ **[A]**

Q.39 Let $\vec{A}$ be a unit vector along the axis of rotation of a purely rotating body and $\vec{B}$ be a unit vector along the velocity of a particle P of the body away from the axis. The value of $\vec{A}.\vec{B}$ is-

(A) 1 (B) –1

(C) 0 (D) none of these **[C]**

Q.40 A body is in pure rotation. The linear speed v of a particle, the distance r of the particle from the axis and the angular velocity ω of the body are related as $\omega = \frac{v}{r}$. Thus-

(A) $\omega \propto \frac{1}{r}$

(B) $\omega \propto r$

(C) $\omega = 0$

(D) ω is independent of r. **[D]**

Q.41 A particle, moving along a circular path has equal magnitudes of linear and angular acceleration. The diameter of the path is : (in meters) -

(A) 1 (B) π

(C) 2 (D) 2π **[C]**

Q.42 A stone of mass 4 kg is whirled in a horizontal circle of radius 1m and makes 2 rev/sec. The moment of inertia of the stone about the axis of rotation is-

(A) 64 kg × m^2 (B) 4 kg × m^2

(C) 16 kg × m^2 (D) 1 kg × m^2 **[B]**

Q.43 In an arrangement four particles, each of mass 2 gram are situated at the coordinate points (3, 2, 0), (1, –1, 0), (0, 0, 0) and (–1, 1, 0). The moment of inertia of this arrangement about the Z-axis will be-

(A) 8 units (B) 16 units

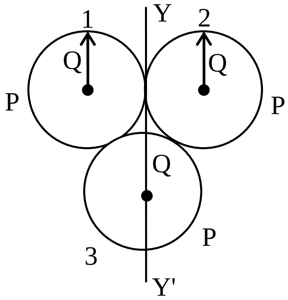
(C) 43 units (D) 34 units **[D]**

Q.44 Two discs have same mass and thickness. Their materials are of densities ρ_1 and ρ_2. The ratio of their moment of inertia about central axis will be -

(A) $\rho_1 : \rho_2$ (B) $\rho_1\rho_2 : 1$

(C) $1 : \rho_1\rho_2$ (D) $\rho_2 : \rho_1$ **[D]**

Q.45 Three rings, each of mass P and radius Q are arranged as shown in the figure. The moment of inertia of the arrangement about YY' axis will be-



(A) $\frac{7}{2}PQ^2$ (B) $\frac{2}{7}PQ^2$

(C) $\frac{2}{5}PQ^2$ (D) $\frac{5}{2}PQ^2$ **[A]**

Q.46 The moment of inertia depends upon-

(A) angular velocity of the body

(B) angular acceleration of the body

(C) only mass of the body

(D) distribution of mass and the axis of rotation of the body **[D]**

Q.47 Three thin uniform rods each of mass M and length L and placed along the three axis of a Cartesian coordinate system with one end of each rod at the origin. The M.I. of the system about z-axis is-

(A) $\frac{ML^2}{3}$ (B) $\frac{2ML^2}{3}$ (C) $\frac{ML^2}{6}$ (D) ML^2

[B]

Q.48 A circular disc A of radius r is made from an iron plate of thickness t and another circular disc B of radius 4r is made from an iron plate of thickness t/4. The relation between the moments of inertia I_A and I_B is-

(A) $I_A > I_B$

(B) $I_A = I_B$

(C) $I_A < I_B$

(D) depends on the actual values of t and r. **[C]**

Q.49 A flywheel has moment of inertia 4 kg-m^2 and has kinetic energy of 200 J. Calculate the number of revolutions is makes before coming to rest if a constant opposing couple of 5N-m is applied to the flywheel -

(A) 12.8 rev (B) 24 rev

(C) 6.4 rev (D) 16 rev

Sol. [C] W = ΔKE or $\tau(\theta) = \frac{1}{2}I\omega^2$

$\tau(2\pi n) = \frac{1}{2}I\omega^2$

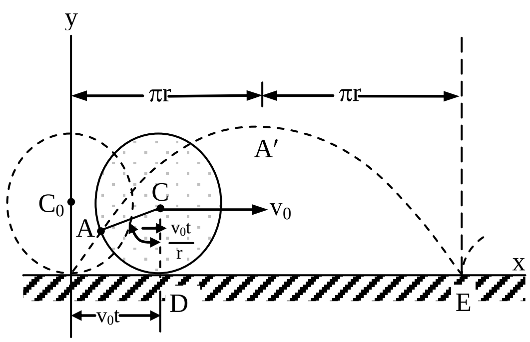
Q.50 A rigid body is rotating about a vertical axis at n rotations per minute, If the axis slowly becomes horizontal in t seconds and the body keeps on rotating at n rotations per minute then the torque acting on the body will be, if the moment of inertia of the body about axis of rotation is I -

(A) zero (B) $\frac{2\pi nI}{60\,t}$

(C) $\frac{2\sqrt{2}\pi nI}{60\,t}$ (D) $\frac{4\pi nI}{60\,t}$ **[C]**

Q.1 A wheel of radius **r** rolls without slip along the x axis with constant speed. Investigate the motion of a point A on the rim of wheel which starts from the origin O.

Sol.



After a laps of time t, the center C of the wheel will have traveled a distance as shown, and since it rolls without slip, the arc DA will also the length v_0t. Thus the angle DCA will be v_ot/r. Then from the geometry of the figure, we can express the coordinates x and y of the point A flows :

$$x = v_0t - r\sin\frac{v_0t}{r}$$

$$y = r - r\cos\frac{v_0t}{r} \qquad \ldots\ldots(o)$$

with t as a parameter, these two equations define in rectangular coordinates path of point A which is called a cycloid.

Differentiating Eqs.(o) with respect to time gives the velocity-time equations as follows :

$$\dot{x} = v_0\left(1-\cos\frac{v_0t}{r}\right)$$

$$\dot{y} = v_0\sin\frac{v_0t}{r} \qquad \ldots\ldots(p)$$

Using now the first of Eqs. (d) for the magnitude of the resultant velocity, we find

$$v = v_0\sqrt{2\left(1-\cos\frac{v_0t}{r}\right)} = 2v_0\sin\frac{v_0t}{2r} \ \ldots\ldots(q)$$

From this expression we see that the maximum speed of point A is 2 v_0 when t = $\pi r/v_0$, that is, when point A is a the top A′ of its path (see in figure)

Differentiating Eqs.(p) again with respect to time, we obtain the acceleration-time equations

$$\ddot{x} = \frac{v_0^2}{r}\sin\frac{v_0t}{r}$$

$$\ddot{y} = \frac{v_0^2}{r}\cos\frac{v_0t}{r} \qquad \ldots\ldots(r)$$

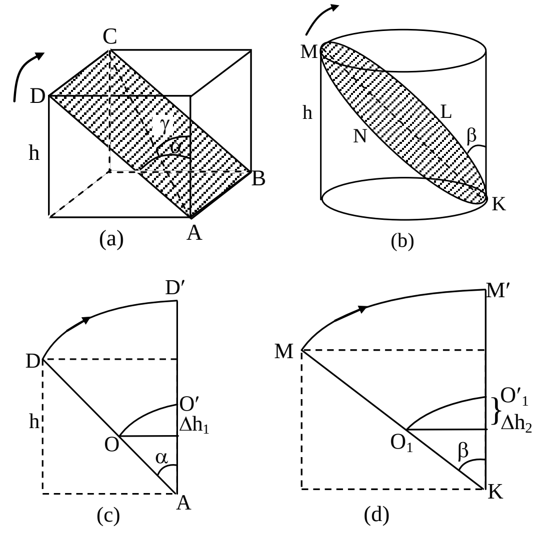
Then using the first of eq. (g) for the magnitude of the resultant acceleration

$$a = \frac{v_0^2}{r} \qquad \ldots\ldots(s)$$

Thus the point A has acceleration of constant magnitude always directed toward the center C of the rolling wheel, as can be established from the last two of Eqs. (g)

Q.2 A cylinder and a cube of the same material, the same height and the same weight stand upright on a horizontal plane. Which of the two bodies is it harder to overturn ?

Sol. To overturn a cube about one edge (e.g. AB) or a cylinder, either must be turned so that their diagonal planes ABCD or KLMN **(Fig. a and b)** occupy a vertical position. For this work must be done to raise the centre of gravity of the body and the work will be greater the higher the centre of gravity has to be raised (since the weight of cube and cylinder are the same).



If the diagonal plane of the cube or cylinder is to occupy a vertical position, the diagonal AD must be rotated through an angle α about the edge AB and the diagonal plane of the cylinder must be turned through an angle β; the cube's centre of gravity will then rise

$$\Delta h_1 = \frac{h}{2}\left(\frac{1}{\cos\alpha} - 1\right),$$

and the cylinder's centre of gravity will rise

$$\Delta h_2 = \frac{h}{2}\left(\frac{1}{\cos\beta} - 1\right)$$

(Fig. c and d). Since the heights and weights of the cube and cylinder are equal and their material is the same, their base areas are also equal, i.e.,

$h^2 = \pi r^2$,

where r is the radius of the base of the cylinder. Plainly for the cube $\alpha = 45^\circ$, For the cylinder

$2r = h \tan\beta$

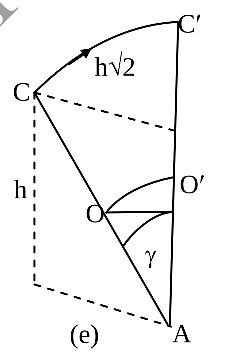
or $4r^2 = h^2\tan^2\beta$.

Substituting for h^2, we get

$4r^2 = \pi r^2 \tan^2\beta$

or $\tan^2\beta = \frac{4}{\pi} > 1$,

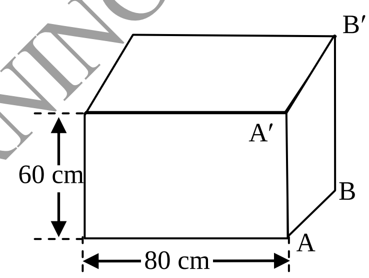
i.e., $\beta > 45^\circ$. Therefore $\cos\beta < \cos\alpha$, and therefore $\Delta h_2 > \Delta h_1$ and **it is harder to overturn the cylinder than the cube about one edge.**



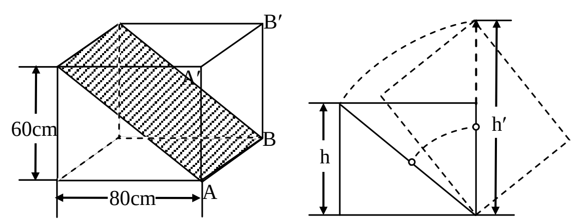
(e)

If the attempt was made to overturn the cube about a corner (instead of about the edge), then it would have to be turned so that the diagonal AC took up a vertical position. For this the cube must be turned through an angle **γ**, formed by this diagonal and the cube's height **(Fig. e)** Then $\tan^2\gamma = 2$. Since $\tan^2\beta = 4/\pi < 2$, $\gamma > \beta$. Therefore $\cos\beta > \cos\gamma$ and **it is harder to overturn a cube about one of its corners than to overturn a cylinder.**

Q.3 Calculate the minimum amount of work necessary to overturn a crate of weight 1 ton, first about edge AB, then about edge A′B′. The dimensions of the crate are given in Fig.



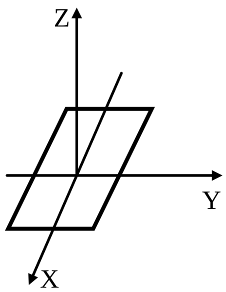
Sol. To overturn a crate about edge AB, the crate must be turned so that the diagonal surface shaded in Fig. becomes vertical (see previous problem) from this point the crate will fall as a result of the force of gravity. Thus work must be done in raising the centre of gravity of the crate to the corresponding height. This work equals



$$W = mg\left(\frac{h'}{2} - \frac{h}{2}\right).$$

In overturning the crate about edge AB we have that $h = 0.6$m, $h' = 1$ m, $W = 0.2$ ton/m. In overturning the crate about edge A′B′ we have that $h = 0.8$ m, $h' = 1$ m, $W = 0.1$ ton/m. **The total work done is 0.2 + 0.1 = 0.3 ton/m.**

Q.4 Given a thin square lamina of a size **a × a** and mass **M** (see fig.)



(a) Find the moment of inertia around the x and y axes.

(b) Find the moment of inertia around the z axis.

(c) Verify the perpendicular axes theorem.

Sol. (a) Denote by A the area of the square. By definition, in our case, we have :

$$I_x = \int_A y^2 dm = \int_A \sigma y^2 dx dy \quad .....(i)$$

where σ is the mass per unit area. Therefore,

$$I_x = \int_{-\frac{a}{2}}^{\frac{a}{2}} dx \int_{-\frac{a}{2}}^{\frac{a}{2}} \sigma y^2 dy = \sigma a \left. \frac{y^3}{3} \right|_{-\frac{a}{2}}^{\frac{a}{2}} = \frac{\sigma a^4}{12} = \frac{Ma^2}{12} \quad ....(ii)$$

Since $M = \sigma a^2$. As the square is symmetric under rotation by 90º, we have

$$I_y = I_x \quad .....(iii)$$

(b) The moment of inertia around the z axis is

$$I_z = \int \sigma(x^2 + y^2) dx dy$$

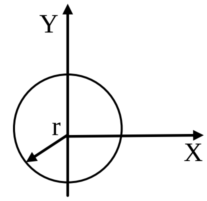
$$= \int_{-\frac{a}{2}}^{\frac{a}{2}} dx \int_{-\frac{a}{2}}^{\frac{a}{2}} \sigma y^2 dy + \int_{-\frac{a}{2}}^{\frac{a}{2}} dy \int_{-\frac{a}{2}}^{\frac{a}{2}} x^2 dx$$

$$= \sigma a \left. \frac{x^3}{3} \right|_{-\frac{a}{2}}^{\frac{a}{2}} + \sigma a \left. \frac{y^3}{3} \right|_{-\frac{a}{2}}^{\frac{a}{2}} = \frac{\sigma a^4}{6} = \frac{Ma^2}{6} \quad .....(iv)$$

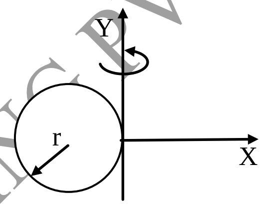
(c) We have : $I_z = I_x + I_y = \frac{Ma^2}{6}$ .....(v)

Hence, the perpendicular axes theorem is verified.

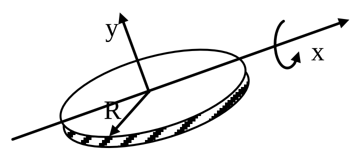
Q.2 (a) Find the moment of inertia of a ring of mass **m** and radius **r** around the diameter **y** (see figure)



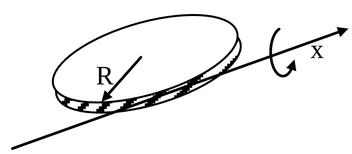
(b) Find the moment of inertia of a ring of mass **m** and radius **r** around an axis tangent to it (see figure).



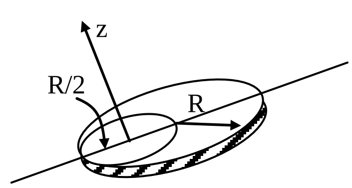
(c) Find the momenta of inertia of a thin disc of mass **m** and radius **R** around the diameter **x** (see figure)



Find the moment of inertia of a thin disc of mass m and radius R around an axis tangent to it (see figure).



(e) Find the moment of inertia of a thin disc of mass m and radius R with a hole of radius $\frac{R}{2}$ around the z axis (see figure).



Sol. (a) By definition,

$$I_y = \int x^2 dm = \int_0^{2\pi} x^2 \lambda r d\theta = \lambda r^3 \int_0^{2\pi} \cos^2\theta d\theta \quad \text{.....(i)}$$

Where λ is the mass per unit length of the ring. Thus,

$$I_y = \lambda r^3 \left(\frac{\theta}{2} + \frac{\sin 2\theta}{2}\right)\Bigg|_0^{2\pi} = \pi\lambda r^3 = \frac{mr^2}{2} \quad \text{.....(ii)}$$

Since the mass of the ring is $m = 2\pi\lambda r$.

(b) Using the parallel axis theorem and Eq. (ii) we obtain :

$$I_{y'} = I_y + mr^2 = \frac{3}{2} mr^2 \quad \text{.....(iii)}$$

(c) By definition,

$$I_x = \int y^2 dm = \sigma \int_A (r^2 \sin^2\theta) r \, dr \, d\theta \quad \text{.....(iv)}$$

Where σ is the mass per unit area. So,

$$I_x = s \int_0^R r^3 dr \int_0^{2\pi} \sin^2\theta d\theta = \frac{\pi\sigma R^4}{4} = \frac{mR^2}{4} \quad \text{.....(v)}$$

Since $m = \pi\sigma R^2$.

(d) From the parallel axis theorem and Eq.(v), it is clear that :

$$I_{x'} = I_x + mR^2 = \frac{5mR^2}{4} \quad \text{.....(vi)}$$

(e) The easiest way is to "fill" the hole, and then subtract the contribution of the hole. The area of the full disc is $\boldsymbol{\pi R^2}$. The area of the hole is $\boldsymbol{\frac{\pi R^2}{4}}$. The area of the disc with the hole is $\boldsymbol{\frac{3\pi\pi^2}{4}}$. Thus by filling the hole, one increases the mass by a factor :

$$\frac{\pi R^2}{3\pi R^2/4} = \frac{4}{3}$$

The moment of inertia of a full disc with a mass of $\boldsymbol{\frac{4}{3}}$ **m** around the z axis is derived from the parallel axis theorem, by :

$$I_{full} = \frac{1}{2} \cdot \frac{4}{3} mR^2 + \frac{4}{3} m \left(\frac{R}{2}\right)^2 = mR^2 \quad \text{.....(vii)}$$

Similarly, the mass of the small disc which fills the hole is given by :

$$m_1 = \frac{\frac{\pi m R^2}{4}}{\frac{3\pi R^2}{4}} = \frac{1}{3} m \quad \text{.....(viii)}$$

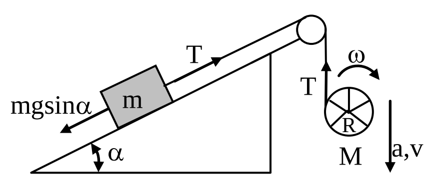
Its moment of inertia around the axis is

$$I_{hole} = \frac{1}{2} \cdot \frac{1}{3} m \left(\frac{R}{2}\right)^2 = \frac{1}{24} mR^2 \quad \text{.....(ix)}$$

Thus, we obtain :

$$I_{total} = I_{full} - I_{hole} = \frac{23}{24} mR^2 \quad \text{.....(x)}$$

Q.3 The system described in **figure** is composed of a wheel of mass **M** and radius **R**, a massless rope which is wrapped at one end around the wheel and is connected at the other end to a block of mass **m**. The block is placed on a slanted plane with a lift angle $\boldsymbol{\alpha}$, and the rope is threaded through a massless pulley connected to the higher end of the incline. Initially, the system is held at rest. As the system is released, the block remains at rest, while the wheel unwraps the rope while rolling down. The wheel is made up of five identical rods of length **R** which are connected at the center of the wheel and distributed evenly over its massless circumference.



(a) Calculate the moment of inertia of the wheel.

(b) Find the angular acceleration of the wheel.

(c) Find the mass of the block.

(d) Calculate the kinetic energy of the system, after the center of mass of the wheel descends a distance **h**.

Sol. (a) The mass of each cord is $\frac{\mathbf{M}}{\mathbf{5}}$. The moment of inertia of each about one of its ends is $\frac{1}{3}\left(\frac{M}{5}R^2\right)$. Therefore, the total moment of inertia of the wheel about its center is :

$$I = 5\frac{1}{3}\ \frac{MR^2}{5} = \frac{1}{3}MR^2 \quad \text{.....(i)}$$

Note: As per the definition of the moment of inertia, I = $\sum_i m_i r_i^2$, we can sum the moments of inertial about the given axis of each.

(b) Since the block is at rest, we have v = ωR and a = R $\frac{d\omega}{dt}$ = Rα, where **a** is the acceleration of the center of mass and **α** is the angular acceleration of the wheel. The equations of motion are therefore,

$$\begin{cases} Mg - T = Ma \text{ equation for force} \\ TR = I\alpha \quad \text{equation for torques} \end{cases} \quad \text{.....(ii)}$$

This set of equations a= $\frac{3}{4}$ g, α= $\frac{3g}{4R}$ and T = $\frac{1}{4}$ Mg.

(c) Using the relation T = mg sin α (m is at rest), and substituting in the value of T found in the previous section, we obtain:

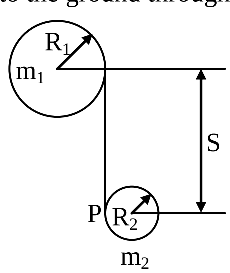
$$m = \frac{M}{4\sin\alpha} \quad \text{.....(iii)}$$

(d) The wheel advance a distance **h**; therefore, the potential energy difference is –Mgh. The principle of conservation of energy implies that ΔK = –ΔU, so,

K = Mgh .....(iv)

Where K is the kinetic energy.

Q.4 A cylinder of mass $\mathbf{m_1}$ is forced to rotate about a fixed axis by a rotating round weight of mass $\mathbf{m_2}$ (see figure). Assume that the string remains vertical to the ground throughout.



(a) Find the center of mass acceleration of $\mathbf{m_2}$.

(b) Calculate the angular acceleration of $\mathbf{m_1}$ and $\mathbf{m_2}$.

(c) Compute the tension of the string.

Sol. Let us write the equations of motion for $\mathbf{m_1}$ and $\mathbf{m_2}$. First, the torque equations are

$$\begin{cases} m_1 : TR_1 = J = I_{m_1}\alpha_1 = \frac{1}{2}m_1R_1^2\alpha_1 \\ m_2 : TR_2 = \frac{1}{2}m_2R_2^2\alpha_2 \end{cases} \quad \text{.....(i)}$$

Now, the sum of forces of m_2 is given by :

$m_2g - T = m_2a$ .....(ii)

Another equation is obtained using the relation linking the angular accelerations, α_1 and α_2

$\alpha = \alpha_1R_1 + \alpha_2R_2$ .....(iii)

Note that $\mathbf{m_2}$ rotates in the same direction as $\mathbf{m_1}$, and therefore, the linear accelerations sum. Now we have a set of four equations for four variables. Solving the set we find :

(a) for the linear acceleration :

$$a = \frac{m_1 + m_2}{\frac{3}{2}m_1 + m_2} \quad \text{.....(iv)}$$

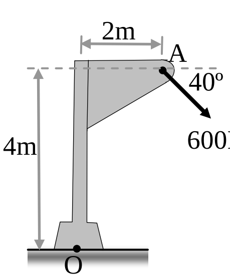
(b) for the angular accelerations :

$$\begin{cases} \alpha_1 = \dfrac{m_2}{\frac{3}{2}m_1 + m_2}\ \dfrac{g}{R_1} \\ \alpha_2 = \dfrac{m_1}{\frac{3}{2}m_1 + m_2R_2}\ \dfrac{g}{R_2} \end{cases} \quad \text{.....(v)}$$

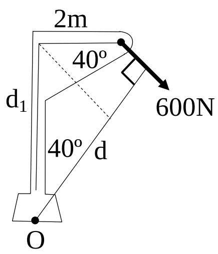
(c) and for the tension in the string:

$$T = \frac{m_1m_2}{3m_1 + 2m_2}g \quad \text{.....(vi)}$$

Q.5 Calculate the magnitude of the moment about base point O of the 600-N force in five different ways.



Sol. (i)



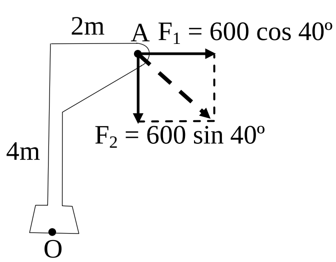
The moment arm to the 600-N force is

$d = 4 \cos 40º + 2 \sin 40º = 4.35$ m

By M = Fd the moment is clockwise and has the magnitude

$M_O = 600\ (4.35) = 2610$ N.m **Ans.**

(ii)



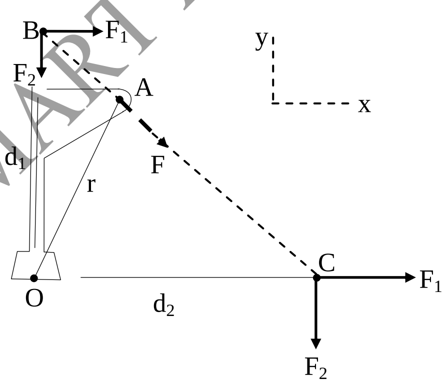
Replace the force by its rectangular components at A

$F_1 = 600 \cos 40º = 460$N, $F_2 = 600 \sin 40º = 386$ N

By Varigonon's theorem, the moment becomes

$M_O = 460(4) + 386(2) = 2610$ N.m

(iii)



By the principle of transmissibility, move the 600-N force along its line of action to point B, which eliminates the moment of the component F_2. The moment arm of F_1 becomes

$d_1 = 4 + 2 \tan 40º = 5.68$ m

and the moment is $M_O = 460(5.68)$

$= 2610$N.m **Ans.**

(iv) Moving the force to point C eliminates the moment of the component F_1. The moment arm of F_2 becomes

$d_2 = 2 + 4 \cot 40º = 6.77$ m

and the moment is $M_O = 386(6.77)$

$= 2610$ N.m**Ans.**

(v) By the vector expression for a moment, and by using the coordinate system indicated on the figure together with the procedures for evaluating cross products, we have

$\mathbf{M_0} = \mathbf{r} \times \mathbf{F} = (2i + 4j) \times 600\ (i \cos 40º - j \sin 40º)$

$= - 2610k$ N.m

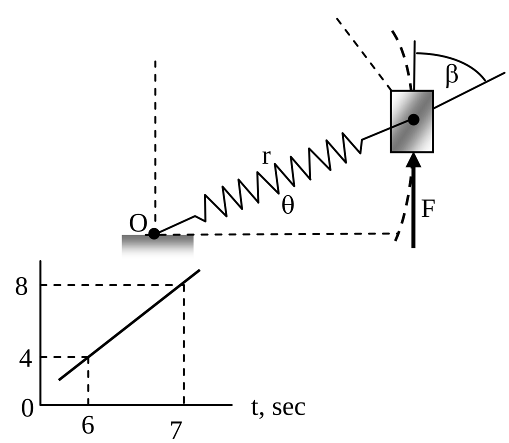
The minus sign indicates that the vector is in the negative z-direction. The magnitude of the vector expression is

$M_O = 2610$ N.m **Ans.**

Helpful Hints:

1. The required geometry here and in similar problems should not cause difficulty if the sketch is carefully drawn.
2. This procedure is frequently the shortest approach.
3. The fact that points B and C are not on the body proper should not cause concern, as the mathematical calculation of the moment of a force does not require that the force be on the body.
4. Alternative choices for the position vector **r** are $\mathbf{r} = d_1\mathbf{j} = 5.68\mathbf{j}$ m and $\mathbf{r} = d_2\mathbf{i} = 6.77\mathbf{i}$ m.

Q.6 The small 3-lb block slides on a smooth horizontal surface under the action of the force in the spring and a force **F**. The angular momentum of the block about O varies with time as shown in the graph. When t = 6.5 sec., it is known that r = 6 in. and β = 60º. Determine **F** for this instant.



Sol. The only moment of the forces about O is due to F since the spring force passes through O. Thus $\Sigma M_O = Fr \sin\beta$. From the graph the time rate of change of H_O for t = 6.5 sec is very nearly (8 – 4) / (7 – 6) or $\dot{H}_O$ = 4 ft-lb. The moment-angular momentum relation gives

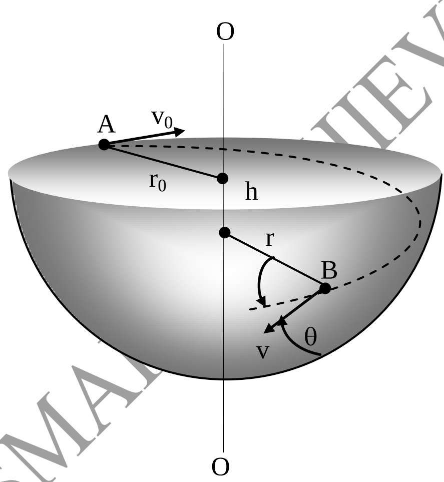
$[\Sigma M_O = \dot{H}_O]$ $\quad F\left(\frac{6}{12}\right)\sin 60^\circ = 4$

F = 9.24 lb **Ans.**

Helpful Hint:

We do not need vector notation here since we have plane motion where since we have plane motion where the direction of the vector H_O does not change.

Q.7 A small mass particle is given an initial velocity $\mathbf{v_0}$ tangent to the horizontal rim of a smooth hemispherical bowl at a radius $\mathbf{r_0}$ from the vertical centerline, as shown at point **A**. As the particle slides past point B, a distance **h** below A and a distance **r** from the vertical centerline, its velocity **v** makes an angle **θ** with the horizontal tangent to the bowl through B. Determine θ.



Sol. The forces on the particle are its weight and the normal reaction exerted by the smooth surface of the bowl. Neither force exerts a moment about the axis O–O, so that angular momentum is conserved about that axis. Thus,

$[(H_O)_1 = (H_O)_2]$

$$mv_0r_0 = mvr\cos\theta$$

Also, energy is conserved so that $E_1 = E_2$. Thus

$[T_1 + V_{g_1} = T_2 + T_{g_2}]$ $\frac{1}{2}mv_0^2 + mgh = \frac{1}{2}mv^2 + 0$

$$v = \sqrt{v_0^2 + 2gh}$$

Eliminating v and substituting $r^2 = r_0^2 - h^2$ give

$$v_0r_0 = \sqrt{v_0^2 + 2gh}\,\sqrt{r_0^2 - h^2}\,\cos\theta$$

$$\theta = \cos^{-1}\frac{1}{\sqrt{1+\frac{2gh}{v_0^2}}\sqrt{1-\frac{h^2}{r_0^2}}} \qquad \textbf{Ans.}$$

Q.8 A flywheel in the form of a uniformly thick disk 4ft in diameter weighs 600lbs and rotates at 1200 rpm. Calculate the constant torque necessary to stop it in 2.0 min.

Sol. The equation of motion for the flywheel is

$$I\ddot{\theta} = -M,$$

Where I is the moment of inertia and M is the stopping torque. Hence

$$\dot{\theta} = \omega_0 - \frac{Mt}{I}.$$

When the flywheel stops at time t, $\dot{\theta} = 0$ and

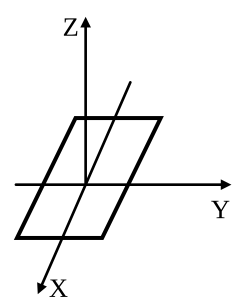
$$M = \frac{I\omega_0}{t}.$$

With $I = \frac{MR^2}{2}$ = 1200 lb ft^2, $\omega_0 = 40\pi$ rad/s,

t = 120s

M = 400π pdl ft = 39 lb ft.

Q.9 Given a thin square lamina of a size **a × a** and mass **M** (see fig.)



(a) Find the moment of inertia around the x and y axes.

(b) Find the moment of inertia around the z axis.

(c) Verify the perpendicular axes theorem.

Sol. (a) Denote by A the area of the square. By definition, in our case, we have :

$$I_x = \int_A y^2 dm = \int_A \sigma y^2 dx dy \quad .....(i)$$

where σ is the mass per unit area. Therefore,

$$I_x = \int_{-\frac{a}{2}}^{\frac{a}{2}} dx \int_{-\frac{a}{2}}^{\frac{a}{2}} \sigma y^2 dy = \sigma a \left.\frac{y^3}{3}\right|_{-\frac{a}{2}}^{\frac{a}{2}} = \frac{\sigma a^4}{12} = \frac{Ma^2}{12} \quad .....(ii)$$

Since $M = \sigma a^2$. As the square is symmetric under rotation by 90º, we have

$$I_y = I_x \quad .....(iii)$$

(b) The moment of inertia around the z axis is

$$I_z = \int \sigma (x^2 + y^2) dx\, dy$$

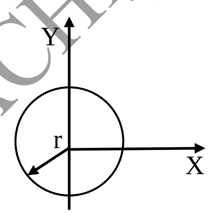
$$= \int_{-\frac{a}{2}}^{\frac{a}{2}} dx \int_{-\frac{a}{2}}^{\frac{a}{2}} \sigma y^2 dy + \int_{-\frac{a}{2}}^{\frac{a}{2}} dy \int_{-\frac{a}{2}}^{\frac{a}{2}} x^2 dx$$

$$= \sigma a \left.\frac{x^3}{3}\right|_{-\frac{a}{2}}^{\frac{a}{2}} + \sigma a \left.\frac{y^3}{3}\right|_{-\frac{a}{2}}^{\frac{a}{2}} = \frac{\sigma a^4}{6} = \frac{Ma^2}{6} \quad .....(iv)$$

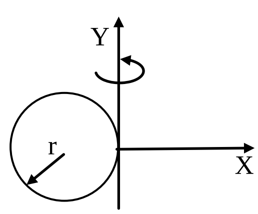
(c) We have : $I_z = I_x + I_y = \frac{Ma^2}{6}$ .....(v)

Hence, the perpendicular axes theorem is verified.

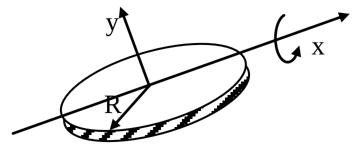
Q.10(a) Find the moment of inertia of a ring of mass **m** and radius **r** around the diameter **y** (see figure)



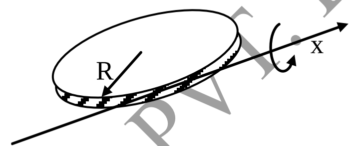
(b) Find the moment of inertia of a ring of mass **m** and radius **r** around an axis tangent to it (see figure).



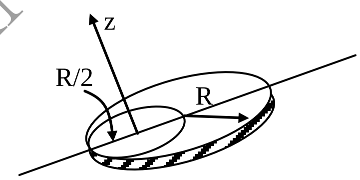
(c) Find the momenta of inertia of a thin disc of mass **m** and radius **R** around the diameter **x** (see figure)



Find the moment of inertia of a thin disc of mass m and radius R around an axis tangent to it (see figure).



(e) Find the moment of inertia of a thin disc of mass m and radius R with a hole of radius $\frac{R}{2}$ around the z axis (see figure).



Sol. (a) By definition,

$$I_y = \int x^2 dm = \int_0^{2\pi} x^2 \lambda r d\theta = \lambda r^3 \int_0^{2\pi} \cos^2\theta d\theta \quad .....(i)$$

Where λ is the mass per unit length of the ring. Thus,

$$I_y = \lambda r^3 \left(\frac{\theta}{2} + \frac{\sin 2\theta}{2}\right)\Bigg|_0^{2\pi} = \pi\lambda r^3 = \frac{mr^2}{2} \quad .....(ii)$$

Since the mass of the ring is $m = 2\pi\lambda r$.

(b) Using the parallel axis theorem and Eq. (ii) we obtain :

$$I_{y'} = I_y + mr^2 = \frac{3}{2} mr^2 \quad .....(iii)$$

(c) By definition,

$$I_x = \int y^2 dm = \sigma \int_A (r^2 \sin^2\theta) r\, dr\, d\theta \quad .....(iv)$$

Where σ is the mass per unit area. So,

$$I_x = s\int_0^R r^3 dr \int_0^{2\pi} \sin^2\theta d\theta = \frac{\pi\sigma R^4}{4} = \frac{mR^2}{4} \quad .....(v)$$

Since $m = \pi\sigma R^2$.

(d) From the parallel axis theorem and Eq.(v), it is clear that :

$$I_{x'} = I_x + mR^2 = \frac{5mR^2}{4} \quad .....(vi)$$

(e) The easiest way is to "fill" the hole, and then subtract the contribution of the hole. The area of the full disc is πR^2. The area of the hole is $\frac{\pi R^2}{4}$. The area of the disc with the hole is $\frac{3\pi\pi^2}{4}$. Thus by filling the hole, one increases the mass by a factor :

$$\frac{\pi R^2}{3\pi R^2/4} = \frac{4}{3}$$

The moment of inertia of a full disc with a mass of $\frac{4}{3}$ **m** around the z axis is derived from the parallel axis theorem, by :

$$I_{full} = \frac{1}{2}\cdot\frac{4}{3}mR^2 + \frac{4}{3}m\left(\frac{R}{2}\right)^2 = mR^2 .....(vii)$$

Similarly, the mass of the small disc which fills the hole is given by :

$$m_1 = \frac{\frac{\pi m R^2}{4}}{\frac{3\pi R^2}{4}} = \frac{1}{3}m \quad .....(viii)$$

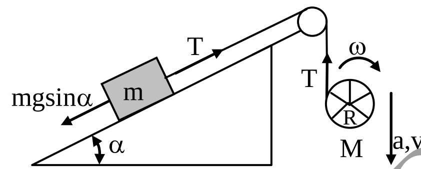
Its moment of inertia around the axis is

$$I_{hole} = \frac{1}{2}\cdot\frac{1}{3}m\left(\frac{R}{2}\right)^2 = \frac{1}{24}mR^2 .....(ix)$$

Thus, we obtain :

$$I_{total} = I_{full} - I_{hole} = \frac{23}{24}mR^2 \quad .....(x)$$

Q.11 The system described in **figure** is composed of a wheel of mass **M** and radius **R**, a massless rope which is wrapped at one end around the wheel and is connected at the other end to a block of mass **m**. The block is placed on a slanted plane with a lift angle **α**, and the rope is threaded through a massless pulley connected to the higher end of the incline. Initially, the system is held at rest. As the system is released, the block remains at rest, while the wheel unwraps the rope while rolling down. The wheel is made up of five identical rods of length **R** which are connected at the center of the wheel and distributed evenly over its massless circumference.



(a) Calculate the moment of inertia of the wheel.

(b) Find the angular acceleration of the wheel.

(c) Find the mass of the block.

(d) Calculate the kinetic energy of the system, after the center of mass of the wheel descends a distance **h**.

Sol. (a) The mass of each cord is $\frac{M}{5}$. The moment of inertia of each about one of its ends is $\frac{1}{3}\left(\frac{M}{5}R^2\right)$. Therefore, the total moment of inertia of the wheel about its center is :

$$I = 5\frac{1}{3}\frac{MR^2}{5} = \frac{1}{3}MR^2 .....(i)$$

Note: As per the definition of the moment of inertia, $I = \sum_i m_i r_i^2$, we can sum the moments of inertial about the given axis of each.

(b) Since the block is at rest, we have v = ωR and a = R $\frac{d\omega}{dt}$ = Rα, where **a** is the acceleration of the center of mass and **α** is the angular acceleration of the wheel. The equations of motion are therefore,

$$\begin{cases} Mg - T = Ma \text{ equation for force} \\ TR = I\alpha \quad \text{ equation for torques} \end{cases} .....(ii)$$

This set of equations $a = \frac{3}{4}g$, $\alpha = \frac{3g}{4R}$ and

$$T = \frac{1}{4}Mg.$$

(c) Using the relation T = mg sin α (m is at rest), and substituting in the value of T found in the previous section, we obtain:

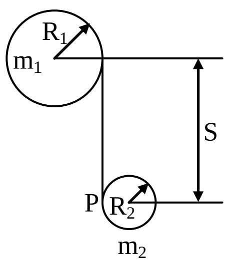
$$m = \frac{M}{4\sin\alpha} .....(iii)$$

(d) The wheel advance a distance **h**; therefore, the potential energy difference is –Mgh. The principle of conservation of energy implies that $\Delta K = -\Delta U$, so,

$$K = Mgh \quad .....(iv)$$

Where K is the kinetic energy.

Q.12 A cylinder of mass $\mathbf{m_1}$ is forced to rotate about a fixed axis by a rotating round weight of mass $\mathbf{m_2}$ (see figure). Assume that the string remains vertical to the ground throughout.



(a) Find the center of mass acceleration of $\mathbf{m_2}$.

(b) Calculate the angular acceleration of $\mathbf{m_1}$ and $\mathbf{m_2}$.

(c) Compute the tension of the string.

Sol. Let us write the equations of motion for $\mathbf{m_1}$ and $\mathbf{m_2}$. First, the torque equations are

$$\begin{cases} m_1 : TR_1 = J = I_{m_1}\alpha_1 = \frac{1}{2}m_1R_1^2\alpha_1 \\ m_2 : TR_2 = \frac{1}{2}m_2R_2^2\alpha_2 \end{cases} \quad .....(i)$$

Now, the sum of forces of m_2 is given by :

$$m_2g - T = m_2a \quad .....(ii)$$

Another equation is obtained using the relation linking the angular accelerations, $\boldsymbol{\alpha_1}$ and $\boldsymbol{\alpha_2}$

$$\alpha = \alpha_1R_1 + \alpha_2R_2 \quad .....(iii)$$

Note that $\mathbf{m_2}$ rotates in the same direction as $\mathbf{m_1}$, and therefore, the linear accelerations sum. Now we have a set of four equations for four variables. Solving the set we find :

(a) for the linear acceleration : $a = \dfrac{m_1 + m_2}{\frac{3}{2}m_1 + m_2}$ ....(iv)

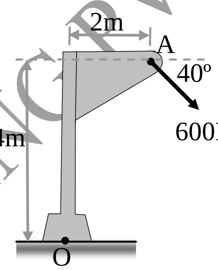
(b) for the angular accelerations :

$$\begin{cases} \alpha_1 = \dfrac{m_2}{\frac{3}{2}m_1 + m_2}\dfrac{g}{R_1} \\ \alpha_2 = \dfrac{m_1}{\frac{3}{2}m_1 + m_2R_2}\dfrac{g}{R_2} \end{cases} \quad .....(v)$$

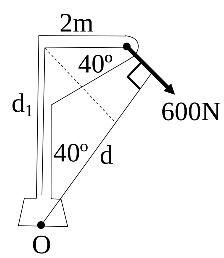
(c) and for the tension in the string:

$$T = \frac{m_1m_2}{3m_1 + 2m_2}g \quad .....(vi)$$

Q.13 Calculate the magnitude of the moment about base point O of the 600-N force in five different ways.



Sol. (i)



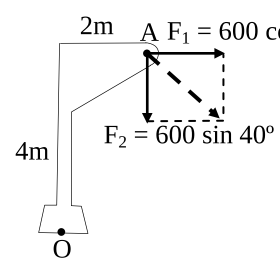
The moment arm to the 600-N force is

$$d = 4\cos 40^\circ + 2\sin 40^\circ = 4.35 \text{ m}$$

By M = Fd the moment is clockwise and has the magnitude

$M_O = 600\,(4.35) = 2610$ N.m **Ans.**

(ii)



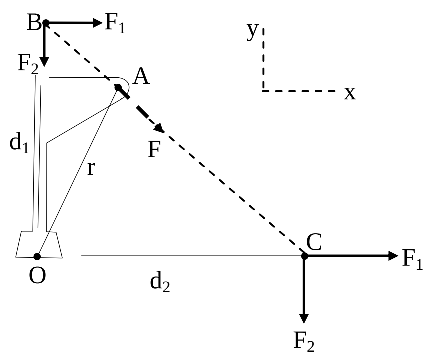
Replace the force by its rectangular components at A

$F_1 = 600 \cos 40^\circ = 460 N$, $F_2 = 600 \sin 40^\circ = 386$ N

By Varigonon's theorem, the moment becomes

$$M_O = 460(4) + 386(2) = 2610 \text{ N.m}$$

(iii)



By the principle of transmissibility, move the 600-N force along its line of action to point B, which eliminates the moment of the component F_2. The moment arm of F_1 becomes

$$d_1 = 4 + 2 \tan 40^\circ = 5.68 \text{ m}$$

and the moment is $M_O = 460(5.68)$

$= 2610$N.m **Ans.**

(iv) Moving the force to point C eliminates the moment of the component F_1. The moment arm of F_2 becomes

$$d_2 = 2 + 4 \cot 40^\circ = 6.77 \text{ m}$$

and the moment is $M_O = 386(6.77)$

$= 2610$ N.m **Ans.**

(v) By the vector expression for a moment, and by using the coordinate system indicated on the figure together with the procedures for evaluating cross products, we have

$\mathbf{M_0} = \mathbf{r} \times \mathbf{F} = (2i + 4j) \times 600\,(i \cos 40^\circ - j \sin 40^\circ)$

$= -2610k$ N.m

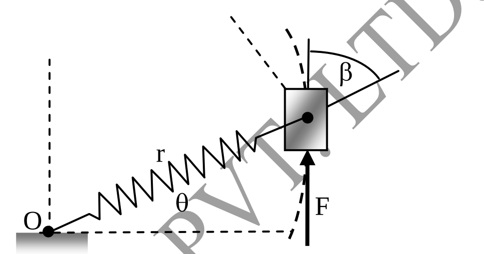
The minus sign indicates that the vector is in the negative z-direction. The magnitude of the vector expression is

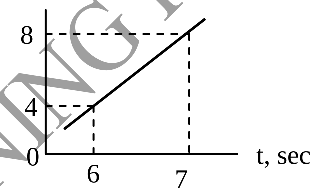
$$M_O = 2610 \text{ N.m}$$

Helpful Hints:

1. The required geometry here and in similar problems should not cause difficulty if the sketch is carefully drawn.
2. This procedure is frequently the shortest approach.
3. The fact that points B and C are not on the body proper should not cause concern, as the mathematical calculation of the moment of a force does not require that the force be on the body.
4. Alternative choices for the position vector **r** are $\mathbf{r} = d_1\mathbf{j} = 5.68\mathbf{j}$ m and $\mathbf{r} = d_2\mathbf{i} = 6.77\mathbf{i}$ m.

Q.14 The small 3-lb block slides on a smooth horizontal surface under the action of the force in the spring and a force **F**. The angular momentum of the block about O varies with time as shown in the graph. When t = 6.5 sec., it is known that r = 6 in. and $\beta = 60^\circ$. Determine **F** for this instant.





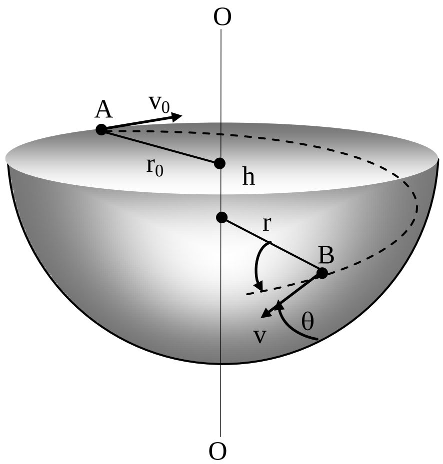
Sol. The only moment of the forces about O is due to F since the spring force passes through O. Thus $\Sigma M_O = Fr \sin \beta$. From the graph the time rate of change of H_O for t = 6.5 sec is very nearly (8 – 4) / (7 – 6) or $H_O = 4$ ft-lb. The moment-angular momentum relation gives

$$[\Sigma M_O = H_O]\ F\left(\frac{6}{12}\right) \sin 60^\circ = 4F = 9.24 \text{ lb} \quad \textbf{Ans.}$$

Helpful Hint:

We do not need vector notation here since we have plane motion where since we have plane motion where the direction of the vector H_O does not change. **Ans.**

Q.15 A small mass particle is given an initial velocity $\mathbf{v_0}$ tangent to the horizontal rim of a smooth hemispherical bowl at a radius $\mathbf{r_0}$ from the vertical centerline, as shown at point **A**. As the particle slides past point B, a distance **h** below A and a distance **r** from the vertical centerline, its velocity **v** makes an angle **θ** with the horizontal tangent to the bowl through B. Determine θ.



Sol. The forces on the particle are its weight and the normal reaction exerted by the smooth surface of the bowl. Neither force exerts a moment about the axis O–O, so that angular momentum is conserved about that axis. Thus,

$[(H_O)_1 = (H_O)_2]$

$$mv_0r_0 = mvr\cos\theta$$

Also, energy is conserved so that $E_1 = E_2$. Thus

$[T_1 + V_{g_1} = T_2 + T_{g_2}]\ \frac{1}{2}mv_0^2 + mgh = \frac{1}{2}mv^2 + 0$

$v = \sqrt{v_0^2 + 2gh}$

Eliminating v and substituting $r^2 = r_0^2 - h^2$ give

$v_0r_0 = \sqrt{v_0^2 + 2gh}\ \sqrt{r_0^2 - h^2}\cos\theta$

$\theta = \cos^{-1}\dfrac{1}{\sqrt{1+\dfrac{2gh}{v_0^2}}\sqrt{1-\dfrac{h^2}{r_0^2}}}$ **Ans.**

Q.16 A flywheel in the form of a uniformly thick disk 4ft in diameter weighs 600lbs and rotates at 1200 rpm. Calculate the constant torque necessary to stop it in 2.0 min.

Sol. The equation of motion for the flywheel is

$$I\ddot{\theta} = -M,$$

Where I is the moment of inertia and M is the stopping torque. Hence

$$\dot{\theta} = \omega_0 - \frac{Mt}{I}.$$

When the flywheel stops at time t, $\dot{\theta} = 0$ and

$$M = \frac{I\omega_0}{t}.$$

With $I = \frac{MR^2}{2} = 1200$ lb ft^2, $\omega_0 = 40\pi$ rad/s, t=120s

M = 400π pdl ft = 39 lb ft.

Q.17 A flywheel rotating freely at 1800 rev/min clockwise is subjected to a variable counterclockwise torque which is first applied at time t = 0. The torque produces a counterclockwise angular acceleration α = 4t rad/s^2, where t is the time in seconds during which the torque is applied. Determine **(a)** The time required for the flywheel to reduce its clockwise angular speed to 900 rev/min, **(b)** The time required for the flywheel to reverse its direction of rotation, and **(c)** The total number of revolutions, clockwise plus counterclockwise, turned by the flywheel during the first 14 seconds of torque application.

Sol. The counterclockwise direction will be taken arbitrarily as positive.

(a) Since α is a known function of the time, we may integrate it to obtain angular velocity. With the initial angular velocity of $-1800(2\pi)/60 = -60\pi$ rad/s, we have

$[d\omega = \alpha\, dt]\ \int_{-60\pi}^{\omega} d\omega = \int_0^t 4t\, dt\ \ \omega = -60\pi + 2t^2$

Substituting the clockwise angular speed of 900 rev/min or $\omega = -900(2\pi)\ 60 = 30\pi$ rad/s gives

$-30\pi = -60\pi + 2t^2$

$t^2 = 15\pi$ **t = 6.86s Ans.**

(b) The flywheel changes direction when its angular velocity is momentarily zero. Thus,

$0 = -60\pi + 2t^2$ $t^2 = 30\pi$ **t = 9.71s Ans.**

(c) The total number of revolutions through which the flywheel turns during 14 seconds is the number of clockwise turns N_1 during the first 9.71 seconds, plus the number of counterclockwise turns N_2 during the remainder of the interval. Integrating the expression for **ω** in terms of **t** gives us the angular displacement in radians. Thus, for the first interval

$[d\theta = \omega\, dt]$

$$\int_{-60\pi}^{\theta_1} d\theta = \int_0^{9.71} (-60\pi + 2t^2)\, dt$$

$$\theta_1 = \left[-60\pi t + \frac{2}{3}t^3\right]_0^{9.71} = -1220 \text{ rad}$$

or $N_1 = 1220/2\pi = 194.2$ revolutions clockwise.

For the second interval

$$\int_0^{\theta_2} d\theta = \int_{9.71}^{14} (-60\pi + 2t^2)\, dt$$

$$\theta = \left[-60\pi t + \frac{2}{3}t^3\right]_{9.71}^{1.4} = 410 \text{ rad}$$

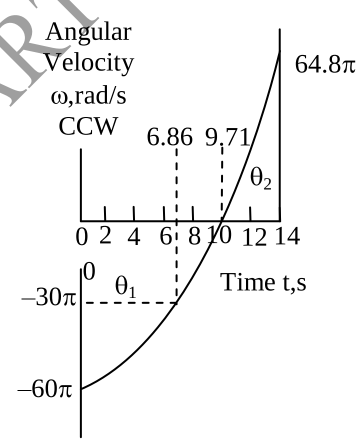
or $N_2 = 410/2\pi = 65.3$ revolutions counterclockwise. Thus, the total number of revolutions turned during the 14 seconds is

$N = N_1 + N_2 = 194.2 + 6.53 =$ **259 rev**

We have plotted **ω** versus **t** and we see that $\boldsymbol{\theta_1}$ is represented by the negative area and $\boldsymbol{\theta_2}$ by the positive area. If we had integrated over the entire interval in one step, we would have obtained $|\theta_2| - |\theta_1|$.

Helpful :

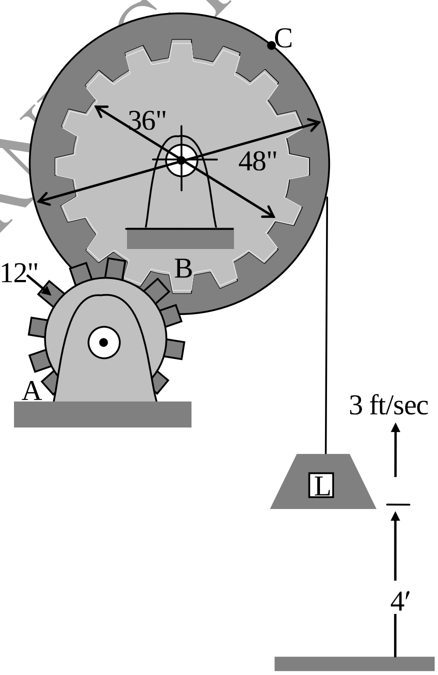
(i) We must be very careful to be consistent with our algebraic signs. The lower limit is the negative (clockwise) value of the initial angular velocity. Also we must convert revolutions of radians since α is in radian units.



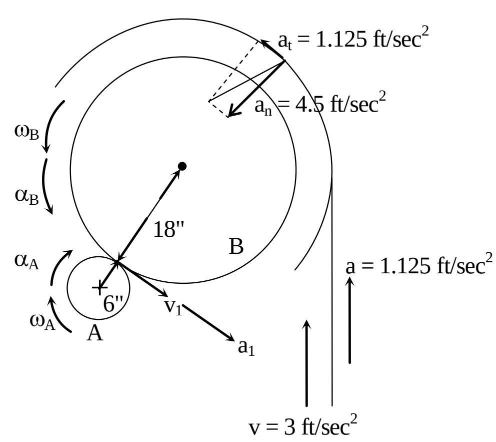
(ii) Again note that the minus sign signifies clockwise in this problem.

(iii) We could have converted the original expression for α into the units of rev/s^2, in which case our integrals would have come out directly in revolutions.

Q.18 The pinion A of the hoist motor drives gear B, which is attached to the hoisting drum. The load L is lifted from its rest position and acquires an upwards velocity of 3ft/sec in a vertical rise of 4 ft with constant acceleration. As the load passes this position, compute **(a)** the acceleration of point C on the cable in contact with the drum and **(b)** the angular velocity and angular acceleration of the pinion A.



Sol.



(a) If the cable does not slip on, the drum, the vertical velocity and acceleration of the load L are, of necessity, the same as the tangential velocity **v** and tangential acceleration $\mathbf{a_t}$ of point C. For the rectilinear motion of L with constant acceleration, the n- and t-components of the acceleration of C become

$[v^2 = 2as]$ $a = a_t = v^2/2s = 3^2/[2(4)] = 1.125$ ft/sec^2

$[a_n = v^2/r]$ $a_n = 3^2/(24/12) = 4.5$ ft/sec^2

$[a = \sqrt{a_n^2 + a_t^2}\,]$ $a_c = \sqrt{(4.5)^2 + (1.125)^2}$

$= 4.64$ ft/sec^2. **Ans.**

(b) The angular motion of gear A is determined from the angular motion of gear B by the velocity $\mathbf{v_1}$ and tangential acceleration $\mathbf{a_1}$ of their common point of contact. First, the angular motion of gear B is determined from the motion of point C on the attached drum. Thus,

$[v = r\omega]$ $\omega_B = v/r = 3/(24/12) = 1.5$ rad/sec

$[a_t = r\alpha]$ $\alpha_B = \alpha_t/r = 1.125/(24/12) = 0.562$ rad/sec^2

Then from $v_1 = r_A\omega_A = r_B\omega_B$ and $a_1 = r_A\alpha_A = r_B\alpha_B$, we have

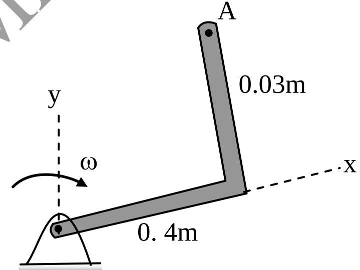
$\omega_A = \dfrac{r_B}{r_A}\omega_B = \dfrac{18/12}{6/12}1.5 =$ **4.5 rad/sec CW**

$\alpha_A = \dfrac{r_B}{r_A}\alpha_B = \dfrac{18/12}{6/12}0.562 =$ **1.688 rad/sec² CW**

Helpful Hint:

Recognize that a point on the cable changes the direction of its velocity after it contacts the drum and acquires a normal component of acceleration.

Q.19 The right-angle bar rotates clockwise with an angular velocity which is decreasing at the rate of 4rad/s^2. Write the vector expressions for the velocity and acceleration of point A when $\omega = 2$ rad/s.



Sol. Using the right-hand rule gives

$\omega = -2\mathbf{k}$ rad/s and $\alpha = +4\mathbf{k}$ rad/s^2

The velocity and acceleration of A become

$[v = \omega \times r]$ $v = -2k \times (0.4i + 0.3j)$

$=$ **0.6i – 8j m/s** **Ans.**

$[a_n = \omega \times (\omega \times r)]$ $a_n = -2k \times (0.6i - 0.8j)$

$= -1.6i - 1.2j$ m/s^2

$[a_t = \alpha \times r]$ $a_t = 4k \times (0.4i + 0.3j)$

$= -1.2i + 1.6j$ m/s^2

$[a = a_n + a_t]$ $a = -2.8i + 0.4j$ m/s^2 **Ans.**

The magnitudes of **v** and **a** are

$v = \sqrt{0.6^2 + 0.8^2} = 1$ m/s and

$a = \sqrt{2.8^2 + 0.4^2} = 2.83$ m/s^2

Q.20 A wheel of radius **r** rolls on a flat surface without slipping. Determine the angular motion of the wheel in terms of the linear motion of its center O. Also determine the acceleration of a point on the rim of the wheel as the point comes into contact with the surface on which the wheel rolls.

Ans.

Ans.

Sol. The figure shows the wheel rolling to the right from the dashed to the full position without slipping. The linear displacement of the center O is **s,** radial line CO rotates to the new position C′O′ through the angle θ, where θ is measured from the vertical direction. If the wheel does not slip, the are CA must equal to distance **s**. Thus the displacement relationship and its two time derivatives give

$$\mathbf{s = r\theta,\ v_O = r\omega,\ a_O = r\alpha} \quad \textbf{Ans}$$

where $v_O = \dot{s}$, $a_O = \dot{v}_O = \ddot{s}$, $\omega = \dot{\theta}$, and $\alpha = \dot{\omega} = \ddot{\theta}$. The angle θ, of course, must be in radians. The acceleration $\mathbf{a_O}$ will be directed in the sense opposite to that of $\mathbf{v_O}$ if the wheel is slowing down. In this event, the angular acceleration α will have the sence opposite to that of ω.

The origin of fixed coordinates is taken arbitrarily but conveniently at the point of contact between C on the rim of the wheel and the ground. When point C has moved along its cycloidal path to C′, its new coordinates and their time derivatives become

$x = s – r \sin\theta = r(\theta – \sin\theta)$

$y = r – r\cos\theta = r(1 – \cos\theta)$

$\dot{x} = r\dot{\theta}(1 – \cos\theta) = v_0(1 – \cos\theta)$

$\dot{y} = r\dot{\theta}\sin\theta = v_0\sin\theta$

$\ddot{x} = \dot{v}_O(1 – \cos\theta) + v_O\dot{\theta}\sin\theta$

$\ddot{y} = \dot{v}_O\sin\theta) + v_O\dot{\theta}\cos\theta$

$= a_O(1 – \cos\theta)\, r\omega^2\sin\theta$

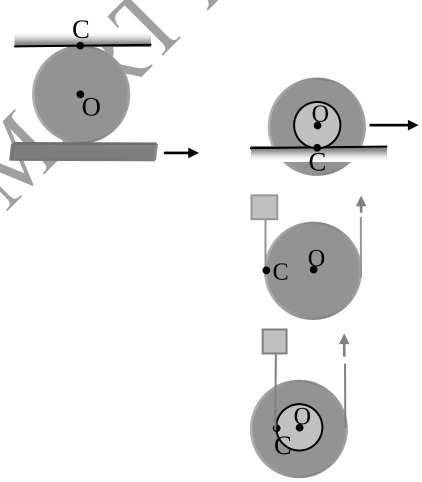
$= a_O\sin\theta + r\omega^2\cos\theta$

For the desired instant of contact, $\theta = 0$ and

$\ddot{x} = 0$ **and** $\ddot{y} = r\omega^2$ **Ans.**

Thus, the acceleration of the point C on the rim at the instant of contact with the ground depends only on **r** and **ω** and is directed toward the center of the wheel. If desired, the velocity and acceleration of C at any position θ may be obtained by writing the expressions $v = \dot{x}i + \dot{y}j$ and $a = \ddot{x}i + \ddot{y}j$.

Application of the kinematic relationships for a wheel which rolls without slipping should be recognized for various configurations of rolling wheels such as those illustrated on the right. If a wheel slips as it rolls, the foregoing relations are no longer valid.



Helpful Hints :

(i) These three relations are not entirely unfamiliar at this point, and their application to the rolling wheel should be mastered thoroughly.

(ii) Clearly, when $\theta = 0$, the point of contact has zero velocity so that $\dot{x} = \dot{y} = 0$. The acceleration of the contact point on the wheel will also be obtained by the principles of relative motion.