PHYSICS

The following questions consists of two statements each, printed as Assertion and Reason. While answering these questions you are to choose any one of the following four responses.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true but Reason is not correct explanation of the Assertion.

(C) If Assertion is true but the Reason is false.

(D) If Assertion is false but Reason is true.

Q.1 **Assertion :** Where two vibrating tuning forks having frequencies 256 Hz and 512 Hz are held near each other, beats cannot be heard.

Reason : The principle of superposition is valid only if the frequencies of the oscillations are nearly equal . **[C]**

Q.2 **Assertion :** In open organ pipe position of pressure node is a little distance out from the opening of tube.

Reason : Sound waves are reaching at the opening of tube are only partially reflected. **[C]**

Sol. Sound waves are not reflected exactly from the opening but from a little distance away from opening.

Q.3 **Assertion :** Sound would travel faster on a hot summer day than on a cold winter day.

Reason : Velocity of sound is directly proportional to the square of its absolute temperature. **[C]**

Q.4 **Assertion** : Two persons on the surface of moon cannot talk to each other.

Reason : There is no atmosphere on moon. **[A]**

Q.5 **Assertion** : Sound passes through air in the form of longitudinal waves.

Reason : Longitudinal waves are easier to propagate. **[C]**

Q.6 **Assertion**: Compression and rarefaction involve changes in density and pressure.

Reason : When particles are compressed, density of medium increases and when they are rarefied, density of medium decreases. **[A]**

Q.7 **Assertion** : Sound travels faster on a rainy day than on a dry day.

Reason : Moisture increases the pressure. **[C]**

Q.8 **Assertion** : For a closed pipe, the first resonance length is 60 cm. The second resonance position will be obtained at 120 cm.

Reason : In a closed pipe $\ell_2 = 3\ell_1$. **[D]**

Q.9 **Assertion** : In stationary waves, energy is confined within the wave region.

Reason : Everything is stationary in a stationary wave. **[C]**

Q.10 **Assertion** : When two vibrating tuning forks having frequencies 256 and 512 are held near each other, beats cannot be heard.

Reason : The principle of superposition is valid only if frequencies of oscillators are nearly equal. **[C]**

Q.11 **Assertion:** If two waves of same amplitude produce a resultant wave of same amplitude, then the phase difference between them will be 120°.

Reason : The resultant amplitude of two waves is equal to sum of amplitudes of two waves.

[C]

Q.12 **Assertion :** If oil of density higher than that of water is used in place of water in a resonance tube, the frequency decreases.

Reason : Frequency does not depends on change in medium in resonance tube. **[D]**

Q.13 **Statement 1 :** The velocity of sound in the air increases due to presence of moisture in it.

Statement 2 : The presence of moisture in air lowers the density of air.

(A) Both Statement 1 & Statement 2 are correct & Statement 2 is correct explanation of Statement 1

(B) Both Statement 1 & Statement 2 are correct & Statement 2 is not correct explanation of Statement 1

(C) Statement 1 is correct & Statement 2 is wrong

(D) Statement 2 is correct & Statement 1 is wrong

Q. 14 **Statement –I :** For a closed pipe, the first resonance length is 60cm. The second resonance position will be obtained at 120 cm.

Statement-II : In a closed pipe, $\ell_2 = 3\ell_1$.

(A) Statement (I) and (II) are correct, and Statement (II) is the correct explanation of Statement (I).

(B) Statement (I) and (II) are correct but Statement (II) is not the correct explanation of Statement (I).

(C) Statement (I) is correct but Statement (II) is wrong.

(D) Statement (I) is wrong but Statement (II) is correct. **[D]**

Sol If first resonance is at $\frac{\lambda}{4}$ = 60 cm

Next resonance will be at $\frac{3\lambda}{4}$ = 180cm

So statement (1) is wrong.

Q.15 **Assertion :** A closed organ pipe vibrates its fundamental mode. The pressure variation is maximum at the open end.

Reason : This is so because the open end is free and the gas pressure is close to the atmospheric pressure. **[D]**

Sol. (A) is false but (R) is true.

Q.16 **Assertion :** A sound wave can be studied as any of the three waves, namely; pressure wave, displacement wave or density wave.

Reason : In a sound wave, pressure, displacement and density change simultaneously to a maximum or minimum.

Sol. **[C]**

A sound wave can be studied as any of the three waves namely pressure wave, displacement wave or density wave. All the three quantities are never at their maximum or minimum simultaneously.

Q.17 **Statement - I :** When two sounds of equal frequencies and slightly different intensities are heard together, beats are heard.

Statement - II : Beats are caused by alternate constructive and destructive interferences between two sounds.

Sol. **[D]**

Q.18 **Statement-I :** Equation of a stationary wave can be given by: $y = 20\,mm \sin\frac{\pi x}{4}.\cos\omega t$.

Statement-II : Distance between two consecutive antinodes is $\lambda/2 = \frac{\pi}{8}$ m.

Sol. **[C]** Statement-I is true but II is false.

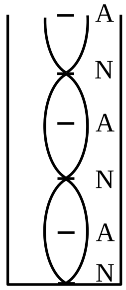
$$d = \frac{\lambda}{2} = \frac{2\pi}{2k} = \frac{\pi}{k} = \frac{\pi}{\pi/4} = 4m.$$

Q.1 For a closed organ pipe, match the following :

Column - I	Column - II
(A) Third overtone frequency is x times the fundamental frequency. Here, x is equal to	(P) 3
(B) Number of nodes in second overtone	(Q) 4
(C) Number of antinodes in second overtone	(R) 5
	(S) None

Sol. **(A) → S; (B) → P; (C) → P**

Third overtone frequency of closed pipe means seventh harmonic. Which is 7 times the fundamental frequency.



Second overtone of closed pipe.

Q.2 Match the following :

Column - I	Column - II
(A) In refraction	(P) Speed of wave does not change
(B) In reflection	(Q) Wavelength is decreased
(C) In refraction from rarer to denser medium	(R) Frequency does not change
(D) In reflection from a denser medium	(S) Phase change of π takes place

Sol. **(A) → R;** **(B) → P, R;**
(C) → Q, R; **(D) → P, R, S**

Q.3 For a closed organ pipe, match the following :

Column-I	Column-II
(A) Third overtone frequency is x times the fundamental frequency. Here, x is equal to	(P) 3
(B) Number of nodes in second overtone	(Q) 4
(C) Number of antinodes in second overtone	(R) 5
	(S) none

Ans. A → S ; B → P ; C → P ;

Q.4 Regarding speed of sound in gas match the following :

Column-I	Column-II
(A) Temperature of gas is made 4 times and pressure 2 times	(P) speed becomes $2\sqrt{2}$ times
(B) Only pressure is made 4 time without change in temperature	(Q) speed becomes 2 times
(C) Only temperature is changed to 4 times	(R) speed remains unchanged
(D) Molecular mass of the gas is made 4 times	(S) speed remains half

Ans. **A → Q ; B → R ; C → Q ; D → S**

Q.5 Fundamental frequency of closed pipe is 100 Hz and that of an open pipe is 200 Hz. Match the following (v_s = 330 m/s) :

Column-I	Column-II
(A) Length of closed pipe	(P) 0.825 m
(B) Length of open pipe	(Q) 1.65 m
(C) Lowest harmonic of closed pipe which is equal to any of the harmonic of open	(R) 5
	(S) none

Ans. **A → P ; B → P ; C → S**

Q.6 Match the following -

Column-I	Column-II
(A) Phase difference between any two particles can have any value between 0 to 2π	(P) Stationary waves
(B) Energy is transferred from one place to other place	(Q) Travelling waves
(C) Phase difference between any two particles is either zero or π	(R) Sound waves
(D) Amplitude of vibration of all particles are equal	(S) Standing waves in an open organ pipe

Sol. **A → Q,R ; B → Q,R ; C → P,S ; D → Q,R**

Q.7 Match the following -

Column-I	Column-II
(A) Energy is transferred from one place to other place	(P) Transverse wave
(B) Amplitude of vibration of all particles are equal	(Q) Longitudinal wave
(C) The phenomenon of interference can take place in	(R) Longitudinal standing wave
(D) The waves created in string is	(S) Transverse standing wave

Sol. A → P,Q ; B → P, Q ; C → P,Q,R,S; D → P,S

Q.8 Column matching –

Column-I	Column-II
(i) $y = 5\text{mm}\left(30\pi t + \frac{\pi}{3}\right)$	(P) Equation of stationary waves
(ii) $y = 5\text{ mm}\sin\left(30\pi t - 3\pi x + \frac{\pi}{3}\right)$	(Q) Equation of SHM
(iii) $y = 10\text{ mm}\sin 30\pi t\cos 3\pi x$	(R) equation of beats
(iv) $y = 6\text{ mm}\sin(30\pi t - 3\pi x) + 6\text{mm}\sin(28\pi t - 3.2\pi x)$	(S) equation of progressive wave

(A) (i) → Q, (ii) → S, (iii) → P, (iv) → R

(B) (i) → S, (ii) → Q, (iii) → P, (iv) → R

(C) (i) → Q, (ii) → S, (iii) → R, (iv) → P

(D) (i) → R, (ii) → S, (iii) → P, (iv) → Q

Sol. **[A]**

(i) in SHM displacement is a function of time but not position

(ii) in progressive wave displacement is a function of position and time and there is a term of linear motion

(iii) In stationary wave, there is no term of linear motion

(iv) In beats frequency of superimposing waves are different

Q.1 Velocity of sound in air is 320 m/s. A pipe closed at one end has a length of 1 m. Neglecting end corrections, the air column in the pipe can resonate for sound of frequency –

(A) 80 Hz (B) 240 Hz
(C) 320 Hz (D) 400 Hz

Sol. **[A,B,D]**

For closed pipe,

$f = n\left(\frac{v}{4\ell}\right)$ n = 1, 3, 5, . . .

For n = 1, $f_1 = \left(\frac{v}{4\ell}\right)$ n = 1,3, 5. . . .

For n = 1, $f_1 = \frac{v}{4\ell} = \frac{320}{4\times1}$ = 80 Hz

For n = 3, $f_3 = 3f_1$ = 240 Hz

For n = 5, $f_5 = 5f_1$ = 400 Hz

Q.2 Given are two tuning forks near one another, one of them is of unknown frequency and the other is of frequency 591 Hz. We can hear beats of maximal intensity I_0, 5 times each second. Then -

(A) Unknown frequency is 596 Hz
(B) Unknown frequency is 586 Hz
(C) Difference in frequency of two tuning forks is
5 Hz
(D) Intensity at time $t = \frac{27}{20}$ sec is $\frac{I_0}{2}$

Sol. **[A,B,C,D]**

Beat frequency : $f_1 - f_2 = \pm 5$

$\Rightarrow f_1 = f_2 \pm 5$

$f_1 = f_2 + 5$ or $f_1 = f_2 - 5$

$f_1 = 596$ Hz or $f_1 = 586$ Hz

Intensity : $I = I_0 \cos^2\left(\frac{\omega_1 - \omega_2}{2}\right)t$

$= I_0 \cos^2 \frac{2\pi}{2}(f_1 - f_2)\,t$

$= I_0 \cos^2 \pi \times 5 \times \frac{27}{20}$

$= I_0 \cos^2 \frac{9\pi}{4} = \frac{I_0}{2}$

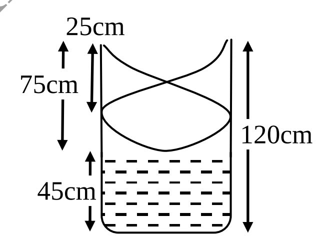
Q.3 In a resonance tube experiment, a close organ pipe of length 120 cm resonates when tuned with a tuning fork of frequency 340 Hz. If water is poured in the pipe then (given v_{air} = 340 m/sec.) :

(A) minimum length of water column to have the resonance is 45 cm.
(B) the distance between two successive nodes is 50 cm.
(C) the maximum length of water column to create the resonance is 95 cm
(D) None of these

Sol. **[A,B,C]**

As $V = \nu\lambda$

$\lambda = \frac{V}{\nu} = \frac{340}{340}$ = 1 m



first Resonance length

$R_1 = \frac{\lambda}{4} = \frac{1}{4}$ m = 25 cm

$\therefore R_2 = \frac{3\lambda}{4} = \frac{3}{4}$ m = 75 cm

$\therefore R_3 = \frac{5\lambda}{4} = \frac{5}{4}$ m = 125 cm

i.e. third resonance does not establish now H_2O is poured

$\therefore$ minimum length of H_2O column to have the resonance = 45 cm

$\therefore$ Distance between two successive nodes = $\frac{\lambda}{2}$

$= \frac{1}{2}$ m = 50 cm

and maximum length of H_2O column to create resonance i.e. 120 – 25 = 95 cm.

Q.4 In wave Motion :-

(A) Energy is transmitted through medium with the net displacement of particles

(B) Particle velocity is less than wave velocity

(C) Rate of transport of potential energy has double the frequency as compared to frequency of oscillation of particle

(D) Rate of transport of kinetic energy is equal to the power transported in the medium

[B,C]

Q.5 A closed organ pipe of length 1.2 m vibrates in its first overtone mode. The pressure variation is maximum at –

(A) 0.8 m from the open end

(B) 0.4 m from the open end

(C) closed end

(D) 1.0 m from the open end **[B, C]**

Sol. $l = \frac{3\lambda}{4}$

$\therefore \frac{\lambda}{4} = \frac{l}{3} = 0.4$ m

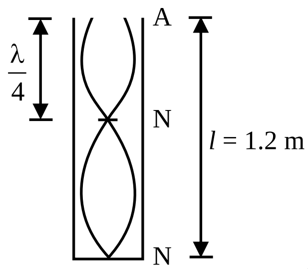


Fig.

Pressure variation will be maximum at displacement nodes i.e., at 0.4 m from the open end and at closed end.

Q.6 Which of the following functions of x and t represents a progressive wave –

(A) y = sin (4t – 3x)

(B) $y = \frac{1}{4 + (4t - 3x)^2}$

(C) $y = \frac{1}{4t + 3x}$

(D) All of these **[A, B]**

Sol. Any function y = f(at ± bx) represents a wave if it is finite everywhere and at all times. The function

$y = \frac{1}{4t + 3x}$

is not defined at x = 0 and t = 0.

Q.7 Sound wave is traveling along positive x-direction. Displacement (y) of particles from their mean position at position x is shows in figure. Choose the correct alternative(s) –

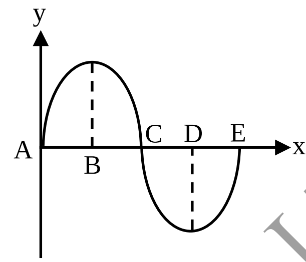


Fig.

(A) Particle located at E has its velocity in negative x-direction

(B) Particle located at D has zero velocity

(C) Particles located near C are under compression

(D) Change in pressure at D is zero

[A, B, C, D]

Sol. Velocity of particle is given by $v_p = -v\left(\frac{dy}{dx}\right)$

Here, v is wave speed and $\frac{dy}{dx}$ the slope.

At point E slope is positive, therefore, v_p will be along negative x-direction. Similarly, slope at D is zero.

Therefore, v_p at D will be zero.

Excess pressure $dP = -B.\frac{dy}{dx}$

At C slope is negative. Therefore, dP is positive i.e., particles located near C are under compression.

At point D, slope is zero i.e., dP = 0

Q.8 The figure shows an instantaneous profile of a rope carrying a progressive wave moving from left to right, then –

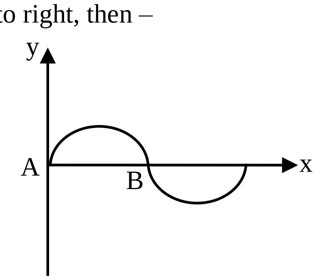


Fig.

(A) the phase at A is greater than the phase at B

(B) the phase at B is greater than the phase at A

(C) A is moving upwards

(D) B is moving upwards **[A, D]**

Sol. The wave travels from left to right. Therefore, points lying leftwards are always ahead in phase. Further:

Particle velocity = – (wave speed) slope

Slope at A is positive, while at B is negative i.e., particle velocity at A is negative and at B is positive. Therefore, A is moving downwards while B is moving upwards.

Q.9 A sound wave passes from a medium A to a medium B. The velocity of sound in B is greater than that in A. Assume that there is no absorption or reflection at the boundary. As the wave moves across the boundary –

(A) the frequency of sound will not change

(B) the wavelength will increase

(C) the wavelength will decrease

(D) the intensity of sound will not change

[A, B, D]

Q.10 When an open organ pipe resonates in its fundamental mode then at the centre of the pipe–

(A) the gas molecules undergo vibrations of maximum amplitude

(B) the gas molecules are at rest

(C) the pressure of the gas is constant

(D) the pressure of the gas undergoes maximum variation **[B, D]**

Q.11 Sounds from two identical sources S_1 and S_2 reach a point P. When the sounds reach directly, and in the same phase, the intensity at P is I_0. The power of S_1 is now reduced by 64%, and the phase difference between S_1 and S_2 is varied continuously. The maximum and minimum intensities recorded at P are now I_{max} and I_{min} –

(A) $I_{max} = 0.64\ I_0$

(B) $I_{min} = 0.36\ I_0$

(C) $I_{max}/I_{min} = 16$

(D) $I_{max}/I_{min} = 1.64/0.36$ **[A, C]**

Sol. Let a = initial amplitude due to S_1 and S_2 each.

$I_0 = k(4a^2)$, where k is a constant.

After reduction of power of S_1, amplitude due to S_1 0.6a.

Due to superposition, $a_{max} = a + 0.6a = 1.6a$, and

$a_{min} = a - 0.6a = 0.4a$

$I_{max}/I_{min} = (a_{max}/a_{min})^2 = \frac{(1.6a)^2}{(0.4a)^2} = 16.$

Q.12 You have three tuning forks A, B and C. Fork B has a frequency of 440 Hz. When A and B are sounded together a beat frequency of 3 Hz is heard. When B and C are sounded together the beat frequency is 4 Hz. Select the correct statements –

(A) The possible frequencies of C are 437 Hz and 443 Hz

(B) The possible frequencies of C are 436 Hz and 444 Hz

(C) The possible beat frequencies when A and C are sounded together are 1 Hz and 7 Hz

(D) The possible beat frequencies when A and C are sounded together are 2 Hz and 8 Hz

[B, C]

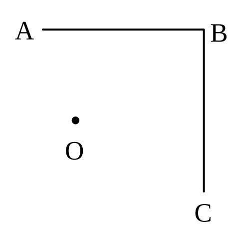
Sol. $f_C = f_B \pm 4\ H\zeta$

$\Rightarrow f_C = 436$ Hz or 444 Hz

$f_A = f_B \pm 3$ Hz

$\Rightarrow f_A = 437$ Hz or 443 Hz

Q.13 An observer is standing in front of two cliff AB and BC making an angle 90° with each-other as shown in figure. He blows a siren and hears echo after 3 sec and 4 sec. Velocity of sound is 300 m/s, then -



(A) Distance of nearest cliff is 900 m

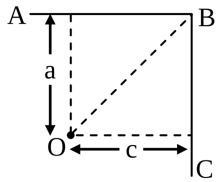
(B) Distance of farthest cliff is 600 m

(C) Distance of junction of cliff (B) is 750 m

(D) Observer may hear more than two echo

[B,C]

Sol.



Let a < b

$2a = 3 \times 300 \Rightarrow a = 450$ m

$2b = 4 \times 300 \Rightarrow b = 600$ m

$\therefore$ OB = $\sqrt{a^2 + b^2}$ = 750 m

Q.14 Sound waves from different source are interfering and pressure at same point in space is given as –

$P = P_0 \sin(200\pi t + \pi/2) \cos(3\pi t + \pi/3)$

(A) Maximum number of different sound waves interfering can be two only

(B) Beat frequency is 6 Hz

(C) Beat frequency is 200 Hz

(D) Beat frequency can be 6 Hz as well as 400 Hz

[A,B]

Sol. $P = P_0 \sin(200\pi t + \frac{\pi}{2}) \cos(3\pi t + \frac{\pi}{3})$ can be rewritten as :

$P = \frac{P_0}{2}\sin(203\pi t + \frac{5\pi}{6}) + \frac{P_0}{2}\sin(197\pi t + \frac{\pi}{6})$

Which is combination of two different wave. Lower frequency corresponds to beat frequency

$\therefore$ Beat frequency = $2 \times 3 = 6$ Hz

Q.15 The velocity of sound is affected by change in–

(A) Temperature (B) Medium

(C) Pressure (D) Wavelength

[A,B]

Q.16 In which case will there be no change in phase of displacement wave ?

(A) wave propagating from denser to rarer medium

(B) wave propagating from rarer to denser medium

(C) wave is reflected from a denser boundary

(D) wave is reflected from a rarer boundary

[A,B,D]

Q.17 A wave travelling in a solid–

(A) must be longitudinal

(B) may be longitudinal

(C) must be transverse

(D) may be transverse **[B,D]**

Q.18 Mark correct statement(s) -

(A) Maximum pressure variation takes place at nodes.

(B) In case of stationary wave, relative deformation at a point is given by $\Delta P = \frac{u}{v}$, where u is particle's velocity at that point.

(C) When a stationary wave is established, maximum intensity is obtained at antinodes.

(D) None of these **[A,B]**

Q.19 Which of the following statements are correct about intensity of sound ?

(A) It depends only on amplitude of wave

(B) It depends both on amplitude and frequency of wave

(C) Its practical unit is decibel

(D) Its practical unit is phon **[B,C]**

Q.20 In case of interference of two waves each of intensity I_0, the intensity at a point of constructive interference will be–

(A) $4I_0$ for coherent sources

(B) $2I_0$ for coherent sources

(C) $4I_0$ for incoherent sources

(D) $2I_0$ for incoherent sources **[A,D]**

Q.1 In a quink tube experiment a tuning fork of frequency 300 Hz is vibrated at one end. It is observed that intensity decreases from maximum to 50 % of its maximum value as tube is moved by 6.25 cm. Velocity of sound (in m/s) is. **[300m/s]**

Sol. $I = I_m \cos^2\phi$

(ϕ = Phase difference, I_m = maximum intensity)

$I = \frac{I_m}{2}$

$\Rightarrow \cos^2\phi = \frac{1}{2}$

$\Rightarrow \cos\phi = \frac{1}{\sqrt{2}}$

$\Rightarrow K\,\Delta x = \frac{\pi}{4}$

$\Rightarrow K = \frac{\pi}{4} \times \frac{1}{2\times 6.25}$

$\Rightarrow \lambda = 1$ m

$\therefore v = \nu\lambda = 300$ m/s

Q.2 A tuning fork of frequency 200 Hz is vibrating with a sonometer wire to produce 10 beats/sec. When the tension in sonometer wire is increased beat frequency decreases. Original frequency of sonometer wire in Hz is.

Sol. $\nu_{\text{tuning fork}} - \nu_{\text{sonometer wire}} = 10$

$\therefore \nu_{\text{sonometer wire}} = 200 - 10 = 190$ Hz

Q.3 A tuning fork is in unison with a sonometer wire vibrating in its fourth overtone. Mass hanged with wire is 9 kg. When additional mass is hanged wire vibrates in unison with tuning fork in its 3rd harmonic. Additional mass hanged in kg is. **[0016]**

Sol. For sonometer wire

$\nu_n = \frac{n\sqrt{\frac{F}{\mu}}}{2\ell}$

$\Rightarrow n\sqrt{F}$ = constant

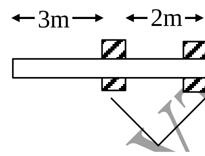
[$\therefore \nu, \mu, \ell$ are constant for two cases of comparison]

$\Rightarrow F_2 = \frac{n_1^2}{n_2^2} . F_1$

$\Rightarrow m_2 = 25$ kg

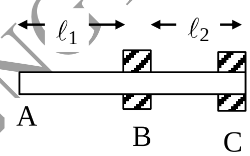
$\therefore$ Additional mass = 16 kg

Q.4 A metal rod of length 5 m is clamped by two rigid support separation between which is 2 m as shown in figure. Longitudinal standing wave are set up in the rod using a device having frequency range 10 Hz – 10 kHz. Velocity of wave in rod is 4000 m/s. Numbers of natural longitudinal oscillation that can be setup in rod is.



[0005]

Sol.



Let rod oscillate in with $(n_1 - 1)$ loop in AB and n_2 loop in BC

$\therefore (2n_1 - 1)\frac{v}{4\ell_1} = n_2\frac{v}{2\ell_2}$

$\Rightarrow n_2 = \frac{(2n_1 - 1)}{3}$

$\therefore$ Possible values of n_2 = 1, 3, 5, 7, 9, 11, 13

Those lying in range 10 Hz – 10 kHz are equal to 5.

Q.5 Three plane sources of sound of frequency $n_1 = 400$ Hz, $n_2 = 401$ Hz and $n_3 = 402$ Hz of equal amplitude 'a' each are sounded together. A detector receives waves from all the three sources simultaneously. Then the period in sec. of one complete cycle of intensity received by detector is

Sol. [1] $y = y_1 + y_2 + y_3 = a\,[\sin 800\,\pi t + \sin 802\,\pi t + \sin 804\,\pi t] \Rightarrow y = a\,(1 + \cos 2\pi t)\sin 802\pi t$

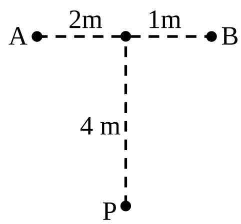
$\therefore A = a\,(1 + \cos 2\pi t)$

Q.6 Due to a point isotropic sonic source, loudness at a point is L = 40 dB. If density of air is $\rho = \frac{15}{11}$ kg/m^3 and velocity of sound in air V = 330 m/s then the pressure oscillation amplitude in 10^{-3} N/m^2 at the point of observation is (assume $I_0 = 10^{-2}\ Wm^{-2}$)

Sol. [3] $(\Delta P)_{max} = \sqrt{2I\rho v} = 3 \times 10^{-3}$ N/m

where $I = I_0\ \text{antilog}_{10}\left(\frac{L}{10}\right)$

Q.7 Two small sound sources A and B emit pure sinusoidal waves in phase. If the speed of sound is 350 m/s, for what minimum frequency does destructive interference occur at point P. Answer is in the form of $n \times 10^2$ Hz. What is n ?



Sol.[5] AP = 4.47 m
BP = 4.12 m
d = 0.35 m
destructive

$d = \frac{\lambda}{2}$

$\lambda = 2d$

$f = \frac{v}{2d} = 500$ Hz.

$= 5 \times 10^2$ Hz

$\therefore$ n = 5

Q.8 An observer at a distance of 800 m from a sound source heard first the sound signal which travelled through water and 1.785 later the signal which travelled through air. The velocity of sound in water is $(x \cdot y) \times 10^3$ m/s. Where x and y is the single digit non zero number, find x. The air temperature is 17ºC –

Sol.[1]

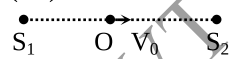
Q.9 A long spring such as slinky is often used to demonstrate longitudinal waves. If mass of spring is m, length L and force constant K, then find the speed of longitudinal waves on the spring where m = 0.250 kg, L = 2.00 m K = 1.50 N/m.

Sol.[5] $v = L\sqrt{\frac{K}{m}}$

v = 4.9 m/s

≈ 5 m/s

Q.10 Two identical s tationary sound sources, emit sound waves of frequency 10 Hz, and speed 300 m/sec as shown. An observer is moving between the sources with a velocity 30 m/sec. Find the beat frequency as recorded by the observer (Hz).



Sol. **[2]**

$f_1 = \frac{300+30}{300}.f$

$f_2 = \frac{300-30}{300}.f$

$\nu = f_1 - f_2 = 2$ Hz

PHYSICS

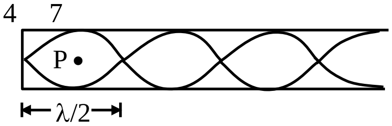
Q.1 A closed organ pipe has length 'ℓ' The air in it is vibrating in 3rd overtone with maximum amplitude 'a'. The amplitude at a distance of $\ell/7$ from closed end of the pipe is equal to–

(A) a (B) a/2

(C) $\frac{a\sqrt{3}}{2}$ (D) zero **[A]**

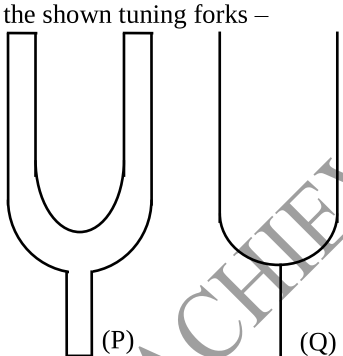
Sol. The figure shows variation of displacement of particles in a closed organ pipe for 3rd overtone. For third overtone $\ell = \frac{7\lambda}{4}$ or $\lambda = \frac{4\ell}{7}$

or $\frac{\lambda}{4} = \frac{\ell}{7}$



Hence the amplitude at P at a distance $\ell/7$ from closed end is 'a' because there is an antinode at that point

Q.2 Which relation is giving the correct information for the shown tuning forks –



(A) $n_P > n_Q$ (B) $n_P < n_Q$

(C) $n_P = n_Q$ (D) None of these

[A]

Sol. $n \propto \frac{t}{\ell^2}$ so $n_P > n_Q$

Q.3 The speed of sound in air at N.T.P is 300 m/s. If pressure of air is increased to four times keeping the temperature constant, the speed of sound will becomes –

(A) 150 m/s (B) 300 m/s

(C) 600 m/s (D) 1200 m/s **[B]**

Sol. Since velocity is independent of pressure so no change

Q.4 In a resonance pipe the first and second resonance are obtained at lengths 22.7 cm and 70.2 cm respectively. What will be the end correction -

(A) 1.05 cm (B) 115.5 cm

(C) 92.5 cm (D) 113.5 cm

[A]

Sol. $e = \frac{\ell_2 - 3\ell_1}{2}$

$e = \frac{70.2 - 3\times 22.7}{2} = \frac{70.2 - 68.1}{2} = \frac{2.1}{2}$

= 1.05 cm

Q.5 An unknown fork produces 4 beats per second with a tuning fork of frequency 288 Hz. When unknown fork is loaded with wax it again produces 4 beats per second. The unknown frequency of tuning fork is -

(A) 284 Hz (B) 292 Hz

(C) 290 Hz (D) 288 Hz **[B]**

Sol.

Unknown	known	Beats
292 or 284	288	4
284		4

Q.6 A wave of frequency ν = 1000 Hz, propagates at a velocity v = 700 m/sec along x-axis. Phase difference at a given point x during a time interval $\Delta t = 0.5 \times 10^{-3}$ sec is -

(A) π (B) $\pi/2$

(C) $3\pi/2$ (D) 2π **[A]**

Sol. $y = A \sin (kx - \omega t)$

ϕ = phase = $kx - \omega t$

$\phi_1 = kx - \omega t_1$

$\phi_2 = kx - \omega t_2$

Phase difference : $\Delta\phi = \phi_2 - \phi_1 = \omega(t_1 - t_2)$

$\Delta\phi = \phi_2 - \phi_1 = -\omega(t_2 - t_1)$

$\Delta\phi = -\omega\Delta t$

$= -2\pi \times 10^3 \times 0.5 \times 10^{-3}$

$= -2\pi \times \frac{1}{2} = -\pi$

Q.7 Consider a plane standing sound wave of frequency 10^3 Hz in air at 300 K. Suppose the amplitude of pressure variation associated with this wave is 1 dyne/cm^2. The equilibrium pressure is 10^6 dyne/cm^2. The amplitude of displacement of air molecules associated with this wave is :
(Given speed of sound : 340 m/s
Molar mass of air : 29×10^{-3} kg/mol)

(A) 4×10^{-6} m (B) 40×10^{-6} m
(C) 400×10^{-6} m (D) 40000×10^{-6} m

[C]

Sol. $y = A \sin(\omega t - kx)$

$$\Delta P = -B \frac{\partial y}{\partial x} = + BAk \cos(\omega t - kx)$$

$$\Delta P = \Delta P_m \cos(\omega t - kx)$$

$$\Delta P_m = BAk = \frac{BA\omega}{v}$$

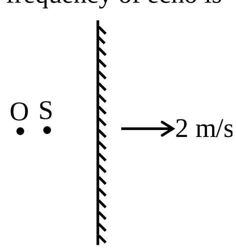
$$A = \frac{\Delta P_m v}{B\omega} = \frac{\Delta P_m v}{\rho v^2 \omega}$$

$$A = \frac{\Delta P_m}{\rho v 2\pi f}$$

$f = 10^3$ Hz
$\Delta P_m = 1$ dyne/m^2
$v = 340$ m/s
$M = 29 \times 10^{-3}$ kg/mole
$T = 300$ K

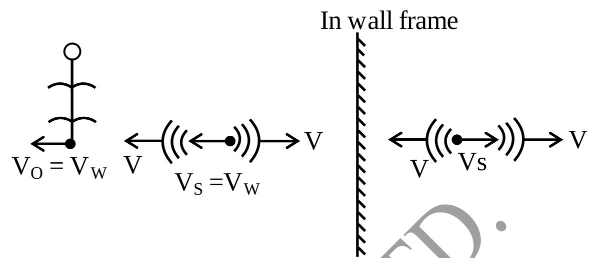
where $\boxed{\rho = \frac{PM}{RT}}$

Q.8 A stationary sound source S of frequency 334 Hz and a stationary observer O are placed near reflecting surface moving away from the source with velocity 2m/s as shown in figure. Velocity of sound waves in air v = 330 m/s. The apparent frequency of echo is –



(A) 332 Hz (B) 326 Hz
(C) 334 Hz (D) 330 Hz

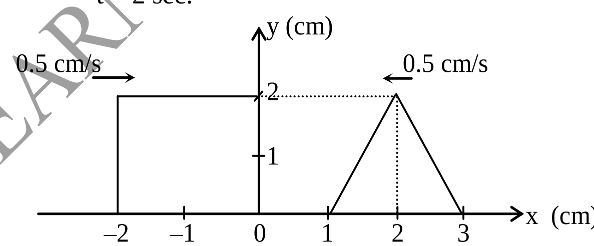
Sol. **[D]**



$$F_{echo} = f_{ac}\left[\frac{V - V_0}{V + V_S}\right] = 334\left[\frac{330 - 2}{330 + 2}\right]$$

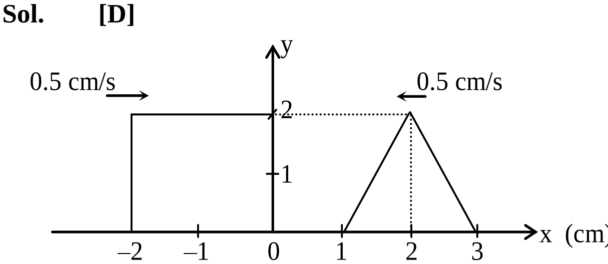
$$= 334 \times \frac{328}{332} = 330 \text{ Hz}$$

Q.9 Figure shows a rectangular pulse and a triangular pulse approaching each other along x-axis. The pulse speed is 0.5 cm/s. What is the resultant displacement of medium particles due to superposition of waves at x = 0.5 cm and t = 2 sec.

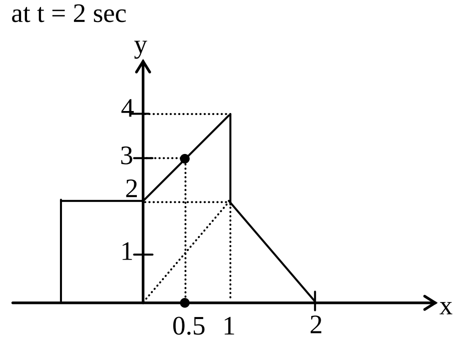


(A) 3.5 cm (B) 2.5 cm
(C) 4 cm (D) 3 cm

Sol. **[D]**



$x_1 = 0.5 \times t = 0.5 \times 2 = 1$cm
$x_2 = 0.5 \times t = 0.5 \times 2 = 1$cm
at t = 2 sec



at x = 0.5 cm, y = 3 cm

Q.10 A sine wave is travelling in a medium. The minimum distance between the two particles, always having same speed, is -

(A) $\lambda/4$ (B) $\lambda/3$

(C) $\lambda/2$ (D) λ **[C]**

Sol. Particle which vibrate in opposite phase having different velocity but having same speed.

Q.11 A balloon filled with CO_2, then for sound wave this will behave as a -

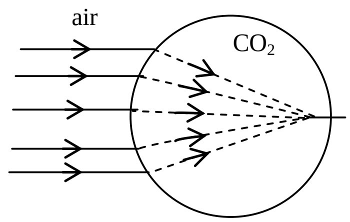
(A) converging lens

(B) diverging lens

(C) both of the above

(D) none of the above **[A]**

Sol.



$V_s = \sqrt{\frac{\gamma R T}{M_w}}$

$M_{wco_2} > M_{wair}$

$V_{CO_2} < V_{air}$, velocity of sound decrease when sound propagate from air to CO_2 gas means CO_2 behave as a denser medium So wave bends towards normal, and CO_2 gas balloon behave as converging lens.

Q.12 A big explosion on the moon cannot be heard on the earth because -

(A) the explosion produces high frequency sound waves which are inaudible

(B) sound waves require a material medium for propagation

(C) sound waves are absorbed in the atmosphere of moon

(D) sound waves are absorbed in the earth's atmosphere **[B]**

Sol. As the sound waves are mechanical waves they requires medium for propagation.

Q.13 A boat at anchor is rocked by waves whose crests are 100 m apart and velocity is 25 m/s. The boat bounces up once in every -

(A) 2500 sec (B) 75 sec

(C) 4 sec (D) 0.25 sec **[C]**

Sol. Wavelength $\rightarrow$ Distance between the crests

so $\lambda = 100$ m , v = 25 m/sec

$v = n\lambda$

or $25 = n(100)$ $\therefore$ $n = \frac{1}{4}$ per sec

$T = \frac{1}{n} = 4$ sec

Q.14 A tuning fork and an air column in resonance tube whose temperature is 51°C produces 4 beats in 1 second when sounded together. When the temperature of the air column decreases, the number of beats per second decreases. When the temperature remains 16°C, only 1 beat per second is produced. Then the frequency of the tuning fork is -

(A) 55 Hz

(B) 50 Hz

(C) 68 Hz

(D) none of the above **[B]**

Sol. $v \propto n \propto \sqrt{T}$ because λ = constant

$\frac{N+4}{N+1} = \sqrt{\frac{324}{289}} = \frac{18}{17}$

$17N + 68 = 18N + 18$

$50 = N$

Q.15 A closed organ pipe and an open organ pipe of same length produce four beats in their fundamental mode when sounded together. If length of the open organ pipe is increased, then the number of beats will -

(A) increase (B) decrease

(C) remain constant

(D) may increase or decrease

Sol. **[D]** $n_o - n_c = 4$

where $n_o = \frac{V}{2L}$, $n_c = \frac{V}{4L}$ so if length of open organ pipe increases its frequency $\downarrow$

so no. of beats also decreases

Q.16 The path difference between the two waves :

$y_1 = a_1 \sin(\omega t - kx)$

and $y_2 = a_2 \cos(\omega t - kx + \phi)$ is -

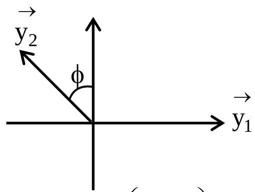
(A) $(\lambda/2\pi)\phi$ (B) $\lambda\left(\frac{\phi + (\pi/2)}{2\pi}\right)$

(C) $\frac{2\pi}{\lambda}\left(\phi - \frac{\pi}{2}\right)$ (D) $\left(\frac{2\pi}{\lambda}\right)\phi$ **[B]**

Sol. Relation between phase difference and path difference

$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

$y_1 = a_1 \sin(\omega t - kx)$
$y_2 = a_2 \cos(\omega t - kx + \phi)$
From phasor diagram : -



$$\Delta\phi = \left(\frac{\pi}{2} + \phi\right)$$

$$\Delta x = \frac{\Delta\phi}{2\pi} \times \lambda$$

$$= \frac{1}{2\pi}\left(\frac{\pi}{2} + \phi\right)\lambda$$

Q.17 An observer standing at the seacoast observes 54 waves reaching the coast per minute. If the wavelength of the waves is 10 m, its velocity is -

(A) 90 m/s (B) 90 cm/s
(C) 9 m/s (D) 900 m/s **[C]**

Sol. Frequency of waves $n = \frac{54}{60}$ per second

$\lambda = 10\,m$

$\therefore v = n\lambda$

$= \frac{54}{60} \times 10 = 9$ m/sec.

Q.18 If fundamental frequency of closed pipe is 50 Hz. then frequency of 2[an] overtone is :

(A) 100 Hz (B) 50 Hz
(C) 250 Hz (D) 150 Hz **[C]**

Q.19 Tube A has both ends open while tube B has one end closed, otherwise they are identical. The ratio of fundamental frequency of tube A and B is :

(A) 1 : 2 (B) 1 : 4
(C) 2 : 1 (D) 4 : 1 **[C]**

Q.20 In one meter long open pipe what is the harmonic of resonance obtained with a tuning fork of frequency 480 Hz

(A) First (B) Second
(C) Third (D) Fourth **[C]**

Q.21 Fundamental frequency of an open pipe of length 0.5 m is equal to the frequency of the first overtone of a closed pipe of length ℓ_c . The value of ℓ_c is (m)

(A) 1.5 (B) 0.75
(C) 2 (D) 1 **[B]**

Q.22 What is the base frequency if a pipe gives notes of frequencies 425, 255 and 595 and decide whether it is closed at one end or open at both ends :

(A) 17, closed (B) 85, closed
(C) 17, open (D) 85, open **[B]**

Q.23 A closed organ pipe and an open organ pipe are tuned to the same fundamental frequency. What is the ratio of lengths :

(A) 1 : 2 (B) 2 : 1
(C) 2 : 3 (D) 4 : 3 **[A]**

Q.24 Consider the three waves z_1, z_2 and z_3 as

$z_1 = A \sin(kx - \omega t)$, $z_2 = A \sin(kx + \omega t)$

and $z_3 = A \sin(ky - \omega t)$. Which of the following represents a standing wave :

(A) $z_1 + z_2$ (B) $z_2 + z_3$
(C) $z_3 + z_1$ (D) $z_1 + z_2 + z_3$ **[A]**

Q.25 An open pipe of length 33 cm resonates with frequency of 100 Hz. If the speed of sound is 330 m/s, then this frequency is :

(A) Fundamental frequency of the pipe
(B) Third harmonic of the pipe
(C) Second harmonic of the pipe
(D) Fourth harmonic of the pipe **[C]**

Q.26 Stationary waves are set up in air column. Velocity of sound in air is 330 m/s and frequency is 165 Hz. Then distance between the nodes is -

(A) 2m (B) 1m (C) 0.5 m (D) 4m

Sol. **[B]** Distance between the nodes = $\lambda/2$

$v = \nu\lambda \Rightarrow \lambda = \frac{330}{165} = 2$

$\therefore$ 1m

Q.27 In open organ pipe, if fundamental frequency is n then the other frequencies are :

(A) n, 2n, 3n, 4n (B) n, 3n, 5n

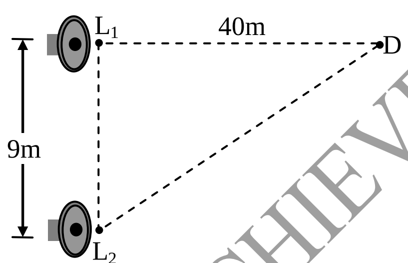
(C) n, 2n, 4n, 8n (D) None of these **[A]**

Q.28 In a resonance pipe the first and second resonances are obtained at depths 22.7 cm and 70.2 cm respectively. What will be the end correction :

(A) 1.05 cm (B) 115.5 cm

(C) 92.5 cm (D) 113.5 cm **[A]**

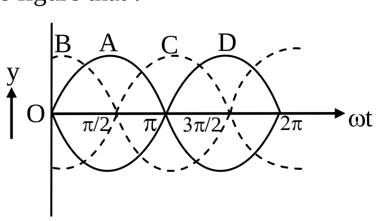
Q.29 Two loudspeakers L_1 and L_2 driven by a common oscillator and amplifier, are arranged as shown. The frequency of the oscillator is gradually increased from zero and the detector at D records a series of maxima and minima. If the speed of sound is 330 ms^{-1} then the frequency at which the first maximum is observed is :



(A) 165 Hz (B) 330 Hz

(C) 496 Hz (D) 660 Hz **[B]**

Q.30 The figure shows four progressive waves A, B, C and D with their phases expressed with respect to the wave A. If can be concluded from the figure that :



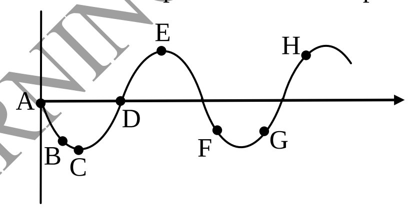
(A) The wave C is ahead by a phase angle of $\pi/2$ and the wave B lags behind by a phase angle of $\pi/2$

(B) The wave C lags behind by a phase angle of $\pi/2$ and the wave B is ahead by a phase angle of $\pi/2$

(C) The wave C is ahead by a phase angle of π and the wave B lags behind by a phase angle of π

(D) The wave C lags behind by a phase angle of π and the B ahead by a phase of π **[B]**

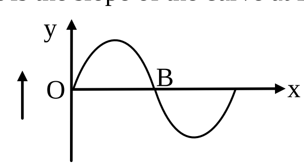
Q.31 The diagram below shows the propagation of a wave. Which points are in same phase :



(A) F, G (B) C and E

(C) B and G (D) B and F **[D]**

Q.32 Fig. below shows the wave y = A sin(ωt – kx) at any instant traveling in the +ve x-direction. What is the slope of the curve at B



(A) ω/a (B) k/A

(C) kA (D) ωA **[A]**

Q.33 The absolute temperature of air in a region linearly increases from 0°C to 819°C in a space of width 'd'. Time taken by sound wave to travel through this space is: [Velocity of sound at 0°C is v_0]

(A) $\frac{2d}{\sqrt{5}v}$ (B) $\frac{6d}{v}$

(C) $\frac{2d}{3v}$ (D) None of these **[C]**

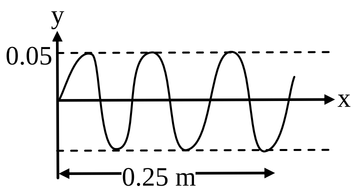
Sol. Velocity of sound at a distance 'x' is given by

$$v(x) = \sqrt{\frac{273+\frac{x}{d}\times 819}{273}}\,.v$$

$\therefore$ Time taken

$$t = \int_0^d \frac{dx}{v(x)} = \frac{2d}{3v}$$

Q.34 If the speed of the wave shown in the figure is 330 m/s in the given medium, then the equation of the wave propagating in the positive x-direction will be (all quantities are in M.K.S units) :



(A) y = 0.05 sin2π(4000t– 12.5 x)
(B) y = 0.05 sin2π (4000t –122.5 x)
(C) y = 0.05 sin2π(3300t– 10 x)
(D) y = 0.05 sin 2π(3300 x – 10t) **[C]**

Q.35 In a resonance tube the first resonance with a tuning fork occurs at 16 cm and second at 49 cm. If the velocity of sound is 330 m/s, the frequency of tuning fork is :

(A) 500 (B) 300
(C) 330 (D) 165 **[A]**

Q.36 An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by 100 Hz, then the fundamental frequency of open pipe is :

(A) 480 Hz (B) 300 Hz
(C) 240 Hz (D) 200 Hz **[D]**

Q.37 Velocity of sound in He at certain temperature is 'v_0'. Velocity of sound in N_2 at that temperature will be -

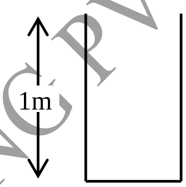
(A) $\frac{\sqrt{3}}{5}\ v_0$ (B) $\frac{\sqrt{3}}{7}\ v_0$
(C) $\frac{1}{\sqrt{7}}\ v_0$ (D) $\sqrt{\frac{3}{7}}\ v_0$ **[A]**

Sol. $\frac{v_2}{v_1} = \sqrt{\frac{\gamma_2}{\gamma_1}.\frac{m_1}{m_2}}$

v_2 = velocity in nitrogen
v_1 = velocity in helium

$\Rightarrow v_2 = \frac{\sqrt{3}}{5}\ v_0$

Q.38 Velocity of sound in air is 320 ms^{-1}. The pipe is shown in figure can not vibrate with a sound of frequency –



(Fig.)

(A) 80 Hz (B) 240 Hz
(C) 320 Hz (D) 400 Hz **[C]**

Sol. Fundamental frequency $n = \frac{v}{4L} = \frac{320}{4\times 1} = 80$ Hz

frequency which can produces from this pipe is = n, 3n, 5n, 7n ……..
= 80, 240, 400 Hz…….

Q.39 Two closed end pipes when sounded together produce 5 beat per second. If their length are in the ratio 100 : 101, then fundamental notes produced by them are –

(A) 245, 250 (B) 250, 255
(C) 495, 500 (D) 500, 505 **[D]**

Sol. $\frac{N}{N+5} = \frac{100}{101}$

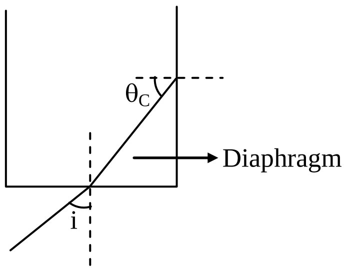
101 N = 100 N + 500
N = 500 Hz
N + 5 = 505 Hz.

Q.40 One end of a thin metal tube is closed by thin diaphragm of latex and the tube is lower in water with closed end downward. The tube is filled with a liquid 'x'. A plane progressive wave inside water hits the diaphragm making an angle 'θ' with its normal. Assuming Snell's law to hold true for sound. Maximum angle 'θ' for which sound is not transmitted through the walls of tube is (velocity of sound in liquid x = $740\sqrt{3}$ m/s and in water = 1480 m/s)

(A) $\sin^{-1}\left(\frac{2}{3}\right)$ (B) $\sin^{-1}\left(\frac{\sqrt{2}}{\sqrt{3}}\right)$

(C) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (D) $\sin^{-1}\left(\frac{1}{2}\right)$ **[C]**

Sol. Figure shows condition for just transmission of sound wave through the wall of tube.



$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

[v_1 = velocity of sound in water
v_2 = velocity of sound in liquid]

$\Rightarrow \sin i = \frac{1480}{740\sqrt{3}} . \sin(90° - \theta_C)$

$\Rightarrow i = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Q.41 A wave is represented by $y = A \sin^2 (kx - \omega t + \phi)$. The amplitude and wavelength of wave is given by -

(A) $2A, \frac{2\pi}{k}$ (B) $A, \frac{2\pi}{k}$

(C) $\frac{A}{2}, \frac{2\pi}{k}$ (D) $\frac{A}{2}, \frac{\pi}{k}$ **[D]**

Sol. $y = A \sin^2 (kx - \omega t + \phi)$ can be rewritten as

$y = \frac{A}{2} - \frac{A}{2} \cos (2kx - 2\omega t + 2\phi)$

Q.42 Four waves are represented by $y_1 = A_1 \sin \pi t$, $y_2 = A_2 \sin (\pi t + \pi/2)$, $y_3 = A_1 \sin (2\pi t + \pi/2)$ and $y_4 = A_2 \sin (\pi t - \pi/3)$. Interference will happen with –

(A) y_1, y_2 and y_3 only (B) y_1, y_2 and y_4 only

(C) y_1 and y_3 only (D) y_1, y_2, y_3 and y_4

[D]

Sol. Interference is phenomena of more than one wave reaching at same point in space simultaneously.

Q.43 Intensity and phase of three sound wave reaching at some point in space is I_0, $4 I_0$, I_0 and 10°, 130° and 250° respectively. Resultant intensity at that point will be –

(A) $6 I_0$ (B) $2I_0$

(C) I_0 (D) $\left(\frac{2+\sqrt{3}}{\sqrt{2}}\right)I_0$ **[C]**

Sol. Amplitude of the three sound wave would be in ratio 1 : 2 : 1. Let amplitude of first wave be A.

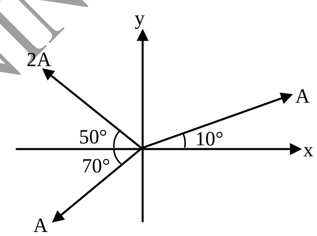


Figure 1

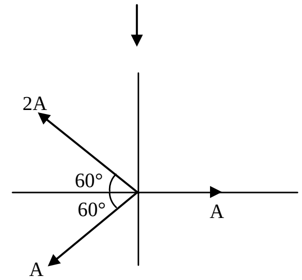


Figure 2

$\therefore A_R = A$

$\Rightarrow I_R \propto A^2$

$\therefore I_R = I_0$

Q.44 The nature of sound waves in gases is-

(A) Transverse

(B) Longitudinal

(C) Transverse and longitudinal

(D) None of these **[B]**

Q.45 Sound waves of wavelength greater than that of audible sound are called-

(A) Seismic waves (B) Sonic waves

(C) Ultrasonic waves (D) Infrasonic waves

[D]

Q.46 The wavelength of ultrasonic waves in air is of the order of-

(A) 5×10^{-5} cm (B) 5×10^{-8} cm

(C) 5×10^{5} cm (D) 5×10^{8} cm **[A]**

Q.47 Two tuning forks A and B are in unison with strings of length 0.96 m and 0.97 m respectively produces 2 beats per half second. The frequency of A and B are in (Hz) –

(A) 384, 388 (B) 384, 386

(C) 388, 384 (D) 388, 386 **[C]**

Sol. For natural frequency of string

$$\nu_n \propto \frac{1}{L}$$

$$\Rightarrow \frac{\nu_A}{\nu_B} = \frac{97}{96} \quad \text{... (i)}$$

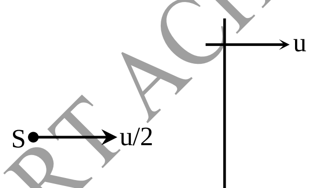
Also, $\nu_A - \nu_B = 4$ ... (ii)

$\because$ Beat frequency = 4

From (i) and (ii),

$\nu_A = 388$, $\nu_B = 384$

Q.48 A wall is moving with velocity u and a source of sound moves with velocity u/2 in the same direction as shown in the figure. Assuming that the sound travels with velocity 10u. The ratio of incident sound wavelength on the wall to the reflected sound wavelength by the wall, is equal to -



(A) 9 : 11 (B) 11 : 9

(C) 4 : 5 (D) 5 : 4 **[A]**

Sol. λ_i = wavelength of the incident sound

$$= \frac{10u - \frac{u}{2}}{f} = \frac{19u}{2f}$$

f_i = frequency of the incident sound

$$= \frac{10u - u}{10u - \frac{u}{2}} f = \frac{18}{19} f = f = f_r$$ = frequency of the reflected sound

λ_r = wavelength of the reflected sound = $\frac{10u + u}{f_r} = \frac{11u}{18f} \times 19 = \frac{11 \times 19}{18} \cdot \frac{u}{f}$

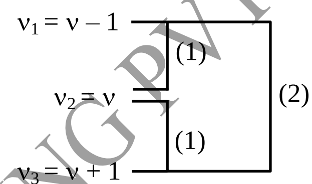
$$\frac{\lambda_i}{\lambda_r} = \frac{19u}{2f} \times \frac{18f}{11 \times 19u} = \frac{9}{11}$$

Q.49 Three sound waves of equal amplitudes have frequencies $(v - 1)$, v, $(v + 1)$. They superpose to give beats. The number of beats produced per second will be –

(A) 4 (B) 3

(C) 2 (D) 1 **[C]**

Sol.



Three sound wave of equal amplitude superpose and produce "2" beats.

Q.50 A point source is emitting sound in all directions. The ratio of distance of two points from the point source where the difference in loudness levels is 3dB, is ($\log_{10} 2 = 0.3$) -

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{4}$ (D) $\frac{2}{3}$ **[B]**

Sol. $$dB = 10 \log \left[\frac{I}{I_0}\right] = 10 \log \left[\frac{K/r^2}{I_0}\right]$$

$= 10 [\log (K') - 2 \log r]$

$dB_1 = 10 (\log K' - 2 \log r_1)$

$dB_2 = 10 (\log K' - 2 \log r_2)$

$3 = dB_1 - dB_2 = 20 \log \left[\frac{r_2}{r_1}\right] \Rightarrow (0.3) = \log \left[\frac{r_2}{r_1}\right]^2$

$$\Rightarrow \left(\frac{r_1}{r_2}\right) = \frac{1}{\sqrt{2}}$$

Q.1 A long spring such as a Slinky is often used to demonstrate longitudinal waves.

a) Show that if a spring that obeys Hooke's law has mass m, length L, and force constant k′, the speed of longitudinal waves on the spring is $v = L\sqrt{k'/m}$.

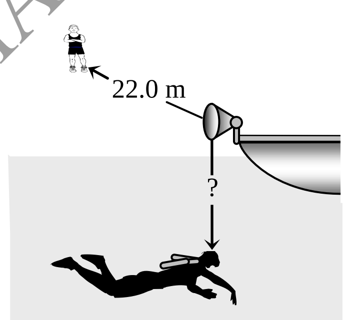
b) Evaluate v for a spring with m = 0.250 kg, L = 2.00 m, and k′ = 1.50 N/m.

Sol. **a)** Consider the derivation of the speed of a longitudinal wave. Instead of the bulk modulus B, the quantity of interest is the change in force per fractional length change. The force constant k′ is the change in force per length change, so the force change per fractional length change is k′ L, the applied force at one end is F = (k′ L)(v_y/v) and the longitudinal impulse when this force is applied for a time t is k′ Ltv_y/v. The change in longitudinal momentum is ((vt)m/L)v_y and equating the expressions, canceling a factor of t and solving for v gives $v^2 = L^2k'/m$.

An equivalent method is use, which relates the force constant k′ and the "Young's modulus" of the Slinky, k′ = YA/L, or Y = k′L/A. The mass density is ρ = m/(AL).

b) $(2.00\text{m})\sqrt{(1.50\text{N/m})/(0.250\text{kg})}$ = 4.90 m/s.

Q.2 A submerged scuba diver hears the sound of a boat horn directly above her on the surface of the lake. At the same time, a friend on dry land 22.0 m from the boat also hears the horn (Fig.).



The horn is 1.2 m above the surface of the water. What is the distance (labeled by "?" in Fig.) from the horn to the diver? Both air and water are at 20ºC.

Sol. Use v_{water} = 1482 m/s at 20ºC, the sound wave travels in water for the same time as the wave travels a distance 22.0 m – 1.20 m = 20.8 m in air, and so the depth of the diver is

$$(20.8\text{ m})\frac{v_{water}}{v_{air}} = (20.8\text{ m})\frac{1482\text{ m/s}}{344\text{ m/s}} = 89.6\text{ m}.$$

This is the depth of the diver; the distance from the horn is 90.8 m.

Q.3 At a temperature of 27.0ºC, what is the speed of longitudinal waves in

a) hydrogen (molar mass 2.02 g/mol)?

b) helium (molar mass 4.00 g/mol)?

c) argon (molar mass 39.9 g/mol)?

d) Compare your answers for parts (a), (b) and (c) with the speed in air at the same temperature.

Sol. **a), b), c)** Using Eq. $v = \sqrt{\frac{\gamma RT}{M}}$ (speed of sound in an ideal gas),

$$v_{H_2} = \sqrt{\frac{(1.41)(8.3145\text{J/mol.K})(300.15\text{K})}{(2.02\times10^{-3}\text{kg/mol})}} = 1.32 \times 10^3 \text{ m/s}$$

$$v_{He} = \sqrt{\frac{(1.67)(8.3145\text{J/mol.K})(300.15\text{K})}{(4.00\times10^{-3}\text{kg/mol})}} = 1.02 \times 10^3 \text{ m/s}$$

$$v_{Ar} = \sqrt{\frac{(1.67)(8.3145\text{J/mol.K})(300.15\text{K})}{(39.9\times10^{-3}\text{kg/mol})}} = 323 \text{ m/s}.$$

d) At T = 300.15 K gives v_{air} = 348 m/s, and so v_{H_2} = 3.80 v_{air}, v_{He} = 2.94 v_{air} and v_{Ar} = 0.928 v_{air}.

Q.4 A jet airliner is cruising at high altitude at a speed of 850 km/h (about 530 mi/h). This is equal to 0.85 times the speed of sound at that altitude (also called "Mach 0.85").

a) What is the air temperature at this altitude?

b) With this data, can you determine the air pressure at this altitude? Explain.

Sol. **a)** Solving Eq. $v = \sqrt{\frac{\gamma RT}{M}}$ (speed of sound in an ideal gas) for the temperature,

$$= \frac{Mv^2}{\gamma R} = $$

$$\frac{(28.8\times10^{-3}\,kg/mol)\left(\left(\frac{850\,km/h}{0.85}\right)\left(\frac{1\,m/s}{3.6\,km/hr}\right)\right)^2}{(1.40)(8.3145\ J/mol.K)}$$

= 191 K,

or – 82ºC.

b) The variation of atmospheric pressure with altitude, assuming a non-constant temperature.

If we know the altitude, $p = p_0\left(1-\frac{\alpha y}{T_0}\right)^{\left(\frac{Mg}{R\alpha}\right)}$.

Since $T = T_0 - \alpha y$,

for T = 191 K, $\alpha = .6 \times 10^{-2}$ ºC/m, and T_0 = 273 K, y = 13,667 m (44,840 ft.). Although a very high altitude for commercial aircraft, some military aircraft fly this high. This result assumes a uniform decrease in temperature that is solely due to the increasing altitude. Then, if we use this altitude, the pressure can be found:

$$p = p_0\left(1-\frac{(.6\times10^{-2}\,{}^{o}C/m)(13{,}667\,m)}{273\,K}\right)^{\left(\frac{(28.8\times10^{-3}\,kg/mol)(9.8\,m/s^2)}{(8.315\,J/molK)(.6\times10^{-2}\,{}^{o}C/m)}\right)}$$

and $p = p_0(.70)^{5.66} = .13\,p_0$, or about .13 atm. Using $p = .18p_0$, which overestimates the pressure due to the assumption of an isothermal atmosphere.

Q.5 An 80.0-m-long brass rod is struck at one end. A person at the other end hears two sounds as a result of two longitudinal waves, one traveling in the metal rod and the other traveling in the air. What is the time interval between the two sounds? The speed of sound in air is 344 m/s.

Sol. The speed of longitudinal waves in brass is much higher than in air, and so the sound that travels through the metal arrives first. The time difference is

$$\Delta t = \frac{L}{v_{air}} - \frac{L}{v_{brass}} = \frac{80.0\,m}{344\,m/s} - \frac{80.0\,m}{\sqrt{(0.90\times10^{11}\,Pa)/(8600\,kg/m^3)}} = 0.208\text{ s.}$$

Q.6 What must be the stress (F/A) in a stretched wire of a material whose Young's modulus is Y for the speed of longitudinal waves to equal 30 times the speed of transverse waves?

Sol. The mass per unit length μ is related to the density (assumed uniform) and the cross-section area A by $\mu = A\rho$, so combining Eq. $v = \sqrt{\frac{F}{\mu}}$ (speed of a transverse wave on a string) and Eq. $v = \sqrt{\frac{Y}{\rho}}$ (speed of a longitudinal wave in a solid rod)

with the given relations between the speeds,

$$\frac{Y}{\rho} = 900\frac{F}{A\rho}\text{ so } F/A = \frac{Y}{900}.$$

Q.7 A longitudinal wave of frequency 220 Hz travels down a copper rod of radius 8.00 mm. The average power in the wave is 6.50 μW.

a) Find the wavelength of the wave.

b) Find the amplitude A of the wave.

c) Find the maximum longitudinal velocity of a particle in the rod.

Sol. **a)**

$$\lambda = \frac{v}{f} = \frac{\sqrt{Y/\rho}}{f} = \frac{\sqrt{(11.0\times10^{10}\text{Pa})/(8.9\times10^{3}\text{kg/m}^3)}}{220\text{ Hz}}$$

= 16.0 m.

b) Solving for the amplitude A (as opposed to the area a = πr^2) in terms of the average power P_{av} = Ia,

$$A = \sqrt{\frac{(2P_{av}/a)}{\sqrt{\rho Y\omega^2}}}$$

$$= \sqrt{\frac{2(6.50\times10^{-6})W)/(\pi(0.800\times10^{-2}\text{m})^2)}{\sqrt{(8.9\times10^3\text{kg/m}^3)(11.0\times10^{10}\text{Pa})}(2\pi(220\text{Hz}))^2}}$$

= 3.29 × 10^{-8} m.

c) $\omega A = 2\pi f A = 2\pi(220\text{ Hz})(3.289\times10^{-8}\text{ m})$ = 4.55 × 10^{-5} m/s.

Q.8 **a)** A longitudinal wave propagating in a water-filled pipe has intensity 3.00 × 10^{-6} W/m² and frequency 3400 Hz. Find the amplitude A and wavelength λ of the wave. Water has density 1000 kg/m³ and bulk modulus 2.18 × 10^9 Pa.

b) If the pipe is filled with air at pressure 1.00 × 10^5 Pa and density 1.20 kg/m³, what will be the amplitude A and wavelength λ of a longitudinal wave with the same intensity and frequency as in part (a)?

c) In which fluid is the amplitude larger, water or air? What is the ratio of the two amplitudes? Why is this ratio so different from one?

Sol. **a)** The amplitude is

$$A = \sqrt{\frac{2I}{\sqrt{\rho B\omega^2}}}$$

$$= \sqrt{\frac{2(3.00\times10^{-6}\text{W/m}^2)}{\sqrt{(1000\text{ kg/m}^3)(2.18\times10^9\text{Pa})}(2\pi(3400\text{ Hz}))^2}}$$

= 9.44 × 10^{-11} m.

The wavelength is

$$\lambda = \frac{v}{f} = \frac{\sqrt{B/\rho}}{f} = \frac{\sqrt{(2.18\times10^9\text{Pa})/(1000\text{ kg/m}^3)}}{3400\text{ Hz}}$$

= 0.434 m.

b) Repeating the above with B = γp = 1.40 × 10^5 Pa and the density of air gives A = 5.66 × 10^{-9} m and λ = 0.100 m.

c) The amplitude is larger in air, by a factor of about 60.0 For a given frequency, the much less dense air molecules must have a larger amplitude to transfer the same amount of energy.

Q.9 **a)** What is the sound intensity level in a car when the sound intensity is 0.500 μW/m²?

b) What is the sound intensity level in the air near a jackhammer when the pressure amplitude of the sound is 0.150 Pa and the temperature is 20.0ºC?

Sol. **a)** The sound level is

β = (10 dB) $\log\frac{I}{I_0}$, so β = (10 dB) $\log\frac{0.500\mu\text{W/m}^2}{10^{-12}\text{ W/m}^2}$, or β = 57 dB.

b) First find v, the speed of sound at 20.0ºC, v = 344 m/s.

The density of air at that temperature is 1.20 kg/m³. Using Equation $I = \frac{p_{max}^2}{2\rho v} = \frac{p_{max}^2}{2\sqrt{\rho B}}$ (intensity of a sinusoidal sound wave).

$$I = \frac{p_{max}^2}{2\rho v} = \frac{(0.150\text{ N/m}^2)^2}{2(1.20\text{kg/m}^3)(344\text{ m/s})}, \text{ or I}$$

= 2.73 × 10^{-5} W/m². Using this in Equation β = (10 dB) $\log\frac{I}{I_0}$

(definition of sound intensity level),

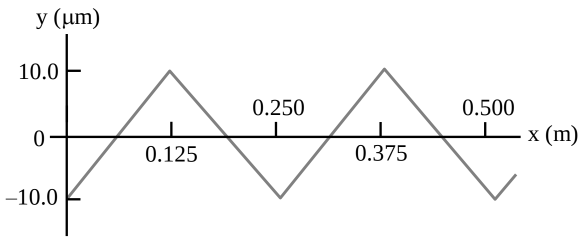
$$\beta = (10\text{ dB})\log\frac{2.73\times10^{-5}\text{W/m}^2}{10^{-12}\text{W/m}^2},$$

or β = 74.4 dB.

Q.10 **a)** Defend the following statement: "In a sinusoidal sound wave, the pressure variation given by Eq. p(x, t) = BkA sin(kx – ωt) is greatest where the displacement given by Eq. y(x, t) = A cos (kx – ωt) is zero."

b) For a sinusoidal sound wave given by Eq. y(x, t) = A cos (kx – ωt) with amplitude A = 10.0 μm and wavelength λ = 0.250 m, graph the displacement y and pressure fluctuation p as functions of x at time t = 0. Show at least two wavelengths of the wave on your graph.

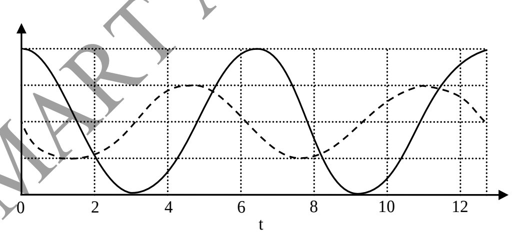
c) The displacement y in a nonsinusoidal sound wave is shown in Fig. as a function of x for t = 0. Draw a graph showing the pressure fluctuation p in this wave as a function of x at t = 0. This sound wave has the same 10.0-μm amplitude as the wave in part (b). Does it have the same pressure amplitude? Why or why not?



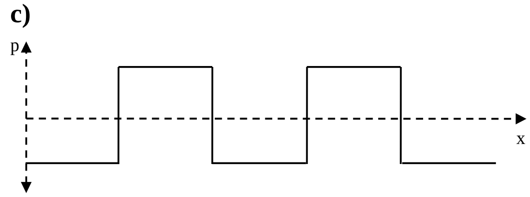
d) Is the statement in part (a) necessarily true if the sound wave is not sinusoidal? Explain your reasoning.

Sol. **a)** Mathematically, the waves given by Eq. y(x, t) = A cos (kx – ωt) and Eq. p(x, t) = BkA sin(kx – ωt) are out of phase. Physically, at a displacement node, the air is most compressed or rarefied on either side of the node, and the pressure gradient is zero. Thus, displacement nodes are pressure antinodes.

b) (This is the same as Fig. p(x, t) = $-B\dfrac{\partial y(x,t)}{\partial x}$). The solid cure is the pressure and the dashed curve is the displacement.



c)



The pressure amplitude is not the same. The pressure gradient is either zero or undefined. At the places where the pressure gradient is undefined mathematically (the "cusps" of the y-x plot), the particles go from moving at uniform speed in one direction to moving at the same speed in the other direction. In the limit that Fig. shows given in question is an accurate depiction, this would happen in a vanishing small time, hence requiring a very large force, which would result from a very large pressure gradient.

d) The statement is true, but incomplete. The pressure is indeed greatest where the displacement is zero, but the pressure is equal to its largest value at points other than those where the displacement is zero.

Q.11 Many airports have noise ordinances restricting the maximum sound intensity that an aircraft may produce when it takes off. At one California airport the maximum allowable sound intensity level is 98.5 dB as measured by a microphone at the end of the 1740-m-long runway. A certain airliner produces a sound intensity level of 100.0 dB on the ground when it flies over at an altitude of 100 m. On takeoff, this airliner rolls for 1200 m along the runway before leaving the ground, at which point it climbs at a 15º angle. Does this airliner violate the noise ordinance? You can ignore any effects due to reflection of the sound waves from the ground.

Sol. The altitude of the plane when it passes over the end of the runway is (1740 m – 1200 m)tan 15º = 145 m, and so the sound intensity is $1/(1.45)^2$ of what the intensity would be at 100 m. The intensity level is then

$100.0\text{ dB} - 10 \times \log[(1.45)^2] = 96.8\text{ dB}$,

so the airliner is not in violation of the ordinance.

Q.12 A common lecture demonstration is as follows: hold or clamp a one meter long thin aluminium bar at the center, strike one end longitudinally (i.e. parallel to the axis of the bar) with a hammer, and the result is a sound wave of frequency 2500 Hz.

a) From this experiment, calculate the speed of sound in air.

b) From this experiment, calculate the speed of sound in aluminium.

c) Where might you hold the bar to excite a frequency of 3750 Hz?

Explain. Does it matter which end of the bar is struck? Explain.

d) Suppose you hold the bar at the center as before, but strike the bar transverse to its length, rather longitudinally. Qualitatively explain why the resultant sound wave is of lower frequency than before.

Sol. **a)** The point where the bar is struck is an antinode and the point where it is held a node. With the bar held at the center and its one end struck, the wavelength λ is related to its length L by $\lambda = 2L$. Hence the speed of sound propagation in the aluminium bar is

$v_{A1} = \nu\lambda = 2\nu L = 2 \times 2500 \times 1 = 5000$ m/s.

The speed of sound in a solid is

$$v = \sqrt{\frac{Y}{\rho}},$$

where Y is the Young's modulus of its material and ρ its density. The speed of sound in a fluid is

$$v = \sqrt{\frac{M}{\rho}},$$

where M is its bulk modulus and ρ its density. For adiabatic compression of a gas, $M = \gamma p$, where p is its pressure and γ the ratio of its principal specific heats; $\gamma = 1.4$ for air, a diatomic gas. Hence

$$\frac{v_{air}}{v_{Al}} = \sqrt{\frac{1.4p\rho_{A1}}{Y\rho_{air}}}.$$

With

$p = 1.013 \times 10^6$ dyn/cm^2 (standard atmosphere),

$Y = 7.05 \times 10^{11}$ dyn/cm^2,

$\rho_{A1} = 2.7$ g/cm^3,

$\rho_{air} = 1.165 \times 10^{-3}$ g/cm^3 (at 30ºC),

$v_{air} = 6.83 \times 10^{-2} \times 5000 = 341$ m/s.

b) $v_{A1} = 5000$ m/s.

c) Suppose the bar is held at distance x from the struck end. We have

$$x = \frac{\lambda}{4} = \frac{v}{4\nu} = \frac{5000}{4 \times 3750} = \frac{1}{3} \text{ m.}$$

Hence the bar is to be held at $\frac{1}{3}$ m from the struck end. If it is so held but struck at the other end, we would have

$$\frac{2}{3} = \frac{v}{4\nu}$$

and the frequency would become 1875 Hz.

d) If the bar is struck transversely, the wave generated will be transverse, not compressional, and the velocity of propagation is then given by

$$v = \sqrt{\frac{N}{\rho}},$$

where N is the shear modulus. As the shear modulus of a solid is generally smaller than its bulk modulus, v is now smaller. And as

$$\nu = \sqrt{\frac{v}{2L}}$$

the frequency generated is lower.

Q.13 Longitudinal standing waves can be produced in a solid rod by holding it at some point between the fingers of one hand and stroking it with the other hand. The rod oscillates with antinodes at both ends.

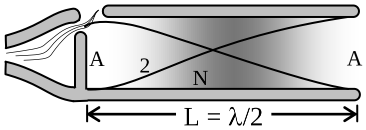
a) Why are the ends antinodes and not nodes?

b) The fundamental frequency can be obtained by stroking the rod while it is held at its center. Explain why this is the only place to hold the rod to obtain the fundamental.

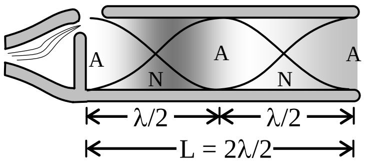
c) Calculate the fundamental frequency of a steel rod of length 1.50 m.

d) What is the next possible standing-wave frequency of this rod? Where should the rod be held to excite a standing wave of this frequency?

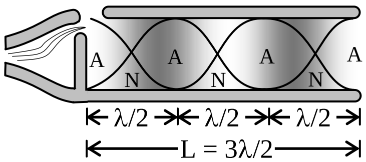
Sol. The steel rod has standing waves much like a pipe open at both ends, as shown in Figure.



(a) $f_1 = \frac{v}{2L}$



(b) $f_2 = 2\frac{v}{2L} = 2f_1$



c) $f_3 = 3\frac{v}{2L} = 3f_1$

An integral number of half wavelengths must fit on the rod, that is, $f_n = \frac{nv}{2L}$.

a) The ends of the rod are antinodes because the ends of the rod are free to oscillate.

b) The fundamental can be produced when the rod is held at the middle because a node is located there.

c) $f_1 = \frac{(1)(5941 \text{ m/s})}{2(1.50 \text{ m})} = 1980$ Hz.

d) The next harmonic is n = 2, or $f_2 = 3961$ Hz. We would need to hold the rod at an n = 2 node, which is located at L/4 from either end, or at 0.375 m from either end.

Q.14 A long tube contains air at a pressure of 1.00 atm and a temperature of 77.0ºC. The tube is open at one end closed at the other by a movable piston. A tuning fork near the open end is vibrating with a frequency of 500 Hz. Resonance is produced when the piston is at distance 18.0, 55.5, and 93.0 cm from the open end.

a) From these measurements, what is the speed of sound in air at 77.0ºC?

b) From the result of part (a), what is the value of γ?

c) These data show that a displacement antinode is slightly outside of the open end of the tube. How far outside is it?

Sol. **a)** The second distance is midway between the first and third, and if there are no other distances for which resonance occurs, the difference between the first and third positions is the wavelength λ = 0.750 m. (This would give the first distance as λ/4 = 18.75 cm, but at the end of the pipe, where the air is not longer constrained to move along the tube axis, the pressure node and displacement antinode will not coincide exactly with the end). The speed of sound in the air is then v = fλ = (500 Hz)(0.750 m) = 375 m/s.

b) Solving Eq. $v = \sqrt{\frac{\gamma RT}{M}}$ (speed of sound in an ideal gas) for γ,

$$\gamma = \frac{Mv^2}{RT} = \frac{(28.8\times10^{-3}\text{kg/mol})(375 \text{ m/s})^2}{(8.3145 \text{ J/mol.K})(350.15 \text{ K})} =$$

1.39.

c) Since the first resonance should occur at τ/4 = 0.875 m but actually occurs at 0.18 m, the difference is 0.0075 m.

Q.15 A platinum wire (density 21.4 g/cm^3) is 225 μm in diameter and 0.450 m long. One end of the wire is attached to the ceiling, while a 420-g mass is attached to the other end so that the wire hangs vertically under tension. If a vibrating tuning fork of just the right frequency is held next to the wire, the wire begins to vibrate as well.

a) What tuning-fork frequency will cause this to happen? You may assume that the bottom end of the wire (to which the mass is attached) is essentially stationary, and that the tension in the wire is essentially constant along its length.

b) Justify the assumptions made in part (a).

Sol. **a)** From Eq. $f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$ (string fixed at both ends), with m the mass of the string and M the suspended mass,

$$f_1 = \sqrt{\frac{F}{4mL}} = \sqrt{\frac{Mg}{\pi d^2 L^2 \rho}} =$$

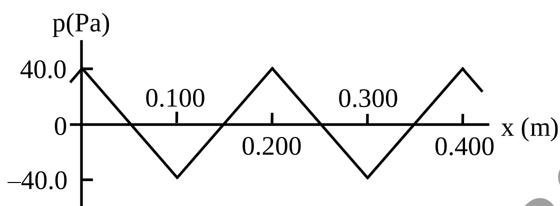
$$\sqrt{\frac{(420.0\times10^{-3}\,\text{kg})(9.80\,\text{m/s}^2)}{\pi(255\times10^{-6}\,\text{m})^2(0.45\,\text{m})^2(21.4\times10^3\,\text{kg/m}^3)}} = 77.3$$

Hz

and the tuning fork frequencies for which the fork would vibrate are integer multiples of 77.3 Hz.

b) The ratio $m/M \approx 9 \times 10^{-4}$, so the tension does not vary appreciably along the string.

Q.16 Figure shows the pressure fluctuation p of a non-sinusoidal sound wave as a function of x for t = 0. The wave is traveling in the +x-direction.



a) Graph the pressure fluctuation p as a function of t for x = 0. Show at least two cycles of oscillation.

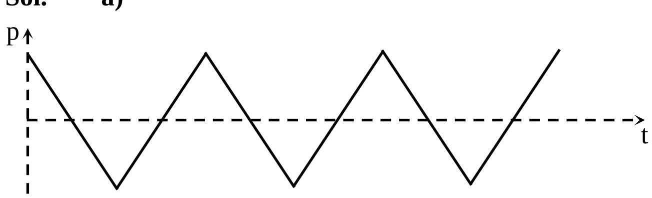
b) Graph the displacement y in this sound wave as a function of x at t = 0. At x = 0, the displacement at t = 0 is zero. Show at least two wavelengths of the wave.

c) Graph the displacement y as a function of t for x = 0. Show at least two cycles of oscillation.

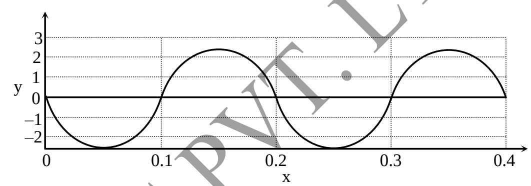
d) Calculate the maximum velocity and the maximum acceleration of an element of the air through which this sound wave is traveling.

e) Describe how the cone of a loudspeaker must move as a function of time to produce the sound wave in this problem.

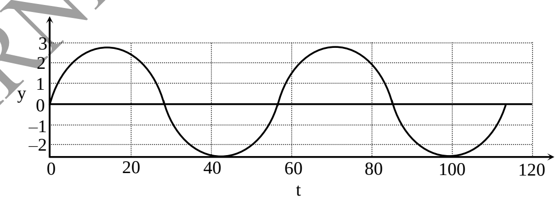
Sol. **a)**



b) From Eq. $p(x, t) = BkA \sin(kx - \omega t)$, the function that has the given p(x, 0) at t = 0 is given graphically as shown. Each section is a parabola, not a portion of a sine curve. The period is $\lambda/v = (0.200\text{ m})/(344\text{ m/s}) = 5.81 \times 10^{-4}$ s and the amplitude is equal to the area under the p – x curve between x = 0 and x = 0 and x = 0.0500 m divided by B, or 7.04×10^{-6} m.



c) Assuming a wave moving in the +x-direction, y(0, t) is as shown.



d) The maximum velocity of a particle occurs when a particle is moving throughout the origin, and the particle speed is $v_y = -\frac{\partial y}{\partial x} v = \frac{pv}{B}$. The maximum velocity is found from the maximum pressure, and v_{ymax} = (40 Pa)(344 m/s)/(1.42 × 10^5 Pa) = 9.69 cm/s. The maximum acceleration is the maximum pressure gradient divided by the density,

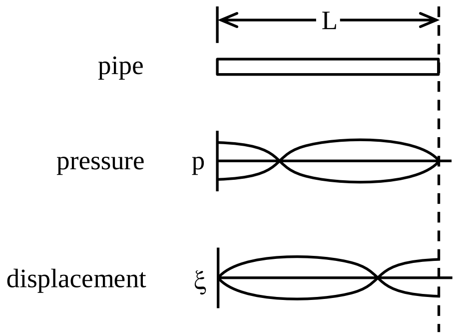
$$a_{max} = \frac{(80.0\,\text{Pa})/(0.100\,\text{m})}{(1.20\,\text{kg/m}^3)} = 6.67 \times 10^2\ \text{m/s}^2.$$

e) The speaker cone moves with the displacement as found in part (c); the speaker cone alternates between moving forward and backward with constant magnitude of acceleration (but changing sign). The acceleration as a function of time is a square wave with amplitude 667 m/s^2 and frequency f = v/λ = (344 m/s)/(0.200 m) = 1.72 kHz.

Q.17 **a)** Plot the pressure and air displacement diagrams along a pipe closed at one end for the second mode.

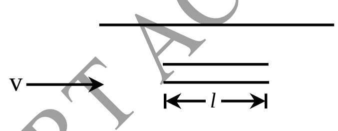
b) What is the frequency of this mode relative to the fundamental?

Sol. **a)** The pressure and air displacement as functions of distance from the closed end are sketched in Fig.



b) For this mode, $L = 3\lambda/4$, while for the fundamental mode, $L = \lambda/4$. Hence if ω_0 is the fundamental frequency, the frequency of this mode is $3\omega_0$.

Q.18 An organ pipe of length l open on both ends is used in a subsonic wind tunnel to measure the Mach number v/c of air in the tunnel as shown in Fig.



The pipe when fixed in the tunnel is observed to resonate with a fundamental period t. If v/c = 1/2, calculate the ratio of periods t/t_0 where t_0 is the fundamental period of the pipe in still air.

Sol. As the organ pipe is open at both ends, the fundamental wavelength of sound in resonance with it is given by $\lambda/2 = l$. The corresponding period is

$$t = \frac{\lambda}{v} = \frac{2l}{v},$$

where v is the velocity of sound relative to the pipe.

When the air in the pipe is still, v is equal to the velocity of sound in still air, c, and the fundamental period is

$$t_0 = \frac{2l}{c}.$$

When the air in the pipe moves with velocity c/2, the pipe can be considered to move with velocity –c/2 in still air. Thus v = c – (– c/2) = 3c/2 and the period is

$$t = \frac{2l}{\frac{3c}{2}} = \frac{4l}{3c}.$$

Hence we have the ratio

$$\frac{t}{t_0} = \frac{2}{3}.$$

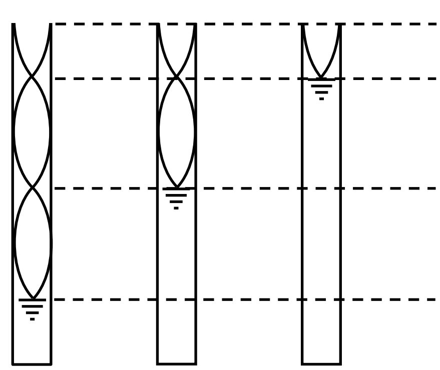
Q.19 A vertical cylindrical pipe, open at the top, can be partially filled with water. Successive resonances of the column with a 512 sec^{-1} tuning fork are observed when the distance from the water surface to the top of the pipe is 15.95 cm, 48.95 cm, and 80.95 cm.

a) Calculate the speed of sound in air.

b) Locate precisely the antinode near the top of the pipe.

c) The above measurements are presented to you by a team of sophomore lab students. How would you criticize their work?

Sol. **a)** The wave forms of the successive resonances in the air column are shown in Fig.



It is seen that for successive resonances, the air columns differ in height by half a wavelength: $d = \lambda/2$. As

$d = 48.45 - 15.95 = 80.95 - 48.45 = 32.50$ cm,

$\lambda = 2d = 65.00$ cm.

The velocity in air is then

$v = \lambda\nu = 0.6500 \times 512 = 330$ m/s.

b) As $\lambda/4 = 16.25$ cm and 16.25 cm – 15.95 cm = 0.30 cm, the uppermost antinode is located at 0.30 cm above the top of the pipe.

c) This method of measuring sound velocity in air is rather inaccurate as the human ear is not sensitive enough to detect precisely small variations in the intensity of sound, and the accuracy of measurement is rather limited. Still, the data obtained are consistent and give a good result. The students ought to be commended for their careful work.

Q.20 Consider a plane standing sound wave of frequency 10^3 Hz in air at 300 K. Suppose the amplitude of the pressure variation associated with this wave is 1 dyn/cm^2 (compared with the ambient pressure of 10^6 dyn/cm^2). Estimate (order of magnitude) the amplitude of the displacement of the air molecules associated with this wave.

Sol. The longitudinal displacement ξ from equilibrium of a point in a plane stationary compressional wave in the x direction can be expressed as

$\xi = \xi_0 \sin(kx)e^{-i\omega t}$,

with $k = n\pi/l$, l being the thickness of the gas and n = 1, 2 ….. The velocity of the wave is

$$v = \sqrt{\frac{M}{\rho}} .$$

Here the bulk modulus M is by definition

$$M = -p\left(\frac{\Delta V}{V}\right)^{-1},$$

p being the excess pressure and V the original volume. Consider a cylinder of the gas of cross-sectional area A and length Δx. We have

$$\frac{\Delta V}{V} = \frac{A\Delta\xi}{A\Delta x} \approx \frac{\partial\xi}{\partial x} .$$

Then

$$p = -M\frac{\partial\xi}{\partial x}$$

$= -M k\xi_0 \cos(kx)e^{-i\omega t}$

$= -p_0 \cos(kx)e^{-i\omega t}$,

where $p_0 = M k\xi_0 = \rho v^2 k\xi_0$ is the amplitude of the excess pressure. Hence

$$\xi_0 = \frac{p_0}{\rho v^2 k} .$$

For the lowest mode

$n = 1, \quad \lambda = 2l$,

$$k = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{v},$$

ν being the frequency of the sound wave. Thus

$$\xi_0 = \frac{p_0}{2\pi\rho v\nu} .$$

For an ideal gas

$$p_a V = \frac{m}{M}RT,$$

giving

$$\rho = \frac{m}{M} = \frac{p_a M}{RT},$$

where p_a, T are the ambient pressure and temperature respectively. As $p_0 = 1$ dyn/cm^2 = 10^{-1} N/m^2, $p_a = 10^6$ dyn/cm^2 = 10^5 N/m^2, $M = 29 \times 10^{-3}$ kg/mol, R = 8.31 J/mol/K, T = 300 K, v = 340 m/s, $\nu = 10^3$ Hz, we find $\xi_0 = 4 \times 10^{-8}$ m as the amplitude of the displacement of the air molecules.