Quadratic Equations & Theory of Equations

Single Correct Answer Type

1. Let α and β be the roots of $x^2-6x-2=0$ with $\alpha>\beta$ if $a_n=\alpha^n-\beta^n$ for $n\geq 1$ then the value of $\dfrac{a_{10}-2a_8}{3a_9}=$

   1) 1 2) 2
   3) 3 4) 4

Key. 2

Sol. $\alpha^2-6\alpha-2=0$ $\beta^2-6\beta-2=0$

$\Rightarrow \alpha^{10}-6\alpha^9-2\alpha^8=0$ ....(1)

$\Rightarrow \beta^{10}-6\beta^9-2\beta^8=0$ ....(2)

subtract (2) from (1)

2. If a,b,c are positive real numbers such that $a+b+c=1$ then the least value of $\dfrac{(1+a)(1+b)(1+c)}{(1-a)(1-b)(1-c)}$ is

   1) 16 2) 8
   3) 4 4) 5

Key. 2

Sol. $a=1-b-c$

$\Rightarrow 1+a=(1-b)+(1-c)\geq 2\sqrt{(1-b)(1-c)}$

$\therefore (1+a)(1+b)(1+c)\geq 8(1-a)(1-b)(1-c)$

3. The range of values of $'a'$ for which all the roots of the equation $(a-1)(1+x+x^2)^2=(a+1)(1+x^2+x^4)$ are imaginary is

   1) $(-\infty,-2]$ 2) $(2,\infty)$
   3) $(-2,2)$ 4) $[2,\infty)$

Key. 3

Sol. The given equation can be written as $(x^2+x+1)(x^2-ax+1)=0$

4. If α,β are the roots of the equation $ax^2+bx+c=0$ and $S_n=\alpha^n+\beta^n$ then $aS_{n+1}+bS_n+cS_{n-1}=$ $(n\geq 2)$

   1) 0 2) $a+b+c$
   3) $(a+b+c)n$ 4) $n^2\ abc$

Key. 1

Sol. $S_{n+1}=\alpha^{n+1}+\beta^{n+1}$

$=(\alpha+\beta)(\alpha^n+\beta^n)-\alpha\beta(\alpha^{n-1}+\beta^{n-1})$

$= -\dfrac{b}{a}.S_n-\dfrac{c}{a}.S_{n-1}$

5. A group of students decided to buy a Alarm Clock priced between Rs. 170 to Rs 195. But at the last moment, two students backed out of the decision so that the remaining students had to pay 1 Rupee more than they had planned. If the students paid equal shares, the price of the Alarm Clock is

1) 190 2) 196
3) 180 4) 171

Key. 3

Sol. Let cost of clock = x
number of students $= n$

then $\frac{x}{n-2} = \frac{x}{n} + 1 \Rightarrow x = \frac{n^2 - 2n}{2}$

$\Rightarrow 170 \le \frac{n^2 - 2n}{2} \le 195$

6. If $\tan A, \tan B$ are the roots of $x^2 - Px + Q = 0$ the value of $\sin^2(A+B) =$ $(where\ P, Q \in R)$

1) $\frac{P^2}{P^2 + (1-Q)^2}$ 2) $\frac{P^2}{P^2 + Q^2}$

3) $\frac{Q^2}{P^2 + (1-Q)^2}$ 4) $\frac{P^2}{(P+Q)^2}$

Key. 1

Sol. $\tan(A+B) = \frac{P}{1-Q}$ then $\sin^2(A+B) = \frac{\tan^2(A+B)}{1+\tan^2(A+B)}$

7. The number of solutions of $\left|[x] - 2x\right| = 4$ where $[x]$ is the greatest integer $\le x$ is

1) 2 2) 4
3) 1 4) Infinite

Key. 2

Sol. If $x = n \in Z,\ \ |n - 2n| = 4 \Rightarrow n = \pm 4$

If $x = n + K$ where $0 < K < 1$ then $|n - 2(n+k)| = 4$, it is possible if $K = \frac{1}{2}$

$\Rightarrow |-n-1| = 4$
$\therefore n = 3, -5$

8. Let a, b and c be real numbers such that $a + 2b + c = 4$ then the maximum value of $ab + bc + ca$ is

1) 1 2) 2 3) 3 4) 4

Key. 4

Sol. Let $ab + bc + ca = x$
$\Rightarrow 2b^2 + 2(c-2)b - 4c + c^2 + x = 0$
Since $b \in R$,
$\therefore c^2 - 4c + 2x - 4 \le 0$
Since $c \in R$

$\therefore x \le 4$

9. For the equation $3x^2 + px + 3 = 0, p > 0$, if one root is the square of the other then value of P is

1) $\frac{1}{3}$ 2) 1

3) 3 4) $\frac{2}{3}$

Key. 3

Sol. $\alpha + \alpha^2 = -\frac{p}{3}$

$\alpha^3 = 1$

10. If the equations $2x^2 + kx - 5 = 0$ and $x^2 - 3x - 4 = 0$ have a common root, then the value of k is

1) -2 2) -3

3) $\frac{27}{4}$ 4) $-\frac{1}{4}$

Key. 2

Sol. If 'α' is the common root then $2\alpha^2 + k\alpha - 5 = 0,\ \alpha^2 - 3\alpha - 4 = 0$ solve the equations.

11. If α and β are the roots of the equation $x^2 - x + 1 = 0$ then $\alpha^{2009} + \beta^{2009} =$

1) 1 2) 2

3) -1 4) -2

Key. 1

Sol. $x = \frac{1 \pm i\sqrt{3}}{2}$

$\therefore \alpha = -\omega,\ \beta = -\omega^2$

12. If $P(Q-r)x^2 + Q(r-P)x + r(P-Q) = 0$ has equal roots then $\frac{2}{Q} =$

$(where\ P, Q, r \in R)$

1) $\frac{1}{P} + \frac{1}{r}$ 2) $\frac{1}{P} - \frac{1}{r}$

3) $P + r$ 4) Pr

Key. 1

Sol. Product of the roots $= 1$

13. If $(1+K)\tan^2 x - 4\tan x - 1 + K = 0$ has real roots $\tan x_1$ and $\tan x_2$ then

1) $k^2 \le 5$ 2) $k^2 \ge 6$

3) $k = 3$ 4) $k > 10$

Key. 1

Sol. Discriminate ≥ 0

14. α,β are the roots of $ax^2+bx+c=0$ and γ,δ are the roots of $px^2+qx+r=0$ and D_1, D_2 be the respective discriminants of these equations. If α,β,γ and δ are in A.P. then $D_1:D_2=$ (where $\alpha,\beta,\gamma,\delta\in R$ & $a,b,c,p,q,r\in R$)

1) $a^2:p^2$ 2) $a^2:b^2$

3) $c^2:r^2$ 4) $a^2:r^2$

Key. 1

Sol. $\beta=\alpha+d,\ \gamma=\alpha+2d,\ \delta=\alpha+3d$

$$d^2=\frac{D_1}{a^2}=\frac{D_2}{p^2}$$

15. If $x^2+4y^2-8x+12=0$ is satisfied by real values of x and y then $'y'\in$

1) $[2,6]$ 2) $[2,5]$

3) $[-1,1]$ 4) $[-2,-1]$

Key. 3

Sol. $x^2-8x+\left(4y^2+12\right)=0$ is a quadratic in $'x'$, $'x'$ is real then discriminate ≥ 0

16. For $x>0, 0\le t\le 2\pi, K>\frac{3}{2}+\sqrt{2}$, K being a fixed real number the minimum value of $x^2+\frac{K^2}{x^2}-2\left\{(1+\cos t)x+\frac{K(1+\sin t)}{x}\right\}+3+2\cos t+2\sin t$ is

a) $\left\{\sqrt{K}-\left(1+\frac{1}{\sqrt{2}}\right)\right\}^2$ b) $\frac{1}{2}\left\{\sqrt{K}-\left(1+\frac{1}{\sqrt{2}}\right)\right\}^2$

c) $3\left\{\sqrt{K}-\left(1+\frac{1}{\sqrt{2}}\right)\right\}^2$ d) $2\left\{\sqrt{K}-\left(1+\frac{1}{\sqrt{2}}\right)\right\}^2$

Key. D

Sol. Given expansion = $\{x-(1+\cos t)\}^2+\left\{\frac{K}{x}-(1+\sin t)\right\}^2$

17. Let $\phi(x)=\frac{(x-b)(x-c)}{(a-b)(a-c)}f(a)+\frac{(x-c)(x-a)}{(b-c)(b-a)}f(b)+\frac{(x-a)(x-b)}{(c-a)(c-b)}f(c)-f(x)$

Where $a<c<b$ and $f^{11}(x)$ exists at all points in (a,b). Then, there exists a real number $\mu, a<\mu<b$ such that

$$\frac{f(a)}{(a-b)(a-c)}+\frac{f(b)}{(b-c)(b-a)}+\frac{f(c)}{(c-a)(c-b)}=$$

a) $f^{11}(\mu)$ b) $2f^{11}(\mu)$ c) $\frac{1}{2}f^{11}(\mu)$ d) $\frac{1}{3}f^{111}(\mu)$

Key. C

Sol. Apply RT's, twice

18. If α,β,γ are the roots of the equation $x^3 + px + q = 0$, then the value of the determinant $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is

(A) 4 (B)2 (C)0 (D) –2

Key. C

Sol. Since α,β,γ are the roots of $x^3 + px + q = 0$

$\therefore \quad \alpha+\beta+\gamma = 0$

Applying $C_1 \to C_1 + C_2 + C_3$, then

$$\begin{vmatrix} \alpha+\beta+\gamma & \beta & \gamma \\ \alpha+\beta+\gamma & \gamma & \alpha \\ \alpha+\beta+\gamma & \alpha & \beta \end{vmatrix} = \begin{vmatrix} 0 & \beta & \gamma \\ 0 & \gamma & \alpha \\ 0 & \alpha & \beta \end{vmatrix} = 0$$

19. The number of points (p, q) such that $p, q \in \{1,2,3,4\}$ and the equation $px^2 + qx + 1 = 0$ has real roots is

A. 7 B. 8 C. 9 D. None of these

Key. A

Sol. $px^2 + qx + 1 = 0$ has real roots if $q^2 - 4p \geq 0$ or $q^2 \geq 4p$

Since $p, q \in \{1,2,3,4\}$

The required points are(1,2), (1,3),(1, 4), (2,3),(2,4),(3,4),(4,4)

So the required number is 7

20. The value of *b* and *c* for which the identity $f(x+1) - f(x) = 8x + 3$ is satisfied, where $f(x) = bx^2 + cx + d$ are

(A) b = 2, c = 1 (B) b = 4, c = –1

(C) b = – 1, c = 4 (D) b = – 1, c = 1

Key. B

Sol. $\because f(x+1) - f(x) = 8x + 3$

$\Rightarrow \quad \{b(x+1)^2 + c(x+1) + d\} - \{bx^2 + cx + d\} = 8x + 3$

$\Rightarrow \quad b\{(x+1)^2 - x^2\} + c = 8x + 3$

$\Rightarrow \quad b(2x+1) + c = 8x + 3$ on comparing

2b = 8 and b + c = 3

Then, b = 4 and c = –1

21. Let $f(x) = ax^2 + bx + c$, $g(x) = ax^2 + px + q$ where a, b, c, q, p, $\in$ R and $b \neq p$. If their discriminants are equal and f(x) = g(x) has a root α, then

1) α will be A.M. of the roots of f(x) = 0, g(x) = 0
2) α will be G.M of all the roots of f(x) = 0, g(x) = 0
3) α will be A.M of the roots of f(x) = 0 or g(x) = 0
4) α will be G.M of the roots of f(x) = 0 or g(x) = 0

Key. 1

Sol. $a\alpha^2+b\alpha+c=a\alpha^2+p\alpha+q \Rightarrow \alpha=\dfrac{q-c}{b-p} \to (i)$

And $b^2-4ac=p^2-4aq$

$\Rightarrow b^2-p^2=4a(c-q)$

$\Rightarrow b+p=\dfrac{4a(c-q)}{b-p}=-4a\alpha \qquad (\text{from}(i))$

$\alpha=\dfrac{-(b+p)}{4a}=\dfrac{\frac{-b}{a}-\frac{p}{a}}{4}$ which is A.M of all the roots of f(x) = 0 and g(x) = 0

22. If the equations $x^2+2\lambda x+\lambda^2+1=0,\ \lambda\in R$ and $ax^2+bx+c=0$ where a, b, c are lengths of sides of triangle have a common root, then the possible range of values of λ is

1) (0, 2) 2) $(\sqrt{3},3)$ 3) $(2\sqrt{2},3\sqrt{2})$ 4) $(0,\infty)$

Key. 1

Sol. $(x+\lambda)^2+1=0$ has clearly imaginary roots

So, both roots of the equations are common

$\therefore \dfrac{a}{1}=\dfrac{b}{2\lambda}=\dfrac{c}{\lambda^2+1}=k(say)$

Then a = k, b = $2\lambda k$, c = (λ^2+1) k

As a, b, c are sides of triangle

$a+b>c \Rightarrow 2\lambda+1>\lambda^2+1 \Rightarrow \lambda^2-2\lambda<0$

$\Rightarrow \lambda\in(0,2)$

The other conditions also imply same relation.

23. The number of real or complex solutions of $x^2-6|x|+8=0$ is

1) 6 2) 7 3) 8 4) 9

Key. 1

Sol. If x is real, $x^2-6|x|+8=0 \Rightarrow |x|^2-6|x|+8=0 \Rightarrow |x|=2,4 \Rightarrow x=\pm2,\pm4$

If x is non – real, say $x=\alpha+i\beta$, then

$(\alpha+i\beta)^2-6\sqrt{\alpha^2+\beta^2}+8=0 \qquad \left(|\alpha+i\beta|=\sqrt{\alpha^2+\beta^2}\right)$

$\left(\alpha^2-\beta^2+8-6\sqrt{\alpha^2+\beta^2}\right)+2i\alpha\beta=0$

Comparing real and imaginary parts,

$\alpha\beta=0 \Rightarrow \alpha=0$ (if $\beta=0$ then x is real.)

& $-\beta^2+8-6\sqrt{\beta^2}=0$

$\beta^2\pm6\beta-8=0 \Rightarrow \beta=\dfrac{\mp6\pm\sqrt{68}}{2}$

ie., $\beta=\pm(3-\sqrt{17})$

Hence $\pm(3-\sqrt{17})i$ are non-real roots.

24. If $x_1, x_2\left(x_1 > x_2\right)$ are abscissae of points P, Q lying on $y = 2x^2 - 4x - 5$ such that the tangents drawn at these points pass through the point (0, -7), then $3x_1 - 2x_2$ equals to

1) 4 2) 5 3) 6 4) 7

Key. 2

Sol. Let (α, β) be point on the curve such that the tangent drawn at (α, β) passes through (0, 7)

$y^1 = 4x - 4 \Rightarrow y^1_{(\alpha,\beta)} = 4\alpha - 4$

Tangent at (α, β) is $y - \beta = (4\alpha - 4)(x - \alpha)$ pass through (0, -7) $\Rightarrow -7 - \beta = (4\alpha - 4)(0 - \alpha)$

But $\beta = 2\alpha^2 - 4\alpha - 5 \therefore$ It follows that α^2 = 1

$\Rightarrow \alpha = \pm 1$

So, $x_1 = 1,\ x_2 = -1$

So, $3x_1 - 2x_2 = 5$.

25. Let $f(x) = x^2 + 5x + 6$, then the number of real roots of $\left(f(x)\right)^2 + 5f(x) + 6 - x = 0$ is

1) 1 2) 2 3) 3 4) 0

Key. 4

Sol. Use "f(x) = x has non real roots $\Rightarrow$ f(f(x)) = x also has non-real roots"

26. Sum of the roots of the equation is $4^x - 3\left(2^{x+3}\right) + 128 = 0$

1) 5 2) 6 3) 7 4) 8

Key. 3

Sol. Put $2^x = y.$ Equation becomes

$y^2 - 3(8y) + 128 = 0 \Rightarrow y^2 - 24y + 128 = 0$

$\Rightarrow (y-8)(y-16) = 0 \Rightarrow y = 16, 8$

$\Rightarrow 2^x = 16, 8 \Rightarrow x = 4, 3$

$\therefore$ Sum of the roots is 7.

27. The number of solutions of $\sqrt{3x^2 + x + 5} = x - 3$ is

1) 0 2) 1 3) 2 4) 4

Key. 1

Sol. Note that we must have $3x^2 + x + 5 \geq 0$ and $x - 3 \geq 0$ or $x \geq 3.$

$\sqrt{3x^2 + x + 5} = x - 3$.... (1)

Squaring both sides of (1), we get

$3x^2 + x + 5 = x^2 - 6x + 9$

$\Rightarrow 2x^2 + 7x - 4 = 0 \Rightarrow (2x-1)(x+4) = 0$

$\Rightarrow x = 1/2, -4$

None of these satisfy the inequality $x \geq 3.$ Thus, (1) has no solution.

28. The value of a for which one root of the quadratic equation.

$\left(a^2 - 5a + 3\right)x^2 + (3a-1)x + 2 = 0$ is twice as large as other, is

1) $-2/3$ 2) $1/3$ 3) $-1/3$ 4) $2/3$

Key. 4

Sol. $\left(a^2 - 5a + 3a\right)x^2 + (3a-1)x + 2 = 0$...... (1)

Let α and 2α be the roots of (1), then

$\left(a^2-5a+3\right)\alpha^2+\left(3a-1\right)\alpha+2=0$ .... (2)

and $\left(a^2-5a+3\right)\left(4\alpha^2\right)+\left(3a-1\right)\left(2\alpha\right)+2=0$ ..... (3)

Multiplying (2) by 4 and subtracting it form (3) we get $\left(3a-1\right)\left(2\alpha\right)+6=0$

Clearly $a\neq 1/3$. Therefore, $\alpha=-3/\left(3a-1\right)$

Putting this value in (2) we get

$\left(a^2-5a+3\right)\left(9\right)-\left(3a-1\right)^2\left(3\right)+2\left(3a-1\right)^2=0$

$\Rightarrow 9a^2-45a+27-\left(9a^2-6a+1\right)=0 \Rightarrow -39a+26=0$

$\Rightarrow a=2/3.$

For $x=2/3$, the equation becomes $x^2+9x+18=0$, whose roots are $-3,-6$.

29. If $f\left(x\right)=x^2+2bx+2c^2$ and $g\left(x\right)=-x^2-2cx+b^2$ are such that $\min f\left(x\right)>\max g\left(x\right)$, then relation between b and c, is

1) no relation
2) $0<c<b/2$
3) $|c|<\dfrac{|b|}{\sqrt{2}}$
4) $|c|>\sqrt{2}|b|$

Key. 4

Sol. $f\left(x\right)=\left(x+b\right)^2+2c^2-b^2$

$\Rightarrow \min f\left(x\right)=2c^2-b^2$

Also $g\left(x\right)=-x^2-2cx+b^2=b^2+c^2-\left(x+c\right)^2$

$\Rightarrow \max g\left(x\right)=b^2+c^2$

As $\min f\left(x\right)>\max g\left(x\right)$, we get $2c^2-b^2>b^2+c^2$

$\Rightarrow c^2>2b^2 \Rightarrow |c|>\sqrt{2}|b|$

30. The equation $\left(\cos p-1\right)x^2+\left(\cos p\right)x+\sin p=0$ in variable x has real roots, if p belongs to the interval

1) $\left(0,2\pi\right)$
2) $\left(-\pi,0\right)$
3) $\left(-\pi/2,\pi/2\right)$
4) $\left(0,\pi\right)$

Key. 4

Sol. $\left(\cos p-1\right)x^2+\left(\cos p\right)x+\sin p=0$ ...... (1)

Discriminant of (1) is given by

$D=\cos^2 p-4\left(\cos p-1\right)\sin p=\cos^2 p+4\left(1-\cos p\right)\sin p$

Note that $\cos^2 p\geq 0, 1-\cos p\geq 0$. Thus, $D\geq 0$ if $\sin p\geq 0$ *i.e.* if $p\in\left(0,\pi\right)$.

31. If $x^2+2ax+10-3a>0$ for each $x\in R$, then

1) $a<-5$
2) $-5<a<2$
3) $a>5$
4) $2<a<5$

Key. 2

Sol. $x^2+2ax+10-3a>0 \forall x\in R$

$\Rightarrow \left(x+a\right)^2-\left(a^2+10-3a\right)>0 \forall x\in R$

$\Rightarrow a^2+3a-10<0$

$\Rightarrow (a+5)(a-2)<0$

$\Rightarrow -5<a<2$

32. Sum of all the values of x satisfying the equation $\log_{17}\log_{11}\left(\sqrt{x+11}+\sqrt{x}\right)=0$ is

1) 25 2) 36 3) 171 4) 0

Key. 1

Sol. $\log_{17}\log_{11}\left(\sqrt{x+1}+\sqrt{x}\right)=0$ …….. (1)

Equation (1) is defined if $x\ge 0$.

We can rewrite (1) as $\log_{11}\left(\sqrt{x+11}+\sqrt{x}\right)=17^0=1$

$\Rightarrow \sqrt{x+11}+\sqrt{x}=11^1=11$

$\Rightarrow \sqrt{x+11}=11-\sqrt{x}$

Squaring both sides we get $x+11=121-22\sqrt{x}+x$

$\Rightarrow 22\sqrt{x}=110 \Rightarrow \sqrt{x}=5$ or $x=25$

This clearly satisfies (1). Thus, sum of all the values satisfying (1) is 25.

33. The number of solutions of the equations of the equation $x^2+[x]-4x+3=0$ is Where [ ] denotes G.I.F.

1) 0 2) 1 3) 2 4) 3

Key. 1

Sol. Given equation can be written as $(x^2-3x+3)-f=O$ where $f=x-[x]$ *and* $O\le f<1$

$\therefore O\le x^2-3x+3<1$

solving $x^2-3x+3=O$; roots are Imaginary

$\therefore x^2-3x+3\ge O\forall x\in R$

solving $x^2-3x+3<1\Rightarrow 1<x<2$

if $1<x<2;[x]=1.$

putting $[x]=1$ in the given equation and solving we get $x=2$. But $1<x<2$ $\therefore$ the given equation has no solution.

34. The number of values of $'a'$ for which the equation $(x-1)^2=|x-a|$ has exactly three solutions is

1) 1 2) 2 3) 3 4) 4

Key. 3

Sol. $|x-a|=(x-1)^2$ Iff $a=x\pm(x-1)^2$

No of solutions = no of intersection its between

$y=a$; $f(x)=x^2-x+1$ and $g(x)=-x^2+3x-1$. clearly the graphs of $f(x),g(x)$ are tangents to each other at $A(1,1)$. The line $y=a$ intersects the two graphs at three points

Iff it passes through one of the three pts A,B, C. Here $B=\left(\frac{1}{2},\frac{3}{4}\right)$ vertex of f

and $C=\left(\frac{3}{2},\frac{5}{4}\right)$ vertex of 'g' i.e if $a\in\left\{\frac{3}{4},\frac{5}{4},1\right\}$

35. If *a, b, c* are positive numbers such that a>b>c and the equation $(a+b-2c)x^2+(b+c-2a)x+(c+a-2b)=0$ has a root in the interval $(-1,0)$, then

A) b cannot be the G.M. of a, c B) b may be the G.M. of a, c
C) b is the G.M. of a, c D) none of these

Key. A

Sol. Let $f(x)=(a+b-2c)x^2+(b+c-2a)x+(c+a-2b)$

According to the given condition, we have

$$f(0)f(-1)<0$$

i.e. $(c+a-2b)(2a-b-c)<0$

i.e. $(c+a-2b)(a-b+a-c)<0$

i.e. $c+a-2b<0$ $\quad [a>b>c,\ given \Rightarrow a-b>0, a-c>0]$

i.e. $b>\frac{a+c}{2}$

$\Rightarrow$ *b* cannot be the G.M. of *a, c,* since G.M < A.M. always.

36. Let α, β (a < b) be the roots of the equation $ax^2+bx+c=0$. If $\lim_{x\to m}\frac{|ax^2+bx+c|}{ax^2+bx+c}=1$, then

A) $\frac{|a|}{a}=-1, m<\alpha$ B) $a>0, \alpha<m<\beta$ C) $\frac{|a|}{a}=1, m>\beta$ D) $a<0, m>\beta$

Key. C

Sol. According to the given condition, we have

$$|am^2+bm+c|=am^2+bm+c$$

i.e. $am^2+bm+c>0$

$\Rightarrow$ if *a< 0,* the *m* lies in (α,β)

and if *a>0,* then *m* does not lies in (α,β)

Hence, option (c) is correct, since

$$\frac{|a|}{a}=1\Rightarrow a>0$$

And in that case *m* does not lie in (α,β).

37. Let $f(x)$ be a function such that $f(x)=x-[x]$, where $[x]$ is the greatest integer less than or equal to x. Then the number of solutions of the equation $f(x)+f\left(\frac{1}{x}\right)=1$ is (are)

A) 0 B) 1 C) 2 D) infinite

Key. D

Sol. Given, $f(x)=x-[x],\ x\in R-\{0\}$

Now $f(x)+f\left(\frac{1}{x}\right)=1$ $\quad\therefore\quad$ $x-[x]+\frac{1}{x}-\left[\frac{1}{x}\right]=1$

$\Rightarrow\left(x+\frac{1}{x}\right)-\left([x]+\left[\frac{1}{x}\right]\right)=1$ $\quad$ $\Rightarrow\left(x+\frac{1}{x}\right)=[x]+\left[\frac{1}{x}\right]+1$ ...(i)

Clearly ,R.H.S is an integer $\quad\therefore$ L. H. S. is also an integer

Let $x+\frac{1}{x}=k$ an integer $\quad\Rightarrow x^2-kx+1=0$

$\therefore x = \dfrac{k \pm \sqrt{k^2 - 4}}{2}$

For real values of *x*, $k^2 - 4 \ge 0 \Rightarrow k \ge 2$ or $k \le -2$

We also observe that *k=2* and -2 does not satisfy equation *(i)*

$\therefore$ The equation *(i)* will have solutions if $k > 2$ or $k < -2$, where $k \in z$.

Hence equation *(i)* has infinite number of solutions.

38. If both the roots of $(2a-4)9^x - (2a-3)3^x + 1 = 0$ are non-negative, then

A) $0 < a < 2$ B) $2 < a < \dfrac{5}{2}$ C) $a < \dfrac{5}{4}$ D) $a > 3$

Key. B

Sol. Putting $3^x = y$, we have

$$(2a-4)y^2 - (2a-3)y + 1 = 0$$

This equation must have real solution

$\Rightarrow \quad (2a-3)^2 - 4(2a-4) \ge 0$

$\Rightarrow \quad 4a^2 - 20a + 25 \ge 0$

$\Rightarrow \quad (2a-5)^2 \ge 0.$ This is true.

$y = 1$ satisfies the equation

Since 3^x is positive and $3^x \ge 3^0$, $y \ge 1$

Product of the roots $= 1 \times y > 1$

$\Rightarrow \quad \dfrac{1}{2a-4} > 1$

$\Rightarrow \quad 2a - 4 < 1 \Rightarrow a < \dfrac{5}{2}$

Sum of the roots $= \dfrac{2a-3}{2a-4} > 1$

$\Rightarrow \quad \dfrac{(2a-3)-(2a-4)}{2a-4} > 0$

$\Rightarrow \quad \dfrac{1}{2a-4} > 0 \Rightarrow a > 2$

$\Rightarrow \quad 2 < a < \dfrac{5}{2}$

39. If the equation $x^2 + 9y^2 - 4x + 3 = 0$ is satisfied for real values of x and y then

A) $x \in [1,3], y \in [1,3]$ B) $x \in [1,3], y \in \left[\dfrac{-1}{3}, \dfrac{1}{3}\right]$

C) $x \in \left[\dfrac{-1}{3}, \dfrac{1}{3}\right], y \in [1,3]$ D) $x \in \left[\dfrac{-1}{3}, \dfrac{1}{3}\right], y \in \left[\dfrac{-1}{3}, \dfrac{1}{3}\right]$

Key. B

Sol. Given equation is $x^2 + 9y^2 - 4x + 3 = 0$ …(i)

Or, $x^2-4x+9y^2+3=0.$

Since x is real $\therefore \ (-4)^2-4(9y^2+3)\ge 0$

Or, $16-4(9y^2+3)\ge 0$ or, $4-9y^2-3\ge 0$

Or, $9y^2-1\le 0$ or, $9y^2\le 1$ or, $y^2\le \frac{1}{9}$

Now $y^2\le \frac{1}{9}\Leftrightarrow -\frac{1}{3}\le y\le \frac{1}{3}$ ...(ii)

Equation (i) can also be written as

$$9y^2+0y+x^2-4x+3=0 \quad \text{...(iii)}$$

Since y is real $\therefore 0^2-4.9(x^2-4x+3)\ge 0$

Or, $x^2-4x+3\le 0$

$\Rightarrow x\in[1,3]$

40. The equation $a_8x^8+a_7x^7+a_6x^6+...+a_0=0$ has all its roots positive and real $\left(where\ a_8=1, a_7=-4, a_0=1/2^8\right)$, then

A) $a_1=\frac{1}{2^8}$ B) $a_1=-\frac{1}{2^4}$ C) $a_2=\frac{7}{2^5}$ D) $a_2=\frac{7}{2^8}$

Key. B

Sol. Let the roots be $\alpha_1,\alpha_2,....,\alpha_8$

$\Rightarrow \quad \alpha_1+\alpha_2+....+\alpha_8=4$

$\alpha_1\alpha_2.....\alpha_8=\frac{1}{2^8}$

$\Rightarrow \quad (\alpha_1\alpha_2.....\alpha_8)^{1/8}=\frac{1}{2}=\frac{\alpha_1+\alpha_2+...+\alpha_8}{8}$

$\Rightarrow$ AM=GM$\Rightarrow$ all the roots are equal to $\frac{1}{2}$.

$\Rightarrow \quad a_1=-{}^8C_7\left(\frac{1}{2}\right)^7=-\frac{1}{2^4}$

$a_2={}^8C_6\left(\frac{1}{2}\right)^6=-\frac{7}{2^4}$

$a_3=-{}^8C_5\left(\frac{1}{2}\right)^5$

41. If every root of a polynomial equation (of degree ‘n’) $f(x)=0$ with leading coefficient “1” is real and distinct, then the equation $f''(x)f(x)-\{f'(x)\}^2=0$ has.

(A) at least one real root (B) no real root
(C) at most one real root (D) exactly two real roots

Key. B

Sol. Let $f(x)=(x-a_1)(x-a_2)..............(x-an)$ where $a_1,a_2.........a_{n\in R}$ take log both sides and differentiate. Then

$$\frac{f'(x)}{f(x)}=\frac{1}{x-a_1}+\frac{1}{x-a_2}+.........+\frac{1}{x-a_n}$$

Again diff w.r.t. 'x'

$$\frac{f\ f''-(f')^2}{f^2}=-\left[\frac{1}{(x-a_1)^2}+\frac{1}{(x-a_2)^2}+......\frac{1}{(x-a_n)^2}\right]$$

$$<0\forall x\in R$$

$\Rightarrow f\ f''-(f')^2=0$ has no real root

42. If $f(x)$ is a polynomial of least degree such that $f(r)=\frac{1}{r}$, $r=1,2,3,$ ___9, then $f(10)$ = ___

A. 1 B. $\frac{1}{2}$ C. $\frac{1}{10}$ D. $\frac{1}{5}$

Key. D

Sol. $xf(x)-1=0$ has roots 1,2,3 ____9

$xf(x)-1=A(x-1)(x-2)$ _____x-9

Put $x=0\Rightarrow A=\frac{1}{9!}$

Put $x=10\Rightarrow 10f(10)-1=1\Rightarrow f(10)=\frac{1}{5}$

43. The number of ordered pairs of integers (x, y) satisfying the equation $x^2+6x+y^2=4$ is

A. 2 B. 8 C. 6 D. 10

Key. B

Sol. $(x+3)^2+y^2=13$

$x+3=\pm2, y=\pm3\, or\, x+3=\pm3, y=\pm2$

44. The number of non-negative integer solutions of $x+y+2z=20$ is

A. 76 B. 84 C. 112 D. 121

Key. D

Sol. $x+y=20-2Z,\ Z=0,1,2,...10$

The number of solutions (non –ve) is $\sum_{Z=0}^{10}(20-2Z+1)_{C_1}=121$

45 If $a+b+c=0$ for $a,b,c\in R$, then the equation $3ax^2+2bx+c=0$ has

A. Atleast one root in $[0,1]$

B. One root in $[2,3]$ and another root in $[-2,-1]$

C. Imaginary roots

D. Atleast one root in $[1,2]$

Key. A

Sol. Let $f(x)=ax^3+bx^2+cx$. Then f is continuous and differentiable in $[0,1]$, $f(0)=f(1)=0$. Hence by Rolle's theorem there exists $k\in(0,1)$ such that $3ak^2+2bk+c=0$

46. If a,b,c be the sides of a triangle ABC and if roots of the equation a(b – c)x² + b(c – a) x + c(a – b) = 0 are equal, then $\sin^2\left(\frac{A}{2}\right),\sin^2\left(\frac{B}{2}\right),\sin^2\left(\frac{C}{2}\right)$ are in

(A) AP (B)GP (C) HP (D) AGP

Key. C

Sol. ∵ a(b – c) + b(c – a) + c (a – b) = 0

∴ x = 1 is a root of the equation

a(b – c)x² + b(c – a)x + c(a – b) =0

Then, other root = 1 (∵ roots are equal)

∴ $\alpha\times\beta=\frac{c(a-b)}{a(b-c)}$

⇒ ab – ac = ca – bc

∴ $b=\frac{2ac}{a+c}$

∴ a, b, c are in HP

Then, $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in AP.

⇒ $\frac{s}{a},\frac{s}{b},\frac{s}{c}$ are in AP

⇒ $\frac{s}{a}-1,\frac{s}{b}-1,\frac{s}{c}-1$ are in AP.

⇒ $\frac{(s-a)}{a},\frac{(s-b)}{b},\frac{(s-c)}{c}$ are in AP.

Multiplying in each by $\frac{abc}{(s-a)(s-b)(s-c)}$

Then $\frac{bc}{(s-b)(s-c)},\frac{ca}{(s-c)(s-a)},\frac{ab}{(s-a)(s-b)}$ are in AP.

⇒ $\frac{(s-b)(s-c)}{bc},\frac{(s-c)(s-a)}{ca},\frac{(s-a)(s-b)}{ab}$ are in HP.

Or $\sin^2\left(\frac{A}{2}\right), \sin^2\left(\frac{B}{2}\right), \sin^2\left(\frac{C}{2}\right)$ are in HP

47. If α, β, γ are the roots of the equation $x^3 + px + q = 0$, then the value of the determinant $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is

(A) 4 (B)2 (C)0 (D) –2

Key. C

Sol. Since α, β, γ are the roots of $x^3 + px + q = 0$

$\therefore \quad \alpha + \beta + \gamma = 0$

Applying $C_1 \to C_1 + C_2 + C_3$, then

$$\begin{vmatrix} \alpha+\beta+\gamma & \beta & \gamma \\ \alpha+\beta+\gamma & \gamma & \alpha \\ \alpha+\beta+\gamma & \alpha & \beta \end{vmatrix} = \begin{vmatrix} 0 & \beta & \gamma \\ 0 & \gamma & \alpha \\ 0 & \alpha & \beta \end{vmatrix} = 0$$

48. The value of *b* and *c* for which the identity $f(x+1) - f(x) = 8x + 3$ is satisfied, where $f(x) = bx^2 + cx + d$ are

(A) b = 2, c = 1 (B) b = 4, c = –1

(C) b = – 1, c = 4 (D) b = – 1, c = 1

Key. B

Sol. $\because f(x+1) - f(x) = 8x + 3$

$\Rightarrow \quad \left\{b(x+1)^2 + c(x+1) + d\right\} - \left\{bx^2 + cx + d\right\} = 8x + 3$

$\Rightarrow \quad b\left\{(x+1)^2 - x^2\right\} + c = 8x + 3$

$\Rightarrow \quad b(2x+1) + c = 8x + 3$ on comparing

2b = 8 and b + c = 3

Then, b = 4 and c = –1

49. If *a, b, c* are positive numbers such that a>b>c and the equation $(a+b-2c)x^2 + (b+c-2a)x + (c+a-2b) = 0$ has a root in the interval $(-1, 0)$, then

A) b cannot be the G.M. of a, c B) b may be the G.M. of a, c

C) b is the G.M. of a, c D) none of these

Key. A

Sol. Let $f(x) = (a+b-2c)x^2 + (b+c-2a)x + (c+a-2b)$

According to the given condition, we have

$f(0)f(-1) < 0$

i.e. $(c+a-2b)(2a-b-c) < 0$

i.e. $(c+a-2b)(a-b+a-c) < 0$

i.e. $c+a-2b < 0 \quad \left[a > b > c,\ given \Rightarrow a-b > 0, a-c > 0\right]$

i.e. $b > \frac{a+c}{2}$

$\Rightarrow$ *b* cannot be the G.M. of *a, c,* since G.M < A.M. always.

50. The values of 'a' for which the quadratic expression $ax^2+(a-2)x-2$ is negative for exactly two integral values of x, belongs to

(A) $[-1,1]$ (B) $[1,2)$

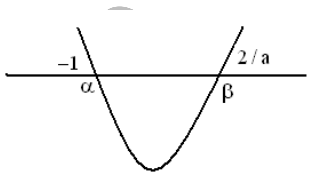
(C) $[3,4]$ (D) $[-2,-1)$

Key. B

Sol. Let $f(x)=ax^2+(a-2)x-2$

$f(x)$ is negative for two integral values of x, so graph should be vertically upward parabola i.e., $a>0$

Let two roots of $f(x)=0$ are α and β then $\alpha,\beta=\dfrac{-(a-2)\pm(a+2)}{2a}$



$\Rightarrow \alpha=-1, \beta=\dfrac{2}{a}\Rightarrow 1<\beta\le 2\Rightarrow 1<\dfrac{2}{a}\le 2\Rightarrow a\in[1,2]$

51. Let $f(x)$ be a function such that $f(x)=x-[x]$, where $[x]$ is the greatest integer less than or equal to x. Then the number of solutions of the equation $f(x)+f\left(\dfrac{1}{x}\right)=1$ is (are)

A) 0 B) 1 C) 2 D) infinite

Key. D

Sol. Given, $f(x)=x-[x],\ x\in R-\{0\}$

Now $f(x)+f\left(\dfrac{1}{x}\right)=1$ $\therefore$ $x-[x]+\dfrac{1}{x}-\left[\dfrac{1}{x}\right]=1$

$\Rightarrow\left(x+\dfrac{1}{x}\right)-\left([x]+\left[\dfrac{1}{x}\right]\right)=1$ $\Rightarrow\left(x+\dfrac{1}{x}\right)=[x]+\left[\dfrac{1}{x}\right]+1$ ...(i)

Clearly ,R.H.S is an integer $\therefore$ L. H. S. is also an integer

Let $x+\dfrac{1}{x}=k$ an integer $\Rightarrow x^2-kx+1=0$

$\therefore x=\dfrac{k\pm\sqrt{k^2-4}}{2}$

For real values of *x*, $k^2-4\ge 0\Rightarrow k\ge 2$ or $k\le -2$

We also observe that *k=2* and -2 does not satisfy equation *(i)*

$\therefore$ The equation *(i)* will have solutions if $k>2$ or $k<-2$,where $k\in z$.

Hence equation *(i)* has infinite number of solutions.

52. If both the roots of $(2a-4)9^x-(2a-3)3^x+1=0$ are non-negative, then

A) $0<a<2$ B) $2<a<\dfrac{5}{2}$ C) $a<\dfrac{5}{4}$ D) $a>3$

Key. B

Sol. Putting $3^x=y$, we have

$$(2a-4)y^2-(2a-3)y+1=0$$

This equation must have real solution

$\Rightarrow \quad (2a-3)^2 - 4(2a-4) \ge 0$

$\Rightarrow \quad 4a^2 - 20a + 25 \ge 0$

$\Rightarrow \quad (2a-5)^2 \ge 0.$ This is true.

$y = 1$ satisfies the equation

Since 3^x is positive and $3^x \ge 3^0$, $y \ge 1$

Product of the roots $= 1 \times y > 1$

$\Rightarrow \quad \frac{1}{2a-4} > 1$

$\Rightarrow \quad 2a-4 < 1 \Rightarrow a < \frac{5}{2}$

Sum of the roots $= \frac{2a-3}{2a-4} > 1$

$\Rightarrow \quad \frac{(2a-3)-(2a-4)}{2a-4} > 0$

$\Rightarrow \quad \frac{1}{2a-4} > 0 \Rightarrow a > 2$

$\Rightarrow \quad 2 < a < \frac{5}{2}$

53. If the equation $x^2 + 9y^2 - 4x + 3 = 0$ is satisfied for real values of x and y then

A) $x \in [1,3], y \in [1,3]$ B) $x \in [1,3], y \in \left[\frac{-1}{3}, \frac{1}{3}\right]$

C) $x \in \left[\frac{-1}{3}, \frac{1}{3}\right], y \in [1,3]$ D) $x \in \left[\frac{-1}{3}, \frac{1}{3}\right], y \in \left[\frac{-1}{3}, \frac{1}{3}\right]$

Key. B

Sol. Given equation is $x^2 + 9y^2 - 4x + 3 = 0$ …(i)

Or, $x^2 - 4x + 9y^2 + 3 = 0.$

Since x is real $\therefore (-4)^2 - 4(9y^2 + 3) \ge 0$

Or, $16 - 4(9y^2 + 3) \ge 0$ or, $4 - 9y^2 - 3 \ge 0$

Or, $9y^2 - 1 \le 0$ or, $9y^2 \le 1$ or, $y^2 \le \frac{1}{9}$

Now $y^2 \le \frac{1}{9} \Leftrightarrow -\frac{1}{3} \le y \le \frac{1}{3}$ …(ii)

Equation (i) can also be written as

$9y^2 + 0y + x^2 - 4x + 3 = 0$ …(iii)

Since y is real $\therefore 0^2 - 4.9(x^2 - 4x + 3) \ge 0$

Or, $x^2 - 4x + 3 \le 0$

$\Rightarrow x \in [1,3]$

54. The equation $a_8x^8 + a_7x^7 + a_6x^6 + ... + a_0 = 0$ has all its roots positive and real $\left(where\ a_8 = 1, a_7 = -4, a_0 = 1/2^8\right)$, then

A) $a_1 = \frac{1}{2^8}$ B) $a_1 = -\frac{1}{2^4}$ C) $a_2 = \frac{7}{2^5}$ D) $a_2 = \frac{7}{2^8}$

Key. B

Sol. Let the roots be $\alpha_1, \alpha_2, ..., \alpha_8$

$$\Rightarrow \quad \alpha_1 + \alpha_2 + ... + \alpha_8 = 4$$

$$\alpha_1\alpha_2.....\alpha_8 = \frac{1}{2^8}$$

$$\Rightarrow \quad (\alpha_1\alpha_2.....\alpha_8)^{1/8} = \frac{1}{2} = \frac{\alpha_1 + \alpha_2 + ... + \alpha_8}{8}$$

$\Rightarrow$ AM=GM $\Rightarrow$ all the roots are equal to $\frac{1}{2}$.

$$\Rightarrow \quad a_1 = -{}^8C_7\left(\frac{1}{2}\right)^7 = -\frac{1}{2^4}$$

$$a_2 = {}^8C_6\left(\frac{1}{2}\right)^6 = -\frac{7}{2^4}$$

$$a_3 = -{}^8C_5\left(\frac{1}{2}\right)^5$$

55. If f(x) = $\prod_{i=1}^{i=3}(x - a_i) + \sum_{i=1}^{3} a_i - 3x$, where $a_i < a_{i+1}$, then f(x) = 0 has

(A) only one real root

(B) three real roots of which two of them are equal

(C) three distinct real roots

(D) three equal roots

KEY : C

SOL : f(x) = $(x - a_1)(x - a_2)(x - a_3) + (a_1 - x) + (a_2 - x) + (a_3 - x)$

Now f(x) $\rightarrow -\infty$ as x $\rightarrow -\infty$ and f(x) $\rightarrow \infty$ are x $\rightarrow \infty$.

Again $f(a_1) = (a_2 - a_1) + (a_3 - a_1) > 0$ $\quad [\because a_1 < a_2 < a_3]$

$\Rightarrow$ One root belongs to $(-\infty, a_1)$

Also, $f(a_3) = (a_1 - a_3) + (a_2 - a_3) < 0$

$\Rightarrow$ One root belongs to (a_1, a_3)

So f(x) = 0 has three distinct real roots.

56. If a, b and c are numbers for which the equation $\dfrac{x^2+10x-36}{x(x-3)^2} = \dfrac{a}{x}+\dfrac{b}{x-3}+\dfrac{c}{(x-3)^2}$ is an identity, then a + b + c equals

(A) 2 (B) 3 (C) 10 (D) 8

Key. A

Sol. =

hence $x^2 + 10x - 36 = a(x-3)^2 + b(x-3)x + cx$

put x = 0; $-36 = 9a \Rightarrow a = -4$

$x^2 + 10x - 36 = x^2(-4 + b) + x(24 - 3b + c) + (-36)$

comparing coefficients

also, $-4 + b = 1 \Rightarrow b = 5$ $24 - 15 + c = 10 \Rightarrow 9 + c = 10 \Rightarrow c = 1$

$a = -4$; $b = 5$; $c = 1$ i.e. $a + b + c = 2$

57. If one root of equation $x^2 - 4ax + a + f(a) = 0$ is three times of the other then minimum value of $f(a)$ is

A) $\dfrac{-1}{6}$ B) $\dfrac{-1}{10}$ C) $\dfrac{-1}{5}$ D) $\dfrac{-1}{12}$

Key. D

Sol. Let roots are α and 3α, then $4\alpha = 4a \Rightarrow \alpha = a$ and

$a^2 - 4a^2 + f(a) = 0 \Rightarrow f(a) = 3a^2 - a$

$f'(a) = 6a - 1,\ f''(a) = 6$, then minimum value of $f'(a) = 6a - 1,\ f''(a) = 6$

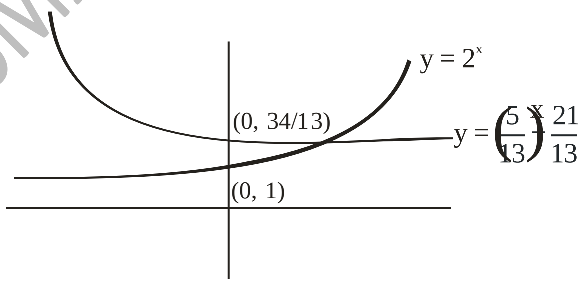
58. The number of real roots of $\left(\dfrac{5}{13}\right)^x + \dfrac{21}{13} = 2^x$ is

(A) Two (B) Infinitely many

(C) only one (D) zero

Key. C

Sol.



Both graphs cut at only one point

59. For a non zero polynomial P, the equation $|P(x)| = e^x$ has

(A) At least one solution (B) No solution

(C) Exactly 2 solution (D) Exactly 1 solution

Key. A

Sol. $\lim\limits_{x\to\infty} e^{-x}|P(x)| = 0$

and $\underset{x\to-\infty}{\text{Lt}}\ e^{-x}|P(x)| = \infty$

consequently there is an $x_0 \in R$ such that $e^{-x_0}|P(x_0)| = 1$

60. A continuous function $y = f(x)$ is defined in a closed interval $[-7,5]$. $A(-7,-4)$, $B(-2,6)$, $C(0,0)$, $D(1,6)$, $E(5,-6)$ are consecutive points on the graph of $'f'$ and AB, BC, CD, DE are line segments. The minimum number of real roots of the equation $f[f(x)] = 6$ is

A) 6 B) 4 C) 2 D) 0

Key. A

Sol. $f[f(x)] = 6 \Rightarrow f(x) = -2$ (or) $f(x) = 1$

$f(x) = -2$, has two roots and $f(x) = 1$ has four roots.

61. If $f(x) = -3x + \prod_{i=1}^{3}(x - a_i) + \sum_{i=1}^{3} a_i$, where $a_i < a_{i+1}$, then $f(x) = 0$ has

A) Only one real root

B) Three real roots of which two of them are equal

C) Three distinct real roots

D) Three equal roots

Key. C

Sol. $f(x) = (x - a_1)(x - a_2)(x - a_3) + (a_1 - x) + (a_2 - x) + (a_3 - x)$

Now, $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ are $x \to \infty$

Again $f(a_1) = (a_2 - a_1) + (a_3 - a_1) > 0\ [\because a_1 < a_2 < a_3]$

$\Rightarrow$ One root belongs to $(-\infty, a_1)$

Also, $f(a_3) = (a_1 - a_3) + (a_2 - a_3) < 0$

$\Rightarrow$ One root belongs to (a_1, a_3)

So, $f(x) = 0$ has three distinct real roots.

62. The number of real values of $'m'$ from for which the equation $z^3+(3+i)z^2-3z-(m+i)=0$ has atleast one real root is

A) 1 B) 3 C) Infinite D) 2

Key. D

Sol. $z^3+(3+i)z^2-3z-(m+i)=0$

$\left(z^3+3z^2-3z-m\right)+i\left(z^2-1\right)=0$

If $'z'$ is a real root, then $z^3+3z^2-3z-m=0$ and $z^2-1=0$

$\therefore\ z=\pm 1$

$z=1 \Rightarrow m=1$

$z=-1 \Rightarrow m=5$

63. Number of all integral values of x, so that $x^2 + 19x + 89$ is a perfect square is

a) 0 b) 1 c) 2 d) 3

Key : C

Sol. Let $x^2+19x+89=\lambda^2$

$\Rightarrow x^2+19x+\left(89-\lambda^2\right)=0$ should have integral roots

$\therefore$ D should be a perfect square.

$\Rightarrow \quad (19)^2-4\left(89-\lambda^2\right)=\text{Perfect square}$

$\Rightarrow \quad (19)^2-4\left(89-\lambda^2\right)=\text{Perfect square}$

$\Rightarrow \quad \left(m^2-4\lambda^2\right)=5 \Rightarrow (m-2\lambda)(m+2\lambda)=5$

$\therefore \quad (m-2\lambda=5, m+2\lambda=1)$

or $\quad (m-2\lambda=-5, m+2\lambda=-1)$

$\Rightarrow \quad (m-2\lambda=-5, m+2\lambda=-1)$

$\Rightarrow \quad m=3,-3, \lambda=1,-1$

For $\lambda=\pm 1$ equation becomes $x^2+19x+88=0$

$\Rightarrow \quad (x+11)(x+8)=0$

$\Rightarrow \quad x=-8,-11.$

Thus, required values of x are –8, –11.

64. Let $f(x) = x^2 + bx + c$, b is negative odd integer, f(x) = 0 has two distinct prime number as roots, and b + c = 15, then least value of f(x) is

(A) $\frac{-233}{4}$ (B) $\frac{233}{4}$

(C) $-\frac{225}{4}$ (D) none of these

Key: C

Hint: $f(x) = (\sin^2\theta)x^3 + \frac{1}{2}\sin2\theta\, x^2 - 2\sin^2\theta.\, x - \sin2\theta$

$f'(x) = (3\sin^2\theta)x^2 + \sin2\theta x - 2\sin^2\theta$

Then D > 0 and product of roots < 0

So f(x) has local maxima at some $x \in R^-$

and local minima at some $x \in R^+$

65. Let $f(x) = x^2 + \lambda x + \mu\cos x$, λ being an integer and μ a real number. The number of ordered pairs (λ, μ) for which the equations $f(x) = 0$ and $f(f(x)) = 0$ have the same (non empty) set of real roots is

(A) 4 (B) 6

(C) 8 (D) infinite

Key: A

Hint: Let α be a root of $f(x) = 0$, so we have $f(\alpha) = 0$ and thus $f(f(\alpha)) = 0$,

$\Rightarrow f(0) = 0 \Rightarrow \mu = 0$.

We then have $f(x) = x(x+\lambda)$ and thus $\alpha = 0, -\lambda$.

$f(f(x)) = x(x+\lambda)(x^2 + \lambda x + \lambda)$

We want λ such that $x^2 + \lambda x + \lambda$ has no real roots besides 0 and $-\lambda$. We can easily find that $0 \le \lambda < 4$.

66. If $ax^2 + bx + c$; $a, b, c \in R$ has no real zeroes, and if c < 0, then

(a) a < 0 (b) a + b + c > 0 (c) 4a + 2b + c > 0 (d) a – b + c > 0

Key: a

Hint: Let $f(x) = ax^2 + bx + c$. Since f(x) has no real zeroes, either f(x) > 0 or f (x) < 0 for all $x \in R$. since f(0) = c < 0, we get f(x) < 0 for all $x \in R$. Therefore, a < 0 as the parabola y = f(x) must open downward. Obviously f(1), f(–1) and f(2) < 0.

67. The quadratic equation $(4 + \cos\theta)\, x^2 - (2\sin\theta)\, x + (3 - \cos\theta) = 0$ has

(A) Real and distinct roots for all θ

(B) Real or complex roots for depending upon θ

(C) Equal roots for all θ

(D) Complex roots for all θ

Key : D

Sol : Discriminant $= 4\sin^2\theta - 4(4 + \cos\theta)(3 - \cos\theta)$

$= 4[\sin^2\theta - (12 - \cos\theta - \cos^2\theta)]$

$= 4[-11 + \cos\theta] < 0 \quad \forall\theta \in R$.

68. If $\alpha_1, \alpha_2, \ldots \alpha_n$ are roots of the equation $x^n + ax + b = 0$, then $(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)(\alpha_1 - \alpha_4) \ldots (\alpha_1 - \alpha_n)$ is equal to

(A) n (B) $n\alpha_1^{n-1}$

(C) $n\alpha_1 + b$ (D) $n\alpha_1^{n-1} + a$

KEY : D

SOL : $x^n + ax + b = (x - \alpha_1)(x - \alpha_2) \ldots (x - \alpha_n)$

differentiate both sides w.r.t. x

$nx^{n-1} + a = (x - \alpha_2) \ldots (x - \alpha_n) + (x - \alpha_1)\left(\frac{d}{dx}(x - \alpha_2) \ldots (x - \alpha_n)\right)$

put $x = \alpha_1$ $\quad$ $n\alpha_1^{n-1} + a = (\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3) \ldots (\alpha_1 - \alpha_n)$

69. The equation $|2ax-3|+|ax+1|+|5-ax|=\frac{1}{2}$ possesses

(A) infinite number of real solution for some $a \in R$

(B) finite number of real solutions for some $a \in R$

(C) no real solution for some $a \in R$

(D) no real solution for all $a \in R$

Key: D

Hint: The equation $|2ax-3|+|ax+1|+|5-ax|$......

$$|2ax-3|+|ax+1|+|5-ax| \ge |2ax-3+(-ax-1)+5-ax| \ge 1$$

So no solution for $\frac{1}{2}$

70. Let P(x) be a polynomial with degree 2009 and leading co-efficient unity such that P(0)=2008, P(1)=2007, P(2)=2006,....P(2008)=0 then the value of P(2009)= $(\underline{|n})-a$ where n and a are natural number then value of $(n+a)$

(A) 2010 (B) 2009

(C) 2011 (D) 2008

Key: A

Hint: $P(x)-2008+x = x(x-1)(x-2)(x-3).....(x-2008)$

Put x = 2009

$P(2009)+1=(2009)!$

71. (L-2)If $f(x)=ax^2+bx+c=0$ has real roots and its coefficients are odd positive integers then

a) f(x) = 0 always has irrational roots

b) $\left|f\left(\frac{p}{q}\right)\right| \ge \frac{1}{q^2}$ where $p, q \in I$

c) If a.c = 1, then equation must have exactly one root α such that $[\alpha]=-1,$ where [.] is greatest integer function

d) equation has rational roots

Key ; a, b

Sol : An equation with odd coefficients cannot have rational roots

$\therefore f(x)=0$ has irrational roots.

$f\left(\frac{p}{q}\right)=\frac{ap^2+bpq+cq^2}{a^2}\ge\frac{1}{a^2}$ ($\therefore$ a, b, c are odd integers p, q are integers)

72. (L-1)Let a, b, c be real numbers with $a\ne 0$ and let α,β be the roots of the equation $ax^2+bx+c=0$. Then one of the roots of the equation $a^3x^2+abcx+c^3=0$ in terms of α,β are

a) $\frac{\alpha^2}{\beta}$

b) α^3

c) β^3

d) $\alpha\beta^2$

Key: d

Sol : We have $\alpha+\beta=-\frac{b}{a},\ \alpha\beta=\frac{c}{a}$

Let γ,δ be the roots of $a^3x^2+abcx+c^3=0$.

Then $\gamma,\delta=\frac{-abc\pm\sqrt{(abc)^2-4a^3c^3}}{2a^3}=\frac{ac\left\{-b\pm\sqrt{b^2-4ac}\right\}}{2a^3}=\frac{c}{2a}\left\{-\frac{b}{a}\pm\sqrt{\left(\frac{b}{a}\right)^2-4\frac{c}{a}}\right\}$

$=\frac{1}{2}(\alpha\beta)\left\{(\alpha+\beta)\pm\sqrt{(\alpha+\beta)^2-4\alpha\beta}\right\}$

$=\frac{1}{2}(\alpha\beta)\left\{(\alpha+\beta)\pm(\alpha-\beta)\right\}=\alpha^2\beta,\alpha\beta^2$

Thus, roots of $a^3x^2+abcx+c^3=0$ are $\alpha^2\beta$ and $\alpha\beta^2$

73. (L-2)If α,β are the roots of $x^2-3x+\lambda=0\ (\lambda\in R)$ and $\alpha<1<\beta$, then the true set of values of λ equals

a) $\lambda\in\left(2,\frac{9}{4}\right]$

b) $\lambda\in\left(-\infty,\frac{9}{4}\right]$

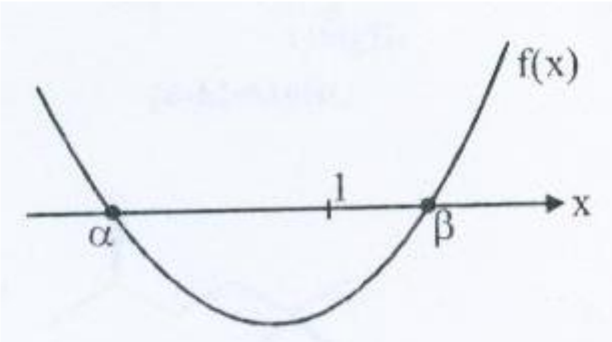
c) $\lambda\in(2,\infty)$

d) $\lambda\in(-\infty,2)$

Key : d

Sol : Let $f(x)=x^2-3x+\lambda$

Clearly f(1) < 0



$\Rightarrow 1-3+\lambda<0 \Rightarrow \lambda<2 \Rightarrow \lambda \in (-\infty, 2)$

74. (L-1)Let $2^{y-x}(x+y)=1$ and $(x+y)^{x-y}=2$ then ordered pair (x, y) can be

a) $\left(\frac{3}{2}, \frac{1}{2}\right)$

b) $\left(-\frac{1}{4}, \frac{3}{4}\right)$

c) $\left(\frac{3}{2}, \frac{3}{4}\right)$

d) $\left(-\frac{1}{4}, \frac{1}{2}\right)$

Key : a

Sol : Put x = 3/2, y = ½ in given equations.

75. (L-1)The equation $|2ax-3|+|ax+1|+|5-ax|=\frac{1}{2}$ possesses

a) infinite number of real solution for some $a \in R$

b) finite number of real solutions for some $a \in R$

c) no real solution for some $a \in R$

d) no real solution for all $a \in R$

Key : d

Sol : $|2ax-3|+|ax+1|+|5-ax| \geq |2ax-3-ax-1+5-ax|$

$=1 \neq \frac{1}{2}$

Hence it has no solution

76. (L-1)If $x^2+5=2x-4\cos(a+bx)$ where $a, b \in (0,5)$, is satisfied for at least one real x, then the maximum value of (a + b) is

a) π

b) 2π

c) 3π

d) none of these

Key : c

Sol : $x^2-2x+5=-4\cos(a+bx)$

$-4\cos(a+bx)\geq 4\rightarrow \cos(a+bx)\leq -1$

$\therefore \cos(a+b)=-1$

$\therefore a+b=\pi or 3\pi$

77. (L-2)If the equation $x^n+a_1x^{n-1}+a_2x^{n-2}+.....+a_n=5,$ with integral co-efficients, has four distinct integral roots then the number of integral roots of the equation

a) 0 b) 1 c) 2 d) 4

KEY : a

Sol : Let $\alpha_i\ i=1,2,3,4 the 4$ integral roots of $x^n+a_1x^{n-1}+...+a_n=5$ and let K be an integral root of $x^n+a_1x^{n-1}+..+a_n=7$

$\Rightarrow(x-\alpha_1)(x-\alpha_2)(x-\alpha_3)(x-\alpha_4)=2$ has an integral root K.

$\Rightarrow(K-\alpha_1)(K-\alpha_2)(K-\alpha_3)(K-\alpha_4)=2$

$K-\alpha_i$, i = 1,2,3,4 are all integers and are distinct which is impossible

($\because$ product of 4 district integers cannot be 2).

Hence $x^n+a_1x^{n-1}+a_2x^{n-2}+...+a_n=7$ has no integral roots.

24. (L-1)The set of values of 'a' for which

$x^2+ax+\sin^{-1}(x^2-4x+5)+\cos^{-1}(x^2-4x+5)=0$ has at least one real solution is given by

a) $(-\infty,-\sqrt{2\pi}]\cup[\sqrt{2\pi},\infty)$ b) $\frac{-\pi-8}{4}$

c) R d) $\frac{\pi-8}{4}$

Key : b

Sol : Charly $x^2-4x+5=(x-2)^2+1, lies\ b|w-1,1.$ $\Rightarrow x=2$ is the only point of the domain, It must be the solution. $\therefore 4+2a+\frac{\pi}{2}=0\Rightarrow a\Rightarrow\frac{-\pi-8}{4}$

78. (L-1)If $ax^2 + bx + c = 0$ and $5x^2 + 6x + 12 = 0$ have a common root where a , b and c are sides of a triangle ABC, then

a) Δ ABC is obtuse angled b) Δ ABC is acute angled

c) Δ ABC is right angled d) none of these

Key : d

sol : $5x^2 + 6x + 12 = 0$

(has complex roots only)

79. (L-1)If 0 < a < 5, 0 < b < 5 and $\frac{x^2+5}{2}=x-2\cos(a+bx)$ is satisfied for atleast one real x, then value of a + b may be equal to

a) π b) $\frac{\pi}{2}$ c) 3π d) 4π

Key : a

sol : cos (a + bx) = $-1-\frac{(x-1)^2}{4}$ exists only when x = 1

at x = 1; a + b = π

cos (a + bx) = $\frac{-(x^2-2x+5)}{4}=-1-\frac{(x-1)^2}{4}$

$\Rightarrow$ x = 1

$\Rightarrow$ a + b = 5

80. (L-1)Number of integral values of x satisfying $3x^2+8x<2\sin^{-1}\sin 4-\cos^{-1}\cos 4$ is

a) one b) two

c) three d) infinite

Key : a

Sol : $3x^2+8x<2\sin^{-1}\sin 4-\cos^{-1}\cos 4$

$3x^2+8x<2(\pi-4)-(2\pi-4)$

$<2\pi-8-2\pi+4$

<-4

$\Rightarrow 3x^2+8x+4<0$ has one solution

81. The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other, is

(A) $\frac{2}{3}$ (B) $-\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $-\frac{1}{3}$

Key. A

Sol. Let the roots are α and 2α

$\Rightarrow \alpha+2\alpha=\frac{1-3a}{a^2-5a+3}$ and $\alpha.2\alpha=\frac{2}{a^2-5a+3}$

$\Rightarrow 2\left[\frac{1}{9}\frac{(1-3a)^2}{(a^2-5a+3)^2}\right]=\frac{2}{a^2-5a+3}$

$\Rightarrow$ $9a^2 - 6a + 1 = 9a^2 - 45a + 27$

$\Rightarrow$ 39a = 26

$\Rightarrow$ $\frac{2}{3}$

82. (L-1)If a, b and c are each positive, and a + b + c = 6 then the minimum value of

$\left(a+\frac{1}{b}\right)^2+\left(b+\frac{1}{c}\right)^2+\left(c+\frac{1}{a}\right)^2$ is

a) $\frac{75}{2}$ b) $\frac{75}{4}$

c) $\frac{65}{4}$ d) $\frac{65}{2}$

Key: b

Sol: Using the $AM \geq HM$ of $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ we get, $\frac{\frac{1}{a},\frac{1}{b},\frac{1}{c}}{3} \geq \frac{3}{a+b+c}=\frac{3}{6}=\frac{1}{2}$

So, $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \geq \frac{3}{2}$

Now,

$$\frac{\left(a+\frac{1}{b}\right)^2+\left(b+\frac{1}{c}\right)^2+\left(c+\frac{1}{a}\right)^2}{3} \geq \left(\frac{a+\frac{1}{b}+b+\frac{1}{c}+c+\frac{1}{a}}{3}\right)^2 \geq \left(\frac{6+\frac{3}{2}}{3}\right)^2=\left(\frac{5}{2}\right)^2=\frac{25}{4}$$

$$\therefore \left(a+\frac{1}{b}\right)^2+\left(b+\frac{1}{c}\right)^2+\left(c+\frac{1}{a}\right)^2 \geq \frac{75}{4}$$

83. (L-2)Given positive real numbers a, b and c such that a + b + c = 1, then maximum value of $a^a b^b c^c + a^b b^c a^a + a^c b^a c^b$ is

a) 1 b) 2 c) 3 d) 4

Key: a

Sol : Using the weighted AM – GM in equality we get,

$$\frac{c.a+a.b+b.c}{c+a+b} \geq \left(a^c b^a c^b\right)^{\frac{1}{a+b+c}}$$

$$\frac{b.a+c.b+a.c}{b+c+a} \geq \left(a^b.b^c.c^a\right)^{\frac{1}{a+b+c}}$$

$$\frac{a.a+b.b+c.c}{a+b+c} \geq \left(a^a b^b c^c\right)^{\frac{1}{a+b+c}}$$

Adding these inequalities together we get,

$$\frac{a^2+b^2+c^2+2(ab+bc+ca)}{a+b+c} \geq \left(a^a.b^b.c^c\right)+\left(a^cb^ac^b\right)+\left(a^bb^cc^a\right)[\because a+b+c=1]$$

$$1=\frac{(a+b+c)^2}{a+b+c} \geq \left(a^a.b^b.c^c\right)+\left(a^c.b^a.c^b\right)+\left(a^bb^cc^a\right)$$

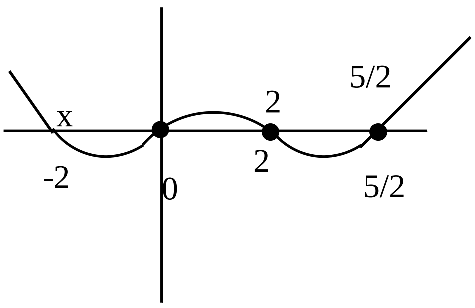
84. (L-2)The solution of $\left|\frac{x^2-5x+4}{x^2-4}\right| \leq 1$ is

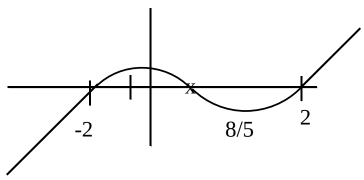
a) $\left[0,\frac{8}{5}\right] \cup \left[\frac{5}{2},+\infty\right)$ b) $\left[0,\frac{5}{8}\right] \cup \left[\frac{5}{2},+\infty\right)$ c) $\left[0,\frac{5}{8}\right] \cup \left[\frac{8}{5},\infty\right)$ d) None of these

Key:A

Hint: $-1 \leq \frac{x^2-5x+4}{x^2-4} \leq 1$

$\frac{x^2-5x+4}{x^2-4}+1 \geq 0$





$\frac{2x^2-5x}{x^2-4} \geq 0$

$\frac{x^2-5x+4}{x^2-4}-1 \leq 0$

$x\left(x-\pi_2\right)(x-2)(x+2) \geq 0$

$\frac{x^2-5x+4-x^2+4}{x^2-4} \leq 0$

$\frac{8-5x}{x^2-4} \leq 0$

$(8-5x)\left(x^2-4\right) \leq 0$

$(x+2)(5x-8)(x-2) \geq 0$

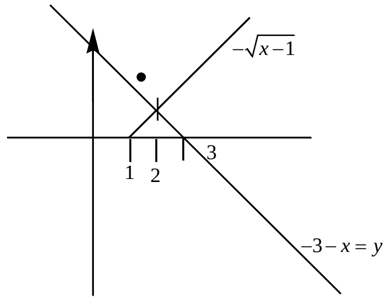
85. (L-2)Complete solution set of the inequation $\sqrt{x-1} \geq 3-x$ is

a) $2 \leq x \leq 5$ b) $2 \leq x \leq 3$

c) $1 \leq x \leq 3$ d) $x \leq 2$

Key: B

Hint:



86. (L-2) The least value of k such that the equation (ln x) + k = e^{x-k} has a solution is

a) e

b) $\frac{1}{e}$

c) 1

d) none of these

Key : c

Sol : $f(x)=e^{x-k}$ then inverse of $f(x)$; $f^{-1}(x)=(\ln x)+k$

and also both functions are increasing, therefore

$f(x)=f^{-1}(x)$ is equivalent to $f(x)=f^{-1}(x)=x$

$\Rightarrow \ln x+k=x$ should have a solution

$\Rightarrow k=x-\ln x$

Now, let g(x) = x – ln x

has least value 1 as $g'(x)=1-\frac{1}{x}$ has a minimum at x = 1

and $\lim_{x\to 0^+} g(x),\ \lim_{x\to\infty} g(x)$ both approach to ∞.

87. (L-2)f(x) be a polynomial of degree n and $f(x)=x^n f\left(\frac{1}{x}\right)$ then f(x) = 0

a) a reciprocal equation of second type

b) not a reciprocal equation

c) a reciprocal equation of first type

d) nothing can be say.

Key : c

Sol : Let $f(x)=a_0x^n+a_1x^{n-1}+....+a_n$

Then $x^n f\left(\frac{1}{x}\right)=x^n\left(\frac{a_0}{x^n}+\frac{a_1}{x^{n-1}}+....+a_n\right)$

$=a_0+a_1x+....+a_nx^n$

Since, $f(x) = x^n f\left(\frac{1}{x}\right)$,

$\therefore a_0 = a_n, a_1 = a_{n-1}, ..., a_n = a_0$

$\therefore$ f(x) = 0 is a reciprocal equation of first type.

88. (L-2)Reduced the equation $3x^6 + x^5 - 27x^4 + 27x^2 - x - 3 = 0$ in standard reciprocal form is

a) $3x^4 + x^3 - 24x^2 + x + 3 = 0$ b) $3x^4 + x^3 + 24x^2 + x + 3 = 0$

c) $3x^4 - x^3 + 24x^2 - x + 3 = 0$ d) none of these

Key ; a

Sol : $\therefore 3x^6 + x^5 - 27x^4 + 27x^2 - x - 3 = 0$

This can be written as,

$3(x^6 - 1) + x(x^4 - 1) - 27x^2(x^2 - 1) = 0$

or, $(x^2 - 1)\{3(x^4 + x^2 + 1) + x(x^2 + 1) - 27x^2\} = 0$

or, $(x^2 - 1)\{3x^4 - 24x^2 + x^3 + x + 3\} = 0$

or, $(x^2 - 1)\{3x^4 + x^3 - 24x^2 + x + 3\} = 0$

So, $3x^4 + x^3 - 24x^2 + x + 3 = 0$ is a reciprocal equation of even degree (i.e. 4) and first type

Hence it is standard form of reciprocal equation.

89. (L-2)The polynomial $x^3 - 3x^2 - 9x + c$ can be written in the form $(x - \alpha)^2 (x - \beta)$ if value of c is

a) 5 b) -7 c) 25 d) 27

Key: d

Sol: The polynomial $x^3 - 3x^2 - 9x + c$ can be written in the form of $(x - \alpha)^2 (x - \beta)$ if the equation $x^3 - 3x^2 - 9x + c = 0$ has two equal roots. Let these be α, α, β.

We have $\alpha + \alpha + \beta = 3$ or $2\alpha + \beta = 3$ …. (1)

$\alpha\alpha + \alpha\beta + \alpha\beta = -9$ or $2\alpha\beta + \alpha^2 = -9$ …. (2)

Putting value of β in (2) we get

$2\alpha(3 - 2\alpha) + \alpha^2 = -9$

or $6\alpha - 3\alpha^2 = -9$

$\Rightarrow \alpha^2 - 2\alpha - 3 = 0$

$\Rightarrow (\alpha - 3)(\alpha + 1) = 0 \Rightarrow \alpha = -1, 3$

When $\alpha = -1, \beta = 5$ and when $\alpha = 3, \beta = -3$. We also have $\alpha^2\beta = -c$

When $\alpha = -1, \beta = 5, c = -5$ when $\alpha = 3, \beta = -3, c = 27$

90. (L-1)The smallest positive value of p for which the equation cos (p sin α) = (p cos α) has a solution $\forall \alpha \in [0, 2\pi]$ is

a) $\frac{\pi}{\sqrt{2}}$ b) $\pi\sqrt{2}$ c) $\frac{\pi\sqrt{2}}{4}$ d) $\frac{\pi}{4\sqrt{2}}$

Key : c

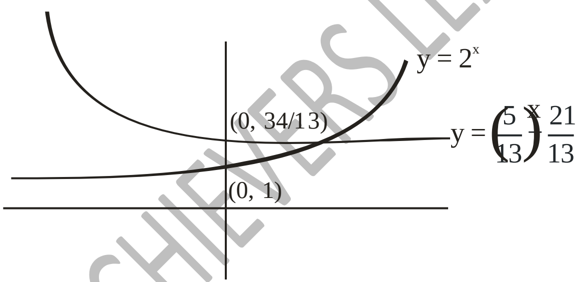
Sol : $\sin\left(\pi + \frac{\pi}{4}\right) = 1 \Rightarrow$ P is minimum

$\Rightarrow P = \frac{\pi}{2\sqrt{2}}$

91. The number of real roots of $\left(\frac{5}{13}\right)^x + \frac{21}{13} = 2^x$ is

(A) Two (B) Infinitely many
(C) only one (D) zero

Key. C



Sol.
Both graphs cut at only one point

92. For a non zero polynomial P, the equation $|P(x)| = e^x$ has

(A) At least one solution (B) No solution
(C) Exactly 2 solution (D) Exactly 1 solution

Key. A

Sol. $\lim_{x \to \infty} e^{-x} |P(x)| = 0$

and $\operatorname{Lt}_{x \to -\infty} e^{-x} |P(x)| = \infty$

consequently there is an $x_0 \in R$ such that $e^{-x_0} |P(x_0)| = 1$

93. Number of rational roots of the equation $\left|x^2 - 2x - 3\right| + 4x = 0$ is

a) 0 b) 1 c)2 d) 4

Key. B

Sol.
$x^2 - 2x - 3 \geq 0 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow x = -3$

$x^2 - 2x - 3 < 0 \Rightarrow x^2 - 6x - 3 = 0$ *no rational root*

94. If the equations $2x^2 - 7x + 1 = 0$ and $ax^2 + bx + 2 = 0$ have a common root, then

a) a=2,b=-7 b) $a = \frac{-7}{2}, b = 1$ c) a = 4, b = –14 d) a = –4, b = 1

Key. C

Sol. First equation has irrational roots.:. both roots common

95. If p,q,r I R and the quadratic equation $px^2 + qx + r = 0$ has no real root, then

a) $p(p+q+r) > 0$ b) $p(p+q+r) < 0$

c) $q(p+q+r) > 0$ d) $q(p+q+r) < 0$

Key. A

Sol. $p\left(px^2 + qx + r\right) > 0 \quad for \; x \in R.$ Take x=1

96. For $x^2 - (\alpha + 2)|x| + 9 = 0$ to have real solutions, the range of ‘α’ is

(A) $(-\infty, 4]$ (B) $[4, \infty)$

(C) $(-\infty, 7] \cup [11, \infty)$ (D) $[-4, \infty)$

Key. B

Sol. $\alpha = \frac{x^2 + 9}{|x|} - 2 = |x| + \frac{9}{|x|} - 2$

$\alpha \geq 4.$

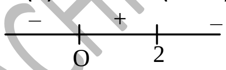
97. The number of solution(s) of the equations $e^x = x^2$ and $e^x = x^3$ are respectively

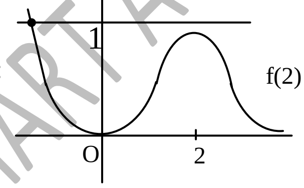
(A) 1 and 2 (B) 1 and 0

(C) 3 and 2 (D) 2 and 1

Key. A

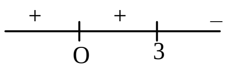
Sol. Let $f(x) = e^{-x} x^k$, $f'(x) = e^{-x} x^{k-1} (k - x)$

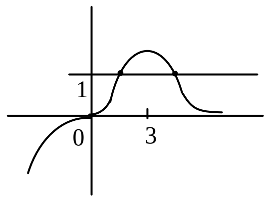
For k = 2, f′(x) : 

f(x) : 

$f(2) = \left(\frac{2}{e}\right)^2 < 1$

So, one solution.

For k = 3, f′(x) : 

f(x) : 

$f(3) = \left(\frac{3}{e}\right)^3 > 1$

So, two solutions.

98. If a,b,c,d are four positive numbers in G.P. then the minimum value of $\frac{c+d}{b}$ is

(A) $\frac{3b^{\frac{1}{3}}c^{\frac{1}{3}}+a^{2/3}}{a^{2/3}}$ (B) $\frac{3(bc)^{\frac{1}{3}}-2a^{2/3}}{a^{2/3}}$

(C) $\frac{3(bc)^{\frac{1}{3}}+3a^{2/3}}{a^{2/3}}$ (D) $\frac{3b^{\frac{1}{3}}c^{\frac{1}{3}}-a^{2/3}}{a^{2/3}}$

Key. D

Sol. Let b = ar, c = ar^2 , d = ar^3

$$\frac{c+d}{b}=r+r^2$$

$$\frac{3b^{\frac{1}{3}}c^{\frac{1}{3}}-a^{2/3}}{a^{2/3}}=3r-1$$

Since $(r-1)^2 \geq r^2-2r+1 \geq 0 \Rightarrow r^2+r \geq 3r-1 \Rightarrow \frac{c+d}{b} \geq \frac{3b^{\frac{1}{3}}c^{\frac{1}{3}}-a^{2/3}}{a^{2/3}}$

99. Three distinct positive real numbers a, b, c are in H.P. then for the quadratic equation $x^2 - kx + 2b^{101} - a^{101} - c^{101} = 0$, $k \in R$ has

(a) roots of same sign (b) roots of opposite sign

(c) roots of imaginary (d) roots are real and equal

Key. B

SOL. IF α, β ARE ROOTS

THEN $\alpha\beta = 2B^{101} - A^{101} - C^{101}$

NOW $\frac{a^{101}+c^{101}}{2} \geq (\sqrt{ac})^{101} \geq b^{101}$

$\Rightarrow$ $2B^{101} - A^{101} - C^{101} < 0$

$\Rightarrow$ $\alpha\beta < 0$

$\Rightarrow$ roots are opposite in sign.

100. If α and β, α and γ , α and δ are the roots of the equations $ax^2+2bx+c=0, 2bx^2+cx+a=0$ and $cx^2+ax+2b=0$ respectively where a, b, c are positive real numbers, then $\alpha+\alpha^2=$

a) -1 b) 1 c) 0 d) abc

Key. A

Sol. $a\alpha^2+2b\alpha+c=0$

$a+2b\alpha^2+c\alpha=0$ then $(a+2b+c)(1+\alpha+\alpha^2)=0$

$a\alpha+2b+c\alpha^2=0$

$\because$ a, b, c$\in R^+$ then $\alpha+\alpha^2=-1$

101. If a, b, c are in geometric progression and the roots of the equations $ax^2+2bx+c=0$ are α and β and those of $cx^2+2bx+a=0$ are γ and δ then

a) $\alpha \neq \beta \neq \gamma \neq \delta$ b) $\alpha \neq \beta$ and $\gamma \neq \delta$

c) $a\alpha = a\beta = c\gamma = c\delta$ d) $\alpha = \beta; \gamma \neq \delta$

Key. C

Sol. $\because b^2 = ac;$ the roots of both the equations are equal.

$\therefore \alpha = \beta;$ and $\gamma = \delta.$ But $\gamma = \frac{1}{\alpha}:\ \delta = \frac{1}{\beta}$ as the given equations are reciprocal to each other

$$\therefore \gamma\delta = \frac{a}{c} \text{ then } c\gamma = a\beta$$

$$a\alpha = a\beta = c\gamma = c\delta$$

102. If $f(x) = (x^2+3x+2)(x^2-7x+a)$ and $g(x) = (x^2-x-12)(x^2+5x+b)$ then the values of a and b, If $(x+1)(x-4)$ is HCF of $f(x)$ and $g(x)$

a) $a=10:b=6$ b) $a=4:b=12$
c) $a=12:b=4$ d) $a=6:b=10$

Key. C

Sol. x^2-7x+a is divisible by $x-4\ \&\ x^2+5x+b$ is divisible by $x+1$
$\therefore a=12; b=4$

103. The equation $(x^2+3x+4)^2+3(x^2+3x+4)+4=x$ has

a) all its solutions real but not all positive b) only two of its solutions real
c) two of its solutions positive and two negative d) none of solutions real.

Key. D

Sol. $f(x)=ax^2+bx+c$: If $f(x)=x$ has no real solution then $f(f(x))=x$ also has no real solution:

104. Let A be a square Matrix all of whose entries are integers. Then which of the following is True?

a) If $\det A=\pm1,$ then A^{-1} exists but all it entries are not necessarily integers.
b) If $\det A\neq\pm1,$ then A^{-1} exists and all it entries are non integers.
c) If $\det A=\pm1$, then A^{-1} exists ad all its entries are integers.
d) If $\det A=\pm1$, A^{-1} need not exist.

Key. C

Sol. Conceptual

105. The values of a for which the roots of the equation $(a+1)x^2-3ax+4a=0\,(a\neq-1)$ are real and greater than 1

a) $\left[-\frac{10}{7},1\right]$ b) $\left[-\frac{12}{7},0\right]$ c) $\left[-\frac{16}{7},-1\right)$ d) $\left(-\frac{16}{7},0\right)$

Key. C

Sol. $D=9a^2-16a(a+1)\geq 0, x_1>1, x_2>1$

Where $x_1+x_2=\frac{3a}{a+1}, x_1x_2=\frac{4a}{a+1} \Rightarrow x_1+x_2-1>0\ \&\ (x_1-1)(x_2-1)>0$

$$\Rightarrow a(7a+16)\leq 0 \qquad (1)$$

$$\frac{a-2}{a+1}>0 \qquad (2)$$

$$\frac{2a+1}{a+1}>0 \qquad (3)$$

Solving $-\frac{16}{7}\leq a<-1.$

106. If the equation $x^4-4x^3+ax^2+bx+1=0$ has four positive roots then (a, b) is given by
(A) (4, 6) (B) (6, –4)

(C) (–4, –6) (C) (2, 3)

Key. B

Sol. Let the roots of the equation be x_1, x_2, x_3, x_4 then $x_1 + x_2 + x_3 + x_4 = 4$
and $x_1 x_2 x_3 x_4 = 1$
As A.M ≥ G.M and equality sign holds only when numbers are equal.
We have $1 = \frac{x_1 + x_2 + x_3 + x_4}{4} \geq (x_1 x_2 x_3 x_4)^{\frac{1}{4}} = 1$
$\Rightarrow x_1 = x_2 = x_3 = x_4 = 1$
$\Rightarrow x^4 - 4x^3 + ax^2 + bx + 1 = (x-1)^4 \Rightarrow a = 6, b = -4$.

107. If roots of the equation $ax^2 + bx + c = 0$; $a, b, c \in R^+$ be non–real numbers, lying inside the unit circle, centered at origin, then

(A) $b > 0$ (B) $b < a$
(C) $c < a$ (D) none of these

Key. C

Sol. Let z_1 be one of the root, then the other root is $\bar{z}_1$

$|z_1|^2 = \frac{c}{a} \Rightarrow \frac{c}{a} < 1 \Rightarrow c < a$

108. If both the roots of the equation $x^2 + 2bx + \log_3(b^2 - 4b + 4) = 0$ are of opposite sign then 'b' belongs to

(A) (1, 3) (B) $(-\infty, 1) \cup (3, \infty)$
(C) [1, 3] (D) $(1, 2) \cup (2, 3)$

Key. D

Sol. Let $f(x) = x^2 + 2bx + \log_3(b^2 - 4b + 4)$
For both roots to be of opposite sign
$f(0) < 0 \Rightarrow \log_3(b^2 - 4b + 4) < 0$
$\Rightarrow b^2 - 4b + 4 < 1$
$\Rightarrow b^2 - 4b + 3 < 0$
$\Rightarrow (b-1)(b-3) < 0 \Rightarrow 1 < b < 3$
But $b \neq 2$
$\therefore b \in (1, 2) \cup (2, 3)$.

109. Let $f(x) = x^3 + ax^2 + bx + c$ and x_1, x_2 be the roots of $f'(x) = 0$, if $x_1 < x_2$ then $f(x) = 0$ will have

a) No real root if $f(x_1) < 0$ or $f(x_2) > 0$

b) Only one real root if $f(x_1) < 0$ or $f(x_2) > 0$

c) Three real roots if $f(x_1) < 0$ or $f(x_2) > 0$

d) cannot say any thing

Key. B

Sol. Since coefficient of x^3 is Positive .

$\therefore$ local maximum is at x_1 and local minimum is at x_2 . case (i) : If $f(x_1) < 0$ then

$f(x_2) < f(x_1) < 0$ then the only real root will be in (x_2, ∞) case (ii) : If $f(x_2) > 0$ then

$f(x_1) > f(x_2) > 0$ then equation will have only one real root in the interval $(-\infty, x)$.

110. Let $f_1(x)$ and $f_2(x)$ be continuous and differentiable functions. If

$f_1(0)=f_1(2)=f_1(4), f_1(1)+f_1(3)=f_2(0)=f_2(2)=f_2(4)=0$ and if $f_1(x)=0$ and

$f_2^1(x)=0$ do not have common root, then the minimum number of zeros of,

$f_1^1(x)f_2^1(x)+f_1(x)f_2^{11}(x)$ in $[0,4]$, is

a) 2 b) 4 c) 5 d) 3

Key. D

Sol. $f_1(x)=0$ has mini two sols in $[0,4]$

$f_2(x)=0$ has mini 3 sols in [0,4]

$f_2^1(x)=0$ has mini 2 sol in [0,4]

$f_1(x)f_2^1(x)=0$ has minimum 4 sols in [0,4]

$\frac{d}{dx}\left(f_1(x)f_2^1(x)\right)=0$ has mini 3 sols in [0.4]

111. For $x^2-(\alpha+2)|x|+9=0$ to have real solutions, the range of 'α' is

(A) $[-\infty, 4]$ (B) $[4, \infty)$

(C) $(-\infty, 7] \cup [11, \infty)$ (D) $[-4, \infty)$

Key. B

Sol. $\alpha=\frac{x^2+9}{|x|}-2=|x|+\frac{9}{|x|}-2$

$\Rightarrow \alpha \geq 4.$

112. $0<c<b<a$ and α, β are roots of equation $cx^2+bx+a=0$ if α, β are non real then

(A) $\frac{|\alpha|+|\beta|}{2}=|\alpha||\beta|$ (B) $\frac{2}{|\alpha|}=\frac{1}{|\beta|}$

(C) $\frac{1}{|\alpha|}+\frac{1}{|\beta|}<2$ (D) $|\alpha|+\frac{1}{|\beta|}<2$

Key. C

SOL. $\alpha\beta=\frac{a}{c}>1$

$|\alpha||\beta|>1$

$\Rightarrow |\alpha|^2>1$

$\Rightarrow |\alpha|>1$

$\Rightarrow |\beta|>1$

$\Rightarrow \frac{1}{|\alpha|}+\frac{1}{|\beta|}<2$

113. If two roots of the equation $(P-1)(x^2+x+1)^2-(p+1)(x^4+x^2+1)=0$ are real and distinct and $f(x)=\frac{1-x}{1+x}$ then $f(f(x))+f\left(f\left(\frac{1}{x}\right)\right)$ is equal to ___

a) P b) $-P$ c) $2P$ d) $-2P$

Key. A

Sol. $\frac{p+1}{p-1}=\frac{x^2+x+1}{x^2-x+1} \quad \Rightarrow \frac{2p}{2}=\frac{2\left(x^2+1\right)}{2x} \Rightarrow p=x+\frac{1}{x}$

As $f\left(x\right)=\frac{1-x}{1+x} \Rightarrow f\left(f\left(x\right)\right)+f\left(f\left(\frac{1}{x}\right)\right)=x+\frac{1}{x}$

$\Rightarrow f\left(f\left(x\right)\right)+f\left(f\left(\frac{1}{x}\right)\right)=p$

114. If α_1, α_2, …. α_n are roots of the equation $x^n + ax + b = 0$, then $(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)(\alpha_1-\alpha_4)\ldots(\alpha_1-\alpha_n)$ is equal to

(A) n (B) $n\alpha_1^{n-1}$

(C) $n\alpha_1 + b$ (D) $n\alpha_1^{n-1}+a$

Key. D

Sol. $x^n + ax + b = (x-\alpha_1)(x-\alpha_2)\ldots.(x-\alpha_n)$

differentiate both sides w.r.t. x

$nx^{n-1} + a = (x-\alpha_2)\ldots(x-\alpha_n) + (x-\alpha_1)\left(\frac{d}{dx}(x-\alpha_2)\ldots(x-\alpha_n)\right)$

put $x = \alpha_1$

$n\alpha_1^{n-1}+a = (\alpha_1-\alpha_2)(\alpha_1-\alpha_3)\ldots(\alpha_1-\alpha_n)$

115. ω is a non real complex cube root of unity and $a,b\in R$ .If ω,ω^2 are roots of $\frac{1}{a+x}+\frac{1}{b+x}=\frac{3}{x}$ then *a,b* are roots of

a) $3x^2-6x+2=0$ b) $6x^2-3x+2=0$

c) $2x^2-3x+6=0$ d) $6x^2-2x+3=0$

Key. B

Sol. The given equation simplifies $x^2+2x\left(a+b\right)+3ab=0$, whose roots are given table ω,ω^2

Hence $a+b=\frac{1}{2},ab=\frac{1}{3}$ .So a,b are roots of $x^2-x\left(\frac{1}{2}\right)+\frac{1}{3}=0$

116. If the function $f(x)=x^3+3(a-7)x^2+3(a^2-9)x-1$ has a point of maximum at positive values of x then

(a) $a\in\left(-\infty,\frac{29}{7}\right)$ (b) $a\in\left(-\infty,7\right)$

(c) $a\in\left(-\infty,-3\right)\cup\left(3,\frac{29}{7}\right)$ (d) $a\in\left(3,\infty\right)\cup\left(-\infty,-3\right)$

Key. C

Sol. $f(x)=x^3+3(a-7)x^2+3(a^2-9)x-1$

$f^{'}(x)=3x^2+6(a-7)x+3(a^2-7)$

The roots of $f'(x)=0$ positive and distinct which is possible if

(i) $b^2-4ac>0 \Rightarrow 6(a-7)^2-4(3)(3)(a^2-9)>0$

$\Rightarrow a < \frac{29}{7}$

(ii) Product of Roots > 0 $a^2 - 9 > 0$

(iii) Sum of Roots > 0 $a - 7 < 0$

$a < 7$

$\Rightarrow$ From i, ii, iii $a \in (-\infty, -3) \cup (3, \frac{29}{7})$

117. If α, β are the roots of $x^2 - px + q = 0$ then value of $\frac{\alpha^2 + \beta^2}{\alpha^{-2} + \beta^{-2}} =$

(A) p (B) q (C) p^2 (D) q^2

Key. D

Sol. $\alpha^2\beta^2 = q^2$

118. For $p > 0$ and $3x^2 + px + 3 = 0$ one root of above equation is square of the other then p is

(A) – 6 (B) 10 (C) 2 (D) 3

Key. D

Sol. $\alpha + \alpha^2 = \frac{-1}{3}; \alpha^3 = 1$

$\alpha = 1, \omega, \omega^2$

If $\alpha = 1$

P=-6 as P>0 neglected

$if \alpha = \omega; P = 3$

119. If one root or the equation $x^2 - 2x + k = 0$ is 1+2i and $k \in R$ then the value of k is

(A) -3 (B) -5 (C) 5 (D) 3

Key. C

Sol. $b^2 = 4ac \Rightarrow 4m^2 = 4(8m - 15)$

$m^2 - 8m + 15 = 0; m = +3, +5$

120. If $\left|\frac{12x}{4x^2+9}\right| \le 1$ then

(A) $x \in R$ (B) $x \in \phi$

(C) $x \in \{1\}$ (D) $x \in C$ where C is set of complex numbers

Key. A

Sol. $12|x| \le 4x^2 + 9$

$(2x-3)^2 \ge 0$; $x \in R$

121. If α, β are roots of $3x^2 + 2bx + c = 0$ whose descriminant is $\Delta_1; \alpha + \delta, \beta + \delta$ are roots of $9x^2 + 2Bx + C = 0$ whose descriminant is Δ_2 then $\frac{\Delta_1}{\Delta_2}$ is

(A) $\frac{1}{9}$ (B) 9 (C) 3 (D) $\frac{1}{3}$

Key. A

Sol. $\alpha-\beta=\frac{\sqrt{\Delta_1}}{3}$

$(\alpha+\delta)-(\beta+\delta)=\frac{\sqrt{\Delta_2}}{9}$

$\frac{\Delta_1}{9}=\frac{\Delta_2}{81};\frac{\Delta_1}{\Delta_2}=\frac{1}{9}$

122. If the sum of the roots of the equation $5x^2-4x+2+k\left(4x^2-2x-1\right)=0$ is 6 , then k =

(A) $13/17$ (B) $17/13$ (C) $-17/13$ (D) $-13/11$

Key. D

Sol. sum of the roots =6

$\frac{2k+4}{5+4k}=6 => k=\frac{-13}{11}$

123. If $\tan\alpha$, $\tan\beta$, $\tan\gamma$ are the roots of the equation $x^3-px^2-r=0$ then the value of $\left(1+\tan^2\alpha\right)\left(1+\tan^2\beta\right)\left(1+\tan^2\gamma\right)$ is equal to

a) $(p-r)^2$ b) $1+(p-r)^2$ c) $1-(p-r)^2$ d) none

Key. B

Sol. $\left(1+\tan^2\alpha\right)\left(1+\tan^2\beta\right)\left(1+\tan^2\gamma\right)$

$=1+\left(\tan^2\alpha+\tan^2\beta+\tan^2\gamma\right)+\left(\tan^2\alpha\tan^2\beta+\tan^2\beta\tan^2\gamma+\tan^2\gamma\tan^2\alpha\right)+\tan^2\alpha\tan^2\beta\tan^2\gamma$

$=1-(p-r)^2$ $\quad\because x^2y^2+y^2z^2+z^2x^2$

$=(xy+yz+zx)^2-2xyz(x+y+z)$

124. If the equation $x^2+9y^2-4x+3=0$ is satisfied for real values of x and y then

A) $x\in[1,3], y\in[1,3]$ B) $x\in[1,3], y\in\left[\frac{-1}{3},\frac{1}{3}\right]$

C) $x\in\left[\frac{-1}{3},\frac{1}{3}\right], y\in[1,3]$ D) $x\in\left[\frac{-1}{3},\frac{1}{3}\right], y\in\left[\frac{-1}{3},\frac{1}{3}\right]$

Key. B

Sol. (B)Given equation is $x^2+9y^2-4x+3=0$ …(i)

Or, $x^2-4x+9y^2+3=0.$

Since x is real $\therefore$ $(-4)^2-4\left(9y^2+3\right)\geq 0$

Or, $16-4\left(9y^2+3\right)\geq 0$ or, $4-9y^2-3\geq 0$

Or, $9y^2-1\leq 0$ or, $9y^2\leq 1$ or, $y^2\leq\frac{1}{9}$

Now $y^2\leq\frac{1}{9}\Leftrightarrow-\frac{1}{3}\leq y\leq\frac{1}{3}$ …(ii)

Equation (i) can also be written as

$$9y^2+0y+x^2-4x+3=0 \qquad ...(iii)$$

Since y is real $\therefore 0^2-4.9\left(x^2-4x+3\right)\ge 0$

Or, $x^2-4x+3\le 0$

$\Rightarrow x\in[1,3]$

125. The equation $a_8x^8+a_7x^7+a_6x^6+...+a_0=0$ has all its roots positive and real $\left(where\ a_8=1, a_7=-4, a_0=1/2^8\right)$, then

A) $a_1=\frac{1}{2^8}$ B) $a_1=-\frac{1}{2^4}$ C) $a_2=\frac{7}{2^5}$ D) $a_2=\frac{7}{2^8}$

Key. B

Sol. (B) Let the roots be $\alpha_1,\alpha_2,....,\alpha_8$

$\Rightarrow \quad \alpha_1+\alpha_2+....+\alpha_8=4$

$\alpha_1\alpha_2.....\alpha_8=\frac{1}{2^8}$

$\Rightarrow \quad \left(\alpha_1\alpha_2.....\alpha_8\right)^{1/8}=\frac{1}{2}=\frac{\alpha_1+\alpha_2+...+\alpha_8}{8}$

$\Rightarrow$ AM=GM$\Rightarrow$ all the roots are equal to $\frac{1}{2}$.

$\Rightarrow \quad a_1=-{}^8C_7\left(\frac{1}{2}\right)^7=-\frac{1}{2^4}$

$a_2={}^8C_6\left(\frac{1}{2}\right)^6=\frac{7}{2^4}$

$a_3=-{}^8C_5\left(\frac{1}{2}\right)^5$

126. If a, b, c are positive numbers such that a>b>c and the equation $(a+b-2c)x^2+(b+c-2a)x+(c+a-2b)=0$ has a root in the interval $(-1,0)$, then

A) b cannot be the G.M. of a, c B) b may be the G.M. of a, c

C) b is the G.M. of a, c D) none of these

Key. A

Sol. Let $f(x)=(a+b-2c)x^2+(b+c-2a)x+(c+a-2b)$

According to the given condition, we have

$f(0)f(-1)<0$

i.e. $(c+a-2b)(2a-b-c)<0$

i.e. $(c+a-2b)(a-b+a-c)<0$

i.e. $c+a-2b<0 \qquad [a>b>c,\ given \Rightarrow a-b>0, a-c>0]$

i.e. $b>\frac{a+c}{2}$

$\Rightarrow$ b cannot be the G.M. of a, c, since G.M < A.M. always.

127. Let α, β (a < b) be the roots of the equation $ax^2+bx+c=0$. If $\lim_{x\to m}\frac{|ax^2+bx+c|}{ax^2+bx+c}=1$, then

A) $\frac{|a|}{a}=-1, m<\alpha$ B) $a>0, \alpha<m<\beta$ C) $\frac{|a|}{a}=1, m>\beta$ D) $a<0, m>\beta$

Key. C

Sol. According to the given condition, we have

$$|am^2+bm+c|=am^2+bm+c$$

i.e. $am^2+bm+c>0$

$\Rightarrow$ if *a< 0,* the *m* lies in (α,β)

and if *a>0,* then *m* does not lies in (α,β)

Hence, option (c) is correct, since

$$\frac{|a|}{a}=1\Rightarrow a>0$$

And in that case *m* does not lie in (α,β).

128. Let $f(x)$ be a function such that $f(x)=x-[x]$, where $[x]$ is the greatest integer less than or equal to x. Then the number of solutions of the equation $f(x)+f\left(\frac{1}{x}\right)=1$ is (are)

A) 0 B) 1 C) 2 D) infinite

Key. D

Sol. Given, $f(x)=x-[x],\ x\in R-\{0\}$

Now $f(x)+f\left(\frac{1}{x}\right)=1$ $\therefore$ $x-[x]+\frac{1}{x}-\left[\frac{1}{x}\right]=1$

$\Rightarrow\left(x+\frac{1}{x}\right)-\left([x]+\left[\frac{1}{x}\right]\right)=1$ $\Rightarrow\left(x+\frac{1}{x}\right)=[x]+\left[\frac{1}{x}\right]+1$ ...(i)

Clearly ,R.H.S is an integer $\therefore$ L. H. S. is also an integer

Let $x+\frac{1}{x}=k$ an integer $\Rightarrow x^2-kx+1=0$

$$\therefore x=\frac{k\pm\sqrt{k^2-4}}{2}$$

For real values of *x*, $k^2-4\ge0\Rightarrow k\ge2$ or $k\le-2$

We also observe that *k=2* and -2 does not satisfy equation *(i)*

$\therefore$ The equation *(i)* will have solutions if $k>2$ or $k<-2$,where $k\in z$.

Hence equation *(i)* has infinite number of solutions.

129. If both the roots of $(2a-4)9^x-(2a-3)3^x+1=0$ are non-negative, then

A) $0<a<2$ B) $2<a<\frac{5}{2}$ C) $a<\frac{5}{4}$ D) $a>3$

Key. B

Sol. Putting $3^x=y$, we have

$$(2a-4)y^2-(2a-3)y+1=0$$

This equation must have real solution

$\Rightarrow \quad (2a-3)^2 - 4(2a-4) \geq 0$

$\Rightarrow \quad 4a^2 - 20a + 25 \geq 0$

$\Rightarrow \quad (2a-5)^2 \geq 0.$ This is true.

$y = 1$ satisfies the equation

Since 3^x is positive and $3^x \geq 3^0$, $y \geq 1$

Product of the roots $= 1 \times y > 1$

$\Rightarrow \quad \frac{1}{2a-4} > 1$

$\Rightarrow \quad 2a - 4 < 1 \Rightarrow \ a < \frac{5}{2}$

Sum of the roots $= \frac{2a-3}{2a-4} > 1$

$\Rightarrow \quad \frac{(2a-3)-(2a-4)}{2a-4} > 0$

$\Rightarrow \quad \frac{1}{2a-4} > 0 \Rightarrow a > 2$

$\Rightarrow \quad 2 < a < \frac{5}{2}$

130. Let α and β be the roots of $x^2 - 6x - 2 = 0$ with $\alpha > \beta$ if $a_n = \alpha^n - \beta^n$ for $n \geq 1$ then the value of $\frac{a_{10} - 2a_8}{3a_9} =$

1) 1 2) 2 3) 3 4) 4

Key. 2

Sol. $\alpha^2 - 6\alpha - 2 = 0$ $\beta^2 - 6\beta - 2 = 0$

$\Rightarrow \alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0$ ....(1)

$\Rightarrow \beta^{10} - 6\beta^9 - 2\beta^8 = 0$ ....(2)

subtract (2) from (1)

131. If a, b, c are positive real numbers such that $a + b + c = 1$ then the least value of $\frac{(1+a)(1+b)(1+c)}{(1-a)(1-b)(1-c)}$ is

1) 16 2) 8 3) 4 4) 5

Key. 2

Sol. $a = 1 - b - c$

$\Rightarrow 1 + a = (1-b) + (1-c) \geq 2\sqrt{(1-b)(1-c)}$

$\therefore (1+a)(1+b)(1+c) \geq 8(1-a)(1-b)(1-c)$

132. The range of values of $'a'$ for which all the roots of the equation $(a-1)(1+x+x^2)^2 = (a+1)(1+x^2+x^4)$ are imaginary is

1) $(-\infty,-2]$ 2) $(2,\infty)$

3) $(-2,2)$ 4) $[2,\infty)$

Key. 3

Sol. The given equation can be written as $\left(x^2+x+1\right)\left(x^2-ax+1\right)=0$

133. If α,β are the roots of the equation $ax^2+bx+c=0$ and $S_n=\alpha^n+\beta^n$ then $aS_{n+1}+bS_n+cS_{n-1}=\ \ (n\geq 2)$

1) 0 2) $a+b+c$

3) $(a+b+c)n$ 4) $n^2\ abc$

Key. 1

Sol. $S_{n+1}=\alpha^{n+1}+\beta^{n+1}$

$=(\alpha+\beta)\left(\alpha^n+\beta^n\right)-\alpha\beta\left(\alpha^{n-1}+\beta^{n-1}\right)$

$= -\frac{b}{a}.S_n-\frac{c}{a}.S_{n-1}$

134. A group of students decided to buy a Alarm Clock priced between Rs. 170 to Rs 195. But at the last moment, two students backed out of the decision so that the remaining students had to pay 1 Rupee more than they had planned. If the students paid equal shares, the price of the Alarm Clock is

1) 190 2) 196

3) 180 4) 171

Key. 3

Sol. Let cost of clock = x

number of students $=n$

then $\frac{x}{n-2}=\frac{x}{n}+1\Rightarrow x=\frac{n^2-2n}{2}$

$\Rightarrow 170\leq\frac{n^2-2n}{2}\leq 195$

135. If $\tan A,\tan B$ are the roots of $x^2-Px+Q=0$ the value of $\sin^2(A+B)=$ $(where\ P,Q\in R)$

1) $\frac{P^2}{P^2+(1-Q)^2}$ 2) $\frac{P^2}{P^2+Q^2}$

3) $\frac{Q^2}{P^2+(1-Q)^2}$ 4) $\frac{P^2}{(P+Q)^2}$

Key. 1

Sol. $\tan(A+B)=\frac{P}{1-Q}$ then $\sin^2(A+B)=\frac{\tan^2(A+B)}{1+\tan^2(A+B)}$

136. The number of solutions of $\left|[x]-2x\right|=4$ where $[x]$ is the greatest integer $\leq x$ is

1) 2 2) 4

3) 1 4) Infinite

Key. 2

Sol. If $x = n \in Z, \quad |n-2n| = 4 \Rightarrow n = \pm 4$

If $x = n + K$ where $0 < K < 1$ then $|n-2(n+k)| = 4$, it is possible if $K = \frac{1}{2}$

$\Rightarrow |-n-1| = 4$

$\therefore n = 3, -5$

137. Let a, b and c be real numbers such that $a+2b+c=4$ then the maximum value of $ab+bc+ca$ is

1) 1 2) 2 3) 3 4) 4

Key. 4

Sol. Let $ab+bc+ca = x$

$\Rightarrow 2b^2 + 2(c-2)b - 4c + c^2 + x = 0$

Since $b \in R$,

$\therefore c^2 - 4c + 2x - 4 \le 0$

Since $c \in R$

$\therefore x \le 4$

138. For the equation $3x^2 + px + 3 = 0, p > 0$, if one root is the square of the other then value of P is

1) $\frac{1}{3}$ 2) 1 3) 3 4) $\frac{2}{3}$

Key. 3

Sol. $\alpha + \alpha^2 = -\frac{p}{3}$

$\alpha^3 = 1$

139. If the equations $2x^2 + kx - 5 = 0$ and $x^2 - 3x - 4 = 0$ have a common root, then the value of k is

1) -2 2) -3

3) $\frac{27}{4}$ 4) $-\frac{1}{4}$

Key. 2

Sol. If 'α' is the common root then $2\alpha^2 + k\alpha - 5 = 0,\ \alpha^2 - 3\alpha - 4 = 0$ solve the equations.

140. If α and β are the roots of the equation $x^2 - x + 1 = 0$ then $\alpha^{2009} + \beta^{2009} =$

1) 1 2) 2

3) -1 4) -2

Key. 1

Sol. $x=\dfrac{1\pm i\sqrt{3}}{2}$

$\therefore \alpha=-\omega,\ \beta=-\omega^2$

141. If $P(Q-r)x^2+Q(r-P)x+r(P-Q)=0$ has equal roots then $\dfrac{2}{Q}=$

$(where\ P,Q,r\in R)$

1) $\dfrac{1}{P}+\dfrac{1}{r}$ 2) $\dfrac{1}{P}-\dfrac{1}{r}$

3) $P+r$ 4) Pr

Key. 1

Sol. Product of the roots $=1$

142. The solution of the differential equation $y_1y_3=3y_2^2$ is

1) $x=A_1y^2+A_2y+A_3$ 2) $x=A_1y+A_2$

3) $x=A_1y^2+A_2y$ 4)none of these

Key. 1

Sol. $y_1y_3=3y_2^2$

$\dfrac{y_3}{y_2}=3\dfrac{y_2}{y_1}\Rightarrow \ln y_2=3\ln y_1+\ln c$

$y_2=cy_1^3$

$\dfrac{y_2}{y_1^2}=cy_1$

$-\dfrac{1}{y_1}=cy+c_2$

$\dfrac{dx}{dy}=-cy-c_2$

$x=-\dfrac{cy^2}{2}-c_2y+c_3$

$\therefore x=A_1y^2+A_2y+A_3$

143. If $(1+K)\tan^2 x-4\tan x-1+K=0$ has real roots $\tan x_1$ and $\tan x_2$ then

1) $k^2\le 5$ 2) $k^2\ge 6$

3) $k=3$ 4) $k>10$

Key. 1

Sol. Discriminate ≥ 0

144. Let $f(x)$ be a real valued function satisfying $a.f(x)+b\,f(-x)=px^2+qx+r,\forall x\in R$. Where $p,q,r\in R-\{0\}$ and $a,b\in R$ such that $|a|\ne|b|$. Then the condition that $f(x)=0$ will have real roots is

A) $\left(\frac{a+b}{a-b}\right)^2 \le \frac{q^2}{4pr}$ B) $\left(\frac{a+b}{a-b}\right)^2 \le \frac{4pr}{q^2}$

C) $\left(\frac{a+b}{a-b}\right)^2 \ge \frac{q^2}{4pr}$ D) $\left(\frac{a+b}{a-b}\right)^2 \ge \frac{4pr}{q^2}$

Key. D

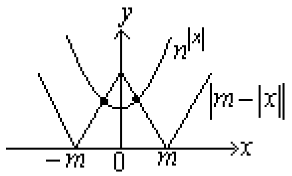
Sol. Using hypothesis we get $f(x)-f(-x)=\frac{2qx}{a-b}$

145. The number of solutions of the equations $n^{-|x|}.\left|m-|x|\right|=1$(where $m,n>1\&n>m$) is

A) 0 B) 1 C) 2 D)4

Key. C

Sol.



•+• = two solutions

146. The values of 'a' for which the equation $x^3+ax+1=0$ and $x^4+ax^2+1=0$ have a common root

A) 2 B) –2 C) 0 D) 1

Key. B

Sol. Let α be a common root

Then $\alpha^3+a\alpha+1=0$ --- (1)

And $\alpha^4=a\alpha^2+1=0$ --- (2)

$\alpha\times(1)-(2)\Rightarrow\alpha-1=0\Rightarrow\alpha=1$

So, from $x^3+ax+1=0\Rightarrow 1+a+1=0\Rightarrow a=-2$

147. If the roots of the equation $ax^2+bx+c=0$ are of the form $\frac{\alpha}{\alpha-1}$ and $\frac{\alpha+1}{\alpha}$, then value of $(a+b+c)^2$ is

A) $2b^2-ac$ B) b^2-2ac D) b^2-4ac D) $4b^2-2ac$

Key. C

Sol. By hypothesis $\frac{\alpha}{\alpha-1}+\frac{\alpha+1}{\alpha}=-\frac{b}{a}$ and $\frac{\alpha}{\alpha-1}.\frac{\alpha+1}{\alpha}=\frac{c}{a}$

$\Rightarrow\frac{2\alpha^2-1}{\alpha^2-\alpha}=-\frac{b}{a}$ and $\alpha=\frac{c+a}{c-a}$

$\Rightarrow(c+a)^2+2b(c+a)+b^2=b^2-4ac\Rightarrow(a+b+c)^2=b^2-4ac$

148. The value of a , for which one root of the equation $(a-5)x^2-2ax+(a-4)=0$ is smaller than 1 and the other greater than 2 is ____________

A) $a\in(5,24)$ B) $a\in\left(\frac{20}{3},\infty\right)$ C) $a\in(5,\infty)$ D) $a\in(-\infty,\infty)$

Key. A

Sol. (i) $D>0$

$4a^2-4(a-5)(a-4)>0$

$9a-20>0 \Rightarrow a>\frac{20}{9} \Rightarrow a \in \left(\frac{20}{9}, \infty\right)$ --- (1)

(ii) $(a-5)f(1)<0;\ (a-5)f(2)<0$

$\Rightarrow (a-5)(a-5-2a+a-4)<0$

$\Rightarrow a>5 \Rightarrow a \in (5,\infty)$ --- (2)

and $(a-5)\{(a-5).4-4a+a-4\}<0$

$\Rightarrow (a-5)(a-24)<0 \Rightarrow 5<a<24$

$\Rightarrow a \in (5,24)$ --- (3)

Using (1) , (2) & (3)

The common condition is $a \in (5,24)$

149. If the equations $ax^2-2bx+c=0, bx^2-2cx+a=0$ and $cx^2-2ax+b=0$ have only positive roots then

A) $a>b>c$ B) $a<b<c$ C) $a=b=c$ D) $a>b; b<c$

Key. C

Sol. Roots of equation $ax^2-2bx+c=0$ are +ve then discriminent $\geq 0 \Rightarrow b^2 \geq ac$

Sum of roots $=\frac{b}{a}>0$, product of roots $=\frac{c}{a}>0$

Similarly for other two equations, we get $c^2 \geq ab \Rightarrow \frac{c}{b}>0, \frac{a}{b}>0$ and

$a^2 \geq bc \Rightarrow \frac{a}{c}>0 \& \frac{b}{c}>0$

Using above conditions a,b,c are all +ve (or) all are –ve.

Multiplying we get $c^2a^2 \geq ab^2c$

$\Rightarrow ac(b^2-ac) \leq 0 \Rightarrow b^2-ac \leq 0 (\because ac>0)$

Also $a^2-bc \leq 0$ & $c^2-ab \leq 0$

And all, we get $a^2+b^2+c^2-ab-bc-ca \leq 0$

$\Rightarrow \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]=0$

$3x^2+px+3=0, p>0,$ (or) P (or) $\frac{1}{3}$

$\alpha+\alpha^2=-\frac{p}{3}$

150. If α is a root of $ax^2+bx+c=0$; β is a root fo $-ax^2+bx+c=0$ and γ is a root of $ax^2+2bx+2c=0$ then

A) $\gamma<\alpha<\beta$ B) $\alpha<\beta<\gamma$ C) $\alpha<\gamma<\beta$ D) $\frac{\alpha}{\beta}<\gamma<\frac{\beta}{\alpha}$

Key. C

Sol. Let $f(x)=ax^2+2bx+2c$

Then, we have $f(\alpha)=a\alpha^2+2b\alpha+2c=-a\alpha^2+2(a\alpha^2+b\alpha+c)$

$=-a\alpha^2$ [$\because \alpha$ is a root of $ax^2+bx+c=0. \therefore a\alpha^2+b\alpha+c=0$]

Also we have, $f(\beta)=a\beta^2+2b\beta+2c=3a\beta^2+2(-a\beta^2+b\beta+c)$

$=3a\beta^2$ [$\because \beta$ is a root of $-ax^2+bx+c=0. \therefore a^2\beta-b\beta-c=0$]

Now. $f(\alpha)f(\beta) = -3a^2\alpha^2\beta^2 < 0$ which implies that $f(\alpha), f(\beta)$ are of opposite signs and hence, proves that the curve represented by $y = f(x)$ cuts the X-axis somewhere between α and β.

In other words $f(x) = 0$ has a root lying between α and β.

151. If for any real x, we have $-1 \le \frac{x^2 + nx - 2}{x^2 - 3x + 4} \le 2$ then the value of n is

A) $n \in \left[-1, \sqrt{40} - 6\right]$ B) $n \in [-1, 3)$ C) $n \in \left[-\sqrt{40} - 6, -1\right]$ D) $n \in \left[1, \sqrt{40} + 6\right]$

Key. A

Sol. $\frac{x^2 + nx - 2}{x^2 - 3x + 4} - 2 \le 0$

$\Rightarrow x^2 - (n+6)x = 10 \ge 0$, true $\forall x \in R$ then

$D \le 0 \Rightarrow (n+6)^2 - 40 \le 0 \Rightarrow -\sqrt{40} - 6 \le n \le \sqrt{40} - 6$ --- (1)

Similarly $\frac{x^2 + nx - 2}{x^2 - 3x + 4} + 1 \ge 0 \Rightarrow 2x^2 + (x-3)x + 2 \ge 0$

$\Rightarrow D \le 0 \Rightarrow (n-3)^2 - 16 \le 0 \Rightarrow -1 \le n \le 7$ --- (2)

Combined (1) & (2) we get $n \in \left[-1, \sqrt{40} - 6\right]$