Q. 1 A uniform spring of normal length ℓ has a force constant k. It is cut into two pieces of lengths ℓ_1, and ℓ_2 such that $\ell_1 = n\ell_2$ where n is an integer. Then the value of k_1 (force constant of spring of length ℓ_1) is -

(A) $\frac{kn}{(n+1)}$ (B) $\frac{k(n+1)}{n}$

(C) $\frac{k(n-1)}{n}$ (D) $\frac{kn}{n-1}$

Sol.[B] $k_1\ell_1 = k_2\ell_2 = k(\ell_1 + \ell_2)$

$k_1 = \frac{k(\ell_1+\ell_2)}{\ell_1}$ or $k_1 = \frac{k(n\ell_2+\ell_2)}{n\ell_2}$

or $k_1 = \frac{k(n+1)}{n}$

Q.2 To study the dissipations of energy student Plots a graph between square root of time and amplitude. The graph would be a -

(A) Straight line (B) hyperbola

(C) Parabola (D) Exponential **[B]**

Sol. $a^2 t$ = constant

$a\sqrt{t}$ = constant

so hyperbola.

Q.3 The pendulum suspended from the ceiling of a train has a period T when the train is at rest. When the train is accelerating with an uniform acceleration, the period of oscillation will -

(A) increase (B) decrease

(C) remain unaffected (D) become infinite

[B]

Sol. Comparing with $y = 2\pi\sqrt{\frac{\ell}{g}}$;

$T' = 2\pi\sqrt{\frac{1}{\sqrt{g^2+a^2}}}$

clearly, T' < T

Q.4 For definite length of wire, if the weight used for applying tension is immersed in water, then frequency will -

(A) become less (B) become more

(C) remain equal (D) become zero **[A]**

Sol. For stretched string

$n \propto \sqrt{T} \propto \sqrt{M.g}$

When weight is dipped in water due to buoyancy force, tension decreases and hence frequency decreases

Q. 5 The amplitude and the time period in SHM are 0.8 cm and 0.2 s respectively. If the initial phase is $\frac{\pi}{2}$ radian, then the equation representing SHM is-

(A) y = 0.8 cos 10πt (B) y = 0.8 sin πt

(C) y = 3 × 0.8 sin πt (D) y = 0.8 sin 10πt

[A]

Sol. $y = a \sin(\omega t + \phi_0)$

$= 0.8 \sin\left[\frac{2\pi}{0.2}t + \frac{\pi}{2}\right]$

$= 0.8 \cos 10\pi t$

Q.6 The time period of a mass suspended from a spring is 5s. The spring is cut into four equal parts and same mass is now suspended from one of its parts. The period is now -

(A) 5s (B) 2.5 s

(C) 1.25 s (D) $\frac{5}{16}$ s **[B]**

Sol. $T = 2\pi\sqrt{\frac{m}{k}}$

Q.7 A particle executes SHM along a straight line so that its period is 12s. The time it takes in traversing a distance equal to half its amplitude from its equilibrium position is -

(A) 6s (B) 4s

(C) 2s (D) 1s **[D]**

Sol. $E = a \sin \omega t$

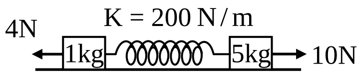
$\frac{a}{2} = a \sin \omega t$

or $\frac{1}{2} = \sin \omega t$

or $\sin\frac{\pi}{6} = \sin\frac{2\pi}{12}t$

t = 1s

Q.8 The maximum elongation in the spring is -



(A) 2 cm (B) 3 cm (C) 4 cm (D) 5 cm

Sol.[D] $x = \frac{2\times10\times1}{200\times6} + \frac{2\times4\times5}{200\times6} = \frac{60}{200\times6} = \frac{1}{20}$ m = 5cm

Q.9 A particle executes SHM along a straight line so that its period is 12 s. The time it takes in traversing a distance equal to half its amplitude from its equilibrium position is-

(A) 6 s (B) 4 s

(C) 2 s (D) 1s **[D]**

Sol. $E = a \sin \omega t \Rightarrow \frac{a}{2} = a\sin\omega t$ or $\frac{1}{2} = \sin\omega t$

or $\sin\frac{\pi}{6} = \sin\frac{2\pi}{12}t \Rightarrow t = 1s$

Q.10 The length of simple pendulum executing SHM is increased by 21%. The percentage increase in the time period of the pendulum is -

(A) 10% (B) 11% (C) 21% (D) 42%

Sol. **[A]**

$\frac{T'}{T} = \sqrt{\frac{121}{100}}$, $T' = \frac{11}{10}T$

$\left(\frac{T'}{T} - 1\right) \times 100\% = \left(\frac{11}{10} - 1\right) \times 100\%$

= 10%

Q.11 A particle executes SHM of amplitude 5cm and period 3s. The velocity of the particle at a distance 4 cm from the mean position-

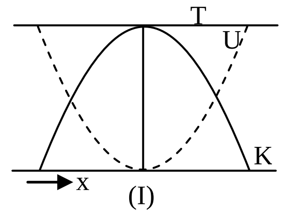
(A) 8 cm/s (B) 12 cm/s

(C) 4 cm/s (D) 6 cm/s

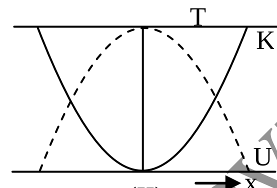
Sol.[D] $v = \omega\sqrt{a^2 - x^2} = \frac{2\pi}{T}\sqrt{a^2 - x^2} = \frac{2\times3}{3}\sqrt{25-16}$

= 6 cm/s

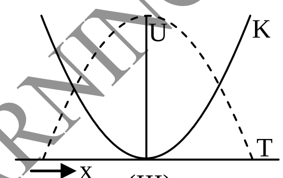
Q.12 A particle is executing S.H.M. along a straight line. The graph showing the variation of kinetic, potential and total energy K, U and T respectively with displacement is



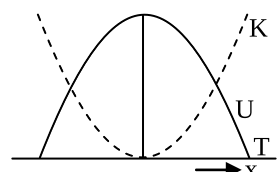
(I)



(II)



(III)

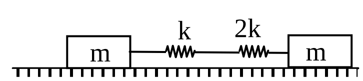


(IV)

(A) I (B) II

(C) III (D) IV **[A]**

Q.13 A system is shown in the figure. The time period for small oscillations of the two blocks will be -



(A) $2\pi\sqrt{\frac{3m}{k}}$ (B) $2\pi\sqrt{\frac{3m}{4k}}$

(C) $2\pi\sqrt{\frac{3m}{8k}}$ (D) $2\pi\sqrt{\frac{3m}{2k}}$

Sol. **[B]**

Both the spring are in series

$K_{eq} = \frac{k(2k)}{k+2k} = \frac{2k}{3}$

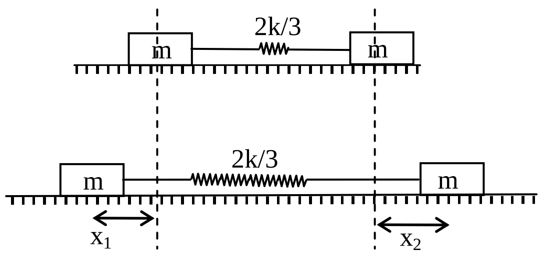
Time period $T = 2\pi\sqrt{\dfrac{\mu}{K_{eq}}}$

where $\mu = \dfrac{m_1.m_2}{m_1 + m_2}$

Here $\mu = \dfrac{m}{2}$

$\therefore T = 2\pi\sqrt{\dfrac{m}{2}.\dfrac{3}{2k}} = 2\pi\sqrt{\dfrac{3m}{4k}}$

Alternative method :



$\therefore mx_1 = mx_2 \Rightarrow x_1 = x_2$

force equation for first block;

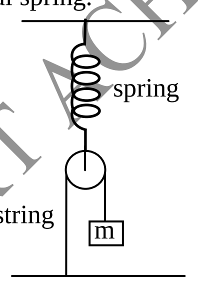
$\dfrac{2k}{3}(x_1+x_2) = m\,\dfrac{d^2x_1}{dt^2}$

Put $x_1 = x_2 \Rightarrow \dfrac{d^2x_1}{dt^2} + \dfrac{4k}{3m}x_1 = 0$

$\Rightarrow \omega^2 = \dfrac{4k}{3m}$

$\therefore T = 2\pi\sqrt{\dfrac{3m}{4k}}$

Q. 14 Find the period of low amplitude vertical vibrations of the system shown The mass of the block is m. The pulley hangs from the ceiling on a spring with a force constant k. The block hangs from an ideal spring.



(A) $2\pi\sqrt{\dfrac{m}{k}}$ (B) $6\pi\sqrt{\dfrac{m}{k}}$

(C) $4\pi\sqrt{\dfrac{m}{k}}$ (D) $8\pi\sqrt{\dfrac{m}{k}}$

Sol. **[C]**

When block is given displacement x spring will stretch by $\dfrac{x}{2}$.

$\therefore$ spring force $= \dfrac{kx}{2}$

Tension in the string $= \dfrac{1}{2} \times$ spring force

$= \dfrac{kx}{4}$

$T = 2\pi\sqrt{\dfrac{m}{\frac{k}{4}}} = 4\pi\sqrt{\dfrac{m}{k}}$

Q. 15 A spring has a force constant k and mass m . The spring hangs vertically and a block of unknown mass is attached to its bottom end: It is known that the mass of the block is much greater than that of the spring. The hanging block stretches the spring the twice its relaxed length. How long (t) would it take for a low amplitude transverse pulse to travel the length of the spring stretched by the hanging block ?

(A) $\sqrt{\dfrac{2m}{k}}$ (B) $\sqrt{\dfrac{m}{k}}$

(C) $\sqrt{\dfrac{m}{2k}}$ (D) $\sqrt{\dfrac{2m}{3k}}$

Sol. **[A]**

Since mass of spring is small compared to the mass m, the tension force is approximately constant

$\therefore T = mg$

$kx = mg \; x = \dfrac{mg}{k}$

When x-acceleration of the spring

x = 2L given

Where L – Relaxed length.

New length = 2L

$= \dfrac{2mg}{k}$

$\therefore$ mass per unit length

$\mu = \dfrac{mk}{2mg} = \dfrac{k}{2g}$

Speed of wave $V = \sqrt{\dfrac{T}{\mu}}$

$= \sqrt{\dfrac{mg \times 2mg}{mk}}$

$$V = g\sqrt{\frac{2m}{k}}$$

$$\text{time} = \frac{2L}{V}$$

$$= \frac{2mg}{k}\,\frac{\sqrt{k}}{\sqrt{2m}}$$

$$\text{time} = \sqrt{\frac{2m}{k}}$$

Q. 16 A particle of mass m is acted upon by a force $F = t^2 - kx$. Initially the particle is at rest at the origin. Then –

(A) Its displacement will be in simple harmonic

(B) Its velocity will be in simple harmonic

(C) Its acceleration will be in simple harmonic

(D) Particle will move with constant velocity

Sol. **[C]**

Conceptual

Q.17 Two particles A and B execute simple harmonic motion with periods of T and $\frac{5T}{4}$ respectively. They start simultaneously from mean position. The phase difference between them when A completes one oscillation will be -

(A) 0 (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$ (D) $\frac{2\pi}{5}$

Sol. **[D]**

$$\Delta\phi = (\omega_1 - \omega_2)t = \left(\frac{2\pi}{T} - \frac{2\pi}{5T/4}\right)T = \frac{2\pi}{5}$$

Q. 18 A simple harmonic oscillator has amplitude 'A' angular frequency ω and mass m. Then average kinetic energy in one time period is -

(A) $\frac{1}{2}m\omega^2A^2$ (B) $\frac{1}{4}m\omega^2A^2$

(C) $m\omega^2A^2$ (D) zero

Sol. **[B]**

$$K_{av} = \frac{1}{T}\int_6^T \frac{1}{2}m\omega^2A^2\cos^2(\omega t + \phi)$$

Q.19 A student says that he had applied a force $F = -k\sqrt{x}$ on a particle and the particle moved in simple harmonic motion. He refuses to tell whether k is a constant or not. Assume that he has worked only with positive x and no other force acted on the particle.

(A) As x increases k increases

(B) As x increases k decreases

(C) As x increases k remains constant

(D) The motion cannot be simple harmonic

[A]

Q.20 When the displacement is half of the amplitude. The ratio of potential energy to the total energy is -

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$

(C) 1 (D) $\frac{1}{8}$

Sol. **[B]** P.E. = $\frac{1}{2}m\omega^2x^2$ & T.E. = $\frac{1}{2}m\omega^2A^2$

So ratio at x = A/2 ⇒ $\frac{\text{P.E.}}{\text{T.E.}} = \frac{1}{4}$

Q.21 The distance moved by a particle in simple harmonic motion in one time period is –

(A) A (B) 2A

(C) 4A (D) zero **[C]**

Q.22 The time period of a particle in simple harmonic motion is equal to the smallest time between the particle acquiring a particular velocity $\vec{v}$. The value of v is –

(A) v_{max}

(B) 0

(C) between 0 and v_{max}

(D) between 0 and $-v_{max}$ **[D]**

Q.23 Which of the following quantities are always positive in a simple harmonic motion ?

(A) $\vec{F}\cdot\vec{a}$ (B) $\vec{v}\cdot\vec{r}$

(C) $\vec{a}\cdot\vec{r}$ (D) $\vec{F}\cdot\vec{r}$ **[A]**

Q.24 A particle moves such that its acceleration is given by

$$a = -\beta(x - 2)$$

Here : β is a positive constant and x is the position from origin. Time period of oscillations is -

(A) $2\pi\sqrt{\beta}$ (B) $2\pi\sqrt{\frac{1}{\beta}}$

(C) $2\pi\sqrt{\beta+2}$ (D) $2\pi\sqrt{\frac{1}{\beta+2}}$

Sol. **[B]**

$a = -\beta(x - 2)$

as $a = -\omega^2(x - x_0)$

$\therefore\ \omega^2 = \beta \Rightarrow T = 2\pi\sqrt{\frac{1}{\beta}}$

Q.25 The displacement of two identical particles executing SHM are represented by equations

$x_1 = 4\sin\left(10t + \frac{\pi}{6}\right)$ and $x_2 = 5\cos\omega t$

For what value of ω energy of both the particles is same ?

(A) 16 unit (B) 6 unit

(C) 4 unit (D) 8 unit

Sol. **[D]**

$E_1 = E_2$

$\therefore\ \frac{1}{2}m_1\omega_1^2A_1^2 = \frac{1}{2}m_2\omega_2^2A_2^2$

but $m_1 = m_2$

$\therefore\ \omega_1^2 \times 16 = \omega_2^2 \times 25$

$\therefore\ 100 \times 16 = \omega^2 \times 25$

$\omega = 8$ units

Q.26 A simple pendulum 4 m long swings with an amplitude of 0.2 m. What is its acceleration at the ends of its path ? ($g = 10\ m/s^2$)

(A) zero (B) $10\ m/s^2$

(C) $0.5\ m/s^2$ (D) $2.5\ m/s^2$

Sol. **[C]**

$\omega = \sqrt{\frac{g}{L}}\ \ \therefore a_{max} = \omega^2 A = \frac{g}{L} \times A = 0.5\ m/s^2$

Q.27 A particle of mass 5×10^{-5} kg is placed at the lowest point of a smooth parabola having the equation $x^2 = 40y$ (x, y in cm). If it is displaced slightly and it moves such that it is constrained to move along the parabola, the angular frequency of oscillation will be, approximately -

(A) $1\ s^{-1}$ (B) $7\ s^{-1}$

(C) $5\ s^{-1}$ (D) None of these

Sol. **[D]**

Restoring force $= F_R = -mg\sin\theta$ where

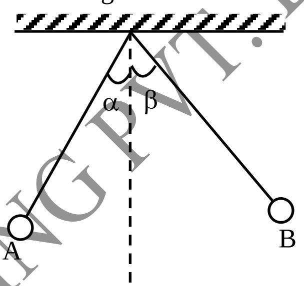
$\tan\theta = \frac{dy}{dx} = \frac{x}{20}$

$\therefore\ F_R = \frac{-mgx}{20}$

$\therefore\ F_R = -m\omega^2 x$

$\therefore\ \omega = \sqrt{\frac{g}{20}}$

Q.28 Two identical simple pendulums A and B are fixed at same point. They are displaced by very small angles α and β ($\beta > \alpha$) and released from rest. Find the time after which B reaches its initial position for the first time. Collisions are elastic and length of strings is ℓ.



(A) $\pi\sqrt{\frac{\ell}{g}}$ (B) $2\pi\sqrt{\frac{\ell}{g}}$

(C) $\frac{\pi\beta}{\alpha}\sqrt{\frac{\ell}{g}}$ (D) $\frac{2\pi\beta}{\alpha}\sqrt{\frac{\ell}{g}}$

Sol. **[B]**

Time period of both A and B $T = 2\pi\sqrt{\frac{\ell}{g}}$

After first collision, B acquires amplitude of A and after second collision it acquires its own amplitude in this process time taken is

$= \frac{T}{4} + \frac{T}{4} + \frac{T}{4} + \frac{T}{4} = T = 2\pi\sqrt{\frac{\ell}{g}}$

Q. 29 If a simple harmonic motion is represented by $\frac{d^2x}{dt^2} + \alpha x = 0$, its time period is -

(A) $\frac{2\pi}{\alpha}$ (B) $\frac{2\pi}{\sqrt{\alpha}}$

(C) $2\pi\alpha$ (D) $2\pi\sqrt{\alpha}$ **[B]**

Q. 30 If the displacement (x) and velocity (v) of a particle executing simple harmonic motion are related through the expression $4v^2 = 25 - x^2$, then its time period is -

(A) π (B) 2π

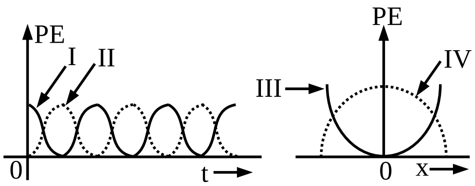
(C) 4π (D) 6π **[C]**

Q. 31 A simple pendulum has a period T. It is taken inside a lift moving up with uniform acceleration of g/3. Now its time period will be

(A) $\frac{\sqrt{2}}{3}T$ (B) $\frac{\sqrt{3}}{2}T$

(C) $\frac{2T}{\sqrt{3}}$ (D) $\frac{3T}{\sqrt{2}}$ **[B]**

Q. 32 For a particle executing simple harmonic motion, the displacement x is given by x = A cos ωt. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement x -



(A) I, III (B) II, IV

(C) II, III (D) I, IV **[A]**

Q. 33 A body of mass 1 kg is executing simple harmonic motion. Its displacement x (in cm) at time t (in second) is given by,

$$x = 6 \sin\left(100t + \frac{\pi}{4}\right)$$

The maximum kinetic energy of the body is -

(A) 6 J (B) 18 J

(C) 24 J (D) 36 J **[B]**

Q. 34 A uniform spring has an unstreched length ℓ and a force constant k. The spring is cut into two parts of unstreched length ℓ_1 and ℓ_2 such that $\ell_1 = \eta\ell_2$ where η is an integer. The corresponding force constants k_1 and k_2 are :

(A) kη and k(η + 1)

(B) $\frac{k(\eta+1)}{\eta}$ and k(η – 1)

(C) $\frac{k(\eta-1)}{\eta}$ and k(η+1)

(D) $\frac{k(\eta+1)}{\eta}$ and k(η+1) **[D]**

Sol. $\ell_1 = \eta\,\ell_2 \Rightarrow \ell_1 : \ell_2 = \eta : 1$

$\Rightarrow \ell_1 = \frac{\eta}{\eta+1}\ell$ & $\ell_2 = \frac{1}{(\eta+1)}\ell$

so $k_1 = \frac{\eta+1}{\eta}k$, $k_2 = (\eta + 1)k$

Q. 35 A particle is vibrating in simple harmonic motion with an amplitude of 4 cm. At what displacement from the equilibrium position is its energy half potential and half kinetic ?

(A) 10 cm (B) $\sqrt{2}$ cm

(C) 2 cm (D) $2\sqrt{2}$ cm

Sol.

[D] The total energy E of a particle vibrating SHM is given by

$E = \frac{1}{2}m\omega^2a^2$ .....(1)

The kinetic energy K is given by

$K = \frac{1}{2}m\omega^2(a^2 - y^2)$

where y = displacements of the particle

but $K = \frac{E}{2} = \frac{1}{2}\left[\frac{1}{2}m\omega^2a^2\right]$

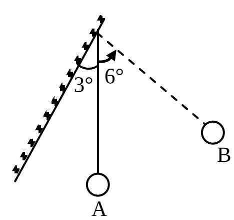
$\therefore \frac{1}{2}\left[\frac{1}{2}m\omega^2a^2\right] = \frac{1}{2}m\omega^2(a^2 - y^2)$

or $\frac{a^2}{2} = a^2 - y^2$ or $y^2 = \frac{a^2}{2}$ $\therefore y = \frac{a}{\sqrt{2}}$

Hence the kinetic energy is half of the total energy when displacement of the particle is $a/\sqrt{2}$. Given that a = 4cm.

$\therefore y = 4/\sqrt{2} = 2\sqrt{2}$.

Q.36 A pendulum of length 10 cm is hanged by wall making an angle 3° with vertical. It is swinged to position B. Time period of pendulum will be



(A) π/5 sec

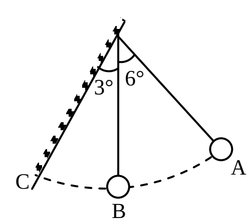
(B) $\frac{2\pi}{15}$ sec

(C) π/6 sec

(D) Subsequent motion will not be periodic

[B]

Sol.



Time taken by pendulum in going from A to B

$= \frac{T}{4}$ where $T = 2\pi\sqrt{\frac{\ell}{g}}$

Time taken by pendulum in going from B to C

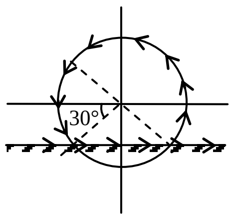
$= \frac{T}{12}$

$\therefore$ Time period of pendulum

$= 2\left(\frac{T}{4} + \frac{T}{12}\right)$

$= \frac{2T}{3} = \frac{2}{3} \cdot \frac{\pi}{5} = \frac{2\pi}{15}$ sec

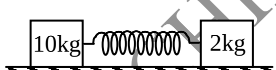
Altier :



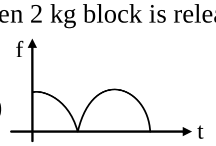
$T' = \frac{240}{360} \cdot T$

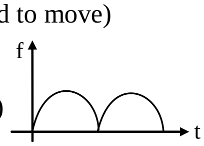
$= \frac{2}{3}T$

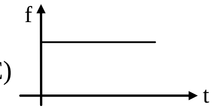
Q.37 Two blocks of mass 10 kg and 2 kg are connected by an ideal spring of spring constant 1000 N/m and the system is placed on a horizontal surface as shown.



The coefficient of friction between 10 kg block and surface is 0.5 but friction is assumed to be absent between 2 kg and surface. Initially blocks are at rest and spring is unstretched then 2 kg block is displaced by 1 cm to elongate the spring then released. Then the graph representing magnitude of frictional force on 10 kg block and time t is : (Time t is measured from that instant when 2 kg block is released to move)

(A) 

(B) 

(C) 

(D) None of these **[A]**

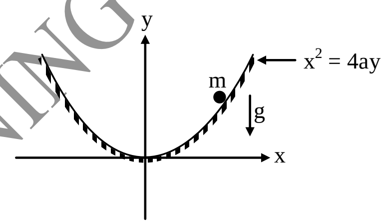
Sol. $f_s = kx$

(where f_s is frictional force on 20 kg block and xis instantaneous elongation or compression in spring)

$f_s = k\,(A \cos \omega t)$

$\therefore \quad |f_s| = kA\,|\cos \omega t|$

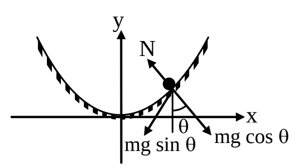
Q.38 A particle of mass m is allowed to oscillate near the minimum point of a vertical parabolic path having the equation $x^2 = 4ay$, then the angular frequency of small oscillations of particle is –



(A) $\sqrt{ga}$ (B) $\sqrt{2ga}$

(C) $\sqrt{\frac{g}{a}}$ (D) $\sqrt{\frac{g}{2a}}$ **[D]**

Sol.



$ma = -\,mg \sin\theta$

$a = -\,g \sin\theta$ or $a = -\,g \tan\theta$ ... (1)

(as θ is small)

Now,

$x^2 = 4ay$

$\therefore \quad \frac{dy}{dx} = \frac{x}{2a}$

$\therefore \quad a = -\,g\frac{x}{2a}$

$-\,\omega^2 x = -\,\frac{gx}{2a}$

$\omega = \sqrt{\frac{g}{2a}}$

Q.39 If the same weight is suspended from three springs having lengths 1 : 3 : 5, the period of oscillations shall be in the ratio of -

(A) 1 : 3 : 5 (B) $1 : \sqrt{3} : \sqrt{5}$

(C) 15 : 5 : 3 (D) $1 : \frac{1}{\sqrt{3}} : \frac{1}{\sqrt{15}}$ **[B]**

Sol. $T \propto \frac{1}{\sqrt{K}}$ and $K \propto \frac{1}{\ell}$

$\therefore T \propto \sqrt{\ell}$

Q.40 The displacement of two identical particles executing SHM are represented by equations

$x_1 = 4 \sin\left(10t + \frac{\pi}{6}\right)$ and $x_2 = 5 \cos \omega t$

For what value of ω energy of both are particles is same ?

(A) 16 unit (B) 6 unit

(C) 4 unit (D) 8 unit **[D]**

Sol. $E = \frac{1}{2} mA^2\omega^2$ i.e., $E \propto (A\omega)^2$

or $(A_1\omega_1)^2 = (A_2\omega_2)^2$

or $A_1\omega_1 = A_2\omega_2$

or $4 \times 10 = 5 \times \omega$

or $\omega = 8$ unit

Q.41 A simple pendulum 4 m long swings with an amplitude of 0.2 m. What is its acceleration at the ends of its path ? ($g = 10$ m/s^2)

(A) zero (B) 10 m/s^2

(C) 0.5 m/s^2 (D) 2.5 m/s^2 **[C]**

Sol.

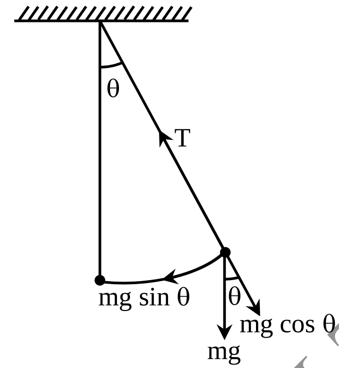


Fig.

$T = mg \cos \theta$

$\therefore F_{net} = mg \sin \theta$

and acceleration = $g \sin \theta = g \tan \theta$ ($\sin \theta = \tan \theta$)

$= (10)\frac{(0.2)}{4} = 0.5$ m/s^2

Q.42 A clock with an Iron Pendulum keeps correct time at 20°C. How much will it lose or gain if temperature changes to 40°C? [Given cubical expansion of iron = 36×10^{-6} °C^{-1}]

(A) 10.368 sec gain (B) 10.368 sec loss

(C) 5.184 sec gain (D) 5.184 sec loss

[B]

Sol. $T = 2\pi\sqrt{\frac{L}{g}}$

and $T' = 2\pi\sqrt{\frac{L'}{g}}$

or $\frac{T'}{T} = \sqrt{\frac{L'}{L}}$

$L' = L(1 \times \alpha \Delta t)$

$\therefore \alpha = \frac{\gamma}{3} = \frac{36 \times 10^{-6}}{3}$

$= L(1 + 12 \times 10^{-6} \times 20) = 12 \times 10^{-6}$ °C^{-1}

$L' = L(1.00024)$

$\frac{T'}{T} = \sqrt{\frac{1.00024 L}{L}}$

or $\frac{T'}{2} = 1.00012$ ($\because$ T = 2 sec)

$T' = 2.00024$

Loss in time per day

$= \frac{(2.00024 - 2) \times 24 \times 60 \times 60}{2}$ sec.

= 10.368 sec. Loss/day

Q.43 A particle of mass 'm' kept at origin is subjected to a force $\vec{F} = (pt - qx)\hat{i}$ where 't' : time elapsed and x : x-coordinate of position of particle. Particle starts its motion at t = 0 with zero initial velocity. If p and q are positive constants, then -

(A) Acceleration of the particle will continuously keep on increasing with time

(B) Particle will execute S.H.M.

(C) Force on particle will have no upper limit

(D) The acceleration of particle varies sinusoidally with time

Sol. **[D]**

F = pt – qx

$\Rightarrow a = \frac{p}{m}t - \frac{q}{m}x$

$\Rightarrow \frac{d^2a}{dt^2} = -\left(\frac{q}{m}\right)a$

Q.44 A particle executes SHM of amplitude 5cm and period 3s. The velocity of the particle at a distance 4 cm from the mean position -

(A) 8 cm/s   (B) 12 cm/s

(C) 4 cm/s   (D) 6 cm/s

Sol. **[D]**

$v = \omega\sqrt{a^2 - x^2}$

$= \frac{2\pi}{T}\sqrt{a^2 - x^2} = \frac{2\times3}{3}\sqrt{25-16} = 6$ cm/s

Q.45 The displacement y of a particle executing periodic motion is given by

$y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000\,t)$

this expression may be considered to be a result of the superposition of ……independent harmonic motions -

(A) Two   (B) Three

(C) Four   (D) Five

Sol. **[B]** $y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000\,t)$

$= 2\{1 + \cos t\}\sin(1000\,t)$

$y = 2\sin(1000\,t) + 2\cos t \sin 1000\,t$

$= 2\sin(1000\,t) + \sin(999\,t) + \sin(1001\,t)$

$\Rightarrow$ it is the super of three

Q.46 A particle undergoes simple harmonic motion having time-period T. The time taken 3/8th oscillation is-

(A) (3/8)T   (B) (5/8) T

(C) (5/12)T   (D) (7/12)T   **[C]**

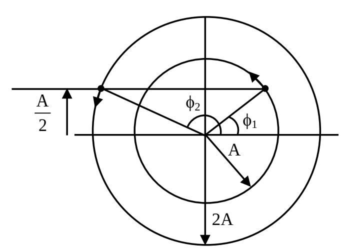
Q.47 Two particles are in SHM with same angular frequency and amplitudes A and 2A respectively along same straight line with same mean position. They cross each other at position $\frac{A}{2}$ distance from mean position in opposite direction. The phase difference between them is

(A) $\frac{5\pi}{6} - \sin^{-1}\left(\frac{1}{4}\right)$ (B) $\frac{\pi}{6} - \sin^{-1}\left(\frac{1}{4}\right)$

(C) $\frac{5\pi}{6} - \cos^{-1}\left(\frac{1}{4}\right)$ (D) $\frac{\pi}{6} - \cos^{-1}\left(\frac{1}{4}\right)$

[A]

Sol.



$\sin\phi_1 = \frac{A/2}{A} = \frac{1}{2}$

$\phi_1 = \frac{\pi}{6}$

$\sin(\pi - \phi_2) = \frac{A/2}{2A} = \frac{1}{4}$

$\phi_2 = \pi - \sin^{-1}\left(\frac{1}{4}\right)$

Phase difference

$$\phi_2 - \phi_1 = \frac{5\pi}{6} - \sin^{-1}\left(\frac{1}{4}\right)$$

Q.48 The velocities of a particle in SHM at displacements x_1 and x_2 from mean position are v_1 and v_2 respectively. Its amplitude will be -

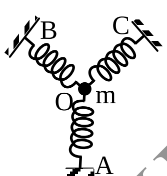
(A) $\left(\frac{x_2^2 v_1^2 - x_1^2 v_2^2}{v_1^2 - v_2^2}\right)^{1/2}$

(B) $\left(\frac{x_2^2 v_2^2 - x_1^2 v_1^2}{v_1^2 - v_2^2}\right)^{1/2}$

(C) $\left(\frac{x_2^2 v_1^2 - x_1^2 v_2^2}{v_2^2 - v_1^2}\right)^{1/2}$

(D) $\left(\frac{x_2^2 v_2^2 - x_1^2 v_1^2}{v_2^2 - v_1^2}\right)^{1/2}$ **[A]**

Q.49 A particle of mass m is attached to three identical springs A, B and C each of force constant k as shown in figure. If the particle of mass m is pushed slightly against the spring A and released, then the time period of oscillation is -



(A) $2\pi\sqrt{\frac{2m}{k}}$ (B) $2\pi\sqrt{\frac{m}{2k}}$

(C) $2\pi\sqrt{\frac{m}{k}}$ (D) $2\pi\sqrt{\frac{m}{3k}}$ **[B]**

Sol. $T = 2\pi\sqrt{\frac{M}{K(1+2\cos^2\theta)}}$

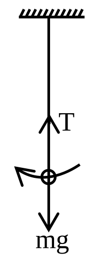
$= 2\pi\sqrt{\frac{M}{K(1+2\cos^2 45)}} = 2\pi\sqrt{\frac{M}{2K}}$

Q.50 A simple pendulum with angular frequency ω oscillates simple harmonically. The tension in the string at lowest point is T. The total acceleration of the bob at its lowest position is -

(A) $\frac{T}{m} - g$ (B) zero

(C) $g - \frac{T}{m}$ (D) $\frac{T}{m} + g$ **[A]**

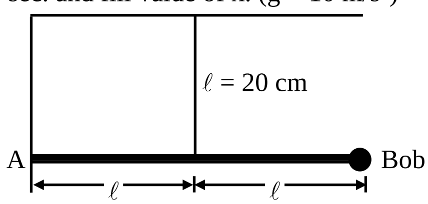
Sol.



$T - mg = m\omega^2 R$

$\frac{T}{m} - g = \omega^2 R = a$

Q.1 A weightless rigid rod with a small iron bob at the end is hinged at point A to the wall so that it can rotate in all directions. The rod is kept in the horizontal position by a vertical inextensible string of length 20 cm, fixed at its mid point. The bob is displaced slightly, perpendicular to the plane of the rod and string. Find period of small oscillations of the system in the form $\frac{\pi x}{10}$ sec. and fill value of x. ($g = 10\ m/s^2$)



Sol. The bob will execute SHM about a stationary axis passing through AB. If its effective length is ℓ' then $T = 2\pi\sqrt{\frac{\ell'}{g'}}$

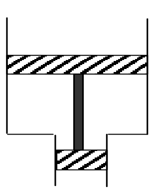
$\ell' = \frac{\ell}{\sin\theta} = \sqrt{2}\ell$

$g' = g\cos\theta = g/\sqrt{2}$

$T = 2\pi\sqrt{\frac{2\ell}{g}} = \frac{2\pi}{5} = \frac{4\pi}{10}$

x = 4.

Q.2 A smooth vertical conducting tube have two different section is open from both ends and equipped with two piston of different areas. Each piston slides in respective tube section. 1 liter of ideal gas at pressure 1.5×10^5 Pa is enclosed between the piston connected with a light rod. The cross section area of upper piston is $10\pi\ cm^2$ greater than lower one. Combined mass of two piston is 1.5 kg. If the piston is displaced slightly. Time period of oscillation will be (in 10^{-1} sec).



[0005]

Sol. Let the piston are displaced by 'x'

Process isothermal

$\therefore \Delta P = \frac{P}{V}\Delta V$

$\therefore a = \frac{\Delta P(A_1 - A_2)}{m}$

[A_1 : Area of upper piston

m = 1.5 kg]

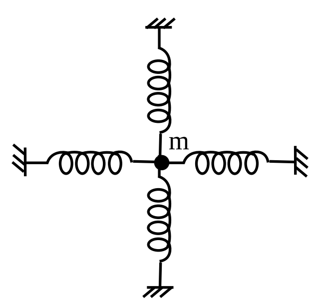
$= \frac{P}{mV}(A_1 - A_2).\Delta V$

$= \frac{P}{mV}(A_1 - A_2)^2 x$

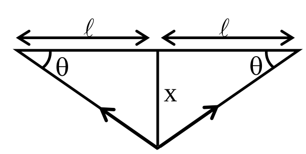
$\Rightarrow \omega = \sqrt{\frac{P}{mV}}\,.\,(A_1 - A_2)$

$\Rightarrow$ T = 0.5 sec

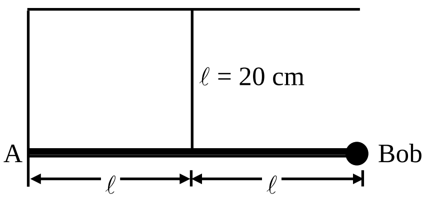
Q.3 As shown in figure a particle of mass m = 100 gm is attached with four identical springs each of length ℓ = 10 cm. Initial tension in each spring is f_0 = 25 N. Neglecting gravity the period of small oscillations of the particle in 10^{-2} sec along a line perpendicular to plane of figure is nearly.



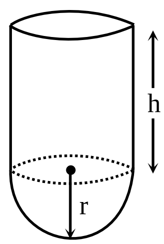
Sol. [6] $F = -4F_0\sin\theta = -\frac{4F_0 x}{\ell}$



Q.4 A weightless rigid rod with a small iron bob at the end is hinged at point A to the wall so that it can rotate in all directions. The rod is kept in the horizontal position by a vertical inextensible string of length 20 cm, fixed at its mid point. The bob is displaced slightly, perpendicular to the plane of the rod and string. Find period of small oscillations of the system in the form $\frac{\pi x}{10}$ sec. and fill value of x. ($g = 10\ m/s^2$)



Q.5 A container consist of hemispherical shell of radius 'r' and cylindrical shell of height 'h' radius of same material and thickness. The maximum value h/r so that container remain stable equilibrium in the position shown (neglect friction) is -



Sol. **[1]**

For stable equilibrium $h \le r \Rightarrow \frac{h}{r} \le 1$

Q.6 A solid uniform sphere of radius r rolls without slipping along the inner surface of a fixed spherical shell of radius R and performs small oscillations and its period is found with an unknown x to be $2\pi\sqrt{\frac{x(R-r)}{5g}}$ then the value of x will be.

Sol. **[7]**

$[P_0 + \rho_2 gh + \rho_1 g(h-20)] = P_0 + \frac{1}{2}\rho_1 V^2$

$\Rightarrow V = \left[\frac{2[\rho_2 gh + \rho_1 g(h-20)]}{\rho_1}\right]^{1/2}$

$= 4$ m/sec