Q. 1 A spherical ball rolls on a table without slipping. Then the fraction of its total energy associated with rotation is -

(A) 2/5 (B) 2/7 (C) 3/5 (D) 3/7 **[B]**

Sol. Total energy

$K = K_R + K_T = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$

$= \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2 + \frac{1}{2} mr^2\omega^2$

$= \frac{1}{5} mr^2\omega^2 + \frac{1}{2} mr^2\omega^2 = \frac{7}{10} mr^2\omega^2$

Now, rotational kinetic energy

$K_R = \frac{1}{2} I\omega^2 = \frac{1}{5} mr^2\omega^2$

$\therefore \frac{K_R}{K} = \frac{\frac{1}{5}mr^2\omega^2}{\frac{7}{10}mr^2\omega^2} = \frac{2}{7}$

Q.2 The moment of inertia of a body about a given axis is 1.2 kg × m^2. Initially, the body is at rest. In order to produce a rotational KE of 1500 joule, an angular acceleration of 25 rad/sec^2 must be applied about that axis for a duration of -

(A) 4 s (B) 2 s

(C) 8 s (D) 10 s **[B]**

Sol. $K_R = \frac{1}{2} I\omega^2 = \frac{1}{2} I(\alpha t)^2 = \frac{1}{2} I\alpha^2 t^2$

$1500 = \frac{1}{2} \times 1.2 \times (25)^2 t^2$

or $t^2 = 4$ or $t = 2s$

Q.3 A body of radius R and mass m is rolling horizontally without slipping with speed v. It then rolls up a hill to a maximum height $h = \frac{3v^2}{4g}$. The body might be a -

(A) solid sphere (B) hollow sphere

(C) disc (D) ring **[C]**

Sol. **[C]**

Let I bt the moment of inertia of the body. Then

total KE = $\frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$

or KE = $\frac{1}{2} mv^2 + \frac{1}{2} I \frac{v^2}{R^2}$ $\left(\omega = \frac{v}{R}\right)$

According to energy conservation loss in KE = gain in PE.

or $\frac{1}{2}\left(m + \frac{I}{R^2}\right)v^2 = mgh = mg\left(\frac{3v^2}{4g}\right)$

Solving this, we get $I = \frac{1}{2} mR^2$

i.e., the solid body is a disc

Q.4 When a wheel of radius R moves a distance smaller than $2\pi R$ making one rotation then -

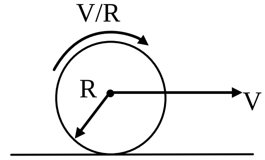
(A) $V_{cm} < R\omega$ (B) $V_{cm} > R\omega$

(C) $V_{cm} \le R\omega$ (D) $V_{cm} \ge R\omega$

Sol. **[A]**

conceptual.

Q.5 A disc is performing pure rolling on a smooth stationary surface with constant angular velocity as shown in figure. At any instant, for the lower most point of the disc.



(A) Velocity is v, acceleration is zero

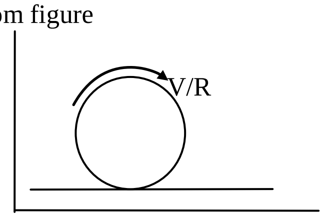
(B) Velocity is zero, acceleration is zero

(C) Velocty is v, acceleration is $\frac{v^2}{R}$

(D) Velocity is zero, acceleration is nonzero

Sol. **[D]**

From figure



$V = R\omega$

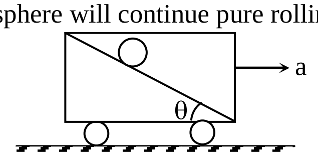
V_{net} (for lowest point) $= v - R\omega = v - v = 0$

and Acceleration $= \dfrac{v^2}{R} + O = \dfrac{v^2}{R}$

(Since linear speed is constant)

Hence (D)

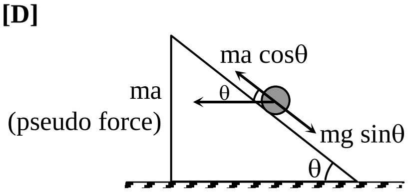
Q.6 Figure shows a smooth inclined plane of inclination θ fixed in a car. A sphere is set in pure rolling on the incline. For what value of 'a' (the acceleration of car in horizontal direction) the sphere will continue pure rolling ?



(A) g cos θ (B) g sin θ

(C) g cot θ (D) g tan θ

Sol. **[D]**

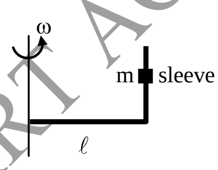


The sphere will continue pure rolling if

mg cos θ = mg sin θ

or a = g tan θ

Q.7 A L shaped rod whose one rod is horizontal and other is vertical is rotating about a vertical axis as shown with angular speed ω. The sleeve shown in figure has mass m and friction coefficient between rod and sleeve is μ. The minimum angular speed ω for which sleeve cannot sleep on rod is –



(A) $\omega = \sqrt{\dfrac{g}{\mu\ell}}$

(B) $\omega = \sqrt{\dfrac{\mu g}{\ell}}$

(C) $\omega = \sqrt{\dfrac{\ell}{\mu g}}$

(D) None of these

Sol. **[A]**

as $f = \mu N = mg$

or, $\mu m\ell\omega^2 = mg \quad \Rightarrow \quad \omega = \sqrt{\dfrac{g}{\mu\ell}}$

Q.8 At any instant, a rolling body may be considered to be in pure rotation about an axis through the point of contact but this axis is translating forward with a speed -

(A) zero

(B) equal to centre of mass

(C) twice of centre of mass

(D) None of these

Sol. **[B]**

Conceptual

Q. 9 The speed of wave traveling on the uniform circular ring, which is rotating about an axis passing through its center and perpendicular to its plane with tangential speed v in gravity free space is -

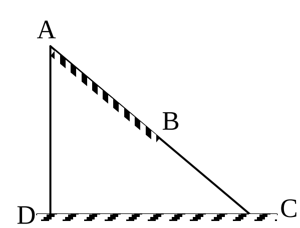
(A) v (B) $\dfrac{v}{2}$

(C) $\dfrac{v}{\sqrt{2}}$ (D) $\sqrt{2}v$

Sol. **[A]**

Tension in rotating ring is $T = \mu v^2$

Q.10 Portion AB of the wedge shown in figure is rough and BC is smooth. A solid cylinder rolls without slipping from A to B. If AB = BC, then ratio of translational kinetic energy to rotational kinetic energy, when the cylinder reaches point C is –



(A) 3/5 (B) 5

(C) 7/5 (D) 8/3

Sol. **[B]**

$K = \beta K_T$

or $K_T + K_R = \beta K_T$

$K_R = (\beta - 1)K_T \Rightarrow K_R = \frac{1}{2} K_T$

At point B : $K_T + K_R = mg \times h$

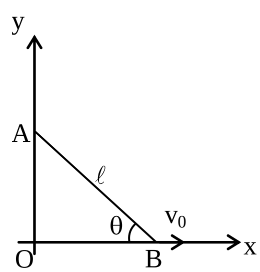
$\therefore K_R = \frac{mgh}{3}$

At point C : $K_T + \frac{mgh}{3} = mg \times 2h$

$K_T = \frac{5mgh}{3}$

$\therefore \frac{K_T}{K_R} = 5$

Q.11 In the figure given below, the end B of the rod AB which makes angle θ with the floor is pulled with a constant velocity v_0 as shown. The length of rod is ℓ. At an instant when $\theta = 37^o$



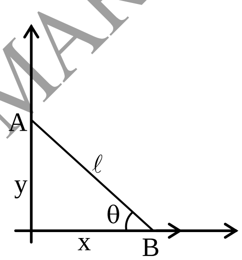
(A) Velocity of end A is $\frac{4v_0}{3}$

(B) angular velocity of rod is $\frac{5v_0}{6\ell}$

(C) angular velocity of rod is constant

(D) velocity of end A is constant

Sol. **[A]**



$x^2 + y^2 = \ell^2$

$\Rightarrow \frac{dy}{dt} = -\left(\frac{x}{y}\right)\frac{dx}{dt}$

$\therefore v_A = -\frac{4}{3}v_0$

Now, $x = \ell \cos\theta$

$\frac{dx}{dt} = -\ell \sin\theta \frac{d\theta}{dt} \Rightarrow \omega = -\frac{5}{3}\left(\frac{v_0}{\ell}\right)$

Q.12 For particle of a purely rotating body, $v = r\omega$, so correct relation will be -

(A) $\omega \propto \frac{1}{r}$ (B) $\omega \propto v$

(C) $v \propto \frac{1}{r}$ (D) ω is independent of r

Sol. **[D]**

Conceptual

Q.13 A ring of mass 100 kg and diameter 2m is rotating at the rate of $\left(\frac{300}{\pi}\right)$ rpm. Then-

(A) moment of inertia is 100 kg – m^2

(B) kinetic energy is 5 kJ

(C) if a retarding torque of 200 N-m starts acting then it will come at rest after 5 sec.

(D) all of these

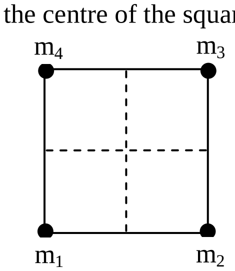
Sol. **[D]**

Moment of inertia = MR^2

k.E of rotation = $\frac{1}{2} I\omega^2$

Torque = $I \propto$ where $\alpha = \frac{\omega_0}{t}$

Q.14 Four particles of mass $m_1 = 2m$, $m_2 = 4m$, $m_3 = m$ and m_4 are placed at four corners of a square. What should be the value of m_4 so that the centre of mass of all the four particles are exactly at the centre of the square ?



(A) 2 m (B) 8 m

(C) 6 m (D) none of these **[D]**

Q.15 Two rings of same radius (r) and mass (m) are placed such that their centres are at a common point and their planes are perpendicular to each other. The moment of inertia of the system about an axis passing through the centre and perpendicular to plane of one of the ring is -

(A) $\frac{1}{2}mr^2$ (B) mr^2

(C) $\frac{3}{2}mr^2$ (D) $2mr^2$ **[C]**

Q.16 A disc is rotating with an angular velocity ω_0. A constant retarding torque is applied on it to stop the disc. The angular velocity becomes $\frac{\omega_0}{2}$ after n rotations. How many more rotations will it make before coming to rest ?

(A) n (B) 2n

(C) $\frac{n}{2}$ (D) $\frac{n}{3}$ **[D]**

Q.17 A particle of mass 1 kg is moving along the line y = x + 2 (here x and y are in metres) with speed 2 m/s. The magnitude of angular momentum of particle about origin is -

(A) 4 kg - m^2/s (B) $2\sqrt{2}$ kg - m^2/s

(C) $4\sqrt{2}$ kg - m^2/s (D) 2 kg - m^2/s **[B]**

Q.18 A circular platform is mounted on a vertical frictionless axle. Its radius is r = 2m and its moment of inertia is I = 200 kg - m^2. It is initially at rest. A 70 kg man stands on the edge of the platform and begins to walk along the edge at speed v_0 = 1.0 m/s relative to the ground. The angular velocity of the platform is -

(A) 1.2 rad/s (B) 0.4 rad/s

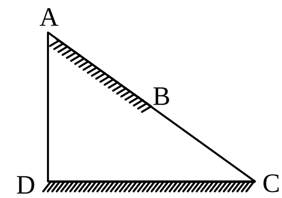
(C) 2.0 rad/s (D) 0.7 rad/s **[D]**

Q.19 The linear velocity of a particle moving with angular velocity $\vec{\omega} = 2\hat{k}$ at position vector $\vec{r} = 2\hat{i} + 2\hat{j}$ is -

(A) $4(\hat{i} - \hat{j})$ (B) $4(\hat{j} - \hat{i})$

(C) $4\hat{i}$ (D) $-4\hat{i}$ **[B]**

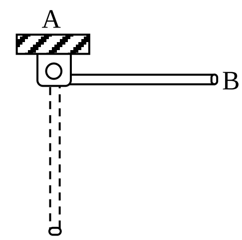
Q.20 Portion AB of the wedge shown in figure is rough and BC is smooth. A solid cylinder rolls without slipping from A to B. If AB = BC, then ratio of translational kinetic energy to rotational kinetic energy, when the cylinder reaches point C is -



(A) 3/5 (B) 5

(C) 7/5 (D) 8/3 **[B]**

Q.21 One end of a uniform rod of length *l* and mass m is hinged at A. It is released from rest from horizontal position AB as shown in figure. The force exerted by the rod on the hinge when it becomes vertical is -



(A) $\frac{3}{2}$ mg (B) $\frac{5}{2}$ mg

(C) 3 mg (D) 5 mg **[B]**

Q.22 A sphere of radius 'R' is rolling over a horizontal surface. All measurement are made with respect to surface over which sphere is rolling. Which of the following strictly confirms pure rolling motion of sphere over horizontal surface ?

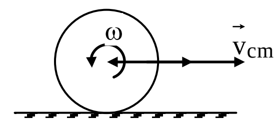
(A) $x_{cm} = R\theta$: x_{cm} & R in meter & 'θ' is in radian

(B) $v_{cm} = R\omega$: R in meter, v_{cm} in m/s, 'ω' in rad/sec

(C) $a_{cm} = R\alpha$: a_{cm} in cm/s^2, R in cm, α in rad/s^2

(D) All of the above **[D]**

Sol. Consider situation shown in figure



Q.23 A square plate is kept in yz-plane. Then according to perpendicular axis theorem -

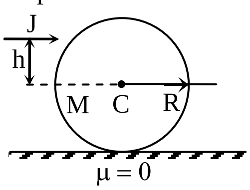
(A) $I_z = I_x + I_y$ (B) $I_x = I_y + I_z$

(C) $I_y = I_x + I_z$ (D) All **[B]**

Sol. For a mass distribution in y-z plane

$I_x = I_y + I_z$

Q.24 A solid sphere of mass M and radius R is placed on a smooth horizontal surface. It is given a horizontal impulse J at a height h above the centre of mass and sphere starts rolling then, the value of h and speed of centre of mass are –



(A) h = $\frac{2}{5}$ R and v = $\frac{J}{M}$

(B) h = $\frac{2}{5}$ R and v = $\frac{2}{5}\frac{J}{M}$

(C) h = $\frac{7}{5}$ R and v = $\frac{7}{5}\frac{J}{M}$

(D) h = $\frac{7}{5}$ R and v = $\frac{J}{M}$ **[A]**

Sol. Let the force producing impulse J is F then

$F \times h = \frac{2}{5} mR^2 \times \alpha$

and F = ma (where a = Rα)

$\therefore$ mah = $\frac{2}{5}$ mRa $\Rightarrow$ h = $\frac{2}{5}$ R

Also impulse = change in momentum

or J = Mv

Q.25 What must be the relation between length 'L' and radius 'R' of the cylinder if its moment of inertia about its axis is equal to that about the equatorial axis ?

(A) L = R (B) L = 2R

(C) L = 3R (D) L = $\sqrt{3}$ R **[D]**

Sol. $\frac{mR^2}{2} = M\left(\frac{L^2}{12} + \frac{R^2}{4}\right)$

or $\frac{R^2}{2} = \frac{L^2}{12} + \frac{R^2}{4}$

or L = $\sqrt{3}$ R

Q.26 A particle performs uniform circular motion with angular momentum 'L'. If the frequency of particles motion is halved and its KE is doubled then the angular momentum becomes –

(A) $\frac{L}{4}$ (B) 4L

(C) 2L (D) L / 2 **[B]**

Sol K.E. = $\frac{1}{2} I\omega^2 = \frac{1}{2}$ (Iω)(ω)

or K.E. = $\frac{1}{2}$ Lω

or L = $\frac{2K.E.}{\omega}$

Now L' = $\frac{2(2K.E.)}{(\omega/2)}$ = 4 L

Q.27 The angular speed of rotating rigid body is increased from 4 ω to 5 ω. The percentage increase in its K.E. is –

(A) 20 % (B) 25 %

(C) 125 % (D) 56 % **[D]**

Sol. K.E. = $\frac{1}{2} I\omega^2$ $\Rightarrow$ K.E. $\propto \omega^2$

% increase K.E. = $\frac{KE_f - KE_i}{KE_i} \times 100$

$= \frac{5^2 - 4^2}{4^2} \times 100$

$= \frac{9}{16} \times 100 = 56\%$

Q.28 Two loops P and Q are made from a uniform wire. The radii of P and Q are r_1 and r_2 respectively, and their moments of inertia are I_1 and I_2 respectively. If $I_2 = 4I_1$, then $\frac{r_2}{r_1}$ equals –

(A) $4^{2/3}$ (B) $4^{1/3}$

(C) $4^{-2/3}$ (D) $4^{-1/3}$ **[B]**

Sol. $I = MR^2 = (2\pi RAd)R^2$

or $I \propto R^3$

or $R \propto I^{1/3}$

or $\frac{R_2}{R_1} = \left(\frac{I_2}{I_1}\right)^{1/3} = \left(\frac{4}{1}\right)^{1/3}$

Q.29 A loop of radius 3 meter and weighs 150 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 15 cm/sec. How much work has to be done to stop it –

(A) 3.375 J (B) 7.375 J

(C) 5.375 J (D) 9.375 J **[A]**

Sol Required work = Total K.E.

$= \frac{1}{2}\ mv^2 \left(1+\frac{k^2}{R^2}\right)$

$= \frac{1}{2} Mv^2\left[1+\frac{k^2}{R^2}\right]$

$= \frac{1}{2} \times 150 \times (0.15)^2\ (1+1) = 3.375$ J

Q.30 A rod of mass **m** and length ***l*** is hinged at one of its end A as shown in figure. A force F is applied at a distance x from A. The acceleration of centre of mass (*a*) varies with x as –

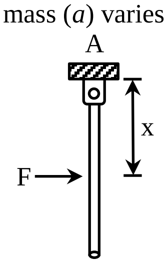
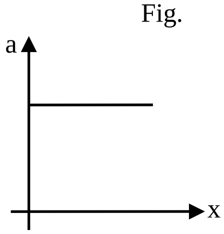
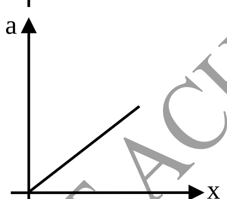


Fig.

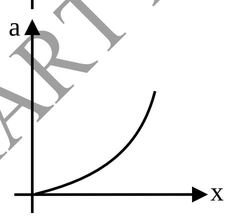
(A)



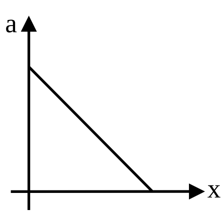
(B)



(C)



(D)



[B]

Sol. The rod will rotate about point A with angular acceleration:

$$\alpha = \frac{\tau}{I} = \frac{Fx}{\frac{ml^2}{3}} = \frac{3Fx}{ml^2}$$

$\therefore\ a = \frac{l}{2}\ \alpha = \frac{3}{2}\ \frac{Fx}{ml}$

or $a \propto x$

i.e., *a*-x graph is a straight line passing through origin.

Q.31 The moment of inertia of a body is I and its coefficient of linear expansion is α if temperature of body rises by a small amount ΔT. Then change in moment of inertia about the same axis –

(A) $\alpha\ I\ \Delta T$ (B) $2\ \alpha\ I\ \Delta T$

(C) $4\ \alpha\ I\ \Delta T$ (D) $\frac{\alpha I \Delta T}{2}$ **[B]**

Sol. Let $I = mr^2$

$\frac{\Delta I}{I} = \frac{2\Delta r}{r} = 2\ \alpha\ \Delta T$

or $\boxed{\Delta I = 2\alpha I \Delta T}$

Q.32 A wheel starts rotating from rest and attains an angular velocity of 60 rad/sec in 5 seconds. The total angular displacement in radians will be-

(A) 60 (B) 80

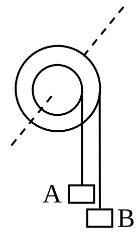
(C) 100 (D) 150 **[D]**

Q.33 A body rotates at 300 rotations per minute. The value in radian of the angle described in 1 sec is-

(A) 5 (B) 5π

(C) 10 (D) 10π **[D]**

Q.34 Figure shows a small wheel fixed coaxially on a bigger one of double the radius. The system rotates about the common axis. The strings supporting A and B do not slip on the wheels. If x and y be the distances travelled by A and B in the same time interval, then-



(A) x = 2y (B) x = y

(C) y = 2x (D) None of these **[C]**

Q.35 A particle is moving with a constant angular velocity about an exterior axis. Its linear velocity will depend upon -

(A) perpendicular distance of the particle form the axis

(B) the mass of particle

(C) angular acceleration of the particle

(D) the linear acceleration of particle **[A]**

Q.36 On account of the earth rotating about its axis-

(A) the linear velocity of objects at equator is greater than at other places

(B) the angular velocity of objects at equator is more than that of objects at poles

(C) the linear velocity of objects at all places at the earth is equal, but angular velocity is different

(D) at all places the angular velocity and linear velocity are uniform **[A]**

Q.37 A chain couples and rotates two wheels in a bicycle. The radii of bigger and smaller wheels in a bicycle. The radii of bigger and smaller wheels are 0.5m and 0.1m respectively. The bigger wheel rotates at the rate of 200 rotations per minute, then the rate of rotation of smaller wheel will be -

(A) 1000 rpm (B) 50/3 rpm

(C) 200 rmp (D) 40 rpm **[A]**

Q.38 If the position vector of a particle is $\hat{r} = (3\hat{i} + 4\hat{j})$ metre and its angular velocity is $\vec{\omega} = (\hat{j} + 2\hat{k})$ rad/sec then its linear velocity is (in m/s)-

(A) $-(8\hat{i} - 6\hat{j} + 3\hat{k})$ (B) $(3\hat{i} + 6\hat{j} + 8\hat{k})$

(C) $-(3\hat{i} + 6\hat{j} + 6\hat{k})$ (D) $(6\hat{i} + 8\hat{j} + 3\hat{k})$ **[A]**

Q.39 Let $\vec{A}$ be a unit vector along the axis of rotation of a purely rotating body and $\vec{B}$ be a unit vector along the velocity of a particle P of the body away from the axis. The value of $\vec{A}.\vec{B}$ is-

(A) 1 (B) –1

(C) 0 (D) none of these **[C]**

Q.40 A body is in pure rotation. The linear speed v of a particle, the distance r of the particle from the axis and the angular velocity ω of the body are related as $\omega = \frac{v}{r}$. Thus-

(A) $\omega \propto \frac{1}{r}$

(B) $\omega \propto r$

(C) $\omega = 0$

(D) ω is independent of r. **[D]**

Q.41 A particle, moving along a circular path has equal magnitudes of linear and angular acceleration. The diameter of the path is : (in meters) -

(A) 1 (B) π

(C) 2 (D) 2π **[C]**

Q.42 A stone of mass 4 kg is whirled in a horizontal circle of radius 1m and makes 2 rev/sec. The moment of inertia of the stone about the axis of rotation is-

(A) 64 kg × m^2 (B) 4 kg × m^2

(C) 16 kg × m^2 (D) 1 kg × m^2 **[B]**

Q.43 In an arrangement four particles, each of mass 2 gram are situated at the coordinate points (3, 2, 0), (1, –1, 0), (0, 0, 0) and (–1, 1, 0). The moment of inertia of this arrangement about the Z-axis will be-

(A) 8 units (B) 16 units

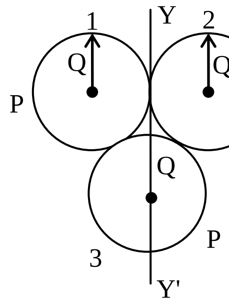
(C) 43 units (D) 34 units **[D]**

Q.44 Two discs have same mass and thickness. Their materials are of densities ρ_1 and ρ_2. The ratio of their moment of inertia about central axis will be -

(A) $\rho_1 : \rho_2$ (B) $\rho_1\rho_2 : 1$

(C) $1 : \rho_1\rho_2$ (D) $\rho_2 : \rho_1$ **[D]**

Q.45 Three rings, each of mass P and radius Q are arranged as shown in the figure. The moment of inertia of the arrangement about YY' axis will be-



(A) $\frac{7}{2}PQ^2$ (B) $\frac{2}{7}PQ^2$

(C) $\frac{2}{5}PQ^2$ (D) $\frac{5}{2}PQ^2$ **[A]**

Q.46 The moment of inertia depends upon-

(A) angular velocity of the body

(B) angular acceleration of the body

(C) only mass of the body

(D) distribution of mass and the axis of rotation of the body **[D]**

Q.47 Three thin uniform rods each of mass M and length L and placed along the three axis of a Cartesian coordinate system with one end of each rod at the origin. The M.I. of the system about z-axis is-

(A) $\frac{ML^2}{3}$ (B) $\frac{2ML^2}{3}$ (C) $\frac{ML^2}{6}$ (D) ML^2

[B]

Q.48 A circular disc A of radius r is made from an iron plate of thickness t and another circular disc B of radius 4r is made from an iron plate of thickness t/4. The relation between the moments of inertia I_A and I_B is-

(A) $I_A > I_B$

(B) $I_A = I_B$

(C) $I_A < I_B$

(D) depends on the actual values of t and r. **[C]**

Q.49 A flywheel has moment of inertia 4 kg-m^2 and has kinetic energy of 200 J. Calculate the number of revolutions is makes before coming to rest if a constant opposing couple of 5N-m is applied to the flywheel -

(A) 12.8 rev (B) 24 rev

(C) 6.4 rev (D) 16 rev

Sol. [C] W = ΔKE or $\tau(\theta) = \frac{1}{2}I\omega^2$

$\tau(2\pi n) = \frac{1}{2}I\omega^2$

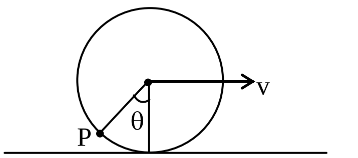
Q.50 A rigid body is rotating about a vertical axis at n rotations per minute, If the axis slowly becomes horizontal in t seconds and the body keeps on rotating at n rotations per minute then the torque acting on the body will be, if the moment of inertia of the body about axis of rotation is I -

(A) zero (B) $\frac{2\pi nI}{60\,t}$

(C) $\frac{2\sqrt{2}\pi nI}{60\,t}$ (D) $\frac{4\pi nI}{60\,t}$ **[C]**

Q. 1 A point P is located on the rim of wheel of radius r = 0.5 m which rolls without slipping along a horizontal surface then the total distance traversed by the point P in meters between two successive moments it touches the surface. **[0004]**

Sol.



$V_P = \sqrt{V^2 + V^2 + 2V^2\cos(\pi - \theta)}$

$\therefore V_P = 2\,V\sin\left(\frac{\theta}{2}\right)$

Now $V_P = \frac{ds}{dt} = \frac{ds}{dv} \cdot \frac{dv}{dt}$

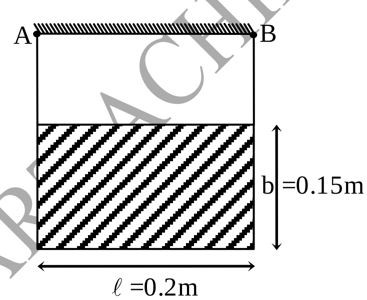
$\therefore V_P = \omega\,\frac{ds}{d\theta} = \frac{V}{R}\,\frac{ds}{d\theta}$

$\therefore \frac{V}{R}\frac{ds}{d\theta} = 2V\sin(\theta/2)$

$\Rightarrow ds = 2R\sin(\theta/2)\,d\theta$

$\therefore S = 2R\int_0^{2\pi}\sin\left(\frac{\theta}{2}\right)d\theta = 8R = 4m$

Q.2 A rectangular plate of mass 20 kg is suspended from points A and B as shown. If the pin B is suddenly removed then the angular acceleration in rad/sec^2 of the plate is : (g = 10 m/s^2).



Sol. Moment of inertia about the axis rotation axis is I then,

$I = I_C + md^2$, where $d^2 = \frac{b^2}{4} + \frac{\ell^2}{4} = \frac{0.0625}{4}$

$\therefore I = \frac{20}{12}[(0.2)^2 + (0.15)^2] + \frac{20 \times 0.0625}{4}$

$= 0.416$ kg-m^2

Now $I\alpha = mg\frac{\ell}{2} \Rightarrow \alpha = \frac{mg\ell}{2I} \approx 48$ rad/sec^2

Q.3 A solid sphere rolls on a rough horizontal surface with a linear speed 7 m/s collides elastically with a fixed smooth vertical wall. Then the speed of the sphere when it has started pure rolling in the backward direction in m/s is. **[0003]**

Sol. Let after time t

Sphere starts pure rolling then,

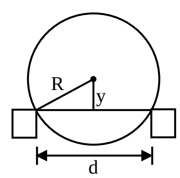
$v = 7 - \mu gt$ ….(1)

and $\frac{v}{r} = \frac{-7}{r} + \frac{5\mu gt}{2r}$ ….(2)

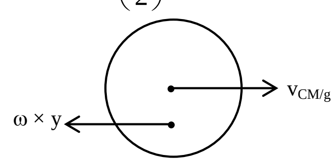
From (1) and (2) v = 3 m/s

Q.4 A uniform ball of radius R = 10 cm rolls without slipping between two rails such that the horizontal distance is d = 16 cm between two contact points of the rail to the ball. If the angular velocity is 5 rad/s, then find the velocity of centre of mass of the ball in cm/s. **[0030]**

Sol.



$y^2 = R^2 - \left(\frac{d}{2}\right)^2$



$v_{CM/g} = \omega \times y$

$= 30$ cm/s

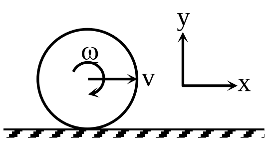
Q.5 A wheel rotating at same angular speed undergoes constant angular retardation. After revolution angular velocity reduces to half its initial value. How many more revolution it will make before stopping ?

5.[3] $\frac{\omega_0^2 - (\omega_{0/2})^2}{\omega_0{}^2} = \frac{9}{\theta_2} \Rightarrow \frac{3}{4} = \frac{9}{\theta_2}$

$\Rightarrow \theta_2 = 12$

$\therefore$ Required No. of revolution = 12 – 9 = 3

Q.6 A disc of radius '5cm' rolls on a horizontal surface with linear velocity $v = 1\hat{i}$ m/s and angular velocity 50 rad/sec. Height of particle from ground on rim of disc which has velocity in vertical direction is (in cm) -

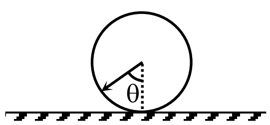


Sol **.[3]**

$v = R\omega \cos\theta$

$\Rightarrow \cos\theta = \dfrac{v}{R\omega} = \dfrac{2}{5}$ cm

$\therefore h = R(1 - \cos\theta) = 3$ cm.



Q.7 A cubical block of mass 6 kg and side 16.1 cm is placed on frictionless horizontal surface. It is hit by a cue at the top as to impart impulse in horizontal direction. Minimum impulse imparted to topple the block must be greater than –

Sol. **[4]**

Let a = side of cube

p = impulse imparted

$\therefore$ After hitting,

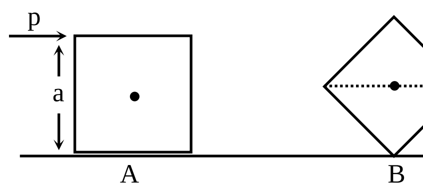
$v_0 = \dfrac{p}{m}$ and $\omega_0 = \dfrac{pa}{2I}$

I : moment of inertial about axis passing through C.M.

For just toppling

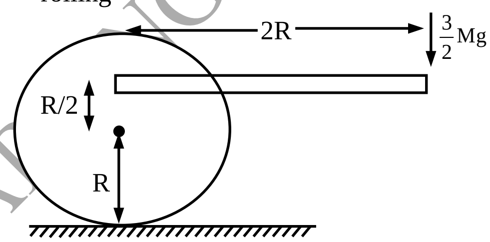
$$\frac{1}{2} I \omega_0^2 = mg\,a\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)$$

(Applying energy conservation between situation A and B)



$\Rightarrow p = \dfrac{2I\omega_0}{a} = 4$ kg m/s.

Q.8 A disc of mass M & radius R is placed a rough horizontal surface with its axis horizontal. A light rod of length '2R' is fixed to the disc at point 'A' as shown in figure and a force $\frac{3}{2}$ Mg is applied at the other end. If disc starts to roll without slipping find the value of "$10\times\mu_{min}$" where μ_{min} is minimum coefficient of friction b/w disc & horizontal surface required for pure rolling -



Sol. **[8]**

F = Ma ……….(i)

$\dfrac{3}{2}$ Mg. 2R – F.R = $\dfrac{MR^2}{2} \cdot \dfrac{a}{R}$ ……...(ii)

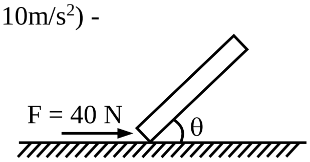
a = 2g

$\therefore F = 2Mg \le \mu N$

Q.9 A homogeneous rod of mass 3 kg is pushed along smooth horizontal surface by a horizontal force

F = 40 N. The angle 'θ' (in degree) for which rod has pure translation motion minus 30 degree is

($g = 10 m/s^2$) -

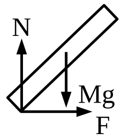


Sol. **[7]**

N = mg

$$F\,\frac{L}{2}\sin\theta = \frac{N^2}{2}\cos\theta$$

$\Rightarrow \tan\theta = \dfrac{3}{4}$



Q.10 Two identical discs are positioned on a vertical axis. The bottom disc is rotating with angular velocity ω_0. The top disc is initially at rest. It is allowed to fall and sticks to the lower disc. Ratio of K.E. before & after collision .

Sol. **[2]** $I\omega_0 = 2I\omega$

(I : M.I. of one disc)

$\therefore K = \dfrac{1}{2}\ 2I\omega^2$

$= \dfrac{1}{2}\left(\dfrac{1}{2} I\omega_0^{\ 2}\right)$

Q.10 A disc is rotating freely about its axis. Percentage change in angular velocity of disc if temperature decreases by 20 °C is (coefficient of linear expansion of material of disc is 5×10^{-4}/°C)

Sol. **[2]**

$I\omega = \text{const.} \Rightarrow \omega \propto \dfrac{1}{I}$

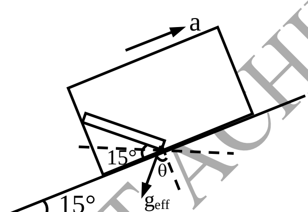
$\therefore$ Percentage change in ω

$= -$ (% change in I)

$= -(2 \times \Delta\theta \times 100)$

$= -(2 \times 5 \times 10^{-4} \times -20 \times 100)$

$= 2$ %



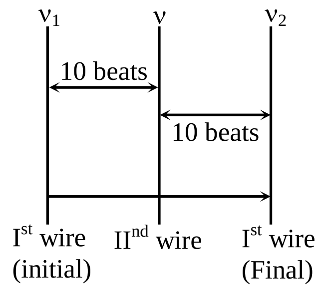
Q.11 Two wires are vibrating together to produce 10 beats/sec. Frequency of one wire is 200Hz. When tension in this wire is increased beat frequency remains unchanged. Frequency (in Hz) of other wire minus 206 Hz is equal to.

Sol. **[4]**

Frequency of first wire is less than that of second wire as upon increasing tension beats frequency remains unchanged.

$\therefore \nu = \nu_1 + 10 = 210$ Hz

$\therefore \nu - 206 = 4$

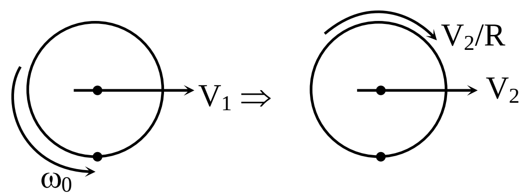


Q.12 A ball of radius R = 20 cm has a mass m = 0.75 kg and moment of inertia about its diameter $I = 0.0125$ kg m^2. The ball rolls without slipping over a rough horizontal floor with velocity $v_0 = 10$ m/s towards a smooth vertical wall. If coefficient of restitution between the wall and ball is e = 0.7 then the velocity of the ball in m/s after long time after collision is (g = 10 m/s^2)

Sol. **[2]**

Let V_1 just after collision and V_2 after long time [ω will not change]

$e = \dfrac{V_1}{V} \Rightarrow V_1 = 0.7 \times 10 = 7$m/s



angular momentum conservation about O

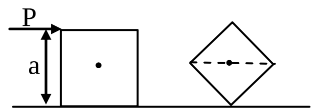
$mV_1R - I\omega_0 = mV_2R + I\ \dfrac{V_2}{R}$

$V_2 = 2$m/s

Q.13 A cubical block of mass 6 kg and side 16.1 cm is placed on frictionless horizontal surface. It is hit by a cue at the top as to impart-impulse in horizontal direction. Minimum impulse imparted to topple the block must be greater than.

Sol.[4] Let, a = side of cube

P = Impulse imparted



∴ After hitting,

$v_0 = \frac{P}{m}$ and $\omega_0 = \frac{Pa}{2I}$

[I = Moment of inertial about axis passing through centre of mass]

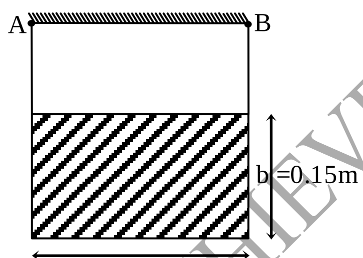
For just toppling

$$\frac{1}{2} I\omega_0^2 = mga \left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)$$

(Applying energy conservation between situation A and B)

$\Rightarrow \quad P = \frac{2I\omega_0}{a} = 4$ kg m/s

Q.14 A rectangular plate of mass 20 kg is suspended from points A and B as shown. If the pin B is suddenly removed then the angular acceleration in rad/sec^2 of the plate divided by 16 is equal to (g = 10 m/s^2)



Sol.[3] Moment of inertia about the axis rotation axis is I then,

$$I = I_C + md^2, \text{ where } d^2 = \frac{b^2}{4} + \frac{\ell^2}{4} = \frac{0.0625}{4}$$

$$\therefore I = \frac{20}{12}[(0.2)^2 + (0.15)^2] + \frac{20 \times 0.0625}{4}$$

$$= 0.416 \text{ kg-m}^2$$

$$\text{Now } I\alpha = mg\frac{\ell}{2} \Rightarrow \alpha = \frac{mg\ell}{2I} \approx 48 \text{ rad/sec}^2$$