PHYSICS

Q.1 A radioactive material has a mean lives of 1620 year and 660 year for α and β emission respectively. The material decay by simultaneous α and β emission. The time in which 1/4th of the material remains intact is -

(A) 4675 year (B) 720 year

(C) 650 year (D) 324 year **[C]**

Sol. $\tau = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} = \frac{1620 \times 660}{2280} = 469$

$\frac{N}{4} = \frac{N}{2^{\frac{t}{T}}}$

t = 2T = 2 τ ℓn 2 = 2 × 0.693 × 469

= 650 years.

Q.2 The ratio activity of an element becomes 1/64th of its original value in 60 sec. Then the half life period is -

(A) 5 sec (B) 10 sec

(C) 20 sec (D) 30 sec **[B]**

Sol. A.P = $\frac{1}{64} = \frac{1}{2^n}$ (n = 6)

$t = n\,T_{1/2} \Rightarrow T_{1/2} = \frac{t}{n} = \frac{60}{6} = 10$ sec

Q.3 The half life period of a radioactive substance is 140 days. After how much time, 15 gm will decay from a 16 gm sample of the substance ?

(A) 140 days (B) 560 days

(C) 420 days (D) 280 days **[B]**

Sol. $\frac{m}{m_0} = \frac{1\,gm}{16\,gm} = \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{\frac{t}{T}}$

$\Rightarrow$ t = 4T = 4 × 140 days = 560 days

Q.4 In free space the intensity of 5 eV neutron beam is reduced by a factor of one half. Half life is $t_{1/2}$ = 12.8 min. The distance travelled by neutron beam is-

(A) 2800 km (B) 23800 km

(C) 28 km (D) 2 km **[B]**

Q.5 90 % of a radio active sample is left undecayed after time t has elapsed. what percentage of the initial sample will decay in a total time 2t. -

(A) 20 % (B) 19 %

(C) 40 % (D) 38 % **[B]**

Sol. $N = N_0\,(0.9)^2$

$N = 0.81\,N_0$

81% of initial value is left

hence % of the initial sample decayed

= 100 – 81 = 19 %

Q.6 The half life of ^{198}Au is 2.7 days. The probability that any ^{198}Au nucleus will decay in one second is -

(A) 10^{-6} (B) 3×10^{-6}

(C) 5×10^{-6} (D) 10^{-5}

Sol. **[B]**

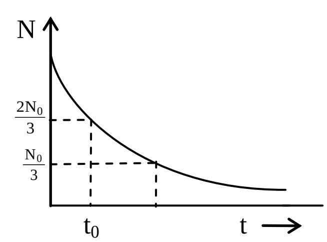
Decay probability per second is just the decay constant

$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{2.7\text{days}}$

$\lambda = \frac{0.693}{2.7 \times 24 \times 60 \times 60}$

$\lambda = 2.97 \times 10^{-6}$/sec

Q.7 Figure shows the variation of the number of radioactive atoms left undecayed with time. The time corresponding to $N = \frac{N_0}{3}$ is -



(A) $3\,t_0$

(B) $t_0\,\log_e 2$

(C) $\frac{t_0 \log_e 3}{\log_e\left(\frac{3}{2}\right)}$

(D) cannot be ascertained because it depends on the decay constant.

Sol. **[C]**

$N = N_0 e^{-\lambda t}$

$\frac{2N_0}{3} = N_0 e^{-\lambda t_0}$ $\quad e^{-\lambda t_0} = \frac{3}{2}$ .... (1)

$\lambda t_0 = \log_e\left(\frac{3}{2}\right)$

Also $\frac{N_0}{3} = N_0\ e^{-\lambda t_1}$

$\lambda t_1 = \log_e 3$

$t_1 = \frac{1}{\lambda}\log_e 3 = \frac{t_0 \log_e 3}{\log_e\left(\frac{3}{2}\right)}$

Q.8 There are two radioactive substances A and B. Decay constant of B is two times that of A. Initially both have equal number of nuclei. After n half lives of A, rate of disintegration of both are equal. The value of n is –

(A) 1 (B) 2

(C) 4 (D) all of these

Sol. **[A]** Let $\lambda_A = \lambda$ and $\lambda_B = 2\lambda$

Initially rate of disintegration of A is λN_0 and that of B is $2\lambda N_0$. After one half life time of A, the rate of disintegration of A becomes $\frac{\lambda N_0}{2}$ and that of B would also be $\frac{\lambda N_0}{2}$ [half-life of B = $\frac{1}{2}$ (half-life of A)]

So, after one half-life of A or two half-lives of B.

$$\left(-\frac{dN}{dt}\right)_A = \left(-\frac{dN}{dt}\right)_B$$

$\therefore$ n = 1

Q.9 The mean lives of a radioactive substance are 1620 year and 405 year for α-emission and β-emission respectively. Find the time during which three-fourth of a sample will decay if it is decaying both by α-emission and β-emission simultaneously.

(A) 249 years (B) 449 years

(C) 133 years (D) 99 years

Sol. **[B]**

The decay constant λ is the reciprocal of the mean life τ.

Thus, $\lambda_\alpha = \frac{1}{1620}$ per year

and $\lambda_\beta = \frac{1}{405}$ per year

$\therefore$ Total decay constant, $\lambda = \lambda_\alpha + \lambda_\beta$

or $\lambda = \frac{1}{1620} + \frac{1}{405} = \frac{1}{324}$ per year

We know that $N = N_0\ e^{-\lambda t}$

When $\frac{3}{4}$ th part of the sample has disintegrated, $N = N_0/4$

$\therefore$ $\frac{N_0}{4} = N_0 e^{-\lambda t}$

or $e^{\lambda t} = 4$

Taking logarithm of both sides, we get

$\lambda t = \log_e 4$

or $t = \frac{1}{\lambda}\log_e 2^2 = \frac{2}{\lambda}\log_e 2$

$= 2 \times 324 \times 0.693 = 449$ year

Q.10 The example of radioactive substance is

(A) Na (B) Mg

(C) He (D) Np **[D]**

Q.11 Radioactivity is not influenced by

(A) pressure

(B) electronic configuration

(C) temperature

(D) all of these **[D]**

Q.12 The parent and the stable product of the Uranium series are respectively, $^{238}_{92}U$ and $^{206}_{82}Pb$. How many α and β- particles respectively are emitted from the parent nucleus to become the stable end product ?

(A) 8, 8 (B) 6, 6

(C) 8, 6 (D) 16, 8 **[C]**

Q.13 In which of the following decays the element does not change ?

(A) α-decay (B) β^+-decay

(C) β^--decay (D) γ-decay **[D]**

Q.14 One of the incomplete nuclear decay process is

$^{228}Th \longrightarrow {}^{224}Ra^* + ...$

The term in the place of blank may be

(A) α (B) β^-

(C) β^+ (D) γ **[A]**

Q.15 The phenomenon in which the masses of a particle and an antiparticle disappear to reappear as energy is called -

(A) Pair production (B) Annihilation

(C) Cerenkov radiation (D) Compton scattering

[B]

Q.16 Which one of the following is not a mode of radioactive decay -

(A) Electron emission (B) Alpha decay

(C) Fusion (D) Gamma emission

[C]

Q.17 In a radioactive decay, neither the atomic number nor mass number changes. Which of the following particles is emitted in the decay ?

(A) proton (B) neutron

(C) electron (D) photon **[D]**

Q.18 The rate of disintegration of fixed quantity of a radioactive element can be increased by -

(A) Increasing the temperature

(B) Increasing the pressure

(C) Chemical reaction

(D) It is not possible **[D]**

Sol. Not change with temperature, pressure or any other chemical reactions

Q.19 An α particle is bombarded on ^{14}N. As a result a ^{17}O nucleus is formed and a particle is emitted. This particle is a–

(A) neutron (B) proton

(C) electron (D) positron **[B]**

Q.20 A radioactive substance X decays into another radioactive substance Y. Initially only X was present. λ_x and λ_y are the disintegration constants of X and Y. N_x and N_y are the number of nuclei of X and Y at any time t. Number of nuclei N_y will be maximum when:

(A) $\frac{N_y}{N_x - N_y} = \frac{\lambda_y}{\lambda_x - \lambda_y}$

(B) $\frac{N_x}{N_x - N_y} = \frac{\lambda_x}{\lambda_x - \lambda_y}$

(C) $\lambda_y N_y = \lambda_x N_x$

(D) $\lambda_y N_x = \lambda_x N_y$ **[C]**

Sol. Net rate of formation of Y at any time t is:

$$\frac{dN_y}{dt} = \lambda_x N_x - \lambda_y N_y$$

N_y is maximum when $\frac{dN_y}{dt} = 0$

or $\lambda_y N_y = \lambda_x N_x$

Q.21 When an electron and positron with equal speeds in opposite direction annihilate each other, they cannot produce just one gamma ray, because that will violate law of–

(A) conservation of charge

(B) conservation of energy

(C) conservation of momentum

(D) conservation of nucleon number **[C]**

Q.22 A radioactive decay counter is switched on at t = 0. A β-active sample is present near the counter. The counter registers the number of β-particles emitted by the sample. The counter registers 1×10^5 β-particles at t = 36 sec and 1.11×10^5 β-particles at t = 108 sec. $T_{1/2}$ of this sample is -

(A) 5.2 sec (B) 10.8 sec

(C) 15.4 sec (D) 20.6 sec **[B]**

Sol. $N = N_0 e^{-\lambda t}$

Decay = $N_0 - N$

$10^5 = N_0 (1 - e^{-36\lambda})$

$1.11 \times 10^5 = N_0(1 - e^{-108\lambda})$

$$\Rightarrow \frac{1-e^{-108\lambda}}{1-e^{-36\lambda}} = 1.11$$

$$\Rightarrow e^{-36\lambda} = 0.1 \Rightarrow \lambda = \frac{\ell n10}{36}$$

$$T_{1/2} = \frac{\ell n2}{\lambda} = \frac{36\,\ell n\,2}{\ell n10} \simeq 10.8$$

Q.23 The compound unstable nucleus $^{236}_{92}U$ often decays in accordance with the following reaction

$^{236}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr +$ other particles. Here the other particles are–

(A) An alpha particle

(B) Two protons

(C) One proton and one neutron

(D) Two neutrons **[D]**

Q.24 Tritium $\left({}^{3}_{1}H\right)$ has a half-life of 12.5y against beta decay. What fraction of a sample of tritium will remain undecayed after 25y ?
(A) 1/4 (B) 3/4
(C) 1/2 (D) 3/8 **[A]**

Q.25 The activity of a certain radionuclide decreases to 15 percent of its original value in 10 days. Find its half-life. [ln (0.15) = – 0.19].
(A) 3.00 days (B) 3.50 days
(C) 3.65 days (D) 3.8 days **[C]**

Q.26 The half life of ^{24}Na is 15.0 h. How long does it take for 80 percent of a sample of this nuclide to decay ?
(A) 30 h (B) 34.8 h
(C) 40 h (D) 32.2 h **[B]**

Q.27 One g of ^{226}Ra has an activity of nearly 1Ci. Determine the half-life of ^{226}Ra.
(A) 1.6×10^2 y (B) 1.6×10^3 y
(C) 1.6 y (D) 8 y **[B]**

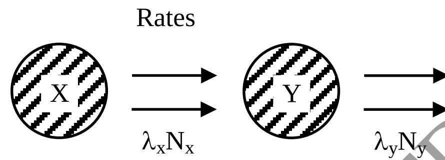
Q.28 The half-life of ${}^{238}_{92}U$ against alpha decay is 4.5×10^9y. Find the activity of 1.0g of ^{238}U.
(A) 1.2×10^4 Bq (B) 1.2×10^3 Bq
(C) 1.2×10^2 Bq (D) 1.2×10^1 Bq **[A]**

Q.29 The activity of a sample of an unknown radionuclide is measured in daily intervals. The results, in MBq, are 33.0, 27.7, 23.3, 19.6 and 16.5. Find the half life of the radionuclide.
(A) 8 days (B) 2 days
(C) 16 days (D) 4 days **[D]**

Q.30 Two radioactive sources A and B of half lives of 1 hour and 2 hours respectively initially contain the same number of radioactive atoms. At the end of two hours, their rates of disintegration are in the ratio of -
(A) 1 : 4 (B) 1 : 3
(C) 1 : 2 (D) 1 : 1 **[D]**

Q.31 A radioactive isotope is being produced at a constant rate X. Half-life of the radioactive substance is Y. After some time the number of radioactive nuclei become constant. The value of this constant is:
(A) $\frac{XY}{\ln(2)}$ (B) XY
(C) (XY) ln (2) (D) $\frac{X}{Y}$ **[A]**

Sol. Number of radio-nuclei become constant, when



$X = \lambda N$

or $N = \frac{X}{\lambda}$

$= \frac{X}{\frac{\ln 2}{Y}} = \frac{XY}{\ln(2)}$

Q.32 In one average life –
(A) half the active nuclei decay
(B) less than half the active nuclie decay
(C) more than half the active nuclie decay
(D) all the nuclie decay **[C]**

Q.33 A freshly prepared radio active source of half life 2 hours emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with this source is–
(A) 6 h (B) 12 h
(C) 24 h (D) 128 h **[B]**

Q.34 The decay constant of a radioactive sample is λ. The half life and the average life of the sample are respectively.
(A) $\frac{1}{\lambda}$ & $\frac{\ell n2}{\lambda}$ (B) $\frac{\ell n2}{\lambda}$ & $\frac{1}{\lambda}$
(C) $\lambda\ \ell n2$ & $\frac{1}{\lambda}$ (D) $\frac{\lambda}{\ell n2}$ & λ **[B]**

Q.35 The mean free path of a 5 eV neutron in vacuum is closest to (Life time of neutron is about 10^3 sec) –
(A) 10 km (B) 100 km
(C) 1,000 km (D) 10,000 km **[D]**

Sol. The mean free path in vacuum is the distance the neutron travels in its lifetime, from generation to decay. Lifetime of the neutron is about 10^3 s. As its energy 5 eV is much less than its rest energy 940 MeV, non-relativistic approximation may be used and its velocity is

$$v = \sqrt{\frac{2E}{m}} = c\sqrt{\frac{2E}{mc^2}} = \sqrt{\frac{2\times5\times10^{-6}}{940}} \times 3 \times 10^8 =$$

10^4 m/s.

Thus $S = vt = 10^4$ km.

Q.36 The activity of a sample of radioactive material is A_1 at time t_1 and A_2 at time $t_2 (t_2 > t_1)$. Its mean life is T then which of the following is correct ?

(A) $A_1t_1 = A_2t_2$ (B) $\frac{A_1 + A_2}{t_2 - t_1}$ = constant

(C) $A_2 = A_1e^{(t_1-t_2)/T}$ (D) $A_2 = A_1e^{(t_1/Tt_2)}$

Sol. **[C]**

$A_1 = A_0e^{-t_1/T}$

$A_2 = A_0e^{-t_2/T}$

$\frac{A_1}{A_2} = e^{(t_1-t_2)/T}$

$A_2 = A_1e^{(t_1-t_2)/T}$

Q.37 Probability that a radioactive nucleus will not decay in time t will be: (given decay constant = λ)

(A) $e^{-\lambda t}$ (B) $1- e^{-\lambda t}$

(C) $e^{\lambda t}$ (D) $1- e^{\lambda t}$ **[A]**

Sol. $N = N_0 e^{-\lambda t}$

$P = \frac{N}{N_0} = e^{-\lambda t}$

Q.38 Half-life of a radioactive substance A is two times the half-life of another radioactive substance B. Initially the number of nuclei of A and B are N_A and N_B respectively. After three half lives of A number of nuclei of both are equal. Then the ratio N_A/N_B is:

(A) 1/4 (B) 1/8

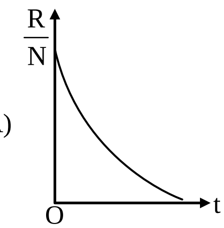
(C) 1/3 (D) 1/6 **[B]**

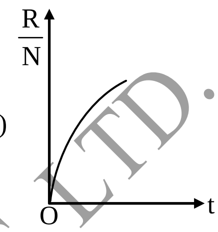
Sol. Three half-lives of A is equivalent to six half-lives of B.

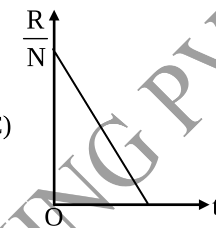
Hence, $N_A\left(\frac{1}{2}\right)^3 = N_B\left(\frac{1}{2}\right)^6$

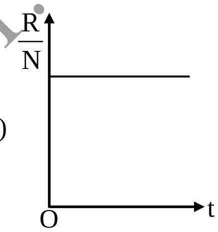
or $\frac{N_A}{N_B} = \frac{1}{8}$

Q.39 A radioactive sample has N_0 active atoms at t = 0. If the rate of disintegration at any time is R the number of atoms is N, then the ratio R/N varies with time as -

(A) 

(B) 

(C) 

(D) 

[D]

Q.40 In free space the intensity of 5 eV neutron beam is reduced by a factor of one half. Half life is $t_{1/2}$ = 12.8 min. The distance travelled by neutron beam is-

(A) 2800 km (B) 23800 km

(C) 28 km (D) 2 km **[B]**

Sol. Speed of the neutrons in beam is

$\frac{1}{2}mv^2 = K = 5eV$

$v = \sqrt{\frac{2(5)\times1.6\times10^{-19}}{1.67\times10^{-27}}}$

Q.41 A sample contains large number of nuclei. The probability that a nucleus in sample will decay after four half lives is-

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$

(C) $\frac{15}{16}$ (D) $\frac{7}{16}$ **[C]**

Sol. Probability that a nucleus will not decay is-

$$\left(\frac{N}{N_0}\right) = \left(\frac{1}{2}\right)^n = q$$

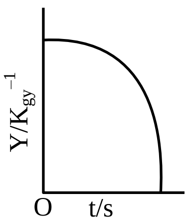
When n is the number of half lives

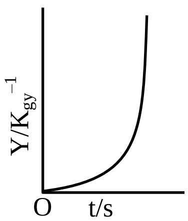
$$q = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

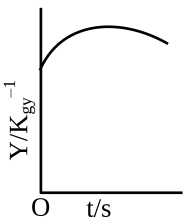
Probability that a nucleus will decay is

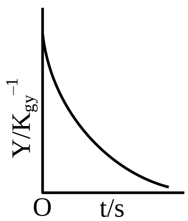
$$p = 1 - q = 1 - \frac{1}{16} = \frac{15}{16}$$

Q.42 The radioactive nucleus of an element X decays to a stable nucleus of elements Y. A graph of the rate of formation of Y against time would look like -

(A) 

(B) 

(C) 

(D) 

[D]

Q.43 There are two radioactive substances A and B. Decay constant of B is two times that of A. Initially both have equal number of nuclei. After n half lives of A rate of disintegration of both are equal. The value of n is:

(A) 1 (B) 2

(C) 4 (D) all of these **[A]**

Sol. Let $\lambda_A = \lambda$ and $\lambda_B = 2\lambda$

Initially rate of disintegration of A is λN_0 and that of B is $2\lambda N_0$.

After one half-life of A, rate of disintegration of A will becomes $\frac{\lambda N_0}{2}$ and that of B would also be $\frac{\lambda N_0}{2}$ (half-life) of B = $\frac{1}{2}$ (half-life of A)

So, after one half-life of A or two half-lives of B.

$$\left(-\frac{dN}{dt}\right)_A = \left(-\frac{dN}{dt}\right)_B$$

$\therefore$ n = 1

Q.44 How would the radio isotope of magnesium with atomic mass 27 undergo radioactive decay ?

(A) Electron capture

(B) Alpha decay

(C) Beta decay

(D) Gamma ray emission **[C]**

Sol. $^{27}_{12}Mg \rightarrow ^{27}_{13}A\ell + e^- + \bar{\nu}$

Beta decay in which isotope $^{27}_{12}Mg$ is converted to an isotope of aluminum $^{27}_{13}A\ell$.

Q.45 A radioactive substance is being produced at a constant rate of 200 nuclei/s. The decay constant of the substance is 1 s^{-1}. After what time the number of radioactive nuclei will become 100. Initially there are no nuclei present ?

(A) 1 s (B) $\frac{1}{\ln(2)}$ s

(C) ln (2) s (D) 2 s **[C]**

Sol. Let N be the number of nuclei at any time t. Then

$$\frac{dN}{dt} = 200 - \lambda N$$

$$\therefore \int_0^N \frac{dN}{200 - \lambda N} = \int_0^t dt$$

$$\text{or } N = \frac{200}{\lambda}(1 - e^{-\lambda t})$$

Given that N = 100 and $\lambda = 1\ s^{-1}$

$\therefore$ 100 = 200 $(1 - e^{-t})$

$$\text{or } e^{-t} = \left(\frac{1}{2}\right)$$

$\therefore$ t = ln (2) sec.

Q.46 The mean lives of a radioactive material for α and β radiations are 1620 years and 520 years respectively. The material decays simultaneously for α and β decay. The time after which one fourth of the material remains undecayed is -

(A) 540 years (B) 324 years

(C) 720 years (D) 840 years **[A]**

Sol. $\tau = \dfrac{\tau_\alpha \tau_\beta}{\tau_\alpha + \tau_\beta} = \dfrac{1620 \times 520}{1620 + 520}$

= 394 years

t = time of decay = $2.303 \log_{10} \dfrac{N_0}{N} \times \tau$

$= 2.303 \times \log_{10} 4 \times 394$

= 540 years

Q.47 Number of nuclei of a radioactive substance at time t = 0 are 1000 and 900 at time t = 2 s. Then number of nuclei at time t = 4 s will be:

(A) 800 (B) 810

(C) 790 (D) 700 **[B]**

Sol. In 2s only 90% nuclei are left behind. Thus, in next 2s 90% of 900 or 810 nuclei will be left.

Q.48 In a sample of a radioactive substance what fraction of the initial number of nuclei will remain undecayed after a time $t = \dfrac{T}{2}$, where T = half-life of radioactive substance:

(A) $\dfrac{1}{\sqrt{2}}$ (B) $\dfrac{1}{2\sqrt{2}}$

(C) $\dfrac{1}{4}$ (D) $\dfrac{1}{\sqrt{2}-1}$ **[A]**

Sol. Fraction of nuclei which remain undecayed is

$f = \dfrac{N}{N_0} = \dfrac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$

$= e^{-\left(\frac{\ln 2}{T}\right)\left(\frac{T}{2}\right)}$

$= \dfrac{1}{e^{\ln\sqrt{2}}} = \dfrac{1}{\sqrt{2}}$

Q.49 β decays, the number of α and β decays undergone is:

(A) 7 and 5 (B) 7 and 7

(C) 5 and 7 (D) 7 and 9 **[B]**

Sol. Let number of α decays are x and number of β decays are y. Then

$92 - 2x + y = 85$

or $2x - y = 7$ … (1)

and $238 - 4x = 210$

$\therefore x = 7$

Substituting this value in Eq. (1), we get

y = 7

Q.50 The half-value period of a radioactive element is 20 seconds. At any instant the number of radioactive nuclei is one million. Ten seconds later, the number of radioactive nuclei left are -

(A) $10^6 \sqrt{2}$ (B) $\dfrac{10^6}{\sqrt{2}}$

(C) 5×10^5 (D) 2.5×10^5 **[B]**

Q. 1 ^{23}Ne decays to ^{23}Na by negative beta emission. Mass of ^{23}Ne is 22.994465 amu mass of ^{23}Na is 22.989768 amu. The maximum kinetic energy of emitted electrons neglecting the kinetic energy of recoiling product nucleus is ....MeV

Sol. **[0004]**

$^{23}_{10}Ne \rightarrow ^{23}_{11}Na + e^- + \bar{\nu}$

$Q = [m(^{23}Ne) - m(^{23}Na)] \times 931.5$ MeV

$A = 4.375$ MeV $= 4.4$ MeV

$Q \simeq 4$ MeV

$Q = KE_y + KE_e + E\bar{\nu}$

KE_y is very very small

$A \approx KE_e + E\bar{\nu}$

when KE_e is maximum $E\bar{\nu}$ is negligible

$KE_e \simeq Q = 4$ MeV

Q.2 In U^{238} ore containing Uranium the ratio of U^{234} to Pb^{206} nuclei is 3. Assuming that all the lead present in the ore is final stable product of U^{238}. Half life of U^{238} to be 4.5×10^9 years and find the age of ore. (in 10^9 years) **[0002]**

Sol. $\frac{N}{N_0 - N} = \frac{3}{1} \Rightarrow \frac{N}{N_0} = \frac{3}{4}$

$\frac{N}{N_0} = \frac{3}{4} = e^{-\lambda t}$

$\lambda t = \log_e 4/3 \Rightarrow t = \left(\frac{\log_e 4/3}{0.693}\right) T_{1/2}$

$t = 4.5 \times 10^9 \left[\frac{.1249}{.3010}\right]$

$= 1.8678 \times 10^9$ years $\approx 2 \times 10^9$ years

Q.3 The radioactivity of an old sample of whisky due to tritium (half life 12.5 years) was found to be only about 4% of that measured in a recently purchased bottle marked 10 years old. Find the age of sample in years. **[0068]**

Sol. $N_1 = N_0 e^{-\lambda.10}$

$N_2 = N_0 e^{-\lambda x}$

$\frac{N_2}{N_1} = \frac{4}{100} = e^{\lambda(10-x)}$

$e^{\lambda(x-10)} = \frac{100}{4}$

$\lambda(x-10) = 2\,\ell n\, 10 - \ell n\, 4$

$\lambda(x-10) = 2(2.3) - 2(0.693)$

$\lambda(x-10) = 3.22$

Now $\lambda = \frac{0.693}{12.5}$ yr^{-1}

$\therefore x - 10 = \frac{12.5}{0.693} \times 3.22 = 58.08$

$x \simeq 68$ years

Q.4 The nuclei of two radioactive isotopes of same substance A^{236} and A^{234} are present in the ratio 4 : 1 in an ore obtained from Mars. Their half lives are 30 min and 60 min respectively. Both isotopes are alpha emitters and the activity of the isotope with half life 30 min is one Rutherford . Calculate after how much time (in min) their activities will become identical. **[0180]**

Sol.

	A^{236}	:	A^{234}
Nuclei	$4N_0$	:	N_0
Half life	30	:	60

Activity $= \lambda N = \lambda N_0 e^{-\lambda t}$

$\lambda_1 4N_0\, e^{-\frac{0.693}{30}t} = \lambda_2 N_0 e^{-\frac{0.693}{60}t}$

$\frac{0.693}{30} \times 4N_0 e^{-\frac{0.693}{30}t} = \frac{0.693}{60} \times N_0 e^{-\frac{0.693}{60}t}$

$8 = e^{0.693t\left(\frac{1}{30} - \frac{1}{60}\right)}$

$8 = e^{+\frac{0.693}{60}t}$

$3 \times 0.693 = \frac{0.693t}{60}$

$t = 180$ min

Q.5 An unstable element is produced in a nuclear reactor at a constant rate. If its half life is 100 years, how much time in years is required to produce 50% of the equilibrium quantity ?

[0100]

Sol. Let rate of production = R

$\therefore \frac{dN}{dt} = R - \lambda N$

$\frac{dN}{dt} + \lambda N = R$

$e^{\lambda t} \frac{dN}{dt} + \lambda N e^{\lambda t} = Re^{\lambda t}$

$\frac{d(Ne^{\lambda t})}{dt} = Re^{\lambda t}$

$Ne^{\lambda t} = \frac{Re^{\lambda t}}{\lambda} + C$

At t = 0, N = 0 $\Rightarrow$ $C = -\frac{R}{\lambda}$

$\therefore N = \frac{R}{\lambda}(1 - e^{-\lambda t})$

At equilibrium quantity $N = \frac{R}{\lambda}$ for $t \to \infty$

$\therefore \frac{R}{2\lambda} = \frac{R}{\lambda}(1 - e^{-\lambda t})$

$\Rightarrow e^{-\lambda t} = \frac{1}{2}$

$t = \frac{\ell n2}{\lambda} = T_{1/2} = 100$ years

Q.6 Find the activity of 0.5 mg of radon-222 in curie. It is known that half-life of radon is 3.8 days. **[0077]**

Sol. $\left|\frac{dN}{dt}\right| = \lambda N_0$

$= \frac{\ell n2}{T_{1/2}} N_0$

$= \frac{0.693 \times 0.5 \times 10^{-3} \times 6.02 \times 10^{23}}{3.8 \times (24 \times 3600) \times 222 \times (3.7 \times 10^{10})}$

= 77.35 mg

$\simeq$ 77 mg

Q.7 A radioactive sample decays with a mean life of 20 millisecond. A capacitor of capacitance 100 μF is charged to some potential and then the plates are connected through a wire of resistance R. What should be the value of R in ohm so that the ratio of the charge on the capacitor to the activity of the radioactive sample remain constant in time ? **[0200]**

Sol. $\frac{Q}{A} = \frac{Q_0 e^{-t/RC}}{A_0 e^{-\lambda t}} = \frac{Q_0}{A_0} e^{(\lambda - \frac{1}{RC})t}$

$\lambda - \frac{1}{RC} = 0$

$\Rightarrow \lambda = \frac{1}{RC}$

$\lambda = \frac{1}{C\lambda} = \frac{T}{C} = \frac{20 \times 10^{-3}}{100 \times 10^{-6}} = 200\ \Omega$

Q.8 The mean lives of a radioactive substance are 1620 and 405 years for α-emission and β-emission respectively. Find out the time (in years) after which three fourth of a sample will decay if it is decaying both by α-emission and β-emission simultaneously. (Take ℓn 2 = 0.693) **[0449]**

Sol. $\frac{1}{T} = \frac{1}{T_\alpha} + \frac{1}{T_\beta}$

$\Rightarrow T = \frac{T_\alpha T_\beta}{T_\alpha + T_\beta} = 324$ years

$\frac{N}{N_0} = e^{-\lambda t}$

$t = \frac{1}{\lambda} \ell n \frac{N_0}{N} = T \ell n \frac{N_0}{N}$

$t = 324 \times 2\ell n\, 2$

t = 449 .06 years

t $\approx$ 449 years

Q.9 If 20 gm of a radioactive substance due to radioactive decay reduces to 10gm in 4 minutes, then in what time (in minutes) 80gm of the same substance will reduce to 20 gm -

Sol. **[8]**

$N = N_0\left(\frac{1}{2}\right)^n$

$20 = 80\left(\frac{1}{2}\right)^n$

$\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^2$ [n = 2]

$t = nT_{1/2} = 2 \times 4 = 8$ minutes

Q.10 Equal masses of two samples of charcoal A and B are burnt separately and the resulting carbon dioxide are collected in two vessels. The radioactivity of ^{14}C is measured for both the samples. The gas from the charcoal A gives 2100 counts per week and gas from the charcoal B gives 1400 counts per week. Find the age difference between two samples. Half life of ^{14}C = 5730 years and [$\log_e\left(\frac{3}{2}\right)$ = 0.4055]

Sol. **[3352]**

Q.11 A polonium ($_{84}P_0^{209}$) nucleus transforms into one of lead ($_{82}Pb^{207}$) by emitting an α-particle, then the kinetic energy of the α-particle in MeV is -

[m (P_0) = 209.98297u ; m (Pb) = 205.97446

m (α-particle) = 4.00260 u]

Q.12 The energy in MeV required to extract a neutron from a carbon nucleus with mass number 13 is -

[m ($_6C^{13}$) = 13.00335u ; m ($_6C^{12}$) = 12.0000u

m_n = 1.00867 u ; m_p = 1.00783u]

Sol. **[5]**

Energy required is equal to difference in binding energy of parent nucleus and daughter nucleus.

Q.13 A nucleus at rest undergoes a decay emitting an α-particle of de-Broglie wavelength 5.76×10^{-15} m. If the mass of daughter nucleus is 223.610 amu and that of α-particle is 4.002 amu. The mass of the parent nucleus is 22X amu then find X appearing in the number 22X.

(1 amu = 931.47 MeV/c^2)

Sol. **[8]**

$\lambda = \frac{h}{P}$ for α-particle

$(K.E.)_\alpha = \frac{P^2}{2m_\alpha}$ and $(K.E.)_{nucleus} = \frac{P^2}{2m_n}$

$E = \frac{P^2}{2}\left[\frac{1}{m_\alpha}+\frac{1}{m_n}\right]$ = 6.25 MeV

$\therefore$ mass of parent nucleus = $\left(m_n + m_\alpha + \frac{E}{c^2}\right)$

= 227.62 amu

Q.14 There are two radio nuclei A and B. A is an alpha emitter and B a beta emitter. Their disintegration constants are in ratio of 1 : 2. The ratio of number of atoms of A and B at any time t so that probabilities of getting alpha and beta particles are same at that instant is -

Sol. **[2]**

$\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$ Probabilities of getting α and β particles are equal. Thus rate of disintegration are equal

$\therefore \lambda_A N_A = \lambda_B N_B$

Q.15 Half life of radioactive substance A is two time that of B. Initially number of nuclei of A and B are N_A and N_B respectively. After three half lives of A number of nuclei of both are equal. Then the ratio $\frac{N_B}{N_A}$ is –

Sol. **[8]**

3 half lives of A is equivalent to 6 half lives of B.

$\therefore N_A\left(\frac{1}{2}\right)^3 = N_B\left(\frac{1}{2}\right)^6$

Q.16 There are two radioactive substances A and B. Decay constant of B is two times that of A. Initially both have equal number of nuclei. After n half lives of A rate of disintegration of both are equal then the value of n is -

Sol. **[1]**

Let $\lambda_A = \lambda$ and $\lambda_B = 2\lambda$

Initially rate of disintegration of A is is λN_0 and that of B is $2\lambda N_0$.

After one half life of A, rate of disintegration of A will becomes $\frac{\lambda N_0}{2}$ and that of B would also be $\frac{\lambda N_0}{2}$ so after one half life of A or two half life of B. $\left(\frac{-dN}{dt}\right)_A = \left(\frac{-dN}{dt}\right)_B$

$\therefore$ n = 1

Q.17 Number of nuclei of a radioactive substance at t = 0 are 1000 and 900 at t = 2 sec. The number of nuclei at t = 4 sec will be x10, then the value of x in number x10 is -

Sol. **[8]**

In 2 sec only 90% of nuclei are left. Thus in next 2 sec. 90% of 900 or 810 nuclei will be left.

Q.18 If 20 gm of a radioactive substance due to radioactive decay reduces to 10 gm in 4 minutes, then in what time (in minutes) 80 gm of the same substance will reduce to 20 gm.

Sol.[4] $N = N_0\left(\frac{1}{2}\right)^n$

$20 = 80\left(\frac{1}{2}\right)^n$

$\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^2$ [n = 2]

$t = n\,T_{1/2} = 2 \times 4 = 8$ minutes