Parabola

Single Correct Answer Type

1. A straight line through A(6, 8) meets the curve $2x^2+y^2=2$ at B and C. P is a point on BC such that AB, AP, AC are in H.P, then the minimum distance of the origin from the locus of 'P' is

A) $\frac{1}{\sqrt{52}}$ B) $\frac{5}{\sqrt{52}}$ C) $\frac{10}{\sqrt{52}}$ D) $\frac{15}{\sqrt{52}}$

Key. A

Sol. $(6+r\cos\theta, 8+r\sin\theta)$ lies on $2x^2+y^2=2$

$\Rightarrow (2\cos^2\theta+\sin^2\theta)r^2+2(12\cos\theta+8\sin\theta)r+134=0$

AB, AP, AC are in H.P $\Rightarrow \frac{2}{r}=\frac{AB+AC}{AB.AC} \Rightarrow \frac{1}{r}=-\frac{(6\cos\theta+4\sin\theta)}{67} \Rightarrow 6x+4y-1=0$

Minimum distance from $'O'=\frac{1}{\sqrt{52}}$

2. Let A (0, 2), B and C are points on parabola $y^2=x+4$ and such that $\underline{|CBA}=\frac{\Pi}{2}$, then the range of ordinate of C is

A) $(-\infty,0)\cup(4,\infty)$ B) $(-\infty,0]\cup[4,\infty)$

C) $[0,4]$ D) $(-\infty,0)\cup[4,\infty)$

Key. B

Sol. $A(0,2)$, $B=(t_1^2-4, t_1)$ $C=(t^2-4,t)$

$\frac{2-t_1}{4-t_1^2}.\frac{t_1-t}{t_1^2-t^2}=-1 \Rightarrow \frac{1}{2+t_1}.\frac{1}{t+t_1}=-1 \Rightarrow t_1^2+(2+t)t_1+(2t+1)=0$

For real t_1, $\Rightarrow (2+t)^2-4(2t+1)=0 \Rightarrow t^2-4t\ge 0 \Rightarrow t\in(-\alpha,0]\cup[4,\alpha)$

3. If $2p^2-3q^2+4pq-p=0$ and a variable line $px+qy=1$ always touches a parabola whose axis is parallel to X-axis, then equation of the parabola is

A) $(y-4)^2=24(x-2)$ B) $(y-3)^2=12(x-1)$

C) $(y-4)^2=12(x-2)$ D) $(y-2)^2=24(x-4)$

Key. C

Sol. The parabola be $(y-a)^2=4b(x-c)$

Equation of tangent is $(y-a)=-\frac{p}{q}(x-c)-\frac{bq}{p}$

Comparing with $px+qy=1$, we get $cp^2-bq^2+apq-p=0$

$\therefore \frac{c}{2}=\frac{b}{3}=\frac{a}{4}=1 \Rightarrow$ the equation is $(y-4)^2=12(x-2)$

4. Consider the parabola $x^2+4y=0$. Let $p=(a,b)$ be any fixed point inside the parabola and let 'S' be the focus of the parabola. Then the minimum value at $SQ+PQ$ as point Q moves on the parabola is

A) $|1-a|$ B) $|ab|+1$ C) $\sqrt{a^2+b^2}$ D) $1-b$

Key. D

Sol. Let foot of perpendicular from Q to the directrix be N

$\Rightarrow SQ+PQ=QN+PQ$ is minimum it $P, Q \& N$ are collinear

So minimum value of $SQ+PQ=PN=1-b$

5. The locus point of intersection of tangents to the parabola $y^2=4ax$, the angle between them being always $45°$ is

A) $x^2-y^2+6ax-a^2=0$ B) $x^2-y^2-6ax+a^2=0$

C) $x^2-y^2+6ax+a^2=0$ D) $x^2-y^2-6ax-a^2=0$

Key. C

Sol. Equation of tangent is $y=mx+\frac{a}{m}$

$\Rightarrow m^2x-my+a=0 \Rightarrow m_1+m_2=\frac{y}{x}, m_1m_2=\frac{a}{x}$

$\tan 45°=\left|\frac{m_1-m_2}{1+m_1m_2}\right| \Rightarrow \left(\frac{y}{x}\right)^2-4\left(\frac{a}{x}\right)=\left(1+\frac{a}{x}\right)^2$

$\Rightarrow x^2-y^2+6ax+a^2=0$

6. The coordinates of the point on the parabola $y=x^2+7x+2$, which is nearest to the straight line $y=3x-3$ are

1) $(-2,-8)$ 2) $(1,10)$ 3) $(2,20)$ 4) $(-1,-4)$

Key. 1

Sol. Hint: Any point on the parabola is (x, x^2+7x+2)

Its distance from the line $y=3x-3$ is given by

$$P=\left|\frac{3x-\left(x^2+7x+2\right)-3}{\sqrt{9+1}}\right|=\left|\frac{x^2+4x+5}{\sqrt{10}}\right|=\frac{x^2+4x+5}{\sqrt{10}}\left(as\,x^2+4x+5>0\,\forall x\in R\right)$$

$\frac{dp}{dx}=0\Rightarrow x=-2$ the required point = $(-2,-8)$

7. The point P on the parabola $y^2=4ax$ for which $|PR-PQ|$ is maximum, where $R=(-a,0), Q=(0,a)$. is

1) $(a,2a)$ 2) $(a,-2a)$ 3) $(4a,4a)$ 4) $(4a,-4a)$

Key. 1

Sol. We know that any side of the triangle is more than the difference of the remaining two sides so that $|PR-PQ|\le RQ$

The required point P will be the point of intersection of the line RQ with parabola which is $(a,2a)$ as PQ is a tangent to the parabola

8. The number of point(s) (x,y)(where x and y both are perfect squares of integers) on the parabola $y^2 = px$, p being a prime number, is

1) zero 2) one 3) two 4) infinite

Key. 2

Sol. If x is a perfect square ,then px will be a perfect square only if p is a perfect square, which is not possible as p is a prime number. Hence y cannot be a perfect square . So number of such points will be only one $(0,0)$

9. The locus of point of intersection of any tangent to the parabola $y^2=4a(x-2)$ with a line perpendicular to it and passing through the focus, is

1) $x=2$ 2) $y=0$ 3) $x=a$ 4) $x=a+2$

Key. 1

Sol. It is well known property of a parabola that a tangent and normal to it from focus intersect at tangent at vertex

10. If the parabola $y=(a-b)x^2+(b-c)x+(c-a)$ touches the $x-axis$ then the line $ax+by+c=0$

1) Always passes through a fixed point 2) represents the family of parallel lines

3) always perpendicular to x-axis 4) always has negative slope

Key. 1

Sol. Solving equation of parabola with x-axis (y=0)

We get $(a-b)x^2+(b-c)x+(c-a)=0,$ which should have two equal values of x, as x-axis touches the parabola $\Rightarrow (b-c)^2-4(a-b)(c-a)=0$

$\Rightarrow (b+c-2a)^2=0 \Rightarrow -2a+b+c=0 \Rightarrow ax+by+c=0$ always passes through $(-2,1)$

11. If one end of the diameter of a circle is $(3,4)$ which touches the $x-axis$ then the locus of other end of the diameter of the circle is

1) Circle 2) parabola 3) ellipse 4) hyperbola

Key. 2

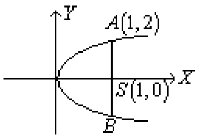
Sol. Let other end of diameter (h,k)

Hence centre is $\sqrt{\left(\frac{3+h}{2}-3\right)^2+\left(\frac{k+4}{2}-4\right)^2}$ gives the equation of parabola

12. The point $(1,2)$ is one extremity of focal chord of parabola $y^2=4x$. The length of this focal chord is

1) 2 2) 4 3) 6 4) none of these

Key. 2

Sol. 

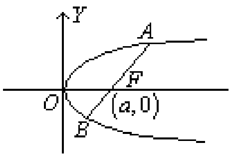
The parabola $y^2=4x,$ here $a=1$ and focus is $(1,0)$

The focal chord is ASB. This is clearly latus rectum of parabola, its value = 4

13. If AFB is a focal chord of the parabola $y^2=4ax \text{ and } AF=4, FB=5$ then the latus-rectum of the parabola is equal to

1) $\frac{80}{9}$ 2) $\frac{9}{80}$ 3) 9 4) 80

Key. 1

Sol. 

FA = 4 , FB = 5

We know that $\frac{1}{a}=\frac{1}{AF}+\frac{1}{FB}$

$\Rightarrow a=\frac{20}{9}\Rightarrow 4a=\frac{80}{9},$

14. If at $x=1, y=2x$ tangent to the parabola $y=ax^2+bx+c,$ then respective values of a,b,c possible are

1) $\frac{1}{2},1,\frac{1}{2}$ 2) $1,\frac{1}{2},\frac{1}{2}$ 3) $\frac{1}{2},\frac{1}{2},1$ 4) $\frac{-1}{2},1,\frac{3}{2}$

Key. 1

Sol. for x =1 , $y=a+b+c$

Tangent at $(1,a+b+c) is \frac{1}{2}(y+a+b+c)=ax+\frac{b}{2}(x+1)+c$

Comparing with $y=2x, c=a, b=2(1-a)$

Which are true for choice (1) only

15. The number of focal chords of length 4/7 in the parabola $7y^2=8x$ is

1) one 2) zero 3) two 4) infinite

Key. 2

Sol. since length of latus –rectum = $\frac{8}{7}$

Latus-rectum is the smallest focal chord

Hence focal chord of length $\frac{4}{7}$ does not exist.

16. The length of the chord of the parabola $x^2=4y$ passing through the vertex and having slope $\cot\alpha$ is

(1) 4 $\cos\alpha . \operatorname{cosec}^2\alpha$ (2) $4\tan\alpha\sec\alpha$ (3) $4\sin\alpha.\sec^2\alpha$ (4) none of these

Key. 1

Sol. Let $A=$vertex, AP = chord of $x^2=4y$ such that slope of AP is $\cot\alpha$

Let $P=\left(2t, t^2\right)$

Slope of $AP=\frac{1}{2}\Rightarrow \cot\alpha=\frac{1}{2}\Rightarrow t=2\cot\alpha$

Now, $AP=\sqrt{4t^2+t^4}=t\sqrt{4+t^2}$

$=4\cot\alpha\operatorname{cosec}\alpha$

$=4\cos\alpha.\operatorname{cosec}^2\alpha$

17. Slope of tangent to $x^2=4y$ from $(-1,-1)$ can be

1) $\frac{-1\pm\sqrt{5}}{2}$ 2) $\frac{-3-\sqrt{5}}{2}$ 3) $\frac{1-\sqrt{5}}{2}$ 4) $\frac{1+\sqrt{5}}{2}$

Key. 1

Sol. $y^1=\frac{x}{2}=m$

$\Rightarrow x=2m \Rightarrow y=m^2$

So equation of tangent is $y-m^2=m(x-2m)$ which passes through $(-1,-1)$

$\Rightarrow -1-m^2=m(-1-2m)$

$\Rightarrow m^2+m-1=0 \quad \Rightarrow m=\frac{-1\pm\sqrt{5}}{2}$

18. If line $y=2x+\frac{1}{4}$ is tangent to $y^2=4ax,$ then a is equal to

1) $\frac{1}{2}$ 2) 1 3) 2 4) None of these

Key. 1

Sol. $c=\frac{a}{m} \Rightarrow a=2\left(\frac{1}{4}\right)=\frac{1}{2}$

19. The Cartesian equation of the curve whose parametric equations are $x=t^2+2t+3$ and $y=t+1$ is

1) $y=(x-1)^2+2(y-1)+3$ 2) $x=(y-1)^2+2(y-1)+5$

3) $x=y^2+2$ 4) none of these

Key. 3

Sol. $x=t^2+2t+3=(t+1)^2+2=y^2+2$

20. If the line $y-\sqrt{3}x+3=0$ cuts the parabola $y^2=x+2$ at A and B, then PA. PB is equal to (where $P\equiv(\sqrt{3},0)$)

1) $\frac{4(\sqrt{3}+2)}{3}$ 2) $\frac{4(2-\sqrt{3})}{3}$ 3) $\frac{4\sqrt{3}}{2}$ 4) $\frac{2(\sqrt{3}+2)}{3}$

Key. 1

Sol. $y-\sqrt{3}x+3=0$ can be rewritten as

$\frac{y-0}{\frac{\sqrt{3}}{2}}=\frac{x-\sqrt{3}}{\frac{1}{2}}=r \quad (1)$

Solving (1) with the parabola $y^2=x+2$

$\frac{3r^2}{4}-\frac{r}{2}-\sqrt{3}-2=0 \Rightarrow PA.PB=r_1r_2=\frac{4(\sqrt{3}+2)}{3}$

21. The equation of the line of the shortest distance between the parabola $y^2=4x$ and the circle $x^2+y^2-4x-2y+4=0$ is.

1) $x+y=3$ 2) $x-y=3$ 3) $2x+y=5$ 4) none of these

Key. 1

Sol. Line of shortest distance is normal for both parabola and circle

Centre of circle is $(2,1)$

Equation of normal to circle is $y-1=m(x-2)\Rightarrow y=mx+(1-2m) \quad (1)$

Equation of normal for a parabola is $y=mx-2am-am^3 \quad (3)$

Comparing (1) and (2)

$am^3=-1\Rightarrow m^3=-1\Rightarrow m=-1 \qquad (a=1)$

Equation is $y-1=-x+2\Rightarrow x+y=3$

22. If $x+k=0$ is equation of directrix to parabola $y^2=8(x+1)$ then $k=$

1) 1 2) 2 3) 3 4) 4

Key. 3

Sol. Focus is $(1,0)$ third vertex is $(-1,0)$. Hence directrix is $x+3=0$

23. If t is the parameter for one end of a focal chord of the parabola $y^2=4ax$, then its length is

1) $a\left(t+\frac{1}{t}\right)^2$ 2) $a\left(t-\frac{1}{t}\right)^2$ 3) $a\left(t+\frac{1}{t}\right)$ 4) $a\left(t-\frac{1}{t}\right)$

Key. 1

Sol. Conceptual

24. The ends of the latus rectum of the conic $x^2+10x-16y+25=0$ are

(1) $(3,-4),(13,4)$ (2) $(-3,-4),(13,-4)$ (3) $(3,4),(-13,4)$ (4) $(5,-8),(-5,8)$

Key. 3

Sol. $(x+5)^2=16y$ comparing it with $x^2=4ay$,

25. If the lines $(y-b)=m_1(x+a)$ and $(y-b)=m_2(x+a)$ are the tangents of $y^2=4ax$ then

1) $m_1+m_2=0$ 2) $m_1m_2=1$ 3) $m_1m_2=-1$ 4) $m_1+m_2=1$

Key. 3

Sol. $y=mx+\frac{a}{m}$

$\Rightarrow m^2x-3y+a=0, m_1.m_2=-1$

26. The equation of a parabola is $y^2=4x.Let\,P(1,3)$ and $Q(1,1)$ are two points in the xy plane. Then, for the parabola

1) P and Q are exterior points
2) P is an interior point while Q is an exterior point
3) P and Q are interior points
4) P is an exterior point while Q is an interior point

Key. 4

Sol. Here, $S\equiv y^2-4x=0$

$S(1,3)=3^2-4.1>0$

$\Rightarrow P(1,3)$ is an exterior point $S(1,1)=1^1-4.1<0$

$\Rightarrow Q(1,1)$ is an interior point

27. If the focus of a parabola is $(-2,1)$ and the directrix has the equation $x+y=3$, then the vertex is:

1) $(0,3)$ 2) $\left(-1,\frac{1}{2}\right)$ 3) $(-1,2)$ 4) $(2,-1)$

Key. 3

Sol. The vertex is the middle point of the perpendicular dropped from the focus to the directrix.

28. The length of the latus-rectum of the parabola $169\left\{(x-1)^2+(y-3)^2\right\}=(5x-12y+17)^2$ is

1) $\frac{12}{13}$ 2) $\frac{14}{13}$ 3) $\frac{28}{13}$ 4) $\frac{31}{13}$

Key. 3

Sol. $(x-1)^2+(y-3)^2=\left(\frac{5x-12y+17}{13}\right)^2$

Length of latus rectum $=4a$

Perpendicular distance from $(1,3)$ to the line $5x-12y+17=0$ is

$$2a=\frac{|5\times 1-12\times 3+17|}{\sqrt{169}}=\frac{14}{13}$$

29. The co-ordinates of a point on the parabola $y^2=8x$ whose focal distance is 4 is

1) $(2,4)$ 2) $(4,2)$ 3) $(2,-6)$ 4) $(4,-2)$

Key. 1

Sol. $a+x=4\Rightarrow 2+x=4\Rightarrow x=2, y=4$

30. Co-ordinate of the focus of the parabola $x^2-4x-8y-4=0$ are

1) $(0,2)$ 2) $(2,1)$ 3) $\left(-3,\frac{-71}{10}\right)$ 4) $(2,-1)$

Key. 2

Sol. $(x-2)^2=8(y+1)$

Focus $x-2=0, y+1=2\Rightarrow x=2, y=1$

Focus (2,1)

31. If focal distance of a point on the parabola $y=x^2-4$ is $\frac{25}{4}$ and points are of the form $(\pm\sqrt{a},b)$ Then $a+b$ is equal to

1) 8 2) 4 3) 2 4) 0

Key. 1

Sol. $y+4=x^2$

$x^2=4.\frac{1}{4}(y+4)$

Focal distance $=\frac{25}{4}$

Distance from directrix $\left(y=\frac{-15}{4}\right)$

Ordinate of points on the parabola whose focal distance is $\frac{25}{4}$

$=\frac{-17}{4}+\frac{25}{4}=2$ points are $(\pm\sqrt{6},2)$ $\Rightarrow a+b=8$

32. Length of side of an equilateral triangle inscribed in a parabola $y^2-2x-2y-3=0$ whose one angular point is vertex of the parabola is

1) $2\sqrt{3}$ 2) $4\sqrt{3}$ 3) $-\sqrt{3}$ 4) $\sqrt{3}$

Key. 2

Sol. Length of side $=8\sqrt{3}a=8\sqrt{3}\frac{1}{2}=4\sqrt{3}$

33. Length of latus rectum of the parabola whose parametric equations are

$x=t^2+t+1, y=t^2-t+1$ where $t\in R$, is equal to

1) 4 2) +1 3) $\sqrt{2}$ 4) 3

Key. 3

Sol. $x+y=2\left(t^2+1\right)$ & $x-y=2t$

$$\therefore\left(x+y-2\right)=2\left(\frac{x-y}{2}\right)^2\Rightarrow\left(\frac{x-y}{\sqrt{2}}\right)^2=\sqrt{2}\left(\frac{x+y-2}{\sqrt{2}}\right)$$

Length of latusrectum = $\sqrt{2}$

34. In the parabola, $y^2-2y+8x-23=0$, the length of double ordinate at a distance of 4 units from its vertex is

1) $4\sqrt{2}$ 2) $8\sqrt{2}$ 3) 6 4) 4

Key. 2

Sol. Length of double ordinate = $8\sqrt{2}$

35. If any point $P(x,y)$ satisfies the relation

$(5x-1)^2+(5y-2)^2=\lambda(3x-4y-1)^2$, represents parabola, then

1) $\lambda=1$ 2) $\lambda<1$ 3) $\lambda>1$ 4) $\lambda>2$

Key. 1

Sol. Conceptual

36. The locus of the vertex of the family of parabolas $y=\frac{a^3x^2}{3}+\frac{a^2x}{2}-2a$

(a is parameter) is

(A) $xy=\frac{105}{64}$ (B) $xy=\frac{3}{4}$ (C) $xy=\frac{35}{16}$ (D) $xy=\frac{64}{105}$

Key. A

Sol. $y=\frac{a^3x^2}{3}+\frac{a^2x}{2}-2a$

$$y=\frac{2a^3}{6}\left(x^2+\frac{3}{2a}x-\frac{12a}{2a^3}\right)$$

$$y=\frac{2a^3}{6}\left(x^2+2.\frac{3}{4a}x+\frac{9}{16a^2}-\frac{9}{16a^2}-\frac{12a}{2a^3}\right)$$

$$y=\frac{2a^3}{6}\left(\left(x+\frac{3}{4a}\right)^2-\frac{1059}{16a^3}\right)$$

$$\left(y+\frac{1059}{48}\right)=\frac{2a^3}{6}\left(x+\frac{3}{4a}\right)^2$$

$$x=\frac{-1059}{48}$$

$$y=\frac{-3}{49}$$

$$xy = \frac{1059}{48} \times \frac{3}{49} = \frac{105}{64}$$

37. Tangents are drawn from the point (-1, 2) to the parabola $y^2 = 4x$. The length of the intercept made by the line x = 2 on these tangents is

(A) 6 (B) $6\sqrt{2}$ (C) $2\sqrt{6}$ (D) none

Key. B

Sol. Equation of pair of tangent is

$$SS_1 = T^2$$
$$\Rightarrow (y^2 - 4x)(8) = 4(y - x + 1)^2$$
$$\Rightarrow y^2 - 2y(1 - x) - (x^2 + 6x + 1) = 0$$
$$Put \quad x = 2$$
$$\Rightarrow y^2 + 2y - 17 = 0$$
$$\Rightarrow |y_1 - y_2| = 6\sqrt{2}$$

38. The given circle $x^2 + y^2 + 2px = 0$, $p \in R$ touches the parabola $y^2 = 4x$ externally, then

(A) p < 0 (B) p > 0 (C) 0 < p < 1 (D) p < - 1

Key. B

Sol. Centre of the circle is (- p, 0), If it touches the parabola, then according to figure only one case is possible.

Hence p > 0

39. The triangle PQR of area A is inscribed in the parabola $y^2 = 4ax$ such that P lies at the vertex of the parabola and base QR is a focal chord. The numerical difference of the ordinates of the points Q & R is

(A) $\frac{A}{2a}$ (B) $\frac{A}{a}$ (C) $\frac{2A}{a}$ (D) $\frac{4A}{a}$

Key. C

Sol. QR is a focal chord

$$\Rightarrow R(at^2, 2at) \,\&\, Q\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$$
$$\Rightarrow d = \left|2at + \frac{2a}{t}\right| = 2a\left|t + \frac{1}{t}\right|$$
$$Now \quad A = \frac{1}{2}\begin{vmatrix} at^2 & 2at & 1 \\ \frac{a}{t^2} & -\frac{2a}{t} & 1 \\ 0 & 0 & 1 \end{vmatrix} = a^2\left|t + \frac{1}{t}\right|$$
$$\Rightarrow 2a\left|t + \frac{1}{t}\right| = \frac{2A}{a}$$

40. Through the vertex O of the parabola $y^2 = 4ax$ two chords OP & OQ are drawn and the circles on OP & OQ as diameter intersect in R. If

$\theta_1, \theta_2 \& \phi$ are the inclinations of the tangents at P & Q on the parabola and the line through O, R respectively, then the value of $\cot\theta_1 + \cot\theta_2$ is

(A) – 2 tan ϕ (B) – 2 tan (π - ϕ) (C) 0 (D) 2 cot ϕ

Key. A

Sol. $Let \quad P(t_1) \& Q(t_2)$

$\Rightarrow$ Slope of tangent at P($\frac{1}{t_1}$) & at Q($\frac{1}{t_2}$) $\quad \Rightarrow \cot\theta_1 = t_1$ *and* $\cot\theta_2 = t_2$

Slope of PQ = $\frac{2}{t_1 + t_2} = \tan\phi$

$\Rightarrow \tan\phi = -\frac{1}{2}(\cot\theta_1 + \cot\theta_2) \quad \Rightarrow \cot\theta_1 + \cot\theta_2 = -2\tan\phi$

41. AB and AC are tangents to the parabola $y^2 = 4ax$. $p_1, p_2 \& p_3$ are perpendiculars from A, B & C respectively on any tangent to the curve (otherthan the tangents at B&C), then $p_1, p_2 \& p_3$ are in

(A) A.P. (B) G.P. (C) H.P (D) none

Key. B

Sol. Let any tangent is tangent at vertex x = 0 and

$Let \quad B(t_1) \& C(t_2)$

$\Rightarrow A(at_1t_2, a(t_1 + t_2))$

$\Rightarrow p_1 = at_1^2; p_2 = at_2^2 \ \& \ p_3 = at_1t_2$

$\Rightarrow p_1, p_2 \ \& \ p$ are in G.P.

42. A tangent to the parabola $x^2 + 4ay = 0$ at the point T cuts the parabola $x^2 = 4by$ at A & B. Then locus of the mid point of AB is

(A) $(b+2a)x^2 = 4b^2y$ (B) $(b+2a)x^2 = 4a^2y$

(C) $(a+2b)y^2 = 4b^2x$ (D) $(a+2b)x^2 = 4b^2y$

Key. D

Sol. Let mid point of AB is M(h, k)

Then equation of AB is $\quad hx - 2b(y+k) = h^2 - 4bk$

$Let\ T(2at, -at^2)$

$\Rightarrow$ *Equation of* tangent(AB) = x$(2at) = -2a(y - at^2)$

Compare these two equations, we get $\frac{h}{2at} = \frac{-2b}{2a} = \frac{h^2 - 2bk}{2a^2t^2}$

By eliminating t and Locus (h, k), we get $(a+2b)x^2 = 4b^2y$

43. A parabola $y = ax^2 + bx + c$ crosses the x-axis at A(p, 0) & B(q, 0) both to the right of origin. A circle also passes through these two points. The length of a tangent from the origin to the circle is

(A) $\sqrt{\frac{bc}{a}}$ (B) ac^2 (C) b/a (D) $\sqrt{\frac{c}{a}}$

Key. D

Sol. Use power of point for the point O figure

$$\Rightarrow OT^2 = OA.OB = pq = \frac{c}{a}$$

$$\Rightarrow OT = \sqrt{\frac{c}{a}}$$

44. The equation of the normal to the parabola $y^2 = 8x$ at the point t is

1. $y - x = t + 2t^2$ 2. $y + tx = 4t + 2t^3$ 3. $x + ty = t + 2t^2$ 4. $y - x = 2t - 3t^3$

Key. 2

Sol. Equation of the normal at 't' is $y + tx = 2(2)t + (2)t^3 \Rightarrow y + tx = 4t + 2t^3$

45. The slope of the normal at $(at^2, 2at)$ of the parabola $y^2 = 4ax$ is

1. $\frac{1}{t}$ 2. t 3. $-t$ 4. $-\frac{1}{t}$

Key. 3

Sol. Slope of the normal at 't' is $-t$.

46. If the normal at the point 't' on a parabola $y^2 = 4ax$ meet it again at t_1, then $t_1 =$

1. t 2. $-t - 1/t$ 3. $-t - 2/t$ 4. None

Key. 3

Sol. Equation of the normal at t is $tx + y = 2at + at^3 \rightarrow (1)$

Equation of the chord passing through t and t_1 is $y(t + t_1) = 2x + 2att_1 \rightarrow (2)$

Comparing (1) and (2) we get $\frac{t}{-2} = \frac{1}{t + t_1} \Rightarrow t + t_1 = -\frac{2}{t} \Rightarrow t_1 = -\frac{2}{t} - t$.

47. If the normal at t_1 on the parabola $y^2 = 4ax$ meet it again at t_2 on the curve, then $t_1(t_1 + t_2) + 2 =$

1. 0 2. 1 3. t_1 4. t_2

Key. 1

Sol. Equation of normal at t_1 is $t_1 x + y = 2at_1 + at_1^3$

It passes through $t_2 \Rightarrow at_1 t_2^2 + 2at_2 = 2at_1 + at_1^3$

$$\Rightarrow t_1(t_2^2 - t_1^2) = 2(t_1 - t_2) \Rightarrow t_1(t_1 + t_2) = -2 \Rightarrow t_1(t_1 + t_2) + 2 = 0$$

48. If the normal at $(1,2)$ on the parabola $y^2 = 4x$ meets the parabola again at the point $(t^2, 2t)$, then the value of t is

1. 1 2. 3 3. -3 4. -1

Key. 3

Sol. $Let(1,2) = (t_1^2, 2t_1) \Rightarrow t_1 = 1$

$t = -t_1 - \frac{2}{t_1} = -1 - \frac{2}{1} = -3$

49. If the normal to parabola $y^2 = 4x$ at $P(1,2)$ meets the parabola again in Q, then Q =

1. $(-6,9)$ 2. $(9,-6)$ 3. $(-9,-6)$ 4. $(-6,-9)$

Key. 2

Sol. $P = (1,2) = (t^2, 2t) \Rightarrow t = 1$

$Q = (t_1^2, 2t_1) \Rightarrow t_1 = -t - 2/t = -1 - 2 = -3 \Rightarrow Q = (9,-6)$.

50. If the normals at the points t_1 and t_2 on $y^2 = 4ax$ intersect at the point t_3 on the parabola, then $t_1 t_2 =$

1. 1 2. 2 3. t_3 4. $2t_3$

Key. 2

Sol. Let the normals at t_1 and t_2 meet at t_3 on the parabola.

The equation of the normal at t_1 is $y + xt_1 = 2at_1 + at_1^3 \rightarrow (1)$

Equation of the chord joining t_1 and t_3 is $y(t_1 + t_3) = 2x + 2at_1t_3 \rightarrow (2)$

(1) and (2) represent the same line.

$\therefore \frac{t_1 + t_3}{1} = \frac{-2}{t_1} \Rightarrow t_3 = -t_1 - \frac{2}{t_1}$. Similarly $t_3 = -t_2 - \frac{2}{t_2}$

$\therefore -t_1 - \frac{2}{t_1} = -t_2 - \frac{2}{t_2} \Rightarrow t_1 - t_2 = \frac{2}{t_2} - \frac{2}{t_1} \Rightarrow t_1 - t_2 = \frac{2(t_1 - t_2)}{t_1 t_2} \Rightarrow t_1 t_2 = 2$

51. The number of normals thWSat can be drawn to the parabola $y^2 = 4x$ form the point $(1,0)$ is

1. 0 2. 1 3. 2 4. 3

Key. 2

Sol. $(1,0)$ lies on the axis between the vertex and focus $\Rightarrow$ number of normals =1.

52. The number of normals that can be drawn through $(-1,4)$ to the parabola

$y^2 - 4x + 6y = 0$ are

1. 4 2. 3 3. 2 4. 1

Key. 4

Sol. $Let\ S \equiv y^2 - 4x + 6y$. $S_{(-1,4)} = 4^2 - 4(-1) + 6(4) = 16 + 4 + 24 = 44 > 0$

$\therefore$ $(-1,4)$ lies out side the parabola and hence one normal can be drawn from $(-1,4)$ to the parabola.

53. If the tangents and normals at the extremities of a focal chord of a parabola intersect at (x_1, y_1) and (x_2, y_2) respectively, then

1. $x_1 = x_2$ 2. $x_1 = y_2$ 3. $y_1 = y_2$ 4. $x_2 = y_1$

Key. 3

Sol. $Let\ A(t_1)\ B(t_2)$ the extremiues of a focal chard of $y^2 = 4ax$

$\therefore\ t_1t_2 = -1$

$(x_1, y_1) = [at_1t_2, a(t_1 + t_2)]; (x_2, y_2) = [a(t_1^2 + t_2^2 + t_1t_2 + 2), at_1t_2(t_1 + t_2)]$

$y_2 = -at_1t_2(t_1 + t_2) = -a(-1)(t_1 + t_2) = a(t_1 + t_2) = y_1$

54. The normals at three points P, Q, R of the parabola $y^2 = 4ax$ meet in (h,k). The centroid of triangle PQR lies on

1. $x = 0$ 2. $y = 0$ 3. $x = -a$ 4. $y = a$

Key. 2

Sol. $Let\ P(t_1), Q(t_2)\ \&\ R(t_3)$

Equation of a normal to $y^2 = 4ax$ is $y + tx = 2at + at^3$

This passes through $(h,k) \Rightarrow k + th = 2at + at^3 \Rightarrow at^3 + (2a - h)t - k = 0$

t_1, t_2, t_3 are the roots of this equation $t_1 + t_2 + t_3 = 0$

Centroid of ΔPQR is $G\left[\frac{a}{3}(t_1^2 + t_2^2 + t_3^2), \frac{2a}{3}(t_1 + t_2 + t_3)\right]$

$t_1 + t_2 + t_3 = 0 \Rightarrow \frac{2a}{3}(t_1 + t_2 + t_3) = 0 \Rightarrow G$ lies on $y = 0$.

55. The ordinate of the centroid of the triangle formed by conormal points on the parabola $y^2 = 4ax$ is

1. 4 2. 0 3. 2 4. 1

Key. 2

Sol. $Let\ t_1, t_2\ \&\ t_3$ be the conormal points drawn from (x_1, y_1) to $y^2 = 4ax$

Equation of the normal at point 't' to $y^2 = 4ax$ is $y + tx = 2at + at^3$

This passes through $(x_1, y_1) \Rightarrow y_1 + tx_1 = 2at + at^3 \Rightarrow at^3 + (2a - x_1)t - y_1 = 0$

t_1, t_2, t_3 are the roots of the equation. $\therefore$ $t_1 + t_2 + t_3 = 0$

The ordinate of the centroid of the triangle formed by the points $t_1, t_2 \;\&\; t_3$ is $\frac{2a}{3}(t_1 + t_2 + t_3) = 0$

56. The normals at two points P and Q of a parabola $y^2 = 4ax$ meet at (x_1, y_1) on the parabola. Then $PQ^2 =$

1. $(x_1 + 4a)(x_1 + 8a)$ 2. $(x_1 + 4a)(x_1 - 8a)$ 3. $(x_1 - 4a)(x_1 + 8a)$ 4. $(x_1 - 4a)(x_1 - 8a)$

Key. 2

Sol. $Let\, P = (at_1^2, 2at_1), Q = (at_2^2, 2at_2)$

Since the normals at P and Q meet on the parabola, $t_1 t_2 = 2$.

Point of intersection of the normals $(x_1, y_1) = \left(a\left[t_1^2 + t_2^2 + t_1 t_2 + 2\right], -at_1 t_2\left[t_1 + t_2\right]\right)$

$\Rightarrow \; x_1 = a(t_1^2 + t_2^2 + t_1 t_2 + 2) = a(t_1^2 + t_2^2 + 4) \Rightarrow a(t_1^2 + t_2^2) = x_1 - 4a$

$PQ^2 = (at_1^2 - at_2^2)^2 + (2at_1 - 2at_2)^2 = a^2(t_1 - t_2)^2[(t_1 + t_2)^2 + 4]$
$= a(t_1^2 + t_2^2 - 4)a(t_1^2 + t_2^2 + 8) = (x_1 - 8a)(x_1 + 4a)$

57. If a normal subtends a right angle at the vertex of the parabola $y^2 = 4ax$, then its length is

1. $\sqrt{5}a$ 2. $3\sqrt{5}a$ 3. $6\sqrt{3}a$ 4. $7\sqrt{5}a$

Key. 3

Sol. $Let a(at_1^2, 2at_1), B(at_2^2, 2at_2)$.

The normal at A cuts the curve again at B. $\therefore$ $t_1 + t_2 = -\frac{2}{t_1}$........(1)

Again AB subtends a right angle at the vertex $0(0,0)$ of the parabola.

Slope $OA = \frac{2at_1}{at_1^2} = \frac{2}{t_1}$, slope of $OB = \frac{2}{t_2}$

$OA \perp OB \Rightarrow \frac{2}{t_1}.\frac{2}{t_2} = -t_1 t_2 = -4$......(2)

Slope of AB is $\frac{2a(t_2 - t_1)}{a(t_2^2 - t_1^2)} = \frac{2}{t_1 + t_2} = -t_1$. [By (1)]

Again from (1) and (2) on putting for t_2, we get $t_1 = \frac{4}{t_1} = -\frac{2}{t_1}$. $\therefore$ $t_1^2 = 2$ or

$t_1 \pm \sqrt{2}$

$$t_2 = \frac{-4}{t_1} = \frac{-4}{(\pm\sqrt{2})} = \pm 2\sqrt{2}. \quad \therefore \quad A = (2a, \pm 2a\sqrt{2}), B = (8a, \pm 4\sqrt{a})$$

$$AB = \sqrt{(2a-8a)^2 + (2a\sqrt{2} + 4\sqrt{2}a)^2} = \sqrt{36a^2 + 72a^2} = \sqrt{108a^2} = 6\sqrt{3}a.$$

58. Three normals with slopes m_1, m_2, m_3 are drawn from any point P not on the axis of the parabola $y^2 = 4x$. If $m_1 m_2 = a$, results in locus of P being a part of parabola, the value of 'a' equals

1. 2   2. -2   3. 4   4. -4

Key. 1

Sol. Equation of normal to $y^2 = 4x$ is $y = mx - 2m - m^3$ ...(i)

It passes through (α, β) $\therefore$ $m_1 m_2 m_3 \beta = m\alpha - 2, -m^3$

$\Rightarrow$ $m^3 + (2-\alpha)\, m + \beta = 0$ ....(*ii*)

(Let m_1, m_2, m_3 are roots)

$\therefore$ $m_1 m_2 m_3 = -\beta$ (as $m_1 m_2 = a$) $\Rightarrow$ $m_3 = -\frac{\beta}{a}$

Now $-\frac{\beta^3}{a^3} - (2-\alpha) \times \frac{\beta}{a} + \beta = 0$

$\Rightarrow$ $\beta^3 + (2-\alpha)a^2\beta - \beta a^3 = 0$

$\Rightarrow$ locus of P is $y^3 + (2-x)ya^2 - ya^3 = 0$

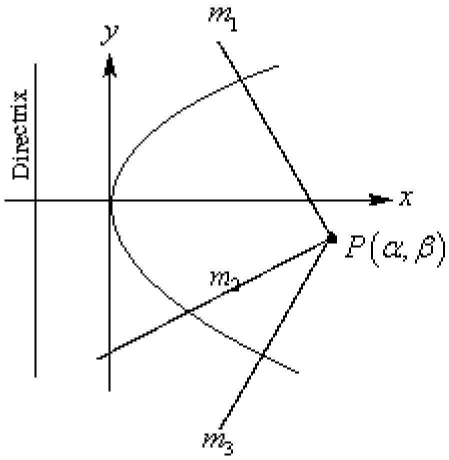
As P is not the axis of parabola

$\Rightarrow$ $y^2 = a^2x - 2a^2 + a^3$ as it is the part of $y^2 = 4x$

$\therefore$ $a^2 = 4$ or $-2a^2 + a^3 = 0$, $a = \pm 2$ or $a^2(a-2) = 0$

$a = \pm 2$ or $a = 0$, $a = 2$

$\Rightarrow$ $a = 2$ is the required value of a



59. The length of the normal chord drawn at one end of the latus rectum of $y^2 = 4ax$ is

1. $2\sqrt{2}a$ 2. $4\sqrt{2}a$ 3. $8\sqrt{2}a$ 4. $10\sqrt{2}a$

Key. 2

Sol. One end of the latus rectum $= (a, 2a)$

Equation of the normal at $(a, 2a)$ is $2a(x-a)+2a(y-2a)=0 \Rightarrow x+y-3a=0$

Solving; $y^2 = 4ax, x+y-3a=0$ we get the ends of normal chord are $(a,2a),(9a,-6a)$.

Length of the chard $= \sqrt{(9a-a)^2+(-6a-2a)^2} = \sqrt{64a^2+64a^2} = 8\sqrt{2}a.$

60. If the line $y = 2x + k$ is normal to the parabola $y^2 = 4x,$ then value of k equals

1. -2 2. -12 3. -3 4. -1/3

Key. 1

Sol. Conceptual

61. The normal chord at a point 't' on the parabola $y^2 = 4ax$ subtends a right angle at the vertex. Then $t^2 =$

1. 4 2. 2 3. 1 4. 3

Key. 2

Sol. Equation of the normal at point 't' is $y + tx = 2at + at^3 \Rightarrow \frac{y+tx}{2at+at^3} = 1$

Homoginising $y^2 = 4ax\left(\frac{y+tx}{2at+at^3}\right) \Rightarrow (2at+at^3)y^2 - 4ax(y+tx) = 0$

These lines re $\perp 1r \Rightarrow$ $2at + at^3 - 4at = 0 \Rightarrow at(t^2-2) = 0 \Rightarrow t^2 = 2$

62. A is a point on the parabola $y^2 = 4ax.$ The normal at A cuts the parabola again at B. If AB subtends a right angle at the vertex of the parabola, then slope of AB is

1. $\sqrt{2}$ 2. 2 3. $\sqrt{3}$ 4. 3

Key. 1

Sol. *Let* $A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$.

The normal at A cuts the curve again at B. $\therefore\ t_1+t_2=-2/t_1$....(1)

Again AB subtends a right angle at the vertex $O(0,0)$ of the parabola.

Slope of $OA=\frac{2at_1}{at_1^2}=\frac{2}{t_1}$, Slope of $OB=\frac{2}{t_2}$

$OA\perp OB\Rightarrow \frac{2}{t_1}.\frac{2}{t_2}=-1\Rightarrow t_1t_2=-4....(2)$

Slope of AB is $\frac{2a(t_2-t_1)}{a(t_2^2-t_1^2)}=\frac{2}{t_1+t_2}=-t_1$ by (1)

Again from (1) and (2) on putting for t_2 we get $t_1-\frac{4}{t_1}=\frac{2}{t_1}$. $\therefore\ t_1^2=2\Rightarrow t_1=\pm\sqrt{2}$.

$\therefore$ Slope $=\pm\sqrt{2}$.

63. If the normal at P meets the axis of the parabola $y^2=4ax$ in G and S is the focus, then SG =

1. SP 2. $2SP$ 3. $\frac{1}{2}SP$ 4. None

Key. 1

Sol. Equation of the normal at $P(at^2,2at)$ is $tx+y=2at+at^3$

Since it meets the axis, $y=0\Rightarrow\ tx=2at+at^3\Rightarrow x=2a+at^2$

$\therefore\ G=(2a+at^2,0)$, Focus $S=(a,0)$

$SG=\sqrt{(2a+at^2-a)^2+(0-0)^2}=\sqrt{(a+at^2)^2}=a+at^2=a(1+t^2)$

$SP=\sqrt{(at^2-a)^2+(2at-0)^2}=\sqrt{(at^2-a)^2+4a^2t^2}=\sqrt{(at^2+a)^2}=at^2+a=a(t^2+1)$

$\therefore\ SG=SP$

64. The normal of a parabola $y^2=4ax$ at (x_1,y_1) subtends right angle at the

1. Focus 2. Vertex 3. End of latus rectum 4. None of these

Key. 1

Sol. Conceptual

65. The normal at P cuts the axis of the parabola $y^2=4ax$ in G and S is the focus of the parabola. If ΔSPG is equilateral then each side is of length

1. a 2. $2a$ 3. $3a$ 4. $4a$

Key. 4

Sol. $Let\ P(at^2,2at)$

Equation of the normal at $P(t)$ is $y+tx=2at+at^3$

Equation to $y-$axis is $x=0$. Solving $G(2a+at^2,0)$

Focus $s(a,0)$

ΔSPG is equilateral $\Rightarrow$ $PG=GS$ $\Rightarrow$ $\sqrt{4a^2+4a^2t^2}=\sqrt{a^2(1+t^2)^2}$

$\Rightarrow$ $4a^2(1+t^2)=a^2(1+t^2)^2 \Rightarrow 4=1+t^2 \Rightarrow t^2=3$

Length of the side $=SG=a(1+t^2)=a(1+3)=4a$

66. If the normals at two points on the parabola $y^2=4ax$ intersect on the parabola, then the product of the abscissa is

1. $4a^2$ 2. $-4a^2$ 3. $2a$ 4. $4a^4$

Key. 1

Sol. *Let* $P(at_1^2,2at_1);Q(at_2^2,2at_2)$

Normals at P & Q on the parabola intersect on the parabola $\Rightarrow t_1t_2=2$

$at_1^2\times at_2^2=a^2(t_1t_2)^2=a^2(2)^2=4a^2$

67. If the normals at two points on the parabola intersects on the curve, then the product of the ordinates of the points is

1. $8a$ 2. $8a^2$ 3. $8a^3$ 4. $8a^4$

Key. 2

Sol. Let the normals at $P(t_1)$ and $Q(t_2)$ intersect on the parabola at $R(t_3)$.

Equation of any noemal is $tx+y=2at+at^3$

Since it passes through Q we get $t.at_3^2+2at_3=2at+at^3$

$\Rightarrow$ $at^3+(2a-at_3^2)t-2at_3=0,$ which is a cubic equation in t and hence its roots are t_1,t_2,t_3.

Product of the roots $=t_1t_2t_3 = \dfrac{-(-2at_3)}{a}=2t_3 \Rightarrow t_1t_2=2$

Product of the absisson of P and Q $=at_1^2.at_2^2=a^2(t_1t_2)^2=a^2(2)^2=4a^2$.

Product of the ordinates of P and Q $=2at_1.2at_2 4a^2.t_1t_2=4a^2(2)=8a^2$

68. The equation of the locus of the point of intersection of two normals to the parabola $y^2 = 4ax$ which are perpendicular to each other is

1. $y^2 = a(x-3a)$ 2. $y^2 = a(x+3a)$ 3. $y^2 = a(x+2a)$ 4. $y^2 = a(x-2a)$

Key. 1

Sol. *Let* $P(x_1, y_1)$ be the point of intersection of the two perpendicular normals at $A(t_1), B(t_2)$ on the parabola $y^2 = 4ax$.

Let t_3 be the foot of the third normal through P.

Equation of a normal at t to the parabola is $y + xt = 2at + at^3$

If this normal passes through P then $y_1 + x_1 t = 2at + at^3 \Rightarrow at^3 + (2a - x_1)t - y_1 = 0 \rightarrow (1)$

Now t_1, t_2, t_3 are the roots of (1). $\therefore$ $t_1 t_2 t_3 = y_1 / a$

Slope of the normal at t_1 is $-t_1$

Slope of the normal at t_2 is $-t_2$.

Normals at t_1 and t_2 are perpendicular $\Rightarrow$ $(-t_1)\ (-t_2) = -1 \Rightarrow t_1 t_2 = -1 \Rightarrow t_1 t_2 t_3 = -t_3$

$\Rightarrow \dfrac{y_1}{a} = -t_3 \Rightarrow t_3 = -\dfrac{y_1}{a}$

t_3 is a root of (1) $\Rightarrow a(-\dfrac{y_1}{a})^3 + (2a - x_1)(-\dfrac{y_1}{a}) - y_1 = 0 \Rightarrow -\dfrac{y_1^3}{a^2} - \dfrac{(2a - x_1)y_1}{a} - y_1 = 0$

$\Rightarrow y_1^2 + a(2a - x_1) + a^2 = y_1^2 = a(x_1 - 3a)$.

$\therefore$ The locus of P is $y^2 = a(x-3a)$

69. The three normals from a point to the parabola $y^2 = 4ax$ cut the axes in points, whose distances from the vertex are in A.P., then the locus of the point is

1. $27ay^2 = 2(x-2a)^3$ 2. $27ay^3 = 2(x-2a)^2$ 3. $9ay^2 = 2(x-2a)^3$ 4. $9ay^3 = 2(x-2a)^2$

Key. 1

Sol. *Let* $P(x_1, y_1)$ be any point.

Equation of any normal is $y = mx - 2am - am^3$

If is passes through P then $y_1 = mx_1 - 2am - am^3$

$\Rightarrow am^3 + (2a - x_1)m_1 + y_1 = 0$, which is cubic in m.

Let m_1, m_2, m_3 be its roots. Then $m_1 + m_2 + m_3 = 0, m_1m_2 + m_2m_3 + m_3m_1 = \frac{2a - x_1}{a}$

Normal meets the axis $(y = 0)$, where $0 = mx - 2am - am^3 \Rightarrow x = 2a + am^2$

$\therefore$ Distances of points from the vertex are $2a + am_1^2, 2a + am_2^2, 2a + am_3^2$

If these are in A.P., then $2(2a + am_2^2) = (2a + am_1^2) + (2a + am_3^2) \Rightarrow 2m_2^2 = m_1^2 + m_3^2$

$\Rightarrow 3m_2^2 = m_1^2 + m_2^2 = (m_1 + m_2 + m_3)^2 - 2(m_1m_2 + m_2m_3 + m_3m_1) = -2(2a - x_1)/a$

$\therefore m_2^2 = 2(x_1 - 2a)/3a$

But $y_1 = m_2(x_1 - 2a - am_2^2) \Rightarrow y_1^2 = m_2^2(x_1 - 2a - am_2^2)^2 = 2(x_1 - 2a)^3/27a$ Locus of P is $27ay^2 = 2(x - 2a)^3$

70. If the normals from any point to the parabola $x^2 = 4y$ cuts the line $y = 2$ in points whose abscissae are in A.P., then the slopes of the tangents at the 3 conormal points are in

1. AP 2. GP 3. HP 4. None

Key. 1

Sol. A point on $x^2 = 4y$ is $(2t, t_2)$ and required point be $P(x_1, y_1)$

Equation of normal at $(2t, t^2)$ is $x + ty = 2t + t^3$.......(1)

Given line equation is $y = 2$........(2)

Solving (1) & (3) $x + t(2) = 2t + t^3 \Rightarrow x = t^3$

This passes through $P(x_1, y_1) \Rightarrow t^3 = x_1$.............(3)

Let $(2t, t_1^2)(2t_2, t_2^2), (2t_3, t_3^2)$ be the co-normal points form P.

$2t_1, 2t_2, 2t_3$ in A.P. $\Rightarrow 4t_2 = 2(t_1 + t_3) \Rightarrow t_2 = \frac{t_1 + t_3}{2}$

$\therefore$ slopes of the tangents t_1, t_2 & t_3 are in A.P.

71. The line $lx+my+n=0$ is normal to the parabola $y^2=4ax$ if

1. $al(l^2+2m^2)+m^2n=0$
2. $al(l^2+2m^2)=m^2n$
3. $al(2l^2+m^2)+m^2n=0$
4. $al(2l^2+m^2)=2m^2n$

Key. 1
Sol. Conceptual

72. The feet of the normals to $y^2=4ax$ from the point $(6a,0)$ are

1. $(0,0)$
2. $(4a,4a)$
3. $(4a,-4a)$
4. $(0,0),(4a,4a),(4a,-4a)$

Key. 4
Sol. Equation of any normal to the parabola $y^2=4ax$ is $y=mx-2am-am^3$

If passes through $(6a,0)$ then $0=6am-2am-am^3 \Rightarrow am^3-4am=0 \Rightarrow am(m^2-4)=0$

$\Rightarrow m=0,\pm 2$.

$\therefore$ Feet of the normals $=(am^2,-2am)=(0,0),(4a,-4a),(4a,4a)$.

73. The condition that parabola $y^2=4ax$ & $y^2=4c(x-b)$ have a common normal other than x-axis is $(a\neq b\neq c)$

1. $\frac{a}{a-c}<2$
2. $\frac{b}{a-c}>2$
3. $\frac{b}{a-c}<1$
4. $\frac{b}{a-c}>1$

Key. 2
Sol. Conceptual

74. Locus of poles of chords of the parabola $y^2=4ax$ which subtends 45^0 at the vertex is $\left(x+4a\right)^2=\lambda\left(y^2-4ax\right)$ then λ =________

1. 1
2. 2
3. 3
4. 4

Key. 4
Sol. Parabola is $y^2=4ax \rightarrow$ ①

Polar of a pole $\left(x_1y_1\right)=yy_1-2ax=2ax_1 \rightarrow$ ②

Making eq ① homogeneous w.r.t ②

$$y^2-4ax\left(\frac{yy_1-2ax}{2ax_1}\right)=0$$

$$x_1y^2-2xyy_1+4ax^2=0$$

Angle between these pair of lines is 45^0

$$\therefore \tan 45^0 = \frac{2\sqrt{y_1^2 - 4ax_1}}{(x_1 + 4a)}$$

Locus of $(x_1 y_1)$ is

$$\Rightarrow (x+4a)^2 = 4(y^2 - 4ax)$$

$$\Rightarrow \lambda = 4$$

75. Length of the latus rectum of the parabola $\sqrt{x} + \sqrt{y} = \sqrt{a}$

1. $a\sqrt{2}$ 2. $\frac{a}{\sqrt{2}}$ 3. a 4. 2a

Key. 1

Sol. $\sqrt{x} = \sqrt{a} - \sqrt{y}$

$$x = a + y - 2\sqrt{ay}$$

$$(x - y - a)^2 = 4ay$$

$$x^2 + (y+a)^2 - 2x(a+y) = 4ay$$

$$x^2 + y^2 - 2xy + 2ay + a^2 - 2ax = 4ay$$

$$x^2 + y^2 - 2xy = 2ax + 2ay - a^2$$

$$(x-y)^2 = 2a\left(x + y - \frac{a}{2}\right)$$

Axis is x-y=0

$$\left(\frac{x-y}{\sqrt{2}}\right)^2 = \frac{2a}{2}\left(\frac{x+y-\frac{a}{2}}{\sqrt{2}}\right) \times \sqrt{2}$$

$$\left(\frac{x-y}{\sqrt{2}}\right)^2 = a\sqrt{2}\left(\frac{x+y-\frac{a}{2}}{\sqrt{2}}\right)$$

$\therefore$ lengthy $L.R = a\sqrt{2}$

76. Equation of common tangent to $x^2 = 32y$ and $y^2 = 32x$

1. $x + y = 8$ 2. $x + y + 8 = 0$ 3. $x - y = 8$ 4. $x - y + 8 = 0$

Key. 2

Sol. Common tangets $y^2 = 4ax$ and $x^2 = 4ay$ is $xa^{\frac{1}{3}} + yb^{\frac{1}{3}} + a^{\frac{2}{3}}b^{\frac{2}{3}} = 0$

Here a=8, b=8

77. The angle subtended at the focus by the normal chord of the point $(\lambda, \lambda),\ \lambda \neq 0$ on the parabola $y^2 = 4ax$ is

A) $\frac{\pi}{4}$ B) $\frac{\pi}{3}$ C) $\frac{\pi}{2}$ D) $\frac{\pi}{6}$

Key. C

Sol. Putting (λ, λ) in $y^2 = 4ax$, gives $\lambda = 4a$

Slope of normal at $(4a, 4a)$ is $-{}^nC_2$

Equation of normal at $(4a, 4a)$ is $y - 4a = -2(x - 4a) \Rightarrow y + 2x - 12a = 0$

The coordinates of intersection points of the above normal,

$y + 2\sum_{k=2}^{n}(k-1) - 12a = 0 \Rightarrow y^2 + 2ay - 24a^2 = 0$

$y = 4a - 6a$ and $x = 4a, 9a,$

Then slope of SA, $m_1 = \frac{n(n-1)}{2} = {}^nC_2$

And slope of SB, $m_2 = \frac{6a}{8a} = \frac{-3}{4}$ $m_1 m_2 = -1$

78. A circle with its centre at the focus of the parabola $y^2 = 4ax$ and touching its directrix intersects the parabola at points A, B. Then length AB is equal to

A) $4a$ B) $2a$ C) a D) $7a$

Key. A

Sol. Centre of circle $(a, 0)$ and radius $2a$

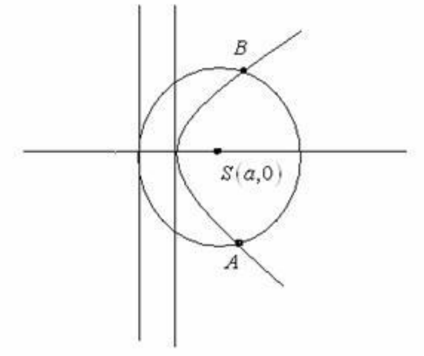
Equation of circle $(x-a)^2 + y^2 = 4a^2$

$x^2 + y^2 - 2ax - 3a^2 = 0$ and $y^2 = 4ax$ solving $x^2 + 4ax - 2ax - 3a^2 = 0$

$x^2 + 2ax - 3a^2 = 0$

$x = -3a, a$ and $y = \pm 2a$

$\therefore$ Length of $AB = 4a$



79. Tangents are drawn to $y^2 = 4ax$ from a variable point P moving on $x + a = 0,$ then the locus of foot of perpendicular drawn from P on the chord of contact of P is

A) $y = 0$ B) $(x-a)^2 + y^2 = a^2$ C) $(x-a)^2 + y^2 = 0$ D) $y(x-a) = 0$

Key. C

Sol. Portion of tangent intercepted between parabola and directrix subtends a right angle at the focus.

80. Three normals are drawn to the curve $y^2 = x$ from a point (c,0).Out of three one is always on x- axis. If two other normals are perpendicular to each other ,then the value of c

a) 3/4 b) 1/2 c) 3/2 d) 2

Key. A

Sol. Normal at $(at^2, 2at)$ is $y + tx = 2at + at^3$ $\left(a = \frac{1}{4}\right)$

If this passes through (c, 0)

We have $ct = 2at + at^3 = \frac{t}{2} + \frac{t^3}{4}$

$\Rightarrow t = 0 \text{ or } t^2 = 4c - 2$

If t = 0, the point at which the normal is drawn is (0, 0) if $t \neq 0$, then the two values of t represents slope of normals through (c, 0)

If these normals are perpendicular

then $(-t_1)(-t_2) = -1 \Rightarrow t_1t_2 = -1 \Rightarrow \left(\sqrt{4c-2}\right)\left(-\sqrt{4c-2}\right) = -1$

C = $\frac{3}{4}$

81. If area of Triangle formed by tangents fom the point (x_1,y_1) to the parabola $y^2 = 4ax$ and their chord of contact is

a) $\frac{\left(y_1{}^2-4ax_1\right)^{3/2}}{2a^2}$ b) $\frac{\left(y_1{}^2-4ax_1\right)^{3/3}}{a^2}$ c) $\frac{\left(y_1^2-4ax_1\right)^{3/2}}{2a}$ d) none of these

Key. C

Sol. Let $A\left(x_1, y_1\right)$ be any point outside the parabola and $B\left(\alpha, \beta\right), C\left(\alpha^1, \beta^1\right)$ be the points of contact of tangents from point A eq of chord BC, $YY_1 = 2a(x+x_1)$

Lengths of $\perp$ from A to BC

$= \frac{2a\left(x_1+x\right)-y_1y}{\sqrt{y^2+4a^2}} = \frac{y_1^2-4ax}{\sqrt{y_1^2+4a^2}}$

Area of $\Delta\, ABC = \frac{1}{2}AL\times BC$

We get $\frac{\left(y_1^2-4ax_1\right)^{3/2}}{2a}$

82. Let 'P' be (1, 0) and Q be any point on the parabola $y^2 = 8x$. The locus of mid point of PQ must be

a) $y^2-4x+2=0$ b) $y^2+4x+2=0$

c) $x^2-4y+2=0$ d) $x^2+4y+2=0$

Key. A

Sol. Let Q be $\left(at^2, 2at\right)$, (for a =2) Q be $\left(2t^2,4t\right)$

Then locus will be eliminant of

x = $\frac{1+2t^2}{2}$, y = $\frac{0+4t}{2}$

We easily get $y^2-4x+2=0$

$\Rightarrow$ (a) is correct

83. Coordinates of the focus of the parabola $\sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}=1$ is

A. $\left(\frac{ab}{a+b},\frac{ab}{a+b}\right)$ B. $\left(\frac{ab^2}{a^2+b^2},\frac{a^2b}{a^2+b^2}\right)$

C. $\left(\frac{a^2b}{a+b},\frac{ab^2}{a+b}\right)$ D. $\left(a,b\right)$

Key. B

Sol. $\sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}=1$

For this parabola x is a tangent at P(a, 0)

Y-axis a tangent Q(0,b)

$\therefore$ O(0,0) is point if inter section perpendicular tagents

$\therefore$ directrix passing through this point

Clearly $\underline{|OSP} = 90^0$

Hence circle on OP as diameter passing though S

i.e., $x^2 + y^2 - ax = 0$ passing through S.

Ily, $\underline{|OSQ} = 90^0$ $\therefore x^2 + y^2 - bx = 0$ passing through S.

Point of intersecting above circle is focus.

$x^2 + y^2 - ax = 0$

$x^2 + y^2 - bx = 0$

$ax - by = 0$

$y = \frac{ax}{b} \Rightarrow x^2 + \frac{a^2x^2}{b^2} = ax$

$x\left(\frac{b^2 + a^2}{b^2}\right) = a$

$x = \frac{ab^2}{a^2 + b^2}$

Ily, $y = \frac{a^2b}{a^2 + b^2}$

Focus $S = \left(\frac{ab^2}{a^2 + b^2}, \frac{a^2b}{a^2 + b^2}\right)$.

84. The Length of Latusrectum of the parabola $x = t^2 + t + 1$, $y = t^2 + 2t + 3$ is

A. $\frac{1}{2}$ B. $\frac{1}{\sqrt{2}}$

C. $\frac{1}{2\sqrt{2}}$ D. $\frac{1}{8}$

Key. C

Sol. $$\left.\begin{array}{l} x = t^2 + t + 1 \Rightarrow t^2 + t + 1 - x = 0 \\ y = t^2 + 2t + 3 \Rightarrow t^2 + 2t + 3 - u = 0 \end{array}\right\} \textit{eliminate } t$$

1	$1-x$	1	1
2	$3-y$	1	1

$$\frac{t^2}{3-y-2+2x} = \frac{t}{1-x-3+y} = \frac{1}{1}$$

$$\left.\begin{array}{l} t = -x + y - 2 \\ t = \dfrac{1-y+2x}{-x+y-2} \end{array}\right\} (x-y+2)^2 = (2x-y+1)$$

$$(x-y)^2 + 4(x-y) + 4 = (2x-y+1)$$

$$(x-y)^2 = -2x + 3y - 3$$

$$\therefore (x-y+\lambda)^2 = -2x + 3y - 3 + 2\lambda(x-y) + \lambda^2$$

$$(x-y+\lambda)^2 = x(2\lambda-2) + y(-2\lambda+3) + \lambda^2 - 3$$

$$\left.\begin{array}{l} \therefore \textit{ slope of } x - y + 1 = 0 \textit{ is } 1 \\ \textit{slope line on RHS is } \dfrac{2-2\lambda}{3-2\lambda} \end{array}\right\} \frac{2-2\lambda}{3-2\lambda} = -1$$

$$2 - 2\lambda = -3 + 2\lambda$$

$$4\lambda = 5 \Rightarrow \lambda = \frac{5}{4}$$

ε of parabola is $\left(x-y+\frac{5}{4}\right)^2=\frac{x}{2}+\frac{y}{2}+\frac{25}{16}-3$

$$\left(x-y+\frac{5}{4}\right)^2=\frac{1}{2}\left(x+y-\frac{23}{16}\right)$$

$$\left(\frac{x-y+\frac{5}{4}}{\frac{1}{\sqrt{2}}}\right)^2=\frac{1}{2\sqrt{2}}\left(\frac{x+y-\frac{23}{16}}{\sqrt{2}}\right) \qquad \text{LR} = \frac{1}{2\sqrt{2}}$$

85. For different values of k and l the two parabolas $y^2=16(x-k)$, $x^2=16(y-l)$ always touch each other then locus of point of contact is

A. $x^2+y^2=64$ B. $xy=8$

C. $y^2=8x$ D. $xy=64$

Key. D

Sol. $y^2=16(x-k)$ $\qquad x^2=16(y-l)$

$$2y\frac{dy}{dx}=16 \qquad 2x=16\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{8}{y}=m_1 \qquad \frac{dy}{dx}=\frac{x}{8}=m_2$$

Since two circle touch each other $m_1=m_2 \Rightarrow \frac{8}{y}=\frac{x}{8}\Rightarrow xy=64$

86. TP and TQ are any two tangents of a parabola $y^2=4ax$ and T is the point of intersection of two tangents. If the tangent at a third point on the parabola meets the above two tangents at P^1 and Q^1. Then $\frac{TP^1}{TP}+\frac{TQ^1}{TQ}$

A. (-1) B. $\frac{1}{2}$ C. $-\frac{1}{2}$ D. 2

Key. A

Sol. $T=(at_1t_2,\ a(t_1+t_2))$

$P^1=\left(at_1t_3\quad a(t_1+t_3)\right)$

$Q^1=\left(at_2t_3\quad a(t_2+t_3)\right)$

$TP^1:TP=\lambda:1$

$\lambda=\dfrac{at_1t_3-at_1t_2}{at_1t_2-at_1^2}$

$=\dfrac{t_3-t_2}{t_2-t_1}$

$\therefore \dfrac{TP^1}{TP}=\dfrac{t_3-t_2}{t_2-t_1}$

Ily, Let $TQ^1:TQ=\mu:1$

$\dfrac{TQ^1}{TQ}=\dfrac{at_2t_3-at_1t_2}{at_1t_2-at_2^2}=\dfrac{t_3-t_1}{t_1-t_2}$

$\therefore \dfrac{TP^1}{TP}+\dfrac{TQ^1}{TQ}=\dfrac{t_3-t_2}{t_2-t_1}+\dfrac{t_3-t_1}{t_1-t_2}=\dfrac{t_1-t_2}{t_2-t_1}=-1$

87. The locus of the Orthocentre of the triangle formed by three tangents of the parabola $(4x-3)^2=-64(2y+1)$ is

A) $y=\dfrac{-5}{2}$ B) $y=1$ C) $x=\dfrac{7}{4}$ D) $y=\dfrac{3}{2}$

Key. D

Sol. The locus is directrix of the parabola

88. A pair of tangents with inclinations α,β are drawn from an external point P to the parabola $y^2=16x$. If the point P varies in such a way that $\tan^2\alpha+\tan^2\beta=4$ then the locus of P is a conic whose eccentricity is

A) $\dfrac{\sqrt{5}}{2}$ B) $\sqrt{5}$ C) 1 D) $\dfrac{\sqrt{3}}{2}$

Key. B

Sol. Let $m_1=\tan\alpha, m_2=\tan\beta$, Let $P=(h,k)$

m_1, m_2 are the roots of $K = mh + \frac{4}{m} \Rightarrow hm^2 - Km + 4 = 0$

$m_1 + m_2 = \frac{K}{h}; \quad m_1 m_2 = \frac{4}{h}$

$m_1^2 + m_2^2 = \frac{K^2}{h^2} - \frac{8}{h} = 4$

Locus of P is $y^2 - 8x = 4x^2 \Rightarrow y^2 = 4(x+1)^2 - 4 \Rightarrow \frac{(x+1)^2}{1} - \frac{y^2}{4} = 1$

89. The length of the latusrectum of a parabola is $4a$. A pair of perpendicular tangents are drawn to the parabola to meet the axis of the parabola at the points A, B. If S is the focus of the parabola then $\frac{1}{|SA|} + \frac{1}{|SB|} =$

A) $2/a$ B) $4/a$ C) $1/a$ D) $2a$

Key. C

Sol. Let $y^2 = 4ax$ be the parabola

$y = mx + \frac{a}{m}$ and $y = \left(-\frac{1}{m}\right)x - am$ are perpendicular tangents

$S = (a, 0), A = \left(-\frac{a}{m^2}, 0\right), B = (-am^2, 0)$

$|SA| = a\left(1 + \frac{1}{m^2}\right) = \frac{a(1+m^2)}{m^2}$

$|SB| = a(1+m^2)$

90. Length of the focal chord of the parabola $(y+3)^2 = -8(x-1)$ which lies at a distance 2 units from the vertex of the parabola is

A) 8 B) $6\sqrt{2}$ C) 9 D) $5\sqrt{3}$

Key. A

Sol. Lengths are invariant under change of axes

consider $y^2 = 8x$. Consider focal chord at $(2t^2, 4t)$

Focus = (2, 0). Equation of focal chord at t is $y = \frac{2t}{t^2-1} 9x - 2) \Rightarrow 2tx + (1-t^2)y - 4t = 0$

$\frac{4|t|^2}{\sqrt{4t^2 + (1-t^2)^2}} = 2 \Rightarrow (|t| - 1)^2 = 0$

Length of focal chord at 't'= $2\left(t + \frac{1}{t}\right)^2 = \frac{2(t^2+1)^2}{t^2} = 8$

91. The slope of normal to the parabola $y=\frac{x^2}{4}-2$ drawn through the point $(10,-1)$

A) -2 B) $-\sqrt{3}$ C) $-1/2$ D) $-5/3$

Key. C

Sol. $x^2=4(y+2)$ is the given parabola

Any normal is $x=m(y+2)-2m-m^3$. If $(10,-1)$ lies on this line then

$10=+m-2m-m^3 \Rightarrow m^3+m+10=0 \Rightarrow m=-2$

Slope of normal = $1/m$.

92. m_1, m_2, m_3 are the slope of normals $(m_1<m_2<m_3)$ drawn through the point $(9,-6)$ to the parabola $y^2=4x$. $A=[a_{ij}]$ is a square matrix of order 3 such that $a_{ij}=1$ if $i\neq j$ and $a_{ij}=m_i$ if $i=j$. Then detA =

A) 6 B) –4 C) –9 D) 8

Key. D

Sol. $y=mx-2m-m^3.(9,-6)$ lies on this

$\therefore -6=9m-2m-m^3 \Rightarrow m^3-7m-6=0$

Roots are $-1,-2,3 \therefore$ $|A|=\begin{vmatrix}-2 & 1 & 1\\ 1 & -1 & 1\\ 1 & 1 & 3\end{vmatrix}=(-2)(-4)-(3-1)+2=8$

93. If parabola of latusrectum 'u' touches a fixed equal parabola, the axes of two curves being parallel, then the locus of the vertex of the moving curve is

(a) A parabola of latusrectum '2u'

(b) A parabola of latusrectum 'u'

(c) An ellipse whose major axis is '2u'

(d) An ellipse whose minor axis is '2u'

Key. A

Sol. Let (α,β) be the vertex of the moving parabola and its equation is

$(y-\beta)^2=-4a(x-\alpha)$ ---------------- (1)

Let the equation of fixed parabola be $y^2=4ax$ ---------- (2) (Here 4a = u)

From (1) & (2) $(y-\beta)^2=-4a\left(\frac{y^2}{4a}-\alpha\right)$

$\Rightarrow 2y^2-2\beta y+\beta^2-4a\alpha=0$

The above is a quadratic equation in y having same roots

$\Rightarrow \Delta=0$ $\Rightarrow \beta^2=8a\alpha$

Hence locus is $y^2=8ax$ i.e., $y^2=2ux$

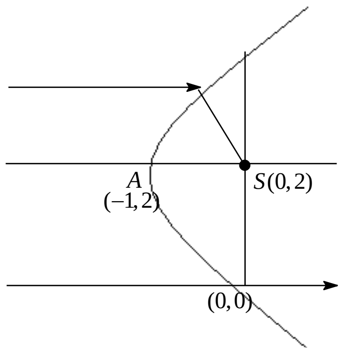
94. A ray of light moving parallel to the x-axis gets reflected form a parabolic mirror whose equation is $(y-2)^2 = 4(x+1)$. After reflection, the ray must pass through the point ……

(a) (0, 2) (b) (2, 0) (c) (0, -2) (d) (-1, 2)

Key. A

Sol. The equation of the axis of the parabola y – 2 = 0

Which is parallel to the x-axis so, a ray parallel to x-axis of parabola. W.K.T any ray parallel to the axis of a parabola passes through this focus after reflection. Here (0, 2) is the focus.



95. If the normal to the parabola $y^2 = 4ax$ at $(at^2, 2at)$ cuts the parabola again at $(aT^2, 2aT)$ then

(a) $-2 \le T \le 2$ (b) $T \in (-\infty, -8) \cup (8, \infty)$

(c) $T^2 < 8$ (d) $T^2 \ge 8$

Key. D

Sol. $T = -t - \dfrac{2}{t}$

$|T| = \left|t + \dfrac{2}{t}\right| \ge 2\sqrt{2}$

$T^2 \ge 8$

96. Let α is the angle which a tangent to $y^2 = 4ax$ makes with its axis, the distance between the tangent and a parallel normal will be

(a) $a\sin^2\alpha\cos^2\alpha$ (b) $a\cos ec\,\alpha \cdot \sec^2\alpha$ (c) $a\tan^2\alpha$ (d) $a\cos^2\alpha \cdot \cos ec^5\alpha$

Key. B

Sol. Equation of Tangent is $yt = x + at^2$

$\therefore Tan\,\alpha = \dfrac{1}{t}; t = \cot\alpha$

Equation of parallel normal is $yt = x + K$

$a \cdot 1^3 + 2a.1.(-t)^2 + (-t)^2 \cdot K = 0$

$K = \dfrac{-(a + 2at^2)}{t^2}$

$$\text{Distance} = \frac{at^2 + \frac{a+2at^2}{t^2}}{\sqrt{1+t^2}} = \frac{at^4 + 2at^2 + a}{t^2\sqrt{1+t^2}} = \frac{a(t^2+1)^{3/2}}{t^2}$$

97. If the normal at a point P on $y^2 = 4ax(a > 0)$ meet it again at Q in such a way that PQ is of minimum length. If 'O' is vertex then ΔOPQ is

(a) a right angled triangle (b) an obtuse angled triangle

(c) an equilateral triangle (d) right angled isosceles triangle

Key. A

Sol. $PQ = 6a\sqrt{3}; OP = 2a\sqrt{3};\ OQ = 4a\sqrt{6}$

98. Coordinates of the focus of the parabola $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$ is

A. $\left(\frac{ab}{a+b}, \frac{ab}{a+b}\right)$

B. $\left(\frac{ab^2}{a^2+b^2}, \frac{a^2b}{a^2+b^2}\right)$

C. $\left(\frac{a^2b}{a+b}, \frac{ab^2}{a+b}\right)$

D. (a,b)

Key. B

Sol. $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$

For this parabola x is a tangent at P(a, 0)

Y-axis a tangent Q(0,b)

$\therefore$ O(0,0) is point if inter section perpendicular tagents

$\therefore$ directrix passing through this point

Clearly $\underline{|OSP} = 90^0$

Hence circle on OP as diameter passing though S

i.e., $x^2 + y^2 - ax = 0$ passing through S.

Illy, $\underline{|OSQ} = 90^0$ $\therefore x^2 + y^2 - bx = 0$ passing through S.

Point of intersecting above circle is focus.

$x^2 + y^2 - ax = 0$

$x^2 + y^2 - bx = 0$

$ax - by = 0$

$y = \frac{ax}{b} \qquad \Rightarrow x^2 + \frac{a^2x^2}{b^2} = ax$

$x\left(\frac{b^2 + a^2}{b^2}\right) = a$

$x = \frac{ab^2}{a^2 + b^2}$

llly, $y = \frac{a^2b}{a^2 + b^2}$

Focus $S = \left(\frac{ab^2}{a^2 + b^2}, \frac{a^2b}{a^2 + b^2}\right)$.

99. The Length of Latusrectum of the parabola $x = t^2 + t + 1$, $y = t^2 + 2t + 3$ is

A. $\frac{1}{2}$ B. $\frac{1}{\sqrt{2}}$

C. $\frac{1}{2\sqrt{2}}$ D. $\frac{1}{8}$

Key. C

Sol. $\left.\begin{matrix} x = t^2 + t + 1 \Rightarrow t^2 + t + 1 - x = 0 \\ y = t^2 + 2t + 3 \Rightarrow t^2 + 2t + 3 - u = 0 \end{matrix}\right\} eliminate\ t$

1 $\quad 1 - x \quad$ 1 $\quad$ 1

2 $\quad 3 - y \quad$ 1 $\quad$ 1

$$\frac{t^2}{3-y-2+2x}=\frac{t}{1-x-3+y}=\frac{1}{1}$$

$$\left.\begin{array}{l} t=-x+y-2 \\ t=\dfrac{1-y+2x}{-x+y-2} \end{array}\right\}(x-y+2)^2=(2x-y+1)$$

$$(x-y)^2+4(x-y)+4=(2x-y+1)$$

$$(x-y)^2=-2x+3y-3$$

$$\therefore (x-y+\lambda)^2=-2x+3y-3+2\lambda(x-y)+\lambda^2$$

$$(x-y+\lambda)^2=x(2\lambda-2)+y(-2\lambda+3)+\lambda^2-3$$

$$\left.\begin{array}{l} \therefore \ \textit{slope of } \ x-y+1=0 \ \textit{ is } 1 \\ \textit{slope line on RHS is } \ \dfrac{2-2\lambda}{3-2\lambda} \end{array}\right\}\frac{2-2\lambda}{3-2\lambda}=-1$$

$$2-2\lambda=-3+2\lambda$$

$$4\lambda=5 \Rightarrow \lambda=\frac{5}{4}$$

ε of parabola is $\left(x-y+\dfrac{5}{4}\right)^2=\dfrac{x}{2}+\dfrac{y}{2}+\dfrac{25}{16}-3$

$$\left(x-y+\frac{5}{4}\right)^2=\frac{1}{2}\left(x+y-\frac{23}{16}\right)$$

$$\left(\frac{x-y+\frac{5}{4}}{\frac{1}{\sqrt{2}}}\right)^2=\frac{1}{2\sqrt{2}}\left(\frac{x+y-\frac{23}{16}}{\sqrt{2}}\right) \qquad \text{LR} = \frac{1}{2\sqrt{2}}$$

100. For different values of k and l the two parabolas $y^2=16(x-k)$, $x^2=16(y-l)$ always touch each other then locus of point of contact is

A. $x^2+y^2=64$ B. $xy=8$

C. $y^2 = 8x$ D. $xy = 64$

Key. D

Sol. $y^2 = 16(x-k)$ $x^2 = 16(y-l)$

$$2y\frac{dy}{dx} = 16 \qquad 2x = 16\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{8}{y} = m_1 \qquad \frac{dy}{dx} = \frac{x}{8} = m_2$$

Since two circle touch each other $m_1 = m_2 \Rightarrow \frac{8}{y} = \frac{x}{8} \Rightarrow xy = 64$

101. TP and TQ are any two tangents of a parabola $y^2 = 4ax$ and T is the point of intersection of two tangents. If the tangent at a third point on the parabola meets the above two tangents at P^1 and Q^1. Then $\frac{TP^1}{TP} + \frac{TQ^1}{TQ}$

A. (-1) B. $\frac{1}{2}$ C. $-\frac{1}{2}$ D. 2

Key. A

Sol. $T = (at_1t_2,\ a(t_1+t_2))$

$$P^1 = \left(at_1t_3 \quad a(t_1+t_3)\right)$$

$$Q^1 = \left(at_2t_3 \quad a(t_2+t_3)\right)$$

$$TP^1 : TP = \lambda : 1$$

$$\lambda = \frac{at_1t_3 - at_1t_2}{at_1t_2 - at_1^2}$$

$$= \frac{t_3 - t_2}{t_2 - t_1}$$

$$\therefore \frac{TP^1}{TP} = \frac{t_3 - t_2}{t_2 - t_1}$$

lly, Let $TQ^1 : TQ = \mu : 1$

$$\frac{TQ^1}{TQ}=\frac{at_2t_3-at_1t_2}{at_1t_2-at_2^2}=\frac{t_3-t_1}{t_1-t_2}$$

$$\therefore \frac{TP^1}{TP}+\frac{TQ^1}{TQ}=\frac{t_3-t_2}{t_2-t_1}+\frac{t_3-t_1}{t_1-t_2}=\frac{t_1-t_2}{t_2-t_1}=-1$$

102. A normal, whose inclination is 30^0, to a parabola cuts it again at an angle of

a) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ b) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ c) $\tan^{-1}(2\sqrt{3})$ d) $\tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$

Key. D

Sol. The normal at $P(at_1{}^2, 2at_1)$ is $y+xt_1=2at_1+at_1{}^3$ with slope say $\tan\alpha=-t_1=\frac{1}{\sqrt{3}}$. If it meets curve at $Q(at_2{}^2, 2at_2)$ then $t_2=-t_1-\frac{2}{t_1}=\frac{7}{\sqrt{3}}$. Then angle θ between parabola (tangent at Q) and normal at P is given by $\tan\theta=\frac{-t_1-\frac{1}{t_2}}{1-\frac{t_1}{t_2}}=\frac{1}{2\sqrt{3}}$

$$\Rightarrow \theta=\tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$$

103. The locus of vertices of family of parabolas, $y=ax^2+2a^2x+1$ is $(a\neq 0)$ a curve passing through

a) (1,0) b) (1,1) c) (0,1) d) (0,0)

Key. C

Sol.

$$y=ax^2+2a^2x+1 \Rightarrow \frac{y-(1-a^3)}{a}=(x+a)^2$$

$$\therefore Vertex=(\alpha,\beta)=(-a,1-a^3)$$

$$\Rightarrow \beta=1+\alpha^3$$

$$\Rightarrow curve\ \ is\ \ y=1+x^3$$

104. Equation of circle of minimum radius which touches both the parabolas $y=x^2+2x+4$ and $x=y^2+2y+4$ is

a) $2x^2+2y^2-11x-11y-13=0$

b) $4x^2+4y^2-11x-11y-13=0$

c) $3x^2+3y^2-11x-11y-13=0$

d) $x^2 + y^2 - 11x - 11y - 13 = 0$

Key. B

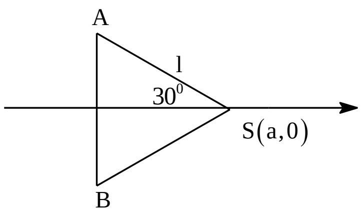
Sol. Circle will be touching both parabolas. Circles centre will be on the common normal

105. An equilateral triangle SAB is inscribed in the parabola $y^2 = 4ax$ having it's focus at 'S'. If the chord AB lies to the left of S, then the length of the side of this triangle is :

a) $3a\left(2-\sqrt{3}\right)$
b) $4a\left(2-\sqrt{3}\right)$
c) $2a\left(2-\sqrt{3}\right)$
d) $8a\left(2-\sqrt{3}\right)$

Key. B

Sol.



$A\left(a - l\cos 30^0, l\sin 30^0\right)$

Point 'A' lies on $y^2 = 4ax$

$\Rightarrow$ a quadratic in 'l'

106. Let the line lx + my = 1 cuts the parabola $y^2 = 4ax$ in the points A & B. Normals at A & B meet at a point C. Normal from C other than these two meet the parabola at a point D, then D =

a) $(a, 2a)$
b) $\left(\frac{4am}{l^2}, \frac{4a}{l}\right)$
c) $\left(\frac{2am^2}{l^2}, \frac{2a}{l}\right)$
d) $\left(\frac{4am^2}{l^2}, \frac{4am}{l}\right)$

Key. D

Sol. Conceptual

107. The normals to the parabola $y^2 = 4ax$ at points Q and R meet the parabola again at P. If T is the intersection point of the tangents to the parabola at Q and R, then the locus of the centroid of ΔTQR, is

a) $y^2 = 3a(x + 2a)$
b) $y^2 = a(2x + 3a)$
c) $y^2 = a(3x + 2a)$
d) $y^2 = 2a(2x + 3a)$

Key. C

Sol. Let $Q = (at_1^2, 2at_1)$

$R = (at_2^2, 2at_2)$

Normals at Q & R meet on parabola

Also $T = (at_1t_2, a(t_1 + t_2))$

Let (α, β) be centroid of ΔQRT

Then $3\alpha = a(t_1^2 + t_2^2 + t_1t_2) \& \beta = a(t_1 + t_2)$

Eliminate $(t_1 + t_2)$

108. The normal at a point P of a parabola $y^2 = 4ax$ meets its axis in G and tangent at its vertex in H. If A is the vertex of the parabola and if the rectangle AGQH is completed, then equation to the locus of vertex Q is

a) $y^2(y-2a) = ax^2$ b) $y^2(y+2a) = ax^2$

c) $x^2(x-2a) = ay^2$ d) $x^2(x+2a) = ay^2$

Key. C

Sol. $A = (a,0), H = (0, 2at + at^3), G = (2at + at^2, 0), Q = (h,k)$

$(h,k) = (2a + at^2, 2at + at^3)$

eliminating 't', $x^3 = 2ax^2 + ay^2$

109. If the focus of the parabola $(y-\beta)^2 = 4(x-\alpha)$ always lies between the lines $x + y = 1$ and $x + y = 3$, then,

a) $3 < \alpha + \beta < 4$ b) $0 < \alpha + \beta < 3$

c) $0 < \alpha + \beta < 2$ d) $-2 < \alpha + \beta < 2$

Key. C

Sol. origin & focus line on off side of $x + y = 1 \Rightarrow \alpha + \beta > 0$

origin & focus line on same side of $x + y = 3 \Rightarrow \alpha + \beta < 2.$

110. Consider the two parabolas $y^2 = 4a(x-\alpha) \& x^2 = 4a(y-\beta)$, where 'a' is the given constant and α, β are variables. If α and β vary in such a way that these parabolas touch each other, then equation to the locus of point of contact

a) circle b) Parabola

c) Ellipse d) Rectangular hyperbola

Key. D

Sol. Let POC be (h,k). Then, tangent at (h,k) to both parabolas represents same line.

111. The points on the axis of the parabola $3y^2 + 4y - 6x + 8 = 0$ from where 3 distinct normals can be drawn is given by

(A) $\left(a, \frac{4}{3}\right); a > \frac{19}{9}$ (B) $\left(a, -\frac{2}{3}\right); a > \frac{19}{9}$

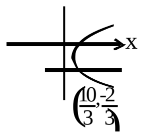
(C) $\left(a, -\frac{2}{3}\right); a > \frac{16}{9}$ (D) $\left(a, -\frac{2}{3}\right); a > \frac{7}{9}$

Key. B

Sol. $3y^2 + 4y = 6x - 8$

$\Rightarrow 3\left(y^2 + \frac{4}{3}y\right) = 6x - 8 \quad \Rightarrow \left(y + \frac{2}{3}\right)^2 = 2x - \frac{8}{3} + \frac{4}{9} \quad \Rightarrow \left(y + \frac{2}{3}\right)^2 = 2\left(x - \frac{10}{9}\right)$

Let any point on the axis $\left(a, -\frac{2}{3}\right)$



$y + \frac{2}{3} = m\left(x - \frac{10}{9}\right) - m - \frac{1}{2}m^3$

$\Rightarrow 0 = m\left[a - \frac{10}{9} - 1 - \frac{1}{2}m^2\right]$

$\Rightarrow a - \frac{19}{9} = \frac{1}{2}m^2 \Rightarrow m^2 = 2\left(a - \frac{19}{9}\right) \qquad \therefore a > \frac{19}{9}$

112. Tangents $\overline{PA}$ and $\overline{PB}$ are drawn to $y^2 = 4ax$. If $m_{\overline{PA}} \,\&\, m_{\overline{PB}}$ are the slopes of the tangents satisfying $\left(m_{\overline{PA}}\right)^2 + \left(m_{\overline{PB}}\right)^2 = 4$ then the locus of P is

(A) $y^2 = 2x(2x + a)$ (B) $y^2 = 2x(2x - a)$

(C) $y^2 = x(x - a)$ (D) None of these

Key. A

Sol. Let $P \equiv (h, k)$

$y = mx + \frac{a}{m}$

$k = mh + \frac{a}{m} \Rightarrow m^2h + a - mk = 0 \qquad \Rightarrow m_{PA} + m_{PB} = \frac{k}{h}$

$m_{\overline{PA}} . m_{PB} = \frac{a}{h}$

$\frac{k^2}{h^2} - \frac{2a}{h} = 4$

$\Rightarrow k^2 - 2ah = 4h^2 \qquad \therefore y^2 = 2ax + 4x^2 = 2x(2x + a)$

113. Minimum distance between $y^2 = 4x$ and $x^2 + y^2 - 12x + 31 = 0$.

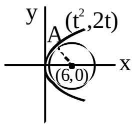
(A) $\sqrt{21}$ (B) $\sqrt{26} - \sqrt{5}$

(C) $\sqrt{20} - \sqrt{5}$ (D) $\sqrt{28} - \sqrt{5}$

Key. C

Sol. $y+tx=2t+t^3$

$6t=2t+t^3$



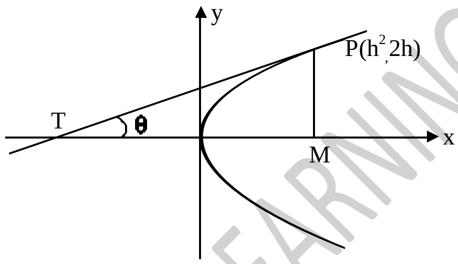
$\Rightarrow \quad t^2+2-6=0$

$t=\pm 2$

$\therefore \quad A\equiv(4,4)$

$\therefore$ Minimum distance $\sqrt{4+16}-\sqrt{5}=\sqrt{20}-\sqrt{5}$.

114. The triangle formed by the tangent to the parabola $y^2=4x$ at the point whose abscissa lies in the interval $[a^2,4a^2]$, the ordinate and the $x-$axis has the greatest area equal to



(A) $12a^3$ (B) $8a^3$

(C) $16a^3$ (D) None

Key. C

Sol. $P\equiv(h^2,2h)$

$\tan\theta=\dfrac{1}{h}$

And $\Delta PTM=\dfrac{1}{2}\times 2h\times 2h\cot\theta=2h^3$

$a^2\le h^2\le 4a^2$

$\therefore$ maximum area = $2(2a)^3=16a^3$

115. Minimum distance between $y^2-4x-8y+40=0$ and $x^2-8x-4y+40=0$

(A) 0 (B) $\sqrt{3}$

(C) $2\sqrt{2}$ (D) $\sqrt{2}$

Key. D

Sol. since two parabolas are symmetrical about $y=x$.

Solving $y=x$ & $y^2-4x-8y+40=0$

$\Rightarrow x^2-12x+40=0$

has no real solution

$\therefore$ They don't intersect

Point on $(x-4)^2 = 4(y-6)$ is (6,7) and the corresponding point on $(y-4)^2 = 4(x-6)$ is (7, 6) minimum distance is $\sqrt{2}$.

116. Minimum distance between the parabolas $y^2 - 4x - 8y + 40 = 0$ and $x^2 - 8x - 4y + 40 = 0$ is

(A) 0 (B) $\sqrt{3}$

(C) $2\sqrt{2}$ (D) $\sqrt{2}$

Key. D

Sol. Since two parabolas are symmetrical about

$y = x$

Minimum distance is distance between tangents to the parabola parallel to y = x.

Differentiating $x^2 - 8x - 4y + 40 = 0$ w.r.t x, we get $2x - 8 - 4y' = 0$

$y' = \frac{x-4}{2} = 1$

$x = 6$ and $y = 7$

Corresponding point on $(y - 4)^2 = 4(x - 6)$

is (7, 6) so minimum distance = $\sqrt{2}$.

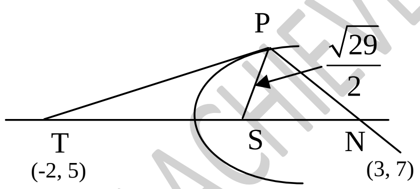
117. If (–2, 5) and (3, 7) are the points of intersection of the tangent and normal at a point on a parabola with the axis of the parabola, then the focal distance of that point is

(A) $\frac{\sqrt{29}}{2}$ (B) $\frac{5}{2}$

(C) $\sqrt{29}$ (D) $\frac{2}{5}$

Key. A

Sol.



118. The locus of the Orthocentre of the triangle formed by three tangents of the parabola $(4x-3)^2 = -64(2y+1)$ is

A) $y = \frac{-5}{2}$ B) $y = 1$ C) $x = \frac{7}{4}$ D) $y = \frac{3}{2}$

Key. D

Sol. The locus is directrix of the parabola

119. A pair of tangents with inclinations α, β are drawn from an external point P to the parabola $y^2 = 16x$. If the point P varies in such a way that $\tan^2\alpha + \tan^2\beta = 4$ then the locus of P is a conic whose eccentricity is

A) $\frac{\sqrt{5}}{2}$ B) $\sqrt{5}$ C) 1 D) $\frac{\sqrt{3}}{2}$

Key. B

Sol. Let $m_1 = \tan\alpha, m_2 = \tan\beta$, Let $P = (h,k)$

m_1, m_2 are the roots of $K = mh + \frac{4}{m} \Rightarrow hm^2 - Km + 4 = 0$

$m_1 + m_2 = \frac{K}{h}; \quad m_1 m_2 = \frac{4}{h}$

$m_1^2 + m_2^2 = \frac{K^2}{h^2} - \frac{8}{h} = 4$

Locus of P is $y^2 - 8x = 4x^2 \Rightarrow y^2 = 4(x+1)^2 - 4 \Rightarrow \frac{(x+1)^2}{1} - \frac{y^2}{4} = 1$

120. The length of the latusrectum of a parabola is $4a$. A pair of perpendicular tangents are drawn to the parabola to meet the axis of the parabola at the points A, B. If S is the focus of the parabola then $\frac{1}{|SA|} + \frac{1}{|SB|} =$

A) $2/a$ B) $4/a$ C) $1/a$ D) $2a$

Key. C

Sol. Let $y^2 = 4ax$ be the parabola

$y = mx + \frac{a}{m}$ and $y = \left(-\frac{1}{m}\right)x - am$ are perpendicular tangents

$S = (a,0), A = \left(-\frac{a}{m^2}, 0\right), B = (-am^2, 0)$

$|SA| = a\left(1 + \frac{1}{m^2}\right) = \frac{a(1+m^2)}{m^2}$

$|SB| = a(1+m^2)$

121. Length of the focal chord of the parabola $(y+3)^2 = -8(x-1)$ which lies at a distance 2 units from the vertex of the parabola is

A) 8 B) $6\sqrt{2}$ C) 9 D) $5\sqrt{3}$

Key. A

Sol. Lengths are invariant under change of axes

consider $y^2 = 8x$. Consider focal chord at $(2t^2, 4t)$

Focus = (2, 0). Equation of focal chord at t is $y = \frac{2t}{t^2 - 1} 9x - 2) \Rightarrow 2tx + (1-t^2)y - 4t = 0$

$$\frac{4|t|^2}{\sqrt{4t^2 + (1-t^2)^2}} = 2 \Rightarrow (|t| - 1)^2 = 0$$

Length of focal chord at 't'= $2\left(t+\frac{1}{t}\right)^2 = \frac{2(t^2+1)^2}{t^2} = 8$

122. The slope of normal to the parabola $y=\frac{x^2}{4}-2$ drawn through the point $(10,-1)$

A) -2 B) $-\sqrt{3}$ C) $-1/2$ D) $-5/3$

Key. C

Sol. $x^2 = 4(y+2)$ is the given parabola

Any normal is $x = m(y+2) - 2m - m^3$. If $(10,-1)$ lies on this line then

$10 = +m - 2m - m^3 \Rightarrow m^3 + m + 10 = 0 \Rightarrow m = -2$

Slope of normal = $1/m$.

123. m_1, m_2, m_3 are the slope of normals $(m_1 < m_2 < m_3)$ drawn through the point $(9,-6)$ to the parabola $y^2 = 4x$. $A = [a_{ij}]$ is a square matrix of order 3 such that $a_{ij} = 1$ if $i \neq j$ and $a_{ij} = m_i$ if $i = j$. Then detA =

A) 6 B) –4 C) –9 D) 8

Key. D

Sol. $y = mx - 2m - m^3.(9,-6)$ lies on this

$\therefore -6 = 9m - 2m - m^3 \Rightarrow m^3 - 7m - 6 = 0$

Roots are $-1,-2,3$ $\therefore$ $|A| = \begin{vmatrix} -2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 3 \end{vmatrix} = (-2)(-4) - (3-1) + 2 = 8$

124. PQ is any focal chord of the parabola $y^2 = 32x$. The length of PQ can never be less than

(A) 8 unit (B) 16 unit

(C) 32 unit (D) 48 unit

Key. C

Sol. Length of focal chord is $a\left(t+\frac{1}{t}\right)^2$, if $(at^2, 2at)$ is one extremity of the parabola $y^2 = 4ax$.

$\therefore\ t+\frac{1}{t} \geq 2$ (AM $\geq$GM)

$\Rightarrow a\left(t+\frac{1}{t}\right)^2 \geq 4a$

Here, 4a = 32

125. PN is the ordinate of any point P on $y^2 = 4x$. The normal at P to the curve meets the axis at G, then

(A) NG = 1 (B) NG = 2

(C) NG = 4 (D) NG = 6

Key. B

Sol. Let P be $(t^2, 2t)$, then the normal at P, is $y + tx = 2t + t^3$ which meets x-axis at G$(2 + t^2, 0)$. Now as N is $(t^2, 0)$.

$\therefore$ NG = 2

126. The coordinates of the focus of the parabola $y^2 = 4(x + y)$, are

(A) (-1, 1) (B) (0, 2)

(C) (2, 1) (D) (2, -1)

Key. B

SOL. $y^2 = 4x + 4y$

$\Rightarrow (y - 2)^2 = 4(x + 1)$

focus (0, 2)

127. The straight line y = mx + c touches the parabola $y^2 = 4a(x + a)$, if

(A) c = am - a/m (B) c = m - a/m

(C) c = am + a/m (D) c = m + am

Key. C

Sol. Putting y = mx + c in parabola $y^2 = 4a(x + a)$

$\Rightarrow (mx + c)^2 = 4a (x + a)$

$\Rightarrow m^2x^2 + 2(mc - 2a) x + (c^2 - 4a^2) = 0$

If roots are equal i.e., D = 0

$\Rightarrow 4(mc - 2a)^2 - 4m^2 (c^2 - 4a^2) = 0$

$\Rightarrow -mc + a + am^2 = 0 \Rightarrow c = am + a/m$

Alternative

Equation of any tangent to the parabola y = m(x + a) = a/m

comparing with y = mx + c

c = am + a/m.

128. Three normals are drawn to the curve $y^2 = x$ from a point (c, 0). Out of three one is always on x-axis. If two other normals are perpendicular to each other, then the value of c is

(A) 3/4 (B) 1/2

(C) 3/2 (D) 2

Key. A

SOL. Normal at $(at^2, 2at)$ is $y + tx = 2at + at^3$ $\left(a = \frac{1}{4}\right)$

if this passes through (c, 0), we have

$$ct = 2at + at^3 = \frac{t}{2} + \frac{t^3}{4}$$

$\Rightarrow t[t^2 + 2 - 4c] = 0$

$\Rightarrow t = 0$ or $t^2 = 4c - 2$

if t = 0 the point at which the normal is drawn is (0, 0).

if $t \neq 0$ then the two values of t represents slope of normals through (c, 0).

if these normals are perpendicular then $(-t_1)(-t_2) = -1$

$\Rightarrow t_1t_2 = -1$

$\Rightarrow (\sqrt{4c-2})(-\sqrt{4c-2}) = -1$

$\Rightarrow \quad c = \frac{3}{4}$

129. Let $y^2 = 4ax$ be a parabola and PQ be a focal chord of parabola. Let T be the point of intersection of tangents at P and Q. Then

(A) area of circumcircle of ΔPQT is $\left(\frac{\pi(PQ)^2}{4}\right)$

(B) orthocenter of ΔPQT will lie on tangent at vertex

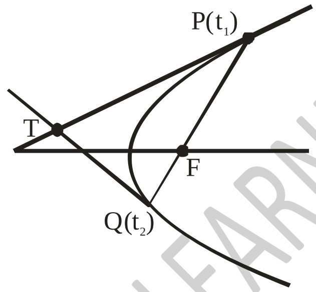
(C) incenter of ΔPQT will be vertex of parabola

(D) incentre of ΔPQT will lie on directrix of parabola

Key. A

Sol. Equation of tangent at P $\rightarrow ty = x + at^2$ .....(i)

Equation of tangent at Q $\rightarrow \frac{-1}{t} y = x + \frac{a}{t^2}$ .....(ii)



$\Rightarrow x = -a.$

$\therefore$ t lies on the directrix and thus ΔPTQ is right angled triangle. thus circle passing through P, Q and T must have P and Q are end points of diameter. thus area of required circle is $\frac{\pi(PQ)^2}{4}$

130. Axis of a parabola is y = x and vertex and focus are at a distance $\sqrt{2}$ and $2\sqrt{2}$ respectively from the origin. Then equation of the parabola is

(A) $(x - y)^2 = 8(x + y - 2)$ (B) $(x + y)^2 = 2(x + y - 2)$

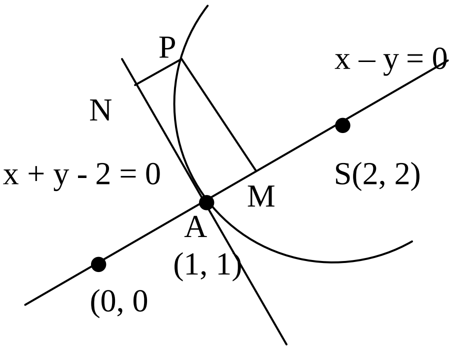
(C) $(x - y)^2 = 4(x + y - 2)$ (D) $(x + y)^2 = 2(x - y + 2)$

Key. A

Sol. $PM^2 = 4a(PN)$

$\frac{(x-y)^2}{2} = 4\sqrt{2}\,\frac{(x+y-2)}{\sqrt{2}}$

$(x - y)^2 = 8(x + y - 2)$



131. If m_1, m_2 are slopes of tangents drawn from (1, 4) to the parabola $y^2 = 4x$, then

(A) $m_1 + m_2 = 4$ (B) $|m_1 - m_2| = 2\sqrt{3}$

(C) $m_1.m_2 = -1$ (D) $m_1 = m_2$

Key. A

Sol. Any tangent of the parabola $y = mx + \frac{a}{m}$

$\Rightarrow 4 = m + \frac{1}{m}$ $\Rightarrow 4m = m^2 + 1$

$\Rightarrow m^2 - 4m + 1 = 0$

$\Rightarrow m_1 + m_2 = 4$ and $m_1m_2 = 1$

132. The locus of point of intersection of two tangents to the parabola $y^2 = 4x$ such that their chord of contact subtends a right angle at the vertex is

A) $x + 4 = 0$ B) $y + 4 = 0$ C) $x - 4 = 0$ D) $y - 4 = 0$

Key : A

Sol. Chord of contact of $(t_1t_2, t_1 + t_2)$ with respect to $y^2 = 4x$ is $(t_1 + t_1)y = 2(x + t_1t_2)$

$\Rightarrow \frac{(t_1 + t_2)y - 2x}{2t_1t_2} = 1 =$ $y^2 = 4x.1 \Rightarrow t_1t_2 + 4 = 0 \Rightarrow t_1t_2 = -4$

$x = -4 \Rightarrow x + 4 = 0$

133. If the line y = x + 2 does not intersect any member of family of parabolas $y^2 = ax$, ($a \in R^+$) at two distinct point, then maximum value of latus rectum of parabola is

(A) 4 (B) 8

(C) 16 (D) 32

KEY : B

HINT

$y^2 = ax$

$y = x + 2$

$(x + 2)^2 - ax = 0$

$x^2 + x(4 - a) + 4 = 0$

$D \le 0$

$a \le 8$

134. Equation of the circle of minimum radius which touches both the parabolas $y = x^2+2x+4$ and $x = y^2+2y+4$ is

A) $2x^2+2y^2-11x-11y-13 = 0$ B) $4x^2+4y^2-11x-11y-13= 0$

C) $3x^2+3y^2-11x-11y-13 = 0$D) $x^2+y^2-11x-11y-13 = 0$

KEY : B

HINT : Given parabolas are symmetric about the line y = x so they have a common normal with slope -1 it meets the parabolas at $\left(\frac{-1}{2},\frac{13}{4}\right),\left(\frac{13}{4},\frac{-1}{2}\right)$ hence the req circles is x^2+y^2

$$-\frac{11}{4}x-\frac{11}{4}y-\frac{13}{4}=0$$

135. The slope of the line which belongs to family of these

$(1 + \lambda)x + (\lambda - 1)y + 2(1 - \lambda) = 0$ and makes shortest intercept on $x^2 = 4y - 4$

(A) $\frac{1}{2}$ (B) 1 (C) 0 (D) 2

Key : C

Hint : Family of lines passes through focus hence latus rectum will makes shortest intercept.

136. If the tangents at two points (1, 2) and (3, 6) as a parabola intersect at the point (– 1, 1), then the slope of the directrix of the parabola is

(A) $\sqrt{2}$ (B)– 2

(C) – 1 (D)none of these

Key : C

Hint : If the tangents at P and Q intersect at T, then axis of parabola is parallel to TR, where R is the mid point of P and Q. So, slope of the axis is 1.

$\therefore$ slope of the directrix = – 1.

137. A variable chord PQ of the parabola $y = 4x^2$ substends a right angle at the vertex. Then the locus of points of intersection of the tangents at P and Q is

a) $4y+1=16x^2$ b) $y+4=0$ c) $4y+4=4x^2$ d) $4y+1=0$

Key: D

Hint: $Let\ P\left(t_1,4t_1^2\right),Q\left(t_2,4t_2^2\right)$

Slope of OP x slope of OQ = -1

$\Rightarrow 4t_1.4t_2=-1$

Eq of tangent at $\left(t_1,4t_1^2\right) is$

$y-4t_1^2=8t_1\left(x-t_1\right)\Rightarrow y+4t_1^2=8t_1x$

Eq of tangent at $\left(t_2,4t_2^2\right) is\ y+4t_2^2=8t_2x$

Let (x_1 , y_1) is the point of intersection

$eq(1)-eq(2)\Rightarrow x_1=\frac{t_1+t_2}{2}$

$y_1=8t_1\left(\frac{t_1+t_2}{2}\right)-4t_1^2=4t_1t_2=\frac{-1}{4}$

$\Rightarrow 4y_1+1=0$

138. Let A ≡ (9, 6), B(4, -4) be two points on parabola $y^2 = 4x$ and P(t^2, 2t), t∈[-2, 3] be a variable point on it such that area of ΔPAB is maximum, then point P will be

(A) (4, 4) (B) (3, $-2\sqrt{3}$)

(C) (4, 1) (D) $\left(\frac{1}{4}, 1\right)$

Key: D

Hint: Let P be (t^2, 2t) area of Δ PAB

$$\frac{1}{2}\begin{vmatrix} t^2 & 2t & 1 \\ 9 & 6 & 1 \\ 4 & -4 & 1 \end{vmatrix} = |5t^2 - 5t - 30|$$

it is maximum at t = 1/2.

139. Let (2, 3) be the focus of a parabola and x + y = 0 and x - y = 0 be its two tangents, then equation of its directrix will be

(A) 2x - 3y = 0 (B) 3x + 4y = 0

(C) x + y = 5 (D) 12x - 5y + 1 = 0

Key: A

Hint: Mirror image of focus in the tangent of parabola lie on its directrix.

140. The line x + y = 6 is a normal to the parabola $y^2 = 8x$ at the point

(a) $(18, -12)$ (b) (4, 2) (c) (2, 4) (d) (3, 3)

Key: c

Hint: Slope of the normal is given to be –1. We know that, foot of the normal is (am^2, –2am). Here a = 2, m = –1. Hence the required point is (2, 4).

141. The tangent and normal at the point P(4, 4) to the parabola, $y^2 = 4x$ intersect the x–axis at the points Q and R respectively. Then the cirucm centre of the ΔPQR is

(A) (2, 0) (B) (2, 1)

(C) (1, 0) (D) (1, 2)

Key : C

Sol : Eq. of tangent 2y = x + 4

∴ Q≡ (– 4, 0)

Eq. of normal is y – 4 = – 2 (x – 4)

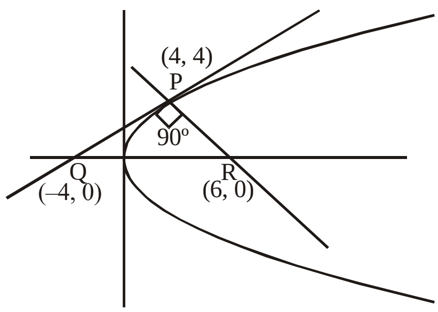
⇒ y + 2x = 12

Clearly QR is diameter of the required circle.

⇒ $(x + 4)(x - 6) + y^2 = 0$

⇒ $x^2 + y^2 - 2x - 24 = 0$

centre (1, 0)



142. The mirror image of the parabola $y^2 = 4x$ in the tangent to the parabola to the point (1,2) is

(A) $(x-1)^2 = 4(y+1)$ (B) $(x+1)^2 = 4(y+1)$

(C) $(x+1)^2 = 4(y-1)$ (D) $(x-1)^2 = 4(y-1)$

Key : C

Sol : Any point on the given parabola is $\left(t^2, 2t\right)$. The equation of the tangent at (1,2) is x-y +1 = 0.

The image (h,k) of the point $(t^2,2t)$ in x-y + 1 = 0 is

given by $\frac{h-t^2}{1}=\frac{k-2t}{-1}=\frac{-2\left(t^2-2t+1\right)}{1+1}$

$\therefore \quad h=t^2-t^2+2t-1=2t-1$

and $\quad k=2t+t^2-2t+1=t^2+1$

Eliminating t from h = 2t – 1 and k = t^2+1

we get, $(h+1)^2=4(k-1)$

The required equation of reflection is $\left(x+1\right)^2=4\left(y-1\right)$

143. $Min\{(x_1-x_2)^2+(12+\sqrt{1-x_1^2}-\sqrt{4x_2})^2\}\forall x_1,x_2\in R$ is

A. $4\sqrt{5}-1$ B. $4\sqrt{5}+1$ C. $\sqrt{5}+1$ D. $\sqrt{5}-1$

Key. A

Sol. Let $y_1=12+\sqrt{1-x_1^2}$ and $y_2=\sqrt{4x_2}$

$(y_1-12)^2=1-x_1^2\Rightarrow x_1^2+(y_1-12)^2=1; y_2^2=4x_2$

Required answer is shortest distance between two curves $x^2+(y-12)^2=1$ and $y^2=4x$

144. The radius of largest circle which passes through focus of parabola $y^2=4(x+y)$ and also contained in it is

A. 4 B. 1 C. 3 D. 2

Key. A

Sol. Parabola is $y^2-4y=4x\Rightarrow(y-2)^2=4(x+1)$

Focus = (0,2)

Let radius of circle = r then centre = (r,2)

Circle is $(x-r)^2+(y-2)^2=r^2$

$\Rightarrow(x-r)^2+4(x+1)=r^2$ has equal roots $\Delta=0\Rightarrow r=4$

145. Length of the latus rectum of the parabola $\sqrt{x}+\sqrt{y}=\sqrt{a}$

1. $a\sqrt{2}$ 2. $\frac{a}{\sqrt{2}}$ 3. a 4. 2a

Key. 1

Sol. $\sqrt{x}=\sqrt{a}-\sqrt{y}$

$x = a + y - 2\sqrt{ay}$

$(x - y - a)^2 = 4ay$

$x^2 + (y + a)^2 - 2x(a + y) = 4ay$

$x^2 + y^2 - 2xy + 2ay + a^2 - 2ax = 4ay$

$x^2 + y^2 - 2xy = 2ax + 2ay - a^2$

$(x - y)^2 = 2a\left(x + y - \frac{a}{2}\right)$

Axis is x-y=0

$$\left(\frac{x-y}{\sqrt{2}}\right)^2 = \frac{2a}{2}\left(\frac{x+y-\frac{a}{2}}{\sqrt{2}}\right)\times\sqrt{2}$$

$$\left(\frac{x-y}{\sqrt{2}}\right)^2 = a\sqrt{2}\left(\frac{x+y-\frac{a}{2}}{\sqrt{2}}\right)$$

$\therefore$ lengthy $L.R = a\sqrt{2}$

146. Equation of common tangent to $x^2 = 32y$ and $y^2 = 32x$

1. $x + y = 8$
2. $x + y + 8 = 0$
3. $x - y = 8$
4. $x - y + 8 = 0$

Key. 2

Sol. Common tangets $y^2 = 4ax$ and $x^2 = 4ay$ is $xa^{\frac{1}{3}} + yb^{\frac{1}{3}} + a^{\frac{2}{3}}b^{\frac{2}{3}} = 0$

Here a=8, b=8

147. Locus of poles of chords of the parabola $y^2 = 4ax$ which subtends 45^0 at the vertex is $(x + 4a)^2 = \lambda(y^2 - 4ax)$ then λ =________

1. 1
2. 2
3. 3
4. 4

Key. 4

Sol. Parabola is $y^2 = 4ax \rightarrow$ ①

Polar of a pole $(x_1 y_1) = yy_1 - 2ax = 2ax_1 \rightarrow$ ②

Making eq ① homogeneous w.r.t ②

$$y^2 - 4ax\left(\frac{yy_1 - 2ax}{2ax_1}\right) = 0$$

$$x_1y^2 - 2xyy_1 + 4ax^2 = 0$$

Angle between these pair of lines is 45^0

$$\therefore \tan 45^0 = \frac{2\sqrt{y_1^2 - 4ax_1}}{(x_1 + 4a)}$$

Locus of (x_1y_1) is

$$\Rightarrow (x+4a)^2 = 4(y^2 - 4ax)$$

$$\Rightarrow \lambda = 4$$

148. The equation of the normal to the parabola $y^2 = 8x$ at the point t is

1. $y - x = t + 2t^2$ 2. $y + tx = 4t + 2t^3$ 3. $x + ty = t + 2t^2$ 4. $y - x = 2t - 3t^3$

Key. 2

Sol. Equation of the normal at 't' is $y + tx = 2(2)t + (2)t^3 \Rightarrow y + tx = 4t + 2t^3$

149. The slope of the normal at $(at^2, 2at)$ of the parabola $y^2 = 4ax$ is

1. $\frac{1}{t}$ 2. t 3. $-t$ 4. $-\frac{1}{t}$

Key. 3

Sol. Slope of the normal at 't' is $-t$.

150. If the normal at the point 't' on a parabola $y^2 = 4ax$ meet it again at t_1, then $t_1 =$

1. t 2. $-t - 1/t$ 3. $-t - 2/t$ 4. None

Key. 3

Sol. Equation of the normal at t is $tx + y = 2at + at^3 \rightarrow (1)$

Equation of the chord passing through t and t_1 is $y(t+t_1) = 2x + 2att_1 \rightarrow (2)$

Comparing (1) and (2) we get $\frac{t}{-2} = \frac{1}{t+t_1} \Rightarrow t + t_1 = -\frac{2}{t} \Rightarrow t_1 = -\frac{2}{t} - t$.

151. If the normal at t_1 on the parabola $y^2 = 4ax$ meet it again at t_2 on the curve, then

$t_1(t_1 + t_2) + 2 =$

1. 0 2. 1 3. t_1 4. t_2

Key. 1

Sol. Equation of normal at t_1 is $t_1x+y=2at_1+at_1^3$

It passes through $t_2 \Rightarrow at_1t_2^2+2at_2=2at_1+at_1^3$

$\Rightarrow t_1(t_2^2-t_1^2)=2(t_1-t_2) \Rightarrow t_1(t_1+t_2)=-2 \Rightarrow t_1(t_1+t_2)+2=0$

152. If the normal at $(1,2)$ on the parabola $y^2=4x$ meets the parabola again at the point $(t^2,2t)$, then the value of t is

1. 1 2. 3 3. -3 4. -1

Key. 3

Sol. $Let(1,2)=(t_1^2,2t_1) \Rightarrow t_1=1$

$t=-t_1-\frac{2}{t_1}=-1-\frac{2}{1}=-3$

153. If the normal to parabola $y^2=4x$ at $P(1,2)$ meets the parabola again in Q, then Q =

1. $(-6,9)$ 2. $(9,-6)$ 3. $(-9,-6)$ 4. $(-6,-9)$

Key. 2

Sol. $P=(1,2)=(t^2,2t) \Rightarrow t=1$

$Q=(t_1^2,2t_1) \Rightarrow t_1=-t-2/t=-1-2=-3 \Rightarrow Q=(9,-6)$.

154. If the normals at the points t_1 and t_2 on $y^2=4ax$ intersect at the point t_3 on the parabola, then $t_1t_2=$

1. 1 2. 2 3. t_3 4. $2t_3$

Key. 2

Sol. Let the normals at t_1 and t_2 meet at t_3 on the parabola.

The equation of the normal at t_1 is $y+xt_1=2at_1+at_1^3 \rightarrow (1)$

Equation of the chord joining t_1 and t_3 is $y(t_1+t_3)=2x+2at_1t_3 \rightarrow (2)$

(1) and (2) represent the same line.

$\therefore \frac{t_1+t_3}{1}=\frac{-2}{t_1} \Rightarrow t_3=-t_1-\frac{2}{t_1}$. Similarly $t_3=-t_2-\frac{2}{t_2}$

$\therefore -t_1-\frac{2}{t_1}=-t_2-\frac{2}{t_2} \Rightarrow t_1-t_2=\frac{2}{t_2}-\frac{2}{t_1} \Rightarrow t_1-t_2=\frac{2(t_1-t_2)}{t_1t_2} \Rightarrow t_1t_2=2$

155. The number of normals thWSat can be drawn to the parabola $y^2 = 4x$ form the point $(1,0)$ is

1. 0 2. 1 3. 2 4. 3

Key. 2

Sol. $(1,0)$ lies on the axis between the vertex and focus $\Rightarrow$ number of normals =1.

156. The number of normals that can be drawn through $(-1,4)$ to the parabola $y^2 - 4x + 6y = 0$ are

1. 4 2. 3 3. 2 4. 1

Key. 4

Sol. $Let\ S \equiv y^2 - 4x + 6y$. $S_{(-1,4)} = 4^2 - 4(-1) + 6(4) = 16 + 4 + 24 = 44 > 0$

$\therefore$ $(-1,4)$ lies out side the parabola and hence one normal can be drawn from $(-1,4)$ to the parabola.

157. If the tangents and normals at the extremities of a focal chord of a parabola intersect at (x_1, y_1) and (x_2, y_2) respectively, then

1. $x_1 = x_2$ 2. $x_1 = y_2$ 3. $y_1 = y_2$ 4. $x_2 = y_1$

Key. 3

Sol. $Let\ A(t_1)\ B(t_2)$ the extremiues of a focal chard of $y^2 = 4ax$

$\therefore\ t_1 t_2 = -1$

$(x_1, y_1) = [at_1t_2, a(t_1 + t_2)]; (x_2, y_2) = [a(t_1^2 + t_2^2 + t_1t_2 + 2), at_1t_2(t_1 + t_2)]$

$y_2 = -at_1t_2(t_1 + t_2) = -a(-1)(t_1 + t_2) = a(t_1 + t_2) = y_1$

158. The normals at three points P, Q, R of the parabola $y^2 = 4ax$ meet in (h,k). The centroid of triangle PQR lies on

1. $x = 0$ 2. $y = 0$ 3. $x = -a$ 4. $y = a$

Key. 2

Sol. $Let\ P(t_1), Q(t_2)\ \&\ R(t_3)$

Equation of a normal to $y^2 = 4ax$ is $y + tx = 2at + at^3$

This passes through $(h,k) \Rightarrow k + th = 2at + at^3 \Rightarrow at^3 + (2a - h)t - k = 0$

t_1, t_2, t_3 are the roots of this equation $t_1 + t_2 + t_3 = 0$

Centroid of ΔPQR is $G\left[\frac{a}{3}(t_1^2+t_2^2+t_3^2), \frac{2a}{3}(t_1+t_2+t_3)\right]$

$t_1+t_2+t_3=0 \Rightarrow \frac{2a}{3}(t_1+t_2+t_3)=0 \Rightarrow G$ lies on $y=0$.

159. The ordinate of the centroid of the triangle formed by conormal points on the parabola $y^2=4ax$ is

1. 4 2. 0 3. 2 4. 1

Key. 2

Sol. $Let\, t_1, t_2 \,\&\, t_3$ be the conormal points drawn from (x_1, y_1) to $y^2=4ax$

Equation of the normal at point 't' to $y^2=4ax$ is $y+tx=2at+at^3$

This passes through $(x_1, y_1) \Rightarrow y_1+tx_1=2at+at^3 \Rightarrow at^3+(2a-x_1)t-y_1=0$

t_1, t_2, t_3 are the roots of the equation. $\therefore\ t_1+t_2+t_3=0$

The ordinate of the centroid of the triangle formed by the points $t_1, t_2 \,\&\, t_3$ is $\frac{2a}{3}(t_1+t_2+t_3)=0$

160. The normals at two points P and Q of a parabola $y^2=4ax$ meet at (x_1, y_1) on the parabola. Then PQ^2 =

1. $(x_1+4a)(x_1+8a)$ 2. $(x_1+4a)(x_1-8a)$ 3. $(x_1-4a)(x_1+8a)$ 4. $(x_1-4a)(x_1-8a)$

Key. 2

Sol. $Let\, P=(at_1^2, 2at_1), Q=(at_2^2, 2at_2)$

Since the normals at P and Q meet on the parabola, $t_1t_2=2$.

Point of intersection of the normals $(x_1, y_1)=\left(a\left[t_1^2+t_2^2+t_1t_2+2\right], -at_1t_2\left[t_1+t_2\right]\right)$

$\Rightarrow\ x_1=a(t_1^2+t_2^2+t_1t_2+2)=a(t_1^2+t_2^2+4) \Rightarrow a(t_1^2+t_2^2)=x_1-4a$

$PQ^2=(at_1^2-at_2^2)^2+(2at_1-2at_2)^2=a^2(t_1-t_2)^2[(t_1+t_2)^2+4]$
$=a(t_1^2+t_2^2-4)a(t_1^2+t_2^2+8)=(x_1-8a)(x_1+4a)$

161. If a normal subtends a right angle at the vertex of the parabola $y^2=4ax$, then its length is

1. $\sqrt{5}a$ 2. $3\sqrt{5}a$ 3. $6\sqrt{3}a$ 4. $7\sqrt{5}a$

Key. 3

Sol. $Let\, a(at_1^2, 2at_1), B(at_2^2, 2at_2)$.

The normal at A cuts the curve again at B. $\therefore \quad t_1 + t_2 = -\dfrac{2}{t_1}$........(1)

Again AB subtends a right angle at the vertex $0(0,0)$ of the parabola.

Slope $OA = \dfrac{2at_1}{at_1^2} = \dfrac{2}{t_1}$, slope of $OB = \dfrac{2}{t_2}$

$OA \perp OB \Rightarrow \dfrac{2}{t_1}.\dfrac{2}{t_2} = -t_1 t_2 = -4$......(2)

Slope of AB is $\dfrac{2a(t_2 - t_1)}{a(t_2^2 - t_1^2)} = \dfrac{2}{t_1 + t_2} = -t_1$. [By (1)]

Again from (1) and (2) on putting for t_2, we get $t_1 = \dfrac{4}{t_1} = -\dfrac{2}{t_1}$. $\quad \therefore \quad t_1^2 = 2$ or

$t_1 \pm \sqrt{2}$

$t_2 = \dfrac{-4}{t_1} = \dfrac{-4}{(\pm\sqrt{2})} = \pm 2\sqrt{2}. \quad \therefore \quad A = (2a, \pm 2a\sqrt{2}), B = (8a, \pm 4\sqrt{a})$

$AB = \sqrt{(2a - 8a)^2 + (2a\sqrt{2} + 4\sqrt{2}a)^2} = \sqrt{36a^2 + 72a^2} = \sqrt{108a^2} = 6\sqrt{3}a.$

162. Three normals with slopes m_1, m_2, m_3 are drawn from any point P not on the axis of the parabola $y^2 = 4x$. If $m_1 m_2 = a$, results in locus of P being a part of parabola, the value of 'a' equals

1. 2   2. -2   3. 4   4. -4

Key. 1

Sol. Equation of normal to $y^2 = 4x$ is $y = mx - 2m - m^3$ ...(i)

It passes through (α, β) $\quad \therefore \quad m_1 m_2 m_3 \beta = m\alpha - 2, -m^3$

$\Rightarrow \; m^3 + (2 - \alpha)\, m + \beta = 0 \qquad \ldots.(ii)$

(Let m_1, m_2, m_3 are roots)

$\therefore \quad m_1 m_2 m_3 = -\beta \qquad$ (as $\; m_1 m_2 = a$) $\quad \Rightarrow \quad m_3 = -\dfrac{\beta}{a}$

Now $-\dfrac{\beta^3}{a^3} - (2 - \alpha) \times \dfrac{\beta}{a} + \beta = 0$

$\Rightarrow \beta^3 + (2-\alpha)a^2\beta - \beta a^3 = 0$

$\Rightarrow$ locus of P is $y^3 + (2-x)ya^2 - ya^3 = 0$

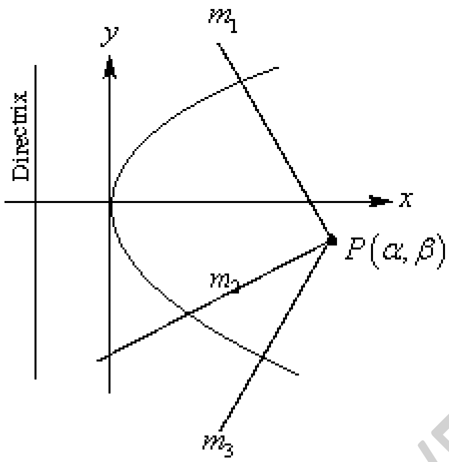
As P is not the axis of parabola

$\Rightarrow$ $y^2 = a^2x - 2a^2 + a^3$ as it is the part of $y^2 = 4x$

$\therefore$ $a^2 = 4$ or $-2a^2 + a^3 = 0$, $a = \pm 2$ or $a^2(a-2) = 0$

$a = \pm 2$ or $a = 0$, $a = 2$

$\Rightarrow$ $a = 2$ is the required value of a



163. The length of the normal chord drawn at one end of the latus rectum of $y^2 = 4ax$ is

1. $2\sqrt{2}a$ 2. $4\sqrt{2}a$ 3. $8\sqrt{2}a$ 4. $10\sqrt{2}a$

Key. 3

Sol. One end of the latus rectum $= (a, 2a)$

Equation of the normal at $(a, 2a)$ is $2a(x-a) + 2a(y-2a) = 0 \Rightarrow x + y - 3a = 0$

Solving; $y^2 = 4ax, x + y - 3a = 0$ we get the ends of normal chord are $(a, 2a), (9a, -6a)$.

Length of the chord $= \sqrt{(9a-a)^2 + (-6a-2a)^2} = \sqrt{64a^2 + 64a^2} = 8\sqrt{2}a.$

164. If the line $y = 2x + k$ is normal to the parabola $y^2 = 4x$, then value of k equals

1. -2 2. -12 3. -3 4. -1/3

Key. 2

Sol. Conceptual

165. The normal chord at a point 't' on the parabola $y^2 = 4ax$ subtends a right angle at the vertex. Then $t^2 =$

1. 4 2. 2 3. 1 4. 3

Key. 2

Sol. Equation of the normal at point 't' is $y + tx = 2at + at^3 \Rightarrow \frac{y+tx}{2at+at^3} = 1$

Homoginising $y^2 = 4ax\left(\frac{y+tx}{2at+at^3}\right) \Rightarrow (2at+at^3)y^2 - 4ax(y+tx) = 0$

These lines re $\perp 1r \Rightarrow$ $2at + at^3 - 4at = 0 \Rightarrow at(t^2-2) = 0 \Rightarrow t^2 = 2$

166. A is a point on the parabola $y^2 = 4ax.$ The normal at A cuts the parabola again at B. If AB subtends a right angle at the vertex of the parabola, then slope of AB is

1. $\sqrt{2}$ 2. 2 3. $\sqrt{3}$ 4. 3

Key. 1

Sol. $Let\ A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$.

The normal at A cuts the curve again at B. $\therefore$ $t_1 + t_2 = -2/t_1 ....(1)$

Again AB subtends a right angle at the vertex $O(0,0)$ of the parabola.

Slope of $OA = \frac{2at_1}{at_1^2} = \frac{2}{t_1}$, Slope of $OB = \frac{2}{t_2}$

$OA \perp OB \Rightarrow \frac{2}{t_1}.\frac{2}{t_2} = -1 \Rightarrow t_1t_2 = -4....(2)$

Slope of AB is $\frac{2a(t_2-t_1)}{a(t_2^2-t_1^2)} = \frac{2}{t_1+t_2} = -t_1$ by (1)

Again from (1) and (2) on putting for t_2 we get $t_1 - \frac{4}{t_1} = \frac{2}{t_1}$. $\therefore$ $t_1^2 = 2 \Rightarrow t_1 = \pm\sqrt{2}$.

$\therefore$ Slope $= \pm\sqrt{2}$.

167. If the normal at P meets the axis of the parabola $y^2 = 4ax$ in G and S is the focus, then SG =

1. SP 2. $2SP$

3. $\frac{1}{2}SP$ 4. None

Key. 1

Sol. Equation of the normal at $P(at^2, 2at)$ is $tx + y = 2at + at^3$

Since it meets the axis, $y=0 \Rightarrow tx = 2at + at^3 \Rightarrow x = 2a + at^2$

$\therefore G = (2a + at^2, 0)$, Focus $S = (a, 0)$

$SG = \sqrt{(2a + at^2 - a)^2 + (0-0)^2} = \sqrt{(a + at^2)^2} = a + at^2 = a(1+t^2)$

$SP = \sqrt{(at^2 - a)^2 + (2at - 0)^2} = \sqrt{(at^2 - a)^2 + 4a^2t^2} = \sqrt{(at^2 + a)^2} = at^2 + a = a(t^2+1)$

$\therefore SG = SP$

168. The normal of a parabola $y^2 = 4ax$ at (x_1, y_1) subtends right angle at the

1. Focus 2. Vertex 3. End of latus rectum 4. None of these

Key. 1

Sol. Conceptual

169. The normal at P cuts the axis of the parabola $y^2 = 4ax$ in G and S is the focus of the parabola. If ΔSPG is equilateral then each side is of length

1. a 2. $2a$ 3. $3a$ 4. $4a$

Key. 4

Sol. $Let\ P(at^2, 2at)$

Equation of the normal at $P(t)$ is $y + tx = 2at + at^3$

Equation to $y-$axis is $x = 0$. Solving $G(2a + at^2, 0)$

Focus $s(a, 0)$

ΔSPG is equilateral $\Rightarrow PG = GS \Rightarrow \sqrt{4a^2 + 4a^2t^2} = \sqrt{a^2(1+t^2)^2}$

$\Rightarrow 4a^2(1+t^2) = a^2(1+t^2)^2 \Rightarrow 4 = 1 + t^2 \Rightarrow t^2 = 3$

Length of the side $= SG = a(1+t^2) = a(1+3) = 4a$

170. If the normals at two points on the parabola $y^2 = 4ax$ intersect on the parabola, then the product of the abscissa is

1. $4a^2$ 2. $-4a^2$ 3. $2a$ 4. $4a^4$

Key. 1

Sol. $Let\ P(at_1^2, 2at_1); Q(at_2^2, 2at_2)$

Normals at $P \& Q$ on the parabola intersect on the parabola $\Rightarrow t_1t_2 = 2$

$at_1^2 \times at_2^2 = a^2(t_1t_2)^2 = a^2(2)^2 = 4a^2$

171. If the normals at two points on the parabola intersects on the curve, then the product of the ordinates of the points is

1. $8a$ 2. $8a^2$ 3. $8a^3$ 4. $8a^4$

Key. 2

Sol. Let the normals at $P(t_1)$ and $Q(t_2)$ intersect on the parabola at $R(t_3)$.

Equation of any noemal is $tx + y = 2at + at^3$

Since it passes through Q we get $t.at_3^2 + 2at_3 = 2at + at^3$

$\Rightarrow at^3 + (2a - at_3^2)t - 2at_3 = 0,$ which is a cubic equation in t and hence its roots are t_1, t_2, t_3.

Product of the roots $= t_1t_2t_3 = \dfrac{-(-2at_3)}{a} = 2t_3 \Rightarrow t_1t_2 = 2$

Product of the absisson of P and Q $= at_1^2.at_2^2 = a^2(t_1t_2)^2 = a^2(2)^2 = 4a^2$.

Product of the ordinates of P and Q $= 2at_1.2at_2 4a^2.t_1t_2 = 4a^2(2) = 8a^2$

172. The equation of the locus of the point of intersection of two normals to the parabola $y^2 = 4ax$ which are perpendicular to each other is

1. $y^2 = a(x-3a)$ 2. $y^2 = a(x+3a)$ 3. $y^2 = a(x+2a)$ 4. $y^2 = a(x-2a)$

Key. 1

Sol. *Let* $P(x_1, y_1)$ be the point of intersection of the two perpendicular normals at $A(t_1), B(t_2)$ on the parabola $y^2 = 4ax$.

Let t_3 be the foot of the third normal through P.

Equation of a normal at t to the parabola is $y + xt = 2at + at^3$

If this normal passes through P then $y_1 + x_1t = 2at + at^3 \Rightarrow at^3 + (2a - x_1)t - y_1 = 0 \rightarrow (1)$

Now t_1, t_2, t_3 are the roots of (1). $\therefore$ $t_1t_2t_3 = y_1 / a$

Slope of the normal at t_1 is $-t_1$

Slope of the normal at t_2 is $-t_2$.

Normals at t_1 and t_2 are perpendicular $\Rightarrow$ $(-t_1)\ (-t_2) = -1 \Rightarrow t_1t_2 = -1 \Rightarrow t_1t_2t_3 = -t_3$

$\Rightarrow \frac{y_1}{a} = -t_3 \Rightarrow t_3 = -\frac{y_1}{a}$

t_3 is a root of (1) $\Rightarrow a(-\frac{y_1}{a})^3 + (2a - x_1)(-\frac{y_1}{a}) - y_1 = 0 \Rightarrow -\frac{y_1^3}{a^2} - \frac{(2a - x_1)y_1}{a} - y_1 = 0$

$\Rightarrow y_1^2 + a(2a - x_1) + a^2 = y_1^2 = a(x_1 - 3a)$.

$\therefore$ The locus of P is $y^2 = a(x - 3a)$

173. The three normals from a point to the parabola $y^2 = 4ax$ cut the axes in points, whose distances from the vertex are in A.P., then the locus of the point is

1. $27ay^2 = 2(x-2a)^3$ 2. $27ay^3 = 2(x-2a)^2$ 3. $9ay^2 = 2(x-2a)^3$ 4. $9ay^3 = 2(x-2a)^2$

Key. 1

Sol. *Let* $P(x_1, y_1)$ be any point.

Equation of any normal is $y = mx - 2am - am^3$

If is passes through P then $y_1 = mx_1 - 2am - am^3$

$\Rightarrow am^3 + (2a - x_1)m_1 + y_1 = 0$, which is cubic in m.

Let m_1, m_2, m_3 be its roots. Then $m_1 + m_2 + m_3 = 0, m_1m_2 + m_2m_3 + m_3m_1 = \frac{2a - x_1}{a}$

Normal meets the axis $(y = 0)$, where $0 = mx - 2am - am^3 \Rightarrow x = 2a + am^2$

$\therefore$ Distances of points from the vertex are $2a + am_1^2, 2a + am_2^2, 2a + am_3^2$

If these are in A.P., then $2(2a + am_2^2) = (2a + am_1^2) + (2a + am_3^2) \Rightarrow 2m_2^2 = m_1^2 + m_3^2$

$\Rightarrow 3m_2^2 = m_1^2 + m_2^2 = (m_1 + m_2 + m_3)^2 - 2(m_1m_2 + m_2m_3 + m_3m_1) = -2(2a - x_1)/a$

$\therefore m_2^2 = 2(x_1 - 2a)/3a$

But $y_1 = m_2(x_1 - 2a - am_2^2) \Rightarrow y_1^2 = m_2^2(x_1 - 2a - am_2^2)^2 = 2(x_1 - 2a)^3/27a$ Locus of P is $27ay^2 = 2(x - 2a)^3$

174. If the normals from any point to the parabola $x^2 = 4y$ cuts the line $y = 2$ in points whose abscissae are in A.P., then the slopes of the tangents at the 3 conormal points are in

1. AP 2. GP 3. HP 4. None

Key. 1

Sol. A point on $x^2 = 4y$ is $(2t, t_2)$ and required point be $P(x_1, y_1)$

Equation of normal at $(2t, t^2)$ is $x + ty = 2t + t^3$.......(1)

Given line equation is $y = 2$........(2)

Solving (1) & (3) $x + t(2) = 2t + t^3 \Rightarrow x = t^3$

This passes through $P(x_1, y_1) \Rightarrow t^3 = x_1$.............(3)

Let $(2t, t_1^2)(2t_2, t_2^2), (2t_3, t_3^2)$ be the co-normal points form P.

$2t_1, 2t_2, 2t_3$ in A.P. $\Rightarrow 4t_2 = 2(t_1 + t_3) \Rightarrow t_2 = \frac{t_1 + t_3}{2}$

$\therefore$ slopes of the tangents t_1, t_2 & t_3 are in A.P.

175. The line $lx + my + n = 0$ is normal to the parabola $y^2 = 4ax$ if

1. $al(l^2 + 2m^2) + m^2 n = 0$
2. $al(l^2 + 2m^2) = m^2 n$
3. $al(2l^2 + m^2) + m^2 n = 0$
4. $al(2l^2 + m^2) = 2m^2 n$

Key. 1

Sol. Conceptual

176. The feet of the normals to $y^2 = 4ax$ from the point $(6a, 0)$ are

1. $(0,0)$
2. $(4a, 4a)$
3. $(4a, -4a)$
4. $(0,0), (4a, 4a), (4a, -4a)$

Key. 4

Sol. Equation of any normal to the parabola $y^2 = 4ax$ is $y = mx - 2am - am^3$

If passes through $(6a, 0)$ then $0 = 6am - 2am - am^3 \Rightarrow am^3 - 4am = 0 \Rightarrow am(m^2 - 4) = 0$

$\Rightarrow m = 0, \pm 2$.

$\therefore$ Feet of the normals $= (am^2, -2am) = (0,0), (4a, -4a), (4a, 4a)$.

177. The condition that parabola $y^2 = 4ax$ & $y^2 = 4c(x - b)$ have a common normal other than x-axis is $(a \neq b \neq c)$

1. $\frac{a}{a-c}<2$ 2. $\frac{b}{a-c}>2$ 3. $\frac{b}{a-c}<1$ 4. $\frac{b}{a-c}>1$

Key. 2
Sol. Conceptual

178. Tangents are drawn from the point (-1, 2) to the parabola $y^2 = 4x$. The length of the intercept made by the line x = 2 on these tangents is
(A) 6 (B) $6\sqrt{2}$ (C) $2\sqrt{6}$ (D) none

Key. B
Sol. Equation of pair of tangent is

$SS_1 = T^2$

$\Rightarrow (y^2 - 4x)(8) = 4(y - x + 1)^2$

$\Rightarrow y^2 - 2y(1-x) - (x^2 + 6x + 1) = 0$

Put $x = 2$

$\Rightarrow y^2 + 2y - 17 = 0$

$\Rightarrow |y_1 - y_2| = 6\sqrt{2}$

179. The given circle $x^2 + y^2 + 2px = 0$, $p \in R$ touches the parabola $y^2 = 4x$ externally, then
(A) p < 0 (B) p > 0 (C) 0 < p < 1 (D) p < - 1

Key. B
Sol. Centre of the circle is (- p, 0), If it touches the parabola, then according to figure only one case is possible.

Hence p > 0

180. The triangle PQR of area A is inscribed in the parabola $y^2 = 4ax$ such that P lies at the vertex of the parabola and base QR is a focal chord. The numerical difference of the ordinates of the points Q & R is
(A) $\frac{A}{2a}$ (B) $\frac{A}{a}$ (C) $\frac{2A}{a}$ (D) $\frac{4A}{a}$

Key. C
Sol. QR is a focal chord

$\Rightarrow R(at^2, 2at) \& Q(\frac{a}{t^2}, -\frac{2a}{t})$

$\Rightarrow d = \left|2at + \frac{2a}{t}\right| = 2a\left|t + \frac{1}{t}\right|$

$$Now \quad A = \frac{1}{2}\begin{vmatrix} at^2 & 2at & 1 \\ \frac{a}{t^2} & -\frac{2a}{t} & 1 \\ 0 & 0 & 1 \end{vmatrix} = a^2\left|t + \frac{1}{t}\right|$$

$\Rightarrow 2a\left|t + \frac{1}{t}\right| = \frac{2A}{a}$

181. Through the vertex O of the parabola $y^2 = 4ax$ two chords OP & OQ are drawn and the circles on OP & OQ as diameter intersect in R. If $\theta_1, \theta_2 \& \phi$ are the inclinations of the tangents at P & Q on the parabola and the line through O, R respectively, then the value of $\cot\theta_1 + \cot\theta_2$ is

(A) – 2 tan ϕ (B) – 2 tan (π - ϕ) (C) 0 (D) 2 cot ϕ

Key. A

Sol. $Let\ \ P(t_1) \& Q(t_2)$

$\Rightarrow$ Slope of tangent at P($\frac{1}{t_1}$) & at Q($\frac{1}{t_2}$) $\Rightarrow \cot\theta_1 = t_1$ and $\cot\theta_2 = t_2$

Slope of PQ = $\frac{2}{t_1 + t_2} = \tan\phi$

$\Rightarrow \tan\phi = -\frac{1}{2}(\cot\theta_1 + \cot\theta_2)$ $\Rightarrow \cot\theta_1 + \cot\theta_2 = -2\tan\phi$

182. AB and AC are tangents to the parabola $y^2 = 4ax$. $p_1, p_2 \& p_3$ are perpendiculars from A, B & C respectively on any tangent to the curve (otherthan the tangents at B&C), then $p_1, p_2 \& p_3$ are in

(A) A.P. (B) G.P. (C) H.P (D) none

Key. B

Sol. Let any tangent is tangent at vertex x = 0 and

$Let\ \ B(t_1) \& C(t_2)$

$\Rightarrow A(at_1t_2, a(t_1 + t_2))$

$\Rightarrow p_1 = at_1^2; p_2 = at_2^2 \ \& \ p_3 = at_1t_2$

$\Rightarrow p_1, p_2 \ \& \ p$ are in G.P.

183. A tangent to the parabola $x^2 + 4ay = 0$ at the point T cuts the parabola $x^2 = 4by$ at A & B. Then locus of the mid point of AB is

(A) $(b + 2a)x^2 = 4b^2y$ (B) $(b + 2a)x^2 = 4a^2y$

(C) $(a + 2b)y^2 = 4b^2x$ (D) $(a + 2b)x^2 = 4b^2y$

Key. D

Sol. Let mid point of AB is M(h, k)

Then equation of AB is $hx - 2b(y + k) = h^2 - 4bk$

$Let\ T(2at, -at^2)$

$\Rightarrow$ *Equation of* tangent(AB) = $x(2at) = -2a(y - at^2)$

Compare these two equations, we get $\frac{h}{2at} = \frac{-2b}{2a} = \frac{h^2 - 2bk}{2a^2t^2}$

By eliminating t and Locus (h, k), we get $(a + 2b)x^2 = 4b^2y$

184. A parabola $y = ax^2 + bx + c$ crosses the x-axis at A(p, 0) & B(q, 0) both to the right of origin. A circle also passes through these two points. The length of a tangent from the origin to the circle is

(A) $\sqrt{\frac{bc}{a}}$ (B) ac^2 (C) b/a (D) $\sqrt{\frac{c}{a}}$

Key. D

Sol. Use power of point for the point O figure

$\Rightarrow OT^2 = OA.OB = pq = \frac{c}{a}$

$\Rightarrow OT = \sqrt{\frac{c}{a}}$

185. The locus of the vertex of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ (a is parameter) is

(A) $xy = \frac{105}{64}$ (B) $xy = \frac{3}{4}$ (C) $xy = \frac{35}{16}$ (D) $xy = \frac{64}{105}$

Key. A

Sol. $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$

$y = \frac{2a^3}{6}\left(x^2 + \frac{3}{2a}x - \frac{12a}{2a^3}\right)$

$y = \frac{2a^3}{6}\left(x^2 + 2.\frac{3}{4a}x + \frac{9}{16a^2} - \frac{9}{16a^2} - \frac{12a}{2a^3}\right)$

$y = \frac{2a^3}{6}\left(\left(x + \frac{3}{4a}\right)^2 - \frac{1059}{16a^3}\right)$

$\left(y + \frac{1059}{48}\right) = \frac{2a^3}{6}\left(x + \frac{3}{4a}\right)^2$

$x = \frac{-1059}{48}$

$y = \frac{-3}{49}$

$xy = \frac{1059}{48} \times \frac{3}{49} = \frac{105}{64}$

186. Equation of a common tangent to the curves $y^2 = 8x$ and $xy = -1$ is

(a) $3y = 9x + 2$ (b) $y = 2x + 1$ (c) $2y = x + 8$ (d) $y = x + 2$

Key. D

Sol. $y^2 = 8k, xy = -1$

Let $P\left(t, \frac{-1}{t}\right)$ be any point on xy = - 1

Equation of the tangent to $xy = -1$ at $P\left(t, \frac{-1}{t}\right)$ is

$$\frac{xy_1 + yx_1}{2} = -1$$

$$\frac{-x}{t} + yt = -2$$

$$y = \frac{x}{t^2} + \left(\frac{-2}{t}\right)\text{.................}(1)$$

If (1) is tangent to the parabola $y^2 = 8x$ then

$$\frac{-2}{t} = \frac{2}{1/t^2} \Rightarrow t^3 = -1$$

$t = -1$

$\therefore$ Common tangent is y = x+2

187. The locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix

1. $x = -a$ 2. $x = -a/2$ 3. $x = 0$ 4. $x = a/2$

Key. 3

Sol. The focus of the parabola $y^2 = 4ax$ is $S(a, 0)$,Let $P(at^2, 2at)$ be any point on the parabola then coordinates of the mid-point of SP are given by

$$x = \frac{a(t^2+1)}{2},\ y = \frac{2at+0}{2}$$

Eliminating 't' we get the locus of the mid-point

As $y^2 = 2ax - a^2$ or $y^2 = 2a(x - a/2)$ (1)

Which is a parabola of the form $Y^2 = 4AX$ (2)

Where $Y = y, X = x - a/2$ and $A = a/2$

Equation of the directrix of (2) is $X = -A$

So equation the directrix of (1) is $x - a/2 = -a/2$

$\Rightarrow$ $x = 0$

188. The tangent at the point $P(x_1, y_1)$ to the parabola $y^2 = 4ax$ meets the parabola $y^2 = 4a(x+b)$ at Q and R, then the coordinates of the mid-point of QR are

1. $(x_1 - a, y_1 + b)$ 2. (x_1, y_1) 3. $(x_1 + b, y_1 + a)$ 4. $(x_1 - b, y_1 - b)$

Key. 2

Sol. Equation of the tangent at $P(x_1, y_1)$ to the parabola $y^2 = 4ax$ is

$yy_1 = 2a(x + x_1)$ Or $2ax - y_1 y + 2ax_1 = 0$ (i)

If $M(h,k)$ is the mid-point of QR , then equation of QR a chord of the parabola $y^2 = 4a(x+b)$ in terms of its mid-point is $ky - 2a(x+h) - 4ab = k^2 - 4a(h+b)$

(using $T = S'$) or $2ax - ky + k^2 - 2ah = 0$ (ii)

Since (i) and (ii) represent the same line, we have

$$\frac{2a}{2a} = \frac{y_1}{k} = \frac{2ax_1}{k^2 - 2ah} \Rightarrow k = y_1 \text{ and } k^2 - 2ah = 2ax_1$$

$$\Rightarrow \quad y_1^2 - 2ah = 2ax_1 \Rightarrow 4ax_1 - 2ax_1 = 2ah$$

(as $P(x_1, y_1)$ lies on the parabola $y^2 = 4ax$)

$\Rightarrow h = x_1$ so that $h = x_1$ $k = y_1$ and the midpoint of QR is (x_1, y_1)

189. Equation of the common tangent touching the circle $(x-3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above the x-axis is

1. $\sqrt{3}y = 3x + 1$ 2. $\sqrt{3}y = -(x+3)$ 3. $\sqrt{3}y = x + 3$ 4. $\sqrt{3}y = -(3x+1)$

Key. 3

Sol. Equation of a tangent to the parabola $y^2 = 4x$ is $y = mx + 1/m.$ it will touch the circle $(x-3) + y^2 = 9$ whose centre is $(3,0)$ and radius is 3 if $\left|\frac{0 + m(3) + (1/m)}{\sqrt{1+m^2}}\right| = 3$

Or if $(3m + 1/m)^2 = 9(1+m^2)$

Or if $9m^2 + 6 + 1/m^2 = 9 + 9m^2$

Or if $m^2 = 1/3, i.e. m = \pm 1/\sqrt{3}$

As the tangent is above the x-axis, we take $m = 1/\sqrt{3}$ and thus the required equation is $\sqrt{3}y = x + 3.$

190. If the normal chord at a point 't' on the parabola $y^2 = 4ax$ subtends a right angle at the vertex, then the value of t is

1. 4 2. $\sqrt{3}$ 3. $\sqrt{2}$ 4. 1

Key. 3

Sol. Equation of the normal at 't' to the parabola $y^2 = 4ax$ is $y = -tx + 2at + at^3$

The joint equation of the lines joining the vertex (origin)to the points of intersection of the parabola and the line (i) is $y^2 = 4ax\left[\frac{y + tx}{2at + at^3}\right]$

$\Rightarrow \quad (2t+t^3)y^2 = 4x(y+tx)$

$\Rightarrow \quad 4tx^2 - (2t+t^3)y^2 + 4xy = 0$

Since these lines are at right angles co efficient of x^2 + coefficient of $y^2 = 0$

$\Rightarrow \quad 4t - 2t - t^3 = 0 \quad \Rightarrow \quad t^2 = 2$

For $t = 0$, the normal line is $y = 0$, *i.e.* axis of the parabola which passes through the vertex $(0,0)$.

191. If the focus of a parabola divides a focal chord of the parabola in segments of length 3 and 2, then the length of the latus rectum of the parabola is

1. 3/2 2. 6/5 3. 12/5 4. 24/5

Key. 4

Sol. Let $y^2 = 4ax$ be the equation of the parabola, then the focus is $S(a,0)$. Let $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ be vertices of a focal chord of the parabola, then $t_1t_2 = -1$. Let $SP = 3 \;\; SQ = 2$

$SP = \sqrt{a^2\left(1-t_1^2\right) + 4a^2t_1^2} = a\left(1+t_1^2\right) = 3 \quad$ (i)

And $\quad SQ = a\left(1+\dfrac{1}{t_1^2}\right) = 2 \quad$ (ii)

From (i) and (ii) we get $t_1^2 = 3/2$ and $a = 6/5$

Hence the length of the latus rectum =24/5.

192. The common tangents to the circle $x^2 + y^2 = a^2/2$ and the parabola $y^2 = 4ax$ intersect at the focus of the parabola

1. $x^2 = 4ay$ 2. $x^2 = -4ay$ 3. $y^2 = -4ax$ 4. $y^2 = 4a(x+a)$

Key. 3

Sol. Equation of a tangent to the parabola $y^2 = 4ax$ is $y = mx + a/m$. If it touches the circle $x^2 + y^2 = a^2/2$

$\dfrac{a}{m} = \left(\dfrac{a}{\sqrt{2}}\right)\sqrt{1+m^2} \;\; \Rightarrow \;\; 2 = m^2(1+m^2)$

$\Rightarrow \quad m^4 + m^2 - 2 = 0 \;\; \Rightarrow \;\; (m^2-1)(m^2+2) = 0$

$\Rightarrow \quad m^2 = 1 \;\Rightarrow\; m = \pm 1$

Hence the common tangents are $y = x + a$ and $y = -x - a$ which intersect at the point $(-a, 0)$ which is the focus of the parabola $y^2 = -4ax$.

193. If $a \neq 0$ and the line $2bx+3cy+4d=0$ passes through the point of intersection of the parabolas $y^2=4ax$ and $x^2=4ay$, then

1. $d^2+(2b-3c)^2=0$ 2. $d^2+(3b+2c)^2=0$ 3. $d^2+(2b+3c)^2=0$ 4. $d^2+(3b-2c)^2=0$

Key. 3

Sol. The pints of intersection of the two parabolas are $(0,0)$ and $(4a,4a)$. If the given line passes through these two points then $d=0$ and $2b+3c=0$ (As $a \neq 0$)so that $d^2(2b+3c)^2=0$.

194. If PQ is a focal chord of the parabola $y^2=4ax$ with focus at S, then $\frac{2SP.SQ}{SP+SQ}=$

1. a 2. $2a$ 3. $4a$ 4. a^2

Key. 2

Sol. Let the coordinates of P be $(at_1^2, 2at_1)$ and of Q be $(at_2^2, 2at_2)$ .Since PQ is a focal chord,

$t_1t_2=-1$

Focus is $S(a,0) \Rightarrow SP=\sqrt{a^2(1-t_1^2)^2+4a^2t_1^2}=a(1+t_1^2)$

And $SQ=a(1+1/t_1^1)=\frac{a(1+t_1^2)}{t_1^2}$

So that $\frac{2SP.SQ}{SP+SQ}=\frac{2a^2(1+t_1^2)^2}{t_1^2 a\left[\left(1+t_1^2\right)+\left(1+\frac{1}{t_1^2}\right)\right]}=2a$

195. If the tangents at the extremities of a chord PQ of a parabola intersect at T, then the distances of the focus of the parabola from the points P,T,Q are in

1. A.P 2. G.P 3. H.P 4. None of these

Key. 2

Sol. Let the equation of the parabola be $y^2=4ax$ and $P(at_1^2, 2at_1)$, $Q(at_2^2, 2at_2)$ be the extremities of the chord PQ. The coordinates of T, the point of intersection of the tangents at P and Q are $(at_1t_2, a(t_1+t_2))$

Now $SP=a\left(1+t_1^2\right)$

$SQ=a\left(1+t_2^2\right)$

And $ST^2=(at_1t_2-a)^2+[a(t_1+t_2)-0]^2$

$=a^2\left(t_1^2+t_2^2+t_1^2t_2^2+1\right)$

$= a^2(1+t_1^2)(1+t_2^2) = SP.SQ$

So that SP, ST, SQ are in $G.P.$

196. If perpendiculars are drawn on any tangent to a parabola $y^2 = 4ax$ from the points $(a \pm k, 0)$ on the axis. The difference of their squares is

1. 4 2. $4a$ 3. $4k$ 4. $4ak$

Key. 4

Sol. Any tangent is $y = mx + a/m.$ Required difference is

$$\left[\frac{m(a+k)+a/m}{\sqrt{1+m^2}}\right]^2 - \left[\frac{m(a-k)+a/m}{\sqrt{1+m^2}}\right]^2$$

$$= \frac{1}{1+m^2} \times 4(ma + a/m)mk = 4ak.$$

197. Which of the following parametric equations does not represent a parabola

1. $x = t^2 + 2t + 1, y = 2t + 2$
2. $x = a(t^2 - 2t + 1), y = 2at - 2a$
3. $x = 3\sin^2 t, y = 6\sin t$
4. $x = a\sin t, y = 2a\cos t$

Key. 4

Sol. $x = aT^2, y = 2aT$ Represents a parabola.

In (a) $a = 1, T = t + 1$, in (b) $a = a, T = (t-1)$

In (c) $a = 3, T + \sin t$ But in (d) if $2aT = 2a\cos t$

$\Rightarrow T = \cos t$ Which does not satisfy $x = aT^2$.

198. $y = -2x + 12a$ is a normal to the parabola $y^2 = 4ax$ at the point whose distance from the directrix of the parabola is

1. $4a$ 2. $5a$ 3. $4\sqrt{2}a$ 4. $8a$

Key. 2

Sol. $y = -2x + 12a$ is a normal at the point $(a(-2)^2, -2a(-2))$ $i, e.,$ $(4a, 4a)$ whose distance from $x = -a$ is $5a$.

199. If the area of the triangle inscribed in the parabola $y^2 = 4ax$ with one vertex at the vertex of the parabola and other two vertices at the extremities of a focal chord is $5a^2/2$,then the length of the focal chord is

1. $3a$ 2. $5a$ 3. $25a/4$ 4. None of these

Key. 3

Sol. Let the vertices be O $(0,0)$, $A(at^2, 2at), B\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$ then

$$\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ at^2 & 2at & 1 \\ \frac{a}{t^2} & \frac{-2a}{t} & 1 \end{vmatrix} = \frac{5a^2}{2} \Rightarrow 2t^2 - 5t + 2 = 0$$

$\Rightarrow$ $t = 2$ or $1/2$ so the vertices of a focal chord are $(4a, 4a)$ and $(a/4, -a)$ (Taking $t = 2$)and length of this focal chord is 25 $a/4$.

200. If the tangents at the extremities of a focal chord of the parabola $x^2 = 4ay$ meet the tangent at the vertex at points whose abcissae are x_1 and x_2 then $x_1x_2 =$

1. a^2
2. $a^2 - 1$
3. $a^2 + 1$
4. $-a^2$

Key. 4

Sol. One extremity of the focal chord be $(2at, at^2)$. Equation of the tangent is $tx = y + at^2$ which meets the tangent at the vertex, $y = 0$ at $x = at$ so $x_1 = at$ and $x_2 = a\left(-\frac{1}{t}\right)$ thus $x_1x_2 = -a^2$.

201. Area of a trapezium whose vertices lie on the parabola $y^2 = 4x$ and its diagonals pass through $(1,0)$ and having length $\frac{25}{4}$ units each is

(A) $\frac{75}{4}$ *sq.units* (B) $\frac{625}{16}$ *sq.units* (C) $\frac{25}{4}$ *sq.units* (D) $\frac{25}{8}$ *sq.units*

Key. 1

Sol. Focus of parabola is $(1,0) \Rightarrow$ diagonals are focal chords

$$AS = 1 + t^2 = CE \quad \frac{1}{C} + \frac{1}{\frac{25}{4} - c} = 1 \quad C = \frac{5}{4}, 5$$

For $C = \frac{5}{4}$ $t = \pm\frac{1}{2}$

$C = 5$ $t = \pm 2$

$$\Rightarrow A = \left(\frac{1}{4}, 1\right) \quad B = (4,4) \quad C = (4,-4) \quad D = \left(\frac{1}{4}, -1\right)$$

$AD = 2 \,\&\, BC = 8$ distance between $AD \,\&\, BC = \dfrac{15}{4}$

Area of trapezium $= \dfrac{75}{4}\, sq.units$

202. Maximum number of common normals of $y^2 = 4ax \,\&\, x^2 = 4by$ may be equal to

(A) 2 (B) 4 (C) 5 (D) 3

Key. 3

Sol. Equation of normal to $y^2 = 4ax$ is $y = mx - 2am - am^3$ & for $x^2 = 4by$ is

$y = mx + 2b + \dfrac{b}{m^2}$

We get $2b + \dfrac{3}{m^2} + 4m + am^3 = 0$

$am^5 + 2am^3 + 2bm^2 + b = 0$

Max 5 normals

203. If the normal to the parabola $y^2 = 4ax$ at a point t_1 cuts the parabola again at t_2, then

(A) $2 \le t_2^2 \le 8$ (B) $t_2^2 \le 2$ (C) $t_2^2 \ge 8$ (D) $t_2^2 \le 1$

Key. 3

Sol. As $t_2 = -t_1 - \dfrac{2}{t_1}$ $\quad t_1 \in R \Rightarrow t_2^2 \ge 8$

204. The normal at a point P of a parabola $y^2 = 4ax$ meets its axis in G and tangent at its vertex in H. If A is the vertex of the parabola and if the rectangle AGQH is completed, then equation to the locus of vertex Q is

a) $y^2(y-2a) = ax^2$ b) $y^2(y+2a) = ax^2$

c) $x^2(x-2a) = ay^2$ d) $x^2(x+2a) = ay^2$

Key. C

Sol. $A = (a,0), H = (0, 2at + at^3), G = (2at + at^2, 0), Q = (h,k)$

$(h,k) = (2a + at^2, 2at + at^3)$

eliminating 't', $x^3 = 2ax^2 + ay^2$

205. If the focus of the parabola $(y-\beta)^2 = 4(x-\alpha)$ always lies between the lines $x + y = 1$ and $x + y = 3$, then,

a) $3 < \alpha + \beta < 4$ b) $0 < \alpha + \beta < 3$

c) $0 < \alpha + \beta < 2$ d) $-2 < \alpha + \beta < 2$

Key. C

Sol. origin & focus line on off side of $x + y = 1 \Rightarrow \alpha + \beta > 0$

origin & focus line on same side of $x + y = 3 \Rightarrow \alpha + \beta < 2.$

206. Consider the two parabolas $y^2 = 4a(x-\alpha) \,\&\, x^2 = 4a(y-\beta)$, where 'a' is the given constant and α, β are variables. If α and β vary in such a way that these parabolas touch each other, then equation to the locus of point of contact

a) circle b) Parabola

c) Ellipse d) Rectangular hyperbola

Key. D

Sol. Let POC be (h,k). Then, tangent at (h,k) to both parabolas represents same line.

207. A parabola y = ax^2 + bx + c crosses x-axis at (α, 0) and (β, 0) both right of origin. A circle passes through these two points. The length of tangent from origin to the circle is

(a) $\sqrt{\frac{bc}{a}}$ (b) ac^2

(c) $\frac{b}{a}$ (d) $\sqrt{\frac{c}{a}}$

Key. D

SOL. ROOTS OF AX^2 + BX + C = 0 ARE α, β

$$\alpha+\beta=-\frac{b}{a},\ \alpha\beta=\frac{c}{a}$$

EQUATION OF CIRCLE THROUGH (α, 0) AND (β, 0)

$$S \equiv (X-\alpha)(X-\beta)+Y^2+\lambda Y=0$$

LENGTH OF TANGENT FROM ORIGIN IS

$$=\sqrt{\alpha\beta}=\sqrt{\frac{c}{a}}$$

208. Equation of the line passing through (α, β), cutting the parabola y^2 = 4ax at two distinct points A and B such that AB subtends right angle at the origin is

(A) $\beta x+(4a-\alpha)y-4a\beta=0$ (B) $2\beta x+(\alpha-4a)y-2a\beta=0$

(C) $\beta x+(\alpha-4a)y-2a\beta=0$ (D) none of these

Key. A

Sol. Any line through (α,β)

$$y-\beta=m(x-\alpha) \quad ..(i)$$

Solving equation (i) with equation of the parabola.

$\Rightarrow\ 2at-\beta=m(at^2-\alpha)$

$\Rightarrow\ amt^2-2at+\beta-m\alpha=0$

$\Rightarrow\ t_1t_2=\frac{\beta-m\alpha}{am}=-4$

$\Rightarrow\ m=\left(\frac{\beta}{\alpha-4a}\right)$

Hence required equation

$$y-\beta=\frac{\beta}{\alpha-4a}(x-\alpha)$$

$\Rightarrow\ y(\alpha-4a)-\alpha\beta+4a\beta=\beta x-\alpha\beta$

$\Rightarrow\ \beta x+(4a-\alpha)y-4a\beta=0$

209. Let 3x – y – 8 = 0 be the equation of tangent to a parabola at the point (7, 13). If the focus of the parabola is at (–1, –1). Its directrix is

(A) x – 8y + 19 = 0 (B) 8x + y + 19 = 0

(C) 8x – y + 19 = 0 (D) x + 8y + 19 = 0

Key. D

Sol. Foot of perpendicular from focus upon tangent is say (α, β). So $\frac{\alpha+1}{3}=\frac{\beta+1}{-1}=\frac{-(-3+1-8)}{3^2+(-1)^2}=1$

$\Rightarrow (\alpha, \beta) \equiv (2, -2)$.

Images of (7, 13) and (–1, –1) w.r.t. (2, –2) will lie on respectively the axis and the directrix of the parabola. The two points are respectively (–3, –17) and (5, –3). Slope of axis = $\frac{-1+17}{-1+3}$ = 8. So equation of directrix: y + 3 = $-\frac{1}{8}(x-5)$

i.e., x + 8y + 19 = 0.

210. A parabola having focus at (2,3) touches both the axes then the equation of its directrix is

a) 2x+3y = 0 b) 3x+2y = 0 c) 2x-3y= 0 d)3x-2y = 0

Key. B

Sol. The foot of the perpendicular from focus (2,3) to the axes are (2,0),(0,3) lie on the tangent at the vertex hence it's slopes $\frac{-3}{2}$. $\therefore$ Equation of directory is 3x+2y = 0

211. Equation of the circle of minimum radius which touches both the parabolas $y = x^2+2x+4$ and $x = y^2+2y+4$ is

a) $2x^2+2y^2-11x-11y-13 = 0$ b) $4x^2+4y^2-11x-11y-13= 0$

c) $3x^2+3y^2-11x-11y-13 = 0$ d) $x^2+y^2-11x-11y-13 = 0$

Key. B

Sol. Given parabolas are symmetric about the line y = x so they have a common normal with slope -1 it meets the parabolas at $\left(\frac{-1}{2},\frac{13}{4}\right),\left(\frac{13}{4},\frac{-1}{2}\right)$ hence the req circles is $x^2+y^2-\frac{11}{4}x-\frac{11}{4}y-\frac{13}{4}=0$

212. If $a_1x + by + c = 0$

$a_2x + by + c = 0$ are two tangents to $y^2 = 8a(x – 2a)$, then

(A) $\left(\frac{a_1}{b}\right)+\frac{a_2}{b}=0$ (B) $1+\frac{a_1}{b}+\frac{a_2}{b}=0$

(C) $a_1a_2 + b^2 = 0$ (D) $a_1a_2 – b^2 = 0$

Key. C

Sol. The tangents are drawn from $\left(0, -\frac{c}{b}\right)$ on. Y-axis which is directrix of the given parabola.

$$\Rightarrow \left(-\frac{a_1}{b}\right)\left(-\frac{a_2}{b}\right)=-1 \Rightarrow a_1a_2+b^2=0$$

213. A normal, whose inclination is 30^0, to a parabola cuts it again at an angle of

a) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ b) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ c) $\tan^{-1}(2\sqrt{3})$ d) $\tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$

Key. D

Sol. The normal at $P(at_1^{\,2}, 2at_1)$ is $y + xt_1 = 2at_1 + at_1^{\,3}$ with slope say $\tan\alpha = -t_1 = \frac{1}{\sqrt{3}}$. If it meets curve at $Q(at_2^{\,2}, 2at_2)$ then $t_2 = -t_1 - \frac{2}{t_1} = \frac{7}{\sqrt{3}}$. Then angle θ between parabola (tangent at Q) and normal at P is given by $\tan\theta = \dfrac{-t_1 - \frac{1}{t_2}}{1 - \frac{t_1}{t_2}} = \dfrac{1}{2\sqrt{3}}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$$

214. The locus of vertices of family of parabolas, $y = ax^2 + 2a^2x + 1$ is $(a \neq 0)$ a curve passing through

a) (1,0) b) (1,1) c) (0,1) d) (0,0)

Key. C

Sol.
$$y = ax^2 + 2a^2x + 1 \Rightarrow \frac{y - (1 - a^3)}{a} = (x + a)^2$$
$$\therefore Vertex = (\alpha, \beta) = (-a, 1 - a^3)$$
$$\Rightarrow \beta = 1 + \alpha^3$$
$$\Rightarrow curve\ \ is\ \ y = 1 + x^3$$

215. The locus of the Orthocentre of the triangle formed by three tangents of the parabola $(4x - 3)^2 = -64(2y + 1)$ is

A) $y = \frac{-5}{2}$ B) $y = 1$ C) $x = \frac{7}{4}$ D) $y = \frac{3}{2}$

Key. D

Sol. The locus is directrix of the parabola

216. A pair of tangents with inclinations α, β are drawn from an external point P to the parabola $y^2 = 16x$. If the point P varies in such a way that $\tan^2\alpha + \tan^2\beta = 4$ then the locus of P is a conic whose eccentricity is

A) $\frac{\sqrt{5}}{2}$ B) $\sqrt{5}$ C) 1 D) $\frac{\sqrt{3}}{2}$

Key. B

Sol. Let $m_1 = \tan\alpha, m_2 = \tan\beta$, Let $P = (h, k)$

m_1, m_2 are the roots of $K = mh + \frac{4}{m} \Rightarrow hm^2 - Km + 4 = 0$

$$m_1 + m_2 = \frac{K}{h}; \quad m_1m_2 = \frac{4}{h}$$

$$m_1^2 + m_2^2 = \frac{K^2}{h^2} - \frac{8}{h} = 4$$

Locus of *P* is $y^2 - 8x = 4x^2 \Rightarrow y^2 = 4(x+1)^2 - 4 \Rightarrow \frac{(x+1)^2}{1} - \frac{y^2}{4} = 1$

217. The length of the latusrectum of a parabola is $4a$. A pair of perpendicular tangents are drawn to the parabola to meet the axis of the parabola at the points A, B. If S is the focus of the parabola then $\frac{1}{|SA|}+\frac{1}{|SB|}=$

A) $2/a$ B) $4/a$ C) $1/a$ D) $2a$

Key. C

Sol. Let $y^2=4ax$ be the parabola

$y=mx+\frac{a}{m}$ and $y=\left(-\frac{1}{m}\right)x-am$ are perpendicular tangents

$S=(a,0), A=\left(-\frac{a}{m^2},0\right), B=(-am^2,0)$

$|SA|=a\left(1+\frac{1}{m^2}\right)=\frac{a(1+m^2)}{m^2}$

$|SB|=a(1+m^2)$

218. Length of the focal chord of the parabola $(y+3)^2=-8(x-1)$ which lies at a distance 2 units from the vertex of the parabola is

A) 8 B) $6\sqrt{2}$ C) 9 D) $5\sqrt{3}$

Key. A

Sol. Lengths are invariant under change of axes

consider $y^2=8x$. Consider focal chord at $(2t^2,4t)$

Focus = (2, 0). Equation of focal chord at *t* is $y=\frac{2t}{t^2-1}9x-2)\Rightarrow 2tx+(1-t^2)y-4t=0$

$\frac{4|t|^2}{\sqrt{4t^2+(1-t^2)^2}}=2\Rightarrow\left(|t|-1\right)^2=0$

Length of focal chord at '*t*'= $2\left(t+\frac{1}{t}\right)^2=\frac{2(t^2+1)^2}{t^2}=8$

219. The slope of normal to the parabola $y=\frac{x^2}{4}-2$ drawn through the point $(10,-1)$

A) -2 B) $-\sqrt{3}$ C) $-1/2$ D) $-5/3$

Key. C

Sol. $x^2=4(y+2)$ is the given parabola

Any normal is $x=m(y+2)-2m-m^3$. If $(10,-1)$ lies on this line then

$10=+m-2m-m^3\Rightarrow m^3+m+10=0\Rightarrow m=-2$

Slope of normal = $1/m$.

220. m_1,m_2,m_3 are the slope of normals $(m_1<m_2<m_3)$ drawn through the point $(9,-6)$ to the parabola $y^2=4x$. $A=[a_{ij}]$ is a square matrix of order 3 such that $a_{ij}=1$ if $i\neq j$ and $a_{ij}=m_i$ if $i=j$. Then det*A* =

A) 6 B) –4 C) –9 D) 8

Key. D

Sol. $y=mx-2m-m^3.(9,-6)$ lies on this

$\therefore -6 = 9m - 2m - m^3 \Rightarrow m^3 - 7m - 6 = 0$

Roots are $-1, -2, 3 \therefore \ |A| = \begin{vmatrix} -2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 3 \end{vmatrix} = (-2)(-4) - (3-1) + 2 = 8$

221. A line L passing through the focus of the parabola $y^2 = 4(x-1)$ intersects the parabola in two distinct points. If '*m*' be the slope of the line L, then

A) $m \in (-1, 1)$ B) $m \in (-\infty, -1) \cup (1, \infty)$

C) $m \in R$ D) $m \in R - \{0\}$

Key. D

Sol. Focus (2, 0)

$y - 0 = m(x-2) \Rightarrow \dfrac{y}{m} + 2 = x \Rightarrow y^2 - \dfrac{4y}{m} - 1 = 0$

$B^2 - 4AC > 0$

$\dfrac{1+m^2}{m^2} > 0 \Rightarrow m \in R - \{0\}$

222. Equation of circle of minimum radius which touches both the parabolas $y = x^2 + 2x + 4$ and $x = y^2 + 2y + 4$ is

a) $2x^2 + 2y^2 - 11x - 11y - 13 = 0$ b) $4x^2 + 4y^2 - 11x - 11y - 13 = 0$

c) $3x^2 + 3y^2 - 11x - 11y - 13 = 0$ d) $x^2 + y^2 - 11x - 11y - 13 = 0$

Key. B

Sol. Circle will be touching both parabolas. Circles centre will be on the common normal

223. If the normal at P(8, 2) on the curve xy = 16 meets the curve again at Q. Then angle subtended by PQ at the origin is

a) $\tan^{-1}\left(\dfrac{15}{4}\right)$ b) $\tan^{-1}\left(\dfrac{4}{15}\right)$ c) $\tan^{-1}\left(\dfrac{261}{55}\right)$ d) $\tan^{-1}\left(\dfrac{55}{261}\right)$

Key. A

Sol. If a normal cuts the hyperbola at point $\left(t, \dfrac{1}{t}\right)$ meets the curve again at $\left(ct^1, \dfrac{C}{t^1}\right)$ then

$t^3 t^1 = -1$

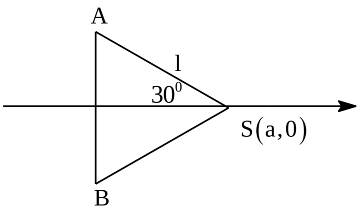
224. An equilateral triangle SAB is inscribed in the parabola $y^2 = 4ax$ having it's focus at 'S'. If the chord AB lies to the left of S, then the length of the side of this triangle is :

a) $3a\left(2 - \sqrt{3}\right)$ b) $4a\left(2 - \sqrt{3}\right)$

c) $2a\left(2 - \sqrt{3}\right)$ d) $8a\left(2 - \sqrt{3}\right)$

Key. B

Sol.



$A\left(a - l\cos 30^0, l\sin 30^0\right)$

Point 'A' lies on $y^2 = 4ax$

$\Rightarrow$ a quadratic in 'l'

225. Let the line lx + my = 1 cuts the parabola $y^2 = 4ax$ in the points A & B. Normals at A & B meet at a point C. Normal from C other than these two meet the parabola at a point D, then D =

a) $(a, 2a)$

b) $\left(\frac{4am}{l^2}, \frac{4a}{l}\right)$

c) $\left(\frac{2am^2}{l^2}, \frac{2a}{l}\right)$

d) $\left(\frac{4am^2}{l^2}, \frac{4am}{l}\right)$

Key. D

Sol. Conceptual

226. The normals to the parabola $y^2 = 4ax$ at points Q and R meet the parabola again at P. If T is the intersection point of the tangents to the parabola at Q and R, then the locus of the centroid of ΔTQR, is

a) $y^2 = 3a(x + 2a)$

b) $y^2 = a(2x + 3a)$

c) $y^2 = a(3x + 2a)$

d) $y^2 = 2a(2x + 3a)$

Key. C

Sol. Let $Q = \left(at_1^2, 2at_1\right)$

$R = \left(at_2^2, 2at_2\right)$

Normals at Q & R meet on parabola

Also $T = \left(at_1t_2, a(t_1 + t_2)\right)$

Let (α, β) be centroid of ΔQRT

Then $3\alpha = a\left(t_1^2 + t_2^2 + t_1t_2\right) \& \beta = a(t_1 + t_2)$

Eliminate $(t_1 + t_2)$

227. The line x –y =1 intersects the parabola y^2 = 4x at A and B. Normals at A and B intersect at C. If D is the point other that A and B at which CD is normal to the parabola then the coordinate of D are

A) (4, 4) B) (4, –4) C) (1, 2) D) (16, –8)

Key. B

Sol. A, B, C be respectively $(t_1^2, 2t_1), (t_2^2, 2t_2), (t_3^2, 2t_3)$ since AB lie on $x - y = 1$

$t_1^2 - 2t_1 = 1$, $t_2^2 - 2t_2 = 1$ subtracting $t_1 + t_2 - 2 = 0$ Now $t_1 + t_2 + t_3 = 0 \Rightarrow t_3 = -2$ so $D(4, -4)$

228. Radius of the largest circle which passes through the focus of the parabola x^2–2x– 4y+5=0 and contained in it is

A) $\sqrt{2} + 1$ B) $4\sqrt{3} + 1$ C) $\sqrt{3} - 1$ D) 4

Key. D

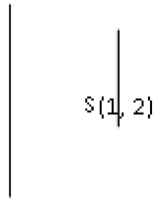
Sol. The parabola is $(x-1)^2 = 4(y-1)$

equation of circle $(x-1)^2 + (y - r - 2)^2 = r^2$

solving with one $y^2 + \{4 - 2(r+2)\}y + 4r = 0$

It has equal roots D=0 $\Rightarrow$ r =4

229. The length of the normal chord at any point on the parabola $y^2 = 4ax$ which subtends a right angle at the vertex of the parabola is



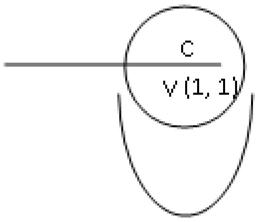
A) $6\sqrt{3}a$ B) $2\sqrt{3}a$ C) $\sqrt{3}a$ D) 2a

Key. A

Sol. $P(at^2, 2at), Q(at_1^{\,2}, 2at_1)$

So $t_1 = -t - \frac{2}{t}$ $\angle POQ = \frac{2}{t}.\frac{2}{t_1} = -1 \Rightarrow t_1 t = -4 \Rightarrow (-t - \frac{2}{t})t + 4 = 0 \Rightarrow t^2 = 2 \Rightarrow t = \sqrt{2}$

$t_1 = -\frac{4}{t} = -2\sqrt{2} \Rightarrow PQ = \sqrt{a^2(t^2 - t_1^{\,2})^2 + 4a^2(t - t_1)^2} = 6\sqrt{3}a$



230. If P is a point (2,4) on the parabola $y^2 = 8x$ and PQ is a focal chord, the coordinate of the mirror image of Q with respect to tangent at P are given by

A) (6,4) B) $(-6,4)$ C) (2, 4) D) (6, 2)

Key. B

Sol. Tangent at extremities of focal chord intersect at right angle at directrix (let R)

$P(2t^2, 4t) \Rightarrow t = 1$

PQ is focal chord $t_1 t_2 = -1 \Rightarrow t_1 = -1 \Rightarrow Q(2, -4)$

Equation of tangent at 'P' ty = x+at^2 $\Rightarrow$ y = x + 2

Coordinate of R (put x = –2 $\Rightarrow$ y =0) $\Rightarrow$ (–2, 0)

R is the mid point of Q & Q^1(mirror image of Q) $\Rightarrow$ $Q^1 = (-6, 4)$

231. The locus of the mid point of chord of the circle $x^2 + y^2 = 9$ such that segment intercepted by the chord on the curve $y^2 - 4x - 4y = 0$ subtends the right angle at the origin.

A) $x^2 + y^2 - 4x - 4y = 0$ B) $x^2 + y^2 + 4x + 4y = 0$ C) $x^2 + 4x + 4y - 9 = 0$ D) None of these

Key. A

Sol. Let the mid point of chord of circle $x^2 + y^2 = 9$ is h, k

equation of chord of circle $hx + ky = h^2 + k^2$

equation of pair of lines joining the point of intersecting of chord and the parabola

with origin is $y^2 - 4(x + y).\frac{(hx + ky)}{(h^2 + k^2)} = 0$

Since the angle between these lines is 90^0 required locus is $x^2 + y^2 = 4(x + y)$

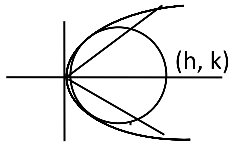
232. The locus of the centre of the circle passing through the vertex and the mid points of perpendicular chords from the vertex of the parabola $y^2 = 4ax$

A) $y^2 = 4a(x - 2a)$ B) $y^2 = a(x - 2a)$ C) $y^2 = 4a(x - a)$ D) $(x - a)^2 + y^2 = a^2$

Key. B

Sol. $t_1 t_2 = -4$ $\quad A(at_1^2, 2at_1)B(at_2^2, 2at_2)$

$P\left(\frac{at_1^2}{2}, at_1\right) \quad Q\left(\frac{at_2^2}{2}, at_2\right)$



C (h, k)

$h = \frac{a}{4}\left(t_1^2 + t_2^2\right), k = \frac{a}{2}(t_1 + t_2) \quad k^2 = \frac{a^2}{4}\left(t_1^2 + t_2^2 + 2t_1t_2\right) = a.\frac{a}{4}\left(t_1^2 + t_2^2\right) - 2a^2$

$k^2 + 2a^2 = a.h \Rightarrow y^2 = a(x - 2a)$

233. Tangents PA and PB are drawn to circle $(x+3)^2 + (y-2)^2 = 1$ from point P lying on $y^2 = 4x$, then the locus of circumcentre of ΔPAB is

A) $(y-1)^2 = 2x-3$ B) $(y+1)^2 = 2x+3$ C) $(y+1)^2 = 2x-3$ D) $(y-1)^2 = 2x+3$

Key. D

Sol. $p(t^2, 2t), C(-3, 2)$

APBC is a cyclic quadrilateral : Circum centre of ΔPAB is the midpoint of CP

$h = \frac{t^2-3}{2} \Rightarrow t^2 = 2h+3$; $\quad k = \frac{2t+2}{2} \Rightarrow t = k-1$; locus $(y-1)^2 = 2x+3$ Q

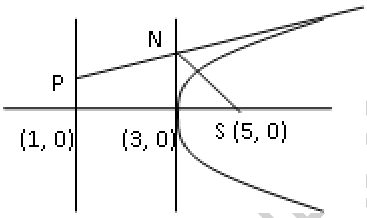
234. From any point P on the straight line $x = 1$ a tangent PQ is drawn to the parabola $y^2 - 8x + 24 = 0$, then the obcissae of N where N is the foot of the perpendicular drawn from A(5, 0) to PQ is

A) 1 B) 2 C) 3 D) 4

Key. C

Sol. $\angle$QNS = 90^0

x-coordinate of N = 3



235. If P(–3, 2) is one end of the focal chord PQ of the parabola y^2+4x+4y = 0 then the slope of the normal at Q is

A) –1/2 B) 1/2 C) 2 D) –2

Key. A

Sol. The equation of the tangent at (–3, 2) to the parabola y^2+4x+4y = 0 is

2y+2(x–3)+2(y+2) = 0 $\Rightarrow$ x+2y–1 = 0

The tangent at one end of the focal chord is parallel to the normal at the other end.

$\Rightarrow$ slope of normal at Q = slope of tangent at P = –1/2

236. The locus of the focus of the family of parabolas having directrix of slope m and touching the lines $x = a \text{ and } y = b$ is

(a) $y + mx = am + b$ (b) $y + mx = am - b$ (c) $y - mx = am + b$ (d) $y - mx = am - b$

Key. A

Sol. Let the focus be (h, k)

Feet of the $\perp$ar from (h , k) on to targets are (a, k) (h, b)

Slope of directrix $=\dfrac{b-k}{h-a}$

$\Rightarrow \dfrac{b-k}{h-a}=m$

The locus is $y+mx=am+b$

237. A circle drawn on any focal chord of the parabola $y^2=4ax$ as diameter cuts the parabola and two points t and t^1 (other than exstremity of a focal chord). Then the value of $tt^1=$

(a) 2 (b) 3 (c) 1 (d) 4

Key. B

Sol. The circle whose diameter ends as $(at^2,2at)\left(\dfrac{a}{t^2},-\dfrac{2a}{t}\right)$ is

$$(x-at^2)\left(x-\frac{a}{t^2}\right)+(y-2at)\left(y+\frac{2a}{t}\right)=0 \quad \to (1)$$

Let t_1,t_2,t_3,t_4 be the points of intersection of (1) and parabola $y^2=4ax$ where t_1,t_2 are the ends of

diameter then $t_1t_2t_3t_4=\dfrac{-3a^2}{a^2}$

$t_3t_4=3$

238. Let S be the set of all possible values of the parameter "a" for which the points of intersection of the parabolas $y^2=3ax$ and $y=\dfrac{1}{2}(x^2+ax+5)$ are concyclic. Then S contains interval

(a) $(-\infty,2)$ (b) $(-2,0)$ (c) $(0,2)$ (d) $(2,\infty)$

Key. D

Sol. The family of curves passing through

The prints of intersection of two parabolas is

$$y^2-3ax+\lambda(x^2+ax+5-2y)=0 \quad \to (1)$$

Since (1) is circle

$a\in(-\infty,-2)\cup(2,\infty)$

239. The line x –y =1 intersects the parabola y^2 = 4x at A and B. Normals at A and B intersect at C. If D is the point other that A and B at which CD is normal to the parabola then the coordinate of D are

A) (4, 4) B) (4, –4) C) (1, 2) D) (16, –8)

Key. B

Sol. A, B, C be respectively $(t_1^2,2t_1),(t_2^2,2t_2),(t_3^2,2t_3)$ since AB lie on $x-y=1$

$t_1^2-2t_1=1$, $t_2^2-2t_2=1$ subtracting $t_1+t_2-2=0$ Now $t_1+t_2+t_3=0\Rightarrow t_3=-2$ so $D(4,-4)$

240. Radius of the largest circle which passes through the focus of the parabola x^2–2x– 4y+5=0 and contained in it is

A) $\sqrt{2}+1$ B) $4\sqrt{3}+1$ C) $\sqrt{3}-1$ D) 4

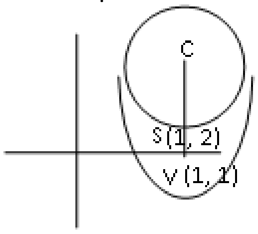
Key. D

Sol. The parabola is $(x-1)^2=4(y-1)$

equation of circle $(x-1)^2+(y-r-2)^2=r^2$

solving with one $y^2+\{4-2(r+2)\}y+4r=0$

It has equal roots D=0 $\Rightarrow$ r =4



241. The length of the normal chord at any point on the parabola $y^2 = 4ax$ which subtends a right angle at the vertex of the parabola is

A) $6\sqrt{3}a$ B) $2\sqrt{3}a$ C) $\sqrt{3}a$ D) 2a

Key. A

Sol. $P(at^2, 2at), Q(at_1^{\,2}, 2at_1)$

So $t_1 = -t - \frac{2}{t}$ $\angle POQ = \frac{2}{t}.\frac{2}{t_1} = -1 \Rightarrow t_1 t = -4 \Rightarrow (-t - \frac{2}{t})t + 4 = 0$ $\Rightarrow$ $t^2 = 2 \Rightarrow t = \sqrt{2}$

$t_1 = -\frac{4}{t} = -2\sqrt{2}$ $\Rightarrow$ $PQ = \sqrt{a^2(t^2 - t_1^{\,2})^2 + 4a^2(t - t_1)^2} = 6\sqrt{3}a$

242. If P is a point (2,4) on the parabola $y^2 = 8x$ and PQ is a focal chord, the coordinate of the mirror image of Q with respect to tangent at P are given by

A) (6,4) B) $(-6,4)$ C) (2, 4) D) (6, 2)

Key. B

Sol. Tangent at extremities of focal chord intersect at right angle at directrix (let R)

$P(2t^2, 4t) \Rightarrow t = 1$

PQ is focal chord $t_1 t_2 = -1 \Rightarrow t_1 = -1 \Rightarrow Q(2, -4)$

Equation of tangent at 'P' ty = x+at^2 $\Rightarrow$ y = x + 2

Coordinate of R (put x = –2 $\Rightarrow$ y =0) $\Rightarrow$ (–2, 0)

R is the mid point of Q & Q^1(mirror image of Q) $\Rightarrow$ $Q^1 = (-6, 4)$

243. The locus of the mid point of chord of the circle $x^2 + y^2 = 9$ such that segment intercepted by the chord on the curve $y^2 - 4x - 4y = 0$ subtends the right angle at the origin.

A) $x^2 + y^2 - 4x - 4y = 0$ B) $x^2 + y^2 + 4x + 4y = 0$ C) $x^2 + 4x + 4y - 9 = 0$ D) None of these

Key. A

Sol. Let the mid point of chord of circle $x^2 + y^2 = 9$ is h, k

equation of chord of circle $hx + ky = h^2 + k^2$

equation of pair of lines joining the point of intersecting of chord and the parabola with origin is $y^2 - 4(x + y).\frac{(hx + ky)}{(h^2 + k^2)} = 0$

Since the angle between these lines is 90^0 required locus is $x^2 + y^2 = 4(x + y)$

244. The locus of the centre of the circle passing through the vertex and the mid points of perpendicular chords from the vertex of the parabola $y^2 = 4ax$

A) $y^2 = 4a(x - 2a)$ B) $y^2 = a(x - 2a)$ C) $y^2 = 4a(x - a)$ D) $(x - a)^2 + y^2 = a^2$

Key. B

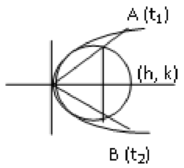
Sol. $t_1 t_2 = -4$ $A(at_1^{\,2}, 2at_1) B(at_2^{\,2}, 2at_2)$

$P\left(\frac{at_1^{\,2}}{2}, at_1\right)$ $Q\left(\frac{at_2^{\,2}}{2}, at_2\right)$

C (h, k)

$h = \frac{a}{4}\left(t_1^2 + t_2^2\right), k = \frac{a}{2}\left(t_1 + t_2\right)$ $\qquad k^2 = \frac{a^2}{4}\left(t_1^2 + t_2^2 + 2t_1t_2\right) = a.\frac{a}{4}\left(t_1^2 + t_2^2\right) - 2a^2$

$k^2 + 2a^2 = a.h \Rightarrow y^2 = a(x - 2a)$



245. Tangents PA and PB are drawn to circle $(x+3)^2 + (y-2)^2 = 1$ from point P lying on $y^2 = 4x$, then the locus of circumcentre of ΔPAB is

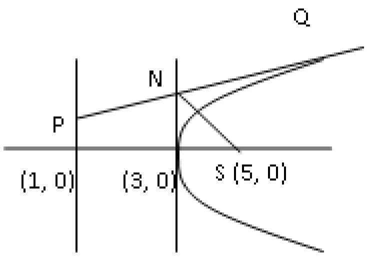
A) $(y-1)^2 = 2x - 3$ B) $(y+1)^2 = 2x + 3$ C) $(y+1)^2 = 2x - 3$ D) $(y-1)^2 = 2x + 3$

Key. D

Sol. $p(t^2, 2t), C(-3, 2)$

APBC is a cyclic quadrilateral : Circum centre of ΔPAB is the midpoint of CP

$h = \frac{t^2 - 3}{2} \Rightarrow t^2 = 2h + 3$; $\quad k = \frac{2t+2}{2} \Rightarrow t = k - 1$; $\quad$ locus $(y-1)^2 = 2x + 3$



246. From any point P on the straight line $x = 1$ a tangent PQ is drawn to the parabola $y^2 - 8x + 24 = 0$, then the obcissae of N where N is the foot of the perpendicular drawn from A(5, 0) to PQ is

A) 1 B) 2 C) 3 D) 4

Key. C

Sol. $\angle QNS = 90^0$

x-coordinate of N = 3

247. If P(–3, 2) is one end of the focal chord PQ of the parabola $y^2+4x+4y = 0$ then the slope of the normal at Q is

A) –1/2 B) 1/2 C) 2 D) –2

Key. A

Sol. The equation of the tangent at (–3, 2) to the parabola $y^2+4x+4y = 0$ is

2y+2(x–3)+2(y+2) = 0 $\Rightarrow$ x+2y–1 = 0

The tangent at one end of the focal chord is parallel to the normal at the other end.

$\Rightarrow$ slope of normal at Q = slope of tangent at P = –1/2

248. A normal whose inclination is 30^0 to a parabola cuts it again at an angle of

(A) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (B) $\tan^{-1}\left(\frac{7}{\sqrt{3}}\right)$ (C) $\tan^{-1}(2\sqrt{3})$ (D) $\tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$

Key. D

Sol. The normal at $P(at_1^2, 2at_1)$ is $y + xt_1 = 2at_1 + at_1^3$ with slope say $\tan\alpha = -t_1 = \frac{1}{\sqrt{3}}$. If it meets curve at $Q(at_2^2, 2at_2)$ then $t_2 = -t_1 - \frac{2}{t_1} = \frac{7}{\sqrt{3}}$. Then angle θ between parabola (tangent at Q) and normal at P is given by $\tan\theta = \frac{-t_1 - \frac{1}{t_2}}{1 - \frac{t_1}{t_2}} = \frac{1}{2\sqrt{3}}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$$

249. The locus of the Orthocentre of the triangle formed by three tangents of the parabola $(4x-3)^2 = -64(2y+1)$ is

(A) $y = \frac{-5}{2}$ (B) $y = 1$ (C) $x = \frac{7}{4}$ (D) $y = \frac{3}{2}$

Key. D

Sol. The locus is directrix of the parabola

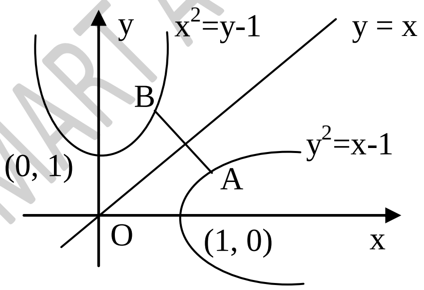
250. Minimum distance between the curves $y^2 = x - 1$ and $x^2 = y - 1$ is equal to

(A) $\frac{3\sqrt{2}}{4}$ (B) $\frac{5\sqrt{2}}{4}$ (C) $\frac{7\sqrt{2}}{4}$ (D) $\frac{\sqrt{2}}{4}$

Key. A

Sol. Both curves are symmetrical about the line y = x. If line AB is the line of shortest distance then at A and B slopes of curves should be equal to one. For $y^2 = x - 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2y} = 1$

$\Rightarrow y = \frac{1}{2}, x = \frac{5}{4}$



$\Rightarrow B = \left(\frac{1}{2}, \frac{5}{4}\right)$, $A = \left(\frac{5}{4}, \frac{1}{2}\right)$

Hence minimum distance AB = $\sqrt{\left(\frac{5}{4} - \frac{1}{2}\right)^2 + \left(\frac{5}{4} - \frac{1}{2}\right)^2} = \frac{3\sqrt{2}}{4}$ units

251. If $(x_1,y_1),(x_2,y_2),(x_3,y_3)$ are the feet of the three normals drawn from a point to the parabola $y^2=4ax$ then $\frac{x_1-x_2}{y_3}+\frac{x_2-x_3}{y_1}+\frac{x_3-x_1}{y_2}=$

(A) 4a (B) 2a (C) a (D) 0

Key. D

Sol. $y_1+y_2+y_3=0$

252. Consider $y^2=8x$. If the normal at a point P on the parabola meets it again at a point Q, then the least distance of Q from the tangent at the vertex of the parabola is.

(A) 16 (B) 8 (C) 4 (D) 2

Key. A

Sol. Let $P(t_1)\&Q(t_2)$ be points on $y^2=8x$. Here $4a=8$ or $a=2$

Required distance = $z=at_2^2=a\left(t_1^2+\frac{4}{t_1^2}+4\right)\quad\left(\because t_2=-t_1-\frac{2}{t_1}\right)$

Z is least if $\frac{dz}{dt_1}=0$ or $t_1^2=2$ Least value of Z = 16

253. A parabola of latusrectum '4a' touches a fixed equal parabola, the axes of the two curves being parallel; the locus of the vertex of moving curve is parabola of latusrectum K then k=

(A) 2a (B) 4a (C) 8a (D) 16a

Key. C

Sol. Let the given parabola be $y^2=4ax$. ----(1)

If the vertex of moving parabola (α,β) its equation is

$(y-\beta)^2=-4a(x-\alpha)-----(2)$

Solving 1 and 2 $2y^2-2\beta y+\beta^2-4a\alpha=0$

Since curve touch each other discriminant=0

$\Rightarrow \beta^2=8a\alpha$ *locus is* $y^2=8ax.$

$\therefore LR=8a$

254. The locus of an end of latus rectum of all ellipses having a given major axis is

(A) A straight line (B) A parabola (C) An ellipse (D) A circle

Key. B

Sol. Let the given major axis have vertices $(-a,0),(a,0)$. If $P(x,y)$ is an end of the latusrectum then

$y=\frac{b^2}{a}=a(1-e^2)$, x = ae

Now eliminate 'e'

255. Given the base of a triangle and the product of the tangents of base angles. Then the locus of the

Third vertex of the triangle is

(A) A straight line (B) A circle

(C) A parabola (D) An ellipse

Key. D

Sol. Take base vertices A (-a, 0) B (a, 0) and vertex C(x, y) given tanA tanB = k

$$\Rightarrow \frac{y}{a+x}.\frac{y}{a-x} = k \Rightarrow \frac{y^2}{a^2-x^2} = k.$$

256. The eccentricity of the conic defined by $\left|\sqrt{(x-1)^2+(y-2)^2}-\sqrt{(x-5)^2+(y-5)^2}\right| = 3$

A) 5/2 B) 5/3 C) $\sqrt{2}$ D) $\sqrt{11}/3$

Key. B

Sol. Hyperbola for which (1, 2) and (5, 5) are foci and length of transverse axis 3.

$2ae = 5$ and $2a = 3$ $\therefore e = 5/3$