Q. 1 A ball of mass m approaches a wall of mass M (>> m) with speed 4 m/s along the normal to the wall. The speed of wall is 1m/s towards the ball. The speed of the ball after an elastic collision with the wall is -

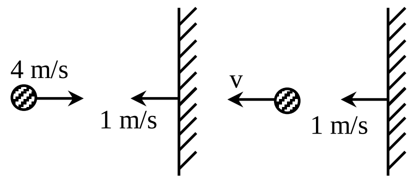
(A) 5 m/s away from the wall

(B) 9 m/s away from the wall

(C) 3 m/s away from the wall

(D) 6 m/s away from the wall **[D]**

Sol.



Before collision After collision

Let v be the velocity of ball after collision, collision is elastic

$\therefore$ e =1

or

relative velocity of separation = relative velocity of approach

$\therefore$ v – 1 = 4 + 1

or v = 6 m/s (away from the wall)

Q.2 One end of a spring of force constant k is fixed to a vertical wall and the other to body of mass m resting on a smooth horizontal surface. There is another wall at a distance x_0 from the body. The spring is then compressed by $2x_0$ and released. The time taken to strike the wall first time is -

(A) $\frac{\pi}{6}\sqrt{\frac{m}{k}}$ (B) $\sqrt{\frac{m}{k}}$

(C) $\frac{2\pi}{3}\sqrt{\frac{m}{k}}$ (D) $\frac{\pi}{4}\sqrt{\frac{m}{k}}$

Sol. **[C]**

The total time from A to C

$t_{AC} = t_{AB} + t_{BC} = \frac{T}{4} + t_{BC}$

Where T = Time period of oscillation of spring-mass system t_{BC} can be given by

$BC = AB \sin\left(\frac{2\pi}{T}\right)t_{BC}$

Putting $\frac{BC}{AB} = \frac{1}{2}$, we get

$t_{BC} = \frac{T}{12}$

$\therefore t_{BC} = \frac{2\pi}{3}\sqrt{\frac{m}{k}}$

Q. 3 A block of mass 2 kg is kept at origin at t = 0 and is having velocity $4\sqrt{5}$ m/s in positive x-direction. The only force acting on it is a conservative and its potential energy is defind as

$U = -x^3 + 6x^2 + 15$ (SI units). Its velocity when its acceleration is minimum after t = 0 is-

(A) 8 m/s (B) 4 m/s

(C) $10\sqrt{24}$ m/s (D) 20 m/s

Sol. **[A]**

$F = -\frac{dU}{dx} \Rightarrow F = 3x^2 - 12x$

Now F = min. $\Rightarrow \frac{dF}{dx} = 0 \Rightarrow x = 2$ m

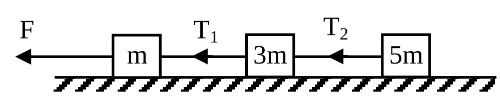
$U_i + K_i = U_f + K_f$

$15 + \frac{1}{2} \times 2 \times 80 = [-(2)^3 + 6 \times (2)^2 + 15] + \frac{1}{2} \times 2 \times v^2$

$v^2 = 64$

v = 8m/sec

Q.4 Figure shows a system of three masses being pulled with a force F. The masses are connected to each other by strings. The horizontal surface is frictionless. The tension T_1 in the first string is 16 N. The acceleration of the system is -



(A) $\frac{1}{m}$ (B) $\frac{2}{m}$

(C) $\frac{3}{m}$ (D) $\frac{4}{m}$ **[B]**

Sol. $T_1 = 8$ ma

$16 = 8\ ma$

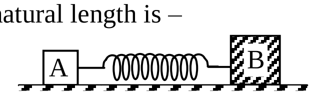
$a = \dfrac{2}{m}$

Q. 5 As observed in the laboratory system, a 6 MeV proton is incident on a stationary ^{12}C target. Velocity of center of mass of the system is- (Take mass of proton to be 1 amu)

(A) 2.6×10^6 m/s (B) 6.2×10^6 m/s

(C) 10×10^6 m/s (D) 10 m/s **[A]**

Q. 6 Two blocks A and B of mass m and 2m are connected by a massless spring of force constant k. They are placed on a smooth horizontal plane. Spring is stretched by an amount x and then released. The relative velocity of the blocks when the spring comes to its natural length is –



(A) $\left(\sqrt{\dfrac{3k}{2m}}\right)x$ (B) $\left(\sqrt{\dfrac{2k}{3m}}\right)x$

(C) $\sqrt{\dfrac{2kx}{m}}$ (D) $\sqrt{\dfrac{3km}{2x}}$

Sol. **[A]**

From conservation of mechanical energy

$\dfrac{1}{2}kx^2 = \dfrac{1}{2}\mu v_r^2$ ... (1)

Here, μ = reduced mass of the blocks

$= \dfrac{(m)(2m)}{m+2m} = \dfrac{2}{3}m$

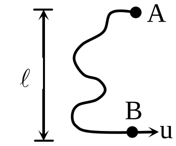
and v_r = relative velocity of the two.

Substituting in Equation (1), we get

$kx^2 = mv_r^2$

$\therefore v_r = \left(\sqrt{\dfrac{3k}{2m}}\right)x$

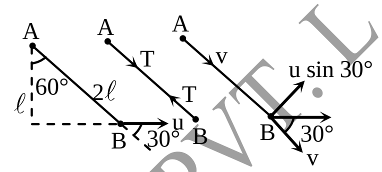
Q.7 Two particles A and B each of mass m are attached by a light inextensible string of length 2ℓ. The whole system lies on a smooth horizontal table with B initially at a distance ℓ from A. The particle at end B is projected across the table with speed u perpendicular to AB. Velocity of ball A just after the string is taut, is –



(A) $\dfrac{u\sqrt{3}}{4}$ (B) $u\sqrt{3}$

(C) $\dfrac{u\sqrt{3}}{2}$ (D) $\dfrac{u}{2}$

Sol. **[A]**



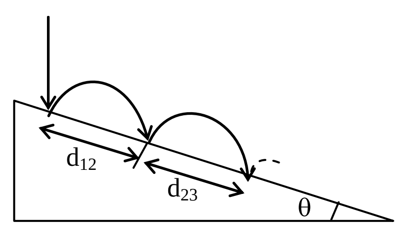
When the string jerk tight both particles begin to move with velocity components v in the direction AB. Using conservation of momentum in the direction AB

mu cos 30° = mv + mv

or $v = \dfrac{u\sqrt{3}}{4}$

Hence, the velocity of ball A just after the jerk is $\dfrac{u\sqrt{3}}{4}$

Q. 8 An elastic ball is dropped on a long inclined plane. If bounces, hits the plane again, bounces and so on. let us Label the distance between the point of the first and second hit d_{12} and the distance between the points of second and the third hit is d_{23}. find the ratio of d_{12}/d_{23} .



(A) $\dfrac{2}{1}$ (B) $\dfrac{1}{2}$

(C) $\dfrac{4}{1}$ (D) $\dfrac{1}{4}$

Sol. **[B]**

If we rotate the coordinate system 80 that the ramp is horizontal then the free fall acceleration will have two components one downward $a_y = -g\cos\theta$ and one horizontal $a_x = g\sin\theta$. In this frame, the initial velocity will have components given by $V_y = V_0\cos\theta$ & $V_x = V_0 \sin\theta$ time for each bounce is given by $t_b = \frac{2V_y}{-ay} = \frac{2V_0}{g}$

The horizontal displacement

$\Delta x = V_x t + 0.5\, a_x t^2$

given these equation

$$d_{12} = V_0 \sin\theta \left(\frac{2v_0}{g}\right) + 0.5g \sin\theta \left(\frac{2V_0}{g}\right)^2$$

$$= \frac{4V_0^2 su\theta}{g}$$

$$d_{13} = V_0\sin\theta\left(\frac{4V_0}{g}\right) + 0.5\ g \sin\theta \left(\frac{4V_0}{g}\right)^2$$

$$= \frac{12V_0^2\ \sin\theta}{g}$$

Since $d_{23} = d_{13} - d_{12} = \frac{8V_0^2\ \sin\theta}{g}$

$$\frac{d_{12}}{d_{23}} = \frac{1}{2}$$

Q.9 In a free space, a rifle of mass M shoots a bullet of mass m at a stationary block of mass M distance D away from it. When the bullet has moved through a distance d towards the block, the centre of mass of the bullet-block system is at a distance of -

(A) $\frac{m(D-d)}{M+m}$ from the block

(B) $\frac{(m+M)d}{M}$ from rifle

(C) $\frac{M(D+d)}{M+m}$ from bullet

(D) None of these

Sol. **[C]**

If x is distance moved by rifle when bullet has traveled through a distance d, then –

$$Mx = md \Rightarrow x = \frac{md}{M}$$

So, distance of bullet from block D – d and distance between block and rifle = D + x

$\therefore$ Distance of C.M from block in

$$r_1 = \frac{Mx(0)+m(D-d)}{m+M} = \frac{(D-d)m}{m+M}$$

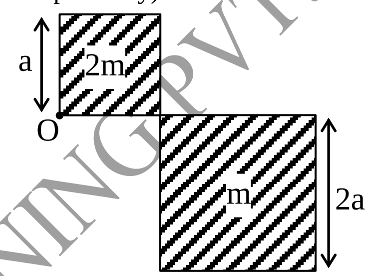
Distance of C.M from rifle

$$= \frac{m(x+d)+M(D+x)}{m+M}$$

Also distance of C.M from bullet

$$= \frac{m\times o+M(D-d)}{M+m}$$

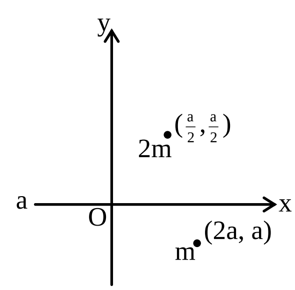
Q.10 The distance of centres of mass of two square plates system a shown from point O. If masses of plates are 2m and m is (their edges are 'a' and '2a' respectively)-



(A) $\frac{a}{2}$ (B) a

(C) $\frac{3a}{2}$ (D) $\frac{2a}{3}$

Sol. **[B]**



$$x_{cm} = \frac{2m\times\frac{a}{2}+m\times 2a}{3m} = a$$

$$y_{cm} = \frac{2m\times\frac{a}{2}-ma}{3m} = 0$$

Q.11 A solid cube of edge 'a' is molten and moulded in eight identical small solid cubes and are placed on one other on a straight line with the edge of bottom cube on the same horizontal plane on which big cube was placed, then the vertical shift in centre of mass is-

(A) $\frac{3a}{2}$ (B) 2a

(C) $\frac{5a}{2}$ (D) 3a

Sol. **[A]**

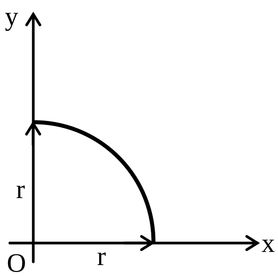
Final hight of centre of mass

$= 4\ell \begin{pmatrix} \text{where } 8\ell^3 = a^3 \\ \Rightarrow \quad \ell = \dfrac{a}{2} \end{pmatrix}$

Initial height of centre of mass $= \dfrac{a}{2}$

$\therefore \quad \Delta h = 4\ell - \dfrac{a}{2} = 2a - \dfrac{a}{2} = \dfrac{3a}{2}$

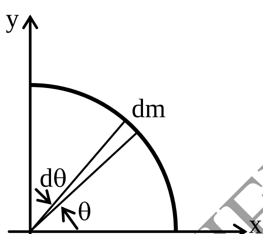
Q.12 The coordinates of centre of mass of the following quarter circular arc is -



(A) $\left(\dfrac{r}{2}, \dfrac{r}{2}\right)$ (B) $\left(\dfrac{2r}{3}, \dfrac{2r}{3}\right)$

(C) $\left(\dfrac{2r}{\pi}, \dfrac{2r}{\pi}\right)$ (D) $\left(\dfrac{4r}{\pi}, \dfrac{4r}{\pi}\right)$

Sol. **[C]**



$dm = \lambda R d\theta = \dfrac{2m}{\pi} d\theta$

$\therefore \quad x_{cm} = \dfrac{1}{m}\int x dm$

$= \dfrac{1}{m}\int_0^{\pi/2} R\cos\theta \dfrac{2m}{\pi} d\theta$

Similarly $y_{cm} = \dfrac{1}{m}\int_0^{\pi/2} R\sin.\theta \dfrac{2m}{\pi} d\theta$

Q.13 A nucleus moving with velocity $\vec{v}$ emits an α-particle. If the velocities of α-particle and the remaining nucleus be $\vec{v_1}$ and $\vec{v_2}$ and their masses be m_1 and m_2, then -

(A) All velocity vectors $\vec{v}$, $\vec{v_1}$ and $\vec{v_2}$ must be parallel

(B) $\vec{v}$ must be parallel to $(\vec{v_1} + \vec{v_2})$

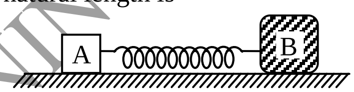
(C) $\vec{v}$ must be parallel to $(m_1\vec{v_1} + m_2\vec{v_2})$

(D) None of above

Sol. **[C]**

Conceptual

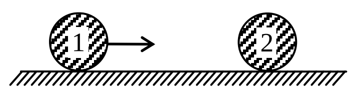
Q.14 Two blocks A and B of mass m and 2m are connected by a massless spring of force constant k. They are placed on a smooth horizontal plane. Spring is stretched by an amount x and then released. The relative velocity of the blocks when the spring comes to its natural length is -



(A) $\left(\sqrt{\dfrac{3k}{2m}}\right)x$ (B) $\left(\sqrt{\dfrac{2k}{3m}}\right)x$

(C) $\sqrt{\dfrac{2kx}{m}}$ (D) $\sqrt{\dfrac{3km}{2x}}$ **[A]**

Q.15 Ball 1 collides with an another identical ball 2 at rest as shown in figure. For what value of coefficient of restitution e, the velocity of second ball becomes two times that of 1 after collision ?



(A) 1/3 (B) 1/2

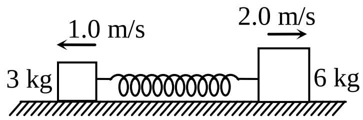
(C) 1/4 (D)1/6 **[A]**

Q.16 After perfectly inelastic collision between two identical particles moving with same speed in different directions, the speed of the particles become half the initial speed. The angle between the velocities of the two before collision is -

(A) 60º (B) 45º

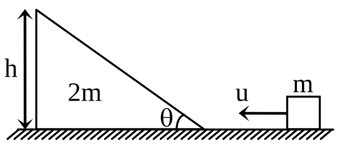
(C) 120º (D) 30º **[C]**

Q.17 Two blocks of mass 3 kg and 6 kg respectively are placed on a smooth horizontal surface. They are connected by a light spring of force constant k = 200 N/m. Initially the spring is unstretched. The indicated velocities are imparted to the blocks. The maximum extension of the spring will be –



(A) 30 cm (B) 25 cm
(C) 20 cm (D) 15 cm **[A]**

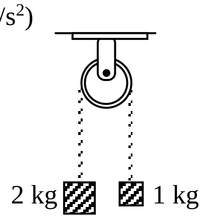
Q.18 A block of mass m is pushed towards a movable wedge of mass 2m and height h with a velocity u. All surfaces are smooth. The minimum value of u for which the block will reach the top of the wedge is –



(A) $2\sqrt{gh}$ (B) $\sqrt{3gh}$
(C) $\sqrt{6gh}$ (D) $\sqrt{\frac{3}{2}gh}$ **[B]**

Q.19 A projectile of mass 3 m explodes at highest point of its path. It breaks into three equal parts. One part retraces its path, the second one comes to rest. The range of the projectile was 100 m if no explosion would have taken place. The distance of the third part from the point of projection when it finally lands on the ground is
(A) 100 m (B) 150 m
(C) 250 m (D) 300 m **[C]**

Q.20 Two blocks of masses 2 kg and 1 kg respectively are tied to the ends of a string which passes over a light frictionless pulley. The masses are held at rest at the same horizontal level and then released. The distance traversed by centre of mass in 2 s is –
($g = 10\ m/s^2$)



(A) 1.42 m (B) 2.22 m
(C) 3.12 m (D) 3.33 m

Sol. **[B]**

Here acceleration of system = $10/9\ m/sec^2$

$S_{CM} = \frac{1}{2} a_{CM} t^2 \Rightarrow 2.22$ m

Q.21 In one dimensional collision of two particles velocities are interchanged when
(i) collision is elastic and mass are equal
(ii) collision is inelastic but masses are unequal
Select the correct alternative
(A) only (i) is correct
(B) only (ii) is correct
(C) both (i) and (ii) are correct
(D) both (i) and (ii) are wrong **[A]**

Q.22 A heavy elastic ball falls freely from point A at a height H_0 onto the smooth horizontal surface of an elastic plate. As the ball strikes the plate another such ball is dropped from the same point A. At what time t, after the second ball is dropped, and at what height will the balls meet ?

(A) $\sqrt{\frac{H_0}{2g}}$; $\frac{3H_0}{4}$ (B) $\sqrt{\frac{2H_0}{g}}$; $\frac{3H_0}{4}$

(C) $\sqrt{\frac{H_0}{2g}}$; $\frac{H_0}{4}$ (D) $\sqrt{\frac{H_0}{g}}$, $\frac{H_0}{4}$ **[A]**

Sol. $t = \sqrt{\frac{H_0}{2g}}$; $h_1 = \frac{3}{4} H_0$

The velocity of the first ball at the moment it strikes the plate will be $v_0 = \sqrt{2gH_0}$. Since the impact is elastic, the ball will begin to rise after the impact with a velocity of the same magnitude v_0. During the time t the first ball will rise to a height

$$h_1 = v_0 t - \frac{gt^2}{2}$$

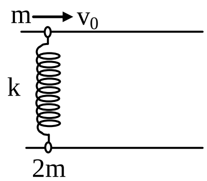
During this time the second ball will move down from a point A a distance

$$h_2 = \frac{gt^2}{2}$$

At the moment the balls meet, $h_1 + h_2 = H_0$. Hence,

$$t = \frac{H_0}{v_0} = \sqrt{\frac{H_0}{2g}} .$$

Q.23 Two ring of mass m and 2 m are connected with a mass less spring and can slips over two frictionless parallel horizontal rails as shown in figure. Ring of mass m is given velocity 'v_0' in the direction shown. Maximum stretch in spring will be -



(A) $\sqrt{\frac{m}{k}}v_0$ (B) $\sqrt{\frac{3m}{k}}v_0$

(C) $\sqrt{\frac{2m}{3k}}v_0$ (D) $\sqrt{\frac{2m}{k}}v_0$ **[C]**

Sol. Maximum expansion in spring is given by

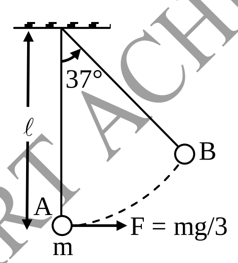
$$\frac{1}{2}kx^2_{max} = \frac{1}{2}\mu v_0^2$$

[μ = Reduced mass]

$$\Rightarrow x_{max} = \sqrt{\frac{\mu}{k}}\,.\,v_0$$

$$= \sqrt{\frac{2m}{3k}}\,v_0$$

Q.24 A pendulum of mass 'm' is pulled from position 'A' by applying a constant horizontal force F = mg/3. Velocity at point 'B' shown in figure –



(A) $\sqrt{\frac{2\ell g}{3}}$ (B) $\sqrt{\frac{3\ell g}{5}}$

(C) $\sqrt{\frac{4}{5}\ell g}$ (D) Zero **[D]**

Sol. $W_{net} = \Delta K$

$$\Rightarrow F\sin\theta.\ell - mg\,\ell\,(1-\cos\theta) = \frac{1}{2}mv^2$$

$$(\therefore\ \theta = 37° \text{ and } F = \frac{mg}{3})$$

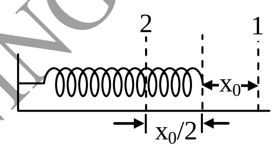
$$\Rightarrow v = \left\{\frac{2\ell}{5m}(3F - mg)\right\}^{1/2} = 0$$

Q.25 Assuming that potential energy of spring is zero when it is stretched by 'x_0', its potential energy when it is compressed by '$x_0/2$' is -

(A) $\frac{3}{8}kx_0^2$ (B) $-\frac{3}{4}kx_0^2$

(C) $-\frac{3}{8}kx_0^2$ (D) $\frac{1}{8}kx_0^2$ **[C]**

Sol. Change in potential energy is independent of reference



$$\therefore U_2 - U_1 = \frac{1}{2}k\left(\frac{x_0}{2}\right)^2 - \frac{1}{2}kx_0^2$$

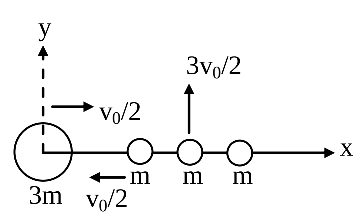
$$= -\frac{3}{8}kx_0^2$$

Q.26 A projectile is fixed with velocity v_0 at an angle 60° with horizontal. At top of its trajectory it explodes into three fragment of equal mass. First fragment retraces the path, second moves vertically upward with speed $\frac{'3v_0'}{2}$. The speed of third fragment –

(A) $\frac{3v_0}{2}$ (B) $\frac{5v_0}{2}$

(C) v_0 (D) $2v_0$ **[B]**

Sol.



Let velocity of third fragment be $\vec{v}$

$$\vec{P}_i = \vec{P}_f$$

$$\Rightarrow 3m\,.\,\frac{v_0}{2}\hat{i} = -\,m\frac{v_0}{2}\hat{i} + m\,.\,\frac{3v_0}{2}\hat{j} + m\vec{v}$$

$$\Rightarrow v = \frac{4v_0}{2}\hat{i} - \frac{3v_0}{2}\hat{j}$$

$$\Rightarrow v = \frac{5v_0}{2}$$

Q.27 A block is hanged from spring in a cage. Elongation in spring is 'x_1' and 'x_2' when cage moves up and down respectively with same acceleration. The expansion in spring when the cage move horizontally with same acceleration -

(A) $\frac{x_1 + x_2}{2}$ (B) $\sqrt{\frac{x_1^2 - x_2^2}{2}}$

(C) $\sqrt{\frac{x_1^2 + x_2^2}{2}}$ (D) $\sqrt{x_1 x_2}$ **[C]**

Sol. $x_1 = \frac{m}{k}(g + a)$

$x_2 = \frac{m}{k}(g - a)$

$x_3 = \frac{m}{k}\sqrt{g^2 + a^2}$

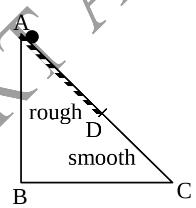
[m = mass of pendulum
k = spring constant]

$\therefore x_1^2 + x_2^2 = \frac{m^2}{k^2}.2(g^2 + a^2)$

$= 2x_3^2$

$\Rightarrow x_3 = \sqrt{\frac{x_1^2 + x_2^2}{2}}$

Q.28 ABC is a fixed incline plane with D mid point of AC. Part AD of incline plane is rough such that when a sphere released from A starts rolling, while the part DC is smooth. The sphere reaches the bottom point C, then –



(A) It is in pure rolling in the part DC

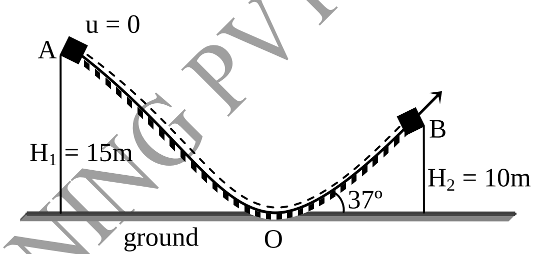
(B) Work done by friction on the sphere is negative when it moves from A to D

(C) Mechanical energy of sphere remains constant for its motion from A to C

(D) All of the above **[C]**

Sol As surface DC is smooth it will slip but as in path AD it is in pure rolling therefore work done by friction is zero. Hence mechanical energy is conserved.

Q. 29 A body starts slipping on a smooth track from point A and leaves the track from point B as shown. The part OB of track is straight at angle 37° with horizontal. The maximum height of body from ground when it is in air is : ($g = 10\ m/s^2$)



(A) 16.8 m (B) 13.6 m

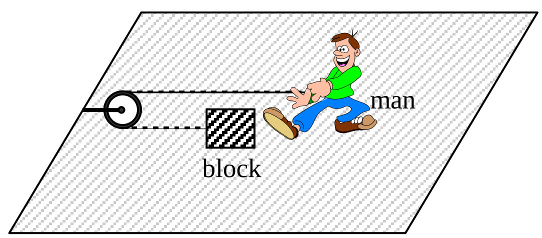
(C) 11.8 m (D) None of these

[C]

Sol. $v_B^2 = 2g\,(H_1 - H_2) = 100\ m^2/s^2$

Now, required height $= H_2 + \frac{v_B^2 \sin^2\theta}{2g} = 11.8$ m

Q. 30 The acceleration of man of mass 40 kg plus block of 10 kg system shown in figure, if the man applies a force 30 N on the string (the plane in fixed, smooth and horizontal, also assume that the string is horizontal)



(A) $0.6\ m/s^2$ (B) Zero

(C) $1.2\ m/s^2$ (D) $2.4\ m/s^2$ **[C]**

Sol. $a_{CM} = \frac{F_{ext}}{\text{Mass of system}} = \frac{60}{50} = 1.2\ m/s^2$

Q.31 A ball falls from a height of 5m and strikes a lift which is moving in the upward direction with a velocity of 1m/s, then the velocity with which the ball rebounds after collision will be -

(A) 11 m/s downwards

(B) 12 m/s upwards

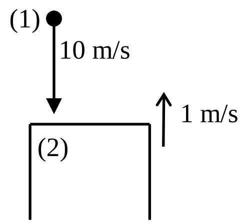
(C) 13 m/s upwards

(D) 12 m/s downwards **[B]**

Sol. Velocity after a fall of 5m = $\sqrt{2\times10\times5}$ = 10 m/s

$\bar{v}_1 - \bar{v}_2 = e(\bar{u}_2 - \bar{u}_1)$

$v_1 - 1 = 1 - (-10) = 12$ m/s



Q.32 A rocket is fired with a speed u = $3\sqrt{gR}$ from the earth surface. What will be its speed at interstellar space ?

(A) zero (B) $\sqrt{2gR}$

(C) $\sqrt{7gR}$ (D) $\sqrt{3gR}$ **[C]**

Sol. From conservation of mechanical energy

$(K.E. + P.E.)_{surface} = (K.E. + P.E.)_{infinity}$

$$\frac{1}{2}m\left(3\sqrt{gR}\right)^2 + \left(-\frac{GMm}{R}\right) = \frac{1}{2}m\,v_\infty^2 + 0$$

or $v_\infty = \sqrt{7gR}$

Q.33 A pulley fixed with celling carries a string with blocks of mass 'm' and '3m' attached to its ends. The masses of string and pulley are negligible. When the system is released, its centre of mass moves with acceleration –

(A) g (B) g/s

(C) g/4 (D) zero **[C]**

Sol. Acceleration of each mass w.r.t. pulley

$$= \frac{3mg - mg}{(3m+m)} = g/2$$

Acceleration of centre of mass

$$= \frac{(3m)g/2 - m\,g/2}{(3m+m)} = g/4$$

Q.34 A ball moving with a velocity v hits a massive wall moving towards the ball with a velocity u. An elastic impact lasts for a time Δt.

(A) The average elastic force acting on the ball is $\frac{m(u+v)}{\Delta t}$

(B) The average elastic force acting on the ball is $\frac{2m(u+v)}{\Delta t}$

(C) The K.E. of the ball increases by mu (u + v)

(D) The K.E. of the ball remains the same after the collision. **[B]**

Sol. $\bar{F} = \frac{m\Delta\bar{v}}{\Delta t} = \frac{m[\bar{v}_f - \bar{v}_i]}{\Delta t}$

$$F = \frac{m[(v+2u)-(-u)]}{\Delta t} = \frac{2m(u+v)}{\Delta t}$$

Q.35 Two masses of 1g and 4g are moving with equal kinetic energies. The ratio of the magnitudes of their momenta is -

(A) 4 : 1 (B) $\sqrt{2}$: 1

(C) 1 : 2 (D) 1 : 16 **[C]**

Sol. $P = \sqrt{2Km}$

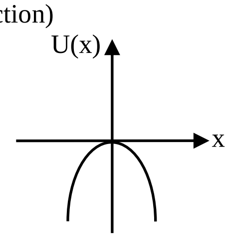
or $P \propto \sqrt{m}$

$\frac{m_1}{m_2} = \frac{1}{4}$

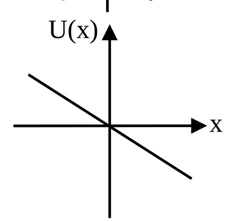
$\therefore \frac{P_1}{P_2} = \frac{1}{2}$

Q.36 A particle is placed at the origin and a force F = kx is acting on it (where k is a positive constant). If U (0) = 0, the graph of U (x) versus x will be: (where U is the potential energy function)

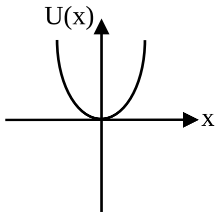
(A)



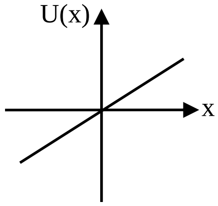
(B)



(C)



(D)



[A]

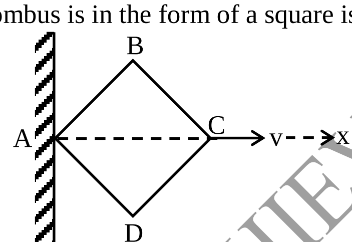
Sol. From $F = -\frac{dU}{dx}$

$$\int_0^{U(x)} dU = -\int_0^x F\,dx = -\int_0^x (kx)\,dx$$

$\therefore\ U(x) = -\frac{kx^2}{2}$

as U (0) = 0

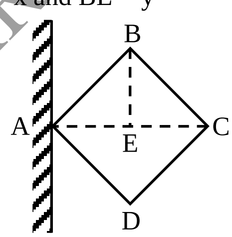
Q.37 Four rods each of length l have been hinged to form a rhombus. Vertex A is fixed to rigid support, vertex C is being moved along the x-axis with a constant velocity v as shown in the figure. The rate at which vertex B is approaching the x-axis at the moment the rhombus is in the form of a square is –



(A) $\frac{v}{4}$ (B) $\frac{v}{3}$

(C) $\frac{v}{2}$ (D) $\frac{v}{\sqrt{2}}$ **[C]**

Sol. Let AC = x and BE = y



Then, $BE^2 + AE^2 = l^2$

or $y^2 + \left(\frac{x}{2}\right)^2 = l^2$

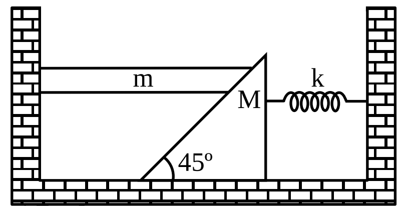
$\therefore\ 2y\left(\frac{dy}{dt}\right) + \frac{x}{2}\cdot\frac{dx}{dt} = 0$

$\therefore\ \left(-\frac{dy}{dt}\right) = \frac{1}{2}\left(\frac{x}{2y}\right)\cdot\frac{dx}{dt}$

x = 2y, when the rhombus is a square.

Hence, $v_B = \frac{1}{2}\ v_c = \frac{v}{2}$

Q.38 All surfaces shown in figure are smooth. System is released with the spring unstretched. In equilibrium, compression in the spring wiil be –



(A) $\frac{mg}{\sqrt{2}k}$ (B) $\frac{2mg}{k}$

(C) $\frac{(M+m)g}{\sqrt{2}k}$ (D) $\frac{mg}{k}$ **[D]**

Sol. Let N be the normal reaction between m and M,

Equilibrium of M

N sin 45º = kx … (i)

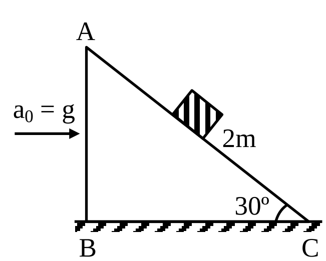
Equilibrium of m in vertical direction gives

N cos 45º = mg … (ii)

From Eqs. (i) and (ii), we get

$x = \frac{mg}{k}$

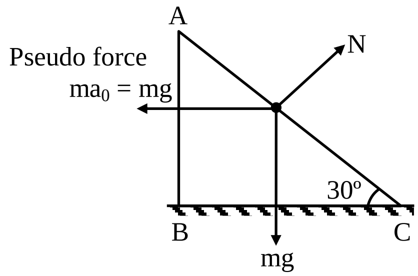
Q.39 A block is placed on an inclined plane moving towards right horizontally with an acceleration a_0 = g. The length of the plane AC = 1 m. Friction is absent everywhere. The time taken by the block to reach from C to A is : (g = 10 m/s²)



(A) 1.2 s (B) 0.74 s

(C) 2.56 s (D) 0.42 s **[B]**

Sol. Drawing free body diagram of block with respect to plane.



Acceleration of the block up the plane is

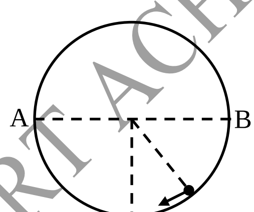
$$a = \frac{mg\cos 30^\circ - mg\sin 30^\circ}{m}$$

$$= \left(\frac{\sqrt{3}-1}{2}\right) g = 3.66\ m/s^2$$

applying $s = \frac{1}{2}at^2$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times 1}{3.66}} = 0.74\ s$$

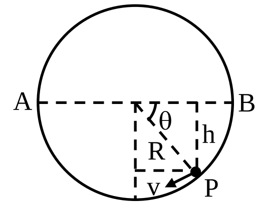
Q.40 A particle of mass m oscillates along the horizontal diameter AB inside a smooth spherical shell of radius R. At any instant the kinetic energy of the particle is K. Then the force applied by particle on the shell at this instant is –



(A) $\frac{K}{R}$ (B) $\frac{2K}{R}$

(C) $\frac{3K}{R}$ (D) $\frac{K}{2R}$ **[C]**

Sol. Let velocity of particle at point P is v.
From conservation of mechanical energy



$$\frac{1}{2}mv^2 = K = mgh$$

Let N be the normal reaction between the particle and the shell at this instant. Then

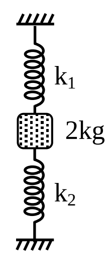
$$N - mg\sin\theta = \frac{mv^2}{R} \quad \left(\frac{mv^2}{R} = \frac{2K}{R}\right)$$

or $$N = mg\left(\frac{h}{R}\right) + \frac{2K}{R} = \frac{K}{R} + \frac{2K}{R}$$

(mgh = K)

$\therefore$ $N = \frac{3K}{R}$ = force on shell

Q.41 A 2 kg block is connected with two springs of force constants k_1 = 100 N/m and k_2 = 300 N/m as shown in figure. The block is released from rest with the springs unstretched. The acceleration of the block in its lowest position is : (g = 10 m/s^2) –



(A) zero (B) 10 m/s^2 upwards

(C) 10 m/s^2 downwards (D) 5 m/s^2 uuwards

[B]

Sol. Let x be the maximum displacement of block downwards. Then from conservation of mechanical energy:
decrease in potential energy of 2 kg block = increase in elastic potential energy of both the springs

$\therefore$ $$mg\,x = \frac{1}{2}(k_1 + k_2)\,x^2$$

or $$x = \frac{2mg}{k_1 + k_2} = \frac{(2)(2)(10)}{100+300} = 0.1\ m$$

Acceleration of block in this position is –

$$a = \frac{(k_1 + k_2)x - mg}{m} \qquad \text{(upwards)}$$

$$= \frac{(400)(0.1) - (2)(10)}{2}$$

$$= 10 \text{ m/s}^2 \qquad \text{(upwards)}$$

Q.42 A U^{238} nucleus initially at rest emits an α-particle and is converted into a Th^{234} nucleus. If the KE of the α-particle is 4.1 MeV what is the recoil energy of the Th nucleus ?

(A) 1 MeV (B) 0.60 MeV
(C) 0.07 MeV (D) 0.005 MeV **[C]**

Q.43 A monkey of mass 20kg rides on a 40kg trolley moving with constant speed of 8m/s along a horizontal track. If the monkey jumps vertically to grab the overhanging branch of a tree, the speed of the trolley after the monkey has jumped off is -

(A) 8 m/s (B) 1 m/s
(C) 4 m/s (D) 12 m/s **[A]**

Q.44 A system of two particle move under the influence of mutual gravitational attraction. If $\vec{p}$ represent the linear momentum and $\vec{J}$ the angular momentum, then the correct formulae are-

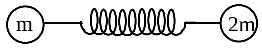
(A) $\Delta \vec{J_1} + \Delta \vec{J_2} = 0$ and $\Delta \vec{p_1} + \Delta \vec{p_2} = 0$

(B) $\Delta \vec{J_1} + \Delta \vec{J_2} \neq 0$ and $\Delta \vec{p_1} + \Delta \vec{p_2} \neq 0$

(C) $\Delta \vec{J_1} + \Delta \vec{J_2} \neq 0$ and $\vec{p_1} + \Delta \vec{p_2} = 0$

(D) $\Delta \vec{J_1} + \Delta \vec{J_2} = 0$ and $\vec{p_1} + \Delta \vec{p_2} \neq 0$ **[A]**

Q.45 Two masses m and 2m are attached to two ends of an ideal spring and the spring is in the compressed state. The energy of spring is 60 joule. If the spring is released, then-



(A) the energy of both bodies will be same
(B) energy of smaller body will be 10J
(C) energy of smaller body will be 20J
(D) energy of smaller body will be 40 J **[D]**

Q.46 A nucleus of mass M emits a X-ray photon of frequency ν, What is the loss of internal energy by nucleus ?

(A) $h\nu$

(B) $\frac{h^2\nu^2}{2MC^2}$

(C) $h\nu\left(1 - \frac{h\nu}{2MC^2}\right)$

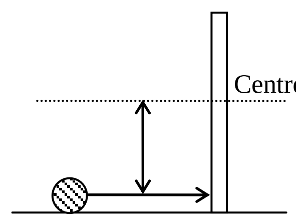
(D) $h\nu\left(1 + \frac{h\nu}{2MC^2}\right)$ **[D]**

Q.47 A nucleus of mass number A originally at rest emits α-particle with speed v. The recoil speed of daughter nucleus is-

(A) $\frac{4v}{A-4}$ (B) $\frac{4v}{A+4}$

(C) $\frac{v}{A-4}$ (D) $\frac{v}{A+4}$ **[A]**

Q.48 A meter scale lying horizontally on a frictionless table is struck by a ball as shown in the diagram. Which of the following quantities is conserved for the ball-scale system-



(A) Linear momentum
(B) Kinetic energy of translation
(C) Angular momentum
(D) Linear momentum and angular momentum both **[D]**

Q.49 Two elastic bodies P and Q having equal masses are moving along the same line with velocities of 16 m/s and 10m/s respectively. Their velocities after the elastic collision will be in m/s -

(A) 0 and 25 (B) 5 and 20
(C) 10 and 16 (D) 20 and 5 **[C]**

Q.50 Two solid balls of rubber A and B whose masses are 200gm and 400gm respectively, are moving in mutually opposite directions. If the velocity A is 0.3 m/s and both the balls come to rest after collision, then the velocity of ball B is-

(A) 0.15 ms^{-1} (B) –0.15 ms^{-1}

(C) 1.5 ms^{-1} (D) none of these **[B]**