Q.1

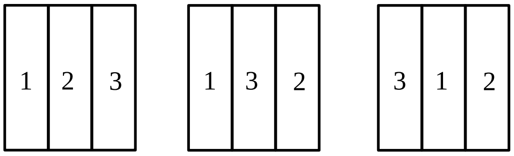


Fig.(a) Fig.(b) Fig.(c)

Figure (a), (b), (c) shows three different arrangements of materials 1, 2 and 3 to form a wall. Thermal conductivities are $K_1 > K_2 > K_3$. The left side of the wall is 20ºC higher than the right side. Temperature difference ΔT across the material 1 has following relation, in three cases:

(A) $\Delta T_a > \Delta T_b > \Delta T_c$

(B) $\Delta T_a = \Delta T_b = \Delta T_c$

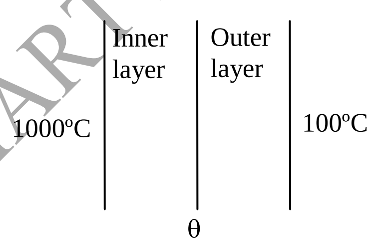
(C) $\Delta T_a = \Delta T_b > \Delta T_c$

(D) $\Delta T_a = \Delta T_b < \Delta T_c$

Sol. **[B]**

All are in series therefore current remains same.
Hence temperature difference = (current × thermal resistance) are equal for every case.

Q.2 The temperature drop through a two layer furnace wall is 900ºC. Each layer is of equal area of cross-section. Which of the following actions will result in lowering the temperature θ of the interface ?



(A) By increasing the thermal conductivity of outer layer
(B) By increasing the thermal conductivity of inner layer
(C) By increasing thickness of outer layer
(D) By decreasing thickness of inner layer

Sol. **[A]**

H = rate of heat flow

$$= \frac{900}{\frac{\ell_i}{K_i A} + \frac{\ell_0}{K_0 A}}$$

$$\text{Now } 100 - \theta = \frac{H\ell_i}{K_i A}$$

$$\text{or } \theta = 1000 - \left[\frac{900}{\frac{\ell_i}{K_i A} + \frac{\ell_0}{K_0 A}}\right]\frac{\ell_i}{K_i A}$$

$$= 100 - \frac{900}{1 + \frac{\ell_0}{K_0}\frac{K_i}{\ell_i}}$$

Now, we can see that θ can be decreased by increasing thermal conductivity of outer layer (K_0) and thickness of inner layer (ℓ_i).

Q.3 A students performs cooling experiment with a solid sphere and hollow sphere of same material and size which are heated to the same temperature. If the temperature difference between each sphere and surroundings is around 30°C , then -

(A) The hollow sphere will cool at a faster rate
(B) The solid sphere will cool at a faster rate
(C) Both spheres will cool at the same rate
(D) Both spheres will cool at the same rate if temp. difference more than 30°C

[A]

Sol. Rate of cooling $\frac{d\theta}{dt} \propto \frac{dQ}{ms}$

dQ is same so $\frac{d\theta}{dt} \propto \frac{1}{m}$

$m_{solid} > m_{hollow}$

hence hollow sphere will cool fast.

Q.4 The ends of the two rods of different materials with their lengths, diameters of cross-section and thermal conductivities all in the ratio 1:2 are maintained at the same temperature difference. The rate of flow of heat in the shorter rod is 1 cal s^{-1}. What is the rate of flow of heat in the larger rod ?

(A) 1 cal s^{-1} (B) 4 cal s^{-1}
(C) 8 cal s^{-1} (D) 16 cal s^{-1} **[B]**

Sol. $\left(\frac{Q}{t}\right)_1 = \frac{KA(\theta_1-\theta_2)}{d} = 1\ \text{cal s}^{-1}$

$\left(\frac{Q}{t}\right)_2 = \frac{2k(4A)(\theta_1-\theta_2)}{2d}$

$= \frac{4kA(\theta_1-\theta_2)}{d}$

$= 4\ \text{cal s}^{-1}$

Q.5 Two metallic spheres P and Q of the same surface area are taken. The weight of P is twice that of Q. Both the spheres are heated to the same temperature and left in a room to cool by radiation. The ratio of the rate of cooling of Q to P is :

(A) $\sqrt{2} : 1$ (B) 2 : 1

(C) 1 : 2 (D) $1 : (2)^{1/3}$

Sol. **[D]**

P and Q have same surface finish and same temperature difference. Hence rate of radiation will depend only on the surface area and mass. That is, rate of radiation will be proportional to mass per unit area $m/4\pi r^2$, i.e., proportional to radius r. Since the mass of P is twice that of Q, $r_P^3 = 2r_Q^3$.

$r_P = (2)^{1/3} r_Q$. where r_P and r_Q are the radii of spheres of P and Q respectively. Hence ratio of rates of cooling is $1 : 2^{1/3}$

Q.6 In preparation for a landing on the bright side of the moon, surface temperature of moon has to be estimated. Assume Lunar surface material is a good insulator. It is given that solar constant is 1353 watts/m^2.
Assume absorptivity and emissivity of moon is same. Approximate surface temperature of moon is
(stefan's constant = $5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$) -

(A) 120ºC (B) 300ºC

(C) 500ºC (D) 720ºC

Sol. **[A]**

At equilibrium

Rate of emission = Rete absorption

$e\sigma T^4 = aI_0$ …(i)

where e = emissivity of moon

a = absorptivity of moon

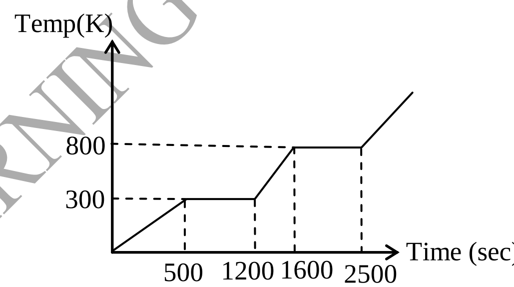
I_0 = solar constant.

given that e = a

$\sigma T^4 = I_0$

$T = \left(\frac{I_0}{\sigma}\right)^{1/4} = \left(\frac{1353}{5.67\times10^{-8}}\right)^{1/4}$ = 393 K = 120º C

Q.7 Temperature variation with time is plotted for an object as shown in figure. The mass of the object is 200 g. Heat is supplied to the object at constant rate of 1 KW. Specific heat of object in liquid phase is -



(A) 3000 J/kg-K (B) 1000 J/kg-K

(C) 4000 J/kg-K (D) 2000 J/kg-K

Sol. **[C]**

$H = \frac{dQ}{dt} = mc\,dT/dt$

$\frac{H}{m\,dT/dt} = C = \frac{1\times10^3}{0.2\times\frac{500}{400}} = \frac{10^4\times2}{5}$

$C = 0.4 \times 10^4$

$C = 4000\ \frac{J}{Kg\,K}$.

Q.8 Consider a black sphere of radius R at temperature T which radiates to distant black surroundings at T = 0 K. The sphere is surrounded by nearby heat shield in the form of black shell whose temperature is determined by radiative equilibrium -

(A) The temperature of the shell is $\frac{T}{\sqrt{2}}$

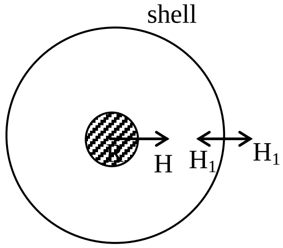
(B) The temperature shell is $\frac{T}{(2)^{1/4}}$

(C) Total power radiated to the surroundings remains the same

(D) Total power radiated to the surroundings is reduces to one fourth of the initial value

[B]

Sol.



Rate of energy absorbed by shell = Rate of energy radiated by shell

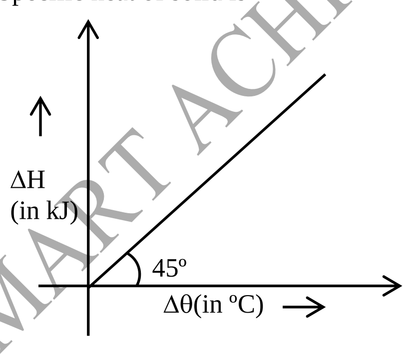
$H - H_1 = H_1$

$H_1 = \frac{H}{2}$; power radiated to the surroundings

$T_1^4 = \frac{T^4}{2}$

$T_1 = \frac{T}{(2)^{1/4}}$

Q.9 A solid of mass 2kg is heated and ΔH (Heat given) vs Δθ (change in temperature) is plotted. Specific heat of solid is –



(A) 1 J/kg/ºC (B) 0.5 J/kg/ºC

(C) 2 kJ/kg/ºC (D) 0.5 kJ/kg/ºC **[D]**

Sol. Slope of ΔH vs Δθ graphs give 'heat capacity'.

$\therefore$ Specific heat capacity = $\frac{\text{heat capacity}}{\text{mass}}$

$= \frac{\tan 45^o}{2}$ kJ/kg/ºC

= 0.5 kJ/kg/ºC

Q.10 Mechanism of heat-transfer involved in freezing of lakes in colder region -

(A) Conduction only (B) Convection only

(C) Radiation only (D) None of these

[D]

Sol. Water of lake gets cooled by conduction till 4ºC after that it is cooled by conduction through ice.

Q.11 A cylinder of radius R made of material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius 3R made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperature. What is the effective thermal conductivity of the system ?

(A) $K_1 + K_2$ (B) $\frac{K_1 + 8K_2}{9}$

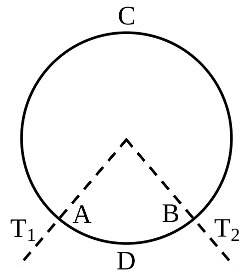
(C) $\frac{K_1K_2}{K_1 + K_2}$ (D) $\frac{8K_1 + K_2}{9}$ **[B]**

Sol. $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

$\frac{K_{eq}(9\pi R^2)}{L} = \frac{K_1\pi R^2}{L} + \frac{K_2 8\pi R^2}{L}$

$K_{eq} = \frac{K_1 + 8K_2}{9}$

Q.12 A ring consisting of two parts ADB and ACB of same conductivity K carries an amount of heat H. The ADB part is now replaced with another metal keeping the temperatures T_1 and T_2 constant. The heat carried increases to 2H. What should be the conductivity of the new ADB part ? (Given $\frac{ACB}{ADB} = 3$)



(A) $\frac{7}{3}$ K (B) 2 K

(C) $\frac{5}{2}$ K (D) 3 K **[A]**

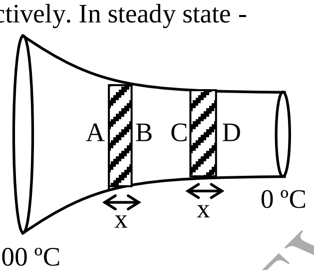
Sol. $H_1 + H_2 = \frac{KA(T_1 - T_2)}{3l} + \frac{KA(T_1 - T_2)}{l} = \frac{4}{3l}$ KA $(T_1 - T_2)$

In later case

$H_2 = 2H - H_1 = \frac{7KA}{3l}(T_1 - T_2) = \frac{K'A}{l}(T_1 - T_2)$

$\Rightarrow K' = \frac{7}{3}K$

Q.13 Two ends of a conducting rod of varying cross-sections are maintained at 200 ºC and 0 ºC respectively. In steady state -



(A) Temperature difference across AB and CD are equal

(B) Temperature difference across AB is greater than that of across CD

(C) Temperature difference across AB is less than that of across CD

(D) Temperature difference may be equal or different depending on the thermal conductivity of the rod

Sol. **[C]**

Sol. Rate of flow of heat $\frac{dQ}{dt}$ or H is equal throughout the rod.

Temperature difference = (H) (thermal resistance)

or Temperature difference $\propto$ thermal resistance (R)

where $R = \frac{l}{KA}$ or $R \propto \frac{1}{A}$

Area across CD is less. Therefore, temperature difference across CD will be more.

Q.14 A cylindrical tube of diameter 3 m and length 20 m is lined with 3 cm of insulating material of conductivity 10^{-4} cal cm^{-1} s^{-10} C^{-1}. It is maintained at 20ºC, although the outer temperature is – 30ºC. What is the rate of heating required to maintain the inner temperature –

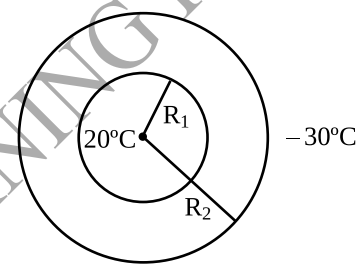


Fig.

(A) 1298 J/s (B) 12980 J/s

(C) 12098 J/s (D) 3100 J/s **[B]**

Sol. R = 1.5 m, length = 20 m.

Consider a cylinder of radius = R and thickness = dR.

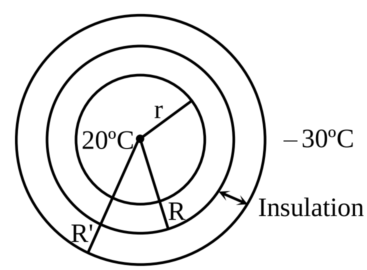


Fig.

Rate of flow of heat

$Q = KA\frac{dt}{dR}$

dt = temp difference in thickness dR.

$A = 2\pi R\ell$, $Q = K(2\pi R)\frac{\ell dt}{dR}$

The rate of flow of heat kept constant to maintain temperature inside.

or $\frac{dR}{R} = \frac{k\,2\pi\ell}{Q}\,dt$

$$\int_R^{R'} \frac{dR}{R} = \frac{k\,2\pi\ell}{Q}\int_{t_1}^{t_2} dt$$

$$[\log R]_R^{R'} = \frac{k\,2\pi\ell}{Q}(t_2 - t_1)$$

$$Q = \frac{k\,2\pi\ell(20-(-30))}{\log\frac{1.53}{1.50}}$$

Q = 3100 cal/sec.

Q = 12980 J/s.

Q.15 A rod of length ℓ and cross section area A has a variable thermal conductivity given by $K = \alpha T$, where α is a positive constant and T is temperature in Kelvin. Two ends of the rod are maintained at temperatures T_1 and T_2 ($T_1 > T_2$). Heat current flowing through the rod will be -

(A) $\frac{A\alpha(T_1^2 - T_2^2)}{3\ell}$ (B) $\frac{A\alpha(T_1^2 + T_2^2)}{\ell}$

(C) $\frac{A\alpha(T_1^2 + T_2^2)}{3\ell}$ (D) $\frac{A\alpha(T_1^2 - T_2^2)}{2\ell}$ **[D]**

Sol. Heat current $i = -KA\,\frac{dT}{dX}$

$$i\,dX = -KA\,dT$$

$$i\int_0^{\ell} dX = -A\alpha\int_{T_1}^{T_2} T\,dT$$

$$\Rightarrow\ i\ell = -A\,\alpha\,\frac{(T_2^2 - T_1^2)}{2}$$

$$\Rightarrow\ i = A\,\alpha\,\frac{(T_1^2 - T_2^2)}{2\ell}$$

Q.16 A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius 2R made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. The effective thermal conductivity of the system is –

(A) $K_1 + K_2$ (B) $(K_1 + 3K_2)/4$

(C) $K_1K_2/(K_1 + K_2)$ (D) $(3K_1 + K_2)/4$ **[B]**

Sol. Parallel combination of cross-section area

πR^2 and $\pi[(2R)^2 - R^2] = 3\pi R^2$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{with } R = \frac{L}{KA}$$

$$\frac{K_{eq}.4\pi R^2}{L} = \frac{K_1\pi R^2}{L} + \frac{K_2(3\pi R^2)}{L}$$

$$\Rightarrow K_{eq} = \frac{K_1 + 3K_2}{4}$$

Q.17 A steel ball of mass m_1 = 1 kg moving with velocity 50 m/sec collides with another steel ball of mass m_2 = 200 gm lying on the ground and both come to rest. During the collision their internal energies changes equally and T_1 and T_2 are the rise in temperature of masses m_1 and m_2 respectively. If s_{steel} = 0.105 cal/gm ºC and J = 4.18, then -

(A) T_1 = 7.1ºC, T_2 = 1.47ºC

(B) T_1 = 1.42ºC, T_2 = 7.1ºC

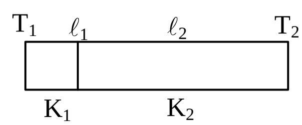
(C) T_1 = 3.4 K, T_2 = 17.0 K

(D) None of these **[B]**

Sol. Half of KE is shared by each ball

$$\frac{1}{2}KE = m_1s_1T_1 = m_2s_2T_2$$

Q.18 One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed of two sections of length ℓ_1 and ℓ_2 and thermal conductivities K_1 and K_2 respectively. The temperature at the interface of the two section is –



(A) $\frac{(K_1\ell_1T_1 + K_2\ell_2T_2)}{(K_1\ell_1 + K_2\ell_2)}$

(B) $\frac{(K_2\ell_2T_1 + K_1\ell_1T_2)}{(K_1\ell_1 + K_2\ell_2)}$

(C) $\frac{(K_2\ell_1T_1 + K_1\ell_2T_2)}{(K_2\ell_1 + K_1\ell_2)}$

(D) $\frac{(K_1\ell_2T_1 + K_2\ell_1T_2)}{(K_1\ell_2 + K_2\ell_1)}$ **[D]**

Sol. $H_1 = H_2 \therefore \dfrac{k_1A[T_1 - T]}{\ell_1} = \dfrac{k_2A[T - T_1]}{\ell_2}$

Q.19 The diameter of a rod is given by $d = d_0(1 + ax)$ where 'a' is a constant and x is distance from one end. If thermal conductivity of material is K. Then the thermal resistance of the rod if its length is ℓ is -

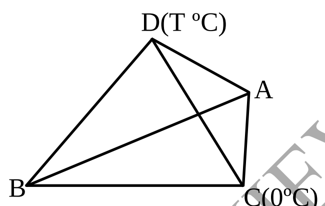
(A) $\dfrac{1}{K\pi d_0^2}$ (B) $\dfrac{4\ell}{K\pi d_0^2(a\ell+1)}$

(C) $\dfrac{2\ell}{K\pi d_0^2(a\ell+1)^2}$ (D) $\dfrac{2\ell}{K\pi d_0^2(a\ell+1)}$ **[B]**

Sol. $dR = \dfrac{dx}{K\dfrac{\pi d^2}{4}} = \dfrac{4dx}{K\pi d_0^2(1+ax)^2}$

$\therefore R = \int dR$

Q.20 Six similar bars each of thermal resistance R are joined to form a regular tetrahedron as shown in the figure. Point D is maintained at a constant temperature T ºC and point C at 0ºC. Temperature of junction A is -



(A) $\dfrac{T}{4}$ (B) $\dfrac{T}{3}$ (C) $\dfrac{T}{2}$ (D) $\dfrac{T}{5}$

Sol. **[C]**

Temperature of junction A is T_A

$\dfrac{KA(T-T_A)}{\ell} = \dfrac{KA(T_A-0)}{\ell}$

$\Rightarrow T - T_A = T_A$

$\Rightarrow T_A = \dfrac{T}{2}$

Q.21 Two rods P and Q of same length and same diameter having thermal conductivity ratio 2 : 3 joined end to end. If temperature at one end of P is 100ºC and at one end of Q 0ºC, then the temperature of the interface is -

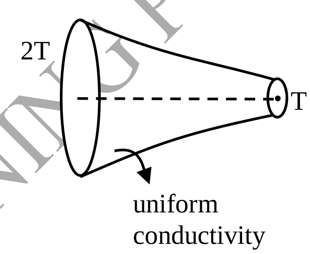
(A) 40ºC (B) 50ºC

(C) 60ºC (D) 70ºC **[A]**

Sol. $\dfrac{K_pA[100-\theta]}{\ell} = \dfrac{K_Q.A[\theta]}{\ell}$

$\therefore \dfrac{K_p}{K_Q} = \dfrac{2}{3} = \dfrac{\theta}{100-\theta}$ OR (θ = 40ºC)

Q.22 In the given conical distorted shape of rod heat is being conducted under steady state. The two ends are maintained at different temperature. Now choose the correct alternative –



(A) The rate of heat flow will not be constant through the rod

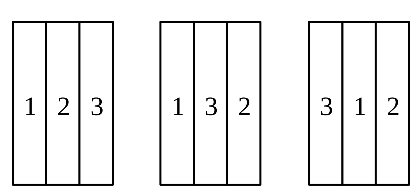
(B) The magnitude of temperature gradient increase from left to right

(C) The temperature at mid point will be $\dfrac{3T}{2}$

(D) The temperature at mid point will be less than $\dfrac{3T}{2}$ **[B]**

Sol. $\dfrac{d\theta}{dt} = KA\left(\dfrac{-dT}{dx}\right)$ so $\left(-\dfrac{dT}{dx}\right) \propto \dfrac{1}{A}$

Q.23 Following figure shows three different arrangements of materials 1, 2 and 3 to form a wall, thermal conductivities are $K_1 > K_2 > K_3$. The left side of wall is 20ºC higher than the right side. Temperature difference ΔT across the material 1 has following relation, in three cases–



(a) (b) (c)

(A) $\Delta T_a > \Delta T_b > \Delta T_c$ (B) $\Delta T_a = \Delta T_b = \Delta T_c$
(C) $\Delta T_a = \Delta T_b > \Delta T_c$ (D) $\Delta T_a = \Delta T_b < \Delta T_c$ **[B]**

Sol. Heat current is same in all cases and resistance of material remain same in all cases.

Q.24 Two rods of same length and transfer a given amount of heat 12 second, when they are joined as shown in figure (i). But when they are joined as shown in figure (ii), then they will transfer same heat in same conditions in

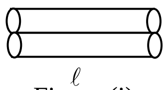


Figure (i)

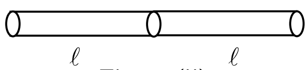
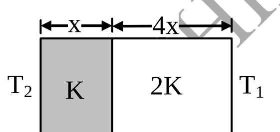


Figure (ii)

(A) 24 s (B) 13 s
(C) 15 s (D) 48 s **[D]**

Sol. $t \propto \dfrac{\ell}{A}$, $t' \propto \dfrac{2\ell}{A/2}$

Q.25 The temperature of the two outer surface of a composite slab consisting of two materials having coefficient of thermal conductivity K and 2K and thickness x and 4x respectively are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab, in steady state is $\left(\dfrac{A(T_2 - T_1)K}{x}\right).f$ with f equal to –



(A) 1 (B) 1/2
(C) 2/3 (D) 1/3 **[D]**

Sol.

Series Req = $R_1 + R_2 = \dfrac{x}{KA} + \dfrac{4x}{2KA} = \dfrac{3x}{KA}$

Rate of heat $\dfrac{dQ}{dt} = \dfrac{T_2 - T_1}{R_{eq}} = \dfrac{T_2 - T_1}{3x}\,KA$

given $\dfrac{dQ}{dt} = \dfrac{A[T_2 - T_1]K}{x} f = \dfrac{T_2 - T_1}{3x}\,KA$

$$f = \frac{1}{3}$$

Q.26 A slab consists of two parallel layers of copper and brass of the same thickness and having thermal conductivities in the ratio 1 : 4. If the free face of brass is at 100ºC and that of copper at 0ºC, the temperature of interface is -

(A) 80ºC (B) 20ºC
(C) 60ºC (D) 40ºC **[A]**

Sol. $4K(100 - \theta) = K(\theta - 0)$
$\Rightarrow 400 - 4\theta = \theta$
$\Rightarrow \theta = 80ºC$

Q.27 Two solid spheres, of radii R_1 and R_2 are made of the same material and have similar surfaces. The spheres are raised to the same temperature and then allowed to cool under identical conditions. Assuming spheres to be perfect conductors of heat, their initial rates of loss of heat are -

(A) R_1^2/R_2^2 (B) R_1/R_2
(C) R_2/R_1 (D) R_2^2/R_1^2 **[A]**

Sol. Rate of hoss of heat

$= \dfrac{\sigma\pi^4 \times A_1 \times e_1}{\sigma\pi^4 \times A_2 \times e_2} = \dfrac{R_1^2}{R_2^2}$ [$\therefore$ $e_1 = e_2$]

Q.28 Two identical rods of copper and iron are coated with wax uniformly. When one end of each is kept at temperature of boiling water, the length upto which wax melts are 8.4 cm and 4.2 cm, respectively. If thermal conductivity of copper is 0.92, then thermal conductivity of iron is -

(A) 0.23 (B) 0.46
(C) 0.115 (D) 0.69 **[A]**

Sol. $\dfrac{K_1}{K_2} = \dfrac{\ell_1^2}{\ell_2^2}$

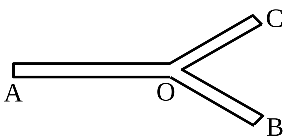
Q.29 Two vessels of different materials are similar in size in every respect. The same quantity of ice filled in them gets melted in 20 min and 35 min, respectively. The ratio of coefficients of thermal conduction of the metals is -

(A) 4 : 7 (B) 7 : 4
(C) 25 : 16 (D) 16 : 25 **[B]**

Sol. $Q_1 = k_1 A \dfrac{\Delta t}{L} . t_1 = K_2 A \dfrac{\Delta t}{L} . t_2$

$\therefore \dfrac{k_1}{k_2} = \dfrac{t_2}{t_1} = \dfrac{7}{4}$

Q.30 Three rods of same material & thickness with $BO = CO = \frac{AO}{2}$. End A & B are maintained at 10°C & 100°C and temperature of C is varied from 65°C to 75°C extremely slowly. Then the correct options is -



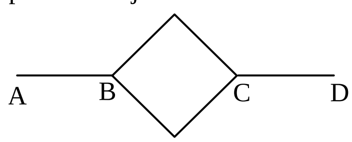
(A) heat always flows from O to C

(B) heat always flows from C to O

(C) heat first flows from O to C & then from C to O (D) heat first flows from C to O & then from O to C **[C]**

Sol. At 70°C there will be equilibrium.

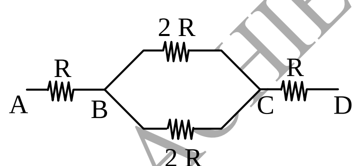
Q.31 Six identical conducting rods are joined as shown in figure. Points A and D are maintained at 200ºC and 20ºC respectively. The temperature of junction B will be –



(A) 120ºC (B) 100ºC

(C) 140ºC (D) 80ºC **[C]**

Sol. Electrical circuit



different between A & D = 200 – 20 = 180ºC.

Resistances for three parts equal $\therefore \frac{180}{3}$ = 60ºC

Temperature B = 200 – 60 = 140ºC

Q.32 The emissive power of a black body at T = 300 K is 100 watt/m^2. Consider a body B of area A = 10 m^2 coefficient of reflectivity r = 0.3 and coefficient of transmission t = 0.5. Its temperature is 300 K. Then which of the following is correct -

(A) The emissive power of B is 20 watt/m^2

(B) The emissive power of B is 200 watt /m^2

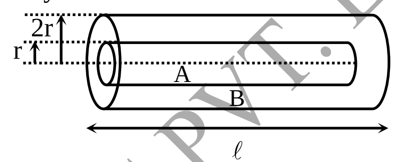
(C) The power emitted by B is 200 watts

(D) The emissivity of B is = 0.2 **[A,C,D]**

Sol. Since, e = a = 0.2 (since a = (1 – r – t) = 0.2 for the body B) E = (100) (0.2) = 20 watt/m^2

Power emitted = e.A = 20 × 10 = 200 watt.

Q.33 A composite cylinder is made of two different materials A and B of thermal conductivities K_A and K_B. The dimensions of the cylinder are as shown in the figure. The thermal resistance of the cylinder between the two end faces is -



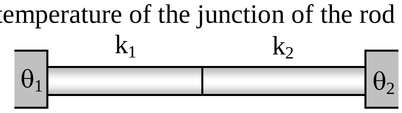
(A) $\frac{\ell}{\pi r^2(K_A + 3K_B)}$ (B) $\frac{\ell}{\pi r^2}\left(\frac{1}{K_A} + \frac{3}{K_B}\right)$

(C) $\frac{\ell}{\pi r^2}\left(\frac{1}{K_A} + \frac{4}{K_B}\right)$ (D) $\frac{\ell}{\pi r^2(K_A + 4K_B)}$

[A]

Sol. $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

Q.34 Two identical rods are made of different materials whose thermal conductivities are k_1 and k_2. They are placed end to end between two heat reservoirs at temperature θ_1 and θ_2. The temperature of the junction of the rod is –



(A) $\frac{\theta_1 + \theta_2}{2}$ (B) $\frac{k_1\theta_1 + k_2\theta_2}{2}$

(C) $\frac{k_1\theta_2 + k_2\theta_1}{k_1 + k_2}$ (D) $\frac{k_1\theta_1 + k_2\theta_2}{k_1 + k_2}$

Sol. **[D]**

In series the rate of heat flow is same

$\frac{\theta_1 - \theta}{\frac{\ell}{k_1 A}} = \frac{\theta - \theta_2}{\frac{\ell}{k_2 A}} \Rightarrow k_1\theta_1 - k_1\theta = k_2\theta - k_2\theta_2$

$\theta = \frac{k_1\theta_1 + k_2\theta_2}{k_1 + k_2}$

Q.35 The ratio of thermal capacities of two spheres A and B, if their diameters are in the ratio 1 : 2, densities in the ratio 2 : 1, and the specific heat in the ratio of 1 : 3, will be -

(A) 1 : 6 (B) 1 : 12

(C) 1 : 3 (D) 1 : 4

Sol. **[B]**

$$\frac{H_1}{H_2} = \frac{m_1s_1}{m_2s_2} = \frac{\rho_1V_1s_1}{\rho_2V_2s_2}$$

Q.36 In a steady state of thermal conduction, temperature of the ends A and B of a 20 cm long rod are 100ºC and 0ºC respectively. What will be the temperature of the rod at a point at a distance of 6 cm from the end A of the rod ?

(A) –30ºC (B) 70ºC

(C) 5ºC (D) None of these

Sol. **[B]**

$$\frac{100-T}{6} = \frac{T-0}{14} \Rightarrow 1400 - 14\,T = 6T$$

$$\Rightarrow 1400 = 20\,T \quad \Rightarrow \quad T = 70\,ºC$$

Q.37 A cylindrical rod with one end in a steam chamber and the other end in ice results in melting of 0.1 g of ice per second. If the rod is replaced by another rod with half the length and double the radius of the first and if the thermal conductivity of material of the second rod is 0.25 times that of first, the rate at which ice melts in g s^{-1} will be -

(A) 0.1 (B) 0.2

(C) 1.6 (D) 3.2

Sol. **[B]**

For first rod $\frac{\Delta Q}{\Delta t} = \frac{\Delta m}{\Delta t}L = \frac{KA(T_1 - T_2)}{L}$

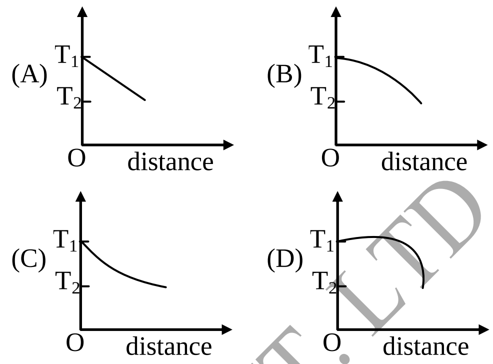
$$0.1 \times L = \frac{KA(T_1 - T_2)}{L} \qquad ....(i)$$

For second rod

$$\frac{\Delta m}{\Delta t}L = \frac{0.25K \times 4A(T_1 - T_2)}{L/2} \quad ....(ii)$$

$$\therefore \frac{\Delta m}{\Delta t} = 0.2 \text{ g/s}$$

Q.38 The ends of a metal bar of constant cross-sectional area are maintained at temperatures T_1 and T_2 which are both higher than the temperature of the surroundings. If the bar is unlagged, which one of the following sketches best represents the variation of temperature with distance along the bar ?



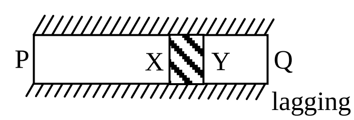
Sol. **[A]**

$$\frac{T_1 - T}{x} = \frac{T - T_2}{L - x}$$

$$\Rightarrow T_1L - T_1x + Tx - TL = Tx - T_2x$$

$$TL = (T_2 - T_1)x + T_1L$$

Q.39 PQ is fully-lagged metal bar, containing a section of XY of a material of lower thermal conductivity. The thermal conductivities of the two materials are independent of temperature. Ends P and Q are maintained at different temperatures.



In the steady state, the temperature difference across XY would be independent of–

(A) the temperature difference between P and Q

(B) the metal of which the bar is made

(C) the thickness of the section XY

(D) the distance of the section XY from the end P

[D]

Q.40 Curved surface of a uniform rod is isolated from surrounding. Ends of the rod are maintained at temperatures T_1 and T_2 ($T_1 > T_2$) for a long time. At an instant, temperature T_1 starts to decrease at a constant and slow rate. If thermal capacity of material of the rod is considered, then which of the following statements is/are correct –

(A) At an instant, rate of heat flow near the hotter end is equal to that near the other end.

(B) Rate of heat flow through the rod starts to decrease near the hotter end and remains constant near the other end.

(C) Rate of heat flow is maximum at mid section of the rod

(D) None of these **[D]**

Q.41 A and B are two points on a uniform metal ring whose centre is C. The angle ACB = θ. A and B are maintained at two different constant temperatures. When θ = 180º, the rate of total heat flow from A to B is 1.2 W. When θ = 90º, this rate will be -

(A) 0.6 W (B) 0.9 W

(C) 1.6 W (D) 1.8 W

Sol. **[C]**

$R_{total} = R$ for $\theta = 180^o$

Two sections of resistance R/2 each are in parallel $R_{eq} = R/4$

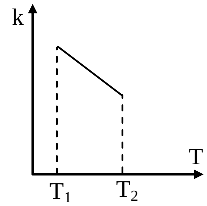
$\therefore$ Rate of heat flow $I_1 = 1.2 = \Delta T/R/4$

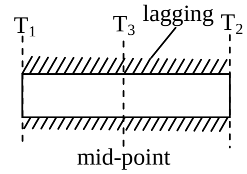
$$\Rightarrow 0.3 = \frac{\Delta T}{R}$$

$\theta = 90^o$ two sections of resistance R/4 & 3R/4 in parallel $R_{eq} = \frac{R/4 . 3R/4}{R/4 + 3R/4} = \frac{3R}{16}$

$\therefore \quad I_2 = \frac{\Delta T}{3R/16} = \frac{16}{3}$ (0.3) = 1.6 watt

Q.42 Over a certain temperature range, the thermal conductivity k of a metal is not constant but varies as indicated in figure. A lagged rod of the metal has its ends maintained at temperatures T_1 and $T_2 (T_2 > T_1)$ as shown in figure–





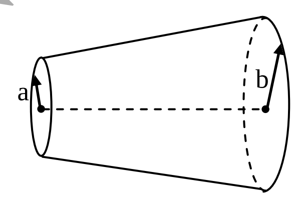
Which one of the following correctly describes how T_3, the temperature at the mid-point of the rod, compares with T_1 and T_2 ?

(A) $T_3 = (T_1 + T_2)/2$ (B) $T_3 = (T_1 - T_2)/2$

(C) $T_3 > (T_1 + T_2)/2$ (D) $T_3 < (T_1 + T_2)/2$

[C]

Q.43 Figure shows a rod of variable cross-section area. Two ends of rod is maintained at different temperature. Let radius of cross-section, where temperature is arithmetic mean of temperature at two ends be r. Then 'r' –



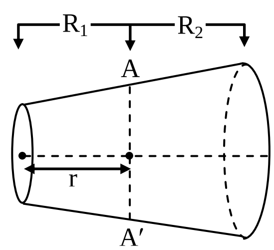
(A) Is equal to arithmetic mean of a and b

(B) Is equal to geometric mean of a and b

(C) Is equal to harmonic mean of a and b

(D) Depends upon length of rod, a and b **[C]**

Sol. Let the cross-section be AA′



$\therefore R_1 = R_2$

[R_1 = Thermal resistance of rod left of cross-section AA′]

$$\Rightarrow \frac{1}{a} - \frac{1}{r} = \frac{1}{r} - \frac{1}{b}$$

$$\Rightarrow r = \frac{2ab}{a+b}$$

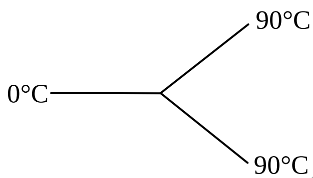
Q.44 Three rods of identical cross-sectional area and made from the same metal form the sides of an isosceles triangle ABC, right-angled at B. The points A and B are maintained at temperature T and $(\sqrt{2})$ T respectively. In the steady state, the temperature of the point C is T_C. Assuming that only heat conduction takes place, Tc/T is-

[IIT – 1995]

(A) $\frac{1}{2(\sqrt{2}-1)}$ (B) $\frac{3}{\sqrt{2}+1}$

(C) $\frac{1}{\sqrt{3}(\sqrt{2}-1)}$ (D) $\frac{1}{\sqrt{2}+1}$ **[B]**

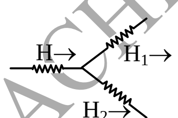
Q.45 Three rods made of the same material and having the same cross section have been joined as shown in figure. The left and right ends are kept at 0°C and 90°C respectively. The temperature of the junction of the three rods will be –



(A) 45°C (B) 60°C
(C) 30°C (D) 20°C

Sol. **[B]**

$H = H_1 + H_2$

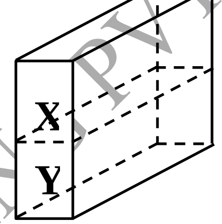


Q.46 An ice cube is floating in water kept in glass. The water in glass will get cooled majorly by –

(A) Convection
(B) Conduction
(C) Radiation
(D) Conduction and convection both **[A]**

Sol. Cooled water being denser will go down displacing hotter water and thereby forming convection cycle.

Q.47 A parallel-sided slab is made of two different materials. The upper half of the slab is made of material X, of thermal conductivity λ ; the lower half is made of material Y, of thermal conductivity 2λ. In the steady state, the left hand face of the composite slab is at a higher, uniform temperature than the right-hand face, and the flow of heat through the slab is parallel to its shortest sides. What fraction of the total heat flow through the slab passes through material X ?



(A) 1/4 (B) 1/3 (C) 1/2 (D) 2/3

Sol. **[B]**

$\frac{\Delta Q}{\Delta t} = -\lambda A \frac{dT}{dx} \Rightarrow \lambda_y = 2\,\lambda_x = 2\lambda$

$\Rightarrow -A\frac{dT}{dx}$ = constant = a, $\frac{\Delta Q_x}{\Delta t} = a\lambda$

$\Rightarrow \frac{\Delta Q_y}{\Delta t} = 2a\lambda \Rightarrow \frac{\Delta Q}{\Delta t} = \frac{\Delta Q_x}{\Delta t} + \frac{\Delta Q_y}{\Delta t}$

$= a\lambda + 2a\lambda = 3a\lambda$

$\therefore$ Fraction $= \frac{\Delta Q_x / \Delta t}{Q/\Delta t} = \frac{1}{3}$

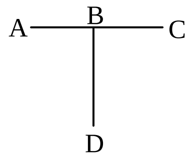
Q.48 The ratio of thermal conductivity of two rods of different materials is 5 : 4. The two rods of same area of cross-section and same thermal resistance will have the lengths in the ratio -

(A) 5 : 4 (B) 1 : 9
(C) 9 : 1 (D) 4 : 5

Sol. **[A]** $\frac{K_1}{K_2} = \frac{5}{4}$ $\because$ R = $\frac{L}{KA}$ $\therefore L \propto K$

$\frac{L_1}{L_2} = \frac{K_1}{K_2} = \frac{5}{4}$

Q.49 Three conducting rods of same material and cross-sectional area are joined as shown. Temperatures at points A, D and C are maintained at 20°C, 90°C and 0°C respectively. The ratio of lengths BD to BC if there is no heat flow in AB is-



(A) $\frac{2}{7}$ (B) $\frac{7}{2}$

(C) $\frac{9}{2}$ (D) $\frac{2}{9}$ **[B]**

Sol. As no heat flows in rod AB

Therefore $T_A = T_B = 20°C$

∴ Rate of heat flow through CB and BD are same.

Hence, $\frac{90-20}{\frac{BC}{KA}} = \frac{20-0}{\frac{BD}{KA}}$

$$\frac{7}{BC} = \frac{2}{BD}$$

$$\frac{BD}{BC} = \frac{7}{2}$$

Q.50 One end of a conducting rod is maintained at temperature 50°C and at the other end ice is maintained at 0°C. The rate of melting of ice is doubled if -

(A) the temperature is made 200°C and the area of cross-section of the rod is doubled

(B) the temperature is made 100°C and length of the rod is made four times

(C) area of cross-section of the rod is halved and length is doubled

(D) the temperature is made 100°C and area of cross-section of rod and length both are doubled **[D]**

Sol. Rate of melting of ice ∝ rate of heat transfer $\left(\frac{dQ}{dt}\right)$

further, $\frac{dQ}{dt} = \frac{\text{temperature difference}}{(\ell/KA)}$

or $\frac{dQ}{dt} \propto \frac{\text{temp diff}}{\ell} \times A$

If temp. diff, A and ℓ are all doubled, then dQ and hence rate of melting of ice are doubled