Exercise set 8.1 (Solution)

1. Find the first five terms of the sequence in which the n^{th} term is given by:

(i) $t_n = 3n - 4$, (ii) $t_n = 2 - 5n$, (iii) $t_n = n^2 - 2n + 3$ for $n \geq 1$.

Sol:-

To find the first five terms, we substitute $n = 1,2,3,4,5$ in each given formula.

(i) $t_n = 3n - 4$

$$t_1 = 3(1) - 4 = -1$$
$$t_2 = 3(2) - 4 = 2$$
$$t_3 = 3(3) - 4 = 5$$
$$t_4 = 3(4) - 4 = 8$$
$$t_5 = 3(5) - 4 = 11$$

First five terms:

$$-1,\ 2,\ 5,\ 8,\ 11$$

(ii) $t_n = 2 - 5n$

$$t_1 = 2 - 5(1) = -3$$
$$t_2 = 2 - 5(2) = -8$$
$$t_3 = 2 - 5(3) = -13$$
$$t_4 = 2 - 5(4) = -18$$
$$t_5 = 2 - 5(5) = -23$$

First five terms:

$$-3,\ -8,\ -13,\ -18,\ -23$$

(iii) $t_n = n^2 - 2n + 3$

$$t_1 = (1)^2 - 2(1) + 3 = 2$$
$$t_2 = (2)^2 - 2(2) + 3 = 3$$
$$t_3 = (3)^2 - 2(3) + 3 = 6$$
$$t_4 = (4)^2 - 2(4) + 3 = 11$$
$$t_5 = (5)^2 - 2(5) + 3 = 18$$

First five terms:

$$2, 3, 6, 11, 18$$

2. Find the 10th and 15th terms of the sequence $t_n = 5n - 3$for $n \geq 1$.

Sol.- Given, $t_n = 5n - 3$

Substitute $n = 10$:

$$t_{10} = 5(10) - 3 = 50 - 3 = 47$$

Substitute $n = 15$:

$$t_{15} = 5(15) - 3 = 75 - 3 = 72$$

3. Determine whether 97 and 172 are terms of the sequence $t_n = 5n - 3$for $n \geq 1$.

Sol:- Given, $t_n = 5n - 3$

For 97:

$$5n - 3 = 97$$
$$5n = 100$$
$$n = 20$$

Since $n$is a natural number, 97 is a term of the sequence.

For 172:

$$5n - 3 = 172$$
$$5n = 175$$
$$n = 35$$

Since $n$is a natural number, 172 is also a term of the sequence.

4. Which term of the sequence $t_n = 5n - 3$ for $n \geq 1$ is 607?

Sol:- Given,

$$t_n = 5n - 3$$

To find the term number, put $t_n = 607$:

$$5n - 3 = 607$$
$$5n = 610$$
$$n = \frac{610}{5} = 122$$

5. A sequence is given by the recursive rule $t_1 = -5,\ t_{n+1} = t_n + 3$ for $n \geq 1$.
Find the first five terms of the sequence. Is 52 a term of this sequence? If so, which term is it?

Sol:- Given, $t_1 = -5$ and each next term is obtained by adding 3.

$$t_1 = -5$$
$$t_2 = t_1 + 3 = -5 + 3 = -2$$
$$t_3 = t_2 + 3 = -2 + 3 = 1$$
$$t_4 = t_3 + 3 = 1 + 3 = 4$$
$$t_5 = t_4 + 3 = 4 + 3 = 7$$

First five terms:

$$-5,\ -2,\ 1,\ 4,\ 7$$

This is an arithmetic sequence with first term -5 and common difference 3.
So,

$$t_n = -5 + (n-1) \times 3 = 3n - 8$$

Now check 52:

$$3n - 8 = 52$$
$$3n = 60$$

$$n = 20$$

First five terms: $-5, \ -2, \ 1, \ 4, \ 7$
Yes, **52 is a term** and it is the **20th term**.

6. Let $T_1 = 1$, $T_2 = 2$, $T_3 = 4$and $T_n = T_{n-1} + T_{n-2} + T_{n-3}$for $n \geq 4$.
Find T_4, T_5, T_6, T_7, T_8.

Sol.-
Given:

$$T_1 = 1, T_2 = 2, T_3 = 4$$
$$T_4 = T_3 + T_2 + T_1 = 4 + 2 + 1 = 7$$
$$T_5 = T_4 + T_3 + T_2 = 7 + 4 + 2 = 13$$
$$T_6 = T_5 + T_4 + T_3 = 13 + 7 + 4 = 24$$
$$T_7 = T_6 + T_5 + T_4 = 24 + 13 + 7 = 44$$
$$T_8 = T_7 + T_6 + T_5 = 44 + 24 + 13 = 81$$

$$T_4 = 7, T_5 = 13, T_6 = 24, T_7 = 44, T_8 = 81$$

Exercise set 8.2 (Solution)

1. Find the 10th and 26th terms of the AP: 3,8,13,18, …

   Sol:-

   Given AP: 3,8,13,18, …
   First term, $a = 3$
   Common difference, $d = 8 - 3 = 5$
   Formula for n^{th}term:

$$a_n = a + (n-1)d$$
$$a_{10} = 3 + (10-1) \times 5 = 3 + 45 = 48$$
$$a_{26} = 3 + (26-1) \times 5 = 3 + 125 = 128$$

2. Which term of the AP : 21, 18, 15, … is – 81? Also, is 0 a term of this AP? Give reasons for your answer.

   Sol.-
   Given AP: 21,18,15, …
   First term, $a = 21$
   Common difference, $d = 18 - 21 = -3$
   Formula for n^{th}*term:*

$$a_n = a + (n-1)d$$

   (i) Find the term which is -81*:*

$$21 + (n-1)(-3) = -81$$
$$21 - 3(n-1) = -81$$
$$21 - 3n + 3 = -81$$
$$24 - 3n = -81$$
$$-3n = -105$$
$$n = 35$$

   So, -81*is the 35th term.*

   (ii) Check whether 0 is a term:

$$21 + (n-1)(-3) = 0$$
$$24 - 3n = 0$$
$$3n = 24$$
$$n = 8$$

   Since n*is a natural number, 0 is a term of the AP.*

3. Find the nth term of the AP: 11, 8, 5, 2 ... Write the recursive rule for this AP.

Sol.-

Given AP: 11, 8, 5, 2, ...

First term, $a = 11$

Common difference, $d = 8 - 11 = -3$

nth term formula:

Using $a_n = a + (n-1)d$

$$a_n = 11 + (n-1)(-3)$$

Simplifying:

$$a_n = 11 - 3(n-1) = 14 - 3n$$

Recursive rule:

$$a_1 = 11$$
$$a_n = a_{n-1} - 3$$

4. An AP consists of 50 terms in which the 3rd term is 12 and the last term is 106. Find the 29th term. (Hint: If 'a' is the first term and 'd' the common difference, then we arrive at the equations a + 2d = 12 and a + 49d = 106. Solve this pair of linear equations for 'a' and 'd'.)

Sol.-

Given that the 3rd term of an AP is 12 and the 50th term is 106.

Let the first term be $a$and common difference be d.

From the definition of an AP:

3rd term $= a + 2d = 12$

50th term $= a + 49d = 106$

These form a pair of linear equations. Subtracting the first from the second eliminates a, giving the value of d. Once $d$is found, substitute it back into one equation to get a.

After finding $a$and d, the required 29th term is obtained using the general formula of an AP:

$$a_n = a + (n-1)d$$

Substitute $n = 29$along with the values of $a$and $d$to get the required term.

Thus, the 29th term of the AP is 64.

5. How many 2-digit numbers are divisible by 3? What is the sum of all these 2-digit numbers?

Sol.-

All 2-digit numbers lie between 10 and 99.

A number is divisible by 3 if it is a multiple of 3.

The smallest 2-digit number divisible by 3 is 12 and the greatest is 99.

Thus, the required numbers form an AP: 12, 15, 18, ..., 99 with first term $a = 12$and common difference $d = 3$.

To find how many such numbers are there, we use the nth term formula of an AP:

$$a_n = a + (n-1)d$$

Putting $a_n = 99$, we get the total number of terms as 30.

To find their sum, we use the sum formula of an AP:

$$S_n = \frac{n}{2}(a + l)$$

Substituting the values, we get the sum = 1665.

Hence, there are 30 two-digit numbers divisible by 3 and their sum is 1665.

6. Harish started work at an annual salary of ` 5,00,000 and received an increment of ` 20,000 each year. After how many years did his income reach ` 7,00,000?

Sol.-

Harish's salaries form an Arithmetic Progression (AP):

First salary $a = 5{,}00{,}000$

Annual increment $d = 20{,}000$

Let his salary become 7,00,000in the nth year.

Using the nth term formula of an AP:

$$a_n = a + (n-1)d$$

Substitute the values:

$$7{,}00{,}000 = 5{,}00{,}000 + (n-1) \cdot 20{,}000$$
$$2{,}00{,}000 = (n-1) \cdot 20{,}000$$
$$n - 1 = 10 \Rightarrow n = 11$$

Thus, his income reaches ₹7,00,000 in the 11th year.

So, the number of years after starting = $11 - 1 = 10$.

7. A child arranges marbles in rows so that the first row has 1 marble, the second has 2 marbles, the third has 3, and so on up to 25 rows. How many marbles does the child use in all?

Sol.-

The marbles form the sequence: 1, 2, 3, ..., 25

This is an AP with first term $a = 1$, common difference $d = 1$, and number of terms $n = 25$.

To find the total number of marbles, we use the sum formula of an AP:

$$S_n = \frac{n}{2}(a + l)$$

Here, last term $l = 25$

$$S_{25} = \frac{25}{2}(1 + 25) = \frac{25}{2} \times 26 = 25 \times 13 = 325$$

Exercise set 8.3 (Solution)

1. Find the 12th term of a GP with common ratio 2, whose 8th term is 192.

Sol.-

In a GP, the nth term is given by:

$$a_n = ar^{n-1}$$

Given:

Common ratio $r = 2$, and 8th term $a_8 = 192$

$$a_8 = a \cdot 2^7 = 192 \Rightarrow a \cdot 128 = 192 \Rightarrow a = \frac{192}{128} = \frac{3}{2}$$

Now find the 12th term:

$$a_{12} = a \cdot 2^{11} = \frac{3}{2} \cdot 2048 = 3 \cdot 1024 = 3072$$

2. Find the 10th and nth terms of the GP: 5, 25, 125, ...

Sol.-

Given GP: 5, 25, 125, ...

First term a $= 5$

Common ratio r $= \frac{25}{5} = 5$

nth term formula of GP:

$$\mathrm{a_n = ar^{n-1}}$$

So,

$$\mathrm{a_n = 5 \cdot 5^{n-1} = 5^n}$$

10th term:

$$\mathrm{a_{10} = 5^{10} = 9765625}$$

3. A sequence is given by the recursive rule $t_1 = 2,\ t_{n+1} = 3t_n - 2$for $n \geq 1$. Which term of the sequence is 730?

Sol:-

Given: $t_1 = 2, t_{n+1} = 3t_n - 2$

Find terms one by one:

$$t_1 = 2$$
$$t_2 = 3(2) - 2 = 4$$
$$t_3 = 3(4) - 2 = 10$$
$$t_4 = 3(10) - 2 = 28$$
$$t_5 = 3(28) - 2 = 82$$
$$t_6 = 3(82) - 2 = 244$$
$$t_7 = 3(244) - 2 = 730$$

Therefore, 730 is obtained at the 7th term.

4. Which term of the GP: 2, 6, 18, … is 4374? Write the explicit formula as well as the recursive formula for the nth term.
Sol.-
Given GP: 2,6,18, ...
First term $a = 2$, common ratio $r = 3$
Explicit formula:
$$a_n = 2 \cdot 3^{n-1}$$
To find which term is 4374:
$$2 \cdot 3^{n-1} = 4374 \Rightarrow 3^{n-1} = 2187 = 3^7 \Rightarrow n - 1 = 7 \Rightarrow n = 8$$

Recursive formula:
$$a_1 = 2$$
$$a_n = 3a_{n-1}$$

5. A ball is dropped from a height of 80 metres. After hitting the ground, it bounces back to 60% of the height from which it fell. It continues bouncing in this way — each time rising to 60% of the previous height.
(i) What height does the ball reach after the 5th bounce?
(ii) What is the total vertical distance the ball has travelled by the time it hits the ground for the 6th time?

Sol:-
Given: Initial height = 80 m, rebound ratio = 60% = 0.6
The heights after each bounce form a GP:
$$80 \times 0.6,\ 80 \times 0.6^2,\ 80 \times 0.6^3, \ldots$$
(i) Height after 5th bounce
$$h_n = 80 \cdot (0.6)^n$$
$$h_5 = 80 \times (0.6)^5 = 80 \times 0.07776 = 6.2208$$

Height ≈ **6.22 m**

(ii) Total distance up to 6th hit on ground

The ball travels:
First fall: 80 m
Then for each bounce: goes up and comes down
So total distance:

$$\text{Distance} = 80 + 2(48 + 28.8 + 17.28 + 10.368 + 6.2208)$$

This is a GP with first term 48, ratio 0.6, 5 terms.
Sum:

$$S = \frac{48(1 - (0.6)^5)}{1 - 0.6} = \frac{48(1 - 0.07776)}{0.4} = 110.6688$$

Total distance:

$$80 + 2 \times 110.6688 = 301.3376$$

6. Which term of the sequence , 22 24 , , … is 128 **?**

Sol.-

This is a GP with first term $a = 2$and common ratio $r = 2$.
nth term formula:

$$a_n = 2 \cdot 2^{n-1}$$
$$a_n = 2^n$$

Now,

$$2^n = 128 = 2^7 \Rightarrow n = 7$$

7. Fig. 8.12 shows Stages 0 to 3 of the Sierpiński square carpet. Stage 0 of this fractal is a square sheet of paper. To construct Stage 1, each side of the square is trisected and the points of trisection of opposite sides are joined to obtain nine smaller squares. The centre square is then removed and the 8 smaller squares are retained, leaving a square hole in the centre. The same process is repeated on the eight smaller shaded squares to obtain Stage 2 and so on.

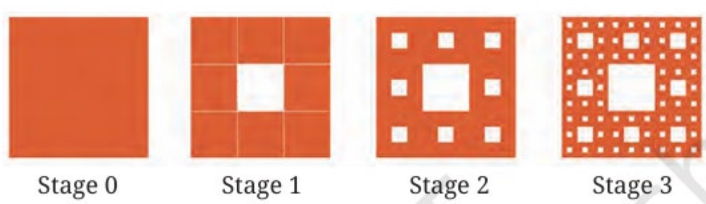

Fig. 8.12: Stages 0, 1, 2 and 3 of the Sierpiński square carpet

Look at Fig. 8.12 and try to answer the following questions.

(i) How many red squares are there in Stages 0 to 3?

(ii) Can you predict the number of red squares in Stages 4 and 5?

(iii) Can you find a rule for the number of red squares at the nth stage? Write the explicit formula as well as the recursive formula for the number of red squares at any stage.

(iv) Suppose the area of the square in Stage 0 is 1 square unit. What is the area of the red region in Stages 1, 2 and 3? What will be the area of the red region in Stages 4 and 5? Find the explicit as well as the recursive formula for the area of the red region at the nth stage. What happens to this area as n, the number of stages, goes on increasing?

Sol.-

(i) In Stage 0, there is 1 red square.

In Stage 1, the square is divided into 9 equal parts and the centre is removed, so 8 squares remain.

In Stage 2, each of these 8 squares again gives 8 smaller squares, so total $8 \times 8 = 64$.

In Stage 3, the number becomes $8 \times 64 = 512$.

Thus, the number of red squares in Stages 0 to 3 are:

$$1,8,64,512$$

(ii) The pattern shows that the number of squares is multiplied by 8 at each stage. So,

Stage 4: $512 \times 8 = 4096$

Stage 5: $4096 \times 8 = 32768$

(iii) Let N_n be the number of red squares at the n^{th} stage.

The sequence is a geometric progression with first term 1 and common ratio 8.

Explicit formula:

$$N_n = 8^n$$

Recursive formula:

$N_0 = 1$,

$$N_n = 8N_{n-1}$$

(iv) Let the area of the square in Stage 0 be 1 square unit.

At each stage, one out of 9 equal parts is removed, so $\frac{8}{9}$ of the area remains.

Thus, the areas form a geometric progression with common ratio $\frac{8}{9}$.

Area in Stage 1 = $\frac{8}{9}$

Area in Stage 2 = $\left(\frac{8}{9}\right)^2 = \frac{64}{81}$

Area in Stage 3 = $\left(\frac{8}{9}\right)^3 = \frac{512}{729}$

Area in Stage 4 = $\left(\frac{8}{9}\right)^4$

Area in Stage 5 = $\left(\frac{8}{9}\right)^5$

Explicit formula:

$$A_n = \left(\frac{8}{9}\right)^n$$

Recursive formula:

$A_0 = 1$,

$$A_n = \frac{8}{9}A_{n-1}$$

As $n$increases, the value of $\left(\frac{8}{9}\right)^n$keeps decreasing and approaches 0.

Hence, the area of the red region tends to 0 as the number of stages increases.

END OF CHAPTER EXERCISES

1. Find the 31st term of an AP whose 11th term is 38 and 16th term is 73.
Sol.-
Let the first term be $a$and common difference be d.
Given:
11th term → $a + 10d = 38$
16th term → $a + 15d = 73$
Subtract the first equation from the second:
$$(a + 15d) - (a + 10d) = 73 - 38$$
$$5d = 35 \Rightarrow d = 7$$
Substitute $d = 7$in $a + 10d = 38$:
$$a + 70 = 38 \Rightarrow a = -32$$
Now find the 31st term using:
$$a_n = a + (n-1)d$$
$$a_{31} = -32 + 30 \times 7 = -32 + 210 = 178$$

2. Determine the AP whose third term is 16 and whose 7th term exceeds the 5th term by 12.
Sol.-
Let the first term be $a$and common difference be d.
Given:
Third term → $a + 2d = 16 \ldots(1)$
Also, 7th term exceeds 5th term by 12:
$$(a + 6d) - (a + 4d) = 12$$
$$2d = 12 \Rightarrow d = 6$$
Substitute $d = 6$in (1):
$$a + 12 = 16 \Rightarrow a = 4$$
Thus, the AP is:
$$4, 10, 16, 22, 28, \ldots$$

3. How many three-digit numbers are divisible by 7?
Sol:-
The smallest three-digit number divisible by 7 is 105 and the largest is 994.
These form an AP: 105, 112, 119, …, 994 with $a = 105$, $d = 7$, $l = 994$.
Using:
$$a_n = a + (n-1)d$$
$$994 = 105 + (n-1) \cdot 7 \Rightarrow 889 = (n-1) \cdot 7 \Rightarrow n - 1 = 127 \Rightarrow n = 128$$

4. How many multiples of 4 lie between 10 and 250?
Sol:-
The smallest multiple of 4 greater than 10 is 12 and the largest less than 250 is 248.
AP: 12, 16, 20, …, 248 with $a = 12, d = 4, l = 248$
$$248 = 12 + (n-1)\cdot 4 \Rightarrow 236 = (n-1)\cdot 4 \Rightarrow n - 1 = 59 \Rightarrow n = 60$$

5. Find a GP for which the sum of the first two terms is –4 and the fifth term is 4 times the third term.
Sol:-
Let first term a, common ratio r
Sum of first two terms:
$$a + ar = -4 \Rightarrow a(1+r) = -4$$

Given:
$$a_5 = ar^4, a_3 = ar^2$$
$$ar^4 = 4(ar^2) \Rightarrow r^2 = 4 \Rightarrow r = 2 \text{ or } -2$$

If $r = 2$:
$$a(1+2) = -4 \Rightarrow a = -\frac{4}{3}$$

If $r = -2$:
$$a(1-2) = -4 \Rightarrow a = 4$$

GPs are:
$$-\frac{4}{3}, -\frac{8}{3}, -\frac{16}{3}, \ldots \text{or } 4, -8, 16, -32, \ldots$$

6. Find all possible ways of expressing 100 as the sum of consecutive natural numbers.
Sol:-
Let 100 be expressed as:
$$n + (n+1) + (n+2) + \cdots + (n+k-1)$$

Sum:
$$100 = \frac{k}{2}[2n + (k-1)]$$
$$200 = k[2n + (k-1)]$$

Trying possible values of k:
For $k = 5$:
$$200 = 5(2n+4) \Rightarrow 40 = 2n + 4 \Rightarrow n = 18$$

Sequence: 18 + 19 + 20 + 21 + 22 = 100

For $k = 8$:

$$200 = 8(2n + 7) \Rightarrow 25 = 2n + 7 \Rightarrow n = 9$$

Sequence: 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 = 100

7. The number of bacteria doubles every hour. Initially 30 bacteria are present. Find the number after 2nd hour, 4th hour and n^{th}hour.

Sol:-

This is a GP with $a = 30, r = 2$

$$a_n = 30 \cdot 2^n$$

After 2 hours: $30 \cdot 2^2 = 120$
After 4 hours: $30 \cdot 2^4 = 480$
After nhours: $30 \cdot 2^n$

8. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

Sol:

$$a + 3d + a + 7d = 24 \Rightarrow 2a + 10d = 24$$
$$a + 5d + a + 9d = 44 \Rightarrow 2a + 14d = 44$$

Subtract:

$$4d = 20 \Rightarrow d = 5$$

Substitute:

$$2a + 50 = 24 \Rightarrow a = -13$$

First three terms: $-13, -8, -3$

9. Find smallest n such that sum of first n natural numbers exceeds 1000.

Sol:-

$$S_n = \frac{n(n+1)}{2}$$

$$\frac{n(n+1)}{2} > 1000 \Rightarrow n(n+1) > 2000$$

$$44 \times 45 = 1980 < 2000$$
$$45 \times 46 = 2070 > 2000$$

So, $n = 45$

10. Which term of GP: 2, 8, 32, … is 131072?

Sol.-

$$\mathrm{a} = 2,\ \mathrm{r} = 4$$

$$2 \cdot 4^{n-1} = 131072 = 2^{17}$$
$$2 \cdot (2^2)^{n-1} = 2^{2n-1} \Rightarrow 2^{2n-1} = 2^{17} \Rightarrow 2n - 1 = 17 \Rightarrow n = 9$$

Explicit: $a_n = 2 \cdot 4^{n-1}$
Recursive: $a_1 = 2,\ a_n = 4a_{n-1}$

11. The sum of the first three terms of a GP is 13 12 and their product is –1. Find the common ratio and the terms.

Sol.-
Let the three consecutive terms of the GP be

$$\frac{a}{r},\ a,\ ar$$

Product of the terms:

$$\frac{a}{r} \cdot a \cdot ar = a^3 = -1 \Rightarrow a = -1$$

Sum of the terms:

$$\frac{a}{r} + a + ar = \frac{13}{12}$$

Substitute $a = -1$:

$$-\frac{1}{r} - 1 - r = \frac{13}{12}$$

Multiply both sides by $12r$:

$$-12 - 12r - 12r^2 = 13r$$

Rearrange:

$$12r^2 + 25r + 12 = 0$$

Factorize:

$$(3r+4)(4r+3)=0$$
$$r=-\frac{4}{3}\text{or}r=-\frac{3}{4}$$

Now find the terms:

If $r=-\frac{4}{3}$:

$$\frac{a}{r}=\frac{-1}{-4/3}=\frac{3}{4}$$

Terms:

$$\frac{3}{4},\ -1,\ \frac{4}{3}$$

If $r=-\frac{3}{4}$:

$$\frac{a}{r}=\frac{-1}{-3/4}=\frac{4}{3}$$

Terms:

$$\frac{4}{3},\ -1,\ \frac{3}{4}$$

Final Answer:

Common ratio:

$$r=-\frac{4}{3}\text{or}r=-\frac{3}{4}$$

Terms:

$$\left(\frac{3}{4},-1,\frac{4}{3}\right)\text{ or }\left(\frac{4}{3},-1,\frac{3}{4}\right)$$

12. If the 4th, 10th and 16th terms of a GP are x, y and z respectively, prove that x, y, z are in GP.

Sol.- Let the GP have first term $a$and common ratio r.

Then,

$$T_n=ar^{n-1}$$

So,

$$x=T_4=ar^3, y=T_{10}=ar^9, z=T_{16}=ar^{15}$$

Now check the condition for three numbers to be in GP:

$$y^2 = xz$$

Compute y^2:

$$y^2 = (ar^9)^2 = a^2r^{18}$$

Compute xz:

$$xz = (ar^3)(ar^{15}) = a^2r^{18}$$

Thus,

$$y^2 = xz$$

Hence, $x, y, z$are in GP.

13. The sum of the first three terms of a geometric progression is 26, and the sum of their squares is 364. Find the terms of the GP.

Sol.- Let the three terms of the GP be

$$\frac{a}{r},\ a,\ ar$$

Sum:

$$\frac{a}{r} + a + ar = 26 \ldots (1)$$

Sum of squares:

$$\frac{a^2}{r^2} + a^2 + a^2r^2 = 364 \ldots (2)$$

Divide (2) by (1):

$$\frac{\frac{a^2}{r^2} + a^2 + a^2r^2}{\frac{a}{r} + a + ar} = \frac{364}{26}$$

$$\Rightarrow \frac{a\left(\frac{1}{r^2}+1+r^2\right)}{\left(\frac{1}{r}+1+r\right)} = 14$$

$$\Rightarrow a \cdot \frac{\frac{1}{r^2} + 1 + r^2}{\frac{1}{r} + 1 + r} = 14$$

Now use identity:

$$\frac{1}{r^2} + 1 + r^2 = \left(\frac{1}{r}+1+r\right)^2 - 2\left(\frac{1}{r}+r\right)$$

After simplification, we get:

$$\frac{1}{r} + 1 + r = \frac{10}{3}$$

Now from (1):

$$a \cdot \frac{10}{3} = 26 \Rightarrow a = \frac{39}{5}$$

Now,

$$\frac{1}{r} + r = \frac{10}{3} - 1 = \frac{7}{3}$$

Multiply by r:

$$r^2 + 1 = \frac{7}{3}r$$
$$3r^2 - 7r + 3 = 0$$
$$r = \frac{7 \pm \sqrt{13}}{6}$$

The three terms are:

$$\frac{a}{r},\ a,\ ar$$
$$\boxed{\left(\frac{39}{5r}, \frac{39}{5}, \frac{39r}{5}\right)}$$

where

$$r = \frac{7+\sqrt{13}}{6} \text{ or } \frac{7-\sqrt{13}}{6}$$

14. Suppose P1 = 1, P2 = 2 and for n > 2, Pn = P1 + P2 + … + Pn–1 + 1. Find the values of P1 , P2 , …, P8 . Can you find a simpler recursive formula for Pn ? Can you give an explicit formula?

Sol.-Given:

$$P_1 = 1,\ P_2 = 2, \text{and for } n > 2,\ P_n = P_1 + P_2 + \cdots + P_{n-1} + 1$$

$$P_3 = P_1 + P_2 + 1 = 1 + 2 + 1 = 4$$
$$P_4 = 1 + 2 + 4 + 1 = 8$$
$$P_5 = 1 + 2 + 4 + 8 + 1 = 16$$
$$P_6 = 1 + 2 + 4 + 8 + 16 + 1 = 32$$
$$P_7 = 1 + 2 + 4 + 8 + 16 + 32 + 1 = 64$$
$$P_8 = 1 + 2 + 4 + 8 + 16 + 32 + 64 + 1 = 128$$

So,

$$\boxed{P_1 = 1,\ P_2 = 2,\ P_3 = 4,\ P_4 = 8,\ P_5 = 16,\ P_6 = 32,\ P_7 = 64,\ P_8 = 128}$$

Observe:

$$P_3 = 2 \times P_2, P_4 = 2 \times P_3, \ldots$$

So,

$$\boxed{P_n = 2P_{n-1} \text{for } n \geq 2}$$

This is a geometric pattern:

$$1,2,4,8,16, \ldots$$

So,

$$\boxed{P_n = 2^{n-1}}$$

$$P_n = 2^{n-1}, \text{and recursive form } P_n = 2P_{n-1}$$

15. Suppose $W_1 = 1,\ W_2 = 2$and for $n > 2,\ W_n = W_1 + W_2 + \cdots + W_{n-2} + 2$.
Find the values of $W_1, W_2, \ldots, W_8$.Do you recognise this sequence?

Sol:-

$W_1 = 1,\ W_2 = 2$
$W_3 = W_1 + 2 = 1 + 2 = 3$
$W_4 = W_1 + W_2 + 2 = 1 + 2 + 2 = 5$
$W_5 = W_1 + W_2 + W_3 + 2 = 1 + 2 + 3 + 2 = 8$
$W_6 = W_1 + W_2 + W_3 + W_4 + 2 = 1 + 2 + 3 + 5 + 2 = 13$
$W_7 = W_1 + W_2 + W_3 + W_4 + W_5 + 2 = 1 + 2 + 3 + 5 + 8 + 2 = 21$
$W_8 = W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + 2 = 1 + 2 + 3 + 5 + 8 + 13 + 2 = 34$
$W_1 = 1,\ W_2 = 2,\ W_3 = 3,\ W_4 = 5,\ W_5 = 8,\ W_6 = 13,\ W_7 = 21,\ W_8 = 34$

Now observe:

$$W_3 = W_1 + W_2, W_4 = W_2 + W_3, W_5 = W_3 + W_4, \text{ and so on}$$

Thus,

$$W_n = W_{n-1} + W_{n-2}$$

Hence, the sequence is the **Fibonacci sequence**.