Exercise set 4.1 (Solution)

1. Using the identity $(a+b)^2 = a^2 + 2ab + b^2$ expand the following:

(i) $(7x+4y)^2$

(ii) $\left(\frac{7}{5}x+\frac{3}{2}y\right)^2$

(iii) $(2.5p+1.5q)^2$

(iv) Expand $\left(\frac{3}{4}s+8t\right)^2$

(v) Expand $\left(x+\frac{1}{2y}\right)^2$

(vi) Expand $\left(\frac{1}{x}+\frac{1}{y}\right)^2$

Sol.- (i) $(7x+4y)^2$

Here,

$a = 7x$ and $b = 4y$

Using the identity:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(7x+4y)^2 = (7x)^2 + 2(7x)(4y) + (4y)^2$$
$$= 49x^2 + 56xy + 16y^2$$

Therefore,

$$(7x+4y)^2 = 49x^2 + 56xy + 16y^2$$

(ii) Expand $\left(\frac{7}{5}x+\frac{3}{2}y\right)^2$

Here,

$$a = \frac{7}{5}x, b = \frac{3}{2}y$$

Using the identity:

$$\left(\frac{7}{5}x+\frac{3}{2}y\right)^2 = \left(\frac{7}{5}x\right)^2 + 2\left(\frac{7}{5}x\right)\left(\frac{3}{2}y\right) + \left(\frac{3}{2}y\right)^2$$
$$= \frac{49}{25}x^2 + \frac{21}{5}xy + \frac{9}{4}y^2$$

Therefore,

$$\left(\frac{7}{5}x+\frac{3}{2}y\right)^2 = \frac{49}{25}x^2 + \frac{21}{5}xy + \frac{9}{4}y^2$$

(iii) Expand $(2.5p+1.5q)^2$

Here,

$$a = 2.5p, b = 1.5q$$

Using the identity:

$$(2.5p + 1.5q)^2 = (2.5p)^2 + 2(2.5p)(1.5q) + (1.5q)^2$$
$$= 6.25p^2 + 7.5pq + 2.25q^2$$

Therefore,

$$(2.5p + 1.5q)^2 = 6.25p^2 + 7.5pq + 2.25q^2$$

(iv) Expand $\left(\frac{3}{4}s+8t\right)^2$

Here,

$$a = \frac{3}{4}s, b = 8t$$

Using the identity

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$\left(\frac{3}{4}s+8t\right)^2 = \left(\frac{3}{4}s\right)^2 + 2\left(\frac{3}{4}s\right)(8t) + (8t)^2$$
$$= \frac{9}{16}s^2 + 12st + 64t^2$$

Therefore,

$$\left(\frac{3}{4}s+8t\right)^2 = \frac{9}{16}s^2 + 12st + 64t^2$$

(v) Expand $\left(x+\frac{1}{2y}\right)^2$

Here,

$$a = x, b = \frac{1}{2y}$$

Using the identity:

$$\left(x+\frac{1}{2y}\right)^2 = x^2 + 2(x)\left(\frac{1}{2y}\right) + \left(\frac{1}{2y}\right)^2$$
$$= x^2 + \frac{x}{y} + \frac{1}{4y^2}$$

Therefore,

$$\left(x+\frac{1}{2y}\right)^2 = x^2 + \frac{x}{y} + \frac{1}{4y^2}$$

(vi) Expand $\left(\frac{1}{x}+\frac{1}{y}\right)^2$

Here,

$$a=\frac{1}{x}, b=\frac{1}{y}$$

Using the identity:

$$\left(\frac{1}{x}+\frac{1}{y}\right)^2=\left(\frac{1}{x}\right)^2+2\left(\frac{1}{x}\right)\left(\frac{1}{y}\right)+\left(\frac{1}{y}\right)^2$$
$$=\frac{1}{x^2}+\frac{2}{xy}+\frac{1}{y^2}$$

Therefore,

$$\left(\frac{1}{x}+\frac{1}{y}\right)^2=\frac{1}{x^2}+\frac{2}{xy}+\frac{1}{y^2}$$

2. Using the same identity, find the values of the following:
(i) $(64)^2$
(ii) $(105)^2$
(iii) $(205)^2$

Sol.-

(i) Find $(64)^2$

$$64=60+4$$

Using the identity:

$$(64)^2=(60+4)^2$$
$$=(60)^2+2(60)(4)+(4)^2$$
$$=3600+480+16$$
$$=4096$$

Therefore,

$$(64)^2=4096$$

(ii) Find $(105)^2$

$$105=100+5$$

Using the identity:

$$(105)^2=(100+5)^2$$

$$= (100)^2 + 2(100)(5) + (5)^2$$
$$= 10000 + 1000 + 25$$
$$= 11025$$

Therefore,

$$(105)^2 = 11025$$

(iii) Find $(205)^2$

$$205 = 200 + 5$$

Using the identity:

$$(205)^2 = (200 + 5)^2$$
$$= (200)^2 + 2(200)(5) + (5)^2$$
$$= 40000 + 2000 + 25$$
$$= 42025$$

Therefore,

$$(205)^2 = 42025$$

Exercise set 4.2 (Solution)

1. Factor completely:

   (i) $9x^2 + 24xy + 16y^2$

   (ii) $4s^2 + 20st + 25t^2$

   (iii) $49x^2 + 28xy + 4y^2$

   (iv) $64p^2 + \frac{32}{3}pq + \frac{4}{9}q^2$

   (v) $3a^2 + 4ab + \frac{4}{3}b^2$

   (vi) $\frac{9}{5}s^2 + 6sv + 5v^2$

Sol.-

(i) Factor $9x^2 + 24xy + 16y^2$

We observe that

$$9x^2 = (3x)^2$$
$$16y^2 = (4y)^2$$

and

$$24xy = 2(3x)(4y)$$

So, the expression is of the form

$$a^2 + 2ab + b^2 = (a+b)^2$$

Therefore,

$$9x^2 + 24xy + 16y^2 = (3x + 4y)^2$$

(ii) Factor $4s^2 + 20st + 25t^2$

We observe that

$$4s^2 = (2s)^2$$
$$25t^2 = (5t)^2$$

and

$$20st = 2(2s)(5t)$$

So, the expression is of the form

$$a^2 + 2ab + b^2 = (a+b)^2$$

Therefore,

$$4s^2 + 20st + 25t^2 = (2s + 5t)^2$$

(iii) Factor $49x^2 + 28xy + 4y^2$

We observe that

$$49x^2 = (7x)^2$$
$$4y^2 = (2y)^2$$

and

$$28xy = 2(7x)(2y)$$

So, the expression is of the form

$$a^2 + 2ab + b^2 = (a+b)^2$$

Therefore,

$$49x^2 + 28xy + 4y^2 = (7x + 2y)^2$$

(iv) Factor $64p^2 + \frac{32}{3}pq + \frac{4}{9}q^2$

We observe that

$$64p^2 = (8p)^2$$
$$\frac{4}{9}q^2 = \left(\frac{2}{3}q\right)^2$$

and

$$\frac{32}{3}pq = 2(8p)\left(\frac{2}{3}q\right)$$

So, the expression is of the form

$$a^2 + 2ab + b^2 = (a+b)^2$$

Therefore,

$$64p^2 + \frac{32}{3}pq + \frac{4}{9}q^2 = \left(8p + \frac{2}{3}q\right)^2$$

(v) Factor $3a^2 + 4ab + \frac{4}{3}b^2$

We observe that

$$3a^2 = (\sqrt{3}a)^2$$
$$\frac{4}{3}b^2 = \left(\frac{2}{\sqrt{3}}b\right)^2$$

and

$$4ab = 2(\sqrt{3}a)\left(\frac{2}{\sqrt{3}}b\right)$$

So, the expression is of the form

$$a^2 + 2ab + b^2 = (a+b)^2$$

Therefore,

$$3a^2 + 4ab + \frac{4}{3}b^2 = \left(\sqrt{3}a + \frac{2}{\sqrt{3}}b\right)^2$$

(vi) Factor $\frac{9}{5}s^2 + 6sv + 5v^2$

We observe that

$$\frac{9}{5}s^2 = \left(\frac{3}{\sqrt{5}}s\right)^2$$

$$5v^2 = (\sqrt{5}v)^2$$

and

$$6sv = 2\left(\frac{3}{\sqrt{5}}s\right)(\sqrt{5}v)$$

So, the expression is of the form

$$a^2 + 2ab + b^2 = (a+b)^2$$

Therefore,

$$\frac{9}{5}s^2 + 6sv + 5v^2 = \left(\frac{3}{\sqrt{5}}s + \sqrt{5}v\right)^2$$

2. Find the values of the following using the identity $(a-b)^2 = a^2 - 2ab + b^2$.

(i) $(79)^2$(ii) $(193)^2$(iii) $(299)^2$

Sol:-

(i) $(79)^2$Take $a = 80, b = 1.$

$$79^2 = (80-1)^2$$
$$= 80^2 - 2 \cdot 80 \cdot 1 + 1^2$$
$$= 6400 - 160 + 1$$
$$= 6241$$

(ii) $(193)^2$Take $a = 200, b = 7.$

$$193^2 = (200-7)^2$$
$$= 200^2 - 2 \cdot 200 \cdot 7 + 7^2$$
$$= 40000 - 2800 + 49$$

$$= 37249$$

(iii) $(299)^2$Take $a = 300$, $b = 1$.

$$299^2 = (300 - 1)^2$$
$$= 300^2 - 2 \cdot 300 \cdot 1 + 1^2$$
$$= 90000 - 600 + 1$$
$$= 89401$$

Exercise set 4.3 (Solution)

1: Find the following squares using one of the above identities. Determine which of these identities will make these calculations easier.

(i) 117^2 (ii) 78^2 (iii) 198^2 (iv) 214^2 (v) 1104^2 (vi) 1120^2

Sol:

(i) 117^2 Take $a = 120, b = 3$.

$$117^2 = (120 - 3)^2$$
$$= 120^2 - 2 \cdot 120 \cdot 3 + 3^2$$
$$= 14400 - 720 + 9$$
$$= 13689$$

(ii) 78^2 Take $a = 80, b = 2$.

$$78^2 = (80 - 2)^2$$
$$= 80^2 - 2 \cdot 80 \cdot 2 + 2^2$$
$$= 6400 - 320 + 4$$
$$= 6084$$

(iii) 198^2 Take $a = 200, b = 2$.

$$198^2 = (200 - 2)^2$$
$$= 200^2 - 2 \cdot 200 \cdot 2 + 2^2$$
$$= 40000 - 800 + 4$$
$$= 39204$$

(iv) 214^2 Take $a = 210, b = 4$.

$$214^2 = (210 + 4)^2$$
$$= 210^2 + 2 \cdot 210 \cdot 4 + 4^2$$
$$= 44100 + 1680 + 16$$
$$= 45796$$

(v) 1104^2 Take $a = 1100, b = 4$.

$$1104^2 = (1100 + 4)^2$$
$$= 1100^2 + 2 \cdot 1100 \cdot 4 + 4^2$$
$$= 1210000 + 8800 + 16$$
$$= 1218816$$

(vi) 1120^2Take $a = 1100, b = 20$.

$$1120^2 = (1100 + 20)^2$$
$$= 1100^2 + 2 \cdot 1100 \cdot 20 + 20^2$$
$$= 1210000 + 44000 + 400$$

2. Factor using suitable identities:

(i) $16y^2 - 24y + 9$
(ii) $\frac{9}{4}s^2 + 6st + 4t^2$
(iii) $\frac{m^2}{9} + \frac{mk}{3} + \frac{k^2}{4} + 3nk + 2mn + 9n^2$
(iv) $\frac{p^2}{16} - 2 + \frac{16}{p^2}$
(v) $9a^2 + 4b^2 + c^2 - 12ab + 6ac - 4bc$

Sol.
(i) $16y^2 - 24y + 9$

$$16y^2 - 24y + 9 = (4y)^2 - 2 \cdot (4y)(3) + 3^2$$

Identity: $(x - y)^2 = x^2 - 2xy + y^2$.
$= (4y - 3)^2 = (4y - 3)(4y - 3)$
(ii) $\frac{9}{4}s^2 + 6st + 4t^2$

$$\frac{9}{4}s^2 + 6st + 4t^2 = \left(\frac{3}{2}s\right)^2 + 2 \cdot \frac{3}{2}s \cdot 2t + (2t)^2$$

Identity: $(x + y)^2$.
$= \left(\frac{3}{2}s+2t\right)^2 = \left(\frac{3}{2}s+2t\right)\left(\frac{3}{2}s+2t\right)$
(iii) $\frac{m^2}{9} + \frac{mk}{3} + \frac{k^2}{4} + 3nk + 2mn + 9n^2$

$$= \left(\frac{m}{3}\right)^2 + 2 \cdot \frac{m}{3} \cdot \frac{k}{2} + \left(\frac{k}{2}\right)^2 + 2 \cdot \frac{m}{3} \cdot 3n + 2 \cdot \frac{k}{2} \cdot 3n + (3n)^2$$

Identity: $(x + y + z)^2$.
$= \left(\frac{m}{3}+\frac{k}{2}+3n\right)^2 = \left(\frac{m}{3}+\frac{k}{2}+3n\right)\left(\frac{m}{3}+\frac{k}{2}+3n\right)$
(iv) $\frac{p^2}{16} - 2 + \frac{16}{p^2}$

$$= \left(\frac{p}{4}\right)^2 + \left(\frac{4}{p}\right)^2 - 2 \cdot \frac{p}{4} \cdot \frac{4}{p}$$

Identity: $(x - y)^2$.
$= \left(\frac{p}{4}-\frac{4}{p}\right)^2 = \left(\frac{p}{4}-\frac{4}{p}\right)\left(\frac{p}{4}-\frac{4}{p}\right)$
(v) $9a^2 + 4b^2 + c^2 - 12ab + 6ac - 4bc$

$$= (3a)^2 + (-2b)^2 + c^2 + 2 \cdot (3a)(-2b) + 2 \cdot (3a)(c) + 2 \cdot (-2b)(c)$$

Identity: $(x + y + z)^2$.
$= (3a - 2b + c)^2 = (3a - 2b + c)(3a - 2b + c)$

3. Expand the following using the identity

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(i) $(p+3q+7r)^2$

(ii) $(3x-2y+4z)^2$

Sol.

(i) $(\boldsymbol{p}+\boldsymbol{3q}+\boldsymbol{7r})^2$

Identity: $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$$(p+3q+7r)^2 = p^2 + (3q)^2 + (7r)^2 + 2(p\cdot 3q) + 2(3q\cdot 7r) + 2(7r\cdot p)$$
$$= p^2 + 9q^2 + 49r^2 + 6pq + 42qr + 14pr$$

(ii) $(\boldsymbol{3x}-\boldsymbol{2y}+\boldsymbol{4z})^{\boldsymbol{2}}$

Identity: $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$$(3x-2y+4z)^2 = (3x)^2 + (-2y)^2 + (4z)^2 + 2(3x\cdot -2y) + 2(-2y\cdot 4z) + 2(4z\cdot 3x)$$
$$= 9x^2 + 4y^2 + 16z^2 - 12xy - 16yz + 24xz$$

4. Is this an identity?

$$(a+b-c)^2 + (a-b+c)^2 + (a-b-c)^2 = 2a^2 + 2b^2 + 2c^2$$

Sol.

Expand each term:

$$(a+b-c)^2 = a^2+b^2+c^2+2ab-2ac-2bc$$
$$(a-b+c)^2 = a^2+b^2+c^2-2ab+2ac-2bc$$
$$(a-b-c)^2 = a^2+b^2+c^2-2ab-2ac+2bc$$

Now add them:

$$= (a^2+b^2+c^2+2ab-2ac-2bc) + (a^2+b^2+c^2-2ab+2ac-2bc) + (a^2+b^2 + c^2-2ab-2ac+2bc)$$
$$= 3a^2+3b^2+3c^2-2ab-2ac-2bc$$

Exercise set 4.4 (Solution)

1. **Fill in the blanks to complete the following identities:**

(i) $s^2 - 11s + 24 = (\_\_\_\_\_)(\_\_\_\_\_)$
(ii) $(\_\_\_\_\_)(x + 1) = (3x^2 - 4x - 7)$
(iii) $10x^2 - 11x - 6 = (2x - \_\_\_\_\_)(\_\_\_\_\_ + 2)$
(iv) $6x^2 + 7x + 2 = (\_\_\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_\_\_)$

Sol.

(i) $s^2 - 11s + 24$

Middle term splitting method:

Product = 24, Sum = -11.

Factors: -3और -8.

$$s^2 - 11s + 24 = s^2 - 3s - 8s + 24$$
$$= (s^2 - 3s) - (8s - 24)$$
$$= s(s - 3) - 8(s - 3)$$
$$= (s - 3)(s - 8)$$

(ii) $(\_\_\_\_\_)(x + 1) = 3x^2 - 4x - 7$

We need to factor RHS:

$$3x^2 - 4x - 7$$

Split middle term: product = -21, sum = -4. Factors: -7 & 3.

$$3x^2 - 4x - 7 = 3x^2 - 7x + 3x - 7$$
$$= (3x^2 - 7x) + (3x - 7)$$
$$= (3x - 7)(x + 1)$$

So blanks: $(3x - 7)(x + 1)$.

(iii) $10x^2 - 11x - 6$

Split middle term: product = -60, sum = -11. Factors: -15 & 4.

$$10x^2 - 11x - 6 = 10x^2 - 15x + 4x - 6$$
$$= (10x^2 - 15x) + (4x - 6)$$
$$= 5x(2x - 3) + 2(2x - 3)$$
$$= (2x - 3)(5x + 2)$$

So blanks: $(2x - 3)(5x + 2)$.

(iv) $6x^2 + 7x + 2$

Split middle term: product = 12, sum = 7. Factors: 3 & 4.

$$6x^2 + 7x + 2 = 6x^2 + 3x + 4x + 2$$
$$= (6x^2 + 3x) + (4x + 2)$$
$$= 3x(2x + 1) + 2(2x + 1)$$
$$= (3x + 2)(2x + 1)$$

2. Select and use the identity that will help you to find the following products without multiplying directly:

(i) $(41)^2$
(ii) $(27)^2$
(iii) (23×17)
(iv) $(135)^2$
(v) $(97)^2$
(vi) (18×29)
(vii) (34×43)
(viii) $(205)^2$
Sol.

(i) $(41)^2$
Identity: $(a+b)^2 = a^2 + 2ab + b^2$. Take $41 = 40 + 1$.

$$(41)^2 = (40+1)^2 = 40^2 + 2(40)(1) + 1^2$$
$$= 1600 + 80 + 1 = 1681$$

(ii) $(27)^2$
Take $27 = 30 - 3$. Identity: $(a-b)^2 = a^2 - 2ab + b^2$.

$$(27)^2 = (30-3)^2 = 30^2 - 2(30)(3) + 3^2$$
$$= 900 - 180 + 9 = 729$$

(iii) (23×17)
Identity: $(a+b)(a-b) = a^2 - b^2$. Here, $23 = 20 + 3$, $17 = 20 - 3$.

$$23 \times 17 = (20+3)(20-3) = 20^2 - 3^2$$
$$= 400 - 9 = 391$$

(iv) $(135)^2$
Take $135 = 100 + 35$. Identity: $(a+b)^2 = a^2 + 2ab + b^2$.

$$(135)^2 = (100+35)^2 = 100^2 + 2(100)(35) + 35^2$$
$$= 10000 + 7000 + 1225 = 18225$$

(v) Find $(97)^2$

$$(97)^2 = (100-3)^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$
$$= (100)^2 - 2(100)(3) + (3)^2$$
$$= 10000 - 600 + 9$$
$$= 9409$$

(vi) Find (18×29)

$$18 \times 29 = 18(30-1)$$

$$= 18 \times 30 - 18 \times 1$$
$$= 540 - 18$$

$= 522$

(vii) Find (34×43)

$$34 \times 43 = 34(40 + 3)$$

$= 34 \times 40 + 34 \times 3$
$= 1360 + 102$
$= 1462$

(viii) Find $(205)^2$

$$(205)^2 = (200 + 5)^2$$

$(a + b)^2 = a^2 + 2ab + b^2$
$= (200)^2 + 2(200)(5) + (5)^2$
$= 40000 + 2000 + 25$
$= 42025$

3. Factor the following:
(i) $9a^2 + b^2 + 4c^2 - 6ab + 12ac - 4bc$
(ii) $16s^2 + 25t^2 - 40st$
(iii) $r^2 - r - 42$
(iv) $49g^2 + 14gh + h^2$
(v) $64u^2 + 121v^2 + 4w^2 - 176uv - 32uw + 44vw$

Sol. (i) Factorise:

$$9a^2 + b^2 + 4c^2 - 6ab + 12ac - 4bc$$

$$= 9a^2 + b^2 + 4c^2 - 6ab + 12ac - 4bc$$

This matches the identity:

$$(x - y + z)^2 = x^2 + y^2 + z^2 - 2xy + 2xz - 2yz$$

Taking,

$$x = 3a, y = b, z = 2c$$

Therefore,

$$= (3a - b + 2c)^2$$

Hence,

$$9a^2 + b^2 + 4c^2 - 6ab + 12ac - 4bc = (3a - b + 2c)^2$$

(ii) Factorise:

$$16s^2 + 25t^2 - 40st$$

Using identity:

$$(a-b)^2 = a^2 - 2ab + b^2$$
$$= (4s)^2 - 2(4s)(5t) + (5t)^2$$
$$= (4s - 5t)^2$$

Hence,

$$16s^2 + 25t^2 - 40st = (4s - 5t)^2$$

(iii) Factorise:

$$r^2 - r - 42$$

We need two numbers whose product is $-42$and sum is -1.

$$-7 \text{ and } 6$$

So,

$$r^2 - r - 42 = r^2 - 7r + 6r - 42$$
$$= r(r-7) + 6(r-7)$$
$$= (r-7)(r+6)$$

Hence,

$$r^2 - r - 42 = (r-7)(r+6)$$

(iv) Factorise:

$$49g^2 + 14gh + h^2$$

Using identity:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$= (7g)^2 + 2(7g)(h) + h^2$$
$$= (7g + h)^2$$

Hence,

$$49g^2 + 14gh + h^2 = (7g + h)^2$$

(v) Factorise:

$$64u^2 + 121v^2 + 4w^2 - 176uv - 32uw + 44vw$$

This matches the identity:

$$(x - y - z)^2 = x^2 + y^2 + z^2 - 2xy - 2xz + 2yz$$

Taking,

$$x = 8u, y = 11v, z = 2w$$

Therefore,

$$= (8u - 11v - 2w)^2$$

Hence,

$$64u^2 + 121v^2 + 4w^2 - 176uv - 32uw + 44vw = (8u - 11v - 2w)^2$$

Exercise set 4.5 (Solution)

1. Simplify the following rational expressions assuming that the expressions in the denominators are not equal to zero:

(i) $\frac{3p^2-3pq-18q^2}{p^2+3pq-10q^2}$

(ii) $\frac{n^3-3n^2m+3nm^2-m^3}{5m^2-10mn+5n^2}$

(iii) $\frac{w^3-v^3+x^3+3wvx}{w^2+v^2+x^2-2wv-2vx+2wx}$

(iv) $\frac{4y^2-20yz+25z^2}{25z^2-4y^2}$

(v) $\frac{(x^2+x-6)(x^2-7x+12)}{(x^2-6x+8)(x^2-9)}$

(vi) $\frac{p^4-16}{p^2-4p+4}$

Sol.-

(i) Simplify:

$$\frac{3p^2-3pq-18q^2}{p^2+3pq-10q^2}$$

Factorising numerator:

$$3p^2-3pq-18q^2 = 3(p^2-pq-6q^2)$$
$$= 3(p-3q)(p+2q)$$

Factorising denominator:

$$p^2+3pq-10q^2 = (p+5q)(p-2q)$$

Therefore,

$$\frac{3p^2-3pq-18q^2}{p^2+3pq-10q^2} = \frac{3(p-3q)(p+2q)}{(p+5q)(p-2q)}$$

(ii) Simplify:

$$\frac{n^3-3n^2m+3nm^2-m^3}{5m^2-10mn+5n^2}$$

Using identity:

$$(a-b)^3 = a^3-3a^2b+3ab^2-b^3$$
$$n^3-3n^2m+3nm^2-m^3 = (n-m)^3$$

Denominator:

$$5m^2-10mn+5n^2 = 5(m^2-2mn+n^2)$$

Using identity:

$$(a-b)^2 = a^2 - 2ab + b^2$$
$$= 5(m-n)^2$$

Since,

$$(m-n)^2 = (n-m)^2$$

Therefore,

$$\frac{(n-m)^3}{5(n-m)^2} = \frac{n-m}{5}$$

Hence,

$$\frac{n^3 - 3n^2m + 3nm^2 - m^3}{5m^2 - 10mn + 5n^2} = \frac{n-m}{5}$$

(iii) Simplify:

$$\frac{w^3 - v^3 + x^3 + 3wvx}{w^2 + v^2 + x^2 - 2wv - 2vx + 2wx}$$

Using identity:

$$a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

Take,

$$a = w, b = x, c = -v$$

Then,

$$w^3 + x^3 - v^3 + 3wvx = (w + x - v)(w^2 + v^2 + x^2 - 2wv - 2vx + 2wx)$$

Therefore,

$$\frac{(w + x - v)(w^2 + v^2 + x^2 - 2wv - 2vx + 2wx)}{w^2 + v^2 + x^2 - 2wv - 2vx + 2wx}$$
$$= w + x - v$$

Hence,

$$\frac{w^3 - v^3 + x^3 + 3wvx}{w^2 + v^2 + x^2 - 2wv - 2vx + 2wx} = w + x - v$$

(iv) Simplify:

$$\frac{4y^2 - 20yz + 25z^2}{25z^2 - 4y^2}$$

Numerator:

$$4y^2 - 20yz + 25z^2 = (2y - 5z)^2$$

Denominator:

$$25z^2 - 4y^2 = (5z - 2y)(5z + 2y)$$

Since,

$$(2y - 5z) = -(5z - 2y)$$

Therefore,

$$\frac{(2y - 5z)^2}{(5z - 2y)(5z + 2y)} = \frac{(5z - 2y)^2}{(5z - 2y)(5z + 2y)}$$
$$= \frac{5z - 2y}{5z + 2y}$$

Hence,

$$\frac{4y^2 - 20yz + 25z^2}{25z^2 - 4y^2} = \frac{5z - 2y}{5z + 2y}$$

(v) Simplify:

$$\frac{(x^2 + x - 6)(x^2 - 7x + 12)}{(x^2 - 6x + 8)(x^2 - 9)}$$

Factorising:

$$x^2 + x - 6 = (x + 3)(x - 2)$$
$$x^2 - 7x + 12 = (x - 3)(x - 4)$$
$$x^2 - 6x + 8 = (x - 2)(x - 4)$$
$$x^2 - 9 = (x - 3)(x + 3)$$

Therefore,

$$\frac{(x + 3)(x - 2)(x - 3)(x - 4)}{(x - 2)(x - 4)(x - 3)(x + 3)}$$

Cancelling common factors,

$$= 1$$

Hence,

$$\frac{(x^2 + x - 6)(x^2 - 7x + 12)}{(x^2 - 6x + 8)(x^2 - 9)} = 1$$

(vi) Simplify:

$$\frac{p^4 - 16}{p^2 - 4p + 4}$$

Factorising numerator:

$$p^4 - 16 = (p^2 - 4)(p^2 + 4)$$

Again,

$$p^2 - 4 = (p - 2)(p + 2)$$

So,

$$p^4 - 16 = (p - 2)(p + 2)(p^2 + 4)$$

Denominator:

$$p^2 - 4p + 4 = (p - 2)^2$$

Therefore,

$$\frac{(p - 2)(p + 2)(p^2 + 4)}{(p - 2)^2}$$

Cancelling one $(p-2)$,

$$= \frac{(p + 2)(p^2 + 4)}{p - 2}$$

Hence,

$$\frac{p^4 - 16}{p^2 - 4p + 4} = \frac{(p + 2)(p^2 + 4)}{p - 2}$$

END OF CHAPTER EXERCISES

1. Use suitable identities to find the following products:

   (i) $(-3x+4)^2$

   (ii) $(2s+7)(2s-7)$

   (iii) $\left(p^2+\frac{1}{2}\right)\left(p^2-\frac{1}{2}\right)$

   (iv) $(2n+7)(2n-7)$

   (v) $(s-2t)(s^2+2st+4t^2)$

   (vi) $\left(\frac{1}{2r}-4r\right)^2$

   (vii) $(-3m+4k-l)^2$

   (viii) $\left(x-\frac{1}{3}y\right)^3$

   (ix) $\left(\frac{7}{2}k-\frac{2}{3}m\right)^3$

Sol.-

(i) Find:

$$(-3x+4)^2$$

Using identity:

$$(a+b)^2 = a^2+2ab+b^2$$
$$= (-3x)^2+2(-3x)(4)+4^2$$
$$= 9x^2-24x+16$$

Hence,

$$(-3x+4)^2 = 9x^2-24x+16$$

(ii) Find:

$$(2s+7)(2s-7)$$

Using identity:

$$(a+b)(a-b) = a^2-b^2$$
$$= (2s)^2-7^2$$
$$= 4s^2-49$$

Hence,

$$(2s+7)(2s-7) = 4s^2-49$$

(iii) Find:

$$\left(p^2+\frac{1}{2}\right)\left(p^2-\frac{1}{2}\right)$$

Using identity:

$$(a+b)(a-b)=a^2-b^2$$
$$=(p^2)^2-\left(\frac{1}{2}\right)^2$$
$$=p^4-\frac{1}{4}$$

Hence,

$$\left(p^2+\frac{1}{2}\right)\left(p^2-\frac{1}{2}\right)=p^4-\frac{1}{4}$$

(iv) Find:

$$(2n+7)(2n-7)$$

Using identity:

$$(a+b)(a-b)=a^2-b^2$$
$$=(2n)^2-7^2$$
$$=4n^2-49$$

Hence,

$$(2n+7)(2n-7)=4n^2-49$$

(v) Find:

$$(s-2t)(s^2+2st+4t^2)$$

Using identity:

$$(a-b)(a^2+ab+b^2)=a^3-b^3$$
$$=s^3-(2t)^3$$
$$=s^3-8t^3$$

Hence,

$$(s-2t)(s^2+2st+4t^2)=s^3-8t^3$$

(vi) Find:

$$\left(\frac{1}{2r}-4r\right)^2$$

Using identity:

$$(a-b)^2 = a^2 - 2ab + b^2$$
$$= \left(\frac{1}{2r}\right)^2 - 2\left(\frac{1}{2r}\right)(4r) + (4r)^2$$
$$= \frac{1}{4r^2} - 4 + 16r^2$$

Hence,

$$\left(\frac{1}{2r}-4r\right)^2 = \frac{1}{4r^2} - 4 + 16r^2$$

(vii) Find:

$$(-3m + 4k - l)^2$$

Using identity:

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$
$$= (-3m)^2 + (4k)^2 + (-l)^2 + 2(-3m)(4k) + 2(4k)(-l) + 2(-3m)(-l)$$
$$= 9m^2 + 16k^2 + l^2 - 24mk - 8kl + 6ml$$

Hence,

$$(-3m + 4k - l)^2 = 9m^2 + 16k^2 + l^2 - 24mk - 8kl + 6ml$$

(viii) Find:

$$\left(x-\frac{1}{3}y\right)^3$$

Using identity:

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$
$$= x^3 - 3x^2\left(\frac{1}{3}y\right) + 3x\left(\frac{1}{3}y\right)^2 - \left(\frac{1}{3}y\right)^3$$
$$= x^3 - x^2y + \frac{1}{3}xy^2 - \frac{1}{27}y^3$$

Hence,

$$\left(x-\frac{1}{3}y\right)^3 = x^3 - x^2y + \frac{1}{3}xy^2 - \frac{1}{27}y^3$$

(ix) Find:

$$\left(\frac{7}{2}k - \frac{2}{3}m\right)^3$$

Using identity:

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$
$$= \left(\frac{7}{2}k\right)^3 - 3\left(\frac{7}{2}k\right)^2\left(\frac{2}{3}m\right) + 3\left(\frac{7}{2}k\right)\left(\frac{2}{3}m\right)^2 - \left(\frac{2}{3}m\right)^3$$
$$= \frac{343}{8}k^3 - \frac{49}{2}k^2m + \frac{14}{3}km^2 - \frac{8}{27}m^3$$

Hence,

$$\left(\frac{7}{2}k - \frac{2}{3}m\right)^3 = \frac{343}{8}k^3 - \frac{49}{2}k^2m + \frac{14}{3}km^2 - \frac{8}{27}m^3$$

2. Find the values using suitable identities:
   (i) 17×21
   (ii) 104×96
   (iii) 24×16
   (iv) 147^3
   (v) 199^3
   (vi) 127^3
   (vii) $(-107)^3$
   (viii) $(-299)^3$

Sol.-
(i) Find:

$$17 \times 21$$
$$= (19-2)(19+2)$$

Using identity:

$$(a-b)(a+b) = a^2 - b^2$$
$$= 19^2 - 2^2$$
$$= 361 - 4$$
$$= 357$$

Hence,

$$17 \times 21 = 357$$

(ii) Find:

$$104 \times 96$$
$$= (100+4)(100-4)$$

Using identity:

$$(a+b)(a-b)=a^2-b^2$$
$$=100^2-4^2$$
$$=10000-16$$
$$=9984$$

Hence,

$$104\times 96=9984$$

(iii) Find:

$$24\times 16$$
$$=(20+4)(20-4)$$

Using identity:

$$(a+b)(a-b)=a^2-b^2$$
$$=20^2-4^2$$
$$=400-16$$
$$=384$$

Hence,

$$24\times 16=384$$

(iv) Find:

$$147^3$$
$$147^3=(150-3)^3$$

Using identity:

$$(a-b)^3=a^3-3a^2b+3ab^2-b^3$$
$$=(150)^3-3(150)^2(3)+3(150)(3)^2-(3)^3$$
$$=3375000-202500+4050-27$$
$$=3176523$$

Hence,

$$147^3=3176523$$

(v) Find:

$$199^3$$
$$199^3=(200-1)^3$$

Using identity:

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$
$$= (200)^3 - 3(200)^2(1) + 3(200)(1)^2 - (1)^3$$
$$= 8000000 - 120000 + 600 - 1$$
$$= 7880599$$

Hence,

$$199^3 = 7880599$$

(vi) Find:

$$127^3$$
$$127^3 = (120+7)^3$$

Using identity:

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$= (120)^3 + 3(120)^2(7) + 3(120)(7)^2 + (7)^3$$
$$= 1728000 + 302400 + 17640 + 343$$
$$= 2048383$$

Hence,

$$127^3 = 2048383$$

(vii) Find:

$$(-107)^3$$
$$= (-100-7)^3$$

Using identity:

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$= (-100)^3 + 3(-100)^2(-7) + 3(-100)(-7)^2 + (-7)^3$$
$$= -1000000 - 210000 - 14700 - 343$$
$$= -1225043$$

Hence,

$$(-107)^3 = -1225043$$

(viii) Find:

$$(-299)^3$$
$$= (-300+1)^3$$

Using identity:

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$= (-300)^3 + 3(-300)^2(1) + 3(-300)(1)^2 + (1)^3$$
$$= -27000000 + 270000 - 900 + 1$$
$$= -26730900 + 1$$
$$= -26730901$$

Hence,

$$(-299)^3 = -26730901$$

3. **Factor the following algebraic expressions:**

(i) $4y^2 + 1 + \frac{1}{16y^2}$

(ii) $9m^2 - \frac{1}{25n^2}$

(iii) $27b^3 - \frac{1}{64b^3}$

(iv) $x^2 + \frac{5x}{6} + \frac{1}{6}$

(v) $27u^3 - \frac{1}{125} - \frac{27u^2}{5} + \frac{9u}{25}$

(vi) $64y^3 + \frac{1}{125}z^3$

(vii) $p^3 + 27q^3 + r^3 - 9pqr$

(viii) $9m^2 - 12m + 4$

(ix) $9x^3 - \frac{8}{3}y^3 + \frac{z^3}{3} + 6xyz$

(x) $4x^2 + 9y^2 + 36z^2 + 12xz + 36yz + 24xy$

(xi) $27u^3 - \frac{1}{216} - \frac{9u^2}{2} + \frac{u}{4}$

Sol.-

(i) $\mathbf{4y^2 + 1 + \frac{1}{16y^2}}$

This matches the identity $(a+b)^2 = a^2 + 2ab + b^2$. Take $a = 2y, b = \frac{1}{4y}$.

$$4y^2 + 1 + \frac{1}{16y^2} = (2y + \frac{1}{4y})^2$$

(ii) $\mathbf{9m^2 - \frac{1}{25n^2}}$

Difference of squares: $a^2 - b^2 = (a-b)(a+b)$. Here $a = 3m, b = \frac{1}{5n}$.

$$= (3m - \frac{1}{5n})(3m + \frac{1}{5n})$$

(iii) $\mathbf{27b^3 - \frac{1}{64b^3}}$

Difference of cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$. Here $a = 3b, b = \frac{1}{4b}$.

$$= (3b - \frac{1}{4b})(9b^2 + \frac{3}{4} + \frac{1}{16b^2})$$

(iv) $\mathbf{x^2 + \frac{5x}{6} + \frac{1}{6}}$

Quadratic trinomial. Factors of $\frac{1}{6}$that add to $\frac{5}{6}$are $\frac{1}{2}, \frac{1}{3}$.

$$= (x+\frac{1}{2})(x+\frac{1}{3})$$

(v) $27u^3 - \frac{1}{125} - \frac{27u^2}{5} + \frac{9u}{25}$

Matches cube expansion: $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$. Here $a = 3u$, $b = \frac{1}{5}$.

$$= (3u - \frac{1}{5})^3$$

(vi) $64y^3 + \frac{1}{125}z^3$

Sum of cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$. Here $a = 4y$, $b = \frac{z}{5}$.

$$= (4y + \frac{z}{5})(16y^2 - \frac{4yz}{5} + \frac{z^2}{25})$$

(vii) $p^3 + 27q^3 + r^3 - 9pqr$

Identity: $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$. Here $a = p$, $b = 3q$, $c = r$.

$$= (p+3q+r)(p^2+9q^2+r^2-3pq-pr-3qr)$$

(viii) $9m^2 - 12m + 4$

Perfect square trinomial.

$$= (3m-2)^2$$

(ix) $9x^3 - \frac{8}{3}y^3 + \frac{z^3}{3} + 6xyz$

This is of the form $a^3 + b^3 + c^3 - 3abc$. Choose $a = 3x$, $b = -\frac{2y}{\sqrt{3}}$, $c = \frac{z}{\sqrt{3}}$.

$$= (3x - \frac{2y}{\sqrt{3}} + \frac{z}{\sqrt{3}})(9x^2 + \frac{4y^2}{3} + \frac{z^2}{3} - 6x \cdot \frac{2y}{\sqrt{3}} - 6x \cdot \frac{z}{\sqrt{3}} + \frac{4yz}{3})$$

(This quadratic factor can be simplified further if needed.)

(x) $4x^2 + 9y^2 + 36z^2 + 12xz + 36yz + 24xy$

Perfect square trinomial in three variables.

$$= (2x+3y+6z)^2$$

(xi) $27u^3 - \frac{1}{216} - \frac{9u^2}{2} + \frac{u}{4}$

Cube identity: $(a-b)^3$. Here $a = 3u$, $b = \frac{1}{6}$.

$$= (3u - \frac{1}{6})^3$$

4. **Simplify the following:**

(i) $\frac{4x^2+4x+1}{4x^2-1}$

(ii) $\frac{9(3a^3-24b^3)}{9a^2-36b^2}$

(iii) $\frac{s^3+125t^3}{s^2-2st-35t^2}$

Sol.-

(i) $\frac{4x^2+4x+1}{4x^2-1}$

Factor numerator: $4x^2 + 4x + 1 = (2x + 1)^2$.

Factor denominator: $4x^2 - 1 = (2x - 1)(2x + 1)$.

$$\frac{(2x+1)^2}{(2x-1)(2x+1)} = \frac{2x+1}{2x-1}$$

(ii) $\frac{9(3a^3-24b^3)}{9a^2-36b^2}$

Simplify numerator: $9(3a^3 - 24b^3) = 27(a^3 - 8b^3)$.

Factor numerator: $a^3 - 8b^3 = (a - 2b)(a^2 + 2ab + 4b^2)$.

Denominator: $9a^2 - 36b^2 = 9(a^2 - 4b^2) = 9(a - 2b)(a + 2b)$. Cancel $(a-2b)$.

$$\frac{27(a-2b)(a^2+2ab+4b^2)}{9(a-2b)(a+2b)} = \frac{3(a^2+2ab+4b^2)}{a+2b}$$

(iii) $\frac{s^3+125t^3}{s^2-2st-35t^2}$

Numerator: sum of cubes, $s^3 + (5t)^3 = (s + 5t)(s^2 - 5st + 25t^2)$.

Denominator: quadratic, $s^2 - 2st - 35t^2 = (s - 7t)(s + 5t)$.

Cancel $(s+5t)$.

$$\frac{(s+5t)(s^2-5st+25t^2)}{(s-7t)(s+5t)} = \frac{s^2-5st+25t^2}{s-7t}$$

5. Find possible expressions for the length and breadth of each of the following rectangles whose areas are given by the following expressions in square units.

(i) $25a^2 - 30ab + 9b^2$

(ii) $36s^2 - 49t^2$

Sol.-

(i) Area = $25a^2 - 30ab + 9b^2$Factorize:

$$25a^2 - 30ab + 9b^2 = (5a - 3b)^2$$

So, possible length and breadth are:

$$(5a - 3b), (5a - 3b)$$

(ii) Area = $36s^2 - 49t^2$Factorize (difference of squares):

$$36s^2 - 49t^2 = (6s - 7t)(6s + 7t)$$

So, possible length and breadth are:

$$(6s - 7t), (6s + 7t)$$

6. Find possible expressions for the length, breadth, and heights of each of the following cuboids whose volumes are given by the following expressions in cubic units.

(i) $6a^2 - 24b^2$
(ii) $3ps^2 - 15ps + 12p$
Sol.-
(i) Volume = $6a^2 - 24b^2$Factorize:

$$6a^2 - 24b^2 = 6(a^2 - 4b^2) = 6(a - 2b)(a + 2b)$$

So, possible dimensions are:

$$\text{Length} = 6, \text{Breadth} = (a - 2b), \text{Height} = (a + 2b)$$

(ii) Volume = $3ps^2 - 15ps + 12p$Factorize:

$$3ps^2 - 15ps + 12p = 3p(s^2 - 5s + 4) = 3p(s - 1)(s - 4)$$

So, possible dimensions are:

$$\text{Length} = 3p, \text{Breadth} = (s - 1), \text{Height} = (s - 4)$$

7. The village playground is shaped as a square of side 40 metres. A path of width s metres is created around the playground for people to walk. Find an expression for the area of the path in terms of s.
Sol.-
Outer square (including path): Side length = $40 + 2s$Area = $(40 + 2s)^2$
Inner square (playground only): Side length = 40Area = $40^2 = 1600$
Area of path = Outer area – Inner area

$$(40 + 2s)^2 - 1600$$

Simplify:

$$= 1600 + 160s + 4s^2 - 1600$$
$$= 160s + 4s^2$$

8. If a number plus its reciprocal equals 10/ 3 , find the number.
Sol.-
Let the number be x.

$$x + \frac{1}{x} = \frac{10}{3}$$

Multiply through by x:

$$x^2 + 1 = \frac{10}{3}x$$

Multiply through by 3:

$$3x^2 + 3 = 10x$$

Rearrange:

$$3x^2 - 10x + 3 = 0$$

Solve quadratic:

$$x = \frac{10 \pm \sqrt{(-10)^2 - 4(3)(3)}}{2 \cdot 3}$$
$$= \frac{10 \pm \sqrt{100 - 36}}{6}$$

$$= \frac{10 \pm \sqrt{64}}{6}$$
$$= \frac{10 \pm 8}{6}$$

So,

$$x = \frac{18}{6} = 3 \text{or} x = \frac{2}{6} = \frac{1}{3}$$

9. A rectangular pool has area 2x2 + 7x + 3 square hastas. If its width is 2x + 1 hastas, find its length. Hasta was a unit used to measure length.

Sol.-

We know:

$$\text{Area} = \text{Length} \times \text{Width}$$

Given:

$$\text{Area} = 2x^2 + 7x + 3, \text{Width} = 2x + 1$$

So,

$$\text{Length} = \frac{\text{Area}}{\text{Width}} = \frac{2x^2 + 7x + 3}{2x + 1}$$

Factorize numerator:

$$2x^2 + 7x + 3 = (2x + 1)(x + 3)$$

Therefore,

$$\text{Length} = \frac{(2x + 1)(x + 3)}{2x + 1} = x + 3$$

10. If both $x - 2$and $x - \frac{1}{2}$are factors of $px^2 + 5x + r$, show that $p = r$.

Sol.

Since $x - 2$is a factor, substituting $x = 2$must make the polynomial zero:

$$p(2)^2 + 5(2) + r = 0$$
$$4p + 10 + r = 0 \Rightarrow 4p + r = -10 \ldots (1)$$

Since $x - \frac{1}{2}$is a factor, substituting $x = \frac{1}{2}$must also make the polynomial zero:

$$p\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + r = 0$$
$$\frac{p}{4} + \frac{5}{2} + r = 0$$

Multiply through by 4:

$$p + 10 + 4r = 0 \Rightarrow p + 4r = -10 \ldots (2)$$

Now solve equations (1) and (2):

$$4p + r = -10$$
$$p + 4r = -10$$

Multiply (2) by 4:

$$4p + 16r = -40$$

Subtract (1):

$$(4p + 16r) - (4p + r) = -40 - (-10)$$
$$15r = -30 \Rightarrow r = -2$$

Substitute $r = -2$into (1):

$$4p - 2 = -10 \Rightarrow 4p = -8 \Rightarrow p = -2$$

Thus,

$$p = r$$

11. If $a + b + c = 5$and $ab + bc + ca = 10$, then prove that
$$a^3 + b^3 + c^3 - 3abc = -25$$

Sol.- We use the identity:

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

Now,

$$a + b + c = 5$$

Also,

$$a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca)$$
$$= 5^2 - 2(10) = 25 - 20 = 5$$

So,

$$a^2 + b^2 + c^2 - (ab + bc + ca) = 5 - 10 = -5$$

Therefore,

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
$$= 5 \times (-5) = -25$$

12. By factoring the expression, check that $n^3 - n$is always divisible by 6 for all natural numbers n. Give reasons.

Sol. We start with:

$$n^3 - n$$

Factorize:

$$n^3 - n = n(n^2 - 1) = n(n - 1)(n + 1)$$

So,

$$n^3 - n = n(n - 1)(n + 1)$$

This is the product of **three consecutive natural numbers**.

Among three consecutive numbers $n, (n - 1), (n + 1)$:

- One of them is always divisible by **2** (since every second number is even).
- One of them is always divisible by **3** (since every third number is a multiple of 3).

Therefore, their product is always divisible by both 2 and 3, i.e., divisible by **6**.

13. Find the value of: **(i)** $x^3 + y^3 - 12xy + 64$, when $x + y = -4$ **(ii)** $x^3 - 8y^3 - 36xy - 216$, when $x = 2y + 6$.

Sol.

(i) We use the identity:

$$x^3 + y^3 = (x + y)^3 - 3xy(x + y)$$

So,

$$x^3 + y^3 - 12xy + 64 = (x + y)^3 - 3xy(x + y) - 12xy + 64$$

Given $x + y = -4$:

$$= (-4)^3 - 3xy(-4) - 12xy + 64$$
$$= -64 + 12xy - 12xy + 64$$
$$= 0$$

(ii) Expression:

$$x^3 - 8y^3 - 36xy - 216$$

Given $x = 2y + 6$. Substitute:

$$= (2y + 6)^3 - 8y^3 - 36(2y + 6)y - 216$$

Expand:

$$= (8y^3 + 72y^2 + 216y + 216) - 8y^3 - (72y^2 + 216y) - 216$$

Simplify:

$$= 8y^3 - 8y^3 + 72y^2 - 72y^2 + 216y - 216y + 216 - 216$$
$$= 0$$