Exercise set 3.1 (Solution)

1. A merchant in the port city of Lothal is exchanging bags of spices for copper ingots. He receives 15 ingots for every 2 bags of spices. If he brings **12** bags of spices to the market, how many copper ingots will he leave with ?

   Sol.- Given that:

   2 bags of spices= 15 ingots

   So, 1 bag of spices= $15/2$ ingots

   Similarly, for 12 bags of spices

   $= 12 \times (15/2)$

   $= 6 \times 15$

   $= 90$

   Therefore, the merchant will leave with 90 copper ingots.

2. Look at the sequence of numbers on one column of the Ishango bone: 11, 13, 17, 19. What do these numbers have in common? List the next three numbers that fit this pattern.

   Sol.- The numbers 11, 13, 17, 19 are all prime numbers (numbers that have only two factors: 1 and itself).

   Next three prime numbers after 19 are 23, 29, 31.

   Therefore, the next three numbers are 23, 29, 31.

3. We know that Natural Numbers are closed under addition (the sum of any two natural numbers is always a natural number). Are they closed under subtraction? Provide a couple of examples to justify your answer.

   Sol.- Natural numbers are NOT closed under subtraction.

   Explanation:

   Closure means the result should also be a natural number.

   Examples:

   (I) $5 - 3 = 2$ (Natural number)

   (ii) $3 - 5 = -2$ (Not a natural number)

   Since subtraction can give a negative number, so natural numbers are not closed under subtraction.

4. Ancient Indians used the joints of their fingers to count, a practice still seen today. Each finger has 3 joints, and the thumb is used to count them. How many can you count on one hand ? How does this relate to the ancient base-12 counting systems?

   Sol.-Each finger (except thumb) has 3 joints.

   Number of fingers used= 4 (excluding thumb)

   Total joints $= 4 \times 3 = 12$

So, we can count up to 12 using one hand.
Relation to base-12 system:
Since counting reaches 12 on one hand, it naturally leads to a base- 12 (duodecimal) counting system used in ancient times.It directly explains how ancient people developed the **base-12 counting system** using finger joints.

Therefore:

- ✓ Total count = 12
- ✓ This explains the origin of base-12 counting system.

Note- **Why base-12 was useful**

Ancient civilizations preferred base-12 because:

- ✓ 12 is divisible by **2, 3, 4, 6** (very practical for trade & measurement)
- ✓ This idea influenced:
  - 12 months in a year
  - 12 hours on a clock
  - 60 minutes/seconds (related to 12 × 5)

Exercise set 3.2 (Solution)

1. The temperature in the high-altitude desert of Ladakh is recorded as 4 °C at noon. By midnight, it drops by 15 °C. What is the midnight temperature?

Sol.- Initial temperature $= 4^{\circ}\mathrm{C}$

Drop $= 1\,s^{\circ}c$

Midnight temperature $= 4 - 15 = -11^{\circ}\mathrm{C}$

Therefore, the midnight temperature is $-11^{\circ}\mathrm{C}$

2. A spice trader takes a loan (debt) of Rs. 850. The next day, he makes a profit (fortune) of Rs.1,200. The following week, he incurs a loss of Rs. 450. Write this sequence as an equation using integers and calculate his final financial standing.

Sol.-Debt = - 850 Rs.

Profit = +1200 Rs.

Loss = − 450 Rs.

Equation: - 850 Rs. +1200 Rs. – 450 Rs.

Step-by-step calculation:

= 350 - 450

= - 100 Rs.

Therefore, his final financial standing is -100 Rs. (Loss of 100 Rs.)

3. Calculate the following using Brahmagupta's laws:

(i) (–12) × 5 (ii) (–8) × (–7) (iii) 0 – (–14) (iv) (–20) ÷ 4

Sol.-As per Brahmagupta's laws:

Debt indicates Negative

Fortune indicates Positive

(i) $(-12) \times 5$

Negative × Positive = Negative [As Debt × Fortune = Debt]

Therefore, $(-12) \times 5 = -60$

(ii) $(-8) \times (-7)$

Negative x Negative = Positive (As Debt x Debt = Fortune]

Therefore, $(-8), (-7) = 55$

(iii) $0 - (-14)$

As per Brahmagupta, zero minus debt is a fortune.
Subtracting a negative is same as adding:
Therefore, $0 - (-14) = 0 + 14 = 14$

(iv) $(-20) \div 4$

Negative + Positive = Negative (As Debt + Fortune = Debt]
Therefore, $(-20) + 4 = -5$.

4. Explain, using a real-world example of debt, why subtracting a negative number is the same as adding a positive number (e.g., 10 – (–5) = 15).

Sol.-

Think of a negative number as debt.

You have Rs 10. You also owe Rs 5, which is written as –5.

Now consider the expression 10 – (–5). Subtracting a negative number means removing a debt. So this situation means you have Rs 10 and your Rs 5 debt is taken away.

Before removing the debt, you effectively have Rs 10 but owe Rs 5. When the debt is removed, you are Rs 5 better off.

So your total becomes 10 + 5 = Rs 15.

This is why subtracting a negative number is the same as adding a positive number.

Note-

Negative number = debt

Subtracting a negative = removing debt

Removing debt = gaining money

Exercise set 3.3 (Solution)

1. Prove that the following rational numbers are equal:

(i) $2/3$ and $4/6$

(ii) $5/4$ and $10/8$

(iii) $-3/5$ and $-6/10$

(iv) $9/3$ and 3

Sol:-

(i) $2/3$ and $4/6$

First Fraction: $2/3 = 2/3$

Second Fraction: $4/6 = 2/3$ (dividing numerator and denominator by 2)

Therefore, $2/3$ and $4/6$ are equal.

(ii) $5/4$ and $10/8$

First Fraction: $5/4 = 5/4$

Second Fraction: $10/8 = 5/4$ (dividing numerator and denominator by 2)

Therefore, $5/4$ and 10/8 are equal.

(iii) $-3/5$ and $-6/10$

First Fraction: $-3/5 = -3/5$

Second Fraction: $-6/10 = -3/5$ (dividing numerator and denominator by 2)

Therefore, $-3/5$ and $-6/10$ are equal.

(iv) $9/3$ and 3

First Fraction: $9/3 = 3$

Hence, 9/3 and 3 are equal.

2. Find the sum:

(i) $2/5 + 3/10$

(ii) $7/12 + 5/8$

(iii) $-4/7 + 3/14$

Sol.-

(i)$2/5 + 3/10$

LCM of 5 and $10 = 10$

Simplifying the first number: $2/5 = 4/10$ [Making the same denominator]

Now, the sum

$= 2/5 + 3/10$
$= 4/10 + 3/10$
$= 7/10$
Therefore, the sum of $2/5 + 3/10$ is $7/10$.

(ii)$7/12 + 5/8$

LCM of 12 and $8 = 24$
Simplifying the first number: $7/12 = 14/24$ [Making the same denominator]
Simplifying the second number: $5/8 = 15/24$ [Making the same denominator]
Now, the sum
$= 7/12 + 5/8$
$= 14/24 + 15/24$
=29/24
Therefore, the sum of $\frac{7}{12} + \frac{5}{8}$ is $29/24$.

(iii) $-4/7 + 3/14$

LCM of 7 and $14 = 14$
Simplifying the first number: $-4/7 = -8/14$ [Making the same denominator]
So, the sum
$= -4/7 + 3/14$
$= -8/14 + 3/14$
$= -\frac{5}{14}$
Therefore, the sum of $-4/7 + 3/14$ is $-5/14$.

3. Find the difference :
(i) $5/6 - 1/4$
(ii) $\frac{11}{8} - \frac{3}{4}$
(iii) $-7/9 - (-2/3)$

Sol.-
(i) $5/6 - 1/4$
LCM of 6 and $4 = 12$
Simplifying the first number: $5/6 = 10/12$ [Making the same denominator]
Simplifying the Second number: $1/4 = 3/12$ [Making the same denominator]
So, the difference
$= 5/6 - 1/4$
$= 10/12 - 3/12$
$= 7/12$
Therefore, the difference is 7/12.

(ii) $\frac{11}{8} - \frac{3}{4}$

LCM of 8 and $4 = 8$

Simplifying the Second number: $3/4 = 6/8$ [Making the same denominator]

So, the difference

$= 11/8 - 3/4$

$= 11/8 - 6/8$

$= \frac{5}{8}$

Therefore, the difference is $5/8$.

(iii) $-7/9 - (-2/3)$

$-7/9 - (-2/3) = -7/9 + 2/3$

LCM of 9 and $3 = 9$

Simplifying the Second number: $2/3 = 6/9$ [Making the same denominator]

So, the difference

$= -7/9 - (-2/3)$

$= -7/9 + 6/9$

$= -1/9$

Therefore, the difference is $-1/9$.

4. Find the product:

(i) $2/3 \times 3/10$

(ii) $7/11 \times 5/8$

(iii) $-4/7 \times 5/14$

Sol:- **(i)** $2/3 \times 3/10$

$= (2 \times 3)/(3 \times 10)$

$= 6/30$

$= 1/5$ [After simplification]

Therefore, the product is $1/5$.

(ii) $7/11 \times 5/8$

$\frac{7}{11} \times \frac{5}{8}$

$= (7 \times 5)/(11 \times 8)$

$= 35/88$

Therefore, the product is $35/88$.

(iii) $-4/7 \times 5/14$

$-4/7 \times 5/14$
$= (4 \times 5)/(7 \times 14)$
$= -20/98$
$= -10/49$ [After simplification]
Therefore, the product is $-10/49$.

5. Find the quotient:
(i) $2/3 \div 3/10$
(ii) $\frac{7}{11} \div \frac{5}{8}$
(iii) $-4/7 \div 5/14$

Sol.-(i)$2/3 \div 3/10$
$= 2/3 \times 10/3$
$= (2 \times 10)/(3 \times 3)$
$= 20/9$

(ii) $\frac{7}{11} \div \frac{5}{8}$
$= 7/11 \times 8/5$
$= (7 \times 8)/(11 \times 5)$
$= 56/55$
Therefore, the quotient is 56/55.
(iii) $-4/7 \div 5/14$
$= -4/7 \times 14/5$
$= (4 \times 14)/(7 \times 5)$
$= -56/35$
$= -8/5$ [After simplification]
Therefore, the quotient is $-8/5$.

6. Show that: $(1/2 + 3/4) \times 8/3 = 1/2 \times 8/3 + \mathbf{3/4} \times \mathbf{8/3}$

Sol.-LHS $= (1/2 + 3/4) \times 8/3$
$= (2/4 + 3/4) \times 8/3$ [First add inside the bracket: $1/2 = 2/4$]
$= (5/4) \times 8/3$ [Since $2/4 + 3/4 = 5/4$)
$= 5/4 \times 8/3$
$= (5 \times 8)/4 \times 3)$
$= 40/12$
$= 10/3$ [After simplification]
$\text{RHS} = 1/2 \times 8/3 + 3/4 \times 8/3$
$= (1 \times 8)/(2 \times 3) + (3 \times 8)/(4 \times 3)$
$= 8/6 + 24/12$
$= 4/3 + 6/3$ [After simplification: $8/6 = 4/3$ and 24/12 =6/3]
$= 10/3$

Since LHS $=$ RHS,
Therefore, $(1/2 + 3/4) \times 8/3 = 1/2 \times 8/3 + 3/4 \times 8/3$
Hence proved.

7. Simplify the following using the distributive property: (7/9)(6/7-3/4).
Sol.-

Using distributive property:
7/9 (6/7-3/4)
$= 7/9 \times 6/7 - 7/9 \times 3/4$
$= 6/9 - 21/36$
$= 2/3 - 7/12$
$= 8/12 - 7/12$ [LCM of 3 and $12 = 12$ and $2/3 = 8/12$]
= 1/12
Therefore, the simplified value of $7/9(6/7 - 3/4)$ is $1/12$.

8. Find the rational number x such that:

$$(5/6)(x + 3/5) = (5/6)x + 1/2$$

Sol.-

Given: $(5/6)(x + 3/5) = (5/6)x + 1/2$

$(5/6)x + (5/6 \times 3/5) = (5/6)x + 1/2$

$(5/6)x + 15/30 = (5/6)x + 1/2$
$15/30 = 1/2$, which is universal truth.
So, $(5/6)(x + 3/5) = (5/6)x + 1/2$ is true for every value of x
Therefore, x can be any rational number.

Exercise set 3.4 (Solution)

1. Represent the rational numbers 2/3, $-5/4$ and $1\frac{1}{2}$ on a single number line.

Sol.-

Convert mixed number:

$$1\frac{1}{2} = 3/2$$

Now to compare the number first convert into decimal:

$-5/4 = -1.25$

$2/3 = 0.67$

$3/2 = 1.5$

On the number line:

$-5/4$ lies between -2 and -1

$2/3$ lies between 0 and 1

3/2 lies between 1 and 2

So, order is:

$-5/4 < 2/3 < 3/2$

So, we can easily Mark these points accordingly on the number line.

2. Find three distinct rational numbers that Lie strictly between $-1/2$ and 1/4.

Sol.-

Given numbers: $-1/2$ and $1/4$

LCM of 2 and $4 = 4$

Converting $-\mathbf{1/2}$ to common denominator:

$-1/2 = -2/4$

Now the given numbers: $-2/4$ and $1/4$

So numbers between $-2/4$ and $1/4$ are $-1/4, 0, 1/8$

Therefore, three rational numbers are $-1/4, 0, 1/8$.

Note: We can find infinite rational number between any two rational numbers.

3. Simplify the expression: (-1/4) + (5/12)

Sol.-

LCM of 4 and $12 = 12$

Converting $-\mathbf{1/4}$ to common denominator:

$-1/4 = -3/12$

So, $-3/12 + 5/12$
$= \frac{2}{12}$
= $1/6$
Therefore, the result of $(-1/4) + (5/12)$ is $1/6$.

4. A tailor has $15\frac{3}{4}$ metres of fine silk. If making one kurta requires $2\frac{1}{4}$ metres of silk, exactly how many kurtas can he make?
Sol.-

Converting improper fractions:

$15\frac{3}{4} = 63/4$

$2\frac{1}{4} = 9/4$

Total amount of silk cloth $= \mathbf{15}\frac{3}{4}$ metres
In $2\frac{1}{4}$ metres of silk, number of kurta $= 1$
In $\mathbf{1}$ metres of silk, number of kurta $= \mathbf{1}/2\frac{1}{4}$
In $\mathbf{15}\frac{\mathbf{3}}{\mathbf{4}}$ metres of silk, number of kurta $= \mathbf{1/2}\frac{\mathbf{1}}{\mathbf{4}} \times \mathbf{15}\frac{\mathbf{3}}{\mathbf{4}}$
$= 1/(9/4)x(63/4)$
$= 4/9x63/4$
$= (4x63)/(9x4)$
$= 63/9 = 7$
Therefore, he can make exactly 7 kurtas from $15\frac{3}{4}$ metres of fine silk.

5. Find three rational numbers between 3.1415 and 3.1416.
Sol.-

We can insert more decimal places:
3.1415 < 3.14151 < 3.14152 < 3.14153 < 3.1416

So, three rational numbers are 3.14151, 3.14152, 3.14153.

6. Can you think of other way(s) to find a rational number between any two rational numbers?
Sol.-

Yes, there are methods:

1. Taking average:

If a and bare two rational numbers, then
$(a + b)/2$ lies between them.

2 By making common denominator:
Convert both numbers to same denominator and choose a number in between.
3. By decimal expansion:

Convert into decimals and pick numbers in between.

Thus, there are infinitely many rational numbers between any two rational numbers.

Exercise set 3.5(Solution)

1. Without performing long division, determine which of the following rational numbers will have terminating decimals and which will be repeating: $\frac{7}{20}, \frac{4}{15}$ and $\frac{13}{250}$. Then check your answers by explicitly performing the long divisions and expressing these rational numbers as decimals.

Sol:-

A rational number p/q (in lowest terms) has a terminating decimal if and only if the denominator q has no prime factors other than 2 and 5.

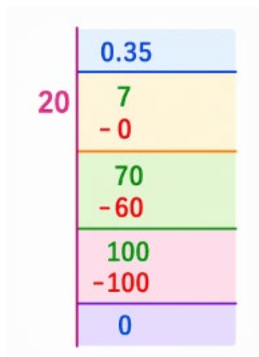
(i) $\frac{7}{20}$

$20 = 2^2 \times 5$

Since the denominator has only the prime factors 2 and 5, the decimal expansion is terminating.

<u>Verification</u>:

$\frac{7}{20} = 0.35$



Hence, the given rational number is a terminating decimal.

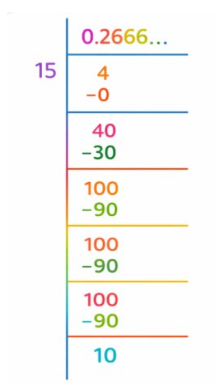
(ii) $\frac{4}{15}$

$15 = 3 \times 5$

Since the denominator has a prime factor 3 (other than 2 or 5), the decimal expansion is non-terminating repeating.

<u>Verification</u>:

$\frac{4}{15} = 0.2666... = 0.2\overline{6}$



Hence, the given rational number is a repeating decimal.

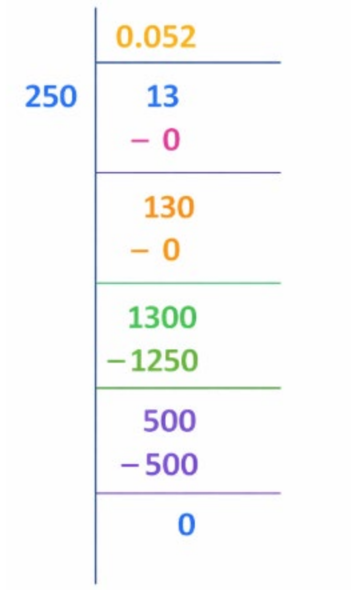
(iii) $\frac{13}{250}$

$250 = 2 \times 5^3$

Since the denominator has only the prime factors 2 and 5, the decimal expansion is terminating.

Verification:

$\frac{13}{250} = 0.052$



Hence, the given rational number is a terminating decimal.

2. Perform the long division for $\frac{1}{13}$. Identify the repeating block of digits. Does it show cyclic properties if you evaluate $\frac{2}{13}$? Now compute $\frac{3}{13}$, $\frac{4}{13}$, etc. What do you notice?

Sol:-

$\frac{1}{13} = 0.\overline{076923}$

Repeating block: 076923 (6 digits)

Now,

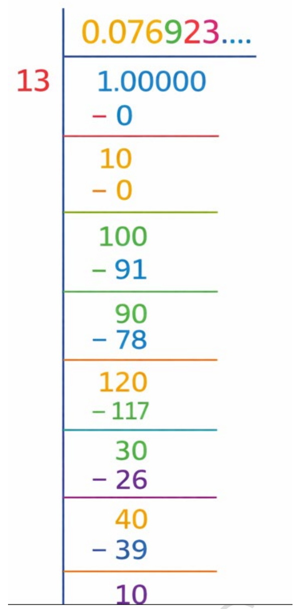
$\frac{2}{13} = 0.\overline{153846}$

$\frac{3}{13} = 0.\overline{230769}$

$\frac{4}{13} = 0.\overline{307692}$

$\frac{5}{13} = 0.\overline{384615}$

$\frac{6}{13} = 0.\overline{461538}$



Observations:

These decimal expansions show a cyclic property. The repeating digits are the same (0, 7, 6, 9, 2, 3) but appear in a different order in each case.

3. Classify the following numbers as rational or irrational:

(i) $\sqrt{81}$

(ii) $\sqrt{12}$

(iii) 0.33333 …

(iv) 0.123451234512345 …

(v) 1.01001000100001 … (Notice the pattern: Is it repeating a single block?)

Sol:-

(i) $\sqrt{81}$

$= \sqrt{9^2} = 9$

Since 9 is rational number.

$\therefore \sqrt{81}$ is rational.

(ii) $\sqrt{12}$

$= \sqrt{3 \times 4}$

$= \sqrt{3 \times 2^2} = 2\sqrt{3}$

Since $\sqrt{3}$ is irrational.

$2\sqrt{3}$ is also irrational.

$\therefore \sqrt{12}$ is irrational.

(iii) 0.33333 …
This is a non-terminating repeating decimal.
∴ it is rational.
(iv) 0.123451234512345 …
= $0.\overline{12345}$
This is a non-terminating repeating decimal.
∴ it is rational.
(v) 1.01001000100001 …
This is a non-terminating, non-repeating decimal.
∴ 1.01001000100001… is an irrational number.

4. The number 0.9 (which means 0.99999 …) is a rational number. Using algebra (let x = 0.9, multiply by 10, and subtract), explain why 0.9 is exactly equal to 1.

Sol:-
Let x = 0.9999… (1)
Multiply both sides by 10:
10x = 9.9999… (2)
Now subtract equation (1) from equation (2):
10x − x = 9.9999… – 0.9999…
9x = 9
x = 1
Since x = 0.9999…, we get:
0.9999… = 1

5. We have seen that the repeating block of $\frac{1}{7}$ is a cyclic number. Try to find more numbers (n) whose reciprocals ($\frac{1}{n}$) produce decimals with repeating blocks that are cyclic.

Sol:-
We know that
$\frac{1}{7} = 0.\overline{142857}$
It has a repeating block whose digits appear in cyclic order.
Similarly, we find other values of n such that $\frac{1}{n}$ gives a cyclic repeating decimal.
$\frac{1}{13} = 0.\overline{076923}$

$$\frac{1}{17} = 0.\overline{0588235294117647}$$
$$\frac{1}{19} = 0.\overline{052631578947368421}$$
$$\frac{1}{23} = 0.\overline{0434782608695652173913}$$

In each case, the digits in the repeating block are the same, and the decimal expansions of

$\frac{2}{n}, \frac{3}{n}, \ldots$ are obtained by cyclic shifts of these digits.

End Of Chapter Exercises

1. Convert the following rational numbers in the form of a terminating decimal or non-terminating and repeating decimal, whichever the case may be, by the process of long division:

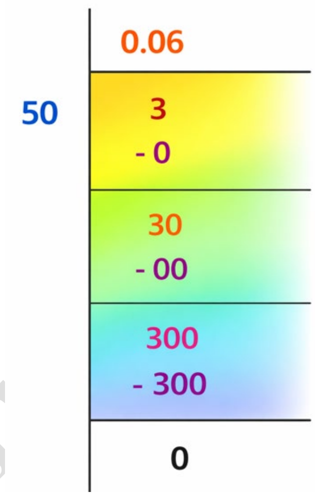
(i) $\frac{3}{50}$

(ii) $\frac{2}{9}$

Solution:

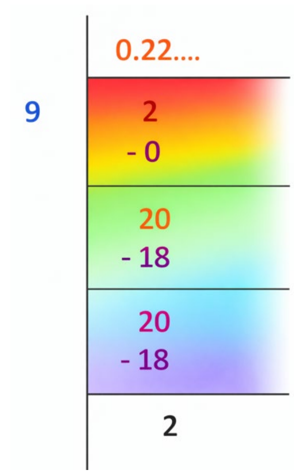
(i) $\frac{3}{50}$ = 0.06

This is a terminating decimal.



(ii) $\frac{2}{9}$ = 0.2222....

This in-terminating and repeating decimal.



2. Prove that $\sqrt{5}$ is an irrational number.

Sol:-

Assume that $\sqrt{5}$ is rational.

Then it can be written in the form

$\sqrt{5} = \frac{a}{b}$

where a and b are integers having no common factor other than 1.

Squaring both sides:

$5 = \frac{a^2}{b^2}$

$5b^2 = a^2$

This shows that a^2 is divisible by 5, so a is divisible by 5.

Let a = 5k, where k is an integer.

Substituting:

$5b^2 = (5k)^2$

$5b^2 = 25k^2$

$b^2 = 5k^2$

This shows that b^2 is divisible by 5, so b is also divisible by 5.

Thus, both a and b are divisible by 5, which contradicts the fact that a and b have no common factor other than 1.

Therefore, our assumption is wrong.

$\sqrt{5}$ is irrational.

3. Convert the following decimal numbers in the form of $\frac{p}{q}$.

(i) 12.6 (ii) 0.0120 (iii) $3.0\overline{52}$

(iv) $1.2\overline{35}$ (v) $0.\overline{23}$ (vi) $2.0\overline{5}$

(vii) $2.12\overline{5}$ (viii) $3.12\overline{5}$ (ix) $2.\overline{1625}$

Sol:-

(i) 12.6

$12.6 = \frac{126}{10} = \frac{63}{5}$

(ii) 0.0120

$0.0120 = \frac{120}{10000} = \frac{3}{250}$

(iii) $3.0\overline{52}$

Let $x = 3.0\overline{52}$ .... (1)

Multiply both sides by 100:

$100x = 305.2\overline{52}$ .... (2)

Now subtract equation (1) from equation (2):

$100x - x = 305.2\overline{52} - 3.0\overline{52}$

$99x = 302.2$

$99x = \frac{3022}{10}$

$x = \frac{3022}{990} = \frac{31511}{495}$

(iv)

$1.2\overline{35}$

Let $x = 1.2\overline{35}$........ (1)

Multiply both sides by 100:

$100x = 123.5\overline{35}$ .... (2)

Now subtract equation (1) from equation (2):

$100x - x = 123.5\overline{35} - 1.2\overline{35}$

$99x = 122.3$

$99x = \frac{1223}{10}$

$x = \frac{1223}{990}$

(v) $0.\overline{23}$

Let $x = 0.\overline{23}$........ (1)

Multiply both sides by 100:

$100x = 23.\overline{23}$ .... (2)

Now subtract equation (1) from equation (2):

$100x - x = 23.\overline{23} - 0.\overline{23}$

$99x = 23$

$x = \frac{23}{99}$

(vi) $2.0\overline{5}$

Let $x = 2.0\overline{5}$........ (1)

Multiply both sides by 10:

$10x = 20.5\overline{5}$ .... (2)

Now subtract equation (1) from equation (2):

$10x - x = 20.5\overline{5} - 2.0\overline{5}$

$9x = 18.5$

$9x = \frac{185}{10}$

$x = \frac{185}{90} = \frac{37}{18}$

(vii) $2.12\overline{5}$

Let $x = 2.12\overline{5}$........ (1)

Multiply both sides by 10:

$10x = 21.25\overline{5}$ .... (2)
Now subtract equation (1) from equation (2):

$10x - x = 21.25\overline{5} - 2.12\overline{5}$
$9x = 19.13$
$9x = \frac{1913}{100}$
$x = \frac{1913}{900}$

(viii) $3.12\overline{5}$

Let $x = 3.12\overline{5}$........ (1)
Multiply both sides by 10:

$10x = 31.25\overline{5}$ .... (2)
Now subtract equation (1) from equation (2):

$10x - x = 31.25\overline{5} - 3.12\overline{5}$
$9x = 28.13$
$9x = \frac{2813}{100}$
$x = \frac{2813}{900}$

(ix) $2.\overline{1625}$

Let $x = 2.\overline{1625}$.......... (1)
Multiply both sides by 10000:

$10000x = 21625.\overline{1625}$............ (2)
Now subtract equation (1) from equation (2):

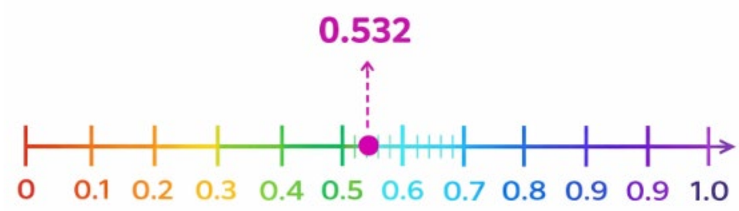
$1000x - x = 21625.\overline{1625} - 2.\overline{1625}$
$9999x = 21623$
$x = \frac{21623}{9999}$

4. Locate the following rational numbers on the number line.

(i) 0.532 (ii) $1.1\overline{5}$

Sol:-
(i) 0.532
Here, $0.532 = \frac{532}{1000}$
To locate on the number line:
Divide the segment from 0 to 1 into 10 equal parts. Each part represents 0.1.
Take the point 0.5.
Now divide the segment from 0.5 to 0.6 into 10 equal parts. Each part represents 0.01.

Take the point 0.53.
Now divide the segment from 0.53 to 0.54 into 10 equal parts. Each part represents 0.001.
Take the second division after 0.53.
So, we get the point 0.532 on the number line.

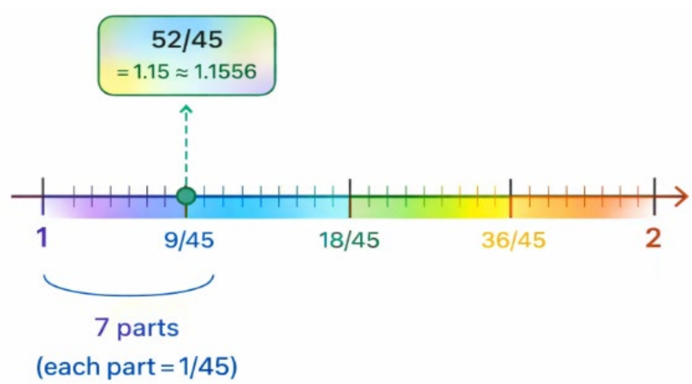


(ii) $1.1\bar{5}$

Let $x = 1.1\bar{5}$........ (1)
Multiply both sides by 10:

$10x = 11.5\bar{5}$ .... (2)
Now subtract equation (1) from equation (2):

$10x - x = 11.5\bar{5} - 1.1\bar{5}$
$9x = 10.4$
$9x = \frac{104}{10}$
$x = \frac{104}{90} = \frac{52}{45}$
To locate on the number line:
$\frac{52}{45} = \frac{45+7}{45} = 1 + \frac{7}{45}$
So it lies between 1 and 2.
Divide the segment from 1 to 2 into 45 equal parts.
Mark the point 7th division to the right of 1.
This point represents $1.1\bar{5}$ on the number line.



5. Find 6 rational numbers between 3 and 4.

Sol:-

Multiply the numerator and denominator by 7,

$3 = \frac{3\times7}{1\times7} = \frac{21}{7}, 4 = \frac{4\times7}{1\times7} = \frac{28}{7}$

Now six rational numbers between $\frac{21}{7}$ and $\frac{28}{7}$:

$\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}.$

So, six rational numbers between 3 and 4 are $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}.$

6. Find 5 rational numbers between $\frac{2}{5}$ and $\frac{3}{5}$.

Sol:-

Convert both fractions to a common denominator:

$\frac{2}{5} = \frac{2\times7}{5\times7} = \frac{14}{35}$ and $\frac{3}{5} = \frac{3\times7}{5\times7} = \frac{21}{35}$

Now pick five numbers between $\frac{14}{35}$ and $\frac{21}{35}$:

$\frac{15}{35}, \frac{16}{35}, \frac{17}{35}, \frac{18}{35}, \frac{19}{35}$

So, five rational numbers between $\frac{2}{5}$ and $\frac{3}{5}$ are

$\frac{15}{35}, \frac{16}{35}, \frac{17}{35}, \frac{18}{35}, \frac{19}{35}.$

7. Find 5 rational numbers between $\frac{1}{6}$ and $\frac{2}{5}$.

Sol:-

Convert both fractions to a common denominator:

$\frac{1}{6} = \frac{1\times5}{6\times5} = \frac{5}{30}$ and $\frac{2}{5} = \frac{2\times6}{5\times6} = \frac{12}{30}$

Now pick five numbers between $\frac{5}{30}$ and $\frac{12}{30}$:

$\frac{6}{30}, \frac{7}{35}, \frac{8}{30}, \frac{9}{30}, \frac{10}{30}$

So, five rational numbers between $\frac{1}{6}$ and $\frac{2}{5}$ are

$\frac{6}{30}, \frac{7}{35}, \frac{8}{30}, \frac{9}{30}, \frac{10}{30}.$

8. If $\frac{x}{3} + \frac{x}{5} = \frac{16}{15}$, find the rational number x.

Sol:-

$\frac{x}{3} + \frac{x}{5} = \frac{16}{15}$

LCM of 3 and 5 = 15

$\frac{x\times5}{3\times5} + \frac{x\times3}{5\times3} = \frac{16}{15}$

$$\frac{5x}{15} + \frac{3x}{15} = \frac{16}{15}$$
$$\frac{5x + 3x}{15} = \frac{16}{15}$$
$$\frac{8x}{15} = \frac{16}{15}$$
x = $\frac{16\times15}{15\times8}$

x = 2.

9. Let a and b be two non-zero rational numbers such that a + $\frac{1}{b}$ = 0. Without assigning any numerical values, determine whether ab is positive or negative. Justify your answer.

Sol:-

a + $\frac{1}{b}$ = 0

a =- $\frac{1}{b}$

ab = a·b =- $\frac{1}{b}$·b = −1

ab = −1

So, ab is negative.

10. A rational number has a terminating decimal expansion whose last non-zero digit occurs in the 4th decimal place. Show that such a number can be written in the form $\frac{p}{10^4}$, where p is an integer not divisible by 10. Is it necessary that the denominator of this rational number, when written in the lowest form, is divisible by 2^4 or 5^4? Give reasons.

Sol:-

Let the rational number be x. Since its decimal expansion terminates and the last non-zero digit is at the 4th decimal place, we can write

x = $0.a_1a_2a_3a_4$, where $a_4 \neq 0$

Multiplying by 10^4, we get

$10^4x = a_1a_2a_3a_4$ = p (an integer)

x = $\frac{a1a2a3a4}{10^4}$

Since $a_4 \neq 0$, p is not divisible by 10.

Now,

$10^4 = (2 \times 5)^4 = 2^4 \times 5^4$

When $\frac{p}{10^4}$ is written in lowest terms, common factors of 2 and/or 5 may cancel.

Therefore, it is necessary that the denominator in the lowest form is divisible by 2^4 or 5^4.

Alternate Solution-

Let the rational number be x.
Since its decimal expansion terminates and the last non-zero digit is at the 4th decimal place, it can be written as

$$x = \frac{p}{10^4}$$

where $p$is an integer.
If p were divisible by 10, then one zero would cancel with the denominator and the decimal would end before the 4th place, which is not possible. Hence, $p$is not divisible by 10.
Now,

$$10^4 = 10000 = 2^4 \times 5^4$$

When the fraction $\frac{p}{10000}$is reduced to lowest form, some common factors of 2 or 5 may cancel with p. Therefore, the denominator in lowest form will still be of the type $2^a \times 5^b$, but the powers may be less than 4.
Thus, it is not necessary

11. Without performing division, determine whether the decimal expansion of $\frac{18}{125}$ is terminating or non-terminating. If it terminates, state the number of decimal places.

Sol:-
Prime factorisation of the denominator:
$125 = 5 \times 5 \times 5 = 5^3$
Since the denominator has only the prime factor 5, the decimal expansion is terminating.
Further,
$\frac{18}{125} = \frac{18\times8}{125\times8} = \frac{144}{1000}$
So, the decimal expansion terminates and has 3 decimal places.

12. A rational number in its lowest form has a denominator $2^3 \times 5$. How many decimal places will its decimal expansion have? Explain your answer.

Sol:-
Given denominator = $2^3 \times 5 = 8 \times 5 = 40$.
Since the denominator has the prime factors 2 and 5, the decimal expansion

is terminating.

To find the number of decimal places, make the denominator a power of 10.

So multiply by 5^2:

$2^3 \times 5 \times 5^2 = 2^3 \times 5^3 = 10^3$

Thus, the denominator becomes 10^3.

Hence, the decimal expansion will terminate and have 3 decimal places.

13. Let a = $\frac{7}{12}$ and b = $\frac{5}{6}$. Express both a and b in the form $\frac{k1}{m}$ and $\frac{k2}{m}$ where k_1, k_2 and m are integers and $k_2 - k_1 > 6$. Using the same denominator *m*, write exactly five distinct rational numbers lying between a and b, keeping an integer numerator. Explain why the condition $k_2 - k_1 > n + 1$ is necessary to find n such rational numbers between the two rational numbers *a* and *b* using this method.

Solution:

Given, a = $\frac{7}{12}$ and b = $\frac{5}{6}$

Convert to a common denominator:

a = $\frac{7}{12} = \frac{7\times2}{12\times2} = \frac{14}{24}$, b = $\frac{5}{6} = \frac{5\times4}{6\times4} = \frac{20}{24}$

Here

$k_1 = 14$, $k_2 = 20$, m = 24

$k_2 - k_1 = 20 - 14 = 6$

So we need a larger common denominator. Scaling up to m = 48

a = $\frac{7}{12} = \frac{14}{24} = \frac{14\times2}{24\times2} = \frac{28}{48}$, b = $\frac{5}{6} = \frac{20}{24} = \frac{20\times2}{24\times2} = \frac{40}{48}$

Now

$k_1 = 28$, $k_2 = 40$, m = 48

$k_2 - k_1 = 40 - 28 = 12 > 6$

Numbers between 28 and 40 are 29, 30, 31, 32, 33.

So, the required rational numbers are $\frac{29}{48}, \frac{30}{48}, \frac{31}{48}, \frac{32}{48}, \frac{33}{48}$.

Explanation:

With a shared denominator, the only candidates are fractions with integer numerators strictly between k_1 and k_2. There are exactly $k_2 - k_1 - 1$ such integers. To fit at least n of them:

$$k_2 - k_1 - 1 \geq n \implies k_2 - k_1 > n + 1$$

14. Three rational numbers x, y, z satisfy x + y + z = 0 and xy + yz + zx = 0. Show that all the rational numbers x, y, z must be simultaneously zero.

Sol:-

Given that

x + y + z = 0 .... (1)

xy + yz + zx = 0 .... (2)
Squaring equation (1),
$(x + y + z)^2 = 0$
$x^2 + y^2 + z^2 + 2(xy + yz + zx) = 0$
Using equation (2)
$x^2 + y^2 + z^2 + 2(0) = 0$
$x^2 + y^2 + z^2 = 0$
Since x, y, z are rational numbers, each of x^2, y^2, z^2 is non-negative.
A sum of non-negative terms equals zero if and only if every term is zero:
$x^2 = 0$, $y^2 = 0$, $z^2 = 0$
$\therefore$ x = 0, y = 0, z = 0

15. Show that the rational number $\frac{(a+b)}{2}$ lies between the rational numbers a and b.

Sol:-
Let us assume a < b.
Since a < b, adding a to both sides:
a + a < a + b
2a < (a + b)
$a < \frac{(a+b)}{2}$
Again, since a < b, adding b to both sides:
a + b < b + b
(a + b) < 2b
$\frac{(a+b)}{2} < b$
Therefore,
$a < \frac{(a+b)}{2} < b$
Hence, $\frac{(a+b)}{2}$ lies between a and b.

16. Find the lengths of the hypotenuses of all the right triangles in Fig. 3.14 which is referred to as the square root spiral.

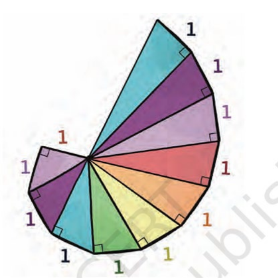


Fig 3. 14: Square root spiral

Sol.-

This is the square root spiral. Each triangle in the spiral is formed by taking one given leg and the previous hypotenuse as the other leg.

<u>1st triangle</u>:

Hypotenuse = $\sqrt{1^2 + 0^2}$= 1

<u>2nd triangle</u>:

Hypotenuse = $\sqrt{1^2 + 1^2}$= $\sqrt{2}$

<u>3rd triangle</u>:

Hypotenuse = $\sqrt{(\sqrt{2})^2 + 1^2}$ = $\sqrt{3}$

<u>4th triangle</u>:

Hypotenuse = $\sqrt{(\sqrt{3})^2 + 1^2}$ = $\sqrt{3 + 1}$= $\sqrt{2}$

Continuing this pattern, each time:

New hypotenuse = $\sqrt{n}$

Therefore, the hypotenuses are:

$$\sqrt{1},\ \sqrt{2},\ \sqrt{3},\ \sqrt{4},\ \sqrt{5},\ \sqrt{6},\ \sqrt{7},\ \sqrt{8},\ \sqrt{9}, \ldots$$