CHAPTER: 7 - The Mathematics of Maybe: Introduction to Probability

Exercise set 7.1 (Solution)

1. Rank the following events on a scale from 0 (Impossible) to 1 (Certain). Label each event: Impossible, less likely, equally likely (even chance), more likely, certain. Give reasons why you gave each event its ranking.

(i) The next Monday will come after Sunday.

(ii)It will snow in Mumbai in July.

(iii) An elephant will walk through your classroom today.

(iv) You will greet at least one friend at school tomorrow

Sol.- (i) The next Monday will come after Sunday

- ✓ Ranking: 1 (Certain)
- ✓ Label: Certain
- ✓ Reason: The calendar system is fixed—Sunday is always followed by Monday. This is a guaranteed event.

(ii) It will snow in Mumbai in July

- ✓ Ranking: 0 (Impossible)
- ✓ Label: Impossible
- ✓ Reason: Mumbai has a tropical climate. July is part of the monsoon season, with heavy rains but no snow. Snowfall in Mumbai is not climatically possible.

(iii) An elephant will walk through your classroom today

- ✓ Ranking: Very close to 0 (Impossible)
- ✓ Label: Impossible
- ✓ Reason: Classrooms are not places where elephants normally appear. While not logically impossible, it is practically impossible in everyday life.

(iv) You will greet at least one friend at school tomorrow

- ✓ Ranking: Around 0.8 (More likely) (It may very depends on person to person)
- ✓ Label: More likely
- ✓ Reason: If you regularly attend school and have friends there, greeting at least one is highly probable. It's not certain (you might be absent or not meet anyone), but it's very likely.

Summary Table-

Event	Probability	Label	Reason
Next Monday after Sunday	1	Certain	Fixed calendar sequence
Snow in Mumbai in July	0	Impossible	Tropical climate, monsoon season
Elephant in classroom today	~0	Impossible	Practically impossible in normal life
Greeting a friend at school tomorrow	~0.8	More likely	Daily routine makes it highly probable

Exercise set 7.2 (Solution)

1. A teacher mixes a large bag of sweets of different colours and randomly selects a sample of 30 sweets. She counts the number of sweets of each colour: 10 red sweets | 8 green sweets | 7 yellow sweets | 5 blue sweets

(i) Calculate the probability that a randomly picked sweet from the sample is green.

(ii) If there are 600 sweets in total in the large bag, estimate how many are likely to be yellow, based on the sample results.

Sol.- Total number of sweets in the sample: 10 (red) + 8 (green) + 7 (yellow) + 5 (blue) = 30

(i) Probability that a randomly picked sweet is green

$$P(\text{green}) = \frac{\text{Number of green sweets}}{\text{Total sweets in sample}}$$

$$P(\text{green}) = \frac{8}{30} = \frac{4}{15}$$

So, the probability is 4/15.

(ii) Estimate of yellow sweets in the large bag (600 sweets total)

The proportion of yellow sweets in the sample is:

$$\frac{7}{30}$$

Expected number of yellow sweets in the large bag:

$$\frac{7}{30} \times 600 = 140$$

So, there are likely 140 yellow sweets in the large bag.

2. A survey is conducted at a school where a random sample of 40 students is asked about their favourite club. The responses are: 14 students: Science Club | 11 students: Arts Club | 9 students: Sports Club | 6 students: Debate Club Assume there are 800 students in the whole school.

(i) What is the probability that a randomly chosen student from the sample prefers the Arts Club?

(ii) Using the sample results, estimate how many students in the whole school are likely to prefer the Sports Club.

Sol.-

Sample Data (40 students)

Science Club = 14

Arts Club = 11

Sports Club = 9

Debate Club = 6

Total = 40

(i) Probability that a randomly chosen student prefers the Arts Club

$$P(\text{Arts Club}) = \frac{\text{Number of Arts Club students}}{\text{Total students in sample}}$$

$$P(\text{Arts Club}) = \frac{11}{40}$$

So, the probability is 11/40.

(ii) Estimate of students in the whole school (800) who prefer the Sports Club

The proportion of Sports Club students in the sample is:

$$\frac{9}{40}$$

Expected number in the whole school:

$$\frac{9}{40} \times 800 = 180$$

So, about 180 students in the school are likely to prefer the Sports Club.

3. Toss a coin 20 times and record the result each time (heads or tails).
(i) How many times did you get heads?
(ii) How many times did you get tails?
(iii) Calculate the experimental probability of getting heads.
(iv) If you toss the coin once more, what is the probability of getting tails?

Sol.-
(i) How many times did you get heads?
This depends on the actual experiment. Suppose in 20 tosses you recorded 12 heads.

(ii) How many times did you get tails?

Since there are 20 tosses in total:

$$\text{Tails} = 20 - \text{Heads} = 20 - 12 = 8$$

So, 8 tails.

(iii) Experimental probability of getting heads

$$P(\text{heads}) = \frac{\text{Number of heads}}{\text{Total tosses}}$$

$$P(\text{heads}) = \frac{12}{20} = \frac{3}{5} = 0.6$$

So, the experimental probability of heads is 0.6.

(iv) Probability of getting tails in one more toss

This is a theoretical probability, since each toss is independent.

$$P(\text{tails}) = \frac{1}{2} = 0.5$$

So, the probability is 0.5.

4. Toss a paper cup into the air 100 times. After each toss record whether the cup lands on its bottom, upside down on its top or on its side (See Fig. 7.5). Assign probabilities to the outcomes by using experimental probability.

Sol.-

Step 1: Conduct the experiment

Toss the paper cup 100 times.

Record each outcome:

- ✓ Lands on bottom (upright)
- ✓ Lands upside down (on its top)
- ✓ Lands on its side

Step 2: Count the frequencies

Suppose after 100 tosses you observed:

- ✓ Bottom = 35 times
- ✓ Upside down = 25 times
- ✓ Side = 40 times

Step 3: Calculate experimental probabilities

Experimental probability is given by:

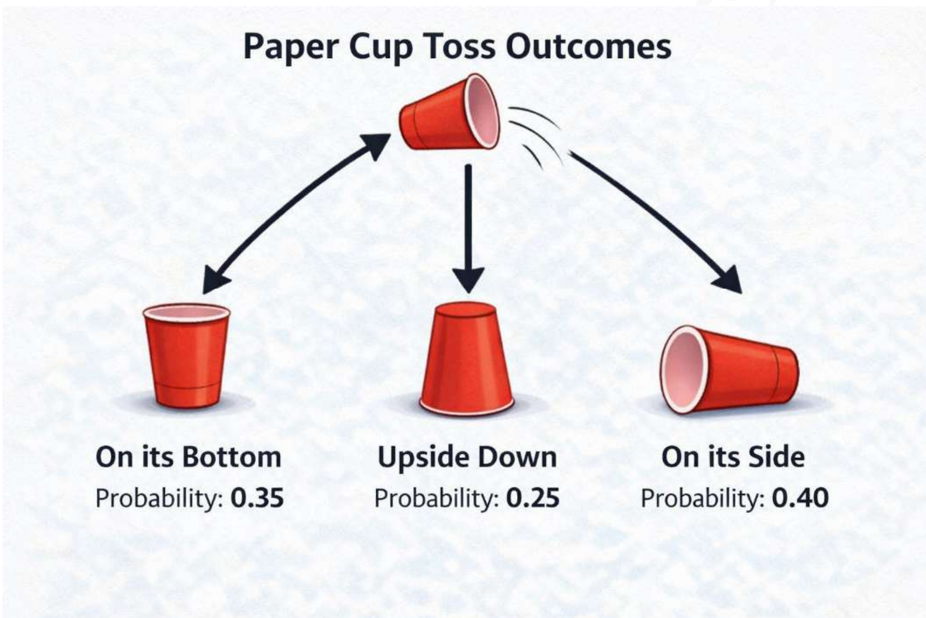
$$P(\text{event}) = \frac{\text{Number of times event occurs}}{\text{Total number of trials}}$$

So:

- ✓ $P(\text{bottom}) = \frac{35}{100} = 0.35$
- ✓ $P(\text{upside down}) = \frac{25}{100} = 0.25$
- ✓ $P(\text{side}) = \frac{40}{100} = 0.40$

Step 4: Interpret

- ✓ These probabilities are based on actual experimental data, not theory.
- ✓ If you repeat the experiment, the numbers may change slightly, but they should stay close to these proportions.
- ✓ The sum of all probabilities = 1 (since one of the outcomes must occur each toss).



5. What is the probability of getting an even number when rolling a fair 6-sided die?

Sol.-
A fair 6-sided die has 6 outcomes: $\{1, 2, 3, 4, 5, 6\}$. The even numbers are $\{2, 4, 6\}$, so there are 3 favorable outcomes.

$$P(\text{even number}) = \frac{3}{6} = \frac{1}{2}$$

So, the probability of rolling an even number is $\frac{1}{2}$or 0.5.

6. Suppose you roll a 6-sided die 12 times and get a '3' three times.
   (i) What is the experimental probability of rolling a '3'?
   (ii) What is the theoretical probability of rolling a '3'?

(iii) Why might these probabilities be different? What would you expect to happen if you roll the die 60, 600, or 6000 times?

Sol.-

(i) Experimental probability of rolling a '3'

You rolled the die 12 times and got a '3' three times.

$$P_{\text{exp}}(3) = \frac{\text{Number of times 3 occurs}}{\text{Total rolls}} = \frac{3}{12} = \frac{1}{4} = 0.25$$

So, the experimental probability is 0.25.

(ii) Theoretical probability of rolling a '3'

On a fair 6-sided die, each face is equally likely.

$$P_{\text{theory}}(3) = \frac{1}{6} \approx 0.1667$$

So, the theoretical probability is $\frac{1}{6}$.

(iii) Why might these probabilities be different? What would you expect to happen if youroll the die 60, 600, or 6000 times?

- ✓ Experimental probability depends on actual outcomes in a limited number of trials.
- ✓ Theoretical probability is based on the assumption of perfect fairness and infinite trials.
- ✓ With only 12 rolls, random variation (chance) can cause differences.
- ✓ As the number of rolls increases (say 60, 600, or 6000), the Law of Large Numbers tells us that the experimental probability will get closer to the theoretical probability.
- ✓ So, with 6000 rolls, you would expect the proportion of '3's to be very close to $\frac{1}{6}$.