CHAPTER: 5 - I’m Up and Down, and Round and Round

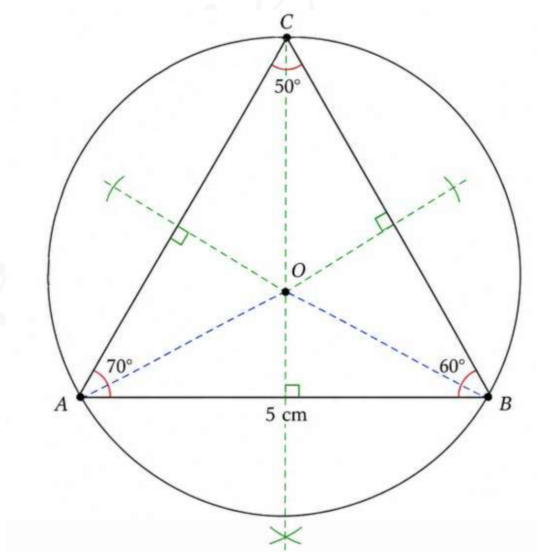
Exercise set 5.1 (Solution)

1. Draw ΔABC with AB = 5 cm, ∠A = 70° and ∠B = 60°. Draw the circumcircle of ΔABC. Is the centre inside or outside the triangle?

Sol.

Construction

- Draw a line segment AB = 5 cm.
- At A, construct an angle ∠BAC = 70°.
- At B, construct an angle ∠ABC = 60° on the same side of AB.
- Thus, ΔABC is constructed.
- Draw the perpendicular bisector of AB.
- Draw the perpendicular bisector of AC.
- Let these perpendicular bisectors meet at O.
  So, O is the circumcentre.
- With O as centre and OA as radius, draw a circle.



This circle passes through A, B, and C, so it is the circumcircle of ΔABC.

Result

The circumcentre O lies inside ΔABC, because ΔABC is an acute-angled triangle.

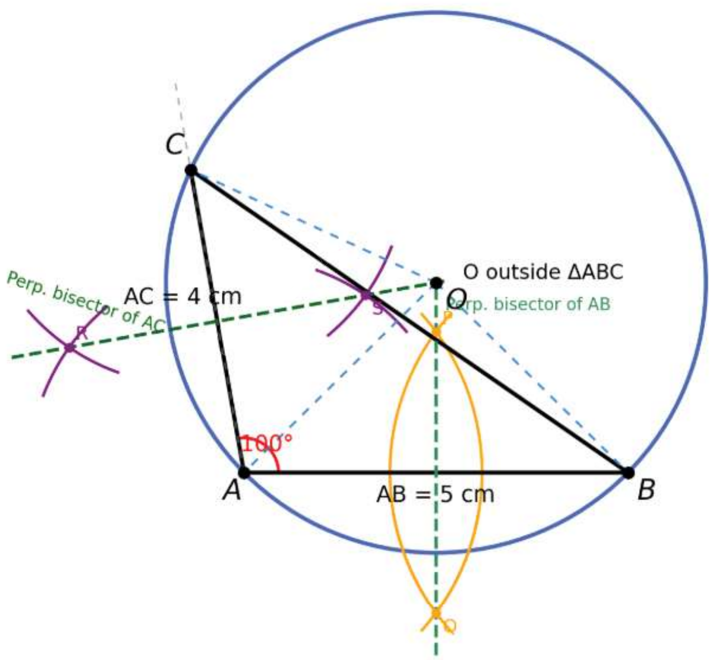
2. Draw ΔABC with AB = 5 cm, ∠A = 100°, AC = 4 cm. Draw the circumcircle of ΔABC. Is the centre inside or outside the triangle?

Sol.

Steps of Construction

- Draw **AB = 5 cm**.
- At point **A**, construct **∠BAX = 100°**.
- On ray **AX**, cut **AC = 4 cm**.
- Join **BC**.
  Thus, **ΔABC** is formed.
- Draw the **perpendicular bisector of AB**.
- Draw the **perpendicular bisector of AC**.
- Let both perpendicular bisectors meet at **O**.
  So, **O** is the circumcentre.
- With **O** as centre and **OA** as radius, draw a circle.

This circle passes through **A, B, and C**, so it is the **circumcircle**.



Result

Since **∠A = 100° > 90°**, ΔABC is obtuse-angled.

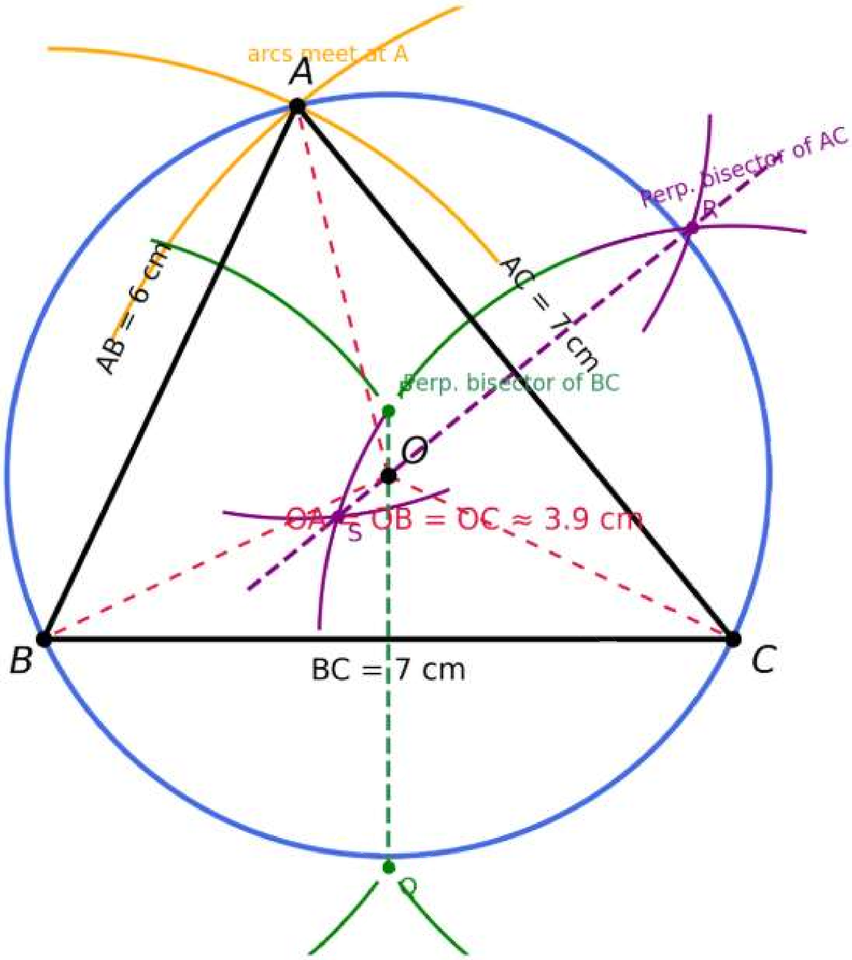
Therefore, the circumcentre lies **outside the triangle**.

3. Draw ΔABC, with AB = 6 cm, BC = 7 cm and CA = 7 cm. Draw the circumcircle of ΔABC. Let the circumcentre be O. Measure OA, OB, OC.

Sol.

Construction of the Diagram

- Draw a line segment **BC = 7 cm**.
- With **B** as centre and radius **6 cm**, draw an arc above BC.
- With **C** as centre and radius **7 cm**, draw another arc cutting the previous arc at **A**.
- Join **AB** and **AC**.
  Thus, **ΔABC** is formed.
- To draw the perpendicular bisector of **BC**:
  With **B** and **C** as centres and equal radius greater than half of BC, draw arcs above and below BC intersecting at points **P** and **Q**.
  Join **PQ**.
- To draw the perpendicular bisector of **AC**:
  With **A** and **C** as centres and equal radius greater than half of AC, draw arcs intersecting at points **R** and **S**.
  Join **RS**.
- Let **PQ** and **RS** intersect at **O**.
  Then **O** is the circumcentre.
- With **O** as centre and radius **OA**, draw a circle passing through **A, B, and C**.
- Measure **OA, OB, and OC**.



Observation

$$OA = OB = OC$$

because all are radii of the circumcircle.

4. What is the least possible radius of a circle through two points A and B?

Sol.
Let the two points be **A** and **B**.
To draw a circle through A and B, the centre of the circle must lie on the **perpendicular bisector of AB**.
The radius will be minimum when the centre is exactly at the midpoint of AB.
In this case,

$$\text{Radius} = \frac{AB}{2}$$

Hence, the **least possible radius** of a circle through two points A and B is:

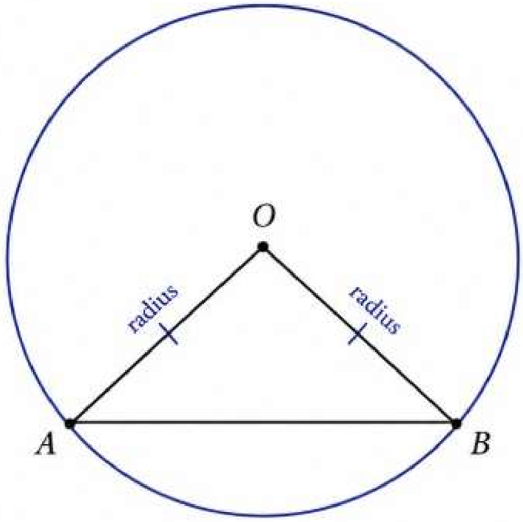
$$\boxed{\frac{AB}{2}}$$

This occurs when **AB is the diameter of the circle**.

Exercise set 5.2 (Solution)

1. Show that the triangle formed by a chord and the centre of the circle is isosceles.

Sol.



Let **AB** be a chord of a circle with centre **O**.

Join **OA** and **OB**.

Since **OA** and **OB** are radii of the same circle,

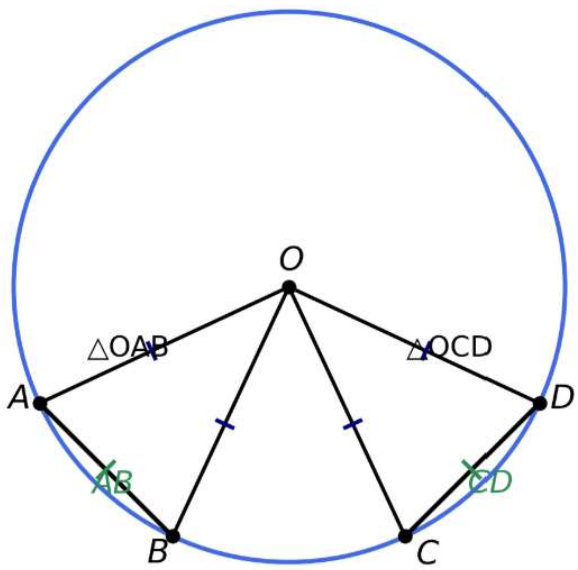
$$OA = OB$$

Therefore, in **ΔOAB**, two sides are equal.

Hence, **ΔOAB is an isosceles triangle**.

2. Show that if two such isosceles triangles (occurring in the previous question) have equal base length, they are congruent to each other.

Sol.



Proof

Let **AB** and **CD** be two equal chords of a circle with centre **O**.

Join **OA, OB, OC, OD**.

In △**OAB** and △**OCD**:

$$OA = OC$$

because both are radii of the same circle.

$$OB = OD$$

because both are radii of the same circle.

Also,

$$AB = CD$$

given, equal base lengths.

Therefore,

$$\triangle OAB \cong \triangle OCD$$

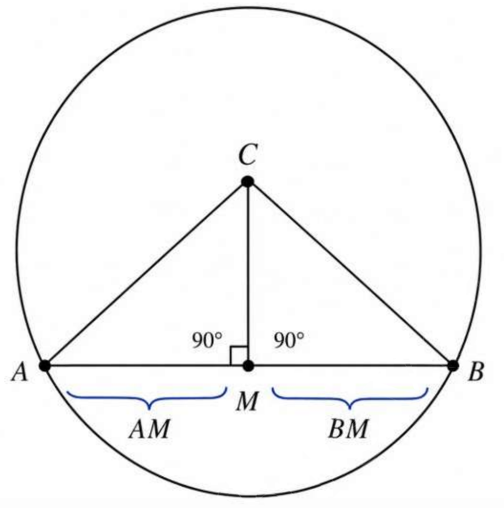
by **SSS congruence criterion**.

Hence proved.

Exercise set 5.2 (Solution)

1. Can you explain why the converse to Theorem 4 is true, i.e., why does the perpendicular from the centre of a circle to a chord of the circle bisect the chord? (Hint: Use Fig. 5.12. You are told that ∠CMA = ∠CMB = 90°. You need to show that AM = BM.)

Sol.



Let **C** be the centre of the circle and **AB** be a chord.

A perpendicular from **C** meets chord **AB** at **M**.

So,

$$\angle CMA = \angle CMB = 90^\circ$$

Now, in triangles **△CMA** and **△CMB**:

$$CA = CB$$

because both are radii of the same circle.

$$CM = CM$$

common side.

$$\angle CMA = \angle CMB$$

each is 90°.

Therefore,

$$\triangle CMA \cong \triangle CMB$$

by **RHS congruence criterion**.

Hence,

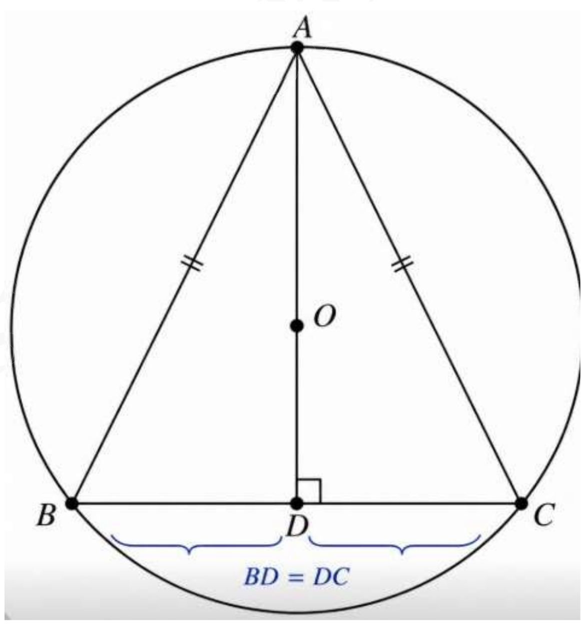
$$AM = BM$$

So, **M is the midpoint of AB**.

Therefore, the perpendicular from the centre of a circle to a chord **bisects the chord**.

2. An isosceles triangle ABC is inscribed in a circle, with AB = AC. Show that the altitude from A to BC passes through the centre of the circle.

Sol.



Proof

Given: **AB = AC** and triangle **ABC** is inscribed in a circle.

Let **AD** be the altitude from **A** to **BC**.

So,

$$AD \perp BC$$

In an isosceles triangle, the altitude from the vertex also bisects the base.

Therefore,

$$BD = DC$$

So, **D is the midpoint of chord BC**.

Now, **AD** is perpendicular to chord **BC** and passes through its midpoint **D**.

Hence, **AD is the perpendicular bisector of chord BC**.

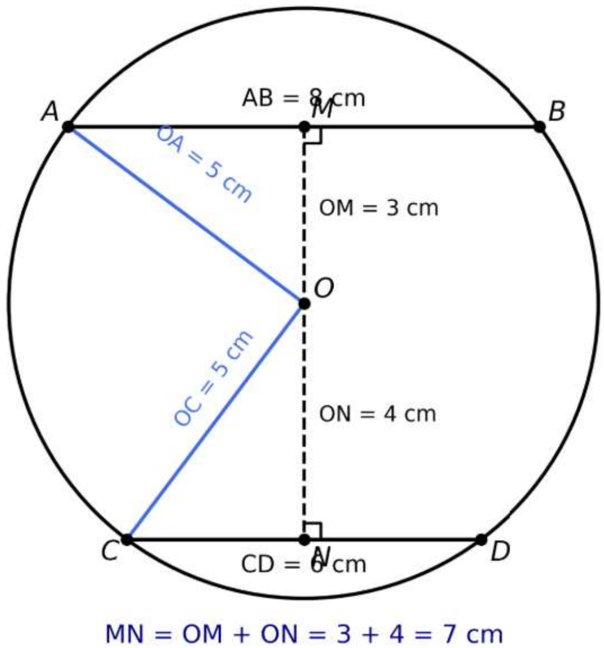
The perpendicular bisector of a chord always passes through the centre of the circle.

Therefore, **AD passes through the centre O**.

Hence proved.

3. Two parallel chords of lengths 6 cm and 8 cm are on opposite sides of the centre of a circle. If the radius of the circle is 5 cm, find the distance between the midpoints of the chords.

Sol.



Let the two parallel chords be **AB = 8 cm** and **CD = 6 cm**.

Radius of the circle:

$$r = 5\text{ cm}$$

Let **O** be the centre of the circle.

Draw perpendiculars from **O** to the chords meeting them at **M** and **N** respectively. Since the perpendicular from the centre bisects the chord,

$$AM = \frac{8}{2} = 4\text{ cm}$$

and

$$CN = \frac{6}{2} = 3\text{ cm}$$

For chord AB:

In right triangle $\triangle OMA$,

$$OM^2 + 4^2 = 5^2$$

$$OM^2 = 25 - 16 = 9$$
$$OM = 3 \text{ cm}$$

For chord CD:

In right triangle $\triangle ONC$,

$$ON^2 + 3^2 = 5^2$$

$$ON^2 = 25 - 9 = 16$$
$$ON = 4 \text{ cm}$$

The chords are on opposite sides of the centre, so the distance between their midpoints is:

$$MN = OM + ON$$
$$MN = 3 + 4 = 7 \text{ cm}$$