CHAPTER: 4 - Exploring Algebraic Identities

Exercise set 4.1 (Solution)

1. Using the identity $(a+b)^2 = a^2 + 2ab + b^2$ expand the following:

(i) $(7x+4y)^2$

(ii) $\left(\frac{7}{5}x+\frac{3}{2}y\right)^2$

(iii) $(2.5p+1.5q)^2$

(iv) Expand $\left(\frac{3}{4}s+8t\right)^2$

(v) Expand $\left(x+\frac{1}{2y}\right)^2$

(vi) Expand $\left(\frac{1}{x}+\frac{1}{y}\right)^2$

Sol.- (i) $(7x+4y)^2$

Here,

$a = 7x$ and $b = 4y$

Using the identity:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(7x+4y)^2 = (7x)^2 + 2(7x)(4y) + (4y)^2$$
$$= 49x^2 + 56xy + 16y^2$$

Therefore,

$$(7x+4y)^2 = 49x^2 + 56xy + 16y^2$$

(ii) Expand $\left(\frac{7}{5}x+\frac{3}{2}y\right)^2$

Here,

$$a = \frac{7}{5}x, b = \frac{3}{2}y$$

Using the identity:

$$\left(\frac{7}{5}x+\frac{3}{2}y\right)^2 = \left(\frac{7}{5}x\right)^2 + 2\left(\frac{7}{5}x\right)\left(\frac{3}{2}y\right) + \left(\frac{3}{2}y\right)^2$$
$$= \frac{49}{25}x^2 + \frac{21}{5}xy + \frac{9}{4}y^2$$

Therefore,

$$\left(\frac{7}{5}x+\frac{3}{2}y\right)^2 = \frac{49}{25}x^2 + \frac{21}{5}xy + \frac{9}{4}y^2$$

(iii) Expand $(2.5p+1.5q)^2$

Here,

$$a = 2.5p, b = 1.5q$$

Using the identity:

$$(2.5p + 1.5q)^2 = (2.5p)^2 + 2(2.5p)(1.5q) + (1.5q)^2$$
$$= 6.25p^2 + 7.5pq + 2.25q^2$$

Therefore,

$$(2.5p + 1.5q)^2 = 6.25p^2 + 7.5pq + 2.25q^2$$

(iv) Expand $\left(\frac{3}{4}s+8t\right)^2$

Here,

$$a = \frac{3}{4}s, b = 8t$$

Using the identity

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$\left(\frac{3}{4}s+8t\right)^2 = \left(\frac{3}{4}s\right)^2 + 2\left(\frac{3}{4}s\right)(8t) + (8t)^2$$
$$= \frac{9}{16}s^2 + 12st + 64t^2$$

Therefore,

$$\left(\frac{3}{4}s+8t\right)^2 = \frac{9}{16}s^2 + 12st + 64t^2$$

(v) Expand $\left(x+\frac{1}{2y}\right)^2$

Here,

$$a = x, b = \frac{1}{2y}$$

Using the identity:

$$\left(x+\frac{1}{2y}\right)^2 = x^2 + 2(x)\left(\frac{1}{2y}\right) + \left(\frac{1}{2y}\right)^2$$
$$= x^2 + \frac{x}{y} + \frac{1}{4y^2}$$

Therefore,

$$\left(x+\frac{1}{2y}\right)^2 = x^2 + \frac{x}{y} + \frac{1}{4y^2}$$

(vi) Expand $\left(\frac{1}{x}+\frac{1}{y}\right)^2$

Here,

$$a = \frac{1}{x}, b = \frac{1}{y}$$

Using the identity:

$$\left(\frac{1}{x}+\frac{1}{y}\right)^2 = \left(\frac{1}{x}\right)^2 + 2\left(\frac{1}{x}\right)\left(\frac{1}{y}\right) + \left(\frac{1}{y}\right)^2$$
$$= \frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2}$$

Therefore,

$$\left(\frac{1}{x}+\frac{1}{y}\right)^2 = \frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2}$$

2. Using the same identity, find the values of the following:
   (i) $(64)^2$
   (ii) $(105)^2$
   (iii) $(205)^2$

   Sol.-

   (i) Find $(64)^2$

$$64 = 60 + 4$$

Using the identity:

$$(64)^2 = (60+4)^2$$
$$= (60)^2 + 2(60)(4) + (4)^2$$
$$= 3600 + 480 + 16$$
$$= 4096$$

Therefore,

$$(64)^2 = 4096$$

(ii) Find $(105)^2$

$$105 = 100 + 5$$

Using the identity:

$$(105)^2 = (100 + 5)^2$$
$$= (100)^2 + 2(100)(5) + (5)^2$$
$$= 10000 + 1000 + 25$$
$$= 11025$$

Therefore,

$$(105)^2 = 11025$$

(iii) Find $(205)^2$

$$205 = 200 + 5$$

Using the identity:

$$(205)^2 = (200 + 5)^2$$
$$= (200)^2 + 2(200)(5) + (5)^2$$
$$= 40000 + 2000 + 25$$
$$= 42025$$

Therefore,

$$(205)^2 = 42025$$

Exercise set 4.2 (Solution)

1. Factor completely:
   (i) $9x^2 + 24xy + 16y^2$
   (ii) $4s^2 + 20st + 25t^2$
   (iii) $49x^2 + 28xy + 4y^2$
   (iv) $64p^2 + \frac{32}{3}pq + \frac{4}{9}q^2$
   (v) $3a^2 + 4ab + \frac{4}{3}b^2$
   (vi) $\frac{9}{5}s^2 + 6sv + 5v^2$

Sol.-

(i) Factor $9x^2 + 24xy + 16y^2$
We observe that

$$9x^2 = (3x)^2$$
$$16y^2 = (4y)^2$$

and

$$24xy = 2(3x)(4y)$$

So, the expression is of the form

$$a^2 + 2ab + b^2 = (a + b)^2$$

Therefore,

$$9x^2 + 24xy + 16y^2 = (3x + 4y)^2$$

(ii) Factor $4s^2 + 20st + 25t^2$
We observe that

$$4s^2 = (2s)^2$$
$$25t^2 = (5t)^2$$

and

$$20st = 2(2s)(5t)$$

So, the expression is of the form

$$a^2 + 2ab + b^2 = (a + b)^2$$

Therefore,

$$4s^2 + 20st + 25t^2 = (2s + 5t)^2$$

(iii) Factor $49x^2 + 28xy + 4y^2$

We observe that

$$49x^2 = (7x)^2$$
$$4y^2 = (2y)^2$$

and

$$28xy = 2(7x)(2y)$$

So, the expression is of the form

$$a^2 + 2ab + b^2 = (a+b)^2$$

Therefore,

$$49x^2 + 28xy + 4y^2 = (7x + 2y)^2$$

(iv) Factor $64p^2 + \frac{32}{3}pq + \frac{4}{9}q^2$

We observe that

$$64p^2 = (8p)^2$$
$$\frac{4}{9}q^2 = \left(\frac{2}{3}q\right)^2$$

and

$$\frac{32}{3}pq = 2(8p)\left(\frac{2}{3}q\right)$$

So, the expression is of the form

$$a^2 + 2ab + b^2 = (a+b)^2$$

Therefore,

$$64p^2 + \frac{32}{3}pq + \frac{4}{9}q^2 = \left(8p + \frac{2}{3}q\right)^2$$

(v) Factor $3a^2 + 4ab + \frac{4}{3}b^2$

We observe that

$$3a^2 = (\sqrt{3}a)^2$$

$$\frac{4}{3}b^2 = \left(\frac{2}{\sqrt{3}}b\right)^2$$

and

$$4ab = 2(\sqrt{3}a)\left(\frac{2}{\sqrt{3}}b\right)$$

So, the expression is of the form

$$a^2 + 2ab + b^2 = (a+b)^2$$

Therefore,

$$3a^2 + 4ab + \frac{4}{3}b^2 = \left(\sqrt{3}a + \frac{2}{\sqrt{3}}b\right)^2$$

(vi) Factor $\frac{9}{5}s^2 + 6sv + 5v^2$

We observe that

$$\frac{9}{5}s^2 = \left(\frac{3}{\sqrt{5}}s\right)^2$$
$$5v^2 = (\sqrt{5}v)^2$$

and

$$6sv = 2\left(\frac{3}{\sqrt{5}}s\right)(\sqrt{5}v)$$

So, the expression is of the form

$$a^2 + 2ab + b^2 = (a+b)^2$$

Therefore,

$$\frac{9}{5}s^2 + 6sv + 5v^2 = \left(\frac{3}{\sqrt{5}}s + \sqrt{5}v\right)^2$$

2. Find the values of the following using the identity $(a-b)^2 = a^2 - 2ab + b^2$.

(i) $(79)^2$(ii) $(193)^2$(iii) $(299)^2$

Sol:-

(i) $(79)^2$Take $a = 80, b = 1$.

$$79^2 = (80-1)^2$$
$$= 80^2 - 2 \cdot 80 \cdot 1 + 1^2$$
$$= 6400 - 160 + 1$$
$$= 6241$$

(ii) $(193)^2$Take $a = 200, b = 7$.

$$193^2 = (200-7)^2$$
$$= 200^2 - 2\cdot 200\cdot 7 + 7^2$$
$$= 40000 - 2800 + 49$$
$$= 37249$$

(iii) $(299)^2$Take $a = 300$, $b = 1$.

$$299^2 = (300-1)^2$$
$$= 300^2 - 2\cdot 300\cdot 1 + 1^2$$
$$= 90000 - 600 + 1$$
$$= 89401$$

Exercise set 4.3 (Solution)

1: Find the following squares using one of the above identities. Determine which of these identities will make these calculations easier.

(i) 117^2 (ii) 78^2 (iii) 198^2 (iv) 214^2 (v) 1104^2 (vi) 1120^2

Sol:

(i) 117^2 Take $a = 120,\ b = 3.$

$$\begin{aligned} 117^2 &= (120 - 3)^2 \\ &= 120^2 - 2 \cdot 120 \cdot 3 + 3^2 \\ &= 14400 - 720 + 9 \\ &= 13689 \end{aligned}$$

(ii) 78^2 Take $a = 80,\ b = 2.$

$$\begin{aligned} 78^2 &= (80 - 2)^2 \\ &= 80^2 - 2 \cdot 80 \cdot 2 + 2^2 \\ &= 6400 - 320 + 4 \\ &= 6084 \end{aligned}$$

(iii) 198^2 Take $a = 200,\ b = 2.$

$$\begin{aligned} 198^2 &= (200 - 2)^2 \\ &= 200^2 - 2 \cdot 200 \cdot 2 + 2^2 \\ &= 40000 - 800 + 4 \\ &= 39204 \end{aligned}$$

(iv) 214^2 Take $a = 210,\ b = 4.$

$$\begin{aligned} 214^2 &= (210 + 4)^2 \\ &= 210^2 + 2 \cdot 210 \cdot 4 + 4^2 \\ &= 44100 + 1680 + 16 \\ &= 45796 \end{aligned}$$

(v) 1104^2 Take $a = 1100,\ b = 4.$

$$\begin{aligned} 1104^2 &= (1100 + 4)^2 \\ &= 1100^2 + 2 \cdot 1100 \cdot 4 + 4^2 \\ &= 1210000 + 8800 + 16 \\ &= 1218816 \end{aligned}$$

(vi) 1120^2Take $a = 1100,\ b = 20.$

$$\begin{aligned} 1120^2 &= (1100 + 20)^2 \\ &= 1100^2 + 2 \cdot 1100 \cdot 20 + 20^2 \\ &= 1210000 + 44000 + 400 \end{aligned}$$

2. **Factor using suitable identities:**

(i) $16y^2 - 24y + 9$

(ii) $\frac{9}{4}s^2 + 6st + 4t^2$

(iii) $\frac{m^2}{9} + \frac{mk}{3} + \frac{k^2}{4} + 3nk + 2mn + 9n^2$

(iv) $\frac{p^2}{16} - 2 + \frac{16}{p^2}$

(v) $9a^2 + 4b^2 + c^2 - 12ab + 6ac - 4bc$

Sol.

(i) $\mathbf{16y^2 - 24y + 9}$

$$16y^2 - 24y + 9 = (4y)^2 - 2 \cdot (4y)(3) + 3^2$$

Identity: $(x-y)^2 = x^2 - 2xy + y^2$.

$= (4y-3)^2 = (4y-3)(4y-3)$

(ii) $\mathbf{\frac{9}{4}s^2 + 6st + 4t^2}$

$$\frac{9}{4}s^2 + 6st + 4t^2 = \left(\frac{3}{2}s\right)^2 + 2 \cdot \frac{3}{2}s \cdot 2t + (2t)^2$$

Identity: $(x+y)^2$.

$= \left(\frac{3}{2}s+2t\right)^2 = \left(\frac{3}{2}s+2t\right)\left(\frac{3}{2}s+2t\right)$

(iii) $\mathbf{\frac{m^2}{9} + \frac{mk}{3} + \frac{k^2}{4} + 3nk + 2mn + 9n^2}$

$$= \left(\frac{m}{3}\right)^2 + 2 \cdot \frac{m}{3} \cdot \frac{k}{2} + \left(\frac{k}{2}\right)^2 + 2 \cdot \frac{m}{3} \cdot 3n + 2 \cdot \frac{k}{2} \cdot 3n + (3n)^2$$

Identity: $(x+y+z)^2$.

$= \left(\frac{m}{3}+\frac{k}{2}+3n\right)^2 = \left(\frac{m}{3}+\frac{k}{2}+3n\right)\left(\frac{m}{3}+\frac{k}{2}+3n\right)$

(iv) $\mathbf{\frac{p^2}{16} - 2 + \frac{16}{p^2}}$

$$= \left(\frac{p}{4}\right)^2 + \left(\frac{4}{p}\right)^2 - 2 \cdot \frac{p}{4} \cdot \frac{4}{p}$$

Identity: $(x-y)^2$.

$= \left(\frac{p}{4}-\frac{4}{p}\right)^2 = \left(\frac{p}{4}-\frac{4}{p}\right)\left(\frac{p}{4}-\frac{4}{p}\right)$

(v) $\mathbf{9a^2 + 4b^2 + c^2 - 12ab + 6ac - 4bc}$

$$= (3a)^2 + (-2b)^2 + c^2 + 2 \cdot (3a)(-2b) + 2 \cdot (3a)(c) + 2 \cdot (-2b)(c)$$

Identity: $(x+y+z)^2$.

$= (3a - 2b + c)^2 = (3a - 2b + c)(3a - 2b + c)$

3. **Expand the following using the identity**

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(i) $(p + 3q + 7r)^2$

(ii) $(3x - 2y + 4z)^2$

Sol.

(i) $(\boldsymbol{p + 3q + 7r})^2$

Identity: $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$(p + 3q + 7r)^2 = p^2 + (3q)^2 + (7r)^2 + 2(p \cdot 3q) + 2(3q \cdot 7r) + 2(7r \cdot p)$$
$$= p^2 + 9q^2 + 49r^2 + 6pq + 42qr + 14pr$$

(ii) $(\boldsymbol{3x - 2y + 4z})^{\boldsymbol{2}}$

Identity: $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$(3x - 2y + 4z)^2 = (3x)^2 + (-2y)^2 + (4z)^2 + 2(3x \cdot -2y) + 2(-2y \cdot 4z) + 2(4z \cdot 3x)$$
$$= 9x^2 + 4y^2 + 16z^2 - 12xy - 16yz + 24xz$$

4. **Is this an identity?**

$$(a + b - c)^2 + (a - b + c)^2 + (a - b - c)^2 = 2a^2 + 2b^2 + 2c^2$$

Sol.

Expand each term:

$$(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2ac - 2bc$$
$$(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2ac - 2bc$$
$$(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$$

Now add them:

$$= (a^2 + b^2 + c^2 + 2ab - 2ac - 2bc) + (a^2 + b^2 + c^2 - 2ab + 2ac - 2bc) + (a^2 + b^2 + c^2 - 2ab - 2ac + 2bc)$$
$$= 3a^2 + 3b^2 + 3c^2 - 2ab - 2ac - 2bc$$

Exercise set 4.4 (Solution)

1. **Fill in the blanks to complete the following identities:**

(i) $s^2 - 11s + 24 = (\_\_\_\_\_)(\_\_\_\_\_)$
(ii) $(\_\_\_\_\_)(x + 1) = (3x^2 - 4x - 7)$
(iii) $10x^2 - 11x - 6 = (2x - \_\_\_\_\_)(\_\_\_\_\_ + 2)$
(iv) $6x^2 + 7x + 2 = (\_\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_\_)$

Sol.

(i) $s^2 - 11s + 24$

Middle term splitting method:

Product = 24, Sum = -11.

Factors: -3और -8.

$$s^2 - 11s + 24 = s^2 - 3s - 8s + 24$$
$$= (s^2 - 3s) - (8s - 24)$$
$$= s(s - 3) - 8(s - 3)$$
$$= (s - 3)(s - 8)$$

(ii) $(\_\_\_\_\_)(x + 1) = 3x^2 - 4x - 7$

We need to factor RHS:

$$3x^2 - 4x - 7$$

Split middle term: product = -21, sum = -4. Factors: -7 & 3.

$$3x^2 - 4x - 7 = 3x^2 - 7x + 3x - 7$$
$$= (3x^2 - 7x) + (3x - 7)$$
$$= (3x - 7)(x + 1)$$

So blanks: $(3x - 7)(x + 1)$.

(iii) $10x^2 - 11x - 6$

Split middle term: product = -60, sum = -11. Factors: -15 & 4.

$$10x^2 - 11x - 6 = 10x^2 - 15x + 4x - 6$$
$$= (10x^2 - 15x) + (4x - 6)$$
$$= 5x(2x - 3) + 2(2x - 3)$$
$$= (2x - 3)(5x + 2)$$

So blanks: $(2x - 3)(5x + 2)$.

(iv) $6x^2 + 7x + 2$

Split middle term: product = 12, sum = 7. Factors: 3 & 4.

$$6x^2 + 7x + 2 = 6x^2 + 3x + 4x + 2$$
$$= (6x^2 + 3x) + (4x + 2)$$
$$= 3x(2x + 1) + 2(2x + 1)$$
$$= (3x + 2)(2x + 1)$$

2. Select and use the identity that will help you to find the following products without multiplying directly:

(i) $(41)^2$
(ii) $(27)^2$
(iii) (23×17)
(iv) $(135)^2$
(v) $(97)^2$
(vi) (18×29)
(vii) (34×43)
(viii) $(205)^2$
Sol.

(i) $(41)^2$
Identity: $(a + b)^2 = a^2 + 2ab + b^2$. Take $41 = 40 + 1$.
$$(41)^2 = (40 + 1)^2 = 40^2 + 2(40)(1) + 1^2$$
$$= 1600 + 80 + 1 = 1681$$
(ii) $(27)^2$
Take $27 = 30 - 3$. Identity: $(a - b)^2 = a^2 - 2ab + b^2$.
$$(27)^2 = (30 - 3)^2 = 30^2 - 2(30)(3) + 3^2$$
$$= 900 - 180 + 9 = 729$$
(iii) (23×17)
Identity: $(a + b)(a - b) = a^2 - b^2$. Here, $23 = 20 + 3$, $17 = 20 - 3$.
$$23 \times 17 = (20 + 3)(20 - 3) = 20^2 - 3^2$$
$$= 400 - 9 = 391$$
(iv) $(135)^2$
Take $135 = 100 + 35$. Identity: $(a + b)^2 = a^2 + 2ab + b^2$.
$$(135)^2 = (100 + 35)^2 = 100^2 + 2(100)(35) + 35^2$$
$$= 10000 + 7000 + 1225 = 18225$$
(v) Find $(97)^2$
$$(97)^2 = (100 - 3)^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$
$$= (100)^2 - 2(100)(3) + (3)^2$$
$$= 10000 - 600 + 9$$
$$= 9409$$

(vi) Find (18×29)

$$18 \times 29 = 18(30 - 1)$$

$$= 18 \times 30 - 18 \times 1$$
$$= 540 - 18$$
$$= 522$$

(vii) Find (34×43)

$$34 \times 43 = 34(40 + 3)$$

$$= 34 \times 40 + 34 \times 3$$
$$= 1360 + 102$$
$$= 1462$$

(viii) Find $(205)^2$

$$(205)^2 = (200 + 5)^2$$

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$= (200)^2 + 2(200)(5) + (5)^2$$
$$= 40000 + 2000 + 25$$
$$= 42025$$

3. Factor the following:

(i) $9a^2 + b^2 + 4c^2 - 6ab + 12ac - 4bc$

(ii) $16s^2 + 25t^2 - 40st$

(iii) $r^2 - r - 42$

(iv) $49g^2 + 14gh + h^2$

(v) $64u^2 + 121v^2 + 4w^2 - 176uv - 32uw + 44vw$

Sol. (i) Factorise:

$$9a^2 + b^2 + 4c^2 - 6ab + 12ac - 4bc$$

$$= 9a^2 + b^2 + 4c^2 - 6ab + 12ac - 4bc$$

This matches the identity:

$$(x - y + z)^2 = x^2 + y^2 + z^2 - 2xy + 2xz - 2yz$$

Taking,

$$x = 3a, y = b, z = 2c$$

Therefore,

$$= (3a - b + 2c)^2$$

Hence,

$$9a^2 + b^2 + 4c^2 - 6ab + 12ac - 4bc = (3a - b + 2c)^2$$

(ii) Factorise:

$$16s^2 + 25t^2 - 40st$$

Using identity:

$$(a - b)^2 = a^2 - 2ab + b^2$$
$$= (4s)^2 - 2(4s)(5t) + (5t)^2$$
$$= (4s - 5t)^2$$

Hence,

$$16s^2 + 25t^2 - 40st = (4s - 5t)^2$$

(iii) Factorise:

$$r^2 - r - 42$$

We need two numbers whose product is $-42$and sum is -1.

-7 and 6

So,

$$r^2 - r - 42 = r^2 - 7r + 6r - 42$$
$$= r(r - 7) + 6(r - 7)$$
$$= (r - 7)(r + 6)$$

Hence,

$$r^2 - r - 42 = (r - 7)(r + 6)$$

(iv) Factorise:

$$49g^2 + 14gh + h^2$$

Using identity:

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$= (7g)^2 + 2(7g)(h) + h^2$$
$$= (7g + h)^2$$

Hence,

$$49g^2 + 14gh + h^2 = (7g + h)^2$$

(v) Factorise:

$$64u^2 + 121v^2 + 4w^2 - 176uv - 32uw + 44vw$$

This matches the identity:

$$(x - y - z)^2 = x^2 + y^2 + z^2 - 2xy - 2xz + 2yz$$

Taking,

$$x = 8u, y = 11v, z = 2w$$

Therefore,

$$= (8u - 11v - 2w)^2$$

Hence,

$$64u^2 + 121v^2 + 4w^2 - 176uv - 32uw + 44vw = (8u - 11v - 2w)^2$$