CHAPTER: 2 - Introduction to Linear Polynomials

Exercise set 2.1 (Solution)

1. Find the degrees of the following polynomials:
(i) $2x^2 - 5x + 3$
(ii) $y^3 + 2y - 1$
(iii) -9
(iv) $4z - 3$

Sol.- Degree of a polynomial is the highest power of the variable.
(i) $2x^2 - 5x + 3$

Highest power of $x = 2$
Degree $= 2$

(II) $y^3 + 2y - 1$

Highest power of $y = 3$
Degree $= 3$

(iii)-9

This is a constant polynomial (no variable)

Degree= 0

(iv) $4z - 3$
Highest power of $z = 1$

Degree $= 1$

2. Write polynomials of degrees 1, 2 and 3.

Sol.- A polynomial of degree 1 (linear polynomial):
Example: $3x + 4$
A polynomial of degree 2 (quadratic polynomial):
Example: $2\mathbf{x^2} - \mathbf{3x} + \mathbf{4}$
A polynomial of degree 3 (cubic polynomial):
Example: $\mathbf{3x^3} - \mathbf{4x^2} + \mathbf{2x} + \mathbf{5}$

3. What are the coefficients of x^2 and x^3 in the polynomial $x^4 - 3x^3 + 6x^2 - 2x + 7$?

Sol.- Given polynomial: $\mathbf{x^4 - 3x^3 + 6x^2 - 2x + 7}$

Coefficient of $x^3 = -3$

Coefficient of $x^2 = 6$

4. What is the coefficient of z in the polynomial $\mathbf{4z^3 + 5z^2}$ - 11?

Sol.-

The given polynomial is $\mathbf{4z^3 + 5z^2}$ - 11 .

There is no term containing z^1 (I.e., z).

Hence, the coefficient of z is 0

5. What is the constant term of the polynomial $9x^3 + 5x^2 - 8x - 10$?

Sol.- In the polynomial $9x^3 + 5x^2 - 8x - 10$, the constant term is -10 .

Exercise set 2.2 (Solution)

1. Find the value of the Linear polynomial 5x-3 if :

   (i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$

   Sol.- Given polynomial: P(x) = $5x - 3$
   (i) $x = 0$
   P(0) $= 5(0) - 3 = -3$
   (ii) $\boldsymbol{x} = -1$
   P(-1) $= 5(-1) - 3 = -5 - 3 = -8$
   (iii) $\mathbf{x} = 2$
   P(2) $= 5(2) - 3 = 10 - 3 = 7$

2. Find the value of the quadratic polynomial $\mathbf{7s^2 - 4s + 6}$ if:
   (i) $\mathrm{s} = 0$ (ii) $\mathrm{s} = -3$ (iii) $\mathrm{s} = 4$

   Sol.- (i) For $\mathrm{s} = 0$
   $\mathbf{7s^2 - 4s + 6}$
   $= 7(0)^2 - 4(0) + 6 = 6$
   (ii) For $\mathrm{s} = -3$
   $7\,s^2 - 4\,s + 6$
   $= 7(-3)^2 - 4(-3) + 6$
   $= 7(9) + 12 + 6$
   $= 63 + 12 + 6 = 81$
   (iii) For $\mathrm{s} = 4$
   $\mathbf{7s^2 - 4s + 6}$
   $= 7(4)^2 - 4(4) + 6$
   $= 7(16) - 16 + 6$
   $= 112 - 16 + 6 = 102$

3. The present age of Salil's mother is three times Salil's present age. After 5 years, their ages will add up to $\mathbf{70}$ years. Find their present ages.

   Sol.- Let Salil's present age = x years

   Mother's present age $= 3x$ years
   After 5 years:
   Salil's age $= x + 5$
   Mother's age $= 3x + 5$

   According to question: $(x + 5) + (3x + 5) = 70$

   $$4x + 10 = 70$$
   $$4x = 60$$
   $$x = 15$$

Therefore, Salil's age $= 15$ years
Mothers age $= 45$ years

4. The difference between two positive integers is 63. The ratio of the two integers is $2:5$. Find the two integers.

Sol.- Let the integers be $2x$ and $5x$
Difference: $5x - 2x = 63$

$$3x = 63$$
$$x = 21$$

Integers: $2x = 42$

$$5x = 105$$

Required integers are 42 and 105.

5. Ruby has $\mathbf{3}$ times as many two-rupee coins as she has five-rupee coins. If she has a total Rs.88, how many coins does she have of each type?

Sol.- Let number of five-rupee coins $= \mathrm{x}$
Number of two-rupee coins $= 3x$

Total value: $5x + 2(3x) = 88$

$$5x + 6x = 88$$
$$11\,x = 88$$
$$x = 8$$

Hence:
Five-rupee coins $= 8$
Two-rupee coins $= 24$

6. A farmer cuts a 300 feet fence into two pieces of different sizes. The longer piece is four times as long as the shorter piece. How long are the two pieces?

Sol.- Let shorter piece $= \mathrm{x}$ feet
Longer piece $= 4\mathrm{x}$ feet

Total: $\mathrm{x} + 4\mathrm{x} = 300$

$$5x = 300$$
$$x = 60$$

7. If the Length of a rectangle is three more than twice its width and its perimeter is $\mathbf{24}$ cm , what are the dimensions of the rectangle?

Sol.- Let width $= \mathrm{x}$ cm
Length $= 2\mathrm{x} + 3$ cm
Perimeter $= 2($ length + width $)$
$2[(2x + 3) + x] = 24$
$2(3x + 3) = 24$

$6x + 6 = 24$
$6x = 18$
$x = 3$
Therefore:
Width $= 3$ cm
Length $= 2(3) + 3 = 9$ cm

Exercise set 2.3 (Solution)

1. A student has Rs. 500 in her savings bank account. She gets Rs. 150 every month as pocket money. How much money will she have at the end of every month from the second month onwards?Find a linear expression to represent the amount she will have in the nth month.

   Sol.- Initial amount in bank = 500 Rs.
   Monthly pocket money = 150 Rs.
   At the end of:
   2nd month $= 500 + 2(150) =$ 800 Rs.
   3rd month $= 500 + 3(150) =$ 950 Rs.
   4th month $= 500 + 4(150) = 1100$ Rs.
   and so on...
   Let the amount in the nth month be A_n ,
   Then, A_n , $= 500 + 150\text{n}$
   Thus, the required linear expression is:
   $$A_n = 150n + 500$$

2. A rally starts with $\mathbf{120}$ members. Each hour, $\mathbf{9}$ members drop out of the group. How many members will remain after $\mathbf{1}, \mathbf{2}, \mathbf{3}$, ... hours? Find a Linear expression to represent the number of members at the end of the $\mathbf{n}^{\text{th}}$ hour.

   Sol.- Initial members = 120
   Members leaving each hour = 9
   After:

   1 hour $= 120 - 9 = 111$
   2 hours $= 120 - 18 = 102$
   3 hours $= 120 - 27 = 93$

   Let number of members after $\mathbf{n}$ hours $= \mathbf{P_n}$
   $\mathbf{P_n} = 120 - 9\text{n}$
   Thus, required linear expression is $\mathbf{P_n} = 120 - 9\text{n}$.

3. Suppose the Length of a rectangle is 13 cm . Find the area if the breadth is (i) 12 cm, (ii) 10 cm, (iii) 8 cm. Find the Linear pattern representing the area of the rectangle.

   Sol.- Length $= 13$ cm
   Area $=$ Length $\cdot$ Breadth
   (i) Breadth $= 12$ cm

   Area $= 13 \times 12 = 156\ \text{cm}^2$
   (ii) Breadth $= 10$ cm

Area $= 13 \times 10 = 130\text{cm}^2$

(iii) Breadth $= 8$ cm

Area $= 13 \times 8 = 104\text{cm}^2$
Let breadth $=$ xcm
Area $= 13x$
Thus, linear pattern is $\mathrm{A} = 13\mathrm{x}$.

4. Suppose the Length of a rectangular box is **7cm** and breadth is **11 cm**. Find the volume if the height is (i) 5 cm , (ii) 9 cm , (iii) 13 cm . Find the Linear pattern representing the volume of the rectangular box.

Sol.- Length $= 7$ cm, Breadth $= 11$ cm
Volume $=$ Length $\times$ Breadth $\times$ Height
$= 7 \times 11 \times \mathrm{h} = 77\ \mathrm{h}$

(i) Height $= 5$ cm

Volume $= 77 \times 5 = 385\ \text{cm}^3$

(ii) Height $= 9$ cm

Volume $= 77 \times 9 = 693\ \text{cm}^3$

(iii) Height=13cm

Volume $= 77 \times 13 = 1001\ \text{cm}^3$
Let height $=$ hcm
Volume $= 77$ h
Thus, linear pattern is $\mathrm{V} = 77\ \mathrm{h}\ \ \text{cm}^3$.

5. Sarita is reading a book of 500 pages. She reads **20** pages every day. How many pages will be Left after 15 days? Express this as a Linear pattern.

Sol.- Total pages $= 500$

Pages read in 15 days $= 20 \cdot 15 = 300$
Pages left $= 500 - 300 = 200$
Let pages left after n days $= P$,
So, $\mathrm{P_n} = 500 - 20\mathrm{n}$
Thus, linear pattern is $\mathrm{P_n} = 500 - 20\mathrm{n}$.

Exercise set 2.4 (Solution)

1. Suppose a plant has height 1.75 feet and it grows by 0.5 feet each month.
(i) Find the height after 7 months.
(ii) Make a table of values for t varying from 0 to 10 months and show how the height, h, increases every month.
(iii) Find an expression that relates h and t, and explain why it represents linear growth.

Sol:-
(i) Initial height = 1.75 ft
Growth per month = 0.5 ft
After 7 months,
Height = 1.75 + (0.5 × 7)
= 1.75 + 3.5 = 5.25 ft
Therefore, height after 7 months is 5.25 feet.

(ii)

Month (t)	Height h (ft)
0	1.75
1	2.25
2	2.75
3	3.25
4	3.75
5	4.25
6	4.75
7	5.25
8	5.75
9	6.25
10	6.75

Here, the height increases by 0.5 ft each month consistently.

(iii) Expression relating h and t:
$h = 1.75 + 0.5t$.
This represents linear growth because the rate of change (0.5 ft/month) is constant.

2. A mobile phone is bought for ₹10,000. Its value decreases by ₹800 every year.
(i) Find the value of the phone after 3 years.
(ii) Make a table of values for t varying from 0 to 8 years and show how the value of the phone, v, depreciates with time.

(iii) Find an expression that relates v and t, and explain why it represents linear decay.

Sol:-(i) Initial value = ₹10,000

Depreciation per year = ₹800

v = 10000 − (800 × 3) = 10000 − 2400 = 7600.

(ii)

Year (t)	Value v (₹)
0	10,000
1	9,200
2	8,400
3	7,600
4	6,800
5	6,000
6	5,200
7	4,400
8	3,600

Thus, the value decreases by ₹800 each year uniformly.

(iii) Expression relating v and t:

v = 10000 − 800t.

This represents linear decay because the rate of change is constant ₹800 per year.

3. The initial population of a village is 750. Every year, 50 people move from a nearby city to the village.

(i) Find the population of the village after 6 years.

(ii) Make a table of values for *t* varying from 0 to 10 years and show how the population, P, increases every year.

(iii) Find an expression that relates *P* and *t,* and explain why it represents linear growth.

Sol:-

(i) Initial population = 750

Increase per year = 50

After 6 years = 750 + (50 × 6)

= 750 + 300 = 1050

Therefore, the population after 6 years is 1050.

(ii)

Year (t)	Population (P)
0	750
1	800
2	850
3	900
4	950
5	1,000
6	1,050
7	1,100
8	1,150
9	1,200
10	1,250

4. A telecom company charges ₹600 for a certain recharge scheme.
This prepaid balance is reduced by ₹15 each day after the recharge.
(i) Write an equation that models the remaining balance $b(x)$ after using the scheme for x days. Explain why it represents linear decay.
(ii) After how many days will the balance run out?
(iii) Make a table of values for x varying from 1 to 10 days and show how the balance $b(x)$, reduces with time.

Sol:-
(i) Initial balance = ₹600
Daily reduction = ₹15
$b(x) = 600 - 15x$
This is a linear decay because the balance decreases at a constant rate of ₹15 per day.

(ii) Set balance to zero:
$600 - 15x = 0$
$15x = 600$
$x = 40$
Thus, the balance runs out after 40 days.

(iii)

Day (x)	Balance b(x) (₹)
1	585
2	570
3	555
4	540
5	525
6	510
7	495
8	480
9	465
10	450

Therefore, the balance decreases by ₹15 each day uniformly, confirming linear decay.

Exercise set 2.5 (Solution)

1. A learning platform charges a fixed monthly fee and an additional cost per digital learning module accessed. A student observes that when she accessed 10 modules, her bill was ₹400. When she accessed 14 modules, her bill was ₹500. If the monthly bill y depends on the number of modules accessed, x, according to the relation y = ax + b, find the values of a and b.

Sol:-
When x = 10, y = 400, then
10a + b = 400 .... (1)
When x = 14, y = 500, then
14a + b = 500 .... (2)
Subtracting (1) from (2),
(14a + b) − (10a + b) = 500 − 400
14a + b – 10a – b = 100
4a = 100
a = $\frac{100}{4}$ = 25
Substituting a = 25 in (1),
10(25) + b = 400
250 + b = 400
b = 400 – 250 = 150.
Hence, a = 25 and b = 150.

2. A gym charges a fixed monthly fee and an additional cost per hour for using the badminton court. A student using the gym observed that when she used the badminton court for 10 hours, her bill was ₹800. When she used it for 15 hours, her bill was ₹1100. If the monthly bill y depends on the hours of the use of the badminton court, x, according to the relation y = ax + b, find the values of a and b.

Sol:-
When x = 10, y = 800, then
10a + b = 800 .... (1)
When x = 15, y = 1100, then
15a + b = 1100 .... (2)
Subtracting (1) from (2),
(15a + b) − (10a + b) = 1100 − 800
5a = 300
a = 60
Substituting a = 60 in (1),

$10(60) + b = 800$
$600 + b = 800$
$b = 200$
Hence, $a = 60$ and $b = 200$.

3. Consider the relationship between temperature measured in degrees Celsius (°C) and degrees Fahrenheit (°F), which is given by °C = a °F + b. Find a and b, given that ice melts at 0 degrees Celsius and 32 degrees Fahrenheit, and water boils at 100 degrees Celsius and 212 degrees Fahrenheit.
(**Hint:** When °C = 0, °F = 32 and when °C = 100, °F = 212. Use this information to find a and b, and thus, the linear relationship between °C and °F.)

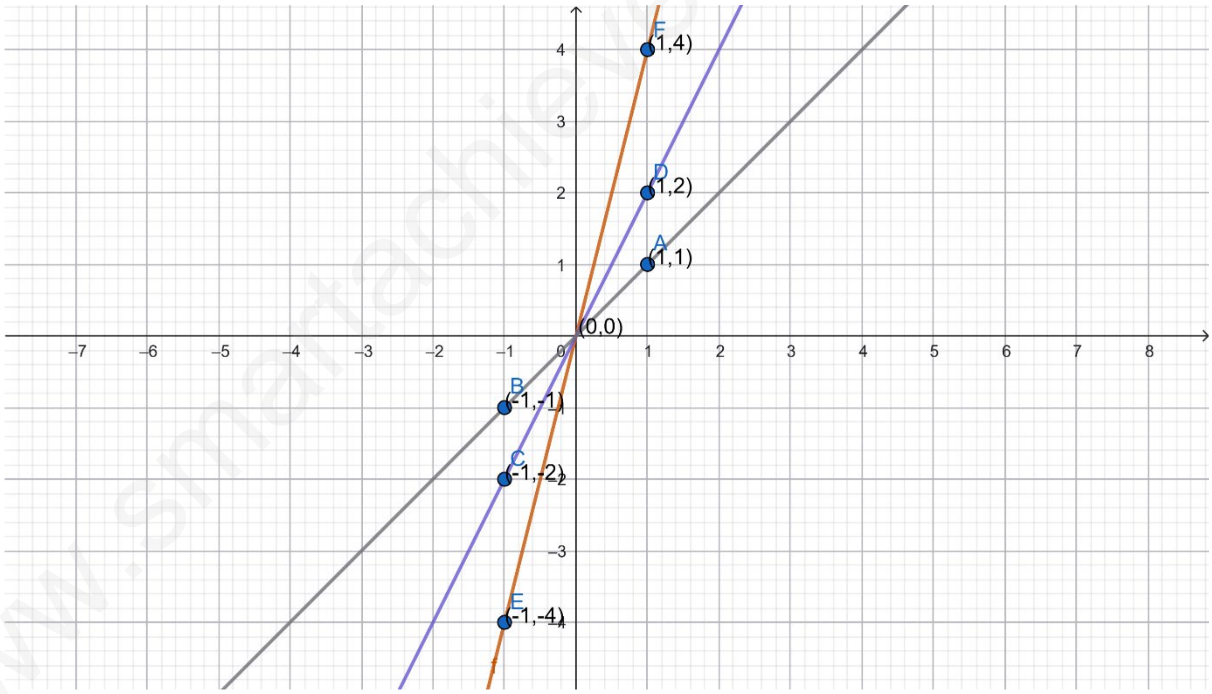
Sol:-
Using the given linear relation,
$C = aF + b$
When $F = 32$, $C = 0$, then
$32a + b = 0$ .... (1)
When $F = 212$, $C = 100$, then
$212a + b = 100$ .... (2)
Subtracting (1) from (2),
$(212a + b) - (32a + b) = 100 - 0$
$180a = 100$
$a = \frac{100}{180} = \frac{5}{9}$
Substitute $a = \frac{5}{9}$ in (1),
$32 \times \frac{5}{9} + b = 0$
$\frac{160}{9} + b = 0$
$b = \frac{-160}{9}$
Thus, $a = \frac{5}{9}$ and $b = \frac{-160}{9}$

Linear relationship: $C = \frac{9}{5}F - \frac{160}{9}$.

Exercise set 2.6 (Solution)

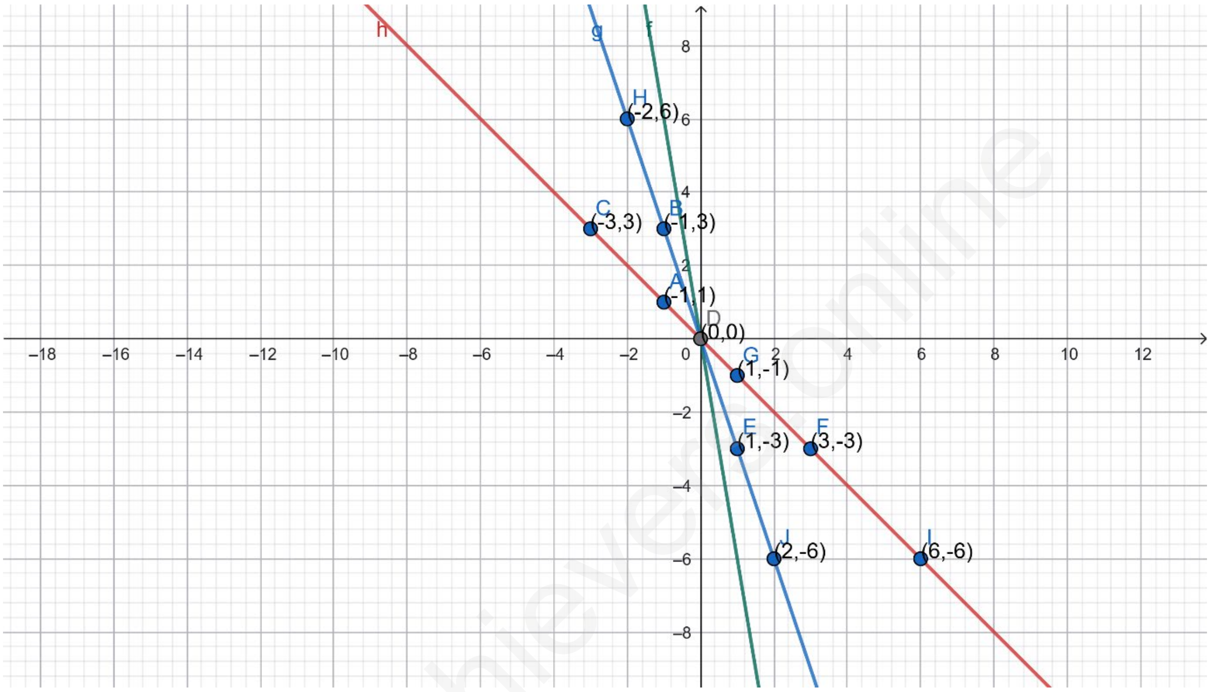
1. Draw the graphs of the following sets of lines. In each case, reflect on the role of 'a' and 'b'.
   (i) $y = 4x$, $y = 2x$, $y = x$
   (ii) $y = -6x$, $y = -3x$, $y = -x$
   (iii) $y = 5x$, $y = -5x$
   (iv) $y = 3x - 1$, $y = 3x$, $y = 3x + 1$
   (v) $y = -2x - 3$, $y = -2x$, $y = 2x + 3$

   Sol.-
   (i) $y = 4x$, $y = 2x$, $y = x$



<u>Observation</u>:
1. All lines pass through because b = 0.
2. Larger a → steeper line.

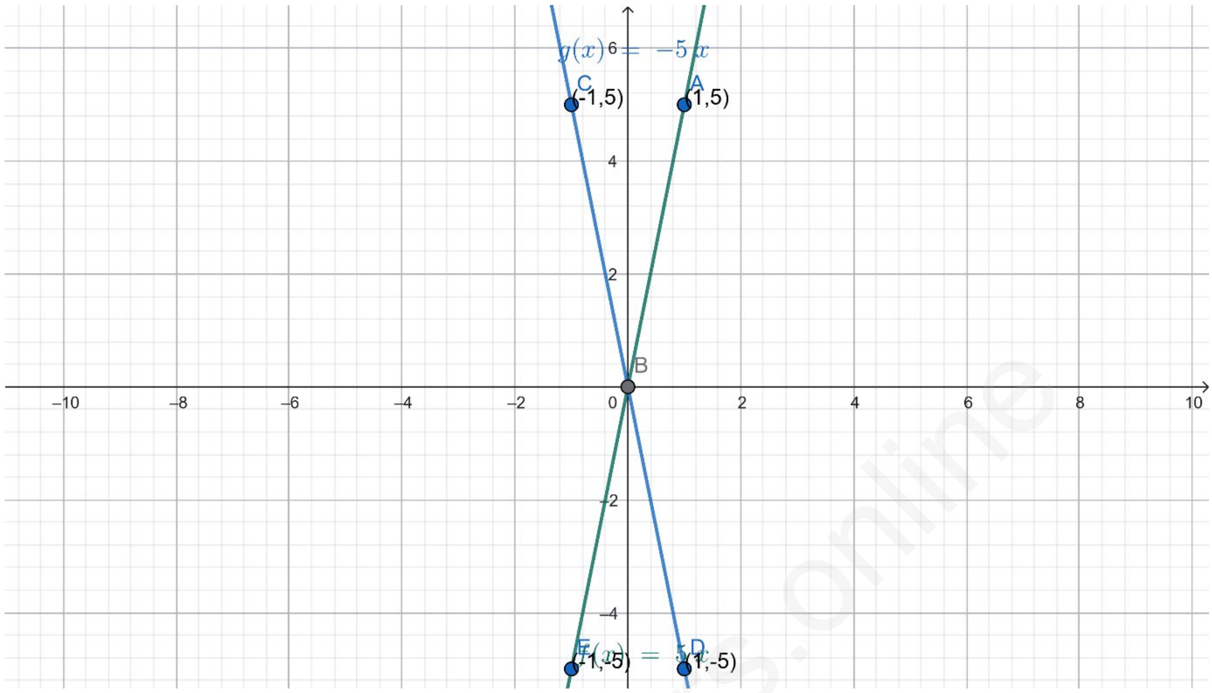
(ii) $y = -6x$, $y = -3x$, $y = -x$



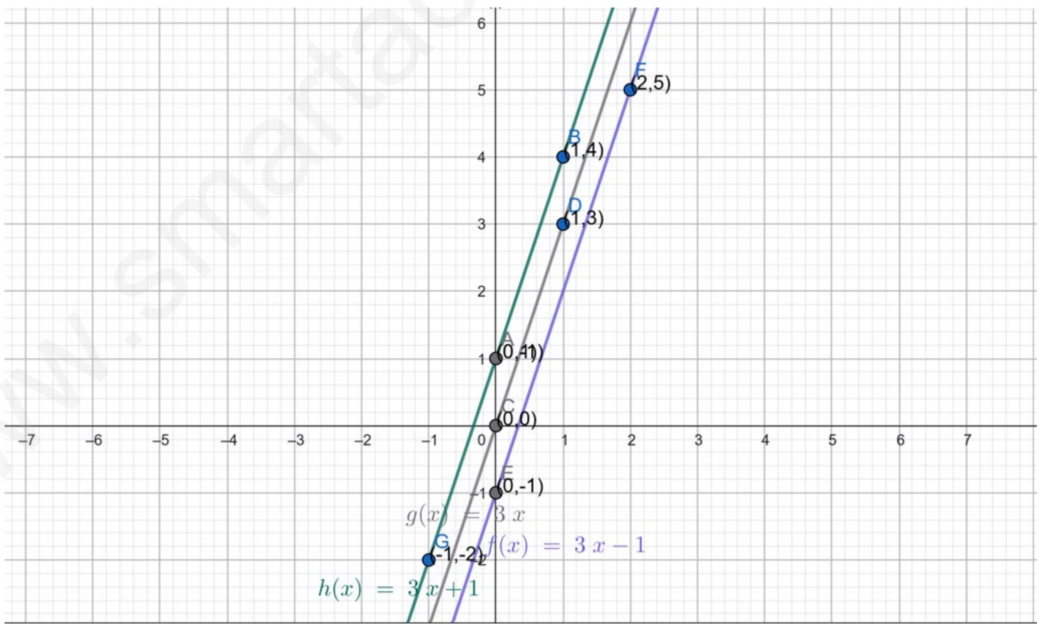
Observation:

1. All lines pass through the origin.
2. Negative a → lines slope downward.
3. Larger magnitude of a → steeper downward slope.

(iii) $y = 5x$, $y = -5x$



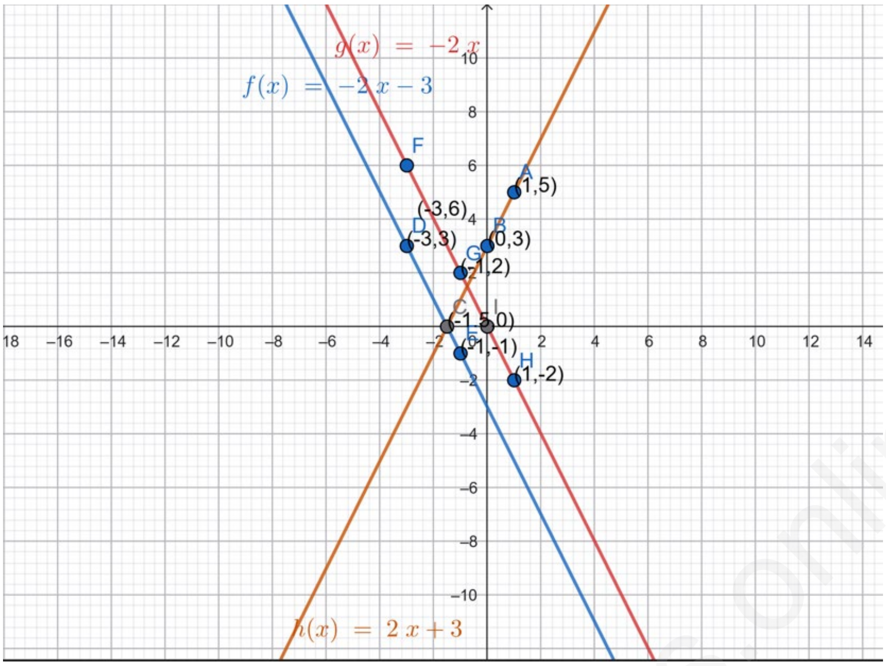
(iv) y = 3x – 1, y = 3x, y = 3x + 1



<u>Observation</u>:

1. Both lines pass through the origin.
2. Same steepness, opposite direction (mirror images).

(v) y = –2x – 3, y = –2x, y = 2x + 3



<u>Observation</u>:

1. First two lines are parallel (same slope 2).
2. Third line has a different slope → different direction.
3. b controls vertical position.

END OF CHAPTER EXERCISES

1. Write a polynomial of degree 3 in the variable x, in which the coefficient of the x^2 term is –7.

Sol:-

A polynomial of degree 3 has the general form:

$ax^3 + bx^2 + cx + d$ $(a \neq 0)$

Given that the coefficient of the x^2 term is –7, so b = -7

One valid example is:

$x^3 - 7x^2 - 8x + 3$.

2. Find the values of the following polynomials at the indicated values of the variables.

(i) $5x^2 - 3x + 7$ if $x = 1$

(ii) $4t^3 - t^2 + 6$ if $t = a$

Sol:-

(i) $5x^2 - 3x + 7$ if $x = 1$

$= 5(1)^2 - 3(1) + 7$

$= 5 - 3 + 7$

$= 12 - 3 = 9$.

(ii) $4t^3 - t^2 + 6$ if $t = a$

$= 4(a)^3 - (a)^2 + 6$

$= 4a^3 - a^2 + 6$

3. If we multiply a number by $\frac{5}{2}$ and add $\frac{2}{3}$ to the product, we get $\frac{-7}{12}$. Find the number.

Sol:-

Let the number be a.

According to the question,

$\frac{5}{2} \times a + \frac{2}{3} = \frac{-7}{12}$

$\frac{5a}{2} = \frac{-7}{12} - \frac{2}{3}$

$\frac{5a}{2} = \frac{-7-8}{12}$

$\frac{5a}{2} = \frac{-15}{12}$

$a = \frac{-15}{12} \times \frac{2}{5} = \frac{-30}{60} = \frac{-1}{2}$

Therefore, the number is $\frac{-1}{2}$.

4. A positive number is 5 times another number. If 21 is added to both the numbers, then one of the new numbers becomes twice the other new number. What are the numbers?

Sol:-

Let the smaller number be x.

Then the other (positive) number is 5x.

After adding 21 to both:

First number = x + 21

Second number = 5x + 21

According to the question,

5x + 21 = 2(x + 21)

5x + 21 = 2x + 42

5x – 2x = 42 – 21

3x = 21

$x = \frac{21}{3} = 7$

Therefore,

Smaller number = x = 7

Larger number = 5x = 35

5. If you have ₹800 and you save ₹250 every month, find the amount you have after (i) 6 months (ii) 2 years. Express this as a linear pattern.

Sol:-

Initial amount = ₹800

Monthly saving = ₹250

Let t = number of months

A = 800 + 250t

(i) After 6 months

A = 800 + 250 × 6

= 800 + 1500 = 2300

(ii) After 2 years (24 months)

A = 800 + 250 × 24

= 800 + 6000 = 6800

Linear Pattern:

A (t months)	B (Amount ₹)
0	800
1	1050
2	1300
3	1550
4	1800
5	2050
6	2300

6. The digits of a two-digit number differ by 3. If the digits are interchanged, and the resulting number is added to the original number, we get 143. Find both the numbers.

Sol:-

Let the two-digit number have:

Tens digit = x

Units digit = y

So the number = 10x + y

Digits differ by 3:

∴ x – y = 3 .... (1)

Interchanged number = 10y + x

Sum of both numbers:

(10x + y) + (10y + x) = 143

11x + 11y = 143

x + y = 13 .... (2)

Adding (1) and (2),

2x = 16

x = 8

Substituting x = 8 in (2),

8 + y = 13

y = 13 – 8

y = 5

Original number = 10x + y = 10(8) + (5) = 80 + 5 = 85.

Interchanged number = 10y + x = 10(5) + 8 = 50 + 8 = 58.

7. Draw the graph of the following equations, and identify their slopes and y-intercepts. Also, find the coordinates of the points where these lines cut the y-axis.

(i) $y = -3x + 4$

(ii) $2y = 4x + 7$

(iii) $5y = 6x - 10$

(iv) $3y = 6x - 11$

Are any of the lines parallel?

Sol:-

(i) $y = -3x + 4$

Comparing with y = ax + b

Slope a = – 3

y-intercept b = 4
Cuts y-axis at (0, 4)

(ii) 2y = 4x + 7
Divide by 2,
y = 2x + $\frac{7}{2}$
Cuts y-axis at (0, $\frac{7}{2}$)
Slope a = 2
y-intercept b = $\frac{7}{2}$

(iii) 5y = 6x – 10
Divide by 5,
y = $\frac{6}{5}$x – 2
Cuts y-axis at (0, –2)
Slope a = $\frac{6}{5}$
y-intercept b = −2

(iv) 3y = 6x − 11
Divide by 3,
y = 2x − $\frac{11}{3}$
Cuts y-axis at (0, $\frac{-11}{3}$)
Slope a = 2
y-intercept b = $\frac{-11}{3}$
Parallel lines: (ii) and (iv) both have slope = 2, so they are parallel to each other

8. If the temperature of a liquid can be measured in Kelvin units as x K and in Fahrenheit units as *y* °F, the relation between the two systems of measurement of temperature is given by the linear equation
y = $\frac{9}{5}$ (*x* – 273) + 32.
(i) Find the temperature of the liquid in Fahrenheit if the temperature of the liquid is 313 K.
(ii) If the temperature is 158 °F, then find the temperature in Kelvin.

Sol:-

Given the relation: y = $\frac{9}{5}$ (x – 273) + 32
(i) When x = 313K
y = $\frac{9}{5}$ (313 – 273) + 32
y = $\frac{9}{5}$ × 40 + 32
y = 9 × 8 + 32
y = 72 + 32 =
y = 104° F

(ii) When y = 158° F

$158 = \frac{9}{5}(x - 273) + 32$

$158 - 32 = \frac{9}{5}(x - 273)$

$126 = \frac{9}{5}(x - 273)$

$126 \times \frac{5}{9} = (x - 273)$

$14 \times 5 = x - 273$

$70 = x - 273$

$x = 70 + 273$

$x = 343$ K

9. The work done by a body on the application of a constant force is the product of the constant force and the distance travelled by the body in the direction of the force. Express this in the form of a linear equation in two variables (work w and distance d), and draw its graph by taking the constant force as 3 units. What is the work done when the distance travelled is 2 units? Verify it by plotting it on the graph.

Sol:-

Given,

Work done = Force × Distance

Let work be w and distance be d.

So,

w = Fd

Given F = 3,

w = 3d

This is the required linear equation.

If d = 2 units, then

w = 3 × 2 = 6 units

Verification:

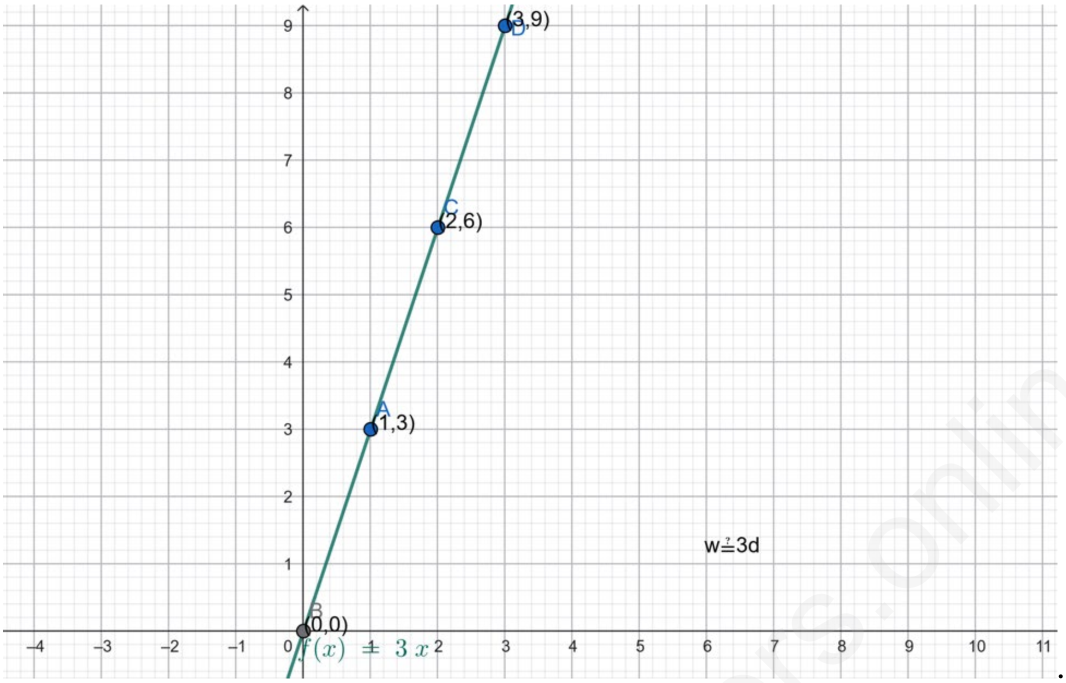
Taking values of d:

d	w = 3d
0	0
1	3
2	6
3	9

Plotting the points (0, 0),(1, 3),(2, 6),(3, 9) on the graph paper and joining them to get a straight line.

On the graph, the point corresponding to d = 2 is (2, 6), which lies on the

Hence, the work done is 6 units, verified



10. The graph of a linear polynomial p(x) passes through the points (1, 5) and (3, 11).

(i) Find the polynomial p(x).

(ii) Find the coordinates where the graph of p(x) cuts the axes.

(iii) Draw the graph of p(x) and verify your answers.

Sol:-

(i) Let p(x) = ax + b

Since the graph passes through (1, 5)

a + b = 5 .... (1)

Since the graph passes through (3, 11)

3a + b = 11 .... (2)

Subtracting (1) from (2),

3a + b – (a + b) = 11 – 5

3a + b – a – b = 6

2a = 6

a = 3

Substituting x = 3 in (1),

3 + b = 5

b = 5 – 3

b = 2

Thus, p(x) = 3x + 2

(ii) Points where the graph cuts the axes:

y-axis: Put x = 0

p(0) = 3(0) + 2

p(0) = 2 ⇒(0, 2)

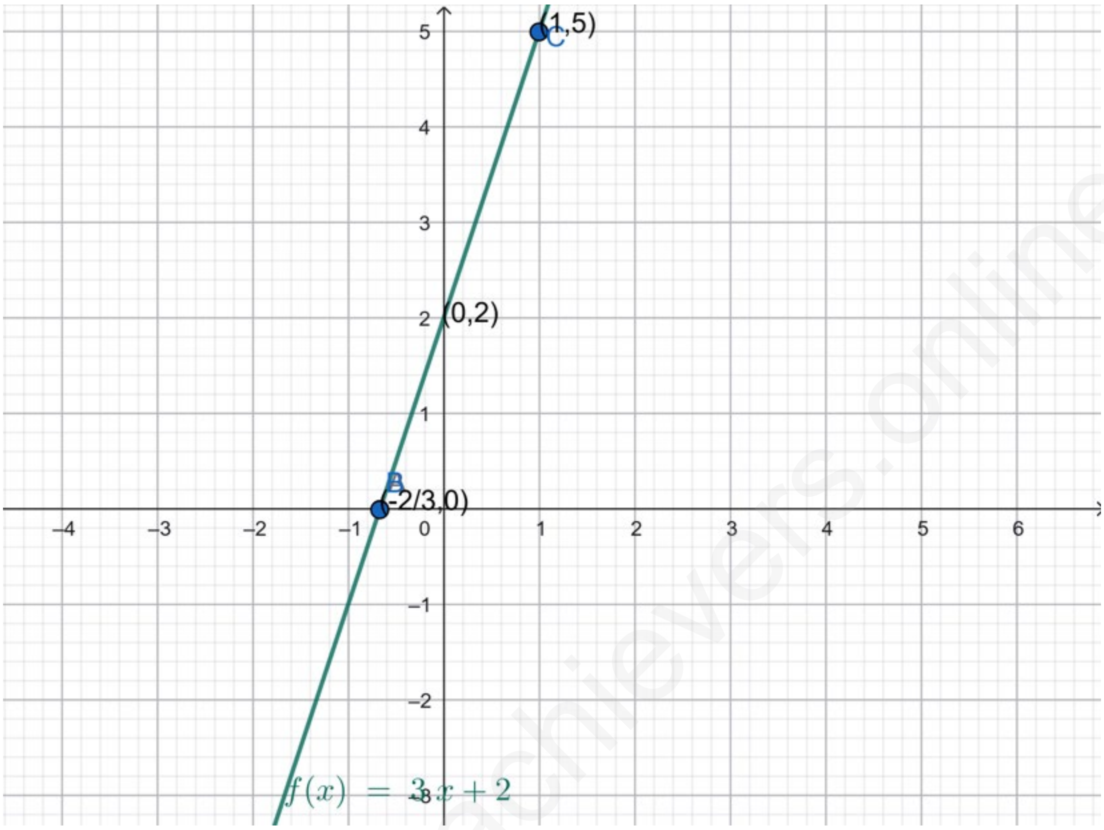
x-axis: Put p(x) = 0

3x + 2 = 0

3x = – 2

x = $\frac{-2}{3} \Rightarrow \left(\frac{-2}{3}, 0\right)$

(iii)



<u>Verification</u>:

1. The line passes through both given points.
2. It cuts the y-axis at (0, 2).
3. It cuts the x-axis at $\left(-\frac{2}{3}, 0\right)$.

11. Let p(x) = ax + b and q(x) = cx + d be two linear polynomials such that:

(i) p(0) = 5.

(ii) The polynomial p(x) – q(x) cuts the x-axis at (3, 0).

(iii) The sum p(x) + q(x) is equal to 6x + 4 for all real x.

Find the polynomials p(x) and q(x).

Sol:-

p(x) = ax + b, q(x) = cx + d

Use p(0) = 5

p(0) = b = 5

So,

p(x) = ax + 5

Use p(x) + q(x) = 6x + 4

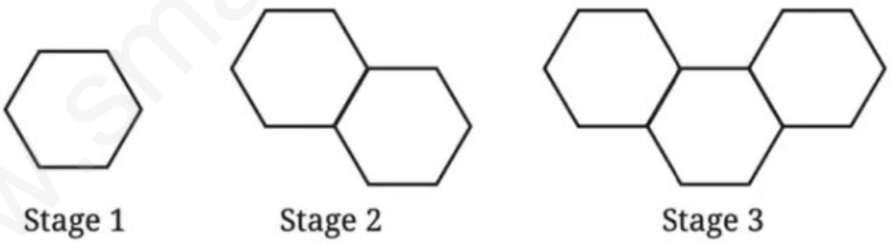
(ax + 5) + (cx + d) = 6x + 4

(a + c)x + (5 + d) = 6x + 4

Equate coefficients:

$a + c = 6$
$5 + d = 4$
$\Rightarrow d = -1$
So,
$q(x) = cx - 1$
Since $p(x) - q(x)$ cuts the x-axis at $(3, 0)$, so:
$p(3) - q(3) = 0$
$(3a + 5) - (3c - 1) = 0$
$3a + 5 - 3c + 1 = 0$
$3a - 3c + 6 = 0$
$a - c = -2$
We have:
$a + c = 6$
$a - c = -2$
Add:
$2a = 4$
$\Rightarrow a = 2$
Then:
$c = 4$
Therefore, $p(x) = 2x + 5$
$q(x) = 4x - 1$

12. Look at the first three stages of a growing pattern of hexagons made using matchsticks. A new hexagon gets added at every stage which shares a side with the last hexagon of the previous stage.



(i) Draw the next two stages of the pattern. How many matchsticks will be required at these stages?
(ii) Complete the following table.

Stage Number	1	2	3	4	5	...	n
Number of matchsticks							

(iii) Find a rule to determine the number of matchsticks required for the n^{th} stage.
(iv) How many matchsticks will be required for the 15^{th} stage of the pattern?
(v) Can 200 matchsticks form a stage in this pattern? Justify your answer.

Sol:-
(i) A hexagon uses 6 matchsticks.
Each new hexagon shares one side, so only 5 additional matchsticks are needed.
Stage 4: 16 + 5 = 21 matchsticks
Stage 5: 21 + 5 = 26 matchsticks
(ii)

Stage Number	1	2	3	4	5	...	n
Number of matchsticks	6	11	16	21	26		5n + 1

(iii) Rule for n^{th} stage = 6 + (n − 1) × 5 = 5n + 1.
(iv) Matchsticks for 15^{th} stage = 5 × 15 + 1 = 76
(v) 5n + 1 = 200
5n = 200 − 1
5n = 199
n = $\frac{199}{5}$ = 39.8
Since nn is not a whole number, 200 matchsticks cannot form a stage.

13. Let p(x) = ax + b and q(x) = cx + d be two linear polynomials such that:
(i) The graph of p(x) passes through the points (2, 3) and (6, 11).
(ii) The graph of q(x) passes through the point (4, –1).
(iii) The graph of q(x) is parallel to the graph of p(x).
Find the polynomials p(x) and q(x). Also, find the coordinates of the point where these lines meet the x-axis.

Sol:-
(i) Starting with p(x) = ax + b.
Since the line passes through (2, 3) and (6, 11).
Slope, a = $\frac{11-3}{6-2}$ = $\frac{8}{4}$ = 2
Now,
3 = 2(2) + b
b = 3 – 4
b = –1

$\therefore$ p(x) = 2x – 1

(ii) Given that q(x) is parallel to p(x), so slopes are equal.
c = 2
Thus, q(x) = 2x + d
It passes through (4, −1):
−1 = 2(4) + d
⇒−1 = 8 + d
⇒d = −9
q(x) = 2x − 9

(iii) x-intercepts
For p(x):
2x − 1 = 0
$\Rightarrow x = \frac{1}{2}$
Point: $\left(\frac{1}{2}, 0\right)$
For q(x):
2x − 9 = 0
$\Rightarrow x = \frac{9}{2}$
Point: $\left(\frac{9}{2}, 0\right)$

14. What do all linear functions of the form f(x) = ax + a, a > 0, have in common?

Sol:-
f(x) = ax + a, a > 0
or f(x) = a(x + 1)
Common properties:
1. Slope:
Slope = a.
Since a > 0, all lines have positive slope.
2. y-intercept:
Putting x = 0:
f(0) = a
So, the y-intercept is (0, a).
Since a > 0, all lines cut the y-axis above the origin.
3. x-intercept:
Put f(x) = 0:
ax + a = 0
a (x + 1) = 0
Since a ≠ 0,
x + 1 = 0

$\Rightarrow x = -1$

So, all lines pass through the fixed point $(-1, 0)$.