Matrices & Determents

Single Correct Answer Type

1. A and B are two non singular matrices so that $A^6 = I$ and $AB^2 = BA(B \neq I)$. A value of K so that $B^K = I$ is

a) 31 b) 32

c) 63 d) 64

Key. C

Sol. $A^5(AB^2) = A^5BA.$

$\Rightarrow B^2 = A^5BA$

$\Rightarrow B^4 = (A^5BA)(A^5BA) = A^5B^2A = A^5(A^5BA)A$

$\Rightarrow B^4 = A^4BA^2$

$\Rightarrow B^8 = (A^4BA^2)(A^4BA^2) = A^4B^2A^2 = A^4(A^5BA)A^2$

$\Rightarrow B^8 = A^3BA^3$

$\Rightarrow B^{16} = (A^3BA^3)(A^3BA^3) = A^3B^2A^3 = A^3(A^5BA)A^3 = A^2BA^4$

$A^{32} = (A^2BA^4)(A^2BA^4) = A^2B^2A^4 = A^2(A^5BA)A^4 = ABA^5$

$A^{64} = (ABA^5)(ABA^5) = AB^2A^5 = A(A^5BA)A^5 = B \Rightarrow A^{63} = I$

2. For each real number x such that – 1 < x < 1, let A (x) be the matrix $(1-x)^{-1}\begin{bmatrix} 1 & -x \\ -x & 1 \end{bmatrix}$ and $z = \frac{x+y}{1+xy}$. Then,

(A) A(z) = A(x) + A(y) (B) A(z) = A(x) $[A(y)]^{-1}$

(C) A(z) = A(x) A(y) (D) A(z) = A(x) – A(y)

Key. C

Sol. $A(z) = A\left(\frac{x+y}{1+xy}\right) = \left[\frac{1+xy}{(1-x)(1-y)}\right]\begin{bmatrix} 1 & -\left(\frac{x+y}{1+xy}\right) \\ -\left(\frac{x+y}{1+xy}\right) & 1 \end{bmatrix}$

$\therefore A(x).A(y) = A(z)$

3. A and B are two non singular matrices so that $A^6 = I$ and $AB^2 = BA(B \neq I)$. A value of K so that $B^K = I$ is

a) 31 b) 32 c) 63 d) 64

Key. C

Sol. $A^5\left(AB^2\right)=A^5BA.$

$\Rightarrow B^2=A^5BA$

$\Rightarrow B^4=\left(A^5BA\right)\left(A^5BA\right)=A^5B^2A=A^5\left(A^5BA\right)A$

$\Rightarrow B^4=A^4BA^2$

$\Rightarrow B^8=\left(A^4BA^2\right)\left(A^4BA^2\right)=A^4B^2A^2=A^4\left(A^5BA\right)A^2$

$\Rightarrow B^8=A^3BA^3$

$\Rightarrow B^{16}=\left(A^3BA^3\right)\left(A^3BA^3\right)=A^3B^2A^3=A^3\left(A^5BA\right)A^3=A^2BA^4$

$A^{32}=\left(A^2BA^4\right)\left(A^2BA^4\right)=A^2B^2A^4=A^2\left(A^5BA\right)A^4=ABA^5$

$A^{64}=\left(ABA^5\right)\left(ABA^5\right)=AB^2A^5=A\left(A^5BA\right)A^5=B\Rightarrow A^{63}=I$

4. If matrix A = $[a_{ij}]_{3\times3}$, matrix B = $[b_{ij}]_{3\times3}$ where $a_{ij}+a_{ji}=0$ and $b_{ij}-b_{ji}=0$, then $A^4 . B^3$ is

(A) skew-symmetric matrix (B) singular

(C) symmetric (D) zero matrix

Key. B

Sol. Since matrix A is skew-symmetric,

$\therefore$ | A | = 0

$\therefore$ $|A^4 . B^3| = 0$

5. If A = $\begin{bmatrix}1 & 2 & -1\\ -1 & 1 & 2\\ 2 & -1 & 1\end{bmatrix}$, then det (Adj (Adj A)) is

(A) $(14)^4$ (B) $(14)^6$ (C) $(14)^9$ (D) $(14)^2$

Key. A

Sol. | A | = = (1 + 2) – 2 (– 1 – 4) – (1 – 2)

= 3 + 10 + 1 = 14

$\therefore$ det (Adj (Adj A)) = $|\text{Adj } A|^2 = |A|^4 = (14)^4$

6. In the expansion of $\left(\sqrt{\frac{q}{p}}+\sqrt[10]{\frac{p^7}{q^3}}\right)^n$, there is a term similar to pq, then that term is equal to

(A) 210 pq (B) 252 pq (C) 120 pq (D) 45 pq

Key. B

7. Let x, y, z be real numbers such that 3x, 4y and 5z form a geometric progression while x, y, z form an H.P. Then the value of $\frac{x}{z}+\frac{z}{x}=\frac{m}{n}$ where *m* and *n* are relatively prime then, (m + n) is equal to

(A) 29 (B) 39 (C) 49 (D) 59

Key. C

8. If A is a square matrix fo order 3 much that $|A| = 2$ then $\left|\left(adj\ A^{-1}\right)^{-1}\right|$ is

(A) 1 (B) 2 (C) 4 (D) 8

Key. C

9. Let A and B be square matrices of same order satisfying AB = A and BA = B. Then A^2B^2 equals, (O being the zero matrix of the same order as B)

(A) A (B) B (C) I (D) O

Key: A

Hint Conceptual

10. If A and B are square matrices of the same order and A is non-singular, then for a positive integer *n,* $(A^{-1} BA)^n$ is equal to

A) $A^{-n} B^n A^n$ B) $A^n B^n A^{-n}$ C) $A^{-1} B^n A$ D) $n(A^{-1} BA)$

Key: C

Hint: $(A^{-1}BA)^2 = (A^{-1}BA)(A^{-1}BA) = A^{-1}B(AA^{-1})BA = A^{-1}BIBA = A^{-1}B^2A$

$\Rightarrow (A^{-1}BA)^3 = (A^{-1}B^2A)(A^{-1}BA) = A^{-1}B^2(AA^{-1})BA = A^{-1}B^2IBA = A^{-1}B^3A$ and so on

$\Rightarrow (A^{-1}BA)^n = A^{-1}B^nA$

11. If A is a skew–symmetric matrix of order 3, then the matrix A^4 is

(A) skew symmetric (B) symmetric (C) diagonal (D) none of those

Key: B

Hint: We have $A^T = -A$

$(A^4)^T = (A.A.A.A.)^T = A^T A^T A^T A^T$

$\Rightarrow (-A) (-A) (-A) (-A)$

$= (-1)^4 A^4 = A^4$

12. If A and B are symmetric matrices of same order and X= AB + BA and Y = AB – BA, then $\left(XY\right)^T$ is equal to

(A) XY (B)YX (C) – YX (D)none of these

Key: C

Hint: X = AB + BA $\Rightarrow X^T = X$

and Y = AB – BA $\Rightarrow Y^T = -Y$

Now, $\left(XY\right)^T = Y^T \times X^T = -YX.$

13. If A and B are any two different square matrices of order n with A – B is non-singular $A^3 = B^3$ and A(AB) = B(BA) , then

(A) $A^2 + B^2 = O$ (B) $A^2 + B^2 = I$ (C) $A^2 + B^3 = I$ (D) $A^3 + B^3 = O$

Key: A

Hint: $A^3 = B^3$.......(i)

$A^2B = B^2A$.................(ii)

$$\left(A^2+B^2\right)(A-B)=0$$

$$\because |A-B| \neq 0$$

$$A^2+B^2=0$$

14. A square matrix A is said to be nilpotent of index m. If $A^m = 0$, now, if for this A $(I - A)^n = I + A + A^2 + ... + A^{m-1}$, then n is equal to

(A) 0 (B) m (C) - m (D) -1

Key: D

Hint: Let $B = I + A + A^2 + ... + A^{m-1}$

$\Rightarrow B (I - A) = (I + A + A^2 + ... + A^{m-1}) (I - A)$

$= I - A^m = I$

$\Rightarrow B = (I - A)^{-1} \Rightarrow n = -1.$

15. If $A=\begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ and $B=\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then A^8 equals

(a) 4B (b)128 B (c) –128 B (d) –64 B

Key: b

Hint: We have A = iB

$\Rightarrow A^2 = (iB)^2 = i^2B^2 = -B^2 = -\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = -2B$

$\Rightarrow A^4 = (-2B)^2 = 4B^2 = 4(2B) = 8B$

$\Rightarrow (A^4)^2 = (8B)^2 \Rightarrow A^8 = 64B^2 = 128B$

16. The number of positive integral solutions of the equation $\begin{vmatrix} y^3+1 & y^2z & y^2x \\ yz^2 & z^3+1 & z^2x \\ yx^2 & x^2z & x^3+1 \end{vmatrix}=11$ is

(A) 1 (B) 2 (C) 3 (D) 4

Key: C

Hint: Multiply by y,z and x in rows 1,2 and 3 respectively and then take common y, z and x from column 1,2 and 3 respectively, then

$$\begin{vmatrix} y^3+1 & y^3 & y^3 \\ z^3 & z^3+1 & z^3 \\ x^3 & x^3 & x^3+1 \end{vmatrix}=11$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & y^3 \\ -1 & 1 & z^3 \\ 0 & -1 & x^3+1 \end{vmatrix}=11 \qquad \left(C_1 \to C_1 - C_2 \text{ and } C_2 \to C_2 - C_3\right)$$

$\Rightarrow 1\left(x^3+1+z^3\right)+y^3(1)=11 \Rightarrow x^3+y^3+z^3=10$

So solution are (1,1,2) , (1,2,1) or (2,1,1)

17. If $a - 2b + c = 1$, then the value of $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ is

(A) x (B) $-x$ (C) -1 (D) 1

Key. C

Sol. $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ $\quad a-2b+c=1$, $(a-b)+(c-b)=1$

Apply the operation,

$R_1 \to R_1 - 2R_2 + R_3$

$R_3 \to R_3 - R_2$, the determinant reduces to

$\begin{vmatrix} 0 & 0 & 1 \\ x+2 & x+3 & x+b \\ 1 & 1 & c-b \end{vmatrix} = -1$

18. If A is involutary matrix, then which of the following is/are correct?

(A) I + A is idempotent (B) I – A is idempotent

(C) (I + A)(I – A) is singular (D) $\frac{I+A}{3}$ is idempotent

Key. C

Sol. $A^2 = I$

$(I + A)(I - A) = I - A^2 = I - I = O$

19. If $A=\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$, $B=\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, $C = ABA^T$, then A^TC^nA equals to $\left(n \in I^+\right)$

(A) $\begin{bmatrix} -n & 1 \\ 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -n \\ 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 1 \\ 1 & -n \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$

Key. D

Sol. $A=\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$

$AA^T = I$ (i)

Now, $C = ABA^T$

$\Rightarrow$ $A^TC = BA^T$ (ii)

Now $A^TC^nA = A^TC.C^{n-1}A = BA^TC^{n-1}A$ (from (ii))

$= BA^TC.C^{n-2}A = B^2A^TC^{n-2}A = \ldots\ldots\ldots$

$= B^{n-1}A^TCA = B^{n-1}BA^TA = B^n = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$

20. If $p+q+r=0$ and $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$, then K=

1) 0 2) abc 3) pqr 4) a+b+c

Key. 3

Sol. $p+q+r=0 \Rightarrow p^3+q^3+r^3=3pqr$

$$\begin{vmatrix} pa & qb & rc \\ qc & rb & pb \\ rb & pc & qa \end{vmatrix} = pqr(a^3+b^3+c^3-3abc)$$

$$pqr\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} \Rightarrow k = pqr$$

21. If $a=\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}$, then $\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix}$ is

1) purely real 2) purely imaginary

3) a complex number 4) a

Key. 2 or 3

Sol. $a=\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}=w^2$

$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & w^2 & w \\ 1 & w & w^2 \end{vmatrix} = 3(w-w^2)$ purely imaginary

22. $\begin{vmatrix} {}^xC_r & {}^xC_{r+1} & {}^xC_{r+2} \\ {}^yC_r & {}^yC_{r+1} & {}^yC_{r+2} \\ {}^zC_r & {}^zC_{r+1} & {}^zC_{r+2} \end{vmatrix} - \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix} =$

1) 0 2) 2^n 3) ${}^{x+y+z}C_r$ 4) ${}^{x+y+z}C_{r+2}$

Key. 1

Sol. $\begin{vmatrix} {}^xC_r & {}^xC_{r+1} & {}^xC_{r+2} \\ {}^yC_r & {}^yC_{r+1} & {}^yC_{r+2} \\ {}^zC_r & {}^zC_{r+1} & {}^zC_{r+2} \end{vmatrix} = \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+1}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+1}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+1}C_{r+2} \end{vmatrix}$

By applying $C_2 \to C_2+C_1, C_3 \to C_3+C_2$

Now apply $C_3 \to C_3+C_2$, $\begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$

$\therefore$ Ans = 0

23. If A is an orthogonal matrix of order n, then the value of |adj.(adj A)| is
(A) 0 (B) ± 1
(C) n (D) n – 2

Key. B

Sol. $AA' = I$
$\Rightarrow$ $|A| = \pm 1$
$\therefore$ |adj. (adj. A)|
$= |A|^{(n-1)^2} = \pm 1.$

24. If a, b, c, d > 0; x ∈ R and $(a^2 + b^2 + c^2)x^2 - 2(ab + bc + cd)x + b^2 + c^2 + d^2 \le 0$, then
$$\begin{vmatrix} 33 & 14 & \log a \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} =$$
(A) 1 (B) – 1
(C) 0 (D) none of these

Key. C

Sol. We have
$(a^2 + b^2 + c^2)x^2 - 2(ab + bc + cd)x + b^2 + c^2 + d^2 \le 0$
$\Rightarrow$ $(ax - b)^2 + (bx - c)^2 + (cx - d)^2 \le 0$
$\Rightarrow$ $(ax - b)^2 + (bx - c)^2 + (cx - d)^2 = 0$
$\Rightarrow$ $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = x$
$\Rightarrow$ $b^2 = ac$ or $2\log b = \log a + \log c$,

Now, $$\begin{vmatrix} 33 & 14 & \log a \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} = \begin{vmatrix} 130 & 54 & \log a + \log c \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix}$$ [Apply $R_1 \to R_1 + R_3$]

$$\begin{vmatrix} 0 & 0 & 0 \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} = 0$$ [Apply $R_1 \to R_1 - 2R_2$]

25. A square matrix P satisfies $P^2 = I - P$, where I is an identity matrix of order as order of P. if $P^n = 5I - 8P$, then n =
(a) 4 (b) 5
(c) 6 (d) 7

Key. C

SOL. SINCE $P^2 = I - P$ (GIVEN) ——(1)
$P^3 = P(I - P)$
$P^3 = P - P^2 = P - (I - P)$ (USING) —— (II)
$P^3 = 2P - I$
SIMILARLY $P^4 = 2P^2 - P = 2I - 3P$ AND $P^5 = 5P - 3I$
$P^6 = 5P^2 - 3P = 5I - 8P$
$\therefore$ $n = 6$

26. If $Y = SX, Z = tX$ all the variables being differentiable functions of x and lower suffices denote the derivative with respect to x and $\begin{vmatrix} X & Y & Z \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix} \div \begin{vmatrix} S_1 & t_1 \\ S_2 & t_2 \end{vmatrix} = X^n$, then $n =$
a) 1 b) 2 c) 3 d) 4

Key. C

Sol. $\Delta = \begin{vmatrix} X & SX & tX \\ X_1 & SX_1 + S_1X & tX_1 + t_1X \\ X_2 & SX_2 + 2S_1X_1 + S_2X & tX_2 + 2t_1X_1 + t_2X \end{vmatrix}$

$$\begin{pmatrix} C_2 \leftarrow C_2 - SC_1 \\ C_3 \leftarrow C_3 - C_1 \end{pmatrix}$$

$= \Delta = \begin{vmatrix} X & 0 & 0 \\ X_1 & S_1X & t_1X \\ X_2 & 2S_1X_1 + S_2X & 2t_1X_1 + t_2X \end{vmatrix}$

$= S^2 \begin{vmatrix} S_1 & t_1 \\ 2S_1X_1 + S_2X & 2t_1X_1 + t_2X \end{vmatrix}$

$= = X^3 \leq \begin{vmatrix} S_1 & t_1 \\ S_2 & t_2 \end{vmatrix} \left(R_2 \leftarrow R_2 - 2X_1R_1\right)$

$\therefore n = 3.$

27. If A and B are two non singular matrices and both are symmetric and commute each other then

a) Both $A^{-1}B$ and $A^{-1}B^{-1}$ are symmetric.
b) $A^{-1}B$ is symmetric but $A^{-1}B^{-1}$ is not symmetric
c) $A^{-1}B^{-1}$ is symmetric but $A^{-1}B$ is not symmetric
d) Neither $A^{-1}B$ nor $A^{-1}B^{-1}$ are symmetric

Key. A

Sol. $AB = BA$

Previous & past multiplying both sides by A^{-1}.

$A^{-1}(AB)A^{-1} = A^{-1}(BA)A^{-1}$

$(A^{-1}A)(BA^{-1}) = A^{-1}B(AA^{-1})$

$\Rightarrow (BA^{-1})^1 = (A^{-1}B)^1 = (A^{-1})^1 B^1 (\text{reversal laws})$

$= A^{-1}B(\text{ as B=B}^1)$

$(A^{-1})^1 = A^{-1} \Rightarrow A^{-1}B$ is symmetric

Similarly for $A^{-1}B^{-1}$.

28. If $f(x) = ax^2 + bx + c \quad a,b,c \in R$ and the equation $f(x) - x = 0$ has imaginary roots α and β and γ and δ be the roots of $f(f(x)) - x = 0$, then $\begin{vmatrix} 2 & \alpha & \delta \\ \beta & 0 & \alpha \\ \gamma & \beta & 1 \end{vmatrix}$ is

a) 0 b) purely real c) purely imaginary d) none of these

Key. B

Sol. $f(x) - x > 0 \text{ or, } f(x) - x < 0 \forall x \in R$

$f(f(x)) - f(x) > 0 \text{ or } f(f(x)) - f(x) < 0$

Adding, $f(f(x)) - x > 0 \text{ or, f}(f(x)) - x < 0$

$\Rightarrow$ roots of $f(f(x)) - x = 0$ are imaginary.

Let $z = \begin{vmatrix} 2 & \alpha & \delta \\ \beta & 0 & \alpha \\ \gamma & \beta & 1 \end{vmatrix}$

$$\bar{z} = \begin{vmatrix} 2 & \bar{\alpha} & \bar{\delta} \\ \bar{\beta} & 0 & \bar{\alpha} \\ \bar{\gamma} & \bar{\beta} & 1 \end{vmatrix} = \begin{vmatrix} 2 & \beta & \gamma \\ \alpha & 0 & \beta \\ \delta & \alpha & 1 \end{vmatrix} = z$$

29. Suppose a Matrix A satisfies $A^2 - 5A + 7I = 0$ If $A^5 = aA + bI$, then the values of $2a + b$ is.

a) -87 b) -105 c) 1453 d) 1155

Key. A

Sol. $A^3 = AA^2 = A(5A - 7I)$

$= 5A^2 - 7A = 5(5A - 7I) - 7A = 18A - 35I$

$A^4 = A.A^3 = A(18A - 35I) = 18(5A - 7I) - 35A$

$A^5 = 149A - 385I \qquad = 55A - 126I$

$A^5 = 149A - 385I$

$a = 149, b = -385$

30. The digits A, B, C are such that the three digit numbers A88, 6B8, 86C are divisible by 72, then the determinant $\begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 8 & 8 & C \end{vmatrix}$ is divisible by

a) 76 b) 144 c) 216 d) 276

Key. B

Sol. $100A + 80 + 8 = 72\lambda_1$

$600 + 10B + 8 = 72\lambda_2 \quad \lambda_1, \lambda_2, \lambda_3 \in I.$

$800 + 60 + C = 72\lambda_3$

$\begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 8 & 8 & C \end{vmatrix} (R_3 \leftarrow R_3 + 10R_2 + 100R_1)$

$= \begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 72\lambda_1 & 72\lambda_2 & 72\lambda_3 \end{vmatrix}$

A88 is div. by 72

$\Rightarrow$ A88 is div. by 9

$\Rightarrow$ A+8+8 is div. by 9

$\therefore A = 2$

$6B8$ is div. by 9 $\Rightarrow B = 4.$

31. If the matrix $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ is invertible, then the planes $a_{11}x + a_{12}y + a_{13}z = 0$, $a_{21}x + a_{22}y + a_{23}z = 0$ and $a_{31}x + a_{32}y + a_{33}z = 0$ ($a_{ij} \in R, \forall i, j$)

(A) intersect in a point (B) intersect in a line
(C) have no common point (D) are same

Key. A

Sol. Given matrix A is invertible $\Rightarrow$ det A $\neq$ 0
$\Rightarrow$ the given system of equation has only one solution
i.e., (0, 0, 0). Hence option (A) is correct.

32. If A is a skew–symmetric matrix of order 3, then the matrix A^4 is
(A) skew symmetric (B) symmetric
(C) diagonal (D) none of those

Key. B

Sol. We have $A^T = -A$
$(A^4)^T = (A.A.A.A.)^T = A^T A^T A^T A^T$
$\Rightarrow (-A)(-A)(-A)(-A)$
$= (-1)^4 A^4 = A^4$

33. If 'α' is a root of $x^4 = 1$ with negative principal argument, then the principal argument of $\Delta(\alpha)$ where

$$\Delta(\alpha) = \begin{vmatrix} 1 & 1 & 1 \\ \alpha^n & \alpha^{n+1} & \alpha^{n+3} \\ \frac{1}{\alpha^{n+1}} & \frac{1}{\alpha^n} & 0 \end{vmatrix} \text{ is}$$

(A) $\frac{5\pi}{14}$ (B) $-\frac{3\pi}{4}$
(C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{4}$

Key. B

Sol. Clearly $\alpha = -i$ where $i^2 = -1$

So $$\Delta(\alpha) = \alpha^n \frac{1}{\alpha^n} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^3 \\ \frac{1}{\alpha} & 1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -i & i \\ i & 1 & 0 \end{vmatrix} = 1(-i) + 1(i^2) + (1 + i^2) = -1 - i$$

So, principal argument of $\Delta(\alpha)$ is $-\frac{3\pi}{4}$

34. If z is a complex number and $l_1, l_2, l_3, m_1, m_2, m_3$ are all real, then

$$\begin{vmatrix} l_1 z + m_1\bar{z} & m_1 z + l_1\bar{z} & m_1 z + l_1 \\ l_2 z + m_2\bar{z} & m_2 z + l_2\bar{z} & m_2 z + l_2 \\ l_3 z + m_3\bar{z} & m_3 z + l_3\bar{z} & m_3 z + l_3 \end{vmatrix} \text{ is equal to}$$

(A) $|z|^2$ (B) 3
(C) $(l_1 l_2 l_3 + m_1 m_2 m_3)^2 |z|^2$ (D) 0

Key. D

Sol. $$\begin{vmatrix} l_1 & m_1 & 0 \\ l_2 & m_2 & 0 \\ l_3 & m_3 & 0 \end{vmatrix} \times \begin{vmatrix} z & \bar{z} & 0 \\ \bar{z} & z & 0 \\ 1 & z & 0 \end{vmatrix} = 0$$

35. Let A, B be square matrix such that A B = 0 and B is non singular then
(A) |A| must be zero but A may non zero (B) A must be zero matrix
(C) nothing can be said in general about A (D) none of these

Key. B

Sol. A. B = 0 $\Rightarrow$ A. B . B^{-1} = 0.B^{-1}
$\Rightarrow$ A . I = 0
$\Rightarrow$ A = 0

36. The value of $\begin{vmatrix} \sqrt{6} & 2i & 3+\sqrt{6} \\ \sqrt{12} & \sqrt{3}+\sqrt{8}i & 3\sqrt{2}+\sqrt{6}i \\ \sqrt{18} & \sqrt{2}+\sqrt{12}i & \sqrt{27}+2i \end{vmatrix}$ is

a) an integer b) a rational number c) an irrational number d) an imaginary number

Key. A

Sol. Take $\sqrt{6}$ common from C_1 and apply $C_3 \to C_3 - 3C_1,\ C_2 \to C_2 - 2iC_1$

37. If p + q + r = 0 and $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = K \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ then the value of K is

a) p + q –r b) p + q + r c) pqr d) - pqr

Key. D

Sol. $pqr\left(a^3+b^3+c^3\right)-abc\left(p^3+q^3+r^3\right)$

$\Rightarrow pqr\left(a^3+b^3+c^3-3abc\right)-abc\left(p^3+q^3+r^3-3pqr\right)$

$\Rightarrow pqr\left(a^3+b^3+c^3-3abc\right)-abc\left(p+q+r\right)\left(p^2+q^2+r^2-pq-qr-rp\right)$

$= pqr\left(a^3+b^3+c^3-3abc\right)$

38. If f(x), g(x), h(x) are polynomials of degree 4 and $\begin{vmatrix} f(x) & g(x) & h(x) \\ a & b & c \\ p & q & r \end{vmatrix} =$

$mx^4+nx^3+rx^2+5x+t$ be an identity in x, then the value of

$\begin{vmatrix} f'''(0)-f''(0) & g'''(0)-g''(0) & h'''(0)-h''(0) \\ a & b & c \\ p & q & r \end{vmatrix}$ is

a) $(3n-r)$ b) $2(3n-r)$ c) $3(3n-r)$ d) $3n+r$

Key. B

Sol. LHS = $(24mx+6n)-(12mx^2+6nx+2r)$

$x=0 \Rightarrow 6n-2r$

$\Rightarrow 2(3n-r)$

39. Let x > 0, y > 0, z > 0 are respectively the 2nd, 3rd, 4th, terms of a G.P and

$$\Delta = \begin{vmatrix} x^k & x^{k+1} & x^{k+2} \\ y^k & y^{k+1} & y^{k+2} \\ z^k & z^{k+1} & z^{k+2} \end{vmatrix} = (r-1)^2\left(1-\frac{1}{r^2}\right)$$ (where r is the common ratio) then

a) $k=-1$ b) $k=1$ c) $k=0$ d) None of these

Key. A

Sol. $x^k y^k z^k \begin{vmatrix} 1 & ar & a^2r^2 \\ 1 & ar^2 & a^2r^4 \\ 1 & ar^3 & a^3r^6 \end{vmatrix}$

$a^{3(k+1)}.r^{3(2k+1)}\left[(r-1)(r^4-1)-(r^2-1)^2\right] \Rightarrow k=-1$

40. If $f(x) = \begin{vmatrix} x^2-4x+6 & 2x^2+4x+10 & 3x^2-2x+16 \\ x-2 & 2x+2 & 3x-1 \\ 1 & 2 & 3 \end{vmatrix}$. Then the value of

$\int_{-3}^{3} \frac{x^2 \sin x}{1+x^6}.f(x)dx$ is

a) 6 b) 3 c) 0 d) $\frac{\pi}{2}$

Key. C

Sol. $f(x)$ is const.

Hence = 0

41. If A and B are square matrices of order 3 such that $|A|=-1, |B|=3$ then $|3AB|$ is equals to

A) –9 B) –81 C) –27 D) 81

Key. B

Sol. $|3AB| = 3^3|A||B| = 27\times -1\times 3 = -81$

42. A = $[a_{ij}]_{n\times n}$ and a_{ij} = i^2-j^2 then A is necessarily

a) a unit matrix b) symmetric matrix c) skew symmetric matrix d) zero matrix

Key. C

Sol. $a_{ji} = j^2 - i^2 = (i^2 - j^2) = -a_{ij}$

43. If $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$ is an orthogonal matrix then value of x+y is equal to

a) –3 b) 0 c) 1 d) 3

Key. A

Sol. $A.A^T = I \Rightarrow \frac{1}{9}\begin{bmatrix} 9 & 0 & x+4+2y \\ 0 & 9 & 2x+2-2y \\ x+4+2y & 2x+2-2y & x^2+y^2+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

x+4+2y = 0, 2x+2–2y = 0 $\Rightarrow$ x = –2, y = –1

44. If $A=\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ then A^{16} =

a) $\begin{bmatrix} 0 & 256 \\ 256 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$

c) $\begin{bmatrix} -16 & 0 \\ 0 & -16 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 16 \\ 16 & 0 \end{bmatrix}$

Key. B

Sol. $A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$, $A^4 = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$ $A^8 = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$, $A^{16} = \begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$

45. Let a, b, c be positive real numbers. Then the following system of equations in x, y, z

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1, \frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1, -\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ has

a) no solution b) unique solution

c) infinite solution d) finitely many solution

Key. D

Sol. Let $\frac{x^2}{a^2}=X$, $\frac{y^2}{b^2}=Y$, $\frac{z^2}{c^2}=Z$

$X+Y-Z=1$, $X-Y+Z=1$, $-X+Y+Z=1$ on solving $X=Y=Z=1$

$\Rightarrow x=\pm a, y=\pm b, z=\pm c$ $\Rightarrow$ 8 solution

46. The value of determinant $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$ is equal to

a) $(1-a^2-b^2)^3$ b) $(a+b+1)^2\,(ab+b+a)$ c) $(1+a^2+b^2)^3$ d) $(1-a^2+b^2)^3$

Key. C

Sol. $\Delta = \frac{1}{ab}\begin{vmatrix} b(1+a^2-b^2) & 2ab^2 & -2b^2 \\ 2a^2b & a(1-a^2+b^2) & 2a^2 \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$ by $(R_1 \times b, R_2 \times a)$

$= \begin{vmatrix} 1+a^2-b^2 & 2b^2 & -2b^2 \\ 2a^2 & 1-a^2+b^2 & 2a^2 \\ 2 & -2 & 1-a^2-b^2 \end{vmatrix}$ by $\left(\frac{C_1}{b}, \frac{C_2}{a}\right)$

$\begin{vmatrix} 1+a^2+b^2 & 0 & -2b^2 \\ 1+a^2+b^2 & 1+a^2+b^2 & 2a^2 \\ 0 & -(1+a^2+b^2) & 1-a^2-b^2 \end{vmatrix}(C_1 \to C_1+C_2, C_2 \to C_2+C_3)$

$=(1+a^2+b^2)^3$

47. If $\begin{vmatrix} a^2+\lambda^2 & ab+c\lambda & ca-b\lambda \\ ab-c\lambda & b^2+\lambda^2 & bc+a\lambda \\ ac+b\lambda & bc-a\lambda & c^2+\lambda^2 \end{vmatrix} \begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix} = (1+a^2+b^2+c^2)^3$ then λ is equal to

a) 0 b) 1 c) –1 d) ± 1

Key. B

Sol. If $\Delta = \begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}$ other determinant (say Δ^1) is the cofactor determinant

$\Delta\Delta^1 = \Delta^3$ (for 3rd order det)

$\Delta = \lambda(\lambda^2+a^2+b^2+c^2)$ by comparing $\lambda = 1$

48. Constant term in f(x) = $\begin{vmatrix} x & (1+\sin x)^3 & \cos x \\ 1 & \ln(1+x) & 2 \\ x^2 & (1+x)^2 & 0 \end{vmatrix}$ when f(x) is expressed polynomial in x, is

a) 0 b) –1 c) 1 d) 2

Key. C

Sol. $f(0) = +1$

49. If A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, B = $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, a, b $\in$ N, then number of matrix 'B' such that AB = BA are

a) 0 b) 1 c) finitely many d) infinite

Key. D

Sol. $AB = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}, BA = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$

AB= BA $\Rightarrow$ a= b

50. If $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then f(α+β) =

a) f(α) + f(β) b) f(α).f(β) c) f(α)–f(β) d) 0

Key. B

Sol. f(α).f(β) = $\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & 0 \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

51. The system of equations x + ky + 3z = 0, 3x + ky – 2z = 0, 2x + 3y – 4z = 0 possess a non-trivial solution over the set of rationals, then 2k is an integral element of the interval

A) [10, 20] B) (20, 30) C) [30, 40] D) (40, 50)

Key. C

Sol. For the given system to have a non-trival solution, we must have

$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0 \quad \Rightarrow \quad \begin{vmatrix} 1 & k & 3 \\ 0 & -2k & -11 \\ 0 & 3-2k & -10 \end{vmatrix} = 0$

[Applying $R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$]

$\Rightarrow \quad 20k+11(3-2k)=0 \quad \Rightarrow \quad k=\frac{33}{2}$

52. If p + q + r = 0 = a + b + c, then the value of the determinant $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$ is

A) 0 B) pq + qb + rc C) 1 D) none of these

Key. A

Sol. $\begin{vmatrix} pq & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = pqr\left(a^3+b^3+c^3\right)-abc\left(p^3+q^3+r^3\right)=pqr(3\,abc)-abc(3\,pqr)=0$

53. Let A and B are two non-singular square matrices, A^T and B^T are the transpose matrices of A and B respectively, then which of the following is correct

A) B^TAB is symmetric matrix if and only if A is symmetric

B) B^TAB is symmetric matrix if and only if B is symmetric

C) B^TAB is skew symmetric matrix for every matrix A

D) B^TAB is skew symmetric matrix if B is skew symmetric

Key. A

Sol. $\left(B^TAB\right)^T = B^TA^T\left(B^T\right)^T = B^TA^TB$

$= B^TAB$ iff A is symmetric

$\therefore$ B^TAB is symmetric iff A is symmetric

Also $\left(B^TAB\right)^T = B^TA^TB = (-B)A^TB$

$\therefore$ B^TAB is not skew symmetric if B is skew symmetric

54. If A and B are two square matrices of order 3 × 3 which satisfy AB = A and BA = B then $(A+B)^7$ is

A) 7(A + B) B) $7.I_{3\times3}$ C) 64(A + B) D) $128I_{3\times3}$

Key. C

Sol. AB = A, BA = B $\Rightarrow$ $A^2 = A$ and $B^2 = B$

$(A+B)^2 = A^2+B^2+AB+BA$

= A + B + A + B = 2 (A + B)

$(A+B)^3 = (A+B)^2(A+B) = 2(A+B)^2 = 2^2(A+B)$

$\therefore$ $(A+B)^7 = 2^6(A+B) = 64(A+B)$

55. $|A_{3\times3}|=3, |B_{3\times3}|=-1$ and $|C_{2\times2}|=+2$ then |2ABC| =

A) $2^3(6)$ B) $2^3(-6)$ C) 2(-6) D) none of these

Key. D

Sol. 2ABC is not defined

$\therefore$ there is no solution

56. If A is a non-diagonal involutory matrix, then

A) A – I = 0 B) A + I = 0

C) A – I is non zero singular D) none of these

Key. C

Sol. $A^2 = I \Rightarrow A^2 - I = O$

$\Rightarrow$ (A + I) (A – I) = O

$\therefore$ either |A + I| = 0 or

|A – I| = 0

If |A – I| $\neq$ 0, then (A + I) (A – I) = O $\Rightarrow$ A + I = O which is not so

$\therefore$ |A – I| = 0 and A – I $\neq$ O.

57. If $A^3 = O$, then $I + A + A^2$ equals

A) I – A B) $(I - A)^{-1}$ C) $(I + A)^{-1}$ D) none of these

Key. B

Sol. $A^3 = 0$

$(I + A + A^2)(I - A) = I - A^3 = I$

$\therefore$ $I + A + A^2 = (I - A)^{-1}$

58. If a determinant of order 3 × 3 is formed by using the numbers 1 or – 1 then minimum value of determinant is

A) – 2 B) – 4 C) 0 D) – 8

Key. B

Sol. Let $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$c_2 \to c_2 - \frac{a_{12}}{a_{11}} C_1$ $\quad C_3 \to \quad C_3 - \frac{a_{13}}{a_{11}} C_1$

$$\begin{vmatrix} a_{11} & 0 & 0 \\ a_{21} & \left(a_{22} - \frac{a_{12}}{a_{11}} \times a_{21}\right) & \left(a_{23} - \frac{a_{13}}{a_{11}} a_{21}\right) \\ a_{31} & \left(a_{32} - \frac{a_{12}}{a_{11}} \times a_{31}\right) & \left(a_{32} - \frac{a_{13}}{a_{11}} \times a_{31}\right) \end{vmatrix}$$ so minimum value = - 4

59. If A is a diagonal matrix of order 3 × 3 is commutative with every square matrix of order 3 × 3 under multiplication and trace (A) = 12, then

A) |A| = 64 B) |A| = 16 C) |A| = 12 D) |A| = 0

Key. A

Sol. A diagonal matrix is commutative with every square matrix if it is scalar matrix so every diagonal element is 4.

$\therefore$ |A| = 64

60. If A is a square matrix of order 3 such that |A| = 2 then $\left|\left(\text{adj } A^{-1}\right)^{-1}\right|$ is

A) 1 B) 2 C) 4 D) 8

Key. C

Sol. $\left|\text{adj}A^{-1}\right| = \left|A^{-1}\right|^2 = \frac{1}{|A|^2}$

$\left|\left(\text{adj}A^{-1}\right)^{-1}\right| = \frac{1}{\left|\text{adj } A^{-1}\right|} = |A|^2 = 2^2 = 4$

61. If A and B are two matrices such that AB = B and BA = A, then

A) $\left(A^6 - B^5\right)^3 = A - B$ B) $\left(A^5 - B^5\right)^3 = A^3 - B^3$

C) A – B is idempotent D) A – B is nilpotent

Key. D

Sol. Since AB = B and BA = A

$\therefore$ A and B both are idempotent

$\left(A - B\right)^2 = A^2 - AB - BA + B^2 = A - B - A + B = 0$

$\therefore$ A – B is nilpotent

62. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ are two matrices such that AB = BA and $c \neq 0$, then value of $\frac{a-d}{3b-c}$ is:

A) 0 B) 2 C) – 2 D) – 1

Key. D

Sol. $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix}$

$BA = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a+3b & 2a+4b \\ c+3d & 2c+4d \end{bmatrix}$

If AB = BA, then a + 2c = a + 3b

$\Rightarrow$ 2c = 3b $\Rightarrow b \neq 0$

b + 2d = 2a + 4b

$\Rightarrow$ 2a – 2d = - 3b

$$\frac{a-d}{3b-c} = \frac{-\frac{3}{2}b}{3b - \frac{3}{2}b} = -1$$

63. Let $f(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $\left(f(\alpha)\right)^{-1}$ is equal to

A) $f(\alpha)$ B) $f(-\alpha)$ C) $f(\alpha - 1)$ D) none

Key. B

Sol. $\left(f(\alpha)\right)^{-1} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(-\alpha)$

64. A and B are square matrices and A is non-singular matrix, $\left(A^{-1}BA\right)^n$, $n \in I^+$, is equal to

A) $A^{-n}B^nA^n$ B) $A^nB^nA^{-n}$ C) $A^{-1}B^nA$ D) $A^{-n}BA^n$

Key. C

Sol. For n = 2 $\Rightarrow$ $\left(A^{-1}BA\right)\left(A^{-1}BA\right) = A^{-1}B^2A$

$\left(A^{-1}B^3A\right)$ and soon

Thus $\left(A^{-1}BA\right)^n = A^{-1}B^nA$

65. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then A =

A) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ D) $-\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Key. A

Sol. $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix} (-1)$

$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

23. If $A = \begin{bmatrix} 5 & -6 \\ 1 & -1 \end{bmatrix}$ then determinant of $A^{1003} - 5A^{1002}$ is

(A) 1 (B) 2

(C) 4 (D) 6

Key. D

Sol. $\left|A^{1003} - 5A^{1002}\right| = \left|A^{1002}\left(A - 5I\right)\right|$

$= \left|A^{1002}\right| \left|A - 5I\right|$

$= \left|A\right|^{1002} \left|A - 5I\right|$

$= 1 \times \begin{vmatrix} 0 & -6 \\ 1 & -6 \end{vmatrix} = 6$