Date: 19/10/2021

Q1. Find the differential equation of the family of curves $y = Ae^{2x} + B \cdot e^{-2x}$.

Q2. Find the differential equation of all non-vertical lines in a plane.

Q3. Find the differential equation of the family of lines through the origin.

Q4. If $\frac{dy}{dx} = e^{-2y}$ and $y = 0$ when $x = 5$, then the value of x when $y = 3$.

Q5. Find the solution of $\frac{dy}{dx} = 2^{y-x}$.

Q6. Find the differential equation of all non-horizontal lines in a plane.

Q7. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$.

Q8. Solve $\frac{dy}{dx} + 2xy = y$.

Q9. Solve the differential equation $\frac{dy}{dx} + 1 = e^{x+y}$.

Q10. Form the differential equation having $y = (\sin^{-1} x)^2 + A \cos^{-1} x + B$, where A and B are arbitrary constants, as its general solution.

Q11. Find the differential equation of system of concentric circles with centre (1, 2).

Q12. Find the general solution of $\frac{dy}{dx} + ay = e^{mx}$.

Q13. Given that $\frac{dy}{dx} = ye^x$ and $x = 0$, when $y = e$. Find the value of y when $x = 1$.

Q14. Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$.

Q15. Find the equation of a curve whose tangent at any point on it, different from origin, has slope $y + \frac{y}{x}$.

Q16. From the differential equation by eliminating A and B in $Ax^2 + By^2 = 1$.

Q17. Solve $ydx - xdy = x^2 ydx$.

Q18. Find the equation of a curve passing through origin and satisfying the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$.

Q19. If $y(x)$ is a solution of $\left(\frac{2 + \sin x}{1 + y}\right)\frac{dy}{dx} = -\cos x$ and $y(0) = 1$, then find the value of $y\left(\frac{\pi}{2}\right)$.

Q20. Find the general solution of $(x + 2y^3)\frac{dy}{dx} = y$.

Q21. Solve the differential equation $\frac{dy}{dx} = 1 + x + y^2 + xy^2$, when $y = 0$ and $x = 0$.

Q22. If $y(t)$ is a solution of $(1+t)\frac{dy}{dt} - ty = 1$ and $y(0) = -1$, then show that $y(1) = -\frac{1}{2}$.

Q23. Find the general solution of $y^2 dx + (x^2 - xy + y^2)\, dy = 0$.

Q24. Find the general solution of the differential equation $(1+y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$.

Q25. Solve $x^2 \frac{dy}{dx} = x^2 + xy + y^2$.

Q26. Solve $(x+y)(dx - dy) = dx + dy$.

Q27. Solve $2(y+3) - xy\frac{dy}{dx} = 0$, given that $y(1) = -2$.

Q28. Solve the differential equation $dy = \cos x\,(2 - y \operatorname{cosec} x)\, dx$ given that $y = 2$, when $x = \frac{\pi}{2}$.

Q29. Solve the differential equation $(1+y^2)\tan^{-1} x\, dx + 2y(1+x^2)\, dy = 0$.

Q30. Find the general solution of $\frac{dy}{dx} - 3y = \sin 2x$.

Q31. Solve $\frac{dy}{dx} = \cos(x+y) + \sin(x+y)$.

Q32. Find the general solution of $(1+\tan y)(dx - dy) + 2x\, dy = 0$.

Q33. Solve $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$.

Q34. Find the equation of a curve passing through (2, 1), if the slope of the tangent to the curve at any point (x, y) is $\frac{x^2+y^2}{2xy}$.

Q35. Find the equation of the curve through the point (1, 0), if the slope of the tangent to the curve at any point (x, y) is $\frac{y-1}{x^2+x}$.

Q36. Find the equation of a curve passing through origin, if the slope of the tangent to the curve at any point (x, y) is equal to the square of the difference of the abcissa and ordinate of the point.

Q37. Find the equation of a curve passing through the point (1, 1), if the tangent drawn at any point $P(x, y)$ on the curve meets the coordinate axes at A and B such that P is the mid-point of AB.

Q38. Solve $x\frac{dy}{dx} = y(\log y - \log x + 1)$.

Q39. Find the equation of a curve passing through $\left(1, \frac{\pi}{4}\right)$ if the slope of the tangent to the curve at any point $P(x, y)$ is $\frac{y}{x} - \cos^2\frac{y}{x}$.

Q40. Find the equation of a curve passing through the point (1, 1) if the perpendicular distance of the origin from the normal at any point $P(x, y)$ of the curve is equal to the distance of P from the x-axis.

Q41. Solve $x^2\frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right)$, $x \neq 0$ and $x = 1$, when $y = \frac{\pi}{2}$.

Q42. Find the solution of differential equation. $xdy - ydx = \sqrt{x^2 + y^2}\, dx$.

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S1. $$y = Ae^{2x} + B.e^{-2x}$$

$$\frac{dy}{dx} = 2Ae^{2x} - 2B.e^{-2x}$$

and $$\frac{d^2y}{dx^2} = 4Ae^{2x} + 4B.e^{-2x}$$

Thus, $$\frac{d^2y}{dx^2} = 4y \quad i.e., \quad \frac{d^2y}{dx^2} - 4y = 0.$$

S2. Since, the family of all non-vertical line is $y = mx + C$, where $m \neq \tan\frac{\pi}{2}$.

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = m$$

Again, differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = 0.$$

S3. Let $y = mx$ be the family of lines through origin.

Therefore, $$\frac{dy}{dx} = m$$

Eliminating m, we get

$$y = \frac{dy}{dx} \cdot x \quad \text{or} \quad x\frac{dy}{dx} - y = 0.$$

S4. Given that, $$\frac{dy}{dx} = e^{-2y} \Rightarrow \frac{dy}{e^{-2y}} = dx$$

$$\Rightarrow \int e^{2y} dy = \int dx \Rightarrow \frac{e^{2y}}{2} = x + C$$

When $x = 5$ and $y = 0$, then substituting these values in Eq. (i), we get

$$\frac{e^0}{2} = 5 + C$$

$$\Rightarrow \frac{1}{2} = 5 + C \Rightarrow C = \frac{1}{2} - 5 = -\frac{9}{2}$$

Eq. (i) becomes $$e^{2y} = 2x - 9$$

When $y = 3$, then $e^6 = 2x - 9 \Rightarrow 2x = e^6 + 9$

$$\therefore \quad x = \frac{(e^6 + 9)}{2}.$$

S5. Given that, $\frac{dy}{dx} = 2^{y-x}$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2^y}{2^x} \qquad \left[\because \; a^{m-n} = \frac{a^m}{a^n}\right]$$

$$\Rightarrow \quad \frac{dy}{2^y} = \frac{dx}{2^x}$$

On integration both sides, we get

$$\int 2^{-y}\, dy = \int 2^{-x}\, dx$$

$$\Rightarrow \quad \frac{-2^{-y}}{\log 2} = \frac{-2^{-x}}{\log 2} + C$$

$$\Rightarrow \quad -2^{-y} + 2^{-x} = +C \log 2$$

$$\Rightarrow \quad 2^{-x} - 2^{-y} = -C \log 2$$

$$\Rightarrow \quad 2^{-x} - 2^{-y} = K. \qquad [\text{where, } K = +C \log 2]$$

S6. The general equation of all non-horizontal lines in a plane in $ax + by = c$, where $a \neq 0$.

Therefore, $a\frac{dx}{dy} + b = 0$

Again, differentiating both sides w.r.t. y, we get

$$a\frac{d^2x}{dy^2} = 0 \quad \Rightarrow \frac{d^2x}{dy^2} = 0.$$

S7. $$\frac{dy}{dx} = \frac{y}{x} \Rightarrow \frac{dy}{y} = \frac{dx}{x} \quad \Rightarrow \quad \int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\Rightarrow \quad \log y = \log x + \log c \quad \Rightarrow \quad y = cx.$$

S8. Given that $\frac{dy}{dx} + 2xy = y$

$$\Rightarrow \quad \frac{dy}{dx} + 2xy - y = 0$$

$$\Rightarrow \quad \frac{dy}{dx} + (2x - 1)y = 0$$

which is linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$$P = (2x - 1), \quad Q = 0$$

$$IF = e^{\int P dx} = e^{\int (2x-1) dx}$$

$$= e^{\left(\frac{2x^2}{2} - x\right)} = e^{x^2 - x}$$

The complete solution is

$$y \cdot e^{x^2 - x} = \int Q \cdot e^{x^2 - x} dx + C$$

$\Rightarrow$ $$y \cdot e^{x^2 - x} = 0 + C$$

$\Rightarrow$ $$y = Ce^{x - x^2}.$$

S9. Given differential equation is

$$\frac{dy}{dx} + 1 = e^{x + y} \quad \text{... (i)}$$

On substituting $x + y = t$, we get

$$1 + \frac{dy}{dx} = \frac{dt}{dx}$$

Eq. (i) becomes $$\frac{dt}{dx} = e^t$$

$\Rightarrow$ $$e^{-t}\, dt = dx$$

$\Rightarrow$ $$-e^{-t} = x + C$$

$\Rightarrow$ $$\frac{-1}{e^{x+y}} = x + C$$

$\Rightarrow$ $$-1 = (x + C)\, e^{x+y}$$

$\Rightarrow$ $$(x + C)\, e^{x+y} + 1 = 0$$

S10. Given that, $$y = (\sin^{-1} x)^2 + A \cos^{-1} x + B$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{2 \sin^{-1} x}{\sqrt{1 - x^2}} + \frac{(-A)}{\sqrt{1 - x^2}}$$

$\Rightarrow$ $$\sqrt{1 - x^2}\, \frac{dy}{dx} = 2 \sin^{-1} x - A$$

Again, differentiating w.r.t. x, we get

$$\sqrt{1 - x^2}\, \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{-2x}{2\sqrt{1 + x^2}} = \frac{2}{\sqrt{1 - x^2}}$$

$\Rightarrow$ $$(1 - x^2)\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1 - x^2}} \cdot \sqrt{1 - x^2}\, \frac{dy}{dx} = 2$$

$\Rightarrow$ $$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$$

$$\Rightarrow \qquad (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$$

which is the required differential equation.

S11. The family of concentric circles with centre (1, 2) and radius a is given by

$$(x-1)^2 + (y-2)^2 = a^2$$

$$\Rightarrow \qquad x^2 + 1 - 2x + y^2 + 4 - 4y = a^2$$

$$\Rightarrow \qquad x^2 + y^2 - 2x - 4y + 5 = a^2 \qquad \ldots \text{(i)}$$

On differentiating Eq. (i) w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} - 2 - 4\frac{dy}{dx} = 0$$

$$\Rightarrow \qquad (2y-4)\frac{dy}{dx} + 2x - 2 = 0$$

$$\Rightarrow \qquad (y-2)\frac{dy}{dx} + (x-1) = 0.$$

S12. Given differential equation is

$$\frac{dy}{dx} + ay = e^{mx}$$

which is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$$P = a, \quad Q = e^{mx}$$

$$IF = e^{\int P dx} = e^{\int a dx} = e^{ax}$$

The general solution is $y \cdot e^{ax} = \int e^{mx} \cdot e^{ax} dx + C$

$$\Rightarrow \qquad y \cdot e^{ax} = \int e^{(m+a)x}\, dx + C$$

$$\Rightarrow \qquad y \cdot e^{ax} = \frac{e^{(m+a)x}}{(m+a)} + C$$

$$\Rightarrow \qquad (m+a)\, y = \frac{e^{(m+a)x}}{e^{ax}} + \frac{(m+a)C}{e^{ax}}$$

$$\Rightarrow \qquad (m+a)\, y = e^{mx} + K e^{-ax}. \qquad [\because K = (m+a)C]$$

S13. $$\frac{dy}{dx} = ye^x \Rightarrow \int \frac{dy}{y} = \int e^x\, dx \Rightarrow \log y = e^x + c$$

Substituting $x = 0$ and $y = e$, we get

$$\log e = e^0 + c, \qquad i.e., \quad c = 0 \qquad [\because \log e = 1]$$

Therefore, $\log y = e^x$.

Now, substituting $x = 1$ in the above, we get

$$\log y = e \quad \Rightarrow \quad y = e^e.$$

S14. The equation is of the type $\frac{dy}{dx} + Py = Q$, which is a linear differential equation.

Now, $$IF = \int \frac{1}{x}\,dx = e^{\log x} = x.$$

Therefore, solution of the given differential equation is

$$y\,.\,x = \int x \cdot x^2\,dx\,, \qquad i.e., \quad yx = \frac{x^4}{4} + c$$

Hence, $$y = \frac{x^3}{4} + \frac{c}{x}.$$

S15. Given $$\frac{dy}{dx} = y + \frac{y}{x} = y\left(1 + \frac{1}{x}\right)$$

$$\Rightarrow \quad \frac{dy}{y} = \left(1 + \frac{1}{x}\right)dx$$

Integrating both sides, we get

$$\log y = x + \log x + c \quad \Rightarrow \quad \log\left(\frac{y}{x}\right) = x + c$$

$$\Rightarrow \quad \frac{y}{x} = e^{x+c} = e^x\,.\,e^c \quad \Rightarrow \quad \frac{y}{x} = k\,.\,e^x$$

$$\Rightarrow \quad y = kx\,.\,e^x.$$

S16. Given, equation is

$$Ax^2 + By^2 = 1$$

On differentiating both sides w.r.t. x, we get

$$2Ax + 2By\frac{dy}{dx} = 0$$

$$\Rightarrow \quad 2By\,\frac{dy}{dx} = -2Ax$$

$$\Rightarrow \quad By\,\frac{dy}{dx} = -Ax \quad \Rightarrow \quad \frac{y}{x} \cdot \frac{dy}{dx} = -\frac{A}{B}$$

Again, differentiating w.r.t. x, we get

$$\frac{y}{x} \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \left(\frac{x\,\frac{dy}{dx} - y}{x^2}\right) = 0$$

$$\Rightarrow \quad \frac{y}{x}\cdot\frac{d^2y}{dx^2}+\frac{x\left(\frac{dy}{dx}\right)^2-y\left(\frac{dy}{dx}\right)}{x^2} = 0$$

$$\Rightarrow \quad xy\,\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\left(\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \quad xy\,y'' + x\,(y')^2 - y\,y' = 0.$$

S17. Given that, $ydx - xdy = x^2ydx$

$$\Rightarrow \quad \frac{1}{x^2}-\frac{1}{xy}\cdot\frac{dy}{dx} = 1 \qquad \text{[Dividing throughout by } x^2ydx\text{]}$$

$$\Rightarrow \quad -\frac{1}{xy}\cdot\frac{dy}{dx}+\frac{1}{x^2}-1 = 0$$

$$\Rightarrow \quad \frac{dy}{dx}-\frac{xy}{x^2}+xy = 0$$

$$\Rightarrow \quad \frac{dy}{dx}-\frac{y}{x}+xy = 0$$

$$\Rightarrow \quad \frac{dy}{dx}+\left(x-\frac{1}{x}\right)y = 0$$

which is a linear differential equation.

On comparing it with $\frac{dy}{dx}+Py = Q$, we get

$$P = \left(x-\frac{1}{x}\right), \quad Q = 0$$

$$IF = e^{\int P\,dx}$$

$$= e^{\int\left(x-\frac{1}{x}\right)dx}$$

$$= e^{\frac{x^2}{2}-\log x}$$

$$= e^{\frac{x^2}{2}}\cdot e^{-\log x}$$

$$= \frac{1}{x}e^{\frac{x^2}{2}}$$

The general solution is

$$y\cdot\frac{1}{x}e^{x^2/2} = \int 0\cdot\frac{1}{x}e^{x^2/2}dx + C$$

$$\Rightarrow \quad y \cdot \frac{1}{x} e^{x^2/2} = C$$

$$\Rightarrow \quad y = Cxe^{-x^2/2}.$$

S18. Given that, $(1 + x^2) \dfrac{dy}{dx} + 2xy = 4x^2$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{4x^2}{1+x^2}$$

which is a linear differential equation.

On comparing it with $\dfrac{dy}{dx} + Py = Q$, we get

$$P = \frac{2x}{1+x^2}, \quad Q = \frac{4x^2}{1+x^2}$$

$$\therefore \quad IF = e^{\int P\,dx} = e^{\int \frac{2x}{1+x^2}dx}$$

Put, $1 + x^2 = t \Rightarrow 2x\,dx = dt$

$$IF = e^{\int \frac{dt}{t}} = e^{\log t} = e^{\log(1+x^2)}$$

The general solution is

$$y \cdot (1 + x^2) = \int \frac{4x^2}{1+x^2}(1+x^2)\,dx + C$$

$$\Rightarrow \quad y \cdot (1 + x^2) = \int 4x^2\,dx + C$$

$$\Rightarrow \quad y \cdot (1 + x^2) = \frac{4x^3}{3} + C \qquad \ldots \text{(i)}$$

Since, the curve passes through origin, then substituting

$$x = 0 \quad \text{and} \quad y = 0$$

In Eq. (i), we get

$$C = 0$$

The required equation of curve is

$$y(1 + x^2) = \frac{4x^3}{3}$$

$$\Rightarrow \quad y = \frac{4x^3}{3(1+x^2)}.$$

S19. Given that, $\left(\dfrac{2 + \sin x}{1+y}\right)\dfrac{dy}{dx} = -\cos x$

$$\Rightarrow \quad \frac{dy}{1+y} = -\frac{\cos x}{2+\sin x}\,dx$$

On integrating both sides, we get

$$\int \frac{1}{1+y}\,dy = -\int \frac{\cos x}{2+\sin x}\,dx$$

$\Rightarrow$ $\log(1+y) = -\log(2+\sin x) + \log C$

$\Rightarrow$ $\log(1+y) + \log(2+\sin x) = \log C$

$\Rightarrow$ $\log\{(1+y)(2+\sin x)\} = \log C$

$\Rightarrow$ $(1+y)(2+\sin x) = C$

$\Rightarrow$ $1 + y = \dfrac{C}{2+\sin x}$

$\Rightarrow$ $y = \dfrac{C}{2+\sin x} - 1$ ... (i)

When $x = 0$ and $y = 1$, then

$$1 = \frac{C}{2} - 1$$

$\Rightarrow$ $C = 4$

On putting $C = 4$ in Eq. (i), we get

$$y = \frac{4}{2+\sin x} - 1$$

$\therefore$ $y\left(\dfrac{\pi}{2}\right) = \dfrac{4}{2+\sin\frac{\pi}{2}} - 1 = \dfrac{4}{2+1} - 1$

$$= \frac{4}{3} - 1 = \frac{1}{3}.$$

S20. Given that, $(x + 2y^3)\dfrac{dy}{dx} = y$

$\Rightarrow$ $y \cdot \dfrac{dx}{dy} = x + 2y^3$

$\Rightarrow$ $\dfrac{dx}{dy} = \dfrac{x}{y} + 2y^2$

$\Rightarrow$ $\dfrac{dx}{dy} - \dfrac{x}{y} = 2y^2$

which is a linear differential equation.

On comparing it with $\frac{dx}{dy} + Px = Q$, we gety

$$P = -\frac{1}{y}, \quad Q = 2y^2$$

$$IF = e^{\int -\frac{1}{y} dy} = e^{-\int \frac{1}{y} dy}$$

$\therefore$ $$= e^{-\log y} = \frac{1}{y}$$

Hence, solution is $$x \cdot \frac{1}{y} = \int 2y^2 \cdot \frac{1}{y} dy + C$$

$\Rightarrow$ $$\frac{x}{y} = \int 2y \, dy + C$$

$\Rightarrow$ $$\frac{x}{y} = y^2 + C$$

$\Rightarrow$ $$x = y^3 + Cy.$$

S21. given that, $$\frac{dy}{dx} = 1 + x + y^2 + xy^2$$

$\Rightarrow$ $$\frac{dy}{dx} = (1 + x) + y^2(1 + x)$$

$\Rightarrow$ $$\frac{dy}{dx} = (1 + y^2)(1 + x)$$

$\Rightarrow$ $$\frac{dy}{1 + y^2} = (1 + x)\, dx$$

On integrating both sides, we get

$$\tan^{-1} y = x + \frac{x^2}{2} + K \quad \text{... (i)}$$

Putting $y = 0$ and $x = 0$, in Eq. (i), we get

$$\tan^{-1}(0) = 0 + 0 + K$$

$\Rightarrow$ $$K = 0$$

$\Rightarrow$ $$\tan^{-1} y = x + \frac{x^2}{2}$$

$\Rightarrow$ $$y = \tan\left(x + \frac{x^2}{2}\right).$$

S22. Given that, $(1+t)\frac{dy}{dt} - ty = 1$

$\Rightarrow$ $$\frac{dy}{dt} - \left(\frac{t}{1+t}\right)y = \frac{1}{1+t}$$

which is a linear differential equation.

On comparing it with $\frac{dy}{dt} + Py = Q$, we get

$$P = -\left(\frac{t}{1+t}\right), \quad Q = \frac{1}{1+t}$$

$$IF = e^{-\int \frac{t}{1+t}dt} = e^{-\int\left(1-\frac{1}{1+t}\right)dt} = e^{-[t-\log(1+t)]}$$
$$= e^{-t}\cdot e^{\log(1+t)}$$
$$= e^{-t}(1+t)$$

The general solution is

$$y\cdot\frac{(1+t)}{e^t} = \int\frac{(1+t)\cdot e^{-t}}{(1+t)}dt + C$$

$\Rightarrow$ $$y = \frac{e^{-t}}{(-1)}\cdot\frac{e^t}{1+t} + C', \quad \text{where} \quad C' = \frac{Ce'}{1+t}$$

$\Rightarrow$ $$y = -\frac{1}{1+t} + C'$$

When $t = 0$ and $y = -1$, then

$$-1 = -1 + C' \Rightarrow C' = 0$$

$$y(t) = -\frac{1}{1+t} \quad \Rightarrow \quad y(1) = -\frac{1}{2}.$$

S23. Given, differential equation is

$$y^2dx + (x^2 - xy + y^2)\,dy = 0$$

$\Rightarrow$ $$y^2dx = -(x^2 - xy + y^2)\,dy$$

$\Rightarrow$ $$y^2\frac{dx}{dy} = -(x^2 - xy + y^2)$$

$\Rightarrow$ $$\frac{dx}{dy} = -\left(\frac{x^2}{y^2} - \frac{x}{y} + 1\right) \qquad \text{... (i)}$$

which is a homogeneous differential equation.

Put $\frac{x}{y} = v$ or $x = vy$

$\Rightarrow$ $\frac{dx}{dy} = v + y\frac{dv}{dy}$

On substituting these values in Eq. (i), we get

$$v + y\frac{dv}{dy} = -[v^2 - v + 1]$$

$\Rightarrow$ $y\frac{dv}{dy} = -v^2 + v - 1 - v$

$\Rightarrow$ $y\frac{dv}{dy} = -v^2 - 1 \Rightarrow \frac{dv}{v^2+1} = -\frac{dy}{y}$

On integrating both sides, we get

$$\tan^{-1}(v) = -\log y + C$$

$\Rightarrow$ $\tan^{-1}\left(\frac{x}{y}\right) + \log y = C$ $\left[\because\ v = \frac{x}{y}\right]$

S24. Given, differential equation is

$$(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$$

$\Rightarrow$ $(1 + y^2) = -(x - e^{\tan^{-1} y})\frac{dy}{dx}$

$$(1 + y^2)\frac{dx}{dy} = -x + e^{\tan^{-1} y}$$

$\Rightarrow$ $(1 + y^2)\frac{dx}{dy} + x = e^{\tan^{-1} y}$

$\Rightarrow$ $\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{\tan^{-1} y}}{1+y^2}$ [Dividing throughout by $(1 + y^2)$]

which is a linear differential equation.

On comparing it with $\frac{dx}{dy} + Px = Q$, we get

$$P = \frac{1}{1+y^2}, \quad Q = \frac{e^{\tan^{-1} y}}{1+y^2}$$

$$IF = e^{\int P\,dy} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$$

The general solution is $x \cdot e^{\tan^{-1} y} = \int \frac{e^{\tan^{-1} y}}{1+y^2} \cdot e^{\tan^{-1} y}\, dy + C$

$$\Rightarrow \quad x \cdot e^{\tan^{-1} y} = \int \frac{(e^{\tan^{-1} y})^2}{1+y^2} \cdot dy + C$$

Put $\tan^{-1} y = t$ $\quad \frac{1}{1+y^2}\, dy = dt$

$$\therefore \quad x \cdot e^{\tan^{-1} y} = \int e^{2t}\, dt + C$$

$$\Rightarrow \quad x \cdot e^{\tan^{-1} y} = \frac{1}{2} e^{2\tan^{-1} y} + C$$

$$\Rightarrow \quad 2x e^{\tan^{-1} y} = e^{2\tan^{-1} y} + 2C$$

$$\Rightarrow \quad 2x e^{\tan^{-1} y} = e^{2\tan^{-1} y} + K \qquad [\because K = 2C]$$

S25. Given that, $\quad x^2 \frac{dy}{dx} = x^2 + xy + y^2$

$$\Rightarrow \quad \frac{dy}{dx} = 1 + \frac{y}{x} + \frac{y^2}{x^2} \qquad \ldots \text{(i)}$$

Let $\quad f(x, y) = 1 + \frac{y}{x} + \frac{y^2}{x^2}$

$$f(\lambda x, \lambda y) = 1 + \frac{\lambda y}{\lambda x} + \frac{\lambda^2 y^2}{\lambda^2 x^2}$$

$$f(\lambda x, \lambda y) = \lambda^0 \left(1 + \frac{y}{x} + \frac{y^2}{x^2}\right)$$

$$= \lambda^0 f(x, y)$$

which is homogeneous expression of degree 0.

Put $\quad y = vx \quad \Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

On substituting these values in Eq. (i), we get

$$\left(v + x\frac{dv}{dx}\right) = 1 + v + v^2$$

$$\Rightarrow \quad x\frac{dv}{dx} = 1 + v + v^2 - v$$

$$\Rightarrow \quad x\frac{dv}{dx} = 1 + v^2$$

$$\Rightarrow \quad \frac{dv}{1+v^2} = \frac{dx}{x}$$

On integrating both sides, we get

$$\tan^{-1} v = \log|x| + C$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{y}{x}\right) = \log|x| + C.$$

S26. Given differential equation is

$$(x + y)(dx - dy) = dx + dy$$

$$\Rightarrow \quad (x+y)\left(1-\frac{dy}{dx}\right) = 1+\frac{dy}{dx} \quad \ldots \text{(i)}$$

Put $x + y = z$

$$\Rightarrow \quad 1+\frac{dy}{dx} = \frac{dz}{dx}$$

On substituting these values in Eq. (i), we get

$$z\left(1-\frac{dz}{dx}+1\right) = \frac{dz}{dx}$$

$$\Rightarrow \quad z\left(2-\frac{dz}{dx}\right) = \frac{dz}{dx}$$

$$\Rightarrow \quad 2z - z\frac{dz}{dx} - \frac{dz}{dx} = 0$$

$$\Rightarrow \quad 2z - (z+1)\frac{dz}{dx} = 0$$

$$\Rightarrow \quad \frac{dz}{dx} = \frac{2z}{z+1}$$

$$\Rightarrow \quad \left(\frac{z+1}{z}\right)dz = 2dx$$

On integrating both sides, we get

$$\int\left(1+\frac{1}{z}\right)dz = 2\int dx$$

$$\Rightarrow \quad z + \log z = 2x - \log C$$

$$\Rightarrow \quad (x + y) + \log(x + y) = 2x - \log C \quad [\because\ z = x + y]$$

$$\Rightarrow \quad 2x - x - y = \log C + \log(x + y)$$

$$\Rightarrow \quad x - y = \log|C(x + y)|$$

$$\Rightarrow \quad e^{x-y} = C(x + y)$$

$$\Rightarrow \quad (x + y) = \frac{1}{C} e^{x-y}$$

$$\Rightarrow \quad x + y = K e^{x-y} \qquad \left[\because \; K = \frac{1}{C}\right]$$

S27. Given that, $2(y + 3) - xy \frac{dy}{dx} = 0$

$$\Rightarrow \quad 2(y + 3) = xy \frac{dy}{dx}$$

$$\Rightarrow \quad 2 \frac{dx}{x} = \left(\frac{y}{y+3}\right) dy$$

$$\Rightarrow \quad 2 \cdot \frac{dx}{x} = \left(\frac{y+3-3}{y+3}\right) dy$$

$$\Rightarrow \quad 2 \cdot \frac{dx}{x} = \left(1 - \frac{3}{y+3}\right) dy$$

On integrating both sides, we get

$$2 \log x = y - 3 \log (y + 3) + C \qquad \dots \text{(i)}$$

When, $x = 1$ and $y = -2$, then

$$2 \log 1 = -2 - 3 \log (-2 + 3) + C$$

$$\Rightarrow \quad 2 \cdot 0 = -2 - 3 \cdot 0 + C$$

$$\Rightarrow \quad C = 2$$

On substituting the value of C in Eq. (i), we get

$$2 \log x = y - 3 \log (y + 3) + 2$$

$$\Rightarrow \quad 2 \log x + 3 \log (y + 3) = y + 2$$

$$\Rightarrow \quad \log x^2 + \log (y + 3)^3 = (y + 2)$$

$$\Rightarrow \quad \log x^2 (y + 3)^3 = y + 2$$

$$\Rightarrow \quad x^2 (y + 3)^3 = e^{y+2}.$$

S28. Given, differential equation $dy = \cos x (2 - y \operatorname{cosec} x) dx$

$$\Rightarrow \quad \frac{dy}{dx} = \cos x (2 - y \operatorname{cosec} x)$$

$$\Rightarrow \quad \frac{dy}{dx} = 2 \cos x - y \operatorname{cosec} x \cdot \cos x$$

$$\Rightarrow \quad \frac{dy}{dx} = 2 \cos x - y \cot x$$

$$\Rightarrow \quad \frac{dy}{dx} + y \cot x = 2 \cos x$$

which is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$$P = \cot x, \quad Q = 2\cos x$$

$$IF = e^{\int P\,dx} = e^{\int \cot x\,dx} = e^{\log \sin x} = \sin x$$

The general solution is

$$y \cdot \sin x = \int 2\cos x \cdot \sin x\,dx + C$$

$$\Rightarrow \quad y \cdot \sin x = \int \sin 2x\,dx + C \qquad [\because \sin 2x = 2\sin x\cos x]$$

$$\Rightarrow \quad y \cdot \sin x = -\frac{\cos 2x}{2} + C \qquad \dots \text{(i)}$$

When $x = \frac{\pi}{2}$ and $y = 2$, then

$$2 \cdot \sin\frac{\pi}{2} = -\frac{\cos\left(2 \times \frac{\pi}{2}\right)}{2} + C$$

$$\Rightarrow \quad 2 \cdot 1 = \frac{1}{2} + C$$

$$\Rightarrow \quad 2 - \frac{1}{2} = C \quad \Rightarrow \quad \frac{4-1}{2} = C$$

$$\Rightarrow \quad C = \frac{3}{2}$$

On substituting the value of C in Eq. (i), we get

$$y\sin x = -\frac{1}{2}\cos 2x + \frac{3}{2}.$$

S29. Given differential equation is

$$(1 + y^2)\tan^{-1} x\,dx + 2y(1 + x^2)\,dy = 0$$

$$\Rightarrow \quad (1 + y^2)\tan^{-1} x\,dx = -2y(1 + x^2)\,dy$$

$$= \quad \frac{\tan^{-1} x\,dx}{1+x^2} = -\frac{2y}{1+y^2}\,dy$$

On integrating both sides, we get

$$\int \frac{\tan^{-1} x}{1+x^2}\,dx = -\int \frac{2y}{1+y^2}\,dy$$

Put, $\tan^{-1} x = t$ in L.H.S., we get

$$\frac{1}{1+x^2}\,dx = dt$$

and put, $1 + y^2 = u$ in R.H.S., we get

$$2y\,dy = du$$

$$\Rightarrow \quad \int t\,dt = -\int \frac{1}{u}\,du \quad \Rightarrow \quad \frac{t^2}{2} = -\log u + C$$

$$\Rightarrow \quad \frac{1}{2}(\tan^{-1} x)^2 = -\log(1 + y^2) + C$$

$$\Rightarrow \quad \frac{1}{2}(\tan^{-1} x)^2 + \log(1 + y^2) = C.$$

S30. Given, $\dfrac{dy}{dx} - 3y = \sin 2x$

which is a linear differential equation.

$$P = -3, \quad Q = \sin 2x$$

$$IF = e^{-3\int dx} = e^{-3x}$$

The general solution is

$$y \cdot e^{-3x} = \int \underset{\text{I}}{\sin 2x}\ \underset{\text{II}}{e^{-3x}}\,dx$$

Let $y \cdot e^{-3x} = I$ ... (i)

$$\therefore \quad I = \int \underset{\text{II}}{e^{-3x}}\ \underset{\text{I}}{\sin 2x}\,dx$$

$$\Rightarrow \quad I = \sin 2x\left(\frac{e^{-3x}}{-3}\right) - \int 2\cos 2x\left(\frac{e^{-3x}}{-3}\right)dx + C_1$$

$$\Rightarrow \quad I = -\frac{1}{3}e^{-3x}\sin 2x + \frac{2}{3}\int \underset{\text{II}}{e^{-3x}}\ \underset{\text{I}}{\cos 2x}\,dx + C_1$$

$$\Rightarrow \quad I = -\frac{1}{3}e^{-3x}\sin 2x + \frac{2}{3}\left(\cos 2x\,\frac{e^{-3x}}{-3} - \int(-2\sin 2x)\frac{e^{-3x}}{-3}\,dx\right) + C_1 + C_2$$

$$\Rightarrow \quad I = -\frac{1}{3}e^{-3x}\sin 2x - \frac{2}{9}\cos 2x\,e^{-3x} - \frac{4}{9}I + C' \qquad [\because\ C' = C_1 + C_2]$$

$$\Rightarrow \quad I + \frac{4I}{9} = +e^{-3x}\left(-\frac{1}{3}\sin 2x - \frac{2}{9}\cos 2x\right) + C'$$

$$\Rightarrow \quad \frac{13}{9}I = e^{-3x}\left(-\frac{1}{3}\sin 2x - \frac{2}{9}\cos 2x\right) + C'$$

$$\Rightarrow \quad I = \frac{9}{13}e^{-3x}\left(-\frac{1}{3}\sin 2x - \frac{2}{9}\cos 2x\right) + C \qquad \left[\text{Where } C = \frac{9C'}{13}\right]$$

$$\Rightarrow \quad I = \frac{3}{13}e^{-3x}\left(-\sin 2x - \frac{2}{3}\cos 2x\right) + C$$

$$\Rightarrow \qquad = \frac{3}{13} e^{-3x} \frac{(-3\sin 2x - 2\cos 2x)}{3} + C$$

$$\Rightarrow \qquad = \frac{e^{-3x}}{13} (-3\sin 2x - 2\cos 2x) + C$$

$$\Rightarrow \qquad I = -\frac{e^{-3x}}{13} (2\cos 2x + 3\sin 2x) + C$$

On substituting the value of I in Eq. (i), we get

$$y \cdot e^{-3x} = -\frac{e^{-3x}}{13} (2\cos 2x + 3\sin 2x) + C$$

$$\Rightarrow \qquad y = -\frac{1}{13} (2\cos 2x + 3\sin 2x) + Ce^{3x}.$$

S31. Given, $$\frac{dy}{dx} = \cos(x+y) + \sin(x+y) \qquad \text{... (i)}$$

Put $$x + y = z$$

$$\Rightarrow \qquad 1 + \frac{dy}{dx} = \frac{dz}{dx}$$

On substituting these values in Eq. (i), we get

$$\left(\frac{dz}{dx} - 1\right) = \cos z + \sin z$$

$$\Rightarrow \qquad \frac{dz}{dx} = (\cos z + \sin z + 1)$$

$$\Rightarrow \qquad \frac{dz}{\cos z + \sin z + 1} = dx$$

On integrating both sides, we get

$$\int \frac{dz}{\cos z + \sin z + 1} = \int 1 dx$$

$$\Rightarrow \qquad \int \frac{dz}{\frac{1-\tan^2 z/2}{1+\tan^2 z/2} + \frac{2\tan z/2}{1+\tan^2 z/2} + 1} = \int dx$$

$$\Rightarrow \qquad \int \frac{dz}{\frac{1-\tan^2 z/2 + 2\tan z/2 + 1 + \tan^2 z/2}{(1+\tan^2 z/2)}} = \int dx$$

$$\Rightarrow \qquad \int \frac{(1+\tan^2 z/2)\,dz}{2 + 2\tan z/2} = \int dx$$

$$\Rightarrow \qquad \int \frac{\sec^2 z/2\,dz}{2(1+\tan z/2)} = \int dx$$

Put 1 + tan $z/2 = t \Rightarrow \left(\frac{1}{2}\sec^2 z/2\right)dz = dt$

$$\Rightarrow \qquad \int \frac{dt}{t} = \int dx$$

$$\Rightarrow \qquad \log|t| = x + C$$

$$\Rightarrow \qquad \log|1 + \tan z/2| = x + C$$

$$\Rightarrow \qquad \log\left|1 + \tan\frac{(x+y)}{2}\right| = x + C.$$

S32. Given differential equation is $(1 + \tan y)(dx - dy) + 2x\,dy = 0$.

On dividing throughout by dy, we get

$$(1 + \tan y)\left(\frac{dx}{dy} - 1\right) + 2x = 0$$

$$\Rightarrow \qquad (1 + \tan y)\frac{dx}{dy} - (1 + \tan y) + 2x = 0$$

$$\Rightarrow \qquad (1 + \tan y)\frac{dx}{dy} + 2x = (1 + \tan y)$$

$$\Rightarrow \qquad \frac{dx}{dy} + \frac{2x}{1 + \tan y} = 1$$

which is a linear differential equation.

On comparing it with $\frac{dx}{dy} + Px = Q$, we get

$$P = \frac{2}{1 + \tan y}, \quad Q = 1$$

$$IF = e^{\int \frac{2}{1+\tan y}dy} = e^{\int \frac{2\cos y}{\cos y + \tan y}dy}$$

$$= e^{\int \frac{\cos y + \sin y + \cos y - \sin y}{\cos y + \sin y}dy}$$

$$= e^{\int \left(1 + \frac{\cos y - \sin y}{\cos y + \sin y}\right)dy} = e^{y + \log(\cos y + \sin y)}$$

$$= e^y \cdot (\cos y + \sin y)$$

The general solution is

$$x \cdot e^y(\cos y + \sin y) = \int 1 \cdot e^y(\cos y + \sin y)\,dy + C$$

$$\Rightarrow \qquad x \cdot e^y(\cos y + \sin y) = \int e^y(\sin y + \cos y)\,dy + C$$

$$\Rightarrow \qquad x \cdot e^y(\cos y + \sin y) = e^y \sin y + C \qquad \left[\because \int e^x\{f(x) + f'(x)\}dx = e^x f(x)\right]$$

$$\Rightarrow \qquad x(\sin y + \cos y) = \sin y + Ce^{-y}.$$

S33. Given differential equation is

$$y + \frac{d}{dx}(xy) = x(\sin x + \log x)$$

$$\Rightarrow \quad y + x\frac{dy}{dx} + y = x(\sin x + \log x)$$

$$\Rightarrow \quad x\frac{dy}{dx} + 2y = x(\sin x + \log x)$$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{2}{x}y = \sin x + \log x$$

which is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$$P = \frac{2}{x}, \quad Q = \sin x + \log x$$

$$IF = e^{\int \frac{2}{x}dx} = e^{2\log x} = x^2$$

The general solution is

$$y \cdot x^2 = \int (\sin x + \log x)\, x^2 dx + C$$

$$\Rightarrow \quad y \cdot x^2 = \int (x^2 \sin x + x^2 \log x)\, dx + C$$

$$\Rightarrow \quad y \cdot x^2 = \int x^2 \sin x\, dx + \int x^2 \log x\, dx + C$$

$$\Rightarrow \quad y \cdot x^2 = I_1 + I_2 + C \qquad \dots \text{(i)}$$

Now, $I_1 = \int x^2 \sin x\, dx$

$$= x^2(-\cos x) + \int 2x \cos x\, dx$$

$$= -x^2 \cos x + \left[2x(\sin x) - \int 2\sin x\, dx\right]$$

$$I_1 = -x^2 \cos x + 2x \sin x + 2\cos x \qquad \dots \text{(ii)}$$

and $I_2 = \int x^2 \log x\, dx$

$$= \log x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3}\, dx$$

$$= \log x \cdot \frac{x^3}{3} - \frac{1}{3}\int x^2 dx$$

$$= \log x \cdot \frac{x^3}{3} - \frac{1}{3} \cdot \frac{x^3}{3} \qquad \dots \text{(iii)}$$

On substituting the value of I_1 and I_2 in Eq. (i), we get

$$y \cdot x^2 = -x^2 \cos x + 2x \sin x + 2 \cos x + \frac{x^3}{3} \log x - \frac{1}{9} x^3 + C$$

$$\therefore \qquad y = -\cos x + \frac{2 \sin x}{x} + \frac{2 \cos x}{x^2} + \frac{x}{3} \log x - \frac{x}{9} + Cx^{-2}$$

S34. It is given that, the slope of tangent to the curve at point (x, y) is $\frac{x^2 + y^2}{2xy}$.

$$\therefore \qquad \left(\frac{dy}{dx}\right)_{(x,\,y)} = \frac{x^2 + y^2}{2xy}$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{1}{2}\left(\frac{x}{y} + \frac{y}{x}\right)$$

which is a homogeneous differential equation.

Put $\qquad y = vx$

$$\Rightarrow \qquad \frac{dy}{dx} = v + x \frac{dv}{dx}$$

On substituting these values in Eq. (i), we get

$$v + x \frac{dv}{dx} = \frac{1}{2}\left(\frac{1}{v} + v\right)$$

$$\Rightarrow \qquad v + x \frac{dv}{dx} = \frac{1}{2}\left(\frac{1 + v^2}{v}\right)$$

$$\Rightarrow \qquad x \frac{dv}{dx} = \frac{1 + v^2}{2v} - v$$

$$\Rightarrow \qquad x \frac{dv}{dx} = \frac{1 + v^2 - 2v^2}{2v}$$

$$\Rightarrow \qquad x \frac{dv}{dx} = \frac{1 - v^2}{2v}$$

$$\Rightarrow \qquad \frac{2v}{1 - v^2}\, dv = \frac{dx}{x}$$

On integrating both sides, we get

$$\int \frac{2v}{1 - v^2}\, dv = \int \frac{dx}{x}$$

Put $1 - v^2 = t$ in L.H.S., we get

$$\Rightarrow \quad -2v\,dv = dt$$

$$\Rightarrow \quad \int \frac{dt}{t} = \int \frac{dx}{x}$$

$$\Rightarrow \quad -\log t = \log x + \log C$$

$$\Rightarrow \quad -\log (1 - v^2) = \log x + \log C$$

$$\Rightarrow \quad -\log\left(1 - \frac{y^2}{x^2}\right) = \log x + \log C$$

$$\Rightarrow \quad -\log\left(\frac{x^2 - y^2}{x^2}\right) = \log x + \log C$$

$$\Rightarrow \quad \log\left(\frac{x^2}{x^2 - y^2}\right) = \log x + \log C$$

$$\Rightarrow \quad \frac{x^2}{x^2 - y^2} = Cx \qquad \dots \text{(i)}$$

Since, the curve passes through the point (2, 1).

$$\therefore \quad \frac{(2)^2}{(2)^2 - (1)^2} = C(2) \quad \Rightarrow \quad C = \frac{2}{3}$$

So, the required solution is $2(x^2 - y^2) = 3x$.

S35. It is given that, slope of tangent to the curve at any point (x, y) is $\frac{y-1}{x^2 + x}$.

$$\therefore \quad \left(\frac{dy}{dx}\right)_{(x,\,y)} = \frac{y-1}{x^2 + x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y-1}{x^2 + x}$$

$$\Rightarrow \quad \frac{dy}{y-1} = \frac{dx}{x^2 + x}$$

On integrating both sides, we get

$$\int \frac{dy}{y-1} = \int \frac{dx}{x^2 + x}$$

$$\Rightarrow \quad \int \frac{dy}{y-1} = \int \frac{dx}{x(x+1)}$$

$$\Rightarrow \quad \int \frac{dy}{y-1} = \int \left(\frac{1}{x} - \frac{1}{x+1}\right) dx$$

$$\Rightarrow \quad \log (y - 1) = \log x - \log (x + 1) + \log C$$

$$\Rightarrow \quad \log (y - 1) = \log\left(\frac{xC}{x+1}\right)$$

$$y - 1 = \frac{xC}{x+1}$$

Since, the given curve passes through point (1, 0).

$\therefore$ $$0 - 1 = \frac{1 \cdot C}{1+1} \quad \Rightarrow \quad C = -2$$

The particular solution is $$y - 1 = \frac{-2x}{x+1}$$

$\Rightarrow$ $$(y-1)(x+1) = -2x$$

$\Rightarrow$ $$(y-1)(x+1) + 2x = 0.$$

S36. Slope of tangent to the curve = $\frac{dy}{dx}$

and difference of abscissa and ordinate = $x - y$

According to the question, $$\frac{dy}{dx} = (x-y)^2 \quad \text{... (i)}$$

Put $$x - y = z$$

$\Rightarrow$ $$1 - \frac{dy}{dx} = \frac{dz}{dx}$$

$\Rightarrow$ $$\frac{dy}{dx} = 1 - \frac{dz}{dx}$$

On substituting these valus in Eq. (i), we get

$$1 - \frac{dz}{dx} = z^2$$

$\Rightarrow$ $$1 - z^2 = \frac{dz}{dx}$$

$\Rightarrow$ $$dx = \frac{dz}{1-z^2}$$

On integrating both sides, we get

$$\int dx = \int \frac{dz}{1-z^2}$$

$\Rightarrow$ $$x = \frac{1}{2} \log \left| \frac{1+z}{1-z} \right| + C$$

$\Rightarrow$ $$x = \frac{1}{2} \log \left| \frac{1+x-y}{1-x+y} \right| + C \quad \text{... (ii)}$$

Since, the curve passes through the origin.

$\therefore$ $$0 = \frac{1}{2}\log\left|\frac{1+0-0}{1-0+0}\right| + C$$

$\Rightarrow$ $$C = 0$$

On substituting the value of C in Eq. (ii), we get

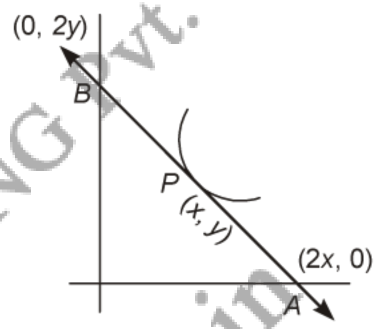
$$x = \frac{1}{2}\log\left|\frac{1+x-y}{1-x+y}\right|$$

$\Rightarrow$ $$2x = \log\left|\frac{1+x-y}{1-x+y}\right|$$

$\Rightarrow$ $$e^{2x} = \left|\frac{1+x-y}{1-x+y}\right|$$

$\Rightarrow$ $$(1 - x + y)\, e^{2x} = 1 + x - y.$$

S37. The below figure obtained by the given information.



Let the coordinate of the point P is (x, y). It is given that, P is mid-point of AB.

So, the coordinates of points A and B are $(2x, 0)$ and $(0, 2y)$, respectively.

$\therefore$ $$\text{Slope of } AB = \frac{0-2y}{2x-0} = -\frac{y}{x}$$

Since, the segment AB is a tangent to the curve at P.

$\therefore$ $$\frac{dy}{dx} = -\frac{y}{x}$$

$\Rightarrow$ $$\frac{dy}{y} = -\frac{dx}{x}$$

On integrating both sides, we get

$$\log y = -\log x + \log C$$

$$\log y = \log\frac{C}{x}$$

Since, the given curve passes through (1, 1).

$\therefore$ $$\log 1 = \log\frac{C}{1}$$

$\Rightarrow$ $$0 = \log C$$

$\Rightarrow$ $$C = 1$$

$\therefore \quad \log y = \log \frac{1}{x}$

$\Rightarrow \quad y = \frac{1}{x}$

$\Rightarrow \quad xy = 1.$

S38. Given, $x \frac{dy}{dx} = y (\log y - \log x + 1)$

$\Rightarrow \quad x \frac{dy}{dx} = y \log\left(\frac{y}{x}\right) + 1$

$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}\left(\log \frac{y}{x} + 1\right)$ ... (i)

which is a homogeneous equation.

Put $\frac{y}{x} = v$ or $y = vx$

$\therefore \quad \frac{dy}{dx} = v + x \frac{dv}{dx}$

On substituting these values in Eq. (i), we get

$$v + x \frac{dv}{dx} = v (\log v + 1)$$

$\Rightarrow \quad x \frac{dv}{dx} = v (\log v + 1 - 1)$

$\Rightarrow \quad x \frac{dv}{dx} = v (\log v)$

$\Rightarrow \quad \frac{dv}{v \log v} = \frac{dx}{x}$

On integrating both sides, we get

$$\int \frac{dv}{v \log v} = \int \frac{dx}{x}$$

On putting $\log v = u$ in L.H.S. integral, we get

$$\frac{1}{v} \cdot dv = du$$

$$\int \frac{du}{u} = \int \frac{dx}{x}$$

$\Rightarrow \qquad \log u = \log x + \log C$

$\Rightarrow \qquad \log u = \log Cx$

$\Rightarrow \qquad u = Cx$

$\Rightarrow \qquad \log v = Cx$

$\Rightarrow \qquad \log\left(\frac{y}{x}\right) = Cx.$

S39. According to the given condition

$$\frac{dy}{dx} = \frac{y}{x} - \cos^2\frac{y}{x} \qquad \text{... (i)}$$

This is a homogeneous differential equation.

Substituting $y = vx$, we get

$$v + x\frac{dv}{dx} = v - \cos^2 v \quad \Rightarrow \quad x\frac{dv}{dx} = -\cos^2 v$$

$\Rightarrow \qquad \sec^2 dv = -\frac{dx}{x} \quad \Rightarrow \quad \tan v = -\log x + c$

$\Rightarrow \qquad \tan\frac{y}{x} + \log x = c \qquad \text{... (ii)}$

Substituting $x = 1$, $y = \frac{\pi}{4}$, we get $c = 1$. Thus, we get

$$\tan\left(\frac{y}{x}\right) + \log x = 1,$$

which is the required equation.

S40. Let the equation of normal of $P(x, y)$ be $Y - y = \frac{-dx}{dy}(X - x)$

$$Y + X\frac{dx}{dy} - \left(y + x\frac{dx}{dy}\right) = 0 \qquad \text{... (i)}$$

Therefore, the length of perpendicular from origin to (i) is

$$\frac{y + x\frac{dx}{dy}}{\sqrt{1 + \left(\frac{dx}{dy}\right)^2}} \qquad \text{... (ii)}$$

Also, distance between P and x-axis is $|y|$. Thus, we get

$$\frac{y + x\frac{dx}{dy}}{\sqrt{1 + \left(\frac{dx}{dy}\right)^2}} = |y|$$

$$\Rightarrow \quad \left(y + x\frac{dx}{dy}\right)^2 = y^2\left[1 + \left(\frac{dx}{dy}\right)^2\right]$$

$$\Rightarrow \quad \frac{dx}{dy}\left[\frac{dx}{dy}(x^2 - y^2) + 2xy\right] = 0 \quad \Rightarrow \quad \frac{dx}{dy} = 0$$

$$\text{or} \quad \frac{dx}{dy} = \frac{2xy}{y^2 - x^2}$$

Case - I: $\frac{dx}{dy} = 0 \quad \Rightarrow \quad x = k \qquad (k \in R)$

Integrating both sides, we get $x = k$, Substituting $x = 1$, we get $k = 1$.

Therefore $x = 1$ is the equation of curve (not possible, so rejected).

Case - II: $\frac{dx}{dy} = \frac{2xy}{y^2 - x^2} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{y^2 - x^2}{2xy},$

Substituting $y = vx$, we get

$$v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2vx^2} \quad \Rightarrow \quad x \cdot \frac{dv}{dx} = \frac{v^2 - 1}{2v} - v$$

$$= \frac{-(1 + v^2)}{2v} \quad \Rightarrow \quad \frac{2v}{1 + v^2}dv = \frac{-dx}{x}$$

Integrating both sides, we get

$$\log(1 + v^2) = -\log x + \log c$$

$$\Rightarrow \quad \log(1 + v^2)(x) = \log c$$

$$\Rightarrow \quad (1 + v^2)x = c$$

$$\Rightarrow \quad x^2 + y^2 = cx.$$

Substituting $x = 1$ and $y = 1$, we get $c = 2$.

Therefore, $x^2 + y^2 - 2x = 0$ is the required equation.

S41. Given equation can be written as

$$x^2\frac{dy}{dx} - xy = 2\cos^2\left(\frac{y}{2x}\right), \quad x \neq 0$$

$$\Rightarrow \quad \frac{x^2\frac{dy}{dx} - xy}{2\cos^2\left(\frac{y}{2x}\right)} = 1 \quad \Rightarrow \quad \frac{\sec^2\left(\frac{y}{2x}\right)}{2}\left[x^2\frac{dy}{dx} - xy\right] = 1$$

Dividing both sides by x^3, we get

$$\frac{\sec^2\left(\frac{y}{2x}\right)}{2}\left[\frac{x\frac{dy}{dx} - y}{x^2}\right] = \frac{1}{x^3} \quad \Rightarrow \quad \frac{d}{dx}\left[\tan\left(\frac{y}{2x}\right)\right] = \frac{1}{x^3}$$

Integrating both sides, we get

$$\tan\left(\frac{y}{2x}\right) = \frac{-1}{2x^2} + k.$$

Substituing $x = 1$, $y = \frac{\pi}{2}$, we get $k = \frac{3}{2}$, therefore, $\tan\left(\frac{y}{2x}\right) = -\frac{1}{2x^2} + \frac{3}{2}$ is the required solution.

S42. Given equation can be written as $xdy = (\sqrt{x^2 + y^2} + y)\, dx$, *i.e.*,

$$\frac{dy}{dx} = \frac{\sqrt{x^2 + y^2} + y}{x} \qquad \text{... (i)}$$

Clearly R.H.S. of (i) is a homogeneous function of degree zero. Therefore, the given equation is a homogeneous differential equation.

Substituting $y = vx$, we get from (i)

$$v + x\frac{dv}{dx} = \frac{\sqrt{x^2 + v^2x^2} + vx}{x} \qquad i.e., \quad v + x\frac{dv}{dx} = \sqrt{1+v^2} + v$$

$$x\frac{dv}{dx} = \sqrt{1+v^2} \quad \Rightarrow \quad \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x} \qquad \text{... (ii)}$$

Integrating both sides of (ii), we get

$$\log(v + \sqrt{1+v^2}) = \log x + \log c \quad \Rightarrow \quad v + \sqrt{1+v^2} = cx$$

$$\Rightarrow \qquad \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = cx \quad \Rightarrow \quad y + \sqrt{x^2 + y^2} = cx^2.$$