Date: 19/10/2021

Q1. Find the area of the region bounded by line $x = 2$ and parabola $y^2 = 8x$.

Q2. Find the area of the region enclosed by the parabola $x^2 = y$ and the line $y = x + 2$.

Q3. Find the area of the region included between $y^2 = 9x$ and $y = x$.

Q4. Find the area of the region bounded by the curve $y^2 = 4x$ and $x^2 = 4y$.

Q5. Find the area of the region bounded by the curve $y = x^3$, $y = x + 6$ and $x = 0$.

Q6. Find the area of the region bounded by the parabola $y^2 = 2px$ and $x^2 = 2py$.

Q7. Find the area of the region bounded by the curves $y^2 = 9x$ and $y = 3x$.

Q8. Find the area enclosed by the curve $y = -x^2$ and the straight line $x + y + 2 = 0$.

Q9. Find the area of the region bounded by $y = \sqrt{x}$ and $y = x$.

Q10. Determine the area under the curve $y = \sqrt{a^2 - x^2}$ included between the lines $x = 0$ and $x = a$.

Q11. Draw a rough sketch of the curve $y = \sqrt{x - 1}$ in the interval [1, 5]. Find the area under the curve and between the lines $x = 1$ and $x = 5$.

Q12. Using integration, find the area of the region bounded by the line $2y = 5x + 7$, X-axis and the lines $x = 2$ and $x = 8$.

Q13. Calculate the area under the curve $y = 2\sqrt{x}$ included between the lines $x = 0$ and $x = 1$.

Q14. Sketch the region $\{(x, 0) : y = \sqrt{4 - x^2}\}$ and X-axis. Find the area of the region using integration.

Q15. Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$.

Q16. Find the area of the region bounded by the curve $y^2 = 2x$ and $x^2 + y^2 = 4x$.

Q17. Find the area bounded by the curve $y = \sqrt{x}$, $x = 2y + 3$ in the first quadrant and X-axis.

Q18. Find the area bounded by the curve $y = 2\cos x$ and the X-axis from $x = 0$ to $x = 2\pi$.

Q19. Find the area bounded by the lines $y = 4x + 5$, $y = 5 - x$ and $4y = x + 5$.

Q20. Compute the area bounded by the lines $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$.

Q21. Draw a rough sketch of the region $\{(x, y) : y^2 \leq 6ax \text{ and } x^2 + y^2 \leq 16a^2\}$. Also, find the area of the region sketched using method of integration.

Q22. Find the area of region bounded by the triangle whose vertices are $(-1, 1)$, $(0, 5)$ and $(3, 2)$, using integration.

Q23. Find the area of the region above the x-axis, included between the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 2ax$.

Q24. Find the area enclosed by the curve $x = 3 \cos t$, $y = 2 \sin t$.

Q25. Find the area of the region bounded by the parabolas $y^2 = 6x$ and $x^2 = 6y$.

Q26. Find the area of the region bounded by the parabola $y^2 = 2x$ and the straight line $x - y = 4$.

Q27. Find the area of the region bounded by the curve $ay^2 = x^3$, the y-axis and the lines $y = a$ and $y = 2a$.

Q28. Find the area of the curve $y = \sin x$ between 0 and π.

Q29. Find the area of the region bounded by the curves $x = at^2$ and $y = 2at$ between the ordinate coresponding to $t = 1$ and $t = 2$.

Q30. Find the area of a minor segment of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{2}$.

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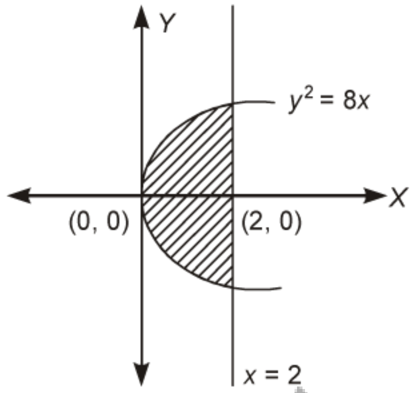
S1. We have, $x = 2$ and $y^2 = 8x$



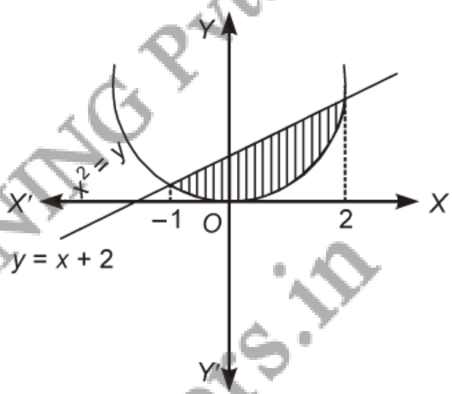
$\therefore$ Area of shaded region, $A = \int_0^2 \sqrt{8x}\,dx = 2 \cdot 2\sqrt{2} \int_0^2 x^{1/2}\,dx$

$$= 4 \cdot \sqrt{2} \cdot \left[2 \cdot \frac{x^{3/2}}{3}\right]_0^2$$

$$= 4\sqrt{2}\left[\frac{2}{3} \cdot 2\sqrt{2} - 0\right]$$

$$= \frac{32}{3} \text{ sq. units.}$$

S2. We have, $x^2 = y$ and $y = x + 2$



$\Rightarrow$ $x^2 = x + 2$

$\Rightarrow$ $x^2 - x - 2 = 0$

$\Rightarrow$ $x^2 - 2x + x - 2 = 0$

$\Rightarrow$ $x(x - 2) + 1(x - 2) = 0$

$\Rightarrow$ $(x + 1)(x - 2) = 0$

$\Rightarrow$ $x = -1, 2$

$\therefore$ Required area of shaded region $= \int_{-1}^{2} (x + 2 - x^2)\,dx$

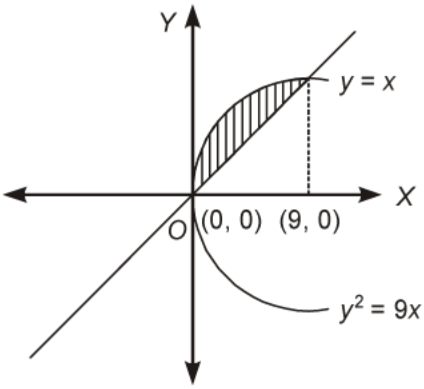
$$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$$

$$= \left[\frac{4}{2} + 4 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3}\right]$$

$$= 6 + \frac{3}{2} - \frac{9}{3} = \frac{36 + 9 - 18}{6}$$

$$= \frac{27}{6} = \frac{9}{2} \text{ sq. units.}$$

S3. We have, $y^2 = 9x$ and $y = x$



$\Rightarrow$ $x^2 - 9x = 0$

$\Rightarrow$ $x(x-9) = 0$

$\Rightarrow$ $x = 0, 9$

$\therefore$ Area of shaded region, $A = \int_0^9 \left(\sqrt{9x} - x\right) dx$

$$= \int_0^9 3x^{1/2}\, dx - \int_0^9 x\, dx$$

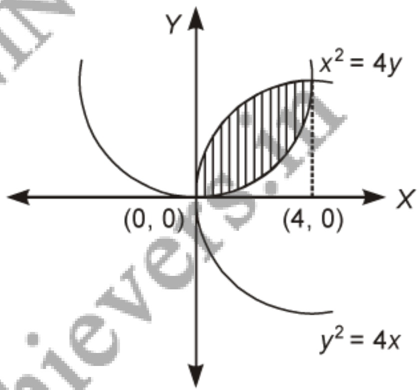
$$= \left[3 \cdot \frac{x^{3/2}}{3} \cdot 2\right]_0^9 - \left[\frac{x^2}{2}\right]_0^9$$

$$= \left[\frac{3 \cdot 3^{\frac{3}{2}\times 2}}{3} \cdot 2 - 0\right] - \left[\frac{81}{2} - 0\right]$$

$$= 54 - \frac{81}{2} = \frac{108-81}{2} = \frac{27}{2} \text{ sq. units.}$$

S4. Given equation of curves are

$$y^2 = 4x \quad \text{and} \quad x^2 = 4y$$



$\Rightarrow$ $\left(\frac{x^2}{4}\right)^2 = 4x$

$\Rightarrow$ $\frac{x^4}{4 \cdot 4} = 4x$

$\Rightarrow$ $x^4 = 64x$

$\Rightarrow$ $x^4 - 64x = 0$

$\Rightarrow$ $x(x^3 - 4^3) = 0$

$\Rightarrow$ $x = 4, 0$

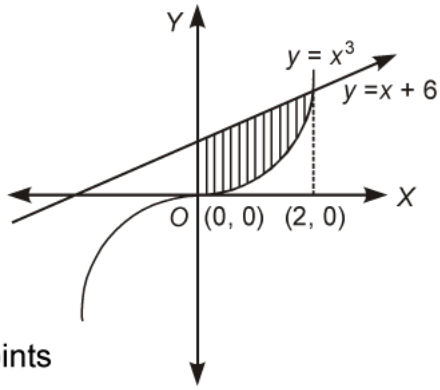
$\therefore$ Area of shaded region, $A = \int_0^4 \left(\sqrt{4x} - \frac{x^2}{4}\right) dx$

$$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx = \left[\frac{2x^{3/2} \cdot 2}{3} - \frac{1}{4} \cdot \frac{x^3}{3}\right]_0^4$$

$$= \frac{2 \cdot 2}{3} \cdot 8 - \frac{1}{4} \cdot \frac{64}{3} - 0$$

$$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3} \text{ sq. units.}$$

S5. We have, $y = x^3, \quad y = x + 6 \quad \text{and} \quad x = 0$

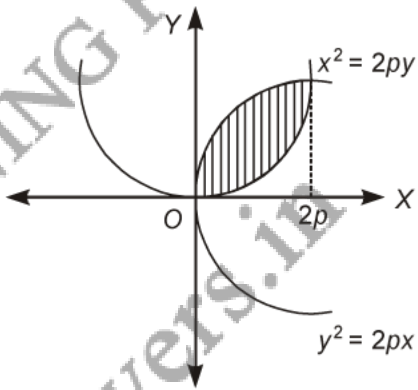


$$\therefore \quad x^3 = x + 6$$
$$\Rightarrow \quad x^3 - x = 6$$
$$\Rightarrow \quad x^3 - x - 6 = 0$$
$$\Rightarrow \quad x^2(x-2) + 2x(x-2) + 3(x-2) = 0$$
$$\Rightarrow \quad (x-2)(x^2 + 2x + 3) = 0$$
$$\Rightarrow \quad x = 2, \quad \text{with two imaginary points}$$

$\therefore$ Required area of shaded region $= \int_0^2 (x + 6 - x^3)\,dx$

$$= \left[\frac{x^2}{2} + 6x - \frac{x^4}{4}\right]_0^2$$
$$= \left[\frac{4}{2} + 12 - \frac{16}{4} - 0\right]$$
$$= [2 + 12 - 4] = 10 \text{ sq. units.}$$

S6. We have, $y^2 = 2px \quad \text{and} \quad x^2 = 2py$



$$\therefore \quad y = \sqrt{2px}$$
$$\Rightarrow \quad x^2 = 2p \cdot \sqrt{2px}$$
$$\Rightarrow \quad x^4 = 4p^2 \cdot (2px)$$
$$\Rightarrow \quad x^4 = 8p^3 x$$
$$\Rightarrow \quad x^4 - 8p^3 x = 0$$
$$\Rightarrow \quad x^3(x - 8p^3) = 0$$
$$\Rightarrow \quad x = 0, 2p$$

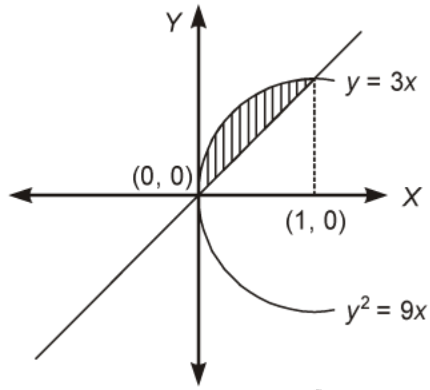
$\therefore$ Required area $= \int_0^{2p} \sqrt{2px}\,dx - \int_0^{2p} \frac{x^2}{2p}\,dx$

$$= \sqrt{2p}\int_0^{2p} x^{1/2}\,dx - \frac{1}{2p}\int_0^{2p} x^2\,dx$$
$$= \sqrt{2p}\left[\frac{2(x)^{3/2}}{3}\right]_0^{2p} - \frac{1}{2p}\left[\frac{x^3}{3}\right]_0^{2p}$$
$$= \sqrt{2p}\left[\frac{2}{3}\cdot(2p)^{3/2} - 0\right] - \frac{1}{2p}\left[\frac{1}{3}(2p)^3 - 0\right]$$
$$= \sqrt{2p}\left(\frac{2}{3}\cdot 2\sqrt{2}\,p^{3/2}\right) - \frac{1}{2p}\left(\frac{1}{3}\,8p^3\right)$$
$$= \sqrt{2}\,p\left(\frac{4\sqrt{2}}{3}\,p^{3/2}\right) - \frac{1}{2p}\left(\frac{8}{3}\,p^3\right)$$

$$= \frac{(16-8)p^2}{6} = \frac{8p^2}{6}$$

$$= \frac{4p^2}{3} \text{ sq. units}.$$

S7. We have, $y^2 = 9x$ and $y = 3x$



$\Rightarrow$ $(3x)^2 = 9x$

$\Rightarrow$ $9x^2 - 9x = 0$

$\Rightarrow$ $9x(x-1) = 0$

$\Rightarrow$ $x = 1, 0$

$\therefore$ Required area, $A = \int_0^1 \sqrt{9x}\, dx - \int_0^1 3x\, dx$

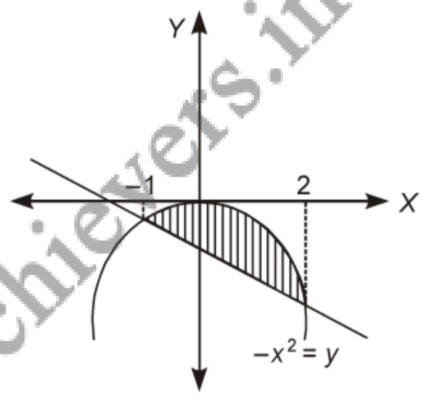
$$= 3\int_0^1 x^{1/2}\, dx - 3\int_0^1 x\, dx$$

$$= 3\left[\frac{x^{3/2}}{3/2}\right]_0^1 - 3\left[\frac{x^2}{2}\right]_0^1$$

$$= 3\left(\frac{2}{3} - 0\right) - 3\left(\frac{1}{2} - 0\right)$$

$$= 2 - \frac{3}{2} = \frac{1}{2} \text{ sq. units}.$$

S8. We have, $y = -x^2$ and $x + y + 2 = 0$



$\Rightarrow$ $-x - 2 = -x^2$

$\Rightarrow$ $x^2 - x - 2 = 0$

$\Rightarrow$ $x^2 + x - 2x - 2 = 0$

$\Rightarrow$ $x(x+1) - 2(x+1) = 0$

$\Rightarrow$ $(x-2)(x+1) = 0$

$\Rightarrow$ $x = 2, -1$

$\therefore$ Area of shaded region, $A = \left|\int_{-1}^{2} (-x - 2 + x^2)\, dx\right| = \left|\int_{-1}^{2} (x^2 - x - 2)\, dx\right|$

$$= \left|\left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_{-1}^{2}\right| = \left|\left[\frac{8}{3} - \frac{4}{2} - 4 + \frac{1}{3} + \frac{1}{2} - 2\right]\right|$$

$$= \left|\frac{16 - 12 - 24 + 2 + 3 - 12}{6}\right| = \left|-\frac{27}{6}\right| = \frac{9}{2} \text{ sq. units}.$$

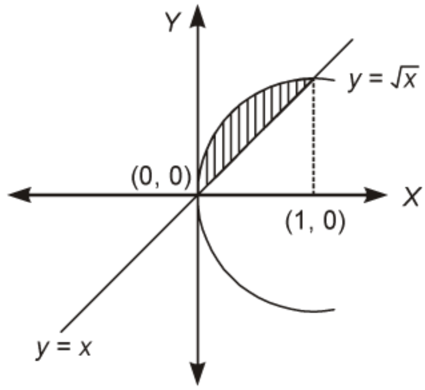
S9. Given equation of vurves are: $y = \sqrt{x}$ and $y = x$.

$\Rightarrow$ $x = \sqrt{x}$ $\Rightarrow$ $x^2 = x$

$\Rightarrow$ $x^2 - x = 0$ $\Rightarrow$ $x(x-1) = 0$

$\Rightarrow$ $x = 0, 1$

$\therefore$ Required area of shaded region



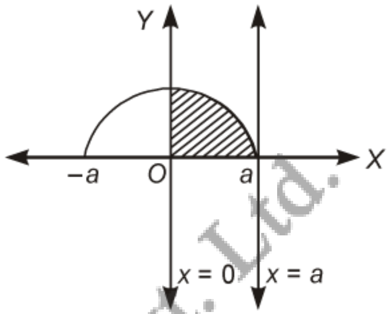
$$A = \int_0^1 (\sqrt{x})\,dx - \int_0^1 x\,dx$$

$$= \left[2 \cdot \frac{x^{3/2}}{3}\right]_0^1 - \left[\frac{x^2}{2}\right]_0^1$$

$$= \frac{2}{3} \cdot 1 - \frac{1}{2}$$

$$= \frac{2}{3} - \frac{1}{2} = \frac{1}{6} \text{ sq. units.}$$

S10. Given equation of the curve is $y = \sqrt{a^2 - x^2}$



$\Rightarrow$ $y^2 = a^2 - x^2$

$\Rightarrow$ $y^2 + x^2 = a^2$

$\therefore$ Area of shaded region, $A = \int_0^a \sqrt{a^2 - x^2}\,dx$

$$= \left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

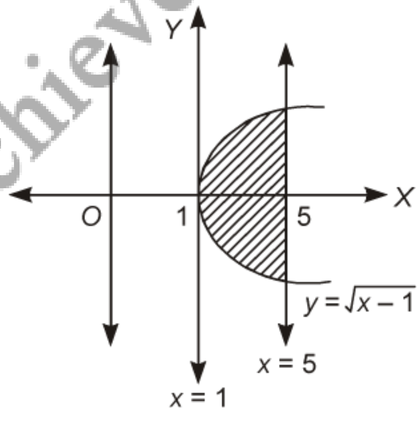
$$= \left[0 + \frac{a^2}{2}\sin^{-1}(1) - \frac{a^2}{2}\sin^{-1} 0\right]$$

$$= \frac{a^2}{2} \cdot \frac{\pi}{2} = \frac{\pi a^2}{4} \text{ sq. units.}$$

S11. Given equation of the curve is $y = \sqrt{x - 1}$

$\Rightarrow$ $y^2 = x - 1$

$\therefore$ Area of shaded region, $A = \int_1^5 (x - 1)^{1/2}\,dx$



$$= \left[\frac{2 \cdot (x - 1)^{3/2}}{3}\right]_1^5$$

$$= \left[\frac{2}{3} \cdot (5 - 1)^{3/2} - 0\right]$$

$$= \frac{16}{3} \text{ sq. units.}$$

S12. We have, $2y = 5x + 7$

$\Rightarrow$ $y = \frac{5x}{2} + \frac{7}{2}$

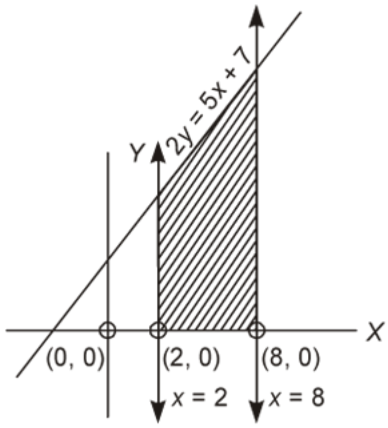
$\therefore$ Area of shaded region, $A = \dfrac{1}{2}\int_2^8 (5x+7)\,dx$

$$= \frac{1}{2}\left[5\cdot\frac{x^2}{2} + 7x\right]_2^8$$

$$= \frac{1}{2}[5\cdot 32 + 7\cdot 8 - 10 - 14]$$

$$= \frac{1}{2}[160 + 56 - 24]$$

$$= \frac{192}{2} = 96 \text{ sq. units.}$$

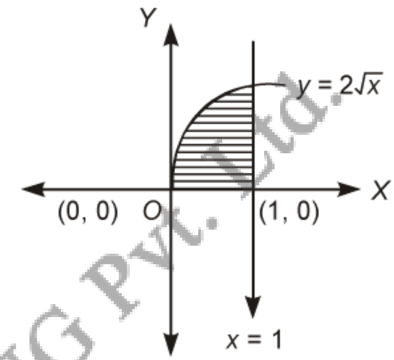


S13. We have, $y = 2\sqrt{x}$, $x = 0$ and $x = 1$

$\therefore$ Area of shaded region, $A = \int_0^1 (2\sqrt{x})\,dx$

$$= 2\cdot\left[\frac{x^{3/2}}{3}\cdot 2\right]_0^1$$

$$= 2\left(\frac{2}{3}\cdot 1 - 0\right) = \frac{4}{3} \text{ sq. units.}$$



S14. Given region $\{(x, 0) : y = \sqrt{4 - x^2}\}$ and X-axis

We have, $y = \sqrt{4 - x^2}$

$\Rightarrow$ $y^2 = 4 - x^2$

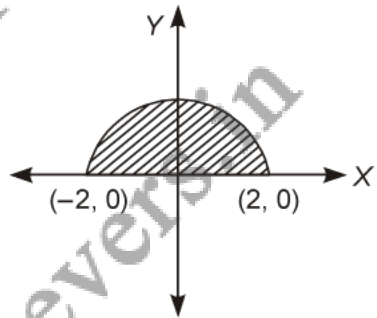
$\Rightarrow$ $x^2 + y^2 = 4$

$\therefore$ Area of shaded region, $A = \int_{-2}^{2} \sqrt{4 - x^2}\,dx = \int_{-2}^{2} \sqrt{2^2 - x^2}\,dx$

$$= \left[\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2}\cdot\sin^{-1}\frac{x}{2}\right]_{-2}^{2}$$

$$= \frac{2}{2}\cdot 0 + 2\cdot\frac{\pi}{2} + \frac{2}{2}\cdot 0 - 2\sin^{-1}(-1)$$

$$= 2\cdot\frac{\pi}{2} + 2\cdot\frac{\pi}{2} = 2\pi \text{ sq. units.}$$



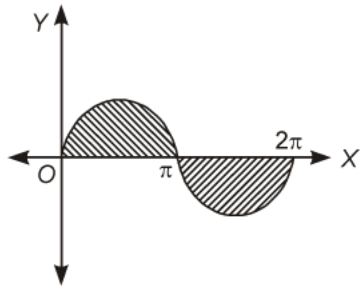
S15. Required area $\int_0^{2\pi} \sin x\,dx = \int_0^{\pi} \sin x\,dx + \left|\int_{\pi}^{2\pi} \sin x\,dx\right|$

$$= -[\cos x]_0^{\pi} + |[-\cos x]_{\pi}^{2\pi}|$$

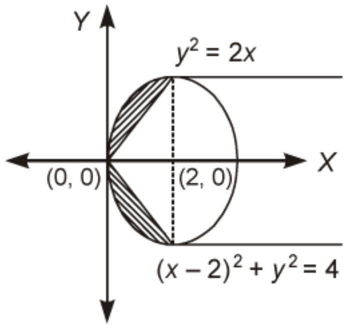
$$= -[\cos \pi - \cos 0] - |[\cos 2\pi - \cos \pi]_{\pi}^{2\pi}|$$

$$= -[-1 - 1] + |-(1 + 1)|$$

$$= 2 + 2 = 4 \text{ sq. units.}$$



S16. We have, $y^2 = 2x$ and $x^2 + y^2 = 4x$



$\Rightarrow$ $x^2 + 2x = 4x$

$\Rightarrow$ $x^2 - 2x = 0$

$\Rightarrow$ $x(x-2) = 0$

$\Rightarrow$ $x = 0, 2$

Also, $x^2 + y^2 = 4x$

$\Rightarrow$ $x^2 - 4x = -y^2$

$\Rightarrow$ $x^2 - 4x + 4 = -y^2 + 4$

$\Rightarrow$ $(x-2)^2 - 2^2 = -y^2$

$\therefore$ Required area $= 2 \cdot \int_0^2 \left[\sqrt{2^2 - (x-2)^2} - \sqrt{2x}\right] dx$

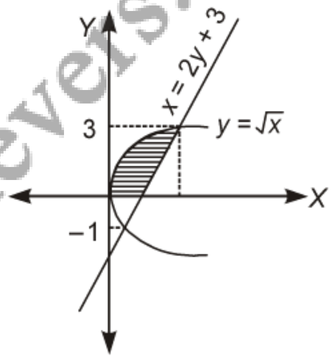
$$= 2\left[\left[\frac{x-2}{2} \cdot \sqrt{2^2 - (x-2)^2} + \frac{2^2}{2} \sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^2 - \left[\sqrt{2} \cdot \frac{x^{3/2}}{3/2}\right]_0^2\right]$$

$$= 2\left[\left(0 + 0 - 1 \cdot 0 + 2 \cdot \frac{\pi}{2}\right) - \frac{2\sqrt{2}}{3}(2^{3/2} - 0)\right]$$

$$= \frac{4\pi}{2} - \frac{8 \cdot 2}{3} = 2\pi - \frac{16}{3} = 2\left(\pi - \frac{8}{3}\right) \text{ sq. units.}$$

S17. Given equation of the curves are $y = \sqrt{x}$ and $x = 2y + 3$ in the first quadrant.

On solving both the equations for y, we get



$y = \sqrt{2y + 3}$

$\Rightarrow$ $y^2 = 2y + 3$

$\Rightarrow$ $y^2 - 2y - 3 = 0$

$\Rightarrow$ $y^2 - 3y + y - 3 = 0$

$\Rightarrow$ $y(y-3) + 1(y-3) = 0$

$\Rightarrow$ $(y+1)(y-3) = 0$

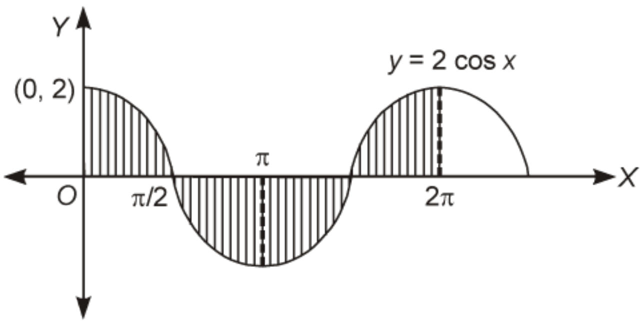
$\Rightarrow$ $y = -1, 3$

$\therefore$ Area of shaded region, $A = \int_0^3 (2y + 3 - y^2)\, dy = \left[\frac{2y^2}{2} + 3y - \frac{y^3}{3}\right]_0^3$

$$= \left[\frac{18}{2} + 9 - 9 - 0\right] = 9 \text{ sq. units.}$$

S18. Required area of shaded region $= \int_0^{2\pi} 2\cos x\, dx$

$$= \int_0^{\pi/2} 2\cos x\, dx + \left|\int_{\pi/2}^{3\pi/2} 2\cos x\, dx\right| + \int_{3\pi/2}^{2\pi} 2\cos x\, dx$$



$$= 2\left[\sin x\right]_0^{\pi/2} + \left|2\left(\sin x\right)_{\pi/2}^{3\pi/2}\right| + 2\left[\sin x\right]_{3\pi/2}^{2\pi}$$

$$= 2 + 4 + 2 = 8 \text{ sq. units.}$$

S19. Given equation of lines are

$$y = 4x + 5 \qquad \text{... (i)}$$

$$y = 5 - x \qquad \text{... (ii)}$$

and $$4y = x + 5 \qquad \text{... (iii)}$$

On solving Eqs. (i) and (ii), we get

$$4x + 5 = 5 - x$$

$\Rightarrow$ $$x = 0$$

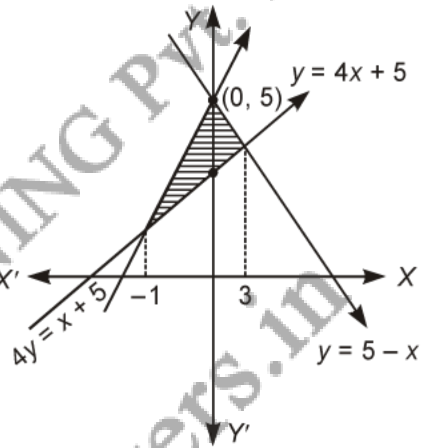
On solving Eqs. (i) and (iii), we get

$$4(4x + 5) = x + 5$$

$\Rightarrow$ $$16x + 20 = x + 5$$

$\Rightarrow$ $$15x = -15$$

$\Rightarrow$ $$x = -1$$



On solving Eqs. (ii) and (iii), we get

$$4(5 - x) = x + 5$$

$\Rightarrow$ $$20 - 4x = x + 5$$

$\Rightarrow$ $$x = 3$$

$\therefore$ Required area $$= \int_{-1}^{0}(4x+5)\,dx + \int_{0}^{3}(5-x)\,dx - \frac{1}{4}\int_{-1}^{3}(x+5)\,dx$$

$$= \left[\frac{4x^2}{2} + 5x\right]_{-1}^{0} + \left[5x - \frac{x^2}{2}\right]_0^3 - \frac{1}{4}\left[\frac{x^2}{2} + 5x\right]_{-1}^{3}$$

$$= [0 - 2 + 5] + \left[15 - \frac{9}{2} - 0\right] - \frac{1}{4}\left[\frac{9}{2} + 15 - \frac{1}{2} + 5\right]$$

$$= 3 + \frac{21}{2} - \frac{1}{4}\cdot 24$$

$$= -3 + \frac{21}{2} = \frac{15}{2} \text{ sq. units}$$

S20. We have $\quad x + 2y = 2 \quad \ldots \text{(i)}$

$y - x = 1 \quad \ldots \text{(ii)}$

and $\quad 2x + y = 7 \quad \ldots \text{(iii)}$

On solving Eqs. (i) and (ii), we get

$$y - (2 - 2y) = 1 \quad \Rightarrow \quad 3y - 2 = 1 \quad \Rightarrow \quad y = 1$$

On solving Eqs. (ii) and (iii), we get

$$2(y - 1) + y = 7$$

$$\Rightarrow \quad 2y - 2 + y = 7$$

$$\Rightarrow \quad y = 3$$

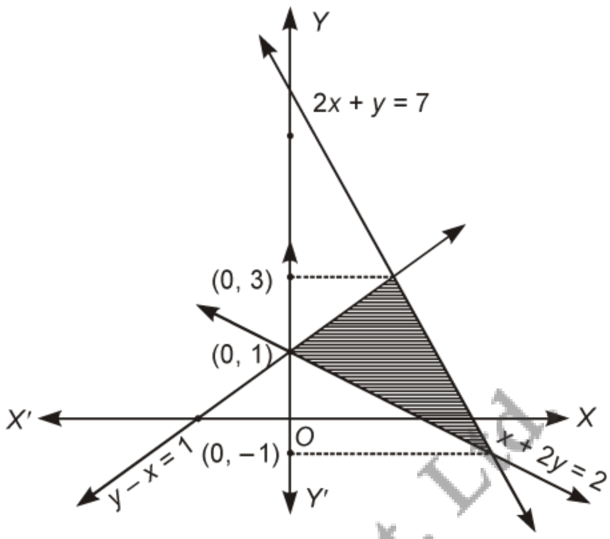
On solving Eqs. (i) and (iii), we get

$$2(2 - 2y) + y = 7$$

$$\Rightarrow \quad 4 - 4y + y = 7$$

$$\Rightarrow \quad -3y = 3$$

$$\Rightarrow \quad y = -1$$



$$\therefore \quad \text{Required area} = \int_{-1}^{1} (2 - 2y)\,dy + \int_{-1}^{3} \frac{(7 - y)}{2}\,dy - \int_{1}^{3} (y - 1)\,dy$$

$$= \left[-2y + \frac{2y^2}{2}\right]_{-1}^{1} + \left[\frac{7y}{2} - \frac{y^2}{2 \cdot 2}\right]_{-1}^{3} - \left[\frac{y^2}{2} - y\right]_{1}^{3}$$

$$= \left[-2 + \frac{2}{2} - 2 - \frac{2}{2}\right] + \left[\frac{21}{2} - \frac{9}{4} + \frac{7}{2} + \frac{1}{4}\right] - \left[\frac{9}{2} - 3 - \frac{1}{2} + 1\right]$$

$$= [-4] + \left[\frac{42 - 9 + 14 + 1}{4}\right] - \left[\frac{9 - 6 - 1 + 2}{2}\right]$$

$$= -4 + 12 - 2 = 6 \text{ sq. units.}$$

S21. We have, $\quad y^2 = 6ax \quad$ and $\quad x^2 + y^2 = 16a^2$

$$\Rightarrow \quad x^2 + 6ax = 16a^2$$

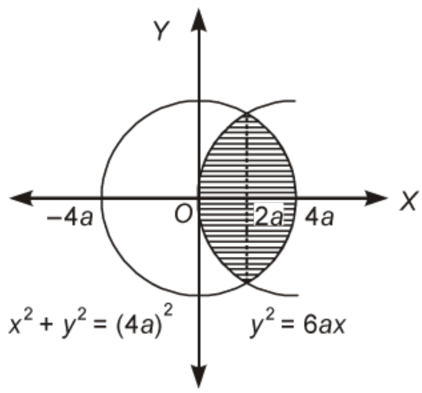
$$\Rightarrow \quad x^2 + 6ax - 16a^2 = 0$$

$$\Rightarrow \quad x^2 + 8ax - 2ax - 16a^2 = 0$$

$$\Rightarrow \quad x(x + 8a) - 2a(x + 8a) = 0$$

$$\Rightarrow \quad (x - 2a)(x + 8a) = 0$$

$$\Rightarrow \quad x = 2a, -8a$$



$$\therefore \quad \text{Area of required region} = 2\left[\int_0^{2a} \sqrt{6ax}\,dx + \int_{2a}^{4a} \sqrt{(4a)^2 - x^2}\,dx\right]$$

$$= 2\left[\int_0^{2a} \sqrt{6a}\,x^{1/2}\,dx + \int_{2a}^{4a} \sqrt{(4a)^2 - x^2}\,dx\right]$$

$$= 2\left[\sqrt{6a}\left[\frac{x^{3/2}}{3/2}\right]_0^{2a} + \left(\frac{x}{2}\sqrt{(4a)^2 - x^2} + \frac{(4a)^2}{2}\sin^{-1}\frac{x}{4a}\right)_{2a}^{4a}\right]$$

$$= 2\left[\sqrt{6a}\cdot\frac{2}{3}((2a)^{3/2} - 0) + \frac{4a}{2}\cdot 0 + \frac{16a^2}{2}\cdot\frac{\pi}{2} - \frac{2a}{2}\sqrt{16a^2 - 4a^2} - \frac{16a^2}{2}\cdot\sin^{-1}\frac{2a}{4a}\right]$$

$$= 2\left[\sqrt{6a}\,\frac{2}{3}\cdot 2\sqrt{2}\,a^{3/2} + 0 + 4\pi a^2 - \frac{2a}{2}\cdot 2\sqrt{3}a - 8a^2\cdot\frac{\pi}{6}\right]$$

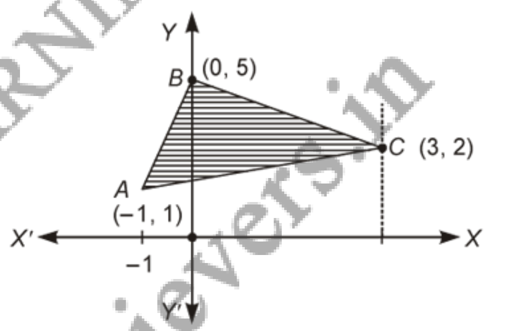
$$= 2\left[\sqrt{12}\cdot\frac{4}{3}a^2 + 4\pi a^2 - 2\sqrt{3}a^2 - \frac{4a^2\pi}{3}\right]$$

$$= 2\left[\frac{8\sqrt{3}a^2 + 12\pi a^2 - 6\sqrt{3}a^2 - 4a^2\pi}{3}\right]$$

$$= \frac{2}{3}a^2[8\sqrt{3} + 12\pi - 6\sqrt{3} - 4\pi]$$

$$= \frac{2}{3}a^2[2\sqrt{3} + 8\pi] = \frac{4}{3}a^2[\sqrt{3} + 4\pi].$$

S22. Let, we have the vertices of a ΔABC as $A(-1, 1)$, $B(0, 5)$ and $C(3, 2)$.



$\therefore$ Equation of AB is $y - 1 = \left(\frac{5-1}{0+1}\right)(x+1)$

$\Rightarrow$ $y - 1 = 4x + 4$

$\Rightarrow$ $y = 4x + 5$ ... (i)

and Equation of BC is $y - 5 = \left(\frac{2-5}{3-0}\right)(x - 0)$

$\Rightarrow$ $y - 5 = \frac{-3}{3}(x)$

$\Rightarrow$ $y = 5 - x$ ... (ii)

Similarly, Equation od AC is $y - 1 = \left(\frac{2-1}{3+1}\right)(x+1)$

$\Rightarrow$ $y - 1 = \frac{1}{4}(x+1)$

$\Rightarrow$ $4y = x + 5$ ... (iii)

$\therefore$ Area of shaded region $= \int_{-1}^{0}(y_1 - y_2)\,dx + \int_{0}^{3}(y_1 - y_2)\,dx$

$$= \int_{-1}^{0}\left[4x+5-\frac{x+5}{4}\right]dx + \int_{0}^{3}\left[5-x-\frac{x+5}{4}\right]dx$$

$$= \left[\frac{4x^2}{2}+5x-\frac{x^2}{8}-\frac{5x}{4}\right]_{-1}^{0} + \left[5x-\frac{x^2}{2}-\frac{x^2}{8}-\frac{5x}{4}\right]_{0}^{3}$$

$$= \left[0-\left(4\cdot\frac{1}{2}+5(-1)-\frac{1}{8}+\frac{5}{4}\right)\right]+\left[\left(15-\frac{9}{2}-\frac{9}{8}-\frac{15}{4}\right)-0\right]$$

$$= \left[-2+5+\frac{1}{8}-\frac{5}{4}+15-\frac{9}{2}-\frac{9}{8}-\frac{15}{4}\right]$$

$$= 18+\left(\frac{1-10-36-9-30}{8}\right)$$

$$= 18+\left(-\frac{84}{8}\right)=18-\frac{21}{2}=\frac{15}{2} \text{ sq. units.}$$

S23. Solving the given equations of curves, we have

$$x^2 + ax = 2ax$$

or $\quad x = 0, \quad x = a \quad$ which give

$$y = 0, \quad y = \pm a$$

From figure, we get

$$\text{Area } ODAB = \int_0^a \left(\sqrt{2ax-x^2}-\sqrt{ax}\right)dx$$

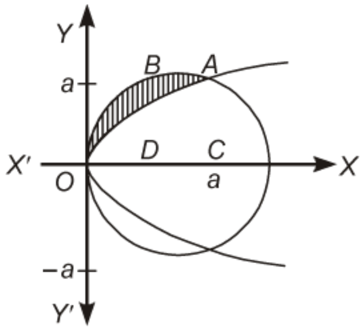
Let $x = 2a\sin^2\theta$. Then $dx = 4a\sin\theta\cos\theta\, d\theta$ and

$$x = 0 \;\Rightarrow\; \theta = 0, \quad x = a \;\Rightarrow\; \theta = \frac{\pi}{4}$$

Again, $$\int_0^a \sqrt{2ax-x^2}\,dx = \int_0^{\frac{\pi}{4}}(2a\sin\theta\cos\theta)(4a\sin\theta\cos\theta)\,d\theta$$

$$= a^2\int_0^{\frac{\pi}{4}}(1-\cos 4\theta)\,d\theta$$

$$= a^2\left(\theta-\frac{\sin 4\theta}{4}\right)_0^{\frac{\pi}{4}} = \frac{\pi}{4}a^2$$



Further more,

$$\int_0^a \sqrt{ax}\,dx = \sqrt{a}\,\frac{2}{3}\left(x^{\frac{3}{2}}\right)_0^a = \frac{2}{3}a^2$$

Thus, Required area = $\frac{\pi}{4}a^2 - \frac{2}{3}a^2$

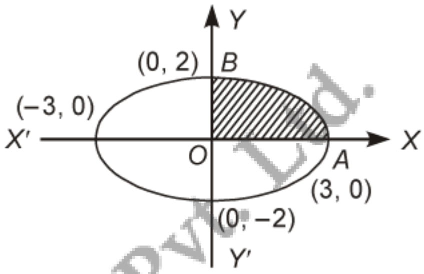
$$= a^2\left(\frac{\pi}{4} - \frac{2}{3}\right) \text{ sq. units}$$

S24. Eliminating t as follows:

$$x = 3\cos t, \quad y = 2\sin t \quad \Rightarrow \quad \frac{x}{3} = \cos t, \quad \frac{y}{2} = \sin t$$

we obtain $\frac{x^2}{9} + \frac{y^2}{4} = 1$,

which is the equation of an ellipse.



From figure, we get

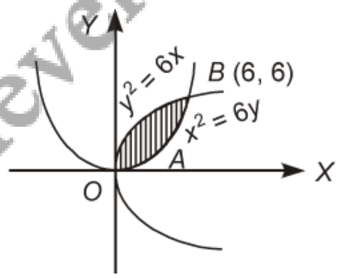
$$\text{Required area} = 4\int_0^3 \frac{2}{3}\sqrt{9-x^2}\,dx$$

$$= \frac{8}{3}\left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3 = 6\pi \text{ sq. units.}$$

S25. The intersecting points of the given parabolas are obtained by solving these equations for x and y, which are $O(0, 0)$ and $(6, 6)$. Hence,

$$\text{Area } OABC = \int_0^6 \left(\sqrt{6x} - \frac{x^2}{6}\right)dx = \left|2\sqrt{6}\,\frac{x^{\frac{3}{2}}}{3} - \frac{x^3}{18}\right|_0^6$$

$$= 2\sqrt{6}\,\frac{(6)^{\frac{3}{2}}}{3} - \frac{(6)^3}{18} = 12 \text{ sq. units}$$



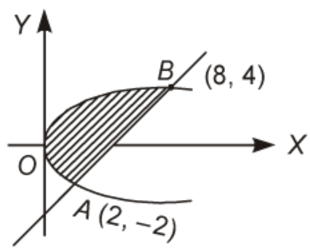
S26. The intersecting points of the given curves are obtained by solving the equations $x - y = 4$ and $y^2 = 2x$ for x and y.

We have, $y^2 = 8 + 2y$

i.e., $(y - 4)(y + 2) = 0$

which gives $y = 4, -2$ and $x = 8, 2$.

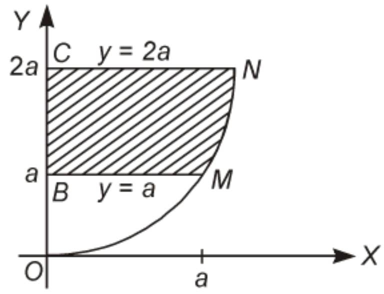
Thus, the points of intersection are $(8, 4)$, $(2, -2)$. Hence,



$$\text{Area} = \int_{-2}^{4}\left(4 + y - \frac{1}{2}y^2\right)dy$$

$$= \left|4y + \frac{y^2}{2} - \frac{1}{6}y^3\right|_{-2}^{4} = 18 \text{ sq. units.}$$

S27. We have,



$$\text{Area } BMNC = \int_a^{2a} x\,dy = \int_a^{2a} a^{\frac{1}{3}} y^{\frac{2}{3}} dy$$

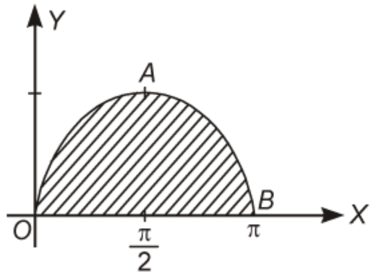
$$= \frac{3a^{\frac{1}{3}}}{5} \left| y^{\frac{5}{3}} \right|_a^{2a}$$

$$= \frac{3a^{\frac{1}{3}}}{5} \left| (2a)^{\frac{5}{3}} - a^{\frac{5}{3}} \right|$$

$$= \frac{3}{5} a^{\frac{1}{3}} a^{\frac{5}{3}} \left| (2)^{\frac{5}{3}} - 1 \right|$$

$$= \frac{3}{5} a^2 \left| 2.2^{\frac{2}{3}} - 1 \right| \text{ sq. units.}$$

S28. We have,



$$\text{Area } OAB = \int_0^{\pi} y\,dx = \int_0^{\pi} \sin x\,dx = \left| -\cos x \right|_0^{\pi}$$

$$= \cos 0 - \cos \pi = 2 \text{ sq. units.}$$

S29. Given that, $x = at^2$ ... (i)

$y = 2at$ ... (ii)

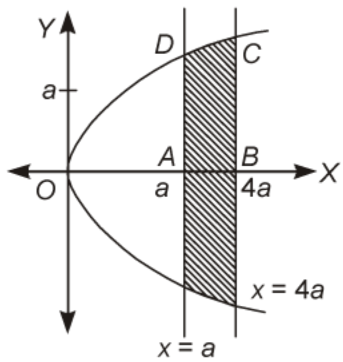
$\Rightarrow$ $t = \dfrac{y}{2a}$

putting the value of t in (i), we get

$$y^2 = 4ax$$

Putting $t = 1$ and $t = 2$ in (i), we get

$$x = a \quad \text{and} \quad x = 4a.$$



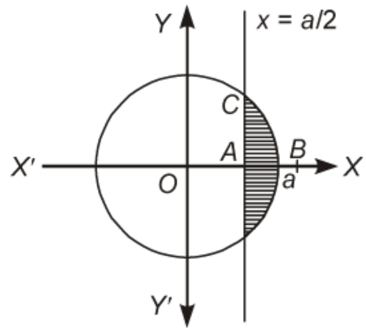
Required area = 2 area of $ABCD$

$$= 2\int_a^{4a} y\,dx = 2 \times 2 \int_a^{4a} \sqrt{ax}\,dx$$

$$= 8\sqrt{a} \left| \frac{(x)^{\frac{3}{2}}}{3} \right|_a^{4a} = \frac{56}{3} a^2 \text{ aq. units.}$$

S30. Solving the equation $x^2 + y^2 = a^2$ and $x = \frac{a}{2}$, we obtain their points of intersection which are $\left(\frac{a}{2}, \sqrt{3}\,\frac{a}{2}\right)$ and $\left(\frac{a}{2}, -\frac{\sqrt{3}a}{2}\right)$.

Hence, from figure, we get



$$\text{Required area} = 2 \text{ area of } OAB = 2\int_{\frac{a}{2}}^{a} \sqrt{a^2 - x^2}\,dx$$

$$= 2\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{\frac{a}{2}}^{a}$$

$$= 2\left[\frac{a}{2}\cdot\frac{\pi}{2} - \frac{a}{4}\cdot a\,\frac{\sqrt{3}}{2} - \frac{a^2}{2}\cdot\frac{\pi}{6}\right]$$

$$= \frac{a^2}{12}\left(6\pi - 3\sqrt{3} - 2\pi\right)$$

$$= \frac{a^2}{12}\left(4\pi - 3\sqrt{3}\right) \text{ sq. units.}$$