Date: 18/10/2021

Q1. If $[2x \quad 3]\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 8 \end{bmatrix} = O$, find the value of x.

Q2. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 3 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $D = \begin{bmatrix} 4 & 6 & 8 \\ 5 & 7 & 9 \end{bmatrix}$, then which of the sums $A + B$, $B + C$, $C + D$ and $B + D$ is defined?

Q3. Construct a matrix $A = [a_{ij}]_{2 \times 2}$ whose elements a_{ij} are given by $a_{ij} = e^{2ix} \sin jx$.

Q4. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$, $a = 4$ and $b = -2$, then show that $(A - B)C = AC - BC$.

Q5. Show by an example that for $A \neq 0$, $B \neq 0$ and $AB = 0$.

Q6. Find non-zero values of x satisfying the matrix equation

$$x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} (x^2+8) & 24 \\ 10 & 6x \end{bmatrix}$$

Q7. Find the values of a and b, if $A = B$, where

$$A = \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}$$

Q8. Construct a 3×2 matrix whose elements are given by $a_{ij} = e^{i.x} \cdot \sin jx$.

Q9. If, $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then show that $A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$.

Q10. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$, $a = 4$ and $b = 2$, then show that $(A - B)^T = A^T - B^T$.

Q11. If possible, find the sum of the matrices A and B where

$$A = \begin{bmatrix} \sqrt{3} & 1 \\ 2 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} x & y & z \\ a & b & c \end{bmatrix}$$

Q12. If a matrix has 28 elements, what are the possible orders it can have? What if it has 13 elements?

Q13. If A is 3×3 invertible matrix, then show that for any scalar k (non-zero), A is invertible and $(kA)^{-1} = \frac{1}{k}A^{-1}$.

Q14. Verify that $A^2 = I$, when $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$.

Q15. If A, B are square matrices of same order and B is a skew-symmetric matrix, then show that $A'BA$ is skew-symmetric.

Q16. Let A and B be square matrices of the order 3×3. Is $(AB)^2 = A^2B^2$? Give reasons?

Q17. Show that $A'A$ and AA' are both symmetric matrices for any matrix A.

Q18. Show that, if A and B are square matrices such that $AB = BA$, then

$$(A + B)^2 = A^2 + 2AB + B^2.$$

Q19. If $A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, then verify $(BA)^2 \neq B^2A^2$.

Q20. Construct $a_{2 \times 2}$ matrix, where

(i) $a_{ij} = \dfrac{(i - 2j)^2}{2}$ (ii) $a_{ij} = |-2i + 3j|$

Q21. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then show that $(A + B)(A - B) \neq A^2 - B^2$.

Q22. If $X = \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$, then find

(i) $X + Y$. (ii) $2X - 3Y$. (iii) a matrix Z such that $X + Y + Z$ is a zero matrix.

Q23. If $A = [3 \quad 5]$ and $B = [7 \quad 3]$, then find a non-zero matrix C such that $AC = BC$.

Q24. If possible, find the value of BA and AB, where

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}$$

Q25. In the matrix $A = \begin{bmatrix} a & 1 & x \\ 2 & \sqrt{3} & x^2 - y \\ 0 & 5 & \frac{-2}{5} \end{bmatrix}$, write

(i) The order of matrix A. (ii) The number of elements. (iii) Elements a_{23}, a_{31} and a_{12}

Q26. If $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, then verify that $A^2 + A = A(A + I)$, where I is 3×3 unit matrix.

Q27. If $\begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$, then find the value of A.

Q28. If $P = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ and $Q = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, then prove that $PQ = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix} = QP$.

Q29. Give an example of matrices A, B and C, such that $AB = AC$, where A is non-zero matrix but $B \neq C$.

Q30. If, $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$, then verify that

(i) $(A')' = A$ (ii) $(AB)' = B'A'$ (iii) $(kA)' = (kA')$

Q31. Solve for x and y, $x\begin{bmatrix} 2 \\ 1 \end{bmatrix} + y\begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = 0$.

Q32. Given, $A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}$, is $(AB)' = B'A'$?

Q33. Find A, if $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$.

Q34. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$, $a = 4$ and $b = -2$, then show that

(i) $A + (B + C) = (A + B) + C$ (ii) $A(BC) = (AB)C$

Q35. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$, then verify that

(i) $(2A + B)' = 2A' + B'$ (ii) $(A - B)' = A' - B'$

Q36. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$, $a = 4$ and $b = -2$, then show that

(i) $(a + b)B = aB + bB$ (ii) $a(C - A) = aC - aA$

Q37. Prove by mathematical induction that $(A')^n = (A^n)'$ when $n \in N$ for any square matrix A.

Q38. If $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $x^2 = -1$, then show that $(A + B)^2 = A^2 + B^2$.

Q39. If $A = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$, $B = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$, then find a matrix C such that $3A + 5B + 2C$ is a null matrix.

Q40. Find the values of a, b, c and d, if

$$3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix} + \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix}$$

Q41. If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that A satisfies the equation $A^3 - 4A^2 - 3A + 11I = 0$.

Q42. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$, then find $A^2 + 2A + 7I$.

Q43. If $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$. Then find the values of x, y, z and w.

Q44. If $AB = BA$ for any two square matrices, then prove by mathematical induction that $(AB)^n = A^nB^n$.

Q45. If A is square matrix sucn that $A^2 = A$, then show that $(I + A)^3 = 7A + I$.

Q46. If matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then find the values of a, b and c.

Q47. Express the matrix $\begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Q48. Find inverse, by elementary row operations (if possible), of the the matrices $\begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}$.

Q49. Find inverse, by elementary row operations (if possible), of the the matrices $\begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix}$.

Q50. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ and $A^{-1} = A'$, then find the value of α.

Q51. If possible, using elementary row transformations, find the inverse of the matrices $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$.

Q52. If possible, using elementary row transformations, find the inverse of the matrices

$$\begin{bmatrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 1 & 1 & -1 \end{bmatrix}$$

Q53. If $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$, verify

(i) $(AB)C = A(BC)$ (ii) $A(B+C) = AB + AC$.

Q54. If $A = [2 \quad 1]$, $B = \begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, then verify that $A(B + C) = (AB + AC)$.

Q55. Find the matrix A satisfying the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Q56. If X and Y are 2×2 matrices, then solve the following matrix equations for X and Y.

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}, \quad 3X + 2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}.$$

Q57. Express the matrix A as the sum of a symmetric and a skew-symmetric matrix, where

$$A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}.$$

Q58. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$. Then show that $A^2 - 4A + 7I = O$. Using this result calculate A^5 also.

Q59. If $P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, then show that $P(x) \cdot P(y) = P(x + y) = P(y) \cdot P(x)$.

Q60. Find the matrix A such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$.

Q61. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then find $A^2 - 5A - 14I$. Hence, obtain A^3.

Q62. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$, $a = 4$ and $b = -2$, then show that

(i) $(A^T)^T = A$ (ii) $(bA)^T = bA^T$ (ii) $(AB)^T = B^T A^T$

Q63. Show that $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ satisfy the equation $A^2 - 3A - 7I = 0$ and hence find the value of A^{-1}.

Q64. Find x, y and z, if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A' = A^{-1}$.

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S1. We have,

$$[2x \quad 3]\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ 8 \end{bmatrix} = O \Rightarrow [2x-9 \quad 4x]\begin{bmatrix} x \\ 8 \end{bmatrix} = [0]$$

or $$[2x^2 - 9x \quad +32x] = [0] \Rightarrow 2x^2 + 23x = 0$$

or $$x(2x + 23) = 0 \Rightarrow x = 0, \quad x = \frac{-23}{2}$$

S2. Only $B + D$ is defined since matrices of the same order can only be added.

S3. For $i = 1,\ j = 1, \quad a_{11} = e^{2x} \sin x$

For $i = 1,\ j = 2, \quad a_{12} = e^{2x} \sin 2x$

For $i = 2,\ j = 1, \quad a_{21} = e^{4x} \sin x$

For $i = 2,\ j = 2, \quad a_{22} = e^{4x} \sin 2x$

Thus, $$A = \begin{bmatrix} e^{2x} \sin x & e^{2x} \sin 2x \\ e^{4x} \sin x & e^{4x} \sin 2x \end{bmatrix}.$$

S4. We have,

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} \text{ and } a = 4, \quad b = -2$$

$$(A - B) = \begin{bmatrix} 1-4 & 2-0 \\ -1-1 & 3-5 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -2 & -2 \end{bmatrix}$$

$$(A - B)\,C = \begin{bmatrix} -3 & 2 \\ -2 & -2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix} \quad \ldots \text{(i)}$$

Now, $$AC = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 1 & -6 \end{bmatrix} \quad \ldots \text{(ii)}$$

and $$BC = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix} \quad \ldots \text{(iii)}$$

$\therefore$ $$(AC - BC) = \begin{bmatrix} 4-8 & -4-0 \\ 1-7 & -6+10 \end{bmatrix} \quad \text{[Using Eqs. (ii) and (iii)]}$$

$$= \begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix}$$

$= (A - B)\,C$ **Hence proved.** [Using Eqs. (i)]

S5. Let,

$$A = \begin{bmatrix} 0 & -4 \\ 0 & 2 \end{bmatrix} \neq 0 \quad \text{and} \quad B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix} \neq 0$$

$\therefore$ $$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$ **Hence proved.**

S6. Given that,

$$x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} (x^2+8) & 24 \\ 10 & 6x \end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix} 2x^2 & 2x \\ 3x & x^2 \end{bmatrix} + \begin{bmatrix} 16 & 10x \\ 8 & 8x \end{bmatrix} = \begin{bmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix} 2x^2+16 & 2x+10x \\ 3x+8 & x^2+8x \end{bmatrix} = \begin{bmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{bmatrix}$$

$\Rightarrow$ $$2x + 10x = 48$$

$\Rightarrow$ $$12x = 48$$

$\therefore$ $$x = \frac{48}{12} = 4.$$

S7. We have,

$$A = \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix}_{2\times 2} \quad \text{and} \quad B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}_{2\times 2}$$

Also, $$A = B$$

$\therefore$ $$a_{11} = b_{11} \Rightarrow a + 4 = 2a + 2 \Rightarrow a = 2$$

$$a_{12} = b_{12} \Rightarrow 3b = b^2 + 2 \Rightarrow b^2 = 3b - 2$$

and $$a_{22} = b_{22} \Rightarrow -6 = b^2 - 5b$$

$\Rightarrow$ $$-6 = 3b - 2 - 5b \qquad [\because\ b^2 = 3b - 2]$$

$\Rightarrow$ $$2b = 4 \Rightarrow b = 2$$

$\therefore$ $$a = 2 \quad \text{and} \quad b = 2.$$

S8. Since, $A = [a_{ij}]_{m\times n}$ $1 \le i \le m$ and $1 \le j \le n$, $i, j \in N$

$\therefore$ $$A = [e^{i.x} \sin jx]_{3\times 2}; \quad 1 \le i \le 3; \quad 1 \le j \le 2$$

$\Rightarrow$ $$a_{11} = e^{1\cdot x} \cdot \sin 1 \cdot x = e^x \sin x$$

$$a_{12} = e^{1\cdot x} \cdot \sin 2 \cdot x = e^x \sin 2x$$

$$a_{21} = e^{2\cdot x} \cdot \sin 1 \cdot x = e^{2x} \sin x$$

$$a_{22} = e^{2\cdot x} \cdot \sin 2 \cdot x = e^{2x} \sin 2x$$

$$a_{31} = e^{3\cdot x} \cdot \sin 1 \cdot x = e^{3x} \sin x$$

$$a_{32} = e^{3\cdot x} \cdot \sin 2 \cdot x = e^{3x} \sin 2x$$

$$A = \begin{bmatrix} e^x \sin x & e^x \sin 2x \\ e^{2x} \sin x & e^{2x} \sin 2x \\ e^{3x} \sin x & e^{3x} \sin 2x \end{bmatrix}_{3 \times 2}$$

S9. We have,

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$\therefore$
$$A^2 = A \cdot A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & \cos\theta \cdot \sin\theta + \sin\theta \cdot \cos\theta \\ -\sin\theta\cos\theta - \cos\theta\sin\theta & -\sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & 2\sin\theta\cos\theta \\ -2\sin\theta & \cos 2\theta \end{bmatrix} \qquad [\because \cos^2\theta - \sin^2\theta = \cos^2\theta]$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix} \qquad [\because \sin 2\theta = 2\sin\theta \cdot \cos\theta]$$ **Hence proved.**

S10. We have,

$$(A - B)^T = \begin{bmatrix} 1-4 & 2-0 \\ -1-1 & 3-5 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 2 \\ -2 & -2 \end{bmatrix}^T = \begin{bmatrix} -3 & -2 \\ 2 & -2 \end{bmatrix}$$

$$A^T - B^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -2 \\ 2 & -2 \end{bmatrix} = (A - B)^T$$ **Hence proved.**

S11. We know that, addition of two or more than two matrices is being possible if and only if they have same order.

Here, $A = \begin{bmatrix} \sqrt{3} & 1 \\ 2 & 3 \end{bmatrix}_{2 \times 2}$ and $B = \begin{bmatrix} x & y & z \\ a & b & c \end{bmatrix}_{2 \times 3}$

Here, A and B are of different orders hence matrix addition is not possible.

S12. Since, a matrix having order $m \times n$ contains mn elements, where m and n are natural numbers.

$\because$ we have, $m \times n = 28 \quad (m, n \in N)$

$\Rightarrow \quad (m, n) = \{(1, 28), (2, 14), (4, 7), (7, 4), (14, 2), (28, 1)\}$

So, the possible orders are:

$$1 \times 28, \quad 2 \times 14, \quad 4 \times 7, \quad 7 \times 4, \quad 14 \times 2, \quad 28 \times 1.$$

Also, if it has 13 elements then $m \times n = 13$

$\therefore \qquad (m, n) = \{(1, 13), (13, 1)\}$

Hence, possible orders are 1×13, 13×1.

S13. We have,

$$(kA)\left(\frac{1}{k}A^{-1}\right) = \left(k \cdot \frac{1}{k}\right)(A \cdot A^{-1}) = 1\,(\text{I}) = I$$

Hence, (kA) is inverse of $\left(\frac{1}{k}A^{-1}\right)$ or $(kA)^{-1} = \frac{1}{k}A^{-1}$.

S14. We have,

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$$

$$\therefore \quad A^2 = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix} \qquad [\because A^2 = A \cdot A]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Hence proved.

S15. Since, A and B are square matrices of same order and B is a skew-symmetric matrix *i.e.*, $B' = -B$.

Now, we have to prove that $A'BA$ is a skew-symmetric matrix.

$$\therefore \quad A'BA' = A'BA' = BA'A' \qquad [\because (AB)' = B'A']$$

$$= A'B'A = A' - BA = -A'BA$$

Hence, $A'BA$ is a skew-symmetrix matrix.

S16. Since multiplication of two matrices is commutative, if they have same order hence for two matrices A and B.

$$AB = BA.$$

Since, A and B are square matrices of order 3×3

$$AB^2 = AB \cdot AB$$

$$= ABAB$$

$$= AABB$$

$$= A^2B^2$$

So, $AB^2 = A^2B^2$ is true when $AB = BA$.

S17. Let $\qquad P = AA'$

$$\therefore \quad P' = (AA')'$$

$$= A'(A')'$$

$$= A'A = P \qquad [\because (AB')' = B'A']$$

So, $A'A$ is symmetric matrix for any matrix A.

Similarly, let $$Q = AA'$$

$$\therefore \quad Q' = (AA')' = (A')'(A)'$$
$$= A(A')' = Q$$

So, AA' is symmetric matrix for any matrix A.

S18. Since A and B are square matrices such that $AB = BA$. Hence, A and B have same orders.

$$(A + B)^2 = (A + B) \cdot (A + B)$$
$$= A^2 + AB + BA + B^2$$
$$= A^2 + AB + AB + B^2$$
$$= A^2 + 2AB + B^2.$$

S19. We have,

$$A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}_{3 \times 2} \quad \text{and} \quad B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2 \times 3}$$

$$\therefore \quad BA = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2 \times 3} \begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}_{3 \times 2}$$

$$= \begin{bmatrix} 6+1+4 & -8+1+0 \\ 3+2+8 & -4+2+0 \end{bmatrix} = \begin{bmatrix} 11 & -7 \\ 13 & -2 \end{bmatrix}$$

and $$(BA) \cdot (BA) = \begin{bmatrix} 11 & -7 \\ 13 & -2 \end{bmatrix} \begin{bmatrix} 11 & -7 \\ 13 & -2 \end{bmatrix}$$

$$\Rightarrow \quad (BA)^2 = \begin{bmatrix} 121-91 & -77+14 \\ 143-26 & -91+4 \end{bmatrix} = \begin{bmatrix} 30 & -63 \\ 117 & -87 \end{bmatrix} \quad \text{... (ii)}$$

Also, $$B^2 = B \cdot B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2 \times 3} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2 \times 3}$$

So, B^2 is not possible, since the B is not a square matrix.

Hence, $(BA)^2 \neq B^2A^2$.

S20. We know that, the notation, namely $A = [a_{ij}]_{m \times n}$ indicates that A is a matrix of order $m \times n$, also $1 \leq i \leq m$, $1 \leq j \leq n$, $i, j \in N$.

(i) Here, $A = [a_{ij}]_{2 \times 2}$

$$\Rightarrow \quad A = \frac{(i - 2j)^2}{2}, \quad 1 \leq i \leq 2, \quad 1 \leq j \leq 2 \quad \text{... (i)}$$

$\therefore \qquad a_{11} = \frac{(1-2)^2}{2} = \frac{1}{2}$

$$a_{12} = \frac{(1-2\times 2)^2}{2} = \frac{9}{2}$$

$$a_{21} = \frac{(2-2\times 1)^2}{2} = 0$$

$$a_{22} = \frac{(2-2\times 2)^2}{2} = 2$$

Thus, $\qquad A = \begin{bmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \end{bmatrix}_{2\times 2}$

(ii) Here, $A = [a_{ij}]_{2\times 2} = |-2i + 3j|, \quad 1 \le i \le 2, \quad 1 \le j \le 2$

$\therefore \qquad a_{11} = |-2 \times 1 + 3 \times 1| = 1$

$$a_{12} = |-2 \times 1 + 3 \times 2| = 4 \qquad [\because \ |-1| = 1]$$

$$a_{21} = |-2 \times 2 + 3 \times 1| = 1$$

$$a_{22} = |-2 \times 2 + 3 \times 2| = 2$$

$\therefore \qquad A = \begin{bmatrix} 1 & 4 \\ 1 & 2 \end{bmatrix}_{2\times 2}$

S21. We have,

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$\therefore \qquad (A + B) = \begin{bmatrix} 0+0 & 1-1 \\ 1+1 & 1+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}_{2\times 2}$

and $\qquad (A - B) = \begin{bmatrix} 0-0 & 1+1 \\ 1-1 & 1-0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

Since, $(A + B) \cdot (A - B)$ is defined, if the number of columns of $(A + B)$ is equal to the number of rows of $(A - B)$, so here multiplication of matrices $(A + B) \cdot (A - B)$ is possible.

Now, $\qquad (A + B)_{2\times 2} \cdot (A - B)_{2\times 2} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 4+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix} \qquad \ldots \text{(i)}$

Also, $\qquad A^2 = A \cdot A$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1 & 0+1 \\ 0+1 & 1+1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

and $$B^2 = B \cdot B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0-1 & 0+0 \\ 0+0 & -1+0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$\Rightarrow$ $$A^2 - B^2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \quad \ldots \text{(ii)}$$

Thus, we see that

$$(A + B) \cdot (A - B) \neq A^2 - B^2 \quad \text{[Using Eqs. (i) and (ii)]}$$

$\Rightarrow$ $$\begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$ **Hence proved.**

S22. We have,

$$X = \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix}_{2 \times 3} \quad \text{and} \quad Y = \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}_{2 \times 3}$$

(i) $$X + Y = \begin{bmatrix} 3+2 & 1+1 & -1-1 \\ 5+7 & -2+2 & -3+4 \end{bmatrix} = \begin{bmatrix} 5 & 2 & -2 \\ 12 & 0 & 1 \end{bmatrix}$$

(ii) $\because$ $$2X = 2\begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 6 & 2 & -2 \\ 10 & -4 & -6 \end{bmatrix}$$

and $$3Y = 3\begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 3 & -3 \\ 21 & 6 & 12 \end{bmatrix}$$

$\therefore$ $$2X - 3Y = \begin{bmatrix} 6-6 & 2-3 & -2+3 \\ 10-21 & -4-6 & -6-12 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ -11 & -10 & -18 \end{bmatrix}$$

(iii) $$X + Y = \begin{bmatrix} 3+2 & 1+1 & -1-1 \\ 5+7 & -2+2 & -3+4 \end{bmatrix} = \begin{bmatrix} 5 & 2 & -2 \\ 12 & 0 & 1 \end{bmatrix}$$

Also, $$X + Y + Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

We see that Z is the additive inverse of $(X + Y)$ or negative of $(X + Y)$.

$\therefore$ $$Z = \begin{bmatrix} -5 & -2 & 2 \\ -12 & 0 & -1 \end{bmatrix} \quad [\because\ Z = -(X + Y)]$$

S23. We have, $$A = [3 \quad 5]_{1 \times 2} \quad \text{and} \quad B = [7 \quad 3]_{1 \times 2}$$

Let $C = \begin{bmatrix} x \\ y \end{bmatrix}_{2 \times 1}$ is a non-zero matrix of order 2×1.

$\therefore \qquad AC = \begin{bmatrix} 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3x + 5y \end{bmatrix}$

and $\qquad BC = \begin{bmatrix} 7 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7x + 3y \end{bmatrix}$

For $AC = BC$, $\qquad [3x + 5y] = [7x + 3y]$

On using equality of matrix, we get

$$3x + 5y = 7x + 3y$$

$\Rightarrow \qquad 4x = 2y$

$\Rightarrow \qquad x = \frac{1}{2} y$

$\Rightarrow \qquad y = 2x$

$\therefore \qquad C = \begin{bmatrix} x \\ 2x \end{bmatrix}$

We see that on taking C of order 2×1, 2×2, 2×3, ..., we get

$$C = \begin{bmatrix} x \\ 2x \end{bmatrix} \begin{bmatrix} x & x \\ 2x & 2x \end{bmatrix} \begin{bmatrix} x & x & x \\ 2x & 2x & 2x \end{bmatrix} \ldots$$

in general, $\qquad C = \begin{bmatrix} k \\ 2k \end{bmatrix} \begin{bmatrix} k & k \\ 2k & 2k \end{bmatrix} etc \ldots$

where, k is any real number.

S24. We have,

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2 \times 3} \quad \text{and} \quad B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}_{3 \times 2}$$

So, AB and BA both are possible.

[Since, in both $A \cdot B$ and $B \cdot A$, the number of columns of first is equal to the number of rows of second.]

$\therefore \qquad AB = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2 \times 3} \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}_{3 \times 2}$

$$= \begin{bmatrix} 8 + 2 + 2 & 2 + 3 + 4 \\ 4 + 4 + 4 & 1 + 6 + 8 \end{bmatrix} = \begin{bmatrix} 12 & 9 \\ 12 & 15 \end{bmatrix}$$

and $\qquad BA = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}_{3 \times 2} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2 \times 3}$

$$= \begin{bmatrix} 4\times 2+1 & 4+2 & 8+4 \\ 4+3 & 2+6 & 4+12 \\ 2+2 & 1+4 & 2+8 \end{bmatrix} = \begin{bmatrix} 9 & 6 & 12 \\ 7 & 8 & 16 \\ 4 & 5 & 10 \end{bmatrix}.$$

S25. We know that, if a matrix is of order $m \times n$ then it contains mn elements where m is number of row and n is number of columns. Hence,

(i) The order of matrix $A = 3 \times 3$

(ii) The number of elements $= 3 \times 3 = 9$.

(iii) $a_{23} = x^2 - y$, $a_{31} = 0$, $a_{12} = 1$

Since, we know that a_{ij} is a representation of element lying in the i^{th} row and j^{th} column.

S26. We have,

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$\therefore$ $$A^2 = A \cdot A$$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -2 \\ 4 & 4 & 4 \\ 2 & 2 & 4 \end{bmatrix}$$

$\therefore$ $$A^2 + A = \begin{bmatrix} 1 & -1 & -2 \\ 4 & 4 & 4 \\ 2 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & -3 \\ 6 & 5 & 7 \\ 2 & 3 & 5 \end{bmatrix} \qquad \text{... (i)}$$

Now, $$A + I = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

and $$A(A + I) = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & -1 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -3 \\ 6 & 5 & 7 \\ 2 & 3 & 5 \end{bmatrix} \qquad \text{... (ii)}$$

Thus, we see that $A^2 + A = A(A + I)$. [Using Eqs. (i) and (ii)]

S27. We have,

$$\begin{bmatrix} 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$$

$\therefore \quad \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = [-2-1+0 \quad 0+1+3 \quad -2+0+3]$

$$= [-3 \quad 4 \quad 1]$$

Now, $\begin{bmatrix} -3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$

$\therefore \quad A = \begin{bmatrix} -3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$$= [-3+0-1] = [-4].$$

S28. $PQ = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix}$ ... (i)

and $QP = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} = \begin{bmatrix} ax & 0 & 0 \\ 0 & by & 0 \\ 0 & 0 & zc \end{bmatrix}$ ... (ii)

Thus, we see that $PQ = QP$ [Using Eqs. (i) and (ii)]

Hence proved.

S29. Let,

$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 3 \\ 4 & 4 \end{bmatrix}$ $[\because \; B \neq C]$

$\therefore \quad AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}$ ... (i)

and $AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}$ ... (ii)

Thus, we see that $AB = AC$ [Using Eqs. (i) and (ii)]

where, A is non-zero matrix but $B \neq C$.

S30. We have,

$$A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$$

(i) We have to verify that. $A' = A$

$\therefore$ $$A' = \begin{bmatrix} 0 & 4 \\ -1 & 3 \\ 2 & -4 \end{bmatrix}$$

and $$(A')' = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix} = A$$ **Hence proved.**

(ii) We have to verify that. $AB' = B'A'$

$\therefore$ $$AB = \begin{bmatrix} 3 & 9 \\ 11 & -15 \end{bmatrix}$$

= $$(AB)' = \begin{bmatrix} 3 & 11 \\ 9 & -15 \end{bmatrix}$$

and $$B'A' = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 6 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ -1 & 3 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 3 & 11 \\ 9 & -15 \end{bmatrix}$$

$$= (AB)'$$ **Hence proved.**

(iii) We have to verify thant, $(kA)' = (kA')$

Now, $$(kA) = \begin{bmatrix} 0 & -k & 2k \\ 4k & 3k & -4k \end{bmatrix}$$

and $$(kA)' = \begin{bmatrix} 0 & 4k \\ -k & 3k \\ 2k & -4k \end{bmatrix}$$

Also, $$kA' = \begin{bmatrix} 0 & 4k \\ -k & 3k \\ 2k & -4k \end{bmatrix} = (kA)'$$ **Hence proved.**

S31. We have, $$x\begin{bmatrix} 2 \\ 1 \end{bmatrix} + y\begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = 0$$

$\Rightarrow$ $$\begin{bmatrix} 2x \\ x \end{bmatrix} + \begin{bmatrix} 3 \cdot y \\ 5 \cdot y \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} 2x+3y-8 \\ x+5y-11 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore \quad 2x + 3y - 8 = 0$

$\Rightarrow \quad 4x + 6y = 16 \qquad \text{... (i)}$

and $\quad x + 5y - 11 = 0$

$\Rightarrow \quad 4x + 20y = 44 \qquad \text{... (ii)}$

On subtracting Eq. (i) from Eq. (ii), we get

$$14y = 28 \quad \Rightarrow \quad y = 2$$

$\therefore \quad 2x + 3 \times 2 - 8 = 0$

$\Rightarrow \quad 2x = 2 \quad \Rightarrow \quad x = 1$

$\therefore \quad x = 1$ and $y = 2$.

S32. We have,

$$A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}_{2\times 3} \quad \text{and} \quad B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}_{3\times 2}$$

$$\therefore \quad AB = \begin{bmatrix} 2+8+0 & 8+32+0 \\ 3+18+6 & 12+72+18 \end{bmatrix} = \begin{bmatrix} 10 & 40 \\ 27 & 102 \end{bmatrix}$$

and $$(AB)' = \begin{bmatrix} 10 & 27 \\ 40 & 102 \end{bmatrix} \qquad \text{... (i)}$$

Also, $$B' = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 8 & 3 \end{bmatrix}_{2\times 3} \quad \text{and} \quad A' = \begin{bmatrix} 2 & 3 \\ 4 & 9 \\ 0 & 6 \end{bmatrix}_{3\times 2}$$

$$\therefore \quad B'A' = \begin{bmatrix} 2+8+0 & 3+18+6 \\ 8+32+0 & 12+72+18 \end{bmatrix} = \begin{bmatrix} 10 & 27 \\ 40 & 102 \end{bmatrix} \qquad \text{... (ii)}$$

Thus, we see that, $(AB)' = B'A'$ [using Eqs. (i) and (ii)]

S33. We have,

$$\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}_{3\times 1} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3\times 3}$$

Let $\quad A = [z \quad y \quad z]$

$$\therefore \quad \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}_{3\times 1} [x \quad y \quad z]_{1\times 3} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3\times 3}$$

$$\Rightarrow \quad \begin{bmatrix} 4x & 4y & 4z \\ x & y & z \\ 3x & 3y & 3z \end{bmatrix} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$$

$\Rightarrow \quad 4x = -4 \quad \Rightarrow \quad x = -1, \quad 4y = 8$

$\Rightarrow \quad y = 2 \quad \text{and} \quad 4z = 4$

$\Rightarrow \quad z = 1$

$\therefore \quad A = [-1 \quad 2 \quad 1].$

S34. We have,

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} \quad \text{and} \quad a = 4, \quad b = -2$$

(i) $$A + (B + C) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$$

and $$(A + B) + C = \begin{bmatrix} 5 & 2 \\ 0 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix} = A + (B + C)$$ **Hence proved.**

(ii) $$(BC) = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix}$$

and $$A(BC) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 8 & 0 \\ 7 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} 8+14 & 0-20 \\ -8+21 & 0-30 \end{bmatrix} = \begin{bmatrix} 22 & -20 \\ 13 & -30 \end{bmatrix}$$

Also, $$(AB) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} . \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix}$$

$$(AB)\,C = \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -20 \\ 13 & -30 \end{bmatrix} = A(BC)$$ **Hence proved.**

S35. We have,

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$$

(i) $\therefore$ $$(2A + B) = \begin{bmatrix} 2 & 4 \\ 8 & 2 \\ 10 & 12 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 14 & 6 \\ 17 & 15 \end{bmatrix}$$

and $$(2A + B)' = \begin{bmatrix} 3 & 14 & 17 \\ 6 & 6 & 15 \end{bmatrix}$$

Also, $$2A' + B' = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 1 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 6 & 7 \\ 2 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 14 & 17 \\ 6 & 6 & 15 \end{bmatrix} = (2A + B)'$$ **Hence proved.**

(ii) $$(A - B) = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -2 & -3 \\ -2 & 3 \end{bmatrix}$$

and $$(A - B)' = \begin{bmatrix} 0 & -2 & -2 \\ 0 & -3 & 3 \end{bmatrix}$$

Also, $$A' - B' = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 1 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 6 & 7 \\ 2 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 & -2 \\ 0 & -3 & 3 \end{bmatrix} = (A - B)'$$ **Hence proved.**

S36. We have,

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} \text{ and } a = 4, \quad b = -2$$

(i) $$(a + b)B = (4 - 2)\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$$ [$\because$ $a = 4$, $b = -2$]

and $$aB + bB = 4B - 2B$$

$$= \begin{bmatrix} 16 & 0 \\ 4 & 20 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix} = (a + b)B$$ **Hence proved.**

(ii) $$(C - A) = \begin{bmatrix} 2-1 & 0-2 \\ 1+1 & -2-3 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 2 & -5 \end{bmatrix}$$

and $$a(C - A) = \begin{bmatrix} 4 & -8 \\ 8 & -20 \end{bmatrix}$$ [$\because$ $a = 4$]

Also, $$aC - aA = \begin{bmatrix} 8 & 0 \\ 4 & -8 \end{bmatrix} - \begin{bmatrix} 4 & 8 \\ -4 & 12 \end{bmatrix} = \begin{bmatrix} 4 & -8 \\ 8 & -20 \end{bmatrix}$$

$$= a(C - A).$$ **Hence proved.**

S37. Let $P(n) : (A')^n = (A^n)'$

$\therefore$ $P(1) : (A)^1 = (A)'$

$\Rightarrow$ $A' = A' \quad \Rightarrow \quad P(1)$ is true.

Now, $P(k) : (A')^k = (A^k)'$

Where $k \in N$

and $P(k + 1) : (A')^{k+1} = (A^{k+1})'$

where $P(k + 1)$ is true whenever $P(k)$ is true.

$\therefore$ $P(k + 1) : (A')^1 \cdot (A')^1 = [A^{k+1}]'$

$$(A^k)' \cdot (A)' = [A^{k+1}]'$$

$$(A \cdot A^k)' = [A^{k+1}]' \qquad [\because\ (A)^k = (Ak) \text{ and } (AB) = BA]$$

$$(A^{k+1})' = [A^{k+1}]'$$

Hence proved.

S38. We have,

$$A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad x^2 = -1$$

$\therefore$
$$(A + B) = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}$$

and
$$(A + B)^2 = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix} \qquad \text{... (i)}$$

Also,
$$A^2 = A \cdot A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}\begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} = \begin{bmatrix} -x^2 & 0 \\ 0 & -x^2 \end{bmatrix}$$

and
$$B^2 = B \cdot B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now,
$$A^2 + B^2 = \begin{bmatrix} -x^2+1 & 0 \\ 0 & -x^2+1 \end{bmatrix} = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix} \qquad \text{[Using Eq. (i)]}$$

$$= (A + B)^2$$

S39. We have,

$$A = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$$

Let
$$C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$\therefore$
$$3A + 5B + 2C = 0$$

$$\Rightarrow \quad \begin{bmatrix} 3 & 15 \\ 21 & 36 \end{bmatrix} + \begin{bmatrix} 45 & 5 \\ 35 & 40 \end{bmatrix} + \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 48+2a & 20+2b \\ 56+2c & 76+2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow$ $2a + 48 = 0 \Rightarrow a = -24$

Also, $20 + 2b = 0 \Rightarrow b = -10$

$56 + 2c = 0 \Rightarrow c = -28$

and $76 + 2d = 0 \Rightarrow d = -38$

$\therefore$ $C = \begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$.

S40. We have,

$$3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix} + \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix} = \begin{bmatrix} a+4 & 6+a+b \\ c+d-1 & 3+2d \end{bmatrix}$$

$\Rightarrow$ $3a = a + 4 \Rightarrow a = 2;$

$3b = 6 + a + b$

$\Rightarrow$ $3b - b = 8 \Rightarrow b = 4;$

$3d = 3 + 2d \Rightarrow d = 3$

and $\Rightarrow$ $3c = c + d - 1 \Rightarrow 2c = d - 1 \Rightarrow 2c = 2 \Rightarrow c = 1$

$\therefore$ $a = 2, \quad b = 4, \quad c = 1$ and $d = 3$.

S41. Since,

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6+2 & 3+0+4 & 2-3+6 \\ 2+0-1 & 6+0-2 & 4+0-3 \\ 1+4+3 & 3+0+6 & 2-2+9 \end{bmatrix} = \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix}$$

and $A^3 = A^2 \times A = \begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix} \times \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 9+14+5 & 27+0+10 & 18-7+15 \\ 1+8+1 & 3+0+2 & 2-4+3 \\ 8+18+9 & 24+0+18 & 16-9+27 \end{bmatrix} = \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix}$$

Now, $$A^3 - 4A^2 - 3A + 11(I) = \begin{bmatrix} 28 & 37 & 26 \\ 10 & 5 & 1 \\ 35 & 42 & 34 \end{bmatrix} - 4\begin{bmatrix} 9 & 7 & 5 \\ 1 & 4 & 1 \\ 8 & 9 & 9 \end{bmatrix} - 3\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 28-36-3+11 & 37-28-9+0 & 26-20-6+0 \\ 10-4-6+0 & 5-16+0+11 & 1-4+3+0 \\ 35-32-3+0 & 42-36-6+0 & 34-36-9+11 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0.$$

S42. Since, A is square matrix hence

$$A^2 = A \times A$$

$$A^2 = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8 & 2+2 \\ 4+4 & 8+1 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ 8 & 9 \end{bmatrix}$$

$\therefore$ $$A^2 + 2A + 7I = \begin{bmatrix} 9 & 4 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 8 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 4 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 8 & 2 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 18 & 8 \\ 16 & 18 \end{bmatrix}.$$

S43. Two matrices are equal if their corresponding elements are equal.

We have, $$\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$$

By equality of matrices

$$x + y = 6, \qquad xy = 8$$

$$x = 6 - y \quad \text{and} \quad (6 - y) \cdot y = 8$$

$\Rightarrow$ $$y^2 - 6y + 8 = 0$$

$\Rightarrow$ $$(y - 2)(y - 4) = 0$$

$\Rightarrow$ $$y = 2 \quad \text{or} \quad y = 4$$

Hence, $x = 2$ or $x = 4$; $z = -6$; $w = 4$

Hence, $x = 2$, $y = 4$ or $x = 4$, $y = 2$

$z = -6$ and $w = 4$.

S44. Let $P(n) : (AB)^n = A^n B^n$

$\therefore \quad P(1) : (AB)^1 = A^1 B^1 \quad \Rightarrow \quad AB = AB$

So, $P(1)$ is true.

Now, $P(k) : (AB)^k = A^k B^k, \quad k \in N$

So, $P(k)$ is true, whenever $P(k+1)$ is true,

$\therefore \quad P(k+1 : AB)^{k+1} = A^{k+1} B^{k+1}$ ... (i)

$\Rightarrow \quad AB^k \cdot AB^1$ $[\because \ AB = BA]$

$\Rightarrow \quad A^k B^k \cdot BA \quad \Rightarrow \quad A^k B^{k+1} A$

$\Rightarrow \quad A^k \cdot A \cdot B^{k+1} \quad \Rightarrow \quad A^{k+1} B^{k+1}$

$\Rightarrow \quad (A \cdot B)^{k+1} = A^{k+1} B^{k+1}$

So, $P(k+1)$ is true for all $n \in N$, whenever $P(k)$ is true.

By mathematical induction $(AB) = A^n B^n$ is true for all $n \in N$.

S45. Since, $A^2 = A$ and $(I + A)\ (I + A) = I^2 + IA + AI + A^2$

$= I^2 + 2AI + A^2$

$= I + 2A + A = I + 3A$

and $(I + A) \cdot (I + A)(I + A) = (I + A)(I + 3A)$

$= I^2 + 3AI + AI + 3A^2$

$= I + 4AI + 3A$

$= I + 7A = 7A + I.$ **Hence proved.**

S46. Let,

$$A = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$$

Since, A is skew-symmetric matrix.

$\therefore \quad A' = -A$

$$\Rightarrow \quad \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & -3 \\ -2 & -b & +1 \\ -c & -1 & 0 \end{bmatrix}$$

By equality of matrices, we get

$$a = -2, \quad c = -3 \quad \text{and} \quad b = -b \quad \Rightarrow \quad b = 0$$

$$a = -2, \quad b = 0 \quad \text{and} \quad c = -3.$$

S47. We have,

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix}$$

$\therefore$ $$A' = \begin{bmatrix} 2 & 1 & 4 \\ 3 & -1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Now, $$\frac{A + A'}{2} = \frac{1}{2}\begin{bmatrix} 4 & 4 & 5 \\ 4 & -2 & 3 \\ 5 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 2 & \frac{5}{2} \\ 2 & -1 & \frac{3}{2} \\ \frac{5}{2} & \frac{3}{2} & 2 \end{bmatrix}$$

and $$\frac{A - A'}{2} = \frac{1}{2}\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & \frac{-3}{2} \\ -1 & 0 & \frac{1}{2} \\ \frac{3}{2} & \frac{-1}{2} & 0 \end{bmatrix}$$

$\therefore$ $$\frac{A + A'}{2} + \frac{A - A'}{2} = \begin{bmatrix} 2 & 2 & \frac{5}{2} \\ 2 & -1 & \frac{3}{2} \\ \frac{5}{2} & \frac{3}{2} & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & \frac{-3}{2} \\ -1 & 0 & \frac{1}{2} \\ \frac{3}{2} & \frac{-1}{2} & 0 \end{bmatrix}$$

which is the required expression.

S48. Let,

$$A = \begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}$$

In order to use elementary row operations, we write $A = IA$.

$\Rightarrow$ $$\begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$\Rightarrow$ $$\begin{bmatrix} 1 & -3 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} A$$ $[\because \ R_2 \to R_2 + 2R_1]$

Since, we obtain all zeros in a row of the matrix A on L.H.S., so A^{-1} does not exist.

S49. Let

$$A = \begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix}$$

In order to use elementary row operations we may write $A = IA$.

$\therefore \quad \begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\Rightarrow \quad \begin{bmatrix} 1 & 3 \\ 0 & 22 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} A \qquad [\because R_2 \to R_2 + 5R_1]$

$\Rightarrow \quad \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5/22 & 1/22 \end{bmatrix} A \qquad \left[\because R_2 \to \frac{1}{22} R_2\right]$

$\Rightarrow \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7/22 & -3/22 \\ 5/22 & 1/22 \end{bmatrix} A \qquad [\because R_1 \to R_1 - 3R_2]$

$\Rightarrow \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{22} \begin{bmatrix} 7 & -3 \\ 5 & 1 \end{bmatrix} A$

$\Rightarrow \quad I = BA$, where B is the inverse of A.

$\therefore \quad B = \frac{1}{22} \begin{bmatrix} 7 & -3 \\ 5 & 1 \end{bmatrix}.$

S50. We have,

$$A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \text{ and } A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Also, $\quad A^{-1} = A'$

$\Rightarrow \quad AA^{-1} = AA'$

$\Rightarrow \quad I = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & 0 \\ 0 & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$

By using equality of matrices, we get

$$\cos^2\alpha + \sin^2\alpha = 1$$

which is true for all real values of α.

S51. For getting the inverse of the given matrix A by row elementary operations we may write the given matrix as:

$$A = IA$$

$$\because \quad \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \quad \begin{bmatrix} 2 & -1 & 3 \\ -3 & 2 & 4 \\ -3 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad [\because\ R_2 \to R_2 + R_1]$$

$$\Rightarrow \quad \begin{bmatrix} 2 & -1 & 3 \\ -3 & 2 & 4 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} A \qquad [\because\ R_3 \to R_3 + R_2]$$

$$\Rightarrow \quad \begin{bmatrix} -1 & 1 & 7 \\ -3 & 2 & 4 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} A \qquad [\because\ R_1 \to R_1 + R_2]$$

$$\Rightarrow \quad \begin{bmatrix} -1 & 1 & 7 \\ 0 & -1 & -17 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ -5 & -2 & 0 \\ -1 & -1 & 1 \end{bmatrix} A \qquad [\because\ R_2 \to R_2 + 3R_1]$$

$$\Rightarrow \quad \begin{bmatrix} -1 & 0 & -10 \\ 0 & -1 & -17 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -1 & 0 \\ -5 & -2 & 0 \\ 1 & 1 & -1 \end{bmatrix} A \qquad [\because\ R_1 \to R_1 + R_2 \text{ and } R_3 \to -1 \cdot R_3]$$

$$\Rightarrow \quad \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 9 & -10 \\ 12 & 15 & -17 \\ 1 & 1 & 1 \end{bmatrix} A \qquad [\because\ R_2 \to R_2 + R_1 \text{ and } R_2 \to R_2 + 17R_3]$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix} A \qquad [\because\ R_2 \to R_2 + R_1 \text{ and } R_2 \to -1R_1]$$

So, the inverse of A is $\begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix}$.

S52. For getting the inverse of the given matrix A by row elementary operations we may write the given matrix as:

$$A = IA$$

$$\therefore \qquad \begin{bmatrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \qquad \begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} A \qquad [\because\ R_2 \to R_2 + R_3 \text{ and } R_1 \to R_1 + 2R_3]$$

$$\Rightarrow \qquad \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} A \qquad [\because\ R_2 \to R_2 + R_1]$$

Since, second row of the matrix A on L.H.S. is containing all zeroes, so we can say that inverse of matrix A does not exist.

S53. We have,

$$A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$$

(i) $$(AB) = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 2+6 & 3-8 \\ -4+3 & -6-4 \end{bmatrix} = \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix}$$

and $$(AB)\,C = \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 8+5 & 0 \\ -1+10 & 0 \end{bmatrix} = \begin{bmatrix} 13 & 0 \\ 9 & 0 \end{bmatrix} \qquad \ldots \text{(i)}$$

Again, $$(BC) = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2-3 & 0 \\ 3+4 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 7 & 0 \end{bmatrix}$$

and $$A\,(BC) = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 7 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1+14 & 0 \\ +2+7 & 0 \end{bmatrix} = \begin{bmatrix} 13 & 0 \\ 9 & 0 \end{bmatrix} \qquad \ldots \text{(ii)}$$

$$\therefore \qquad (AB)\,C = A\,(BC) \qquad \text{[Using Eqs. (i) and (ii)]}$$

(ii) $(B + C) = \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 2 & -4 \end{bmatrix}$

and $$A \cdot (B + C) = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 3 \\ 2 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 3+4 & 3-8 \\ -6+2 & -6-4 \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ -4 & -10 \end{bmatrix} \quad \text{... (iii)}$$

Also, $$AB = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 2+6 & 3-8 \\ -4+3 & -6-4 \end{bmatrix} = \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix}$$

and $$AC = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1-2 & 0 \\ -2-1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ -3 & 0 \end{bmatrix}$$

$\therefore$ $$AB + AC = \begin{bmatrix} 8 & -5 \\ -1 & -10 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ -3 & 0 \end{bmatrix}$$

$\Rightarrow$ $$AB + AC = \begin{bmatrix} 7 & -5 \\ -4 & -10 \end{bmatrix} \quad \text{... (iv)}$$

From Eqs. (iii) and (iv),

$$A(B + C) = AB + AC.$$

S54. We have to verify that, $A(B + C) = (AB + AC)$.

we have, $A = [2 \quad 1]$, $B = \begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

$\therefore$ $$A(B + C) = [2 \quad 1] \begin{bmatrix} 5-1 & 3+2 & 4+1 \\ 8+1 & 7+0 & 6+2 \end{bmatrix}$$

$$= [2 \quad 1] \begin{bmatrix} 4 & 5 & 5 \\ 9 & 7 & 8 \end{bmatrix}$$

$$= [8+9 \quad 10+7 \quad 10+8] \quad \text{... (i)}$$

$$= [17 \quad 17 \quad 18]$$

Also, $$AB = [2 \quad 1] \begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix}$$

$$= [10+8 \quad 6+7 \quad 8+6] = [18 \quad 13 \quad 14]$$

and $$AC = [2 \quad 1]\begin{bmatrix}-1 & 2 & 1\\ 1 & 0 & 2\end{bmatrix}$$

$$= [-2+1 \quad 4+0 \quad 2+2] = [-1 \quad 4 \quad 4]$$

$\therefore$ $$AB + AC = [18 \quad 13 \quad 14] + [-1 \quad 4 \quad 4]$$

$$= [17 \quad 17 \quad 18] \qquad \text{... (ii)}$$

$\therefore$ $$A(B+C) = (AB + AC) \qquad \text{[Using Eqs. (i) and (ii)]}$$

Hence Proved.

S55. We have,

$$\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}_{2\times 2} A\cdot\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}_{2\times 2} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}_{2\times 2}$$

Let $$A = \begin{bmatrix}a & b\\ c & d\end{bmatrix}_{2\times 2}$$

$\therefore$ $$\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix}2a+c & 2b+d\\ 3a+2c & 3b+2d\end{bmatrix}\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix}-6a-3c+10b+5d & 4a+2c-6b-3d\\ -9a-6c+15b+10d & 6a+4c-9b-6d\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$\Rightarrow$ $$-6a - 3c + 10b + 5d = 1 \qquad \text{... (i)}$$

$\Rightarrow$ $$4a + 2c - 6b - 3d = 0 \qquad \text{... (ii)}$$

$\Rightarrow$ $$-9a - 6c + 15b + 10d = 0 \qquad \text{... (iii)}$$

$\Rightarrow$ $$6a + 4c - 9b - 6d = 1 \qquad \text{... (iv)}$$

On adding Eqs. (i) and (ii), we get

$$c + b - d = 2 \quad \Rightarrow \quad d = c - 2 \qquad \text{... (v)}$$

On adding Eqs. (ii) and (iii), we get

$$-5a - 4c + 9b + 7d = 0 \qquad \text{... (vi)}$$

On adding Eqs. (vi) and (iv), we get

$$a + 0 + 0 + d = 1 \quad \Rightarrow \quad d = 1 - a \qquad \text{... (vii)}$$

From Eqs. (vi) and (vii), we get

$$c + b - 2 = 1 - a \quad \Rightarrow \quad a + b + c = 3 \qquad \text{... (viii)}$$

$\Rightarrow$ $$a = 3 - b - c$$

Now, using the values of a and d in Eq. (iii), we get

$$-9(3-b-c)-6c+15b+10(-2+b+c)=0$$

$$\Rightarrow \quad -27+9b+9c-6c+15b-20+10b+10c=0$$

$$\Rightarrow \quad 34b+13c=47 \qquad \dots \text{(ix)}$$

Now, using the values of a and d in Eq. (ii), we get

$$4(3-b-c)+2c-6b-3(b+c-2)=0$$

$$\Rightarrow \quad 12-4b \;\; 4c+2c-6b-3b-3c+6=0$$

$$\Rightarrow \quad -13b-5c=-18 \qquad \dots \text{(x)}$$

On multiplying Eq. (ix) by 5 and Eq. (x) by 13, then adding, we get

$$-169b-65c=-234$$

$$\frac{170b+65c=235}{b=1}$$

$$\Rightarrow \quad -13\times 1-5c=-18 \qquad \text{[From Eq. (x)]}$$

$$\Rightarrow \quad -5c=-18+13=-5 \quad \Rightarrow \quad c=1$$

$$\therefore \quad a=3-1-1 \quad \text{and} \quad d=1-1=0$$

$$\therefore \quad A=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}.$$

S56. We have,

$$2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix} \qquad \dots \text{(i)}$$

and

$$3X+2Y=\begin{bmatrix}-2 & 2\\ 1 & -5\end{bmatrix} \qquad \dots \text{(ii)}$$

On subtracting Eq. (i) from Eq. (ii), we get

$$\Rightarrow \quad (3X+2Y)-(2X+3Y)=\begin{bmatrix}-2-2 & 2-3\\ 1-4 & -5-0\end{bmatrix}$$

$$(X-Y)=\begin{bmatrix}-4 & -1\\ -3 & -5\end{bmatrix} \qquad \dots \text{(iii)}$$

On adding Eqs. (i) and (ii), we get

$$(5X+5Y)=\begin{bmatrix}0 & 5\\ 5 & -5\end{bmatrix}$$

$$(X+Y)=\frac{1}{5}\begin{bmatrix}0 & 5\\ 5 & -5\end{bmatrix}=\begin{bmatrix}0 & 1\\ 1 & -1\end{bmatrix} \qquad \dots \text{(iv)}$$

On adding Eqs. (i) and (ii), we get

$$(X - Y) + (X + Y) = \begin{bmatrix} -4 & 0 \\ -2 & -6 \end{bmatrix}$$

$$\Rightarrow \qquad 2X = 2\begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$$

$$\therefore \qquad X = \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$$

From Eq. (iv), we get

$$\begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix} + Y = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\therefore \qquad Y = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$$

S57. We have,

$$A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}, \quad \text{then} \quad A' = \begin{bmatrix} 2 & 7 & 1 \\ 4 & 3 & -2 \\ -6 & 5 & 4 \end{bmatrix}$$

Hence,

$$\frac{A + A'}{2} = \frac{1}{2}\begin{bmatrix} 4 & 11 & -5 \\ 11 & 6 & 3 \\ -5 & 3 & 8 \end{bmatrix} = \begin{bmatrix} 2 & \frac{11}{2} & \frac{-5}{2} \\ \frac{11}{2} & 3 & \frac{3}{2} \\ \frac{-5}{2} & \frac{3}{2} & 4 \end{bmatrix}$$

and

$$\frac{A - A'}{2} = \frac{1}{2}\begin{bmatrix} 0 & -3 & -7 \\ 3 & 0 & 7 \\ 7 & -7 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-3}{2} & \frac{-7}{2} \\ \frac{3}{2} & 0 & \frac{7}{2} \\ \frac{7}{2} & \frac{-7}{2} & 0 \end{bmatrix}$$

Therefore,

$$\frac{A + A'}{2} + \frac{A - A'}{2} = \begin{bmatrix} 2 & \frac{11}{2} & \frac{-5}{2} \\ \frac{11}{2} & 3 & \frac{3}{2} \\ \frac{-5}{2} & \frac{3}{2} & 4 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-3}{2} & \frac{-7}{2} \\ \frac{3}{2} & 0 & \frac{7}{2} \\ \frac{7}{2} & \frac{-7}{2} & 0 \end{bmatrix} = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix} = A.$$

S58. We have,

$$A^2 = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$$

$$-4A = \begin{bmatrix} -8 & -12 \\ 4 & -8 \end{bmatrix} \quad \text{and} \quad 7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}.$$

Therefore, $$A^2 - 4A + 7I = \begin{bmatrix} 1-8+7 & 12-12+8 \\ -4+4+0 & 1-8+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

$$A^2 = 4A - 7I$$

Thus, $$A^3 = A \cdot A^2 = A(4A - 7I) = 4(4A - 7I) - 7A$$

$$= 16A - 28I - 7A = 9A - 28I$$

and so, $$A^5 = A^3A^2$$

$$= (9A - 28I)(4A - 7I)$$

$$= 36A^2 - 63A - 112A + 196I$$

$$= 36(4A - 7I) - 175A + 196I$$

$$= -31A - 56I$$

$$= -31\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} - 56\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}.$$

S59. We have,

$$P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$\therefore$ $$P(y) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

Now, $$P(x) \cdot P(y) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}\begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

$$= \begin{bmatrix} \cos x \cdot \cos y - \sin x \cdot \sin y & \cos x \cdot \sin y + \sin x \cdot \cos y \\ -\sin x \cdot \cos y - \cos x \cdot \sin y & -\sin x \cdot \sin y + \cos x \cdot \cos y \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} \qquad \ldots \text{(i)}$$

$$\begin{bmatrix} \because \ \cos(x+y) = \cos x \cdot \cos y - \sin x \cdot \sin y \\ \text{and } \sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y \end{bmatrix}$$

and $$P(x+y) = \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} \qquad \ldots \text{(ii)}$$

Also, $P(y)\cdot P(x) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

$$= \begin{bmatrix} \cos y\cdot\cos x - \sin y\cdot\sin x & \cos y\cdot\sin x + \sin y\cdot\cos x \\ -\sin y\cdot\cos x - \sin x\cdot\cos y & -\sin y\cdot\sin x + \cos y\cdot\cos x \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} \qquad \dots \text{(i)}$$

Thus, we see from the Eqs. (i), (ii) and (iii) that,

$$P(x)\cdot P(y) = P(x+y) = P(y)\cdot P(x)$$ **Hence proved.**

S60. We have, $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}_{3\times 2} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}_{3\times 3}$

For the given equation it is clear that order of A should be 2×3.

Let $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

$\therefore \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a-d & 2b-e & 2c-f \\ a+0d & b+0\cdot e & c+0\cdot f \\ -3a+4d & -3b-4e & -3c+4f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a-d & 2b-e & 2c-f \\ a & b & c \\ -3a+4d & -3b-4e & -3c+4f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$

By equality of matrices, we get

$$a = 1, \quad b = -2, \quad c = -5$$

and $2a - d = -1 \Rightarrow d = 2a + 1 = 3;$

$\Rightarrow$ $2b - e = -8 \Rightarrow e = 2(-2) + 8 = 4$

$2c - f = -10 \Rightarrow f = 2c + 10 = 0$

$\therefore A = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}.$

S61. We have,

$$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \qquad \dots \text{(i)}$$

$\therefore A^2 = A\cdot A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\cdot\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} \quad \text{... (ii)}$$

$$\therefore \quad A^2 - 5A - 14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Now, $A^2 - 5A - 14I = 0$

$\Rightarrow \quad A \cdot A^2 - 5A \cdot A - 14AI = 0$

$\Rightarrow \quad A^3 - 5A^2 - 14A = 0$

$\Rightarrow \quad A^3 = 5A^2 + 14A$

$$= 5\begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + 14\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \quad \text{[Using Eqs. (i) and (ii)]}$$

$$= \begin{bmatrix} 145 & -125 \\ -100 & 120 \end{bmatrix} + \begin{bmatrix} 42 & -70 \\ -56 & 28 \end{bmatrix}$$

$$= \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$$

S62. We have,

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} \text{ and } a = 4, \quad b = -2$$

(i) $$A^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

Now, $$(A^T)^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} = A.$$ **Hence proved.**

(ii) $$(bA)^T = \begin{bmatrix} -2 & -4 \\ 2 & -6 \end{bmatrix}^T = \begin{bmatrix} -2 & 2 \\ -4 & -6 \end{bmatrix} \quad [\because \; b = -2]$$

and $$A^T = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$\therefore$ $$bA^T = \begin{bmatrix} -2 & 2 \\ -4 & -6 \end{bmatrix} = (bA)^T$$ **Hence proved.**

(iii) $$AB = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2 & 0+10 \\ -4+3 & 0+15 \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ -1 & 15 \end{bmatrix}$$

$\therefore$ $$(AB)^T = \begin{bmatrix} 6 & -1 \\ 10 & 15 \end{bmatrix}$$

Now, $$B^T A^T = \begin{bmatrix} 4 & 1 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ 10 & 15 \end{bmatrix} = (AB)^T$$ **Hence proved.**

S63. We have,

$$A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

$\therefore$ $$A^2 = A \cdot A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 25-3 & 15-6 \\ -5+2 & -3+4 \end{bmatrix} = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$$

$$3A = 3\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix}$$

and $$7I = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$\therefore$ $$A^2 - 3A - 7I = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 22-15-7 & 9-9-0 \\ -3+3+0 & 1+6-7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0.$$ **Hence proved.**

Since, $A^2 - 3A - 7I = 0$

$\Rightarrow$ $A^{-1}[(A^2) - 3A - 7I] = A^{-1}0$

$\Rightarrow$ $A^{-1}A \cdot A - 3A^{-1}A - 7IA^{-1} = 0$ $[\because\ A^{-1}0 = 0]$

$\Rightarrow$ $IA - 3I - 7A^{-1} = 0$ $[\because\ A^{-1}A = I]$

$\Rightarrow$ $A - 3I - 7A^{-1} = 0$ $[\because\ A^{-1}I = A^{-1}]$

$\Rightarrow$ $-7A^{-1} = -A + 3I$

$$= \begin{bmatrix} -5 & -3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

$\therefore$ $$A^{-1} = \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix} \times \frac{-1}{7} = \begin{bmatrix} -2/7 & -3/7 \\ 1/7 & 5/7 \end{bmatrix}.$$

S64. We have,

$$A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \quad \text{and} \quad A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

By using elementary row transformations, we get

$$A = IA$$

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ 0 & -2y & 2z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} A \qquad [\because\ R_3 \to R_3 - R_2]$$

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & 3y & 0 \\ 0 & 0 & 3z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} A \qquad [\because\ R_3 \to R_3 + R_1 \text{ and } R_2 \to R_2 + R_1]$$

$$\Rightarrow \begin{bmatrix} -x & -y & z \\ x & 3y & 0 \\ 0 & 0 & z \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 1 & 0 \\ \frac{1}{3} & \frac{-1}{3} & \frac{1}{3} \end{bmatrix} A \qquad [\because\ R_1 \to R_1 - R_2 \text{ and } R_3 \to \frac{1}{3} R_3]$$

$$\Rightarrow \begin{bmatrix} -x & -y & 0 \\ x & 3y & 0 \\ 0 & 0 & z \end{bmatrix} = \begin{bmatrix} \frac{-1}{3} & \frac{-2}{3} & \frac{-1}{3} \\ 1 & 1 & 0 \\ \frac{1}{3} & \frac{-1}{3} & \frac{1}{3} \end{bmatrix} A \qquad [\because\ R_1 \to R_1 - R_3]$$

$$\Rightarrow \begin{bmatrix} -x & -y & 0 \\ 0 & 2y & 0 \\ 0 & 0 & z \end{bmatrix} = \begin{bmatrix} \frac{-1}{3} & \frac{-2}{3} & \frac{-1}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{-1}{3} & \frac{1}{3} \end{bmatrix} A \qquad [\because\ R_2 \to R_2 + R_1]$$

$$\Rightarrow \begin{bmatrix} -x & 0 & 0 \\ 0 & 2y & 0 \\ 0 & 0 & z \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-1}{2} \\ \frac{2}{3} & \frac{1}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{-1}{3} & \frac{1}{3} \end{bmatrix} A \qquad [\because\ R_1 \to R_1 + \frac{1}{2} R_2]$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2x} & \frac{1}{2x} \\ \frac{1}{3y} & \frac{1}{6y} & \frac{-1}{6y} \\ \frac{1}{3z} & \frac{-1}{3z} & \frac{1}{3z} \end{bmatrix} A \qquad [\because R_1 \to \frac{-1}{x} R_1,\ R_2 \to \frac{1}{2y} R_2,\ R_3 \to \frac{1}{z} R_3]$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 0 & \frac{1}{2x} & \frac{1}{2x} \\ \frac{1}{3y} & \frac{1}{6y} & \frac{-1}{6y} \\ \frac{1}{3z} & \frac{-1}{3z} & \frac{1}{3z} \end{bmatrix} = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

$$\Rightarrow \quad \frac{1}{2x} = x \quad \Rightarrow \quad x = \pm \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{1}{6y} = y \quad \Rightarrow \quad y = \pm \frac{1}{\sqrt{6}}$$

$$\Rightarrow \quad \frac{1}{3z} = z \quad \Rightarrow \quad x = \pm \frac{1}{\sqrt{3}}$$