Date: 18/10/2021

Q1. Find the value of $\sec\left(\tan^{-1}\frac{y}{2}\right)$.

Q2. Prove that $\tan(\cot^{-1}x) = \cot(\tan^{-1}x)$. State with reason wheather the equality is valid for all values of x.

Q3. Evaluate: $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.

Q4. Find the value of $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$.

Q5. Find the principal value of $\cos^{-1}x$, for $x = \frac{\sqrt{3}}{2}$.

Q6. Find the value of $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$.

Q7. Evaluate $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$.

Q8. Evaluate: $\tan^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right)$.

Q9. Find the value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.

Q10. Evaluate: $\tan(\tan^{-1}(-4))$.

Q11. Evaluate: $\sin^{-1}\left[\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right]$.

Q12. Show that $2\tan^{-1}(-3) = \frac{-\pi}{2} + \tan^{-1}\left(\frac{-4}{3}\right)$.

Q13. Find the value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.

Q14. Find the value of $\sin\left(2\tan^{-1}\frac{1}{3}\right) + \cos(\tan^{-1}2\sqrt{2})$.

Q15. Find the value of $\sin\left[2\cot^{-1}\left(\frac{-5}{12}\right)\right]$.

Q16. Find the value of $\tan(\cos^{-1}x)$ and hence evaluate $\tan\left(\cos^{-1}\frac{8}{17}\right)$.

Q17. If $2\tan^{-1}(\cos\theta) = \tan^{-1}(2\operatorname{cosec}\theta)$, then show that $\theta = \frac{\pi}{4}$, where n is any integer.

Q18. Find the simplified form of $\cos^{-1}\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right)$, where $x \in \left(\frac{-3\pi}{4}, \frac{\pi}{4}\right)$.

Q19. Evaluate: $\cos\left[\sin^{-1}\frac{1}{4} + \sec^{-1}\frac{4}{3}\right]$.

Q20. Which is greater, than 1 or $\tan^{-1}1$?

Q21. Find the value of $\sin\left(2\tan^{-1}\frac{2}{3}\right)+\cos(\tan^{-1}\sqrt{3})$.

Q22. Find the values of x which satisfy the equation

$$\sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x.$$

Q23. Solve for x: $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x,\ x>0$.

Q24. Solve the equation $\sin^{-1}6x+\sin^{-1}6\sqrt{3}x=-\frac{\pi}{2}$.

Q25. Prove that $\cot\left(\frac{\pi}{4}-2\cot^{-1}3\right)=7$.

Q26. Show that $\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}=\tan^{-1}\frac{63}{16}$.

Q27. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$.

Q28. Solve the equation $\cos(\tan^{-1}x)=\sin\left(\cot^{-1}\frac{3}{4}\right)$.

Q29. Show that $\cos\left(2\tan^{-1}\frac{1}{7}\right)=\sin\left(4\tan^{-1}\frac{1}{3}\right)$.

Q30. Find the value of $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$.

Q31. Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$.

Q32. Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)=\frac{4-\sqrt{7}}{3}$ and justify why the other value $\frac{4+\sqrt{7}}{3}$ is ignored?

Q33. Find the value of $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}$.

Q34. Prove that $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\sin^{-1}\frac{1}{\sqrt{5}}$.

Q35. Show that $2\tan^{-1}\left\{\tan\frac{\alpha}{2}\cdot\tan\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right\}=\tan^{-1}\frac{\sin\alpha\cos\beta}{\cos\alpha+\sin\beta}$.

Q36. Prove that $\cot^{-1}7+\cot^{-1}8+\cot^{-1}18=\cot^{-1}3$.

Q37. Prove that $2\sin^{-1}\frac{3}{5}-\tan^{-1}\frac{17}{31}=\frac{\pi}{4}$.

Q38. If $a_1, a_2, a_3, \ldots, a_n$ is an arithmetic progression with common difference d, then evaluate the following expression.

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\tan^{-1}\left(\frac{d}{1+a_3a_4}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right].$$

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S1. Let $\tan^{-1}\dfrac{y}{2}=\theta$, where $\theta\in\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$. So, $\tan\theta=\dfrac{y}{2}$,

which gives $$\sec\theta = \frac{\sqrt{4+y^2}}{2}$$

Therefore, $$\sec\left(\tan^{-1}\frac{y}{2}\right) = \sec\theta=\frac{\sqrt{4+y^2}}{2}.$$

S2. Let $\cot^{-1}x=\theta$, then $\cot\theta = x$

or, $$\tan\left(\frac{\pi}{2}-\theta\right)=x \;\Rightarrow\; \tan^{-1}x=\frac{\pi}{2}-\theta$$

So, $\tan(\cot^{-1}x)=\tan\theta=\cot\left(\dfrac{\pi}{2}-\theta\right)=\cot\left(\dfrac{\pi}{2}-\cot^{-1}x\right)=\cot(\tan^{-1}x)$

The equality is valid for all values of x since $\tan^{-1}x$ and $\cot^{-1}x$ are true for $x\in\mathbf{R}$.

S3. $$\tan^{-1}\sqrt{3}-\sec^{-1}(-2) = \tan^{-1}\sqrt{3}-[\pi-\sec^{-1}2] \qquad [\because\ \sec^{-1}(-\theta)=\pi-\sec^{-1}(\theta)]$$

$$=\frac{\pi}{3}-\pi+\cos^{-1}\left(\frac{1}{2}\right)=-\frac{2\pi}{3}+\frac{\pi}{3}=-\frac{\pi}{3}.$$

S4. $$\tan^{-1}\left(\tan\frac{9\pi}{8}\right) = \tan^{-1}\tan\left(\pi+\frac{\pi}{8}\right)$$

$$=\tan^{-1}\left(\tan\left(\frac{\pi}{8}\right)\right)=\frac{\pi}{8}.$$

S5. If $\cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)=\theta$, then $\cos\theta=\dfrac{\sqrt{3}}{2}$.

Since we are considering principal branch, $a\in[0,\pi]$. Also, since $\dfrac{\sqrt{3}}{2}>0$, θ being in the first quadrant, hence $\cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)=\dfrac{\pi}{6}$.

S6. We have,

$$\tan^{-1}\left(\tan\frac{2\pi}{3}\right) = \tan^{-1}\tan\left(\pi-\frac{\pi}{3}\right) \qquad [\because\ \tan^{-1}(-x)=-\tan^{-1}x]$$

$$=\tan^{-1}\left(-\tan\frac{\pi}{3}\right) \qquad \left[\because\ \tan^{-1}(\tan x)=x,\ x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)\right]$$

$$= -\tan^{-1}\tan\frac{\pi}{3} = -\frac{\pi}{3}$$

S7. We have,

$$\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\frac{\pi}{6}\right] = \cos\left[\cos^{-1}\left(\cos\frac{5\pi}{6}\right)+\frac{\pi}{6}\right] \qquad \left[\because \quad \cos\frac{5\pi}{6} = \frac{-\sqrt{3}}{2}\right]$$

$$= \cos\left(\frac{5\pi}{6}+\frac{\pi}{6}\right) \qquad \{\because \ \cos^{-1}\cos x = x; \ x \in [0, \pi]\}$$

$$= \cos\left(\frac{6\pi}{6}\right)$$

$$= \cos(\pi) = -1.$$

S8. $\because \qquad \sin(-\theta) = -\sin\theta \qquad$ (Odd function)

$\therefore \qquad \tan^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right) = \tan^{-1}\left(-\sin\frac{\pi}{2}\right)$

$$= \tan^{-1}(-1) = \frac{\pi}{4}.$$

S9. D_f: $\quad \cos^{-1}x$ is $[-1, 1]$

$\therefore \qquad \cos\frac{13\pi}{6} = \cos\left(2\pi+\frac{\pi}{6}\right) = \cos\frac{\pi}{6}$

$\therefore \qquad \cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left(\cos\frac{\pi}{6}\right) = \frac{\pi}{6}.$

S10. $\because \qquad \tan(\tan^{-1}x) = x \quad \forall x \in R$

$\therefore \qquad \tan(\tan^{-1}(-4)) = -4.$

S11. $\because \qquad \sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$

$\therefore \qquad \sin^{-1}\left[\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right] = \sin^{-1}\left[\cos\left(\frac{\pi}{3}\right)\right]$

$$= \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}.$$

S12. L.H.S. = $2\tan^{-1}(-3) = -2\tan^{-1}3$

$$= -\left[\cos^{-1}\frac{1-3^2}{1+3^2}\right] \qquad \left[\because\ 2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2}, x \geq 0\right]$$

$$= -\left[\cos^{-1}\left(\frac{-8}{10}\right)\right] = -\left[\cos^{-1}\left(\frac{-4}{5}\right)\right]$$

$$= -\left[\pi - \cos^{-1}\left(\frac{4}{5}\right)\right] \qquad \{\because\ \cos^{-1}(-x) = \pi - \cos^{-1}x,\ x \in [-1, 1]\}$$

$$= -\pi + \cos^{-1}\left(\frac{4}{5}\right) \left[\text{Let} \cos^{-1}\left(\frac{4}{5}\right) = \theta \Rightarrow \cos\theta = \frac{4}{5} \Rightarrow \tan\theta = \frac{3}{4} \Rightarrow \theta = \tan^{-1}\frac{3}{4}\right]$$

$$= -\pi + \tan^{-1}\left(\frac{3}{4}\right) = -\pi\left[\frac{\pi}{2} - \cot^{-1}\left(\frac{3}{4}\right)\right]$$

$$= -\frac{\pi}{2} - \cot^{-1}\frac{3}{4} = -\frac{\pi}{2} - \tan^{-1}\frac{4}{3}$$

$$= -\frac{\pi}{2} + \tan^{-1}\left(\frac{-4}{3}\right) \qquad [\because\ \tan^{-1}(-x) = -\tan^{-1}x]$$

= R.H.S.

Hence proved.

S13. We know that,

$$\tan^{-1}\tan x = x;\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \text{ and } \cos^{-1}\cos x = x;\ x \in [0, \pi]$$

$$\therefore\quad \tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{6}\right)\right] + \cos^{-1}\left[\cos\left(\pi + \frac{7\pi}{6}\right)\right]$$

$$= \tan^{-1}\left(-\tan\frac{\pi}{6}\right) + \cos^{-1}\left(-\cos\frac{7\pi}{6}\right)$$

$$[\because\ \cos(\pi + \theta) = -\cos\theta]$$

$$= -\tan^{-1}\left(\tan\frac{\pi}{6}\right) + \pi - \left[\cos^{-1}\cos\left(\frac{7\pi}{6}\right)\right]$$

$\{\because\ \tan^{-1}(-x) = -\tan^{-1};\quad x \in R\quad$ and
$\cos^{-1}(-x) = \pi - \cos^{-1}x;\ x \in [-1, 1]\}$

$$= -\tan^{-1}\left(\tan\frac{\pi}{6}\right) + \pi - \cos^{-1}\left[\cos\left(\pi + \frac{\pi}{6}\right)\right]$$

$$= -\tan^{-1}\left(\tan\frac{\pi}{6}\right) + \pi - \left[\cos^{-1}\left(-\cos\frac{\pi}{6}\right)\right]$$

$[\because \cos(\pi + \theta) = -\cos\theta]$

$$= -\tan^{-1}\left(\tan\frac{\pi}{6}\right) + \pi - \pi + \cos^{-1}\left(\cos\frac{\pi}{6}\right)$$

$[\because \cos^{-1}(-x) = \pi - \cos^{-1}x]$

$$= -\frac{\pi}{6} + 0 + \frac{\pi}{6} = 0.$$

S14. We have,

$$\sin\left(2\tan^{-1}\frac{1}{3}\right) + \cos(\tan^{-1}2\sqrt{2}) = \sin\left[\sin^{-1}\left\{\frac{2\times\frac{1}{3}}{1+\left(\frac{1}{3}\right)^2}\right\}\right] + \cos\left(\cos^{-1}\frac{1}{3}\right) \quad \left[\because \tan^{-1}x = \cos^{-1}\frac{1}{\sqrt{1+x^2}}\right]$$

$$\left[\because 2\tan^{-1}x = \sin^{-1}\frac{2}{1+x^2}, -1 \le x \le 1 \text{ and } \tan^{-1}(2\sqrt{2}) = \cos^{-1}\frac{1}{3}\right]$$

$$= \sin\left[\sin^{-1}\left\{\frac{\frac{2}{3}}{1+\frac{1}{9}}\right\}\right] + \frac{1}{3} \qquad \{\because \cos(\cos^{-1}x) = x;\ x \in [-1, 1]\}$$

$$= \sin\left[\sin^{-1}\left(\frac{2\times 9}{3\times 10}\right)\right] + \frac{1}{3} = \sin\left[\sin^{-1}\left(\frac{3}{5}\right)\right] + \frac{1}{3} \quad [\because \sin(\sin^{-1}x) = x]$$

$$= \frac{3}{5} + \frac{1}{3} = \frac{9+5}{15} = \frac{14}{15}.$$

S15. Let

$\cot^{-1}\left(\frac{-5}{12}\right) = y$. Then $\cot y = \frac{-5}{12}$.

Now, $\sin\left[2\cot^{-1}\left(\frac{-5}{12}\right)\right] = \sin 2y$

$$= 2\sin y\cos y = 2\left(\frac{12}{13}\right)\left(\frac{-5}{13}\right) \qquad \left[\text{Since, } \cot y < 0, \text{ so } y \in \left(\frac{\pi}{2}, \pi\right)\right]$$

$$= \frac{-120}{169}.$$

S16. Let $\cos^{-1} x = \theta$, then $\cos\theta = x$, where $\theta \in [0, \pi]$.

Therefore, $$\tan(\cos^{-1} x) = \tan\theta = \frac{\sqrt{1-\cos^2\theta}}{\cos\theta} = \frac{\sqrt{1-x^2}}{x}$$

Hence, $$\tan\left(\cos^{-1}\frac{8}{17}\right) = \frac{\sqrt{1-\left(\frac{8}{17}\right)^2}}{\frac{8}{17}} = \frac{15}{8}.$$

S17. We have, $$2\tan^{-1}(\cos\theta) = \tan^{-1}(2\,\text{cosec}\,\theta)$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{2\cos\theta}{1-\cos^2\theta}\right) = \tan^{-1}(2\,\text{cosec}\,\theta) \qquad \left[\because \; 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$\Rightarrow \quad \left(\frac{2\cos\theta}{\sin^2\theta}\right) = (2\,\text{cosec}\,\theta)$$

$$\Rightarrow \quad (\cot\theta \cdot 2\,\text{cosec}\,\theta) = (2\,\text{cosec}\,\theta) \quad \Rightarrow \quad \cot\theta = 1$$

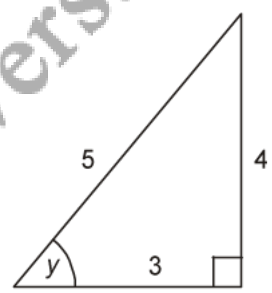
$$\Rightarrow \quad \cot\theta = \cot\frac{\pi}{4} \quad \Rightarrow \quad \theta = \frac{\pi}{4}.$$

S18. We have,

$$\cos^{-1}\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right),\; x \in \left(\frac{-3\pi}{4}, \frac{\pi}{4}\right)$$

Let $$\cos y = \frac{3}{5}$$

$$\Rightarrow \quad \sin y = \frac{4}{5}$$



$$\Rightarrow \quad y = \cos^{-1}\frac{3}{5} = \sin^{-1}\frac{4}{5} = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\therefore \quad \cos^{-1}[\cos y \cdot \cos x + \sin y \cdot \sin x]$$

$$= \cos^{-1}[\cos(y-x)] \quad [\because \; \cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B]$$

$$= y - x = \left(\tan^{-1}\frac{4}{3} - x\right) \qquad \left[\because \; y = \tan^{-1}\frac{4}{3}\right]$$

S19. $$\cos\left[\sin^{-1}\frac{1}{4} + \sec^{-1}\frac{4}{3}\right] = \cos\left[\sin^{-1}\frac{1}{4} + \cos^{-1}\frac{3}{4}\right]$$

$$= \cos\left(\sin^{-1}\frac{1}{4}\right) + \cos\left(\cos^{-1}\frac{3}{4}\right) - \sin\left(\sin^{-1}\frac{1}{4}\right) + \sin\left(\cos^{-1}\frac{3}{4}\right)$$

$$= \frac{3}{4}\sqrt{1-\left(\frac{1}{4}\right)^2} - \frac{1}{4}\sqrt{1-\left(\frac{3}{4}\right)^2}$$

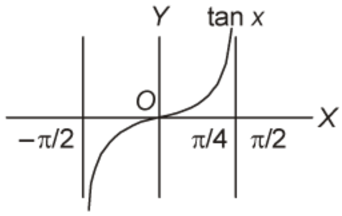
$$= \frac{3}{4}\,\frac{\sqrt{15}}{4} - \frac{1}{4}\,\frac{\sqrt{7}}{4} = \frac{3\sqrt{15}-\sqrt{7}}{16}.$$

S20. We note that from given figure, that tan x is an increasing function in the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, since $1 > \frac{\pi}{4} \Rightarrow \tan 1 > \tan\frac{\pi}{4}$. This gives

$$\tan 1 > 1$$

$$\Rightarrow \quad \tan 1 > 1 > \frac{\pi}{4}$$

$$\Rightarrow \quad \tan 1 > 1 > \tan^{-1}(1).$$



S21. Let $\tan^{-1}\frac{2}{3} = x$ and $\tan^{-1}\sqrt{3} = y$, so that $\tan x = \frac{2}{3}$ and $\tan y = \sqrt{3}$.

Therefore, $\sin\left(2\tan^{-1}\frac{2}{3}\right) + \cos(\tan^{-1}\sqrt{3}) = \sin(2x) + \cos y$

$$= \frac{2\tan x}{1+\tan^2 x} + \frac{1}{\sqrt{1+\tan^2 y}}$$

$$= \frac{2\cdot\frac{2}{3}}{1+\frac{4}{9}} + \frac{1}{\sqrt{1+(\sqrt{3})^2}} = \frac{12}{13} + \frac{1}{2} = \frac{37}{26}.$$

S22. From the given equation, we have

$$\sin(\sin^{-1} x) + \sin^{-1}(1-x)) = \sin(\cos^{-1} x)$$

$$\Rightarrow \quad \sin(\sin^{-1} x)\cos(\sin^{-1}(1-x)) + \cos(\sin^{-1} x)\sin(\sin^{-1}(1-x)) = \sin(\cos^{-1} x)$$

$$\Rightarrow \quad x\sqrt{1-(1-x)^2} + (1-x)\sqrt{1-x^2} = \sqrt{1-x^2}$$

$$\Rightarrow \quad x\sqrt{2x-x^2} + \sqrt{1-x^2}\,(1-x-1) = 0$$

$$\Rightarrow \quad x(\sqrt{2x-x^2} - \sqrt{1-x^2}) = 0$$

$$\Rightarrow \quad x = 0 \quad \text{or} \quad 2x - x^2 = 1 - x^2$$

$$\Rightarrow \quad x = 0 \quad \text{or} \quad x = \frac{1}{2}$$

S23. From given equation, we have

$$2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \tan^{-1} x$$

$$\Rightarrow \quad 2[\tan^{-1} 1 - \tan^{-1} x] = \tan^{-1} x$$

$$\Rightarrow \quad 2\left(\frac{\pi}{4}\right) = 3\tan^{-1}x \quad = \quad \frac{\pi}{6} = \tan^{-1}x$$

$$\Rightarrow \quad x = \frac{1}{\sqrt{3}}.$$

S24. From the given equation, we have $\sin^{-1}6x = -\frac{\pi}{2} - \sin^{-1}6\sqrt{3}x$

$$\Rightarrow \quad \sin(\sin^{-1}6x) = \sin\left(-\frac{\pi}{2} - \sin^{-1}6\sqrt{3}x\right)$$

$$\Rightarrow \quad 6x = -\cos\left(\sin^{-1}6\sqrt{3}x\right)$$

$\Rightarrow \quad 6x = -\sqrt{1-108x^2}$. Squaring, we get

$$36x^2 = 1 - 108x^2$$

$$\Rightarrow \quad 144x^2 = 1 \quad \Rightarrow \quad x = \pm\frac{1}{12}$$

Note that $x = -\frac{1}{12}$ is the only root of the equation as $x = \frac{1}{12}$ does not satisfy it.

S25. We have to prove,

$$\cot\left(\frac{\pi}{4} - 2\cot^{-1}3\right) = 7$$

$$\Rightarrow \quad \left(\frac{\pi}{4} - 2\cot^{-1}3\right) = \cot^{-1}7$$

$$\Rightarrow \quad (2\cot^{-1}3) = \frac{\pi}{4} - \cot^{-1}7$$

$$\Rightarrow \quad 2\tan^{-1}\frac{1}{3} = \frac{\pi}{4} - \tan^{-1}\frac{1}{7}$$

$$\Rightarrow \quad 2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan^{-1}\frac{2/3}{1-(1/3)^2} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan^{-1}\frac{2/3}{8/9} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan^{-1}\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\cdot\frac{1}{7}} = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan^{-1}\frac{(21+4)/28}{(28-3)/28} = \frac{\pi}{4}$$

$$\Rightarrow \qquad \tan^{-1}\frac{25}{25} = \frac{\pi}{4}$$

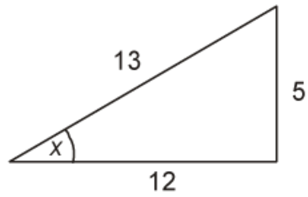
$$\Rightarrow \qquad 1 = \tan\frac{\pi}{4}$$

$$\Rightarrow \qquad 1 = 1$$

$\Rightarrow$ L.H.S. = R.H.S. **Hence proved.**

S26. We have, $\sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5} = \tan^{-1}\frac{63}{16}$ ... (i)

Let $\sin^{-1}\frac{5}{13} = x$



$$\Rightarrow \qquad \sin x = \frac{5}{13}$$

and $\cos^2 x = 1 - \sin^2 x$

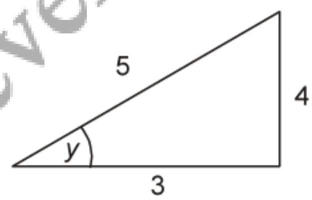
$$= 1 - \frac{25}{169} = \frac{144}{169}$$

$$\Rightarrow \qquad \cos x = \sqrt{\frac{144}{169}} = \frac{12}{13}$$

$\therefore$ $\tan x = \frac{\sin x}{\cos x} = \frac{5/13}{12/13} = \frac{5}{12}$ ... (ii)

$\Rightarrow$ $\tan x = 5/12$ ... (iii)

Again, let $\cos^{-1}\frac{3}{5} = y \quad = \quad \cos y = \frac{3}{5}$



$$\therefore \qquad \sin y = \sqrt{1 - \cos^2 y}$$

$$= \sqrt{1 - \left(\frac{3}{9}\right)^2} = \sqrt{1 - \frac{9}{25}}$$

$\sin y = \sqrt{\frac{16}{25}} = \frac{4}{5}$ ... (iii)

$$\Rightarrow \qquad \tan y = \frac{\sin y}{\cos y} = \frac{4/5}{3/5} = \frac{4}{3}$$

We know that, $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$

$$\Rightarrow \qquad \tan(x + y) = \frac{\frac{5}{12} + \frac{4}{3}}{1 - \frac{5}{12} \cdot \frac{4}{3}} \quad \Rightarrow \quad \tan(x + y) = \frac{\frac{15 + 48}{36}}{\frac{36 - 20}{36}}$$

$$\Rightarrow \quad \tan(x+y) = \frac{63/36}{16/36}$$

$$\Rightarrow \quad \tan(x+y) = \frac{63}{16}$$

$$\Rightarrow \quad x + y = \tan^{-1}\frac{63}{16}$$

$$\Rightarrow \quad \sin^{-1}\frac{5}{12} + \cos^{-1}\frac{4}{3} = \tan^{-1}\frac{63}{16}$$

Hence proved.

S27. We have,

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) = \frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$$

$$\text{L.H.S.} = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right) \quad \text{... (i)}$$

[Let, $x^2 = \cos 2\theta = (\cos^2\theta - \sin^2\theta) = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$]

$$\Rightarrow \quad \cos^{-1} x^2 = 2\theta \quad \Rightarrow \quad \theta = \frac{1}{2}\cos^{-1}x^2$$

$$\therefore \quad \sqrt{1+x^2} = \sqrt{1+\cos 2\theta}$$

$$= \sqrt{1+2\cos^2\theta - 1} = \sqrt{2}\cos\theta$$

and $$\sqrt{1-x^2} = \sqrt{1-\cos 2\theta}$$

$$= \sqrt{1-1+2\sin^2\theta} = \sqrt{2}\sin\theta$$

$$\therefore \quad \text{L.H.S.} = \tan^{-1}\left(\frac{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right) = \tan^{-1}\left(\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\cdot\tan\theta}\right)$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\theta\right)\right] \qquad \left[\because \quad \tan(x+y) = \frac{\tan x + \tan y}{1-\tan x\cdot\tan y}\right]$$

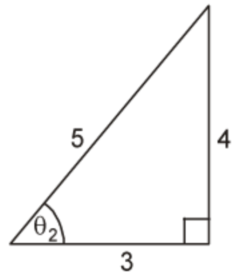
$$= \frac{\pi}{4} + \theta = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$$

$$= \text{R.H.S.}$$

Hence proved.

S28. We have,

$$\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$$



$$\Rightarrow \quad \cos\left(\cos^{-1}\frac{1}{\sqrt{x^2+1}}\right) = \sin\left(\sin^{-1}\frac{4}{5}\right)$$

Let $\quad \tan^{-1} x = \theta_1 \quad \Rightarrow \quad \tan\theta_1 = \frac{x}{1}$

$$\Rightarrow \quad \cos\theta_1 = \frac{1}{\sqrt{x^2+1}} \quad \Rightarrow \quad \theta_1 = \cos^{-1}\frac{1}{\sqrt{x^2+1}}$$

and $\quad \cot^{-1}\frac{3}{4} = \theta_2 \quad \Rightarrow \quad \cot\theta_2 = \frac{3}{4}$

$$\Rightarrow \quad \sin\theta_2 = \frac{4}{5} \quad \Rightarrow \quad \theta_2 = \sin^{-1}\frac{4}{5}$$

$$\frac{1}{\sqrt{x^2+1}} = \frac{4}{5}$$

$$\{\because \cos(\cos^{-1} x) = x;\ x \in [-1, 1] \text{ and } \sin(\sin^{-1} x) = x;\ x \in [-1, 1]\}$$

On squaring both sides, we get

$$16(x^2 + 1) = 25$$

$$\Rightarrow \quad 16x^2 = 9$$

$$\Rightarrow \quad x^2 = \left(\frac{3}{4}\right)^2$$

$$\therefore \quad x = \pm\frac{3}{4} = \frac{-3}{4}, \frac{3}{4}.$$

S29. We have,

$$\cos\left(2\tan^{-1}\frac{1}{7}\right) = \sin\left(4\tan^{-1}\frac{1}{3}\right)$$

$$\Rightarrow \quad \cos\left[\cos^{-1}\left(\frac{1-\left(\frac{1}{7}\right)^2}{1+\left(\frac{1}{7}\right)^2}\right)\right] = \sin\left[2.2\tan^{-1}\frac{1}{3}\right] \qquad \left[\because\ 2\tan^{-1} x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right]$$

$$\Rightarrow \quad \cos\left[\cos^{-1}\left(\frac{\left(\frac{48}{49}\right)}{\left(\frac{50}{49}\right)}\right)\right] = \sin\left[2\cdot\left(\tan^{-1}\frac{\frac{2}{3}}{1-\left(\frac{1}{3}\right)^2}\right)\right] \qquad \left[\because \; 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$\Rightarrow \quad \cos\left[\cos^{-1}\left(\frac{48\times 49}{50\times 49}\right)\right] = \sin\left[2\tan^{-1}\left(\frac{18}{24}\right)\right]$$

$$\Rightarrow \quad \cos\left[\cos^{-1}\left(\frac{24}{25}\right)\right] = \sin\left(2\tan^{-1}\frac{3}{4}\right)$$

$$\Rightarrow \quad \cos\left[\cos^{-1}\left(\frac{24}{25}\right)\right] = \sin\left(\sin^{-1}\frac{2\times\frac{3}{4}}{1+\frac{9}{16}}\right) \qquad \left[\because \; 2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}\right]$$

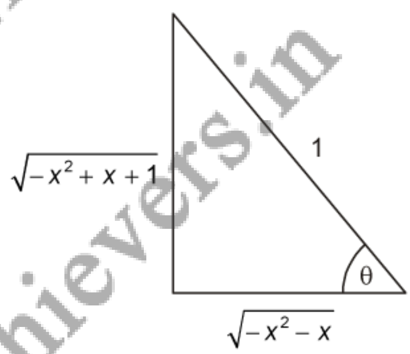
$$\Rightarrow \quad \frac{24}{25} = \sin\left(\sin^{-1}\frac{3/2}{25/16}\right)$$

$$\Rightarrow \quad \frac{24}{25} = \frac{48}{50} \quad \Rightarrow \quad \frac{24}{25} = \frac{24}{25}$$

$\therefore$ L.H.S. = R.H.S. **Hence proved.**

S30. We have,

$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2} \qquad \text{... (i)}$$



Let $\sin^{-1}\sqrt{x^2+x+1} = \theta$

$$\Rightarrow \quad \sin\theta = \frac{\sqrt{x^2+x+1}}{1}$$

$$\Rightarrow \quad \tan\theta = \frac{\sqrt{x^2+x+1}}{\sqrt{-x^2-x}} \qquad \left[\because \; \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$\because \quad \theta = \tan^{-1}\frac{\sqrt{x^2+x+1}}{\sqrt{-x^2-x}}$$

$$= \sin^{-1}\sqrt{x^2+x+1}$$

On putting the value of θ in Eq. (i), we get

$$\tan^{-1}\sqrt{x(x+1)} + \tan^{-1}\frac{\sqrt{x^2+x+1}}{\sqrt{-x^2-x}} = \frac{\pi}{2}$$

We know that, $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right), \; xy < 1$

$$\therefore \quad \tan^{-1}\left[\frac{\sqrt{x(x+1)}+\sqrt{\dfrac{x^2+x+1}{-x^2-x}}}{1-\sqrt{x(x+1)}\cdot\sqrt{\dfrac{x^2+x+1}{-x^2-x}}}\right] = \frac{\pi}{2}$$

$$\Rightarrow \quad \tan^{-1}\left[\frac{\sqrt{x^2+x}+\sqrt{\dfrac{x^2+x+1}{-1(x^2+x)}}}{1-\sqrt{(x^2+x)\cdot\dfrac{x^2+x+1}{-1(x^2+x)}}}\right] = \frac{\pi}{2}$$

$$\Rightarrow \quad \frac{x^2+x+\sqrt{-(x^2+x+1)}}{[1-\sqrt{-(x^2+x+1)}]\sqrt{(x^2+x)}} = \tan\frac{\pi}{2} = \frac{1}{0}$$

$$\Rightarrow \quad [1-\sqrt{-(x^2+x+1)}]\sqrt{(x^2+x)} = 0$$

$$\Rightarrow \quad -(x^2+x+1) = 1 \quad \text{or} \quad x^2+x=0$$

$$\Rightarrow \quad -x^2-x-1) = 1 \quad \text{or} \quad x(x+1)=0$$

$$\Rightarrow \quad x^2+x+2 = 0 \quad \text{or} \quad x(x+1)=0$$

$$\therefore \quad x = \frac{-1\pm\sqrt{1-4\times 2}}{2}$$

$$\Rightarrow \quad x = 0 \quad \text{or} \quad x=-1$$

For real solution, we have $x = 0, -1$.

S31. We have,

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right] = \tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cot^{-1}\left(\cot\frac{\pi}{3}\right)+\tan^{-1}(-1)$$

$$= \tan^{-1}\left[\tan\left(\pi-\frac{\pi}{6}\right)\right]+\cot^{-1}\left[\cot\left(\frac{\pi}{3}\right)\right]+\tan^{-1}\left[\tan\left(\pi-\frac{\pi}{4}\right)\right]$$

$$= \tan^{-1}\left(-\tan\frac{\pi}{6}\right)+\cot^{-1}\left(\cot\frac{\pi}{3}\right)+\tan^{-1}\left(-\tan\frac{\pi}{4}\right)$$

$$\left[\begin{array}{l} \because \quad \tan^{-1}(\tan x) = x, \quad x\in\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right) \\ \qquad \cot^{-1}(\cot x) = x, \quad x\in(0,\pi) \\ \text{and } \tan^{-1}(-x) = -\tan^{-1}x \end{array}\right]$$

$$= -\frac{\pi}{6}+\frac{\pi}{3}-\frac{\pi}{4} = \frac{-2\pi+4\pi-3\pi}{12}$$

$$= \frac{-5\pi+4\pi}{12} = -\frac{\pi}{12}.$$

S32. We have,

$$\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$$

$\therefore$ $$\text{LHS} = \tan\left[\frac{1}{2}\sin^{-1}\left(\frac{3}{4}\right)\right]$$

Let $$\frac{1}{2}\sin^{-1}\frac{3}{4} = \theta \quad \Rightarrow \quad \sin^{-1}\frac{3}{4} = 2\theta$$

$\Rightarrow$ $$\sin 2\theta = \frac{3}{4} \quad \Rightarrow \quad \frac{2\tan\theta}{1+\tan^2\theta} = \frac{3}{4}$$

$\Rightarrow$ $$3 + 3\tan^2\theta = 8\tan\theta$$

$\Rightarrow$ $$3\tan^2\theta - 8\tan\theta + 3 = 0$$

Let $$\tan\theta = y$$

$\therefore$ $$3y^2 - 8y + 3 = 0$$

$\Rightarrow$ $$y = \frac{+8 \pm \sqrt{64 - 4\times 3\times 3}}{2\times 3} = \frac{8\pm\sqrt{28}}{6}$$

$$= \frac{2[4\pm\sqrt{7}]}{2\cdot 3}$$

$\Rightarrow$ $$\tan\theta = \frac{4\pm\sqrt{7}}{3}$$

$\Rightarrow$ $$\theta = \tan^{-1}\left[\frac{4\pm\sqrt{7}}{3}\right]$$

$$\left\{\text{But } \frac{4+\sqrt{7}}{3} > \frac{1}{2}\cdot\frac{\pi}{2}, \text{ since } \max\left[\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)\right] = 1\right\}$$

$\therefore$ $$\text{L.H.S.} = \tan\tan^{-1}\left(\frac{4-\sqrt{7}}{3}\right) = \frac{4-\sqrt{7}}{3} = \text{R.H.S.}$$

Note: Since, $$\frac{-\pi}{2} \le \sin^{-1}\frac{3}{4} \le \pi/2$$

$\Rightarrow$ $$\frac{-\pi}{4} \le \frac{1}{2}\sin^{-1}\frac{3}{4} \le \pi/4$$

$\therefore$ $$\tan\left(\frac{-\pi}{4}\right) \le \tan\frac{1}{2}\left(\sin^{-1}\frac{3}{4}\right) \le \tan\frac{\pi}{4}$$

$\Rightarrow$ $$-1 \le \tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) \le 1$$

S33. We have,

$$4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239} = 2\cdot 2\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}$$

$$= 2\cdot\left[\tan^{-1}\frac{\frac{2}{5}}{1-\left(\frac{1}{5}\right)^2}\right]-\tan^{-1}\frac{1}{239} \quad \left[\because\ 2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$= 2\cdot\left[\tan^{-1}\left(\frac{\frac{2}{5}}{1-\frac{1}{25}}\right)\right]-\tan^{-1}\frac{1}{239}$$

$$= 2\cdot\left[\tan^{-1}\left(\frac{2/5}{24/25}\right)\right]-\tan^{-1}\frac{1}{239}$$

$$= 2\tan^{-1}\frac{5}{12}-\tan^{-1}\frac{1}{239}$$

$$= \tan^{-1}\frac{2\cdot\frac{5}{12}}{1-\left(\frac{5}{12}\right)^2}-\tan^{-1}\frac{1}{239} \quad \left[\because\ 2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

$$= \tan^{-1}\left(\frac{\frac{5}{6}}{1-\frac{25}{144}}\right)-\tan^{-1}\frac{1}{239}$$

$$= \tan^{-1}\left(\frac{144\times 5}{119\times 6}\right)-\tan^{-1}\frac{1}{239}$$

$$= \tan^{-1}\left(\frac{120}{119}\right)-\tan^{-1}\frac{1}{239}$$

$$= \tan^{-1}\left(\frac{\frac{120}{119}-\frac{1}{239}}{1+\frac{120}{119}\cdot\frac{1}{239}}\right) \quad \left[\because\ \tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1-xy}\right)\right]$$

$$= \tan^{-1}\left(\frac{120\times 239-119}{119\times 239+120}\right)$$

$$= \tan^{-1}\left[\frac{28680-119}{28441+120}\right]=\tan^{-1}\frac{28561}{28561}$$

$$= \tan^{-1}(1) = \tan^{-1}\left(\tan\frac{\pi}{4}\right) = \frac{\pi}{4}.$$

S34. We have,

$$\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \sin^{-1}\frac{1}{\sqrt{5}} \quad \text{... (i)}$$

Let $\tan^{-1}\frac{1}{4} = x$

$\Rightarrow \tan x = \frac{1}{4}$

$\Rightarrow \tan^2 x = \frac{1}{16}$

$\Rightarrow \sec^2 x - 1 = \frac{1}{16}$

$\Rightarrow \sec^2 x = 1 + \frac{1}{16} = \frac{17}{16}$

$\Rightarrow \frac{1}{\cos^2 x} = \frac{17}{16}$

$\Rightarrow \cos^2 x = \frac{16}{17}$

$\Rightarrow \cos x = \frac{4}{\sqrt{17}}$

$\Rightarrow \sin^2 x = 1 - \cos^2 x = 1 - \frac{16}{17} = \frac{1}{17}$

$\Rightarrow \sin x = \frac{1}{\sqrt{17}}$ ... (ii)

Again, let $\tan^{-1}\frac{2}{9} = y$

$\Rightarrow \tan y = \frac{2}{9} \quad \Rightarrow \quad \tan^2 y = \frac{4}{81}$

$\Rightarrow \sec^2 y - 1 = \frac{4}{81}$

$\Rightarrow \sec^2 y = \frac{4}{81} + 1 = \frac{85}{81}$

$\Rightarrow \cos^2 y = \frac{81}{85} \quad \Rightarrow \quad \cos y = \frac{9}{\sqrt{85}}$

$\Rightarrow \quad \sin^2 y = 1 - \cos^2 y = 1 - \frac{81}{85} = \frac{4}{85}$

$\Rightarrow \quad \sin y = \frac{2}{\sqrt{85}}$ ... (iii)

We know that, $\sin(x + y) = \sin x \cdot \cos y + \cos x \cdot \sin y$

$\Rightarrow \quad = \frac{1}{\sqrt{17}} \cdot \frac{9}{\sqrt{85}} + \frac{4}{\sqrt{17}} \cdot \frac{2}{\sqrt{85}}$

$\Rightarrow \quad = \frac{17}{\sqrt{17} \cdot \sqrt{85}} = \frac{\sqrt{17}}{\sqrt{17} \cdot \sqrt{5}} = \frac{1}{\sqrt{5}}$

$\Rightarrow \quad (x + y) = \sin^{-1} \frac{1}{\sqrt{5}}$

$\Rightarrow \quad \tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9} = \sin^{-1} \frac{1}{\sqrt{5}}$ **Hence proved.**

S35.

$$\text{L.H.S.} = \tan^{-1} \frac{2 \tan \frac{\alpha}{2} \cdot \tan\left(\frac{\pi}{4} - \frac{\beta}{2}\right)}{1 - \tan^2 \frac{\alpha}{2} \tan^2\left(\frac{\pi}{4} - \frac{\beta}{2}\right)} \qquad \left(\text{since } 2\tan^{-1} x = \tan^{-1} \frac{2x}{1 - x^2}\right)$$

$$= \tan^{-1} \frac{2 \tan \frac{\alpha}{2} \left(\frac{1 - \tan \frac{\beta}{2}}{1 + \tan \frac{\beta}{2}}\right)}{1 - \tan^2 \frac{\alpha}{2} \left(\frac{1 - \tan \frac{\beta}{2}}{1 + \tan \frac{\beta}{2}}\right)^2}$$

$$= \tan^{-1} \frac{2 \tan \frac{\alpha}{2} \cdot \left(1 - \tan^2 \frac{\beta}{2}\right)}{\left(1 + \tan \frac{\beta}{2}\right)^2 - \tan^2 \frac{\alpha}{2} \left(1 - \tan \frac{\beta}{2}\right)^2}$$

$$= \tan^{-1} \frac{2 \tan \frac{\alpha}{2} \left(1 - \tan^2 \frac{\beta}{2}\right)}{\left(1 + \tan^2 \frac{\beta}{2}\right)\left(1 - \tan^2 \frac{\beta}{2}\right) + 2 \tan \frac{\beta}{2} \left(1 + \tan^2 \frac{\alpha}{2}\right)}$$

$$= \tan^{-1} \frac{\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \cdot \frac{1-\tan\frac{\beta}{2}}{1+\tan^2\frac{\beta}{2}}}{\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} + \frac{2\tan\frac{\beta}{2}}{1+\tan^2\frac{\beta}{2}}}$$

$$= \tan^{-1}\left(\frac{\sin\alpha\cos\beta}{\cos\alpha+\sin\beta}\right) = \text{R.H.S.}$$

S36. We have,

$$\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18} \qquad \left(\text{Since, } \cot^{-1} x = \tan^{-1}\frac{1}{x}, \text{ if } x > 0\right)$$

$$= \tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\times\frac{1}{8}}\right) + \tan^{-1}\frac{1}{18} \qquad \left(\text{Since, } x \cdot y = \frac{1}{7}\cdot\frac{1}{8} < 1\right)$$

$$= \tan^{-1}\frac{3}{11} + \tan^{-1}\frac{1}{18} = \tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\times\frac{1}{8}}\right) + \tan^{-1}\frac{1}{18} \qquad (\text{Since, } xy < 1\}$$

$$= \tan^{-1}\frac{65}{195} = \tan^{-1}\frac{1}{3} = \cot^{-1} 3.$$

S37. We have, $a_1 = a, \quad a_2 = a + d, \quad a_3 = a + 2d$

and $d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots = a_n - a_{n-1}$

Given that, $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \tan^{-1}\left(\frac{d}{1+a_3a_4}\right) + \ldots + \tan^{-1}\left(\frac{d}{1+a_{n-1}\cdot a_n}\right)\right]$

Therefore, $2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31} = 2\theta \tan^{-1}\frac{17}{31} = 2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31}$

$$= \tan^{-1}\left(\frac{2\cdot\frac{3}{4}}{1-\frac{9}{16}}\right) - \tan^{-1}\frac{17}{31} = \tan^{-1}\frac{24}{7} - \tan^{-1}\frac{17}{31}$$

$$= \tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\cdot\frac{17}{31}}\right) = \frac{\pi}{4}.$$

S38. We have, $a_1 = a, \quad a_2 = a + d, \quad a_3 = a + 2d$

and $d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots = a_n - a_{n-1}$

Given that, $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\tan^{-1}\left(\frac{d}{1+a_3a_4}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_{n-1}\cdot a_n}\right)\right]$

$$= \tan\left[\tan^{-1}\frac{a_2-a_1}{1+a_2\cdot a_1}+\tan^{-1}\frac{a_3-a_2}{1+a_3\cdot a_2}+\ldots+\tan^{-1}\frac{a_n-a_{n-1}}{1+a_n\cdot a_{n-1}}\right]$$

$$= \tan\left[(\tan^{-1}a_2 - \tan^{-1}a_1) + (\tan^{-1}a_3 - \tan^{-1}a_2) + \ldots + (\tan^{-1}a_n - \tan^{-1}a_{n-1})\right]$$

$$= \tan\left[\tan^{-1}a_n - \tan^{-1}a_1\right]$$

$$= \tan\left[\tan^{-1}\frac{a_n-a_1}{1+a_n\cdot a_1}\right] \qquad \left[\because \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$$

$$= \frac{a_n-a_1}{1+a_n\cdot a_1}. \qquad [\because \tan(\tan^{-1}x) = x]$$