Date: 19/10/2021

Q1. Let E_1 and E_2 be two independent events such that $P(E_1) = P_1$ and $P(E_2) = P_2$. Describe in words of the events whose probailities are:

(i) P_1P_2 (ii) $1 - (1 - P_1)(1 - P_2)$

Q2. Three events A, B and C have probabilities $\frac{2}{5}$, $\frac{1}{3}$ and $\frac{1}{2}$ respectively. If $P(A \cap C) = \frac{1}{5}$ and $P(B \cap C) = \frac{1}{4}$, then find the values of $P(C/B)$ and $P(A' \cap C')$.

Q3. If A and B are two events such that

$$P(A) = \frac{1}{2}, P(B) = \frac{1}{3} \text{ and } P(A \cap B) = \frac{1}{4}, \text{ then find}$$

(i) $P(B/A)$ (ii) $P(A'/B')$

Q4. If A and B are two events such that

$$P(A) = \frac{1}{2}, P(B) = \frac{1}{3} \text{ and } P(A \cap B) = \frac{1}{4}, \text{ then find}$$

(i) $P(A/B)$ (ii) $P(A'/B)$

Q5. The probability that atleast one of the two events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3, evaluate $P(\bar{A}) + P(\bar{B})$.

Q6. The die is thrown two times, Let A and B be the events, 'same number each time and 'a total score is 10 or more', respectively. If the die were fair, determine whether or not the events A and B are independent.

Q7. For a loaded die, the probabilities of outcomes are given as under:

$$P(1) = P(2) = 0.2, \quad P(3) = P(5) = P(6) = 0.1 \quad \text{and} \quad P(4) = 0.3$$

The die is thrown two times, Let A and B be the events, 'same number each time and 'a total score is 10 or more', respectively. Determine whether or not A and B are independent.

Q8. A and B are two candidates seeking admission in a college. The probability that A is selected is 0.7 and the probability that exactly one of them is selected is 0.6. Find the probability that B is selected.

Q9. A committee of 4 students is selected at random from a group consisting 8 boys and 4 girls. Given that there is at least one girl on the committee, calculate the probability that there are exactly 2 girls on the committee.

Q10. The probability of simultaneous occurence of at least one of two events A and B is p. If the probability that exactly one of A, B occurs is q, then prove that $P(A') + P(B') = 2 - 2p + q$.

Q11. Two dice are thrown together and the total score is noted. The events E, F and G are 'a total of 4', 'a total of 9 or more' and 'a total divisible by 5', respectively. Calculate $P(E)$, $P(F)$ and $P(G)$ and decide which pairs of events, if any are independent.

Q12. Two dice are tossed. Find whether the following two events A and B are independent $A = \{(x, y) : x + y = 11\}$ and $B = \{(x, y) : x \neq 5\}$, where (x, y) denotes a typical sample point.

Q13. Two dice are thrown together. Let A be the event 'getting 6 on the first die' and B be the event 'getting 2 on the second die'. Are the events A and B independent?

Q14. Three bags contain a number of red and white balls as follows Bag I : 3 red balls, Bag II : 2 red balls and 1 white ball and Bag III : 3 white balls. The probability that bag I will be chosen and a ball is selected from it is $\frac{i}{6}$, where $i = 1, 2, 3$. If a white ball is selected, what is the probability that it came from

(i) Bag II? (ii) Bag III?

Q15. Three bags contain a number of red and white balls as follows Bag I : 3 red balls, Bag II : 2 red balls and 1 white ball and Bag III : 3 white balls. The probability that bag I will be chosen and a ball is selected from it is $\frac{i}{6}$, where $i = 1, 2, 3$. What is the probability that

(i) a red ball will be selected? (ii) a white ball is selected?

Q16. Three machines E_1, E_2, E_3 in a certain factory produce 50%, 25% and 25%, respectively, of the total daily output of electric tubes. It is known that 4% of the tubes produced one each of machines E_1 and E_2 are defective, and that 5% of those produced on E_3 are defective. If one tube is picked up at random from a day's production. Calculate the probability that the defective tube was produced on machine E_1.

Q17. Three machines E_1, E_2, E_3 in a certain factory produce 50%, 25% and 25%, respectively, of the total daily output of electric tubes. It is known that 4% of the tubes produced one each of machines E_1 and E_2 are defective, and that 5% of those produced on E_3 are defective. If one tube is picked up at random from a day's production, calculate the probability that it is defective.

Q18. 10% of the bulbs produced in a factory are of red colour and 2% are red and defective. If one bulb is picked up at random, determine the probability of its being defective if it is red.

Q19. A shopkeeper sells three types of flower seeds A_1, A_2 and A_3. They are sold as a mixture, where the proportions are 4 : 4 : 2, respectively. The germination rates of the three types of seeds are 45%, 60% and 35%. Calculate the probability

(i) of a randomly chosen seed to germinate.

(ii) that it will not germinate given that the seed is of type A_3.

(iii) that it is of the type A_2 given that a randomly chosen seed does not germinate.

Q20. A letter is known to have come either from 'TATA NAGAR' or from 'CALCUTTA'. On the envelope, just two consecutive letters TA are visible. What is the probability that the letter came from 'TATA NAGAR'?

Q21. The probability distribution of a random variable X is given below:

X	0	1	2	3
$P(X)$	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$

(i) Determine the value of k. (ii) Determine $P(X \leq 2)$ and $P(X > 2)$.

(iii) Find $P(X \leq 2) + P(X > 2)$.

Q22. Consider the probability distribution of a random variable X.

X	0	1	2	3	4
$P(X)$	0.1	0.25	0.3	0.2	0.15

Calculate (i) $V\left(\frac{X}{2}\right)$ and (ii) Variance of X.

Q23. Suppose 10000 tickets are sold in a lottery each for Rs. 1. First prize is of Rs. 3000 and the second prize is of Rs. 2000. There are three third prizes of Rs. 500 each. If you buy one ticket, then what is your expectation?

Q24. In a dice game, a player pays a stake of Rs. 1 for each throw of a die. She receives Rs. 5, if the die shows a 3, Rs. 2, if the die shows a 1 or 6 and nothing otherwise, then what is the player's expected profit per throw over a long series of throws?

Q25. Find the probability distribution of the maximum of the two scores obtained when a die is thrown twice. Determine also the mean of the distribution.

Q26. If two natural numbers r and s are drawn one at a time, without replacement from the set $S = \{1, 2, 3, ..., n\}$, then find $P(r \leq p / s \leq p)$, where $p \in S$.

Q27. Two probability distributions of the discrete random variables X and Y are given below:

X	0	1	2	3
$P(X)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

Y	0	1	2	3
$P(Y)$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{10}$

Prove that $E(Y^2) = 2E(X)$.

Q28. Two biased dice are thrown together. For the first die $P(6) = \frac{1}{2}$, while for the second die $P(1) = \frac{2}{5}$ and the other scores are equally likely. Find the probability distribution of 'the number of one's seen'.

Q29. For the following probability distribution determine standard deviation of the random variable X.

X	2	3	4
$P(X)$	0.2	0.5	0.3

Q30. A die is thrown three times. Let X be the 'number of twos seen', find the expectation of X.

Q31. Find the variance of the following distribution.

X	0	1	2	3	4	5
$P(X)$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

Q32. Find the probability that in 10 throws of a fair die a score which is a multiple of 3 will be obtained in at least 8 of the throws.

Q33. A lot of 100 watches is known to have 10 defective watches. If 8 watches are selected (One by one without replacement) at random, then what is the probability that three wil be atleast one defective watch?

Q34. The probability of a man hitting a target is 0.25. If he shoots 7 times, then what is the probability of his hitting atleast twice?

Q35. If ten coins are tossed, then what is the probability of getting atleast 8 heads?

Q36. If a die is thrown 5 time, then find the probability that an odd number will come up exactly three times.

Q37. Explain why the experiment of tossing a coin three times is said to have Binomial distribution.

Q38. A discrete random variable X has the following probability distribution:

X	1	2	3	4	5	6	7
$P(X)$	C	$2C$	$2C$	$3C$	C^2	$2C^2$	$7C^2 + C$

Find the value of C. Also find the mean of the distribution.

Q39. There are 5 cards numbered 1 to 5, one number on one card. Two cards are drawn at random without replacement. Let X denotes the sum of the numbers on two cards drawn. Find the mean and variance of X.

Q40. The probability distribution of a discrete random variable X is given as under

X	1	2	4	2A	3A	5A
$P(X)$	1/2	1/5	3/25	1/10	1/25	1/25

Calculate (i) the value of A, if $E(X) = 2.94$, (ii) variance of X.

Q41. A die is tossed twice. If a 'success' is getting an even number on a toss, then find the variance of the number of successes.

Q42. Let X be a discrete random variable whose probability distribution is defined as follows.

$$P(X = x) = \begin{cases} k(x+1), & \text{for } x = 1, 2, 3, 4 \\ 2kx, & \text{for } x = 5, 6, 7 \\ 0, & \text{otherwise} \end{cases}$$

where, k is a constant. Calculate

(i) the value of k (ii) $E(X)$ (iii) standard deviation of X

Q43. There are two bags, one of which contains 3 black and 4 white balls while the other contains 4 black and 3 white balls. A die is thrown. If it shows up 1 or 3, a ball is taken from the I^{st} bag but it shows up any other number, a ball is chosen from the II^{nd} bag. Find the probability of choosing a black ball.

Q44. An urn contains m white and n black balls. A ball is drawn at random and is put back into the urn along with k additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. Show that the probability of drawing a white ball now does not depend on k.

Q45. Suppose that 6% of the people with blood group O are left handed and 10% of those with other blood groups are left handed, 30% of the people have blood group O. If a left handed person is selected at random, what is the probability that he/she will have blood group O?

Q46. Suppose you have two coins which appear identical in your pocket. You know that, one is fair and one is 2 headed. If you take one out, toss it and get a head, what is the probability that it was a fair coin?

Q47. A box has 5 blue and 4 red balls. One ball is drawn at random and not replaced. Its colour is also not noted. Then, another ball is drawn at random,. What is the probability of second ball being blue?

Q48. Four cards are successively drawn without replacement from a deck of 52 playing cards. What is the probability that all the four cards are king?

Q49. Bag I contains 3 black and 2 white balls, bag II contains 2 black and 4 white balls. A bag and a ball is selected at random. Determine the probability of selecting a black ball.

Q50. A bag contains 5 red marbles and 3 black marbles. Three marbles are drawn one by one without replacement. What is the probability that atleast one of the three marbles drawn be black, if the first marble is red?

Q51. A bag contains $(2n + 1)$ coins. It is known that n of these coins have a head on both sides whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, then determine the value of n.

Q52. An item is manufactured by three machines A, B and C. Out of the total number of items manufactured during a specified period, 50% are manufactured on A, 30% on B and 20% on C. 2% of the items produced on A and 3% of items produced on B are defective and 3% of these produced on C are defective. All the items are stored at one godown. One item is drawn at random and is found to be defective. What is the probability that it was manufactured on machine A?

Q53. By examining the chest X-ray, the probability that TB is detected when a person is actually suffering is 0.99. The probability of an healthy person diagnosed to have TB is 0.001. In a certain city, 1 in 1000 people suffers from TB. A person is selected at random and is diagnosed to have TB. What is the probability that the actually has TB?

Q54. There are three urns containing 2 white and 3 black balls, 3 white and 2 black balls and 4 white and 1 black balls, respectively. There is an equal probability of each urn being chosen. A ball is drawn at random from the chosen urn and it is founded to be white. Find the probability that the ball drawn was from the second urn.

Q55. Two cards are drawn successively without replacement from a well shuffled deck of cards. Find the mean and standard deviation of the random variable X, where X is the number of aces.

Q56. The random variable X can take only the values 0, 1, 2. If $P(X = 0) = P(X = 1)$ and $E(X^2) = E(X)$, then find the value of p.

Q57. A biased die is such that $P_{(4)} = \frac{1}{10}$ and other scores being equally likely. The die is tossed twice. If X is the 'number of fours seen', then find the variance of the random variable X.

Q58. If X is the number of tails in three tosses of a coin, then determine the standard deviation of X.

Q59. A discrete random variable X has the probability distribution as given below:

X	0.5	1	1.5	2
$P(X)$	k	k^2	$2k^2$	k

(i) Find the value of k (ii) Determine the mean of the distribution

Q60. A factory produces bulbs. The probability that any one bulb is defective is $\frac{1}{50}$ and they are packed in 10 boxes. From a single box, find the probability that

(i) none of the bulbs is defective.

(ii) exactly two bulbs are defective.

(iii) more than 8 bulbs work properly.

Q61. Four balls are to be drawn without replacement from a box containing 8 red and 4 white balls. If X denotes the number of red ball drawn, find the probability distibution of X.

Q62. A car manufacturing factory has two plants, X and Y. Plant X manufactures 70% of cars and plant Y manufactures 30%. 80% of the cars at plant X and 90% of the cars at plant Y are rated of standard quality. A car is chosen at random and is found to be of standard quality. What is the probability that it has come from plant X?

Q63. Determine variance and standard deviation of the number of heads in three tosses of a coin.

Date: 19/10/2021

S1. $$P(E_1) = P_1 \text{ and } P(E_2) = P_2$$

(i) $P_1P_2 = P(E_1)\cdot P(E_2) = P(E_1 \cap E_2)$

So, E_1 and E_2 occur.

(ii) $1-(1-P_1)(1-P_2) = 1 - P(E_1)'\,P(E_2) = 1 - P(E_1' \cap E_2)$

$$= 1 - [1 - P(E_1 \cup E_2)] = 1 - P(E_1 \cup E_2)$$

So, either E_1 or E_2 or both E_1 and E_2 occurs.

S2. Here, $$P(A) = \frac{2}{5},\quad P(B) = \frac{1}{3},\quad P(A \cap C) = \frac{1}{5} \quad\text{and}\quad P(B \cap C) = \frac{1}{4}$$

$\therefore$ $$P(C/B) = \frac{P(B \cap C)}{P(B)} = \frac{1/4}{1/3} = \frac{3}{4}$$

and $$P(A' \cap C') = 1 - P(A \cup C) = 1 - [P(A) + P(C) - P(A \cap C)]$$

$$= 1 - \left[\frac{2}{5} + \frac{1}{2} - \frac{1}{5}\right] = 1 - \left[\frac{4+5-2}{10}\right] = 1 - \frac{7}{10} = \frac{3}{10}.$$

S3. Here, $$P(A) = \frac{1}{2},\ P(B) = \frac{1}{3} \text{ and } P(A \cap B) = \frac{1}{4}$$

(i) $$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/4}{1/2} = \frac{1}{2}.$$

(ii) $$P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{1 - [P(A) + P(B) - P(A \cap B)]}{1 - P(B)}$$

$$= \frac{1 - \left[\frac{1}{2} + \frac{1}{3} - \frac{1}{4}\right]}{1 - \frac{1}{3}} = \frac{1 - \left(\frac{5}{6} - \frac{1}{4}\right)}{\frac{2}{3}}$$

$$= \frac{1 - 14/24}{2/3} = \frac{10/24}{2/3} = \frac{30}{48} = \frac{5}{8}.$$

S4. Here, $$P(A) = \frac{1}{2},\ P(B) = \frac{1}{3} \text{ and } P(A \cap B) = \frac{1}{4}$$

(i) $$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/3} = \frac{3}{4}.$$

(ii) $P(A'/B) = 1 - P(A/B) = 1 - \frac{3}{4} = \frac{1}{4}$

or $P(A'/B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = \frac{\frac{1}{3} - \frac{1}{4}}{\frac{1}{3}} = \frac{\frac{1}{12}}{\frac{1}{3}} = \frac{1}{4}$.

S5. We know that, $A \cup B$ denotes the occurrence of atleast one of A and B and $A \cap B$ denotes the occurrence of both A and B, simultaneously.

Thus, $P(A \cup B) = 0.6$ and $P(A \cap B) = 0.3$

Also, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow$ $0.6 = P(A) + P(B) - 0.3$

$\Rightarrow$ $P(A) + P(B) = 0.9$

$\Rightarrow$ $[1 - P(\bar{A})] + 1 - P(\bar{B})] = 0.9$ $[\because P(A) = 1 - P(\bar{A})$ and $P(B) = 1 - P(\bar{B})]$

$\Rightarrow$ $P(\bar{A}) + P(\bar{B}) = 2 - 0.9 = 1.1$

S6. We have

$A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$\Rightarrow$ $n(A) = 6$ and $n(S) = 6^2 = 36$ [Where, S is sample space]

$\therefore$ $P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

and $B = \{(4, 6), (6, 4), (5, 5), (5, 6), (6, 5), (6, 6)\}$

$\Rightarrow$ $n(B) = 6$ and $n(S) = 6^2 = 36$

$\therefore$ $P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

Also, $A \cap B = \{5, 5), (6, 6)\}$

$\Rightarrow$ $n(A \cap B) = 2$ and $n(S) = 36$

$\therefore$ $P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

Also, $P(A) \cdot P(B) = \frac{1}{36}$

Thus, $P(A \cap B) \neq P(A) \cdot P(B)$ $\left[\because \frac{1}{18} \neq \frac{1}{36}\right]$

So, we can say that both A and B are not independent events.

S7. For a loaded die, it is given that

$P(1) = P(2) = 0.2,$

$P(3) = P(5) = P(6) = 0.1$ and $P(4) = 0.3$

Also, die is thrown two times.

Here, A = Same number each time and B = Total score is 10 or more

$\therefore$ $A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

So, $P(A) = [P(1, 1) + P(2, 2) + P(3, 3) + P(4, 4) + P(5, 5) + P(6, 6)]$

$= [P(1) \cdot P(1) + P(2) \cdot P(2) + P(3) \cdot P(3) + P(4) \cdot P(4) + P(5) \cdot P(5) + P(6) \cdot P(6)]$

$= [0.2 \times 0.2 + 0.2 \times 0.2 + 0.1 \times 0.1 + 0.3 \times 0.3 + 0.1 \times 0.1 + 0.1 \times 0.1]$

$= [0.04 + 0.04 + 0.01 + 0.09 + 0.01 + 0.01 = 0.20$

and $B = \{(4, 6), (6, 4), (5, 5), (5, 6), (6, 5), (6, 6)\}$

$\therefore$ $P(B) = P(4, 6) + P(6, 4) + P(5, 5) + P(5, 6) + P(6, 5) + P(6, 6)$

$= P(4) \cdot P(6) + P(6) \cdot P(4) + P(5) \cdot P(5) + P(5) \cdot P(6) + P(6) \cdot P(5) + P(6) \cdot P(6)$

$= [0.3 \times 0.1 + 0.1 \times 0.3 + 0.1 \times 0.1 + 0.1 \times 0.1 + 0.1 \times 0.1 + 0.1 \times 0.1]$

$= 0.03 + 0.03 + 0.01 + 0.01 + 0.01 + 0.01 = 0.10$

Also, $A \cap B = \{(5, 5), (6, 6)\}$

$\therefore$ $P(A \cap B) = P(5, 5) + P(6, 6) = P(5) \cdot P(5) + P(6) \cdot P(6)$

$= 0.1 \times 0.1 + 0.1 \times 0.1 = 0.01 + 0.01 = 0.02$

We know that, for two events A and B, if $P(A \cap B) = P(A) \cdot P(B)$, then both are indenpndent events.

Here, $P(A \cap B) = 0.02$ and $P(A) \cdot P(B) = 0.20 \times 0.10 = 0.02$

Thus, $P(A \cap B) = P(A) \cdot P(B) = 0.02$

Hence, A and B are independent events.

S8. Let p be the probability that B gets selected.

P(Exactly one of A, B is selected) = 0.6 (given)

$\Rightarrow$ P(A is selected, B is not selected; B is selected, A is not selected) = 0.6

$\Rightarrow$ $P(A \cap B') + P(A' \cap B) = 0.6$

$\Rightarrow$ $P(A) P(B') + P(A') P(B) = 0.6$

$\Rightarrow$ $(0.7)(1 - p) + (0.3)p = 0.6$

$\Rightarrow$ $p = 0.25$

Thus, the probability that B gets selected is 0.25.

S9. Let A denote the event that at least one girl will be chosen, and B the event that exactly 2 girls will be chosen. We require $P(B|A)$.

Since A denotes the event that at least one girl will be chosen, hence A' denotes that no girl is chosen, *i.e.*, 4 boys are chosen. Then,

$$P(A') = \frac{^8C_4}{^{12}C_4} = \frac{70}{495} = \frac{14}{99}$$

$\Rightarrow$ $$P(A) = 1 - \frac{14}{99} = \frac{85}{99}$$

Now, $$P(A\ \ B) = P(\text{2 boys and 2 girls})$$

$$= \frac{^8C_2 \cdot {^4C_2}}{^{12}C_4} = \frac{6 \times 28}{495} = \frac{56}{165}$$

Thus, $$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{56}{165} \times \frac{99}{85} = \frac{168}{425}.$$

S10. Since P(exactly one of A, B occurs) = q (given), we get

$$P(A \cup B) - P(A \cap B) = q$$

$\Rightarrow$ $$p - P(A \cap B) = q$$

$\Rightarrow$ $$P(A \cap B) = p - q$$

$\Rightarrow$ $$1 - P(A' \cup B') = p - q$$

$\Rightarrow$ $$P(A' \cup B') = 1 - p + q$$

$\Rightarrow$ $$P(A') + P(B') - P(A' \cap B') = 1 - p + q$$

$\Rightarrow$ $$P(A') + P(B') = (1 - p + q) + P(A' \cap B')$$

$$= (1 - p + q) + (1 - (P(A \cup B))$$

$$= (1 - p + q) + (1 - p)$$

$$= 2 - 2p + q.$$

S11. Two dice are thrown together *i.e.*, sample space $(S) = 36 \Rightarrow n(S) = 36$

$$E = A \text{ total of } 4 = \{(2, 2), (3, 1), (1, 3)\}$$

$\Rightarrow$ $$n(E) = 3$$

$$F = A \text{ total of 9 or more}$$

$$= \{(3, 6), (6, 3), (4, 5), (4, 6), (5, 4), (6, 4), (5, 5), (5, 6), (6, 5), (6, 6)\}$$

$\Rightarrow$ $$n(F) = 10$$

$$G = \text{a total divisible by } 5 = \{(1, 4), (4, 1), (2, 3), (3, 2), (4, 6), (6, 4), (5, 5)\}$$

$\Rightarrow$ $$n(G) = 7$$

Here, $$(E \cap F) = \phi \quad \text{and} \quad (E \cap G) = \phi$$

Also, $$(F \cap G) = \{(4, 6), (6, 4), (5, 5)\}$$

$\Rightarrow$ $$n(F \cap G) = 3 \quad \text{and} \quad (E \cap F \cap G) = \phi$$

$\therefore$ $$P(E) = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

$$P(F) = \frac{n(F)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$

$$P(G) = \frac{n(G)}{n(S)} = \frac{7}{36}$$

$$P(F \cap G) = \frac{3}{36} = \frac{1}{12}$$

$$P(F) \cdot P(G) = \frac{5}{18} \cdot \frac{7}{36} = \frac{35}{648}$$

Here, we see that $P(F \cap G) \neq P(F) \cdot P(G)$

Hence, there is no pair which is independent.

S12. We have $A = \{(x, y) : x + y = 11\}$ and $B = \{(x, y) : x \neq 5\}$

$A = \{(5, 6), (6, 5)\}$

B = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$\Rightarrow$ $n(A) = 2,\quad n(B) = 30\quad$ and $\quad n(A \cap B) = 1$

$\therefore$ $P(A) = \frac{2}{36} = \frac{1}{18}\quad$ and $\quad P(B) = \frac{30}{36} = \frac{5}{6}$

$\Rightarrow$ $P(A) \cdot P(B) = \frac{5}{108}\quad$ and $\quad P(A \cap B) = \frac{1}{36} \neq P(A) \cdot P(B)$

So, A and B are not independent.

S13.

$A = \{(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$

$B = \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2)\}$

$A \cap B = \{(6, 2)\}$

$$P(A) = \frac{1}{6},\quad P(B) = \frac{1}{6},\quad P(A \cap B) = \frac{1}{36}$$

Events A and B will be independent if

$$P(A \cap B) = P(\text{A})\, P(B)$$

i.e., L.H.S. $= P(A \cap B) = \frac{1}{36}$

R.H.S. $= P(A)\, P(B) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Hence, A and B are independent.

S14. Bag I : 3 red balls and 0 white ball,

Bag II : 2 red balls and 1 white ball,

Bag III : 0 red balls and 3 white balls

Let E_1, E_2 and E_3 be the events that bag I, bag II, and bag III is selected and a ball is chosen from it.

$$P(E_1) = \frac{1}{6}, \quad P(E_2) = \frac{2}{6}, \quad P(E_3) = \frac{3}{6}$$

(i)
$$P(E_2/F) = \frac{P(E_2)\cdot P(F/E_2)}{P(E_1)\cdot P(F/E_1) + P(E_2)\cdot P(F/E_2) + P(E_3)\cdot P(F/E_3)}.$$

$$= \frac{\frac{2}{6}\cdot\frac{1}{3}}{\frac{1}{6}\cdot 0 + \frac{2}{6}\cdot\frac{1}{3} + \frac{3}{6}\cdot 1} = \frac{\frac{2}{18}}{\frac{2}{18}+\frac{3}{6}} = \frac{2/18}{\frac{2+9}{18}} = \frac{2}{11}.$$

(ii)
$$P(E_3/F) = \frac{P(E_3)\cdot P(F/E_3)}{P(E_1)\cdot P(F/E_1) + P(E_2)\cdot P(F/E_2) + P(E_3)\cdot P(F/E_3)}$$

$$= \frac{\frac{3}{6}\cdot 1}{\frac{1}{6}\cdot 0 + \frac{2}{6}\cdot\frac{1}{3} + \frac{3}{6}\cdot 1} = \frac{\frac{3}{6}}{\frac{2}{18}+\frac{3}{6}} = \frac{3/6}{\frac{2}{18}+\frac{9}{18}} = \frac{9}{11}.$$

S15. Bag I : 3 red balls and 0 white ball,

Bag II : 2 red balls and 1 white ball,

Bag III : 0 red balls and 3 white balls

Let E_1, E_2 and E_3 be the events that bag I, bag II, and bag III is selected and a ball is chosen from it.

$$P(E_1) = \frac{1}{6}, \quad P(E_2) = \frac{2}{6}, \quad P(E_3) = \frac{3}{6}$$

(i) Let E be the event that a red ball is selected. Then, probability that red ball will be selected

$$P(E) = P(E_1)\cdot P(E/E_1) + P(E_2)\cdot P(E/E_2) + P(E_3)\cdot P(E/E_3)$$

$$= \frac{1}{6}\cdot\frac{3}{3} + \frac{2}{6}\cdot\frac{2}{3} + \frac{3}{6}\cdot 0$$

$$= \frac{1}{6} + \frac{2}{9} + 0$$

$$= \frac{3+4}{18} = \frac{7}{18}$$

(ii) Let F be the event that a white ball is selected.

$$P(F) = P(E_1) \cdot P(F/E_1) + P(E_2) \cdot P(F/E_2) + P(E_3) \cdot P(F/E_3)$$

$$= \frac{1}{6} \cdot 0 + \frac{2}{6} \cdot \frac{1}{3} + \frac{3}{6} \cdot 1 = \frac{1}{9} + \frac{3}{6} = \frac{11}{18}$$

S16. Let D be the event that the picked up tube is defective.

Let A_1, A_2 and A_3 be the events that the tube is produced on machines A_1, A_2 and A_3, respectively.

$$P(D) = P(A_1)\,P(D|A_1) + P(A_2)\,P(D|A_2) + P(A_3)\,P(D|A_3) \qquad \text{... (i)}$$

$$P(A_1) = \frac{50}{100} = \frac{1}{2}, \quad P(A_2) = \frac{1}{4}, \quad P(A_3) = \frac{1}{4}$$

Also, $$P(D|A_1) = P(D|A_2) = \frac{4}{100} = \frac{1}{25}$$

Now, we have to find $P(A_1/D)$.

$$P(A_1/D) = \frac{P(A_1 \cap D)}{P(D)} = \frac{P(A_1)\,P(D/A_1)}{P(D)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{25}}{\frac{17}{400}} = \frac{8}{17}.$$

S17. Let D be the event that the picked up tube is defective.

Let A_1, A_2 and A_3 be the events that the tube is produced on machines A_1, A_2 and A_3, respectively.

$$P(D) = P(A_1)\,P(D|A_1) + P(A_2)\,P(D|A_2) + P(A_3)\,P(D|A_3) \qquad \text{... (i)}$$

$$P(A_1) = \frac{50}{100} = \frac{1}{2}, \quad P(A_2) = \frac{1}{4}, \quad P(A_3) = \frac{1}{4}$$

Also, $$P(D|A_1) = P(D|A_2) = \frac{4}{100} = \frac{1}{25}$$

$$P(D|A_3) = \frac{5}{100} = \frac{1}{20}$$

Putting these values in Eq. (i), we get

$$P(D) = \frac{1}{2} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{20}$$

$$= \frac{1}{50} + \frac{1}{100} + \frac{1}{80} = \frac{17}{400} = .0425\,.$$

S18. Let A and B be the events that the bulb is red and defective, respectively.

$$P(A) = \frac{10}{100} = \frac{1}{10}$$

$$P(A \cap B) = \frac{2}{100} = \frac{1}{50}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1}{50} \times \frac{10}{1} = \frac{1}{5}$$

Thus the probability of the picked up bulb of its being defective, if it is red, is $\frac{1}{5}$.

S19. We have, $A_1 : A_2 : A_3 = 4 : 4 : 2$

$$P(A_1) = \frac{4}{10}, \quad P(A_2) = \frac{4}{10}, \quad P(A_3) = \frac{2}{10}$$

where, A_1, A_2 and A_3 denote the three types of flower seeds.

Let E be the event that a seed germinates and $\bar{E}$ be the event that a seed does not grrminate.

$$\therefore \quad P(E/A_1) = \frac{45}{100}, \quad P(E/A_2) = \frac{60}{100}, \quad P(E/A_3) = \frac{35}{100}$$

and $$P(\bar{E}/A_1) = \frac{55}{100}, \quad P(\bar{E}/A_2) = \frac{40}{100}, \quad P(\bar{E}/A_3) = \frac{65}{100}$$

(i) $\therefore$ $$P(E) = P(A_1) \cdot P(E/A_1) + P(A_2) \cdot P(E/A_2) + P(A_3) \cdot P(E/A_3)$$

$$= \frac{4}{10} \cdot \frac{45}{100} + \frac{4}{10} \cdot \frac{60}{100} + \frac{2}{10} \cdot \frac{35}{100}$$

$$= \frac{180}{1000} + \frac{240}{1000} + \frac{70}{1000} = \frac{490}{1000} = 0.49$$

(ii) $$P(\bar{E}/A_3) = 1 - P(E/A_3) = 1 - \frac{35}{100} = \frac{65}{100}$$

(iii) $$P(\bar{E}/A_2) = \frac{P(A_2) \cdot P(\bar{E}/A_2)}{P(A_1) \cdot P(\bar{E}/A_1) + P(A_2) \cdot P(\bar{E}/A_2) + P(A_3) \cdot P(\bar{E}/A_3)}$$

$$= \frac{\frac{4}{10} \cdot \frac{40}{100}}{\frac{4}{10} \cdot \frac{55}{100} + \frac{4}{10} \cdot \frac{40}{100} + \frac{2}{10} \cdot \frac{65}{100}} = \frac{\frac{160}{1000}}{\frac{220}{1000} + \frac{160}{1000} + \frac{130}{1000}}$$

$$= \frac{160/1000}{510/1000} = \frac{16}{51} = 0.313725 = 0.314.$$

S20. Let E_1 be the event that letter is from TATA NAGAR and E_2 be the event that letter is from CALCUTTA.

Also, let E_3 be the event that on the letter, two consecutive letters TA are visible.

$$\therefore \quad P(E_1) = \frac{1}{2} \quad \text{and} \quad P(E_1) = \frac{1}{2}$$

and $$P(E_3/E_1) = \frac{2}{8} \quad \text{and} \quad P(E_3/E_2) = \frac{1}{7}$$

[Since, if letter is from TATA NAGAR, we see that the events of two consecutive letters visible are {TA, AT, TA, AN, NA, AG, GA, AR}. So, $P(E_3/E_1) = \frac{2}{8}$ and if letter is from CALCUTTA, we see that the events of two consecutive letters to visible are {CA, AL, LC, CU, UT, TT, TA}. So, $P(E_3/E_2) = \frac{1}{7}$].

$$\therefore \quad P(E_1/E_3) = \frac{P(E_1)\cdot P(E_3/E_1)}{P(E_1)\cdot P(E_3/E_1) + P(E_2)\cdot P(E_3/E_2)}$$

$$= \frac{\frac{1}{2}\cdot\frac{2}{8}}{\frac{1}{2}\cdot\frac{2}{8}+\frac{1}{2}\cdot\frac{1}{7}} = \frac{\frac{1}{8}}{\frac{1}{8}+\frac{1}{14}} = \frac{1/8}{\frac{22}{8\times 14}} = \frac{\frac{1}{8}}{\frac{11}{56}} = \frac{7}{11}.$$

S21. We have,

X	0	1	2	3
$P(X)$	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$

(i) Since, $\sum_{i=1}^{n} P_i = 1, \quad i = 1, 2, ..., n \quad \text{and} \quad P_i \geq 0$

$$\therefore \quad k + \frac{k}{2} + \frac{k}{4} + \frac{k}{8} = 1$$

$$\Rightarrow \quad 8k + 4k + 2k + k = 8$$

$$\therefore \quad k = \frac{8}{15}$$

(ii) $P(X \leq 2) = P(0) + P(1) + P(2) = k + \frac{k}{2} + \frac{k}{4}$

$$= \frac{(4k+2k+k)}{4} = \frac{7k}{4} = \frac{7}{4}\cdot\frac{8}{15} = \frac{14}{15}$$

and $P(X > 2) = P(3) = \frac{k}{8} = \frac{1}{8}\cdot\frac{8}{15} = \frac{1}{15}$

(iii) $P(X \leq 2) + P(X > 2) = \frac{14}{15} + \frac{1}{15} = 1.$

S22. We have,

X	0	1	2	3	4
$P(X)$	0.1	0.25	0.3	0.2	0.15
$XP(X)$	0	0.25	0.6	0.6	0.60
$X^2P(X)$	0	0.25	1.2	1.8	2.40

$$\text{Var } X = E(X^2) - [E(X)]^2$$

where, $$E(X) = \mu = \sum_{i=1}^{n} x_i P_i(x_i)$$

and $$E(X^2) = \sum_{i=1}^{n} x_i^2 P(x_i)$$

∴ $$E(X) = 0 + 0.25 + 0.6 + 0.6 + 0.60 = 2.05$$

$$E(X^2) = 0 + 0.25 + 1.2 + 1.8 + 2.40 = 5.65$$

(i) $$V\left(\frac{X}{2}\right) = \frac{1}{4}V(X) = \frac{1}{4}[5.65 - (2.05)^2]$$

$$= \frac{1}{4}[5.65 - 4.2025] = \frac{1}{4} \times 1.4475 = 0.361875$$

(ii) $V(X) = 1.4475$

S23. Let X is the random variable of profit per throws.

X	0	500	2000	3000
$P(X)$	$\frac{9995}{10000}$	$\frac{3}{10000}$	$\frac{1}{10000}$	$\frac{1}{10000}$

Since, $$E(X) = \sum X P(X)$$

∴ $$E(X) = 0 \times \frac{9995}{10000} + \frac{1500}{10000} + \frac{2000}{10000} + \frac{3000}{10000}$$

$$= \frac{1500 + 2000 + 3000}{10000}$$

$$= \frac{6500}{10000} = \frac{13}{20} = \text{Rs. } 0.65.$$

S24. Let X is the random variable of profit per throws.

X	-1	1	4
$P(X)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

Since, she loss Rs. 1 on getting any of 2, 4 or 5.

So, at $X = -1$, $P(X) = \frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$

Similarly, at $X = 1$, $P(X) = \frac{1}{6}+\frac{1}{6}=\frac{1}{3}$ [If die shows of either 1 or 6]

and at $X = 4$, $P(X) = \frac{1}{6}$ [If die shows a 3]

$\therefore$ Player's expected pofit = $E(X) = \sum X P(X)$

$$= -1\times\frac{1}{2}+1\times\frac{1}{3}+4\times\frac{1}{6}$$

$$= \frac{-3+2+4}{6}=\frac{3}{6}=\frac{1}{2} = \text{Rs. } 0.50.$$

S25. Let X is the random variable score obtained when a die is thrown twice.

$\therefore$ $X = 1, 2, 3, 4, 5, 6$

Here, $S = \{(1, 1), (1, 2), (2, 1), (2,2), (1, 3), (2, 3), (3, 1), (3, 2), (3, 3), ..., (6, 6)\}$

$\therefore$ $P(X = 1) = \frac{1}{6}\cdot\frac{1}{6}=\frac{1}{36}$

$P(X = 2) = \frac{1}{6}\cdot\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}=\frac{3}{36}$

$P(X = 3) = \frac{1}{6}\cdot\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}+\frac{1}{6}\cdot\frac{1}{6}=\frac{5}{36}$

Similarly, $P(X = 4) = \frac{7}{36}$

$P(X = 5) = \frac{9}{36}$

$P(X = 6) = \frac{11}{36}$

So, the required distribution is,

X	**1**	**2**	**3**	**4**	**5**	**6**
P(X)	1/36	3/36	5/36	7/36	9/36	11/36

Also, we know that, Mean $E(X) = \sum X P(X)$

$$= \frac{1}{36}+\frac{6}{36}+\frac{15}{36}+\frac{28}{36}+\frac{45}{36}+\frac{66}{36}=\frac{161}{36}.$$

S26. $\because$ Set $S = \{1, 2, 3, ..., n\}$

$$\therefore \quad P(r \le p / s \le p) = \frac{P(p \cap S)}{P(S)}$$

$$= \frac{p-1}{n} \times \frac{n}{n-1} = \frac{p-1}{n-1}.$$

S27. We have,

X	0	1	2	3
$P(X)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

Y	0	1	2	3
$P(Y)$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{10}$

Since, we have to prove that,

$$E(Y^2) = 2E(X)$$

$$E(X) = \sum X P(X)$$

$$= 0 \cdot \frac{1}{5} + 1 \cdot \frac{2}{5} + 2 \cdot \frac{1}{5} + 3 \cdot \frac{1}{5} = \frac{7}{5}$$

$$\Rightarrow \quad 2E(X) = \frac{14}{5} \qquad \text{... (i)}$$

$$E(Y^2) = \sum Y^2 P(Y).$$

$$= 0 \cdot \frac{1}{5} + 1 \cdot \frac{3}{10} + 4 \cdot \frac{2}{5} + 9 \cdot \frac{1}{10}$$

$$= \frac{3}{10} + \frac{8}{5} + \frac{9}{10} = \frac{28}{10} = \frac{14}{5}$$

$$\Rightarrow \quad E(Y^2) = \frac{14}{5} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$E(Y^2) = 2E(X).$$ **Hence proved.**

S28. For first die, $P(6) = \frac{1}{2}$ and $P(6') = \frac{1}{2}$

$$\Rightarrow \quad P(1) + P(2) + P(3) + P(4) + P(5) = \frac{1}{2}$$

$\Rightarrow \qquad P(1) = \frac{1}{10} \quad \text{and} \quad P(1') = \frac{9}{10}$

$[\because \; P(1) = P(2) = P(3) = P(4) = P(5)]$

For second die, $\qquad P(1) = \frac{2}{5} \quad \text{and} \quad P(1') = 1 - \frac{2}{5} = \frac{3}{5}$

Let, X = Number of one's seen

For $X = 0$, $\qquad P(X = 0) = P(1') \cdot P(1') = \frac{9}{10} \cdot \frac{3}{5} = \frac{25}{50} = 0.54$

$$P(X = 1) = P(1') \cdot P(1) + P(1) \cdot P(1') = \frac{9}{10} \cdot \frac{2}{5} + \frac{1}{10} \cdot \frac{3}{5}$$

$$= \frac{18}{50} + \frac{3}{50} = \frac{21}{50} = 0.42$$

$$P(X = 2) = P(1) \cdot P(1) = \frac{1}{10} \cdot \frac{2}{5} = \frac{2}{50} = 0.04$$

Hence, the required probability distribution is as below:

X	**0**	**1**	**2**
***P*(*X*)**	**0.54**	**0.42**	**0.04**

S29. We have,

X	**2**	**3**	**4**
***P*(*X*)**	0.2	0.5	0.3
***XP*(*X*)**	0.4	1.5	1.2
$X^2P(X)$	0.8	4.5	4.8

We know that, standard deviation of $X = \sqrt{\text{Var}(X)}$

where, $\qquad \text{Var } X = E(X^2) - [E(X)]^2$

$$= \sum_{i=1}^{n} x_i^2 P(x_i) - \left[\sum_{i=1}^{n} x_i P_i\right]^2$$

$\therefore \qquad \text{Var } X = [0.8 + 4.5 + 4.8] - [0.4 + 1.5 + 1.2]^2$

$$= 10.1 - (3)^2 = 10.1 - 9.61 = 0.49$$

$\therefore \qquad$ Standard deviation of $X = \sqrt{\text{Var}(X)} = \sqrt{0.49} = 0.7$

S30. We have, X = Number of twos seen

So, on throwing a die three times, we will have X = 0, 1, 2, 3.

$$P(X=0) = P_{(\text{not }2)} \cdot P_{(\text{not }2)} \cdot P_{(\text{not }2)} = \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{5}{6} = \frac{125}{216}$$

$$P(X=1) = P_{(\text{not }2)} \cdot P_{(\text{not }2)} \cdot P_{(2)} + P_{(\text{not }2)} \cdot P_{(2)} \cdot P_{(\text{not }2)} + P_{(2)} \cdot P_{(\text{not }2)} \cdot P_{(\text{not }2)}$$

$$= \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} + \frac{5}{6}\cdot\frac{1}{6}\cdot\frac{5}{6} + \frac{1}{6}\cdot\frac{5}{6}\cdot\frac{5}{6} = \frac{25}{36}\cdot\frac{3}{6} = \frac{25}{72}$$

$$P(X=2) = P_{(\text{not }2)} \cdot P_{(2)} \cdot P_{(2)} + P_{(2)} \cdot P_{(2)} \cdot P_{(\text{not }2)} + P_{(2)} \cdot P_{(\text{not }2)} + P_{(2)}$$

$$= \frac{5}{6}\cdot\frac{1}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6}\cdot\frac{5}{6} + \frac{1}{6}\cdot\frac{5}{6}\cdot\frac{1}{6}$$

$$= \frac{1}{36}\cdot\left[\frac{15}{6}\right] = \frac{15}{216}$$

$$P(X=3) = P_{(2)} \cdot P_{(2)} \cdot P_{(2)} = \frac{1}{6}\cdot\frac{1}{6}\cdot\frac{1}{6} = \frac{1}{216}$$

We know that, $E(X) = \sum X P(X) = 0\cdot\frac{125}{216} + 1\cdot\frac{25}{72} + 2\cdot\frac{15}{216} + 3\cdot\frac{1}{216}$

$$= \frac{75+30+3}{216} = \frac{108}{216} = \frac{1}{2}.$$

S31. We have,

X	0	1	2	3	4	5
$P(X)$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$
$XP(X)$	0	$\frac{5}{18}$	$\frac{4}{9}$	$\frac{1}{2}$	$\frac{4}{9}$	$\frac{5}{18}$
$X^2P(X)$	0	$\frac{5}{18}$	$\frac{8}{9}$	$\frac{3}{2}$	$\frac{16}{9}$	$\frac{25}{18}$

$\therefore$ Variance $= E(X^2) - [E(X)]^2 = \sum X^2P(X) - [\sum XP(X)]^2$

$$= \left[0 + \frac{5}{18} + \frac{8}{9} + \frac{3}{2} + \frac{16}{9} + \frac{25}{18}\right] - \left[0 + \frac{5}{18} + \frac{4}{9} + \frac{1}{2} + \frac{4}{9} + \frac{5}{18}\right]^2$$

$$= \left[\frac{5+16+27+32+25}{18}\right] - \left[\frac{5+8+9+8+5}{18}\right]^2$$

$$= \frac{105}{18} - \frac{35\cdot 35}{18\cdot 18} = \frac{18\cdot 105 - 35\cdot 35}{18\cdot 18}$$

$$= \frac{35}{18\cdot 18}[54-35] = \frac{19\cdot 35}{324} = \frac{665}{324}.$$

S32. Here, success is a score which is a multiple of 3 *i.e.*, 3 or 6.

Therefore, $p\,(3 \text{ or } 6) = \dfrac{2}{6} = \dfrac{1}{3}$

The probability of r successes in 10 throws is given by

$$P(r) = {}^{10}C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{10-r}$$

Now, P (at least 8 successes) $= P(8) + P(9) + P(10)$

$$= {}^{10}C_8 \left(\frac{1}{3}\right)^8 \left(\frac{2}{3}\right)^2 + {}^{10}C_9 \left(\frac{1}{3}\right)^9 \left(\frac{2}{3}\right)^1 + {}^{10}C_{10} \left(\frac{1}{3}\right)^{10}$$

$$= \frac{1}{3^{10}}[45 \times 4 + 10 \times 2 + 1] = \frac{201}{3^{10}}.$$

S33. Probability of defective watch from a lot of 100 watches $\dfrac{10}{100} = \dfrac{1}{10}$

$\therefore$ $p = 1/10, \quad q = \dfrac{9}{10}, \quad n = 8 \quad \text{and} \quad r \geq 1$

$\therefore$ $P(r \geq 1) = 1 - P(r = 0) = 1 - {}^8C_0 \left(\dfrac{1}{10}\right)^0 \left(\dfrac{9}{10}\right)^{8-0}$

$$= 1 - \frac{8!}{0!8!} \cdot \left(\frac{9}{10}\right)^8 = 1 - \left(\frac{9}{10}\right)^8.$$

S34. Here, $n = 7, \quad p = 0.25 = \dfrac{1}{4}, \quad q = 1 - \dfrac{1}{4} = \dfrac{3}{4}, \quad r \geq 2$

where, $P(X) = {}^nC_r (p)^r (q)^{n-r}$

$\therefore$ $P(X = r) = 1 - [P(r = 0) + P(r = 1)]$

$$= 1 - \left[{}^7C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{7-0} + {}^7C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^{7-1}\right]$$

$$= 1 - \left[\frac{7!}{0!7!}\left(\frac{3}{4}\right)^7 + \frac{7!}{1!6!}\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^6\right]$$

$$= 1 - \left[\left(\frac{3}{4}\right)^6 \left(\frac{3}{4} \cdot 1 + \frac{1}{4} \cdot 7\right)\right]$$

$$= 1 - \left[\frac{3^6}{4^6}\left(\frac{10}{4}\right)\right] = 1 - \left[\frac{3^6 \times 10}{4^7}\right] = 1 - \left[\frac{27 \cdot 27 \cdot 10}{64 \cdot 256}\right]$$

$$= 1 - \left[\frac{7290}{16384}\right] = 1 - \frac{3645}{8192} = \frac{4547}{8192}$$

S35. In this case, we have to find out the probability of getting atleast 8 heads. Let X is the random variable for getting a head.

Here, $n = 10, \quad r \geq 8$

i.e., $r = 8, 9, 10, \quad p = \frac{1}{2}, \quad q = \frac{1}{2}$

We know that, $P(X = r) = {}^nC_r p^r q^{n-r}$

$\therefore$ $P(X = r) = P(r = 8) + P(r = 9) + P(r = 10)$

$$= {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^{10-8} + {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^{10-9} + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^{10-10}$$

$$= \frac{10!}{8!2!}\left(\frac{1}{2}\right)^{10} + \frac{10!}{9!1!}\left(\frac{1}{2}\right)^{10} + \frac{10!}{0!10!}\left(\frac{1}{2}\right)^{10}$$

$$= \left(\frac{1}{2}\right)^{10}\left[\frac{10 \times 9}{2} + 10 + 1\right]$$

$$= \left(\frac{1}{2}\right)^{10} \cdot 56 = \frac{1}{2^7 \cdot 2^3} \cdot 56 = \frac{7}{128}.$$

S36. Here, $n = 5, \quad p = \left(\frac{1}{6} + \frac{1}{6} + \frac{1}{6}\right) = \frac{1}{2}$ and $q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

Also, $r = 3$

$\therefore$ $P(X = r) = {}^nC_r (p)^r (q)^{n-r} = {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3}$

$$= \frac{5!}{3!2!} \cdot \frac{1}{8} \cdot \frac{1}{4} = \frac{10}{32} = \frac{5}{16}.$$

S37. We know that, a random variable X taking values 0, 1, 2, ..., n is said to have a binomial distribution with parameters n and P, if its probability distribution is given by

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

where, $q = 1 - p$

and $r = 0, 1, 2, ..., n$

Similarly, in an experiment of tossing a coin three times, we have $n = 3$ and random variable X can take values $r = 0, 1, 2$ and 3 with $p = \frac{1}{2}$ and $q = \frac{1}{2}$.

X	0	1	2	3
$P(X)$	${}^3C_0 q^3$	${}^3C_1 pq^2$	${}^3C_2 p^2 q$	${}^3C_3 p^3$

So, we see that in the experiment of tossing a coin three times, we have random variable X which can take values 0, 1, 2 and 3 with parameters $n = 3$ and $P = \frac{1}{2}$,

Therefore, it is said to have a Binomial distribution.

S38. Since, $\sum p_i = 1$, we have

$$C + 2C + 2C + 3C + C^2 + 2C^2 + 7C^2 + C = 1$$

i.e., $$10C^2 + 9C - 1 = 0$$

i.e., $$(10C - 1)(C + 1) = 0$$

$\Rightarrow$ $$C = \frac{1}{10}, \qquad C = -1$$

Therefore, the permissible value of $C = \frac{1}{10}$ $[\because\ 0 \le P(X) \le 1]$

$$\text{Mean} = \sum_{i=1}^{n} x_i p_i = \sum_{i=1}^{7} x_i p_i$$

$$= 1\times\frac{1}{10} + 2\times\frac{2}{10} + 3\times\frac{2}{10} + 4\times\frac{3}{10} + 5\times\left(\frac{1}{10}\right)^2 + 6\times 2\left(\frac{1}{10}\right)^2 + 7\left(7\left(\frac{1}{10}\right)^2 + \frac{1}{10}\right)$$

$$= \frac{1}{10} + \frac{4}{10} + \frac{6}{10} + \frac{12}{10} + \frac{5}{100} + \frac{12}{100} + \frac{49}{100} + \frac{7}{10}$$

$= 3.66.$

S39. Here, S = {(1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2), (1, 4), (4, 1), (1, 5), (5, 1), (2, 4), (4, 2), (2, 5), (5, 2), (3, 4), (4, 3), (3, 5), (5, 3),(5, 4), (4, 5)}.

$\Rightarrow$ $n(S) = 20$

Let random variable be X which denotes the sum of the numbers on two cards drawn.

$\therefore$ $X = 3, 4, 5, 6, 7, 8, 9$

At $X = 3$, $P(X) = \frac{2}{20} = \frac{1}{10}$

At $X = 4$, $P(X) = \frac{2}{20} = \frac{1}{10}$

At $X = 5$, $P(X) = \frac{4}{20} = \frac{1}{5}$

At $X = 6$, $P(X) = \frac{4}{20} = \frac{1}{5}$

At $X = 7$, $P(X) = \frac{4}{20} = \frac{1}{5}$

At $X = 8$, $P(X) = \frac{2}{20} = \frac{1}{10}$

At $X = 9$, $P(X) = \frac{2}{20} = \frac{1}{10}$

$\therefore$ Mean $E(X) = \sum X P(X) = \frac{3}{10} + \frac{4}{10} + \frac{5}{5} + \frac{6}{5} + \frac{7}{5} + \frac{8}{10} + \frac{9}{10}$

$$= \frac{3 + 4 + 10 + 12 + 14 + 8 + 9}{10} = 6$$

Also, $\sum X^2 P(X) = \frac{9}{10} + \frac{16}{10} + \frac{25}{5} + \frac{36}{5} + \frac{49}{5} + \frac{64}{10} + \frac{81}{10}$

$$= \frac{9 + 16 + 50 + 72 + 98 + 64 + 81}{10} = 39$$

$\therefore$ $\text{Var}(X) = \sum X^2 P(X) - [\sum X P(X)]^2$

$$= 39 - (6)^2 = 39 - 36 = 3.$$

S40. (i) We have, $\sum X P(X) = \frac{1}{2} + \frac{2}{5} + \frac{12}{25} + \frac{2A}{10} + \frac{3A}{25} + \frac{5A}{25}$

$$= \frac{25 + 20 + 24 + 10A + 6A + 10A}{50} = \frac{69 + 26A}{50}$$

Since, $E(X) = \sum X P(X)$

$\Rightarrow$ $2.94 = \frac{69 + 26A}{50}$

$\Rightarrow$ $26A = 50 \times 2.94 - 69$

$\Rightarrow$ $A = \frac{147 - 69}{26} = \frac{78}{26} = 3$

(ii) We know that,

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$= \sum X^2 P(X) - [\sum X P(X)]^2$$

$$= \frac{1}{2} + \frac{4}{5} + \frac{48}{25} + \frac{4A^2}{10} + \frac{9A^2}{25} + \frac{25A^2}{25} - [E(X)]^2$$

$$= \frac{25 + 40 + 96 + 20A^2 + 18A^2 + 50A^2}{50} - [E(X)]^2$$

$$= \frac{161 + 88A^2}{50} - [E(X)]^2 = \frac{161 + 88 \times (3)^2}{50} - [E(X)]^2 \quad [\because A = 3]$$

$$= \frac{953}{50} - [2.94]^2 \quad [\because E(X) = 2.94]$$

$$= 19.0600 - 8.6436 = 10.4164.$$

S41. Let X be the random variable for a 'sucess' for getting an even number on a toss

$$\therefore \quad X = 0, 1, 2 \quad n = 2, \quad p = \frac{3}{6} = \frac{1}{2} \quad \text{and} \quad q = \frac{1}{2}$$

At $X = 0$, $$P(X = 0) = {}^2C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{2-0} = \frac{1}{4}$$

At $X = 1$, $$P(X = 1) = {}^2C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{2-1} = 2 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$$

At $X = 2$, $$P(X = 2) = {}^2C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{2-2} = \frac{1}{4}$$

Thus

X	**0**	**1**	**2**
$P(X)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
$XP(X)$	0	$\frac{1}{2}$	$\frac{1}{2}$
$X^2P(X)$	0	$\frac{1}{2}$	1

$$\therefore \quad \sum XP(X) = 0 + \frac{1}{2} + \frac{1}{2} = 1 \quad \text{... (i)}$$

and $$\sum X^2P(X) = 0 + \frac{1}{2} + 1 = \frac{3}{2} \quad \text{... (ii)}$$

$$\because \quad \text{Var}(X) = E(X^2) - [E(X)]^2$$
$$= \sum X^2P(X) - [\sum XP(X)]^2$$
$$= \frac{3}{2} - (1)^2 = \frac{1}{2} \quad \text{[Using Eq. (i) and (ii)]}$$

S42. Given,

$$P(X = x) = \begin{cases} k(x+1), & \text{for} \quad x = 1, 2, 3, 4 \\ 2kx, & \text{for} \quad x = 5, 6, 7 \\ 0, & \text{otherwise} \end{cases}$$

Thus, we have following table

X	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**Otherwise**
P(X)	$2k$	$3k$	$4k$	$5k$	$10k$	$12k$	$14k$	0
XP(X)	$2k$	$6k$	$12k$	$20k$	$50k$	$72k$	$98k$	0
$\boldsymbol{X^2P(X)}$	$2k$	$12k$	$36k$	$80k$	$250k$	$432k$	$686k$	0

(i) SInce, $\sum P_i = 1$

$$\Rightarrow \quad k(2 + 3 + 4 + 5 + 10 + 12 + 14) = 1 \quad \Rightarrow \quad k = \frac{1}{50}$$

(ii) $\because$ $E(X) = \sum XP(X)$

$\therefore$ $E(X) = 2k + 6k + 12k + 20k + 50k + 72k + 98k + 0 = 260k$

$$= 260 \times \frac{1}{50} = \frac{26}{5} = 5.2 \qquad \left[\because \; k = \frac{1}{50}\right] \quad \dots \text{(i)}$$

(iii) We know that,

$$\text{Var}(X) = [E(X^2)] - [E(X)]^2 = \sum X^2 P(X) - [\sum \{XP(X)\}]^2$$

$$= [2k + 12k + 36k + 80k + 250k + 432k + 686k + 0] - [5.2]^2 \text{ [Using Eq. (i)]}$$

$$= [1498k] - 27.04 = \left[1498 \times \frac{1}{50}\right] - 27.04 \qquad \left[\because \; k = \frac{1}{50}\right]$$

$$= 29.96 - 27.04 = 2.92$$

We know that, standard deviation of $X = \sqrt{\text{Var}(X)} = \sqrt{2.92} = 1.7088 = 1.7$ (Approx.)

S43. Since, bag I^{st} = {3 black, 4 white balls}, bag II^{nd} = {4 black, 3 white balls}

Let E_1 be the event that bag I^{st} is selected and E_2 be the event that bag II^{nd} is selected.

Let E_3 be the event that black ball is chosen.

$\therefore$ $P(E_1) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$ and $P(E_2) = 1 - \frac{1}{3} = \frac{2}{3}$

and $P(E_3/E_1) = \frac{3}{7}$ and $P(E_3/E_2) = \frac{4}{7}$

$\therefore$ $P(E_3) = P(E_1) \cdot P(E_3/E_1) + P(E_2) \cdot P(E_3/E_2)$

$$= \frac{1}{3} \cdot \frac{3}{7} + \frac{2}{3} \cdot \frac{4}{7} = \frac{11}{21}.$$

S44. Let U = {m white, n black balls}

E_1 = {First ball drawn of white colour}

E_2 = {First ball drawn of black colour}

and E_3 = {Second ball drawn of white colour}

$\therefore$ $P(E_1) = \frac{m}{m+n}$ and $P(E_2) = \frac{n}{m+n}$

Also, $P(E_3/E_1) = \frac{m+k}{m+n+k}$ and $P(E_3/E_2) = \frac{m}{m+n+k}$

$$\therefore \quad P(E_3) = P(E_1) \cdot P(E_3/E_1) + P(E_2) \cdot P(E_3/E_2)$$

$$= \frac{m}{m+n} \cdot \frac{m+k}{m+n+k} + \frac{n}{m+n} \cdot \frac{m}{m+n+k}$$

$$= \frac{m(m+k)+nm}{(m+n+k)(m+n)} = \frac{m^2+mk+nm}{(m+n+k)(m+n)}$$

$$= \frac{m(m+k+n)}{(m+n+k)(m+n)} = \frac{m}{m+n}$$

Hence, the probability of drawing a white ball does not depend on k.

S45.

		Blood group 'O'	**Other than blood group 'O'**
1.	Number of people	30%	70%
2.	Percentage of left handed people	6%	10%

E_1 = Event that the person selected is of blood group O

E_2 = Event that the person selected is of other than blood group O

(E_3) = Event that selected person is left handed

$\therefore \quad P(E_1) = 0.30, \quad P(E_2) = 0.70$

$P(E_3/E_1) = 0.06$ and $P(E_3/E_2) = 0.10$

By using Baye's theorem,

$$P(E_1/E_3) = \frac{P(E_1) \cdot P(E_3/E_1)}{P(E_1) \cdot P(E_3/E_1) + P(E_2) \cdot P(E_3/E_2)}$$

$$= \frac{0.30 \times 0.06}{0.30 \cdot 0.06 + 0.70 \cdot 0.10}$$

$$= \frac{0.0180}{0.0180 + 0.0700}$$

$$= \frac{0.0180}{0.0880} = \frac{180}{880} = \frac{9}{44}.$$

S46. Let

E_1 = Event that fair coin is drawn

E_2 = Event that 2 headed coin is drawn

E = Event that tossed coin get a head

$\therefore \quad P(E_1) = 1/2, \quad P(E_2) = 1/2, \quad P(E/E_1) = 1/2$ and $P(E/E_2) = 1$

Now, using Baye's theorem

$$P(E_1/E) = \frac{P(E_1) \cdot P(E/E_1)}{P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2)}$$

$$= \frac{\frac{1}{2}\cdot\frac{1}{2}}{\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot 1} = \frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{2}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

S47. A box = {5 blue, 4 red}

Let E_1 is the event that first ball drawn is blue, E_2 is the event that first ball drawn is red and E is the event that second ball drawn is blue.

$$\therefore \quad P(E) = P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2)$$

$$= \frac{5}{9}\cdot\frac{4}{8}+\frac{4}{9}\cdot\frac{5}{8} = \frac{5}{9}.$$

S48. Let E_1, E_2, E_3 and E_4 are the events that the first, second, third and fourth card is king, respectively.

$$\therefore \quad P(E_1 \cap E_2 \cap E_3 \cap E_4) = P(E_1) \cdot P(E_2/E_1) \cdot P(E_3/E_1 \cap E_2) \cdot P[(E_4/(E_1 \cap E_2 \cap E_3 \cap E_4)]$$

$$= \frac{4}{52}\cdot\frac{3}{51}\cdot\frac{2}{50}\cdot\frac{1}{49} = \frac{1}{52\cdot 51\cdot 50\cdot 49}$$

$$= \frac{1}{13\cdot 17\cdot 25\cdot 49} = \frac{1}{270725}.$$

S49. Given that, I = {3B, 2W}, Bag II = {2B, 4W}

Let E_1 = Event that bag I is selected

E_2 = Event that bag II is selected

and E = Event that a black ball is selected

$$\Rightarrow \quad P(E_1) = 1/2, \quad P(E_2) = \frac{1}{2}, \quad P(E/E_1) = \frac{3}{5}, \quad P(E/E_2) = \frac{2}{6} = \frac{1}{3}$$

$$\therefore \quad P(E) = P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2)$$

$$= \frac{1}{2}\cdot\frac{3}{5}+\frac{1}{2}\cdot\frac{2}{6} = \frac{3}{10}+\frac{2}{12}$$

$$= \frac{18+10}{60} = \frac{28}{60} = \frac{7}{15}.$$

S50. Let R = {5 red marbles} and B = {3 black marbles}

For atleast one of the three marbles drawn be black, if the first marble is red, then the following three conditions will be followed

(i) Second ball is black and third is red (E_1).

(ii) Second ball is black and third is also black (E_2).

(iii) Second ball is red and third is black (E_3).

$\therefore \quad P(E_1) = P(R_1)\cdot P(B_1/R_1)\cdot P(R_2/R_1B_1) = \frac{5}{8}\cdot\frac{3}{7}\cdot\frac{4}{6} = \frac{60}{336} = \frac{5}{28}$

$P(E_2) = P(R_1)\cdot P(B_1/R_1)\cdot P(B_2/R_1B_1) = \frac{5}{8}\cdot\frac{3}{7}\cdot\frac{2}{6} = \frac{60}{336} = \frac{5}{56}$

and $\quad P(E_3) = P(R_1)\cdot P(R_2/R_1)\cdot P(B_1/R_1R_2) = \frac{5}{8}\cdot\frac{4}{7}\cdot\frac{3}{6} = \frac{60}{336} = \frac{5}{28}$

$\therefore \quad P(E) = P(E_1) + P(E_2) + P(E_3) = \frac{5}{28} + \frac{5}{56} + \frac{5}{28}$

$= \frac{10+5+10}{56} = \frac{25}{56}.$

S51. Given, n coins have head on both sides and $(n + 1)$ coins are fair coins.

Let $\quad E_1$ = Event that an unfair coin is selected.

E_2 = Event that a fair coin is selected.

E_3 = Event that the toss results is a head.

$\therefore \quad P(E_1) = \frac{n}{2n+1}$ and $P(E_2) = \frac{n+1}{2n+1}$

Also, $\quad P\left(\frac{E}{E_1}\right) = 1,$ and $P\left(\frac{E}{E_2}\right) = \frac{1}{2}$

$\therefore \quad P(E) = P(E_1)\cdot P\left(\frac{E}{E_1}\right) + P(E_2)\cdot P\left(\frac{E}{E_2}\right) = \frac{n}{2n+1}\cdot 1 + \frac{n+1}{2n+1}\cdot\frac{1}{2}$

$\Rightarrow \quad \frac{31}{42} = \frac{2n+n+1}{2(2n+1)} \Rightarrow \frac{31}{42} = \frac{3n+1}{4n+2}$

$\Rightarrow \quad 124n + 62 = 126n + 42$

$\Rightarrow \quad 2n = 20 \Rightarrow n = 10.$

S52. Let $\quad E_1$ = Event that item is manufactured on A,

E_2 = Event that an item is manufactured on B,

E_3 = Event that an item is manufactured on C,

Let E be the event that an item is defective.

$\therefore \quad P(E_1) = \frac{50}{100} = \frac{1}{2}$ and $P(E_2) = \frac{30}{100} = \frac{3}{10}$ and $P(E_3) = \frac{20}{100} = \frac{1}{5}$

$P\left(\frac{E}{E_1}\right) = \frac{2}{100} = \frac{1}{50},\quad P\left(\frac{E}{E_2}\right) = \frac{3}{100}$ and $P\left(\frac{E}{E_3}\right) = \frac{3}{100}$

$\therefore \quad P\left(\frac{E_1}{E}\right) = \frac{P(E_1)\cdot P\left(\frac{E}{E_1}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right) + P(E_2)\cdot P\left(\frac{E}{E_2}\right) + P(E_3)\cdot P\left(\frac{E}{E_3}\right)}$

$$= \frac{\frac{1}{2}\cdot\frac{1}{50}}{\frac{1}{2}\cdot\frac{2}{50} + \frac{3}{10}\cdot\frac{3}{100} + \frac{1}{5}\cdot\frac{3}{100}} = \frac{\frac{1}{100}}{\frac{20+9+6}{1000}} = \frac{1}{100}\times\frac{1000}{35} = \frac{2}{7}.$$

S53. Let E_1 = Event that person has TB

E_2 = Event that person does not have TB

E = Event that the person is diagnosed to have TB

$\therefore \quad P(E_1) = \frac{1}{1000} = 0.001, \quad P(E_2) = \frac{990}{1000} = 0.999$

and $P(E/E_1) = 0.99$ and $P(E/E_2) = 0.001$

$\therefore \quad P(E_1/E) = \frac{P(E_1)\cdot P(E/E_1)}{P(E_1)\cdot P(E/E_1) + P(E_2)\cdot P(E/E_2)}$

$$= \frac{0.001\times 0.99}{0.001\times 0.99 + 0.999\times 0.001}$$

$$= \frac{0.000990}{0.000990 + 0.000999}$$

$$= \frac{990}{1989} = \frac{110}{221}.$$

S54. Let U_1 = {2 white, 3 black balls}

U_2 = {3 white, 2 black balls}

and U_3 = {4 white, 1 black balls}

$\therefore \quad P(U_1) = P(U_2) = P(U_3) = \frac{1}{3}$

Let E be the event that a ball is chosen from urn U_1, E_2 be the event that a ball is chosen from urn U_2 and E_3 be the event that a ball is chosen from urn U_3.

Also, $P(E_1) = P(E_2) = P(E_3) = 1/3$

Now, Let E be the event that white ball is drawn.

$\therefore \quad P(E/E_1) = \frac{2}{5}, \quad P(E/E_2) = \frac{3}{5}, \quad P(E/E_3) = \frac{4}{5}$

Now, $P(E_2/E) = \frac{P(E_2)\cdot P(E/E_2)}{P(E_1)\cdot P(E/E_1) + P(E_2)\cdot P(E/E_2) + P(E_3)\cdot P(E/E_3)}$

$$= \frac{\frac{1}{3}\cdot\frac{3}{5}}{\frac{1}{3}\cdot\frac{2}{5}+\frac{1}{3}\cdot\frac{3}{5}+\frac{1}{3}\cdot\frac{4}{5}} = \frac{\frac{3}{15}}{\frac{2}{15}+\frac{3}{15}+\frac{4}{15}} = \frac{3}{9} = \frac{1}{3}.$$

S55. Let X denotes a random variable of number of aces.

$\therefore \quad X = 0, 1, 2$

Now, $P(X = 0) = \frac{48}{52}\cdot\frac{47}{51} = \frac{2256}{2652}$

$$P(X = 1) = \frac{48}{52}\cdot\frac{4}{51}+\frac{4}{52}\cdot\frac{48}{51} = \frac{384}{2652}$$

$$P(X = 2) = \frac{4}{52}\cdot\frac{3}{51} = \frac{12}{2652}$$

X	0	1	2
$P(X)$	$\frac{2256}{2652}$	$\frac{384}{2652}$	$\frac{12}{2652}$
$XP(X)$	0	$\frac{384}{2652}$	$\frac{24}{2652}$
$X^2P(X)$	0	$\frac{384}{2652}$	$\frac{48}{2652}$

We know that, Mean $(\mu) = E(X) = \sum XP(X)$

$$= 0 + \frac{384}{2652}+\frac{24}{2652}$$

$$= \frac{408}{2652} = \frac{2}{13}$$

Also, $\text{Var}(X) = E(X^2) - [E(X)]^2 = \sum X^2P(X) - [E(X)]^2$

$$= \left[0+\frac{384}{2652}+\frac{48}{2652}\right]-\left(\frac{2}{13}\right)^2 \qquad \left[\because\ E(X)=\frac{2}{13}\right]$$

$$= \frac{432}{2652}-\frac{4}{169} = 0.1628 - 0.0236 = 0.1391$$

$\therefore$ Standard deviation $= \sqrt{\text{Var}(X)} = \sqrt{0.1391} = 0.373$ (Approx.)

S56. Since, $X = 0, 1, 2$ and $P(X)$ at $X = 0$ and 1 is p, let at $X = 2$, $P(X)$ is x,

$\Rightarrow \quad p + p + x = 1$

$\Rightarrow \quad x = 1 - 2p$

We get, the following distribution.

X	0	1	2
$P(X)$	p	p	$1-2p$

$\therefore$

$$E(X) = \sum X P(X)$$
$$= 0 \cdot p + 1 \cdot p + 2(1-2p)$$
$$= p + 2 - 4p = 2 - 3p$$

and

$$E(X^2) = \sum X^2 P(X)$$
$$= 0 \cdot p + 1 \cdot p + 4 \cdot (1-2p)$$
$$= p + 4 - 8p = 4 - 7p$$

Also, given that $E(X^2) = E(X)$

$\Rightarrow$ $4 - 7p = 2 - 3p$

$\Rightarrow$ $4p = 2 \Rightarrow p = \frac{1}{2}$.

S57. Since, X = Number of fours seen

On tossing two die, $X = 0, 1, 2.$

Also, $P_{(4)} = \frac{1}{10}$ and $P_{(\text{not } 4)} = \frac{9}{10}$

So,

$$P(X=0) = P_{(\text{not } 4)} \cdot P_{(\text{not } 4)} = \frac{9}{10} \cdot \frac{9}{10} = \frac{81}{100}$$

$$P(X=1) = P_{(\text{not } 4)} \cdot P_{(4)} + P_{(4)} \cdot P_{(\text{not } 4)} = \frac{9}{10} \cdot \frac{1}{10} + \frac{1}{10} \cdot \frac{9}{10} = \frac{18}{100}$$

$$P(X=2) = P_{(4)} \cdot P_{(4)} = \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100}$$

Thus, we get following table

X	0	1	2
$P(X)$	$\frac{81}{100}$	$\frac{18}{100}$	$\frac{1}{100}$
$XP(X)$	0	18/100	2/100
$X^2P(X)$	0	18/100	4/100

$\therefore$

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = \sum X^2 P(X) - [\sum XP(X)]^2$$

$$= \left[0 + \frac{18}{100} + \frac{4}{100}\right] - \left[0 + \frac{18}{100} + \frac{2}{100}\right]^2$$

$$= \frac{22}{100} - \left(\frac{20}{100}\right)^2 = \frac{11}{50} - \frac{1}{25}$$

$$= \frac{11-2}{50} = \frac{9}{50} = \frac{18}{100} = 0.18.$$

S58. Given that, random variable X is the number of tails in three tosses of a coin.

So, $X = 0, 1, 2, 3.$

$\Rightarrow$ $P(X = x) = {}^nC_x (p)^x q^{n-x},$

where $n = 3, \quad p = 1/2, \quad q = 1/2 \quad \text{and} \quad x = 0, 1, 2, 3.$

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
$XP(X)$	0	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{3}{8}$
$X^2P(X)$	0	$\frac{3}{8}$	$\frac{3}{2}$	$\frac{9}{8}$

We know that, $\text{Var}(X) = E(X^2) - [E(X)]^2$ ... (i)

where, $E(X^2) = \sum_{i=1}^{n} x_i^2 P(x_i)$ and $E(X) = \sum_{i=1}^{n} x_i P(x_i)$

$\therefore$ $E(X^2) = \sum_{i=1}^{n} x_i^2 P(X_i) = 0 + \frac{3}{8} + \frac{3}{2} + \frac{9}{8} = \frac{24}{8} = 3$

and $[E(X)]^2 = \left[\sum_{i=1}^{n} x_i P(X_i)\right]^2 = \left[0 + \frac{3}{8} + \frac{3}{4} + \frac{3}{8}\right]^2 = \left[\frac{12}{8}\right]^2 = \frac{9}{4}$

$\therefore$ $\text{Var}(X) = 3 - \frac{9}{4} = \frac{3}{4}$ [Using Eq. (i)]

and standard deviation of $X = \sqrt{\text{Var}(X)} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}.$

S59. We have,

X	0.5	1	1.5	2
$P(X)$	k	k^2	$2k^2$	k

(i) We know that, $\sum_{i=1}^{n} P_i = 1$, where $P_i \geq 0$

$\Rightarrow$ $P_1 + P_2 + P_3 + P_4 = 1$

$\Rightarrow$ $k + k^2 + 2k^2 + k = 1$

$\Rightarrow \quad 3k^2 + 2k - 1 = 0$

$\Rightarrow \quad 3k^2 + 3k - k - 1 = 0$

$\Rightarrow \quad 3k(k + 1) - 1(k + 1) = 0$

$\Rightarrow \quad (3k - 1)(k + 1) = 0$

$\Rightarrow \quad k = 1/3 \quad \Rightarrow \quad k = -1$

Since, $\quad k$ is $\geq 0 \quad \Rightarrow \quad k = 1/3$

(ii) Mean of the distribution $(\mu) = E(X) = \sum_{i=1}^{n} x_i P_i$

$$= 0.5(k) + 1(k^2) + 1.5(2k^2) + 2(k) = 4k^2 + 2.5k$$

$$= 4 \cdot \frac{1}{9} + 2.5 \cdot \frac{1}{3} \qquad \left[\because k = \frac{1}{3}\right]$$

$$= \frac{4 + 7.5}{9} = \frac{23}{18}.$$

S60. Let X is the random variable which denotes that a bulb is defective.

Also, $n = 10$, $p = \frac{1}{50}$ and $q = \frac{49}{50}$ and $P(X = r) = {}^nC_r p^r q^{n-r}$

(i) None of the bulbs is defective *i.e.*, $r = 0$

$$\therefore \quad P(X = r) = P_{(0)} = {}^{10}C_0 \left(\frac{1}{50}\right)^0 \left(\frac{49}{50}\right)^{10-0} = \left(\frac{49}{50}\right)^{10}$$

(ii) Exactly two bulbs are defective *i.e.*, $r = 2$

$$\therefore \quad P(X = r) = P_{(2)} = {}^{10}C_2 \left(\frac{1}{50}\right)^2 \left(\frac{49}{50}\right)^8$$

$$= \frac{10!}{8!2!}\left(\frac{1}{50}\right)^2 \cdot \left(\frac{49}{50}\right)^8 = 45 \times \left(\frac{1}{50}\right)^{10} \times (49)^8$$

(iii) More than 8 bulbs work properly *i.e.*, there is less that 2 bulbs which are defective.

So, $\quad r < 2 \quad \Rightarrow \quad r = 0, 1$

$$\therefore \quad P(X = r) = P(r < 2) = P(0) + P(1)$$

$$= {}^{10}C_0 \left(\frac{1}{50}\right)^0 \left(\frac{49}{50}\right)^{10-0} + {}^{10}C_1 \left(\frac{1}{50}\right)^1 \left(\frac{49}{50}\right)^{10-1}$$

$$= \left(\frac{49}{50}\right)^{10} + \frac{10!}{1!9!} \cdot \frac{1}{50} \cdot \left(\frac{49}{50}\right)^9$$

$$= \left(\frac{49}{50}\right)^{10} + \frac{1}{5}\cdot\left(\frac{49}{50}\right)^{9} = \left(\frac{49}{50}\right)^{9}\left(\frac{49}{50}+\frac{1}{5}\right)$$

$$= \left(\frac{49}{50}\right)^{9}\left(\frac{59}{50}\right) = \frac{59(49)^9}{50^{10}}.$$

S61. Since 4 balls have to be drawn, therefore, X can take the values 0, 1, 2, 3, 4.

$$P(X = 0) = P \text{ (no red ball)} = P\text{(4 white balls)} = \frac{{}^4C_4}{{}^{12}C_4} = \frac{1}{495}$$

$$P(X = 1) = P \text{ (1 red ball and 3 white balls)} = \frac{{}^8C_1 \times {}^4C_3}{{}^{12}C_4} = \frac{32}{495}$$

$$P(X = 2) = P \text{ (2 red balls and 2 white balls)} = \frac{{}^8C_2 \times {}^4C_2}{{}^{12}C_4} = \frac{168}{495}$$

$$P(X = 3) = P \text{ (3 red balls and 1 white ball)} = \frac{{}^8C_3 \times {}^4C_1}{{}^{12}C_4} = \frac{224}{495}$$

$$P(X = 4) = P \text{ (4 red balls)} = \frac{{}^8C_4}{{}^{12}C_4} = \frac{70}{495}$$

Thus, the following is the required probability distribution of X.

S62. Let E be the event that the car is of standard quality. Let B_1 and B_2 be the events that the car is manufactured in plants X and Y, respectively. Now,

$$P(B_1) = \frac{70}{100} = \frac{7}{10}, \quad P(B_2) = \frac{30}{100} = \frac{3}{10}$$

$P(E \mid B_1)$ = Probability that a standard quality car is manufactured in plant X

$$= \frac{80}{100} = \frac{8}{10}$$

$$P(E \mid B_2) = \frac{90}{100} = \frac{9}{10}$$

$P(B_1 \mid E)$ = Probability that a standard quality car has come from plant X

$$= \frac{P(B_1) \times P(E|B_1)}{P(B_1)\cdot P(E|B_1) + P(B_2)\cdot P(E|B_2)}$$

$$= \frac{\frac{7}{10}\times\frac{8}{10}}{\frac{7}{10}\times\frac{8}{10}+\frac{3}{10}\times\frac{9}{10}} = \frac{56}{83}$$

Hence, the required probability is $\frac{56}{83}$.

S63. Let X denote the number of heads tossed. So, X can take the values 0, 1, 2, 3. When a coin is tossed three times, we get

Sample space S = {HHH, HHT, HTH, THH, THT, TTH, TTT, HTT}

$$P(X = 0) = P\text{ (no head)} = P\text{(TTT)} = \frac{1}{8}$$

$$P(X = 1) = P\text{ (one head)} = P\text{(HTT, THT, TTH)} = \frac{3}{8}$$

$$P(X = 2) = P\text{ (two heads)} = P\text{(HHT, HTH, THH)} = \frac{3}{8}$$

$$P(X = 3) = P\text{ (three heads)} = P\text{(HHH)} = \frac{1}{8}$$

Thus the probability distribution of X is:

X	**0**	**1**	**2**	**3**
***P*(*X*)**	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$$\text{Variance of } X = \sigma^2 = \sum x_i^2 p_i - \mu^2, \quad \text{... (i)}$$

where μ is the mean of X given by

$$\mu = \sum x_i p_i = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$$

$$= \frac{3}{2} \quad \text{... (ii)}$$

Now, $$\sum x_i^2 p_i = 0^2 \times \frac{1}{8} + 1^2 \times \frac{3}{8} + 2^2 \times \frac{3}{8} + 3^2 \times \frac{1}{8} = 3 \quad \text{... (iii)}$$

From Eq. (i), (ii) and (iii), we get

$$\sigma^2 = 3 - \left(\frac{3}{2}\right)^2 = \frac{3}{4}$$

$$\text{Standard deviation} = \sqrt{\sigma^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}.$$