Date: 19/10/2021

Q1. Minimise $z = 3x + 5y$
subject to $x + 3y \geq 3$, $x + y \geq 2$, $x, y \geq 0$.

Q2. Solve the following linear programming problem graphically :
Minimise $z = x - 5y + 20$ subject to $x - y \geq 0$, $-x + 2y \geq 2$, $x \geq 3$, $y \leq 4$, $x, y \geq 0$.

Q3. Solve the following linear programming problem graphically:
Maximise $z = -x + 2y$ subject to $-x + 3y \leq 10$, $x + y \leq 6$, $x - y \leq 2$, $x, y \geq 0$.

Q4. Solve the following linear programming problem graphically:
Maximise $z = 6x + 5y$ subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x, y \geq 0$.

Q5. Minimise $z = x + 2y$
subject to $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$.
show that the minimum of z occurs at more than one points.

Q6. Minimise $z = -3x + 4y$
subject to $x + 2y \leq 8$, $3x + 2y \leq 12$, $x \geq 0$, $y \geq 0$.

Q7. Maximize $z = 5x + 3y$
subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

Q8. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two food F_1 and F_2 are available. Food F_1 costs Rs. 4 per unit and F_2 costs Rs. 6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem and find graphically the minimum cost for diet that consists of mixture of these two foods and also meets the minerals nutritional requirements.

Q9. A dietician mix together two kinds of foods in such a way that the mixture contains at least 6 units of vitamin A, 7 units of vitamin B, 11 units of vitamin C and 9 units of vitamin D. The vitamin contents of 1 kg of food X and food Y are given below :

	Vit. A	*Vit. B*	*Vit. C*	*Vit. D*
Food *X*	1	1	1	2
Food *Y*	2	1	3	1

One kg. food X casts Rs. 5 where as one kg. of food Y casts Rs. 8. Find the least cost of the mixture which will produce the diet.

Q10. Minimise and maximise $z = x + 2y$
subject to $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x, y \geq 0$.

Q11. Maximise $z = x + y$
subject to $x - y \leq -1$, $-x + y \leq 0$, $x, y \geq 0$.

Q12. Minimise $z = 5x + 10y$
subject to $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x, y \geq 0$.

Q13. Maximize $z = 3x + 2y$
subject to $x + 2y \leq 10$, $3x + y \leq 15$, $x, y \geq 0$.

Q14. A dietician wants to mix two types of foods in such way that the vitamin contents of the mixture contains at least 8 units of vitamin *A* and 10 units of vitamin *C*. Food I contains 2 units per kg of vitamin *A* and 1 unit per kg of vitamin *C*. Food II contains 1 unit per kg of vitamin *A* and 2 units per kg of vitamin *C*. It costs Rs. 50 per kg to purchase food I and Rs. 70 per kg to purchase food II. Formulate the problem as a linear programming problem to minimise the cost of such mixture and find the minimise cost graphically.

Q15. A diet for a sick person must contains at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods *A* and *B* are available at a cost of Rs. 5 and Rs. 4 per unit, respectively. One unit of the food *A* contains 200units of vitamins, 1 unit of minerals and 40 units of calories, while one unit of the food *B* contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the foods *A* and *B* should be use to have least cost but it must satisfy the requirements of the sick person. Form the question as LPP and solve it graphically.

Q16. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of grinding/cutting machine and a sprayer. It takes 2h on the grinding/cutting machine and 3h on the sprayer to manufacture a pedestal lamp. It takes 1h on the grinding/cutting machine and 2h on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20h and the grinding/cutting machine for at the most 12h. The profit from the sale of a lamp is Rs. 5 and that from a shade is Rs. 3. Assuming that the manufacturer can sale all the lamps and shades that he produces, how should he schedule his daily production in order to maximize his profit? Make an LPP and solve it graphically.

Q17. A factory owner purchases two types of machines *A* and *B* for his factory. The requirements and the limitations for the machines are as follows

Machine	Area occupied	Labour force	Daily Output (in units)
A	1000 m^2	12 men	60
B	1200 m^2	8 men	40

He has maximum area of 9000 m^2 available and 72 skilled laboures who can operate both the machines. How many machines of each type should be brought to maximize the daily output?

Q18. A man has Rs. 1500 for purchasing wheat and rice. A bag of rice and a bag of wheat cost Rs. 180 and Rs. 120, respectively. He has a storage capacity of only 10 bags. He earns a profit of Rs. 11 and Rs. 9 per bag of rice and wheat, respectively. Formulate the problem as an LPP to find the number of bags of each type he should buy for getting maximum profit and solve it graphically.

Q19. Every gram of wheat provides 0.1 g of proteins and 0.25 g of carbohydrates. The corresponding values for rice are 0.05 g and 0.5 g respectively. Wheat costs Rs. 4 per kg and rice Rs. 6 per kg. The minimum daily requirements of proteins and carbohydrates for an average child are 50 grams and 200 grams respectively. In what quantities should wheat and rice be mixed in the daily diet to provide minimum daily requirements of proteins and carbohydrates at minimum cost. Frame an L.P.P. and solve it graphically.

Q20. One kind of cake requires 300 g of flour and 15 g of fat, another kind of cake requires 150 g of flour and 30 g of fat. Find the maximum number of cakes which can be made from 7.5 kg of flour and 600 g of fat, assuming that there is no shortage of other ingredients used in making the cakes. Make it as an LPP and solve it graphically.

Q21. One kind of cake requires 200 g of flour and 25g of fat and another kind of cake requires 100g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat, assuming that there is no shortage of other ingredients used in making the cakes. Formulate the above as a linear programming problem and solve it graphically.

Q22. A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food is given below :

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

One kg of food X costs Rs 16 and one kg of food Y costs Rs 20. Find the least cost of the mixture which will produce the required diet ?

Q23. A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to maximise the amount of vitamin A in the diet ? What is the maximum amount of vitamin A in the diet ?

Q24. Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs. 60/kg and Food Q costs Rs. 80/kg. Food P contains 3 units/kg of vitamin A and 5 units/kg of vitamin B while food Q contains 4 units/kg of vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture.

Q25. A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs. 7 and screws B at a profit of Rs. 10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximise his profit ?" Determine the maximum profit.

Q26. A manufacture produces two types of steel trunks. He has two machines A and B. For completing, the first type of the trunk requires 3 hours on machine A and 2 hours on machine B, whereas the second type of the trunk requires 3 hours on machine A and 3 hours on machine B. Machines A and B can work at the most for 18 hours and 14 hours per day respectively. He earns a profit of Rs. 30 and Rs. 40 per trunk of the first type and the second type respectively. How many trunks of the each type must he make each day to make maximum profit ?

Q27. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type *A* require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type *B* require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is Rs. 5 each for type *A* and Rs. 6 each for type *B* souvenirs. How many souvenirs of each type should the company manufacture in order to maximise the profit?

Q28. A manufacturer considers that men and women workers are equally efficient and so he pays them at the same rate. He has 30 workers (male and female) and 17 units of capital, which he uses to produce two types of goods *A* and *B*. To produce one unit of *A*, 2 workers and 3 units of capital are required while 3 workers and 1 unit of capital is required to produce one unit of *B*. If *A* and *B* are priced at Rs. 100 and Rs. 120 per unit respectively, how many units of goods A and B be produced to maximize the total revenue?
Form the above as an LPP and solve graphically.

Do you agree with this view of the manufacturer that men and women worker are equally efficient and so should be paid at the same rate?

Q29. A manufacturer produces nuts and bolts. It takes 1h of work on machine *A* and 3h on machine *B* to produce package of nuts. It takes 3h on machine *A* and 1h on machine *B* to produce a package of bolts. He earns a profit of Rs. 17.50 per package on nuts and Rs. 7 per package on bolts. How many packages of each should be produced each day so as to maximize his profits, if he operates his machines for almost 12h a day. Formulate above as a Linear Programming Problem (LPP) and solve it graphically.

Q30. An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 1000 is made on each executive class ticket and a profit of Rs. 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit?

Q31. A cooperative society of farmers has 50 hec. of land to grow two crops *A* and *B*. The profits from crops *A* and *B* per hectare are estimated as Rs. 10,500 and Rs. 9,000, respectively. To control weeds, a liquid herbicide has to be used for crops *A* and *B* at the rate of 20 L/hec and 10 L/hec, respectively. Further not more than 800 L of herbicide should be used in order to protect fish and wildlife using a pond which collects drainage from this land. Keeping in mind that the protection of fish and other wildlife is more important than earning profit, how much land should be allocated to each crop so as to maximize the total profit? Form an LPP from the above and solve it graphically. Do you agree with the message that the protection of wildlife is atmost necessary to preserve the balance in environment?

Q32. A library has to accommodate two different types of books on a shelf. The books are 6 cm and 4 cm thick and weight 1 kg and $1\frac{1}{2}$ kg each, respectively. The shelf is 96 cm long and atmost can support a weight of 21 kg. How should the shelf be filled with the books of two types in order to include the greatest number of books? Make it as an LPP and solve it graphically.

Q33. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space for atmost 20 items. A fan costs Rs. 360 and a sewing machine costs Rs. 240. He can sell a fan at a profit of Rs. 22 and a sewing machine at a profit of Rs. 18. Assuming that he can sell all the items that he can buy how should he invest his money in order to maximize his profit? Formulate the problem as and LPP and solve it graphically.

Q34. Two godowns *A* and *B* have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, *D*, *E* and *F* whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table:

To / From	Transportation cost per packet	
	A	*B*
D	6	4
E	3	2
F	2.50	3

How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost?

Q35. A medicine company has factories at two places *A* and *B*. From these places, supply is made to each of its three agencies situated at *P*, *Q* and *R*. The monthly requirement of the agencies are respectively 40, 40 and 50 packets of the medicines, while the production capacity of the factories at *A* and *B* are 60 and 70 packets respectively. The transportation cost per packet from the factories to the agencies are given below.

From / To	Transportation cost per packet	
	A	*B*
P	5	4
Q	4	2
R	3	5

How many packets from each factory to transported to each agency so that the cost of transportation is minimum? Also find the minimum cost.

Q36. A retired person has Rs. 70,000 to invest and two types of bonds are available in the market for investment. First type of bond yields an annual income of 8% on the amount invested and the second type of bond yields 10% annum. As per norms, he has to invest a minimum of Rs. 10,000 in the first type and not more than Rs. 30,000 in the second type. How should he plan his investment, so as to get maximum return, after one year of investment?

Q37. An oil company has two depots *A* and *B* with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps, *D*, *E* and *F* whose requirements are 4500 L, 3000 L and 3500 L respectively. The distances (in km) between the depots and the petrol pumps is given in the following table:

To / From	Transportation cost per packet	
	A	*B*
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost of 10 litres of oil is Rs. 1 per km, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost?

Date: 19/10/2021

S1. The given objective function is $z = 3x + 5y$

Consider the linear constraint defined by the inequality

$$x + 3y \geq 3$$

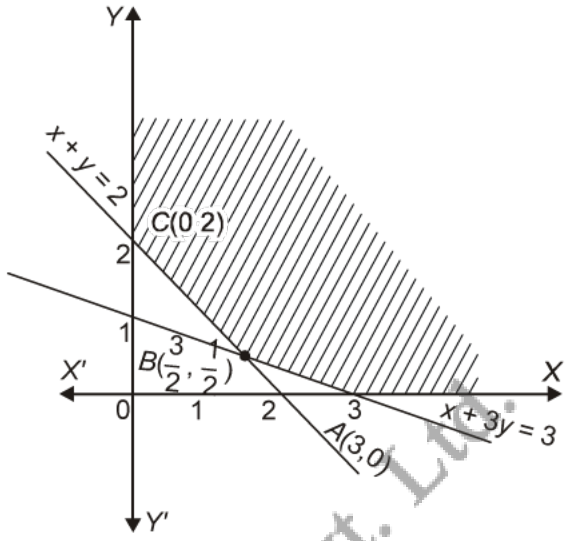
First draw the graph of the line $x + 3y = 3$

x	0	3
y	1	0

Putting (0, 0) in the inequality $x + 3y \geq 3$, we have

$$0 + 3 \times 0 \geq 3 \quad \Rightarrow \quad 0 \geq 3, \text{ which is false}$$

So the half plane of $x + 3y \geq 3$ is away from origin.



Now consider the linear constraint defined by the inequality

$$x + y \geq 2$$

First draw the graph of the line $x + y = 2$

x	0	2
y	2	0

Putting (0, 0) in the inequality $x + y \geq 2$, we have

$$0 + 0 \geq 2 \quad \Rightarrow \quad 0 \geq 2, \text{ which is false}$$

So the half plane of $x + y \geq 2$ is away from origin.

Since $\qquad x, y \geq 0$

So the feasible region lies in the first quadrant.

The coordinates of the corner points of the feasible region are $A(3, 0)$, $B\left(\frac{3}{2}, \frac{1}{2}\right)$, $C(0, 2)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $\qquad z = 3x + 5y$

At $A(3, 0)$ $\qquad z = 3 \times 3 + 5 \times 0 = 9 + 0 = 9$

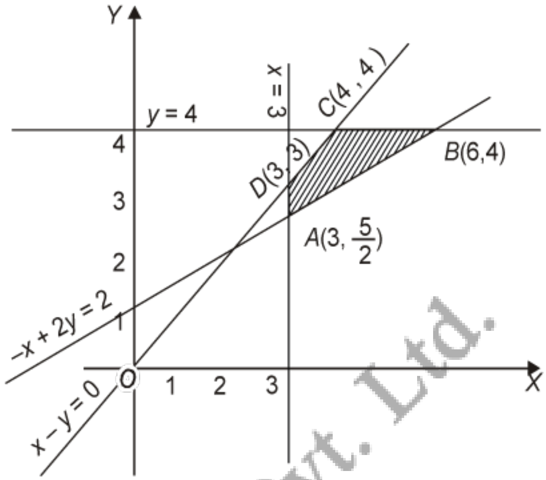
At $B\left(\frac{3}{2}, \frac{1}{2}\right)$ $\qquad z = 3 \times \frac{3}{2} + 5 \times \frac{1}{2} = \frac{9}{2} + \frac{5}{2} = \frac{14}{2} = 7$

At $C(0, 2)$ $z = 3 \times 0 + 5 \times 2 = 0 + 10 = 10$

Thus z is minimum at $\left(\frac{3}{2}, \frac{1}{2}\right)$ and minimum value = 7.

The feasible region is unbounded. So we plot the region $3x + 5y < 7$, which does not have any common part with the feasible region.

S2. The given objective function is $z = x - 5y + 20$



$$x - y \geq 0$$

First draw the graph of the line $x - y = 0$

x	0	1
y	0	1

Putting (3, 0) in the inequality $x - y \geq 0$, we have

$$3 - 0 \geq 0 \quad \Rightarrow \quad 3 \geq 0, \text{ which is true}$$

So the half plane of $x - y \geq 0$ is towards the point (3, 0).

Now consider the linear constraint defined by the inequality

$$-x + 2y \geq 2$$

First draw the graph of the line $-x + 2y = 2$

x	0	2
y	1	2

Putting (0, 0) in the inequality $-x + 2y \geq 2$.

$$-0 + 2 \times 0 \geq 2 \quad \Rightarrow \quad 0 \geq 2, \text{ which is false}$$

So the half plane of $-x + 2y \geq 2$ is away from origin.

Now consider the linear constraint defined by the inequality

$$x \geq 3$$

First draw the graph of the line $x = 3$

Putting (0, 0) in the inequality $x \geq 3$, we have

$$0 \geq 3, \text{ which is false}$$

So the half plane of $x \geq 3$ is away from origin.

Now consider the linear constraint defined by the inequality

$$y \leq 4$$

First draw the graph of the line $y = 4$

Putting (0, 0) in the inequality $y \leq 4$, we have

$0 \le 4$, which is true

So the half plane of $y \le 4$ is towards origin.

Since $\quad x, y \ge 0$

So the feasible region lies in the first quadrant.

The coordinates of the corner points of the feasible region are $A\left(3, \frac{5}{2}\right)$, $B(6, 4)$, $C(4, 4)$ and $D(3, 3)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $\quad z = x - 5y + 20$

At $A\left(3, \frac{5}{2}\right)$ $\quad z = 3 - 5 \times \frac{5}{2} + 20 = \frac{6 - 25 + 40}{2} = \frac{21}{2}$

At $B(6, 4)$ $\quad z = 6 - 5 \times 4 + 20 = 6 - 20 + 20 = 6$

At $C(4, 4)$ $\quad z = 4 - 5 \times 4 + 20 = 4 - 20 + 20 = 4$

At $D(3, 3)$ $\quad z = 3 - 5 \times 3 + 20 = 3 - 15 + 20 = 8$

Thus z is minimum at (4, 4) and minimum value = 4.

S3. The given objective function is $z = -x + 2y$

Consider the linear constraint defined by the inequality

$$-x + 3y \le 10$$

First draw the graph of the line $-x + 3y = 10$

x	–1	2
y	3	4

Putting (0, 0) in the inequality $-x + 3y \le 10$, we have

$$-0 + 3 \times 0 \le 10 \quad \Rightarrow \quad 0 \le 10, \text{ which is true}$$

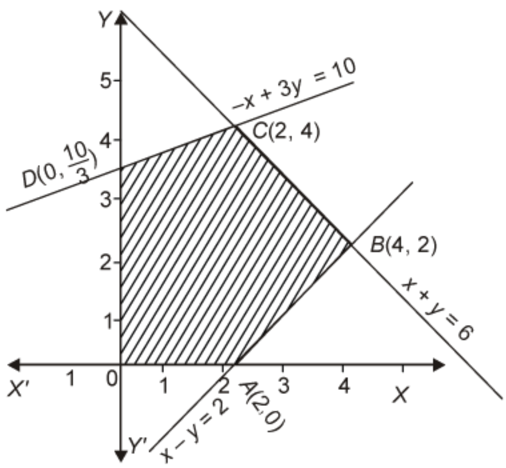
So the half plane of $-x + 3y \le 10$ is towards the origin.

Now consider the linear constraint defined by the inequality

$$x + y \le 6$$

First draw the graph of the line $x + y = 6$

x	1	2
y	5	4



Putting (0, 0) in the inequality $x + y \le 6$, we have

$$0 + 0 \le 6 \quad \Rightarrow \quad 0 \le 6, \text{ which is true}$$

So the half plane of $x + y \le 6$ is towards the origin.

Now consider the linear constraint defined by the inequality

$$x - y \le 2$$

First draw the graph of the line $x - y = 2$

x	3	4
y	1	2

Putting (0, 0) in the inequality $x - y \le 2$, we have

$$0 - 0 \le 2 \quad \Rightarrow \quad 0 \le 2, \text{ which is true}$$

So the half plane of $x - y \le 2$ is towards the origin.

Since $\quad x, y \ge 0$

So the feasible region lies in the first quadrant.

The coordinates of the corner points of the feasible region are $O(0, 0)$, $A(2, 0)$, $B(4, 2)$, $C(2, 4)$ and $D\left(0, \frac{10}{3}\right)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $\quad z = -x + 2y$

At $O(0, 0)$ $\quad z = -0 + 2 \times 0 = 0$

At $A(2, 0)$ $\quad z = -2 + 2 \times 0 = -2 + 0 = -2$

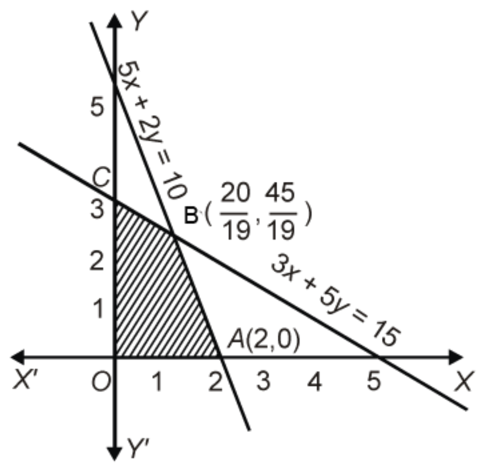
At $B(4, 2)$ $\quad z = -4 + 2 \times 2 = -4 + 4 = 0$

At $C(2, 4)$ $\quad z = -2 + 2 \times 4 = -2 + 8 = 6$

At $D\left(0, \frac{10}{3}\right)$ $\quad z = -0 + 2 \times \frac{10}{3} = \frac{20}{3}$

Thus z is maximum at $\left(0, \frac{10}{3}\right)$ and maximum value $= \frac{20}{3}$.

S4. The given objective function is $z = 6x + 5y$



Consider the linear constraint defined by the inequality

$$3x + 5y \le 15$$

First draw the graph of the line $3x + 5y = 15$

x	0	5
y	3	0

Putting (0, 0) in the inequality $3x + 5y \le 15$, we have

$$3 \times 0 + 5 \times 0 \le 15 \quad \Rightarrow \quad 0 \le 15, \text{ which is true}$$

So the half plane of $3x + 5y \le 15$ is towards the origin.

Now consider the linear constraint defined by the inequality

$$5x + 2y \le 10$$

First draw the graph of the line $5x + 2y = 10$

x	0	2
y	5	0

Putting (0, 0) in the inequality $5x + 2y \le 10$, we have

$$5 \times 0 + 2 \times 0 \le 10 \quad \Rightarrow \quad 0 \le 10, \text{ which is true}$$

So the half plane of $5x + 2y \le 10$ is towards the origin.

Since $\qquad x, y \ge 0$

So the feasible region lies in first quadrant.

The coordinates of the corner points of the feasible region are $O(0, 0)$

$A(2, 0)$, $B\left(\frac{20}{19}, \frac{45}{19}\right)$ and $C(0, 3)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $\qquad z = 6x + 5y$

At $O(0, 0)$ $\qquad z = 6 \times 0 + 5 \times 0 = 0$

At $A(2, 0)$ $\qquad z = 6 \times 2 + 5 \times 0 = 12 + 0 = 12$

At $B\left(\frac{20}{19}, \frac{45}{19}\right)$ $\qquad z = 6 \times \frac{20}{19} + 5 \times \frac{45}{19} = \frac{120}{19} + \frac{225}{19} = \frac{345}{19}$

At $C(0, 3)$ $\qquad z = 6 \times 0 + 5 \times 3 = 0 + 15 = 15$

Thus z is maximum at $\left(\frac{20}{19}, \frac{45}{19}\right)$ and maximum value $= \frac{345}{19}$.

S5. The given objective function is $z = x + 2y$

Consider the linear constraint defined by the inequality

$$2x + y \ge 3$$

First draw the graph of the line $2x + y = 3$

x	0	1
y	3	1

Putting (0, 0) in the inequality $2x + y \geq 3$, we have

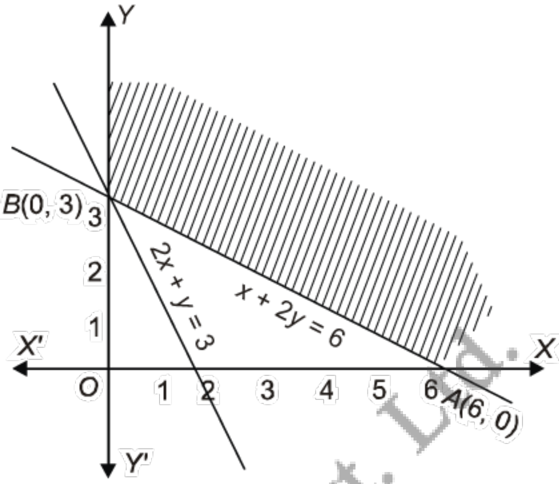
$$2 \times 0 + 0 \geq 3 \quad \Rightarrow \quad 0 \geq 3, \text{ which is false}$$

So the half plane of $2x + y \geq 3$ is away from origin.

Now consider the linear constraint defined by the inequality

$$x + 2y \geq 6$$

First draw the graph of the line $x + 2y = 6$



x	0	2
y	3	2

Putting (0, 0) in the inequality $x + 2y \geq 6$, we have

$$0 + 2 \times 0 \geq 6 \quad \Rightarrow \quad 0 \geq 6, \text{ which is false}$$

So the half plane of $x + 2y \geq 6$ is away from origin.

Since $\quad x, y \geq 0$

So the feasible region lies in the first quadrant.

The coordinate of the corner points of the feasible region are $A(6, 0)$ and $B(0, 3)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $\quad z = x + 2y$

At $A(6, 0)$ $\quad z = 6 + 2 \times 0 = 6 + 0 = 6$

At $B(0, 3)$ $\quad z = 0 + 2 \times 3 = 0 + 6 = 6$

Thus z is minimum at (6, 0) and (0, 3) and minimum value = 6.

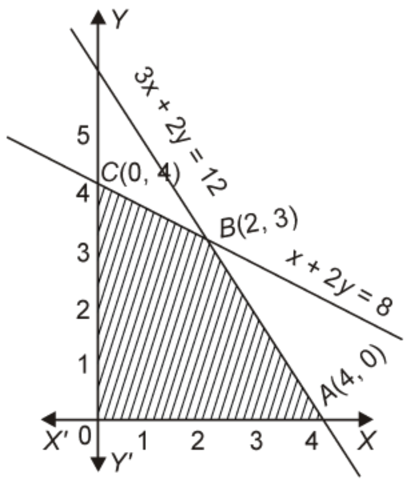
The feasible region is unbounded. So we plot the region $x + 2y < 6$, which does not have any common part with the feasible region.

S6. The given objective function is $z = -3x + 4y$

Consider the linear constraint defined by the inequality

$$x + 2y \leq 8$$

First draw the graph of the line $x + 2y = 8$



x	0	2
y	4	3

Putting (0, 0) in the inequality $x + 2y \leq 8$, we have

$$0 + 2 \times 0 \leq 8 \quad \Rightarrow \quad 0 \leq 8, \text{ which is true}$$

So the half plane of $x + 2y \leq 8$ is towards the origin.

Now consider the linear constraint defined by the inequality

$$3x + 2y \le 12$$

First draw the graph of the line $3x + 2y = 12$

x	0	4
y	6	0

Putting (0, 0) in the inequality $3x + 2y \le 12$, we have

$$3 \times 0 + 2 \times 0 \le 12 \quad \Rightarrow \quad 0 \le 12, \text{ which is true}$$

So the half plane of $3x + 2y \le 12$ is towards the origin.

Since $\quad x, y \ge 0$

So the feasible region lies in the first quadrant.

The coordinates of the corner points of the feasible region are $O(0, 0)$, $A(4, 0)$, $B(2, 3)$ and $C(0, 4)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $\quad z = -3x + 4y$

At $O(0, 0)$ $\quad z = -3 \times 0 + 4 \times 0 = 0$

At $A(4, 0)$ $\quad z = -3 \times 4 + 4 \times 0 = -12 + 0 = -12$

At $B(2, 3)$ $\quad z = -3 \times 2 + 4 \times 3 = -6 + 12 = 6$

At $C(0, 4)$ $\quad z = -3 \times 0 + 4 \times 4 = 0 + 16 = 16$

Thus z is minimum at (4, 0) and minimum value = – 12.

S7. The given objective function is $z = 5x + 3y$

Consider the linear constraint defined by the inequality

$$3x + 5y \le 15$$

First draw the graph of the line $3x + 5y = 15$

x	0	5
y	3	0

Putting (0, 0) in the inequality $3x + 5y \le 15$, we have

$$3 \times 0 + 5 \times 0 \le 15 \quad \Rightarrow \quad 0 \le 15, \text{ which is true}$$

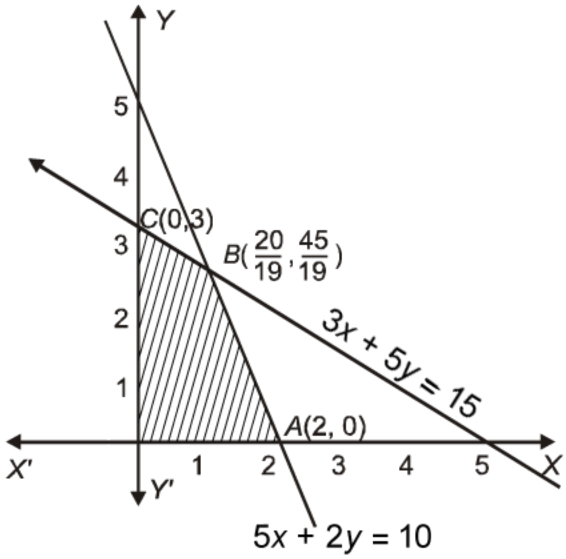
So the half plane of $3x + 5y \le 15$ is towards the origin.

Now consider the linear constraint defined by the inequality

$5x + 2y \le 10$

First draw the graph of the line $5x + 2y = 10$

x	0	2
y	5	0



Putting (0, 0) in the inequality $5x + 2y \le 10$, we have

$5 \times 0 + 2 \times 0 \le 10 \quad \Rightarrow \quad 0 \le 10$, which is true

So the half plane of $5x + 2y \le 10$ is towards the origin.

Since $\quad x,\ y \ge 0$

So the feasible region lies in the first quadrant.

The coordinate of the corner points of the feasible region are $O(0, 0)$, $A(2, 0)$ $B\left(\frac{20}{19}, \frac{45}{19}\right)$ and $C(0, 3)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $\quad z = 5x + 3y$

At $O(0, 0)$ $\quad z = 5 \times 0 + 3 \times 0 = 0$

At $A(2, 0)$ $\quad z = 5 \times 2 + 3 \times 0 = 10 + 0 = 10$

At $B\left(\frac{20}{19}, \frac{45}{19}\right)$ $\quad z = 5 \times \frac{20}{19} + 3 \times \frac{45}{19} = \frac{100}{19} + \frac{135}{19} = \frac{235}{19}$

At $C(0, 3)$ $\quad z = 5 \times 0 + 3 \times 3 = 0 + 9 = 9$

Thus z is maximum at $\left(\frac{20}{19}, \frac{45}{19}\right)$ and maximum value = $\frac{235}{19}$.

S8. Let the diet contains x units of food F_1 and y units of food F_2. The given data can be put in tabular which is as follows

Food type	**Vitamin *A***	**Minerals**	**Cost**
$F_1(x)$	3	4	Rs. 4
$F_2(y)$	6	3	Rs. 6
Total	≥ 80	≥ 100	

Required LPP is

Minimize cost, $Z = 4x + 6y$

Subject to the constraints, $\quad 3x + 6y \ge 80 \quad$ ... (i)

$$4x + 3y \geq 100 \quad \ldots \text{(ii)}$$

$$x, y \geq 0$$

Table for line $3x + 6y = 80$ is

x	0	$\frac{80}{3}$
y	$\frac{40}{3}$	0

$\therefore$ It passes through the points $\left(0, \frac{40}{3}\right)$ and $\left(\frac{80}{3}, 0\right)$.

Table for line $4x + 3y = 100$ is

x	0	25
y	$\frac{100}{3}$	0

$\therefore$ It passes through the points $\left(0, \frac{100}{3}\right)$ and (25, 0).

To find intersection point

Multiplying Eq. (ii) by 2 and subtracting Eq. (ii) from Eq. (i), we get

$$\begin{array}{rl} 3x + 6y = & 80 \\ \underset{-}{}8x \underset{-}{+} 6y = & \underset{-}{}200 \\ \hline -5x \quad\quad = & -120 \end{array}$$

$\Rightarrow$ $x = 24$

Putting $x = 24$ in Eq. (i), we get

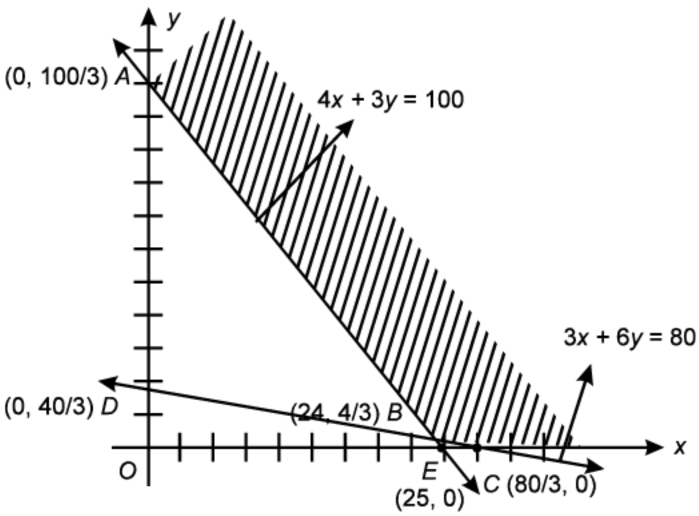
$$3 \times (24) + 6y = 80$$

$\Rightarrow$ $6y = 80 - 72 = 8$

$\Rightarrow$ $y = \frac{8}{6} = \frac{4}{3}$

$\therefore$ The point of intersection is $B\left(24, \frac{4}{3}\right)$.

Now, we draw the graph and find the feasible region.



$\therefore$ The corner points of feasible region are $A\left(0, \frac{100}{3}\right)$, $B\left(24, \frac{4}{3}\right)$ and $C\left(\frac{80}{3}, 0\right)$.

Now, evaluate Z at various corner points.

Corner points	$Z = 4x + 6y$
$A\left(0, \frac{100}{3}\right)$	$0 + 6 \times \frac{100}{3} = 200$
$B\left(24, \frac{4}{3}\right)$	$4 \times 24 + 6 \times \frac{4}{3} = 104$ (minimum)
$C\left(\frac{80}{3}, 0\right)$	$4 \times \frac{80}{3} + 0 = \frac{320}{3} = 106.6$

$\therefore$ The minimum cost is Rs. 104 when $x = 24$ units and $y = \frac{4}{3}$ units

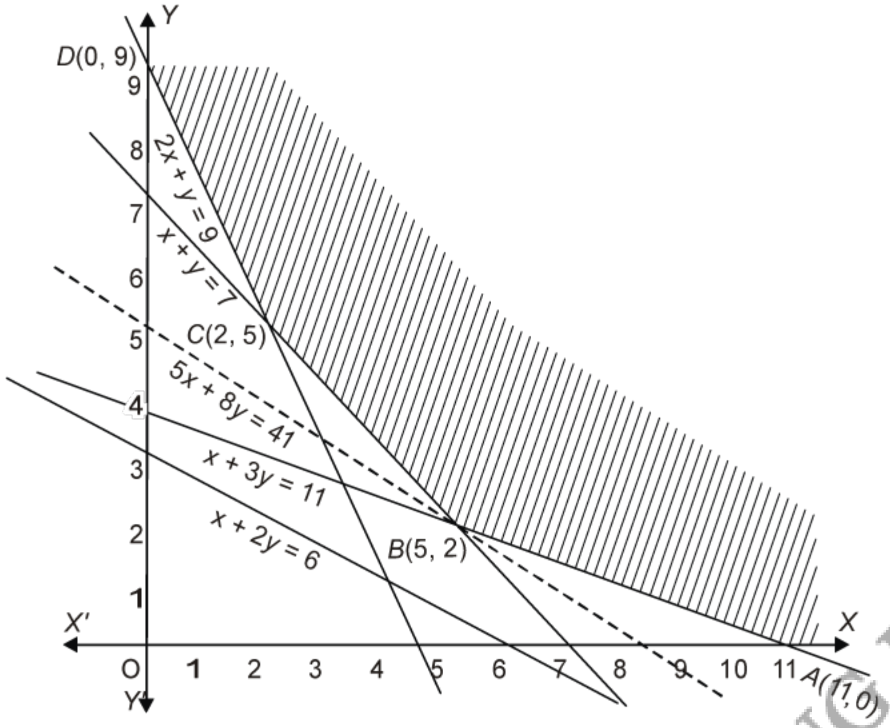
S9. Let the dietician mix x kg of food X and y kg of food Y to make the mixture. Let Z be the total cost of mixture.

	Food X	***Food Y***	***Minimum requirement***
Vitamin A	1	2	6
Vitamin B	1	1	7
Vitamin C	1	3	11
Vitamin D	2	1	9
Cost	Rs. 5	Rs. 8	

Thus the mathematical formulation of the given L.P.P. is as

Minimise $z = 5x + 8y$

subject to $x + 2y \geq 6$, $x + y \geq 7$, $x + 3y \geq 11$, $2x + y \geq 9$, $x, y \geq 0$.



Consider the linear constraint defined by the inequality

$$x + 2y \geq 6$$

First draw the graph of the line $x + 2y = 6$

x	0	2
y	3	2

Putting (0, 0) in the inequality $x + 2y \geq 6$, we have

$$0 + 2 \times 0 \geq 6 \quad \Rightarrow \quad 0 \geq 6, \text{ which is false}$$

So the half plane of $x + 2y \geq 6$ is away from origin.

Now consider the linear constraint defined by the inequality

$$x + y \geq 7$$

First draw the graph of the line $x + y = 7$

x	3	4
y	4	3

Putting (0, 0) in the inequality $x + y \geq 7$, we have

$$0 + 0 \geq 7 \quad \Rightarrow \quad 0 \geq 7, \text{ which is false}$$

So the half plane of $x + y \geq 7$ is away from origin.

Now consider the linear constraint defined by the inequality

$$x + 3y \geq 11$$

First draw the graph of the line $x + 3y = 11$

x	2	5
y	3	2

Putting (0, 0) in the inequality $x + 3y \geq 11$, we have

$$0 + 3 \times 0 \geq 11 \quad \Rightarrow \quad 0 \geq 11, \text{ which is false}$$

So the half plane of $x + 3y \geq 11$ is away from origin.

Now consider the linear constraint defined by the inequality

$$2x + y \geq 9$$

First draw the graph of the line $2x + y = 9$

x	3	4
y	3	1

Putting (0, 0) in the inequality $2x + y \geq 9$, we have

$$2 \times 0 + 0 \geq 9 \quad \Rightarrow \quad 0 \geq 9, \text{ which is false}$$

So the half plane of $2x + y \geq 9$ is away from origin.

Since $\qquad x, y \geq 0$

So the feasible region lies in the first quadrant.

The coordinates of the corner points of the feasible region are $A(11, 0)$, $B(5, 2)$, $C(2, 5)$ and $D(0, 9)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $\qquad z = 5x + 8y$

At $A(11, 0)$ $\qquad z = 5 \times 11 + 8 \times 0 = 55 + 0 = 55$

At $B(5, 2)$ $\qquad z = 5 \times 5 + 8 \times 2 = 25 + 16 = 41$

At $C(2, 5)$ $\qquad z = 5 \times 2 + 8 \times 5 = 10 + 40 = 50$

At $D(0, 9)$ $\qquad z = 5 \times 0 + 8 \times 9 = 0 + 72 = 72$

Thus z is minimum at (5, 2) and minimum value = 41

Since the feasible region is unbounded, so we have to determine whether $z = 41$ is the minimum value or not.

Now consider the linear constraint defined by the inequality

$$5x + 8y < 41$$

First draw the graph of the line $5x + 8y = 41$

x	–3	5
y	7	2

Putting (0, 0) in the inequality $5x + 8y < 41$, we have

$$5 \times 0 + 8 \times 0 < 41 \quad \Rightarrow \quad 0 < 41, \text{ which is true}$$

So the half plane of $5x + 8y < 41$ is towards the origin.

Now the half plane of $5x + 8y < 41$ and feasible region have no common points.

Thus least cost of mixture is Rs 41 when 5 kg of food X and 2 kg of food Y are mixed.

S10. The given objective function is $z = x + 2y$

Consider the linear constraint defined by the inequality

$$x + 2y \geq 100$$

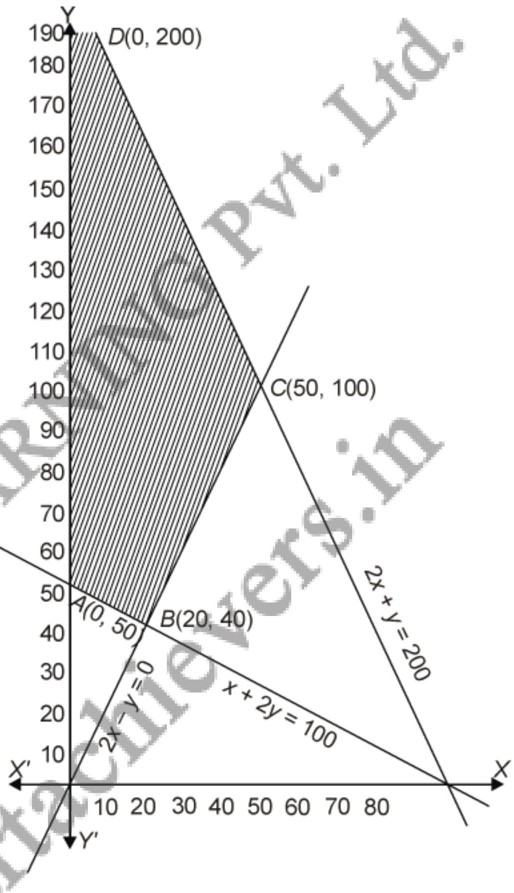
First draw the graph of the line $x + 2y = 100$

x	20	40
y	40	30

Putting (0, 0) in the inequality $x + 2y \geq 100$, we have

$$0 + 2 \times 0 \geq 100 \quad \Rightarrow \quad 0 \geq 100, \text{ which is false}$$

So the half plane of $x + 2y \geq 100$ is away from origin.



Now consider the linear constraint defined by the inequality

$$2x - y \leq 0$$

First draw the graph of the line $2x - y = 0$

x	0	10
y	0	20

Putting (5, 0) in the inequality $2x - y \leq 0$, we have

$$2 \times 5 - 0 \leq 0 \quad \Rightarrow \quad 10 \leq 0, \text{ which is false}$$

So the half plane of $2x - y \leq 0$ is away from the point (5, 0)

Now consider the linear constraint defined by the inequality

$$2x + y \leq 200$$

First draw the graph of the line $2x + y = 200$

x	70	60
y	60	80

Putting (0, 0) in the inequality $2x + y \le 200$, we have

$$2 \times 0 + 0 \le 200 \quad \Rightarrow \quad 0 \le 200, \text{ which is true}$$

So the half plane of $2x + y \le 200$ is towards the origin.

Since $\qquad x, y \ge 0$

So the feasible region lies in the first quadrant.

The coordinate of the corner points of the feasible region are $A(0, 50)$, $B(20, 40)$, $C(50, 100)$ and $D(0, 200)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $\qquad z = x + 2y$

At $A(0, 50)$ $\qquad z = 0 + 2 \times 50 = 0 + 100 = 100$

At $B(20, 40)$ $\qquad z = 20 + 2 \times 40 = 20 + 80 = 100$

At $C(50, 100)$ $\qquad z = 50 + 2 \times 100 = 50 + 200 = 250$

At $D(0, 200)$ $\qquad z = 0 + 2 \times 200 = 0 + 400 = 400$

Thus z is minimum at points (0, 50) and (20, 40) and minimum value = 100, z is maximum at point (0, 200) and maximum value = 400.

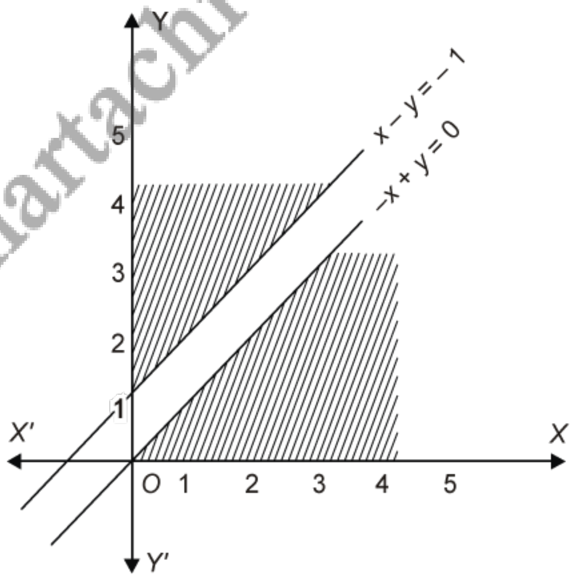
S11. The given objective function is $z = x + y$

Consider the linear constraint defined by the inequality

$$x - y \le -1$$

First draw the graph of the line $x - y = -1$

x	0	1
y	1	2



Putting (0, 0) in the inequality $x - y \le -1$, we have

$$0 - 0 \le -1 \quad \Rightarrow \quad 0 \le -1, \text{ which is false}$$

So the half plane of $x - y \le -1$ is away from origin.

Now consider the linear constraint defined by the inequality

$$-x + y \le 0$$

First draw the graph of the line $-x + y = 0$

x	0	1
y	0	1

Putting (2, 0) in the inequality $-x + y \le 0$, we have

$$-2 + 0 \le 0 \quad \Rightarrow \quad -2 \le 0, \text{ which is true}$$

So the half plane of $-x + y \le 0$ is towards the point (2, 0)

Since $x, y \ge 0$

So the feasible region lies in the first quadrant.

The feasible region is not common

Thus there is no maximum value of z.

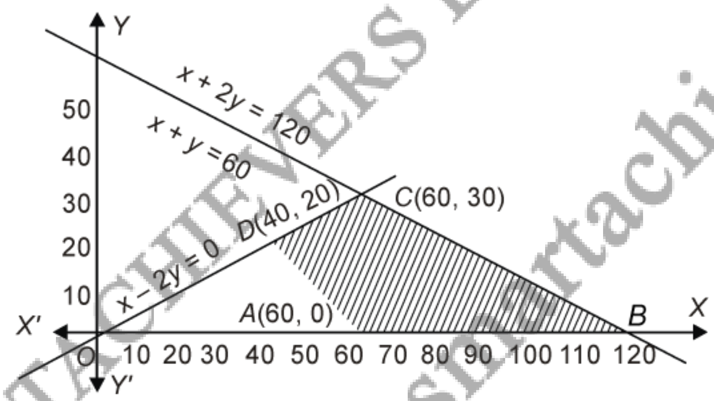
S12. The given objective function is $z = 5x + 10y$

Consider the linear constraint defined by the inequality

$$x + 2y \le 120$$

First draw the graph of the line $x + 2y = 120$

x	30	40
y	45	40



Putting (0, 0) in the inequality $x + 2y \le 120$, we have

$$0 + 2 \times 0 \le 120 \quad \Rightarrow \quad 0 \le 120, \text{ which is true}$$

So the half plane of $x + 2y \le 120$ is towards the origin.

Now consider the linear constraint defined by the inequality

$$x + y \ge 60$$

First draw the graph of the line $x + y = 60$

x	30	40
y	30	20

Putting (0, 0) in the inequality $x + y \geq 60$, we have

$$0 + 0 \geq 60 \quad \Rightarrow \quad 0 \geq 60, \text{ which is false}$$

So the half plane of $x + y \geq 60$ is away from origin.

Now consider the linear constraint defined by the inequality

$$x - 2y \geq 0$$

First draw the graph of the line $x - 2y = 0$

x	0	10
y	0	5

Putting (5, 0) in the inequality $x - 2y \geq 0$, we have

$$5 - 2 \times 0 \geq 0 \quad \Rightarrow \quad 5 \geq 0, \text{ which is true}$$

So the half plane of $x - 2y \geq 0$ is towards the point (5, 0)

Since $\quad x, y \geq 0$

So the feasible region lies in the first quadrant.

The coordinate of the corner points of the feasible region are A (60, 0) B (120, 0), C (60, 30) and D (40, 20). These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $\quad z = 5x + 10y$

At A (60, 0) $\quad z = 5 \times 60 + 10 \times 0 = 300 + 0 = 300$

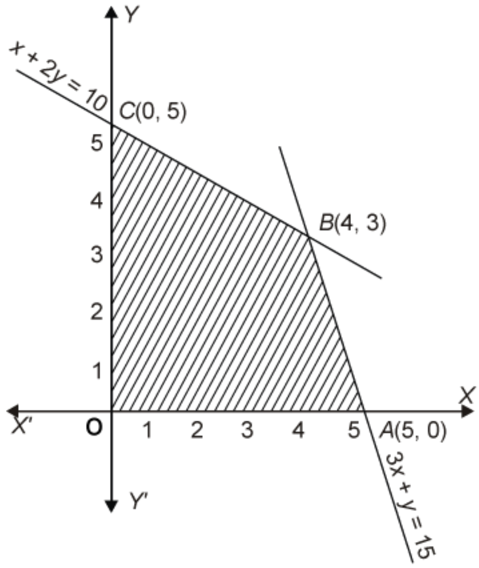
At B (120, 0) $\quad z = 5 \times 120 + 10 \times 0 = 600 + 0 = 600$

At C (60, 30) $\quad z = 5 \times 60 + 10 \times 30 = 300 + 300 = 600$

At D (40, 20) $\quad z = 5 \times 40 + 10 \times 20 = 200 + 200 = 400$

Thus z is minimum at (60, 0) and minimum value = 300.

S13. The given objective function is $z = 3x + 2y$



Consider the linear constraint defined by the inequality

$$x + 2y \le 10$$

First draw the graph of the line $x + 2y = 10$

x	0	2
y	5	4

Putting (0, 0) in the inequality $x + 2y \le 10$, we have

$0 + 2 \times 0 \le 10 \quad \Rightarrow \quad 0 \le 10$, which is true

So the half plane of $x + 2y \le 10$ is towards the origin.

Now consider the linear constraint defined by the inequality

$$3x + y \le 15$$

First draw the graph of the line $3x + y = 15$

x	4	5
y	3	0

Putting (0, 0) in the inequality $3x + y \le 15$, we have

$3 \times 0 + 0 \le 15 \quad \Rightarrow \quad 0 \le 15$, which is true

So the half plane of $3x + y \le 15$ is towards the origin.

Since $\quad x, y \ge 0$

So the feasible region lies in the first quadrant.

The coordinate of the corner points of the feasible region are $O(0, 0)$, $A(5, 0)$ $B(4, 3)$ and $C(0, 5)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $\quad z = 3x + 2y$

At $O(0, 0)$ $\quad z = 3 \times 0 + 2 \times 0 = 0$

At $A(5, 0)$ $\quad z = 3 \times 5 + 2 \times 0 = 15 + 0 = 15$

At $B(4, 3)$ $\quad z = 3 \times 4 + 2 \times 3 = 12 + 6 = 18$

At $C(0, 5)$ $\quad z = 3 \times 0 + 2 \times 5 = 0 + 10 = 10$

Thus z is maximum at (4, 3) and maximum value = 18.

S14. The given data can be put in the tabular form as follows

Food	Vitamin *A*	Vitamin *C*	Cost/Unit
I	2	1	Rs. 50
II	1	2	Rs. 70
Least requirements	8	10	

Suppose the diet contains x units of food I and y units of food II.

Then, the required LPP is Minimize, $Z = 50x + 70y$

Subject to the constraints, $2x + y \geq 8$

$$x + 2y \geq 10$$

$$x \geq 0,\ y \geq 0$$

Let us draw the lines $2x + y = 8$ ... (i)

$x + 2y = 10$ ... (ii)

Line $2x + y = 8$ passes through the points (0, 8) and (4, 0).

$\because$ For $2x + y = 8$

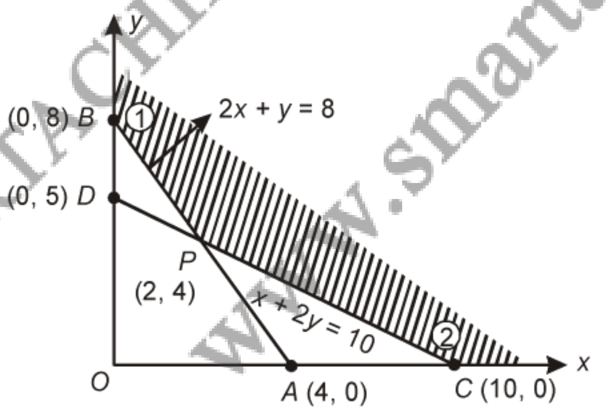
x	0	4
y	8	0

and the line x + 2y = 10 passes through points (10, 0) and (0, 5).

$\because$ For $x + 2y = 10$

x	10	0
y	0	5

Graph of above LPP is given as follows



Multiplying Eq. (i) by 2 and subtracting Eq. (ii) from Eq. (i), we get

$$\begin{aligned} 4x + 2y &= 16 \\ \underline{-\ x + 2y} &= \underline{-\ 10} \\ 3x &= 6 \end{aligned}$$

$\Rightarrow$ $x = 2$

Putting $x = 2$, in Eq. (i), we get

$$2(2) + y = 8 \Rightarrow y = 8 - 4 = 4$$

These lines intersect at $P(2, 4)$.

The solution set is shaded region.

$\therefore$ We have, the table with corner point and value of Z.

Corner points	**Value of the objective function $Z = 50x + 70y$**
$C(10, 0)$	Rs. $50(10) + 70(0) = 500$
$P(2, 4)$	Rs. $50(2) + 70(4) = 380$
$B(0, 8)$	Rs. $50(0) + 70(8) = 560$

Clearly, Z is minimum at (2, 4) *i.e.*, when $x = 2$, $y = 4$

Also, the minimum value = Rs. 380.

S15. Let food $A = x$ unit and food $B = y$ unit

The given data can be put into tabular form as follows :

Food	**Vitamins**	**Minerals**	**Calories**	**Cost**
$A(x)$	200	1	40	5
$B(y)$	100	2	40	4
Requirement	4000	50	1400	

$\therefore$ Required LPP is Minimize cost $Z = 5x + 4y$

Subject to the constraints,

$$200x + 100y \geq 4000 \quad \ldots \text{(i)}$$

$$x + 2y \geq 50 \quad \ldots \text{(ii)}$$

$$40x + 40y \geq 1400 \quad \ldots \text{(iii)}$$

$$x, y \geq 0$$

Table for line $200x + 100y = 4000$ is

x	0	20
y	40	0

$\therefore$ It passes through the points (0, 40) and (20, 0).

Table for line $x + 2y = 50$ is

x	0	50
y	25	0

$\therefore$ It passes through the points (0, 25) and (50, 0).

Table for line $40x + 40y = 1400$ is

x	0	35
y	35	0

$\therefore$ It passes through the points (0, 35) and (35, 0).

To find intersection point

Multiplying Eq. (ii) by 50 and subtracting Eq. (ii) from Eq. (i), we get

$$150x = 1500 \Rightarrow x = 10$$

Putting $x = 10$ in Eq. (ii), we get

$$10 + 2y = 50$$

$\Rightarrow$ $2y = 40$

$\Rightarrow$ $y = 20$

$\therefore$ Point of intersection is (10, 20).

Multiplying Eq. (ii) by 20 and subtracting Eq. (iii) from Eq. (ii), we get

$$-20x = -400 \Rightarrow x = 20$$

Putting $x = 20$ in Eq. (ii), we get

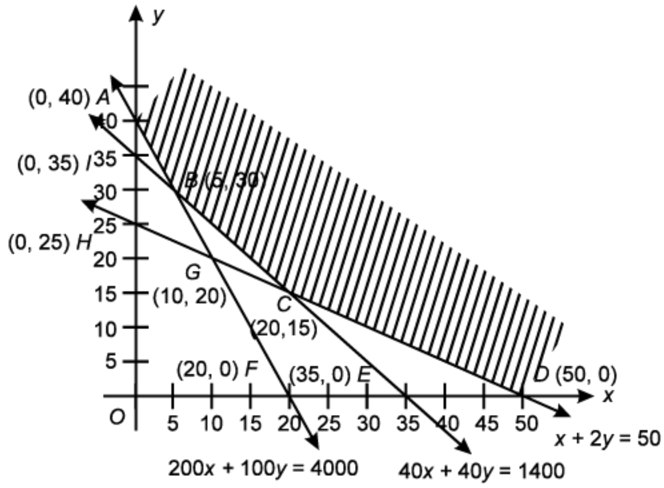
$$20 + 2y = 50$$

$\Rightarrow$ $2y = 30 \Rightarrow y = 15$

$\therefore$ Point of intersection is C(20, 15).

Multiplying Eq. (iii) by 5 and subtracting Eq. (iii) from Eq. (i), we get

$$-100y = -3000 \Rightarrow y = 30$$



Putting $y = 30$ in Eq. (iii), we get

$$40x + 40 \times 30 = 1400$$

$$\Rightarrow \quad 40x = 1400 - 1200$$

$$\Rightarrow \quad 40x = 200 \Rightarrow x = 5$$

$\therefore$ Point of intersection is $B(5, 30)$.

Now, graph of above LPP is

$\therefore$ Corner points of feasible region are

$$A(0, 40), \quad B(5, 30), \quad C(20, 15), \quad D(50, 0).$$

Now, evaluate Z at the corner points

Corner point	$Z = 5x + 4y$
$A(0, 40)$	$0 + 160 = 160$
$B(5, 30)$	$25 + 120 = 145$ (least cost)
$C(20, 15)$	$100 + 60 = 160$
$D(50, 0)$	$250 + 0 = 250$

$\therefore$ Least cost = 145 at $x = 5$, $y = 30$.

S16. Let the number of pedestal lamps sold = x

and the number of wooden shades sold = y

The given data can be put into tabular form as follows

	Pedestal lamps	Wooden shades	Total hours available
Griding/Cutting	2	1	12
Sprayer	3	2	20
Profit	Rs. 5	Rs. 3	

$\therefore$ The required LPP is given as

Maximize $\quad Z = 5x + 3y$

Subject to constraints $\quad 2x + y \le 12$ [Grinding/cutting constraints]

$3x + 2y \le 20$ [Sprayer constraints]

$x, y \ge 0$ ...(ii)

Now, table for line $2x + y = 12$ is

x	6	0
y	0	12

So, it passes through points (6, 0) and (0, 12). Table for line $3x + 2y = 20$ is

x	$\frac{20}{3}$	0
y	0	10

So, it passes through points $\left(\frac{20}{3}, 0\right)$ and (0, 10).

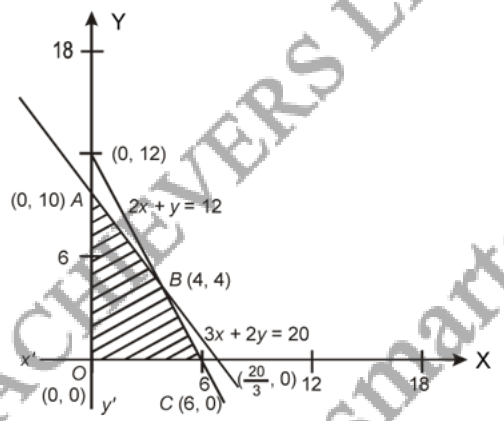
Now, we solve the lines to find their point of intersection.

$$3 \times (2x + y = 12) \Rightarrow 6x + 3y = 36$$
$$2 \times (3x + 2y = 20) \Rightarrow \underset{-}{6x} \underset{-}{+} 4y = \underset{-}{40}$$
$$-y = -4$$

$\Rightarrow$ $y = 4$

Putting, $y = 4$ in $2x + y = 12$, we get $2x = 8$ or $x = 4$

$\therefore$ Lines intersect at points (4, 4). Now, graph of above LPP is follows



From the figure, we see that the region $OABC$ is the feasible region. The corner points of the feasible region are $O(0, 0)$, $A(0, 10)$, $B(4, 4)$, $C(6, 0)$. Now, we find the value of P at the corner points.

Corner points	**$Z = 5x + 3y$**
$O(0, 0)$	$Z = 5(0) + 3(0) = 0$
$A(0, 10)$	$Z = 5(0) + 3(10) = 30$
$B(4, 4)$	$Z = 5(4) + 3(4) = 20 + 12 = 32$ (maximum)
$C(6, 0)$	$Z = 5(6) + 3(0) = 30$

∴ The maximum profit is Rs. 32 at the point $B(4, 4)$ when 4 lamps and 4 shades are sold.

S17. Let the machines of type $A = x$

And the machines of type $B = y$

The given data can be put in the tabular form as follows

Machine	Area occupied	Labour force	Daily output (in units)
A	1000 m^2	12 men	60
B	1200 m^2	8 men	40
Requirement	9000 m^2	72 men	

Maximize daily output $Z = 60x + 40y$

Subject to the constraints,

$$1000x + 1200y \leq 9000 \quad \text{... (i)}$$

$$12x + 8y \leq 72 \quad \text{... (ii)}$$

$$x,\ y \geq 0$$

Table for line $1000x + 1200y = 9000$ is

x	0	9
y	7.5	0

∴ It passes through the points (0, 7.5) and (9, 0).

Table for line $12x + 8y = 72$ is

x	0	6
y	9	0

∴ It passes through the points (0, 9) and (6, 0).

To find intersection point

Multiplying Eq. (ii) by 150 and subtracting Eq. (ii) from Eq. (i), we get

$$\begin{array}{rcl} 1000x + 1200y &=& 9000 \\ \underline{-}\,1800x \,\underline{+}\, 1200y &=& \underline{-}\,10800 \\ \hline -\ 800x &=& -\ 1800 \end{array}$$

$$\Rightarrow \qquad x = \frac{1800}{800} = \frac{9}{4}$$

Putting $x = \frac{9}{4}$ in Eq. (ii), we get

$$12 \times \frac{9}{4} + 8y = 72$$

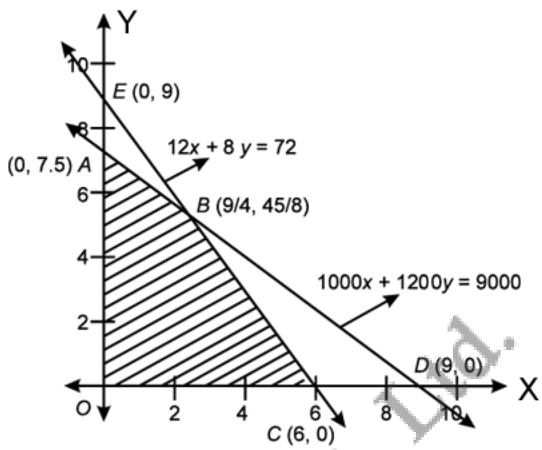
$$\Rightarrow \qquad 8y = 72 - 27 = 45$$

$$\Rightarrow \qquad y = \frac{45}{8}$$

$\therefore$ Point of intersection is $B\left(\frac{9}{4}, \frac{45}{8}\right)$.

Graph of above LPP is as follows:

So, *OABC* is the feasible region.



The corner points are $O(0, 0)$, $A(0, 7.5)$, $B\left(\frac{9}{4}, \frac{45}{8}\right)$ and $C(6, 0)$.

Now, we find Z at various corner points.

Corner points	$Z = 60x + 40y$
$A(0, 7.5)$	$0 + 300 = 300$
$B\left(\frac{9}{4}, \frac{45}{8}\right)$	$135 + 225 = 360$ (maximum)
$C(6, 0)$	$360 + 0 = 360$
$O(0, 0)$	0

The maximum output is at *B* and *C*. But the number of machines cannot be in fraction. Hence, number of machines of type $A = 6$, and number of machines of type $B = 0$.

S18. Let x be the number of rice bags and y be the number of wheat bags.

The given data can be put in the tabular form as follows

Products	Bags	Cost (per bag)	Profit cost
Rice bag	1	Rs. 180	Rs. 11
Wheat bag	1	Rs. 120	Rs. 9
Requirement	10	Rs. 1500	

Maximize profit, $P = 11x + 9y$

Subject to the constraints,

$$180x + 120y \leq 1500 \quad \text{... (i)}$$

$$x + y \leq 10 \quad \text{... (ii)}$$

$$x, y \geq 0$$

Table for line $180x + 120y = 1500$ is

x	0	$\frac{25}{3}$
y	$\frac{50}{4}$	0

∴ It passes through points (0, 12.5) and (8.3, 0).

Table for line $x + y = 10$ is

x	0	10
y	10	0

∴ It passes through (0, 10) and (10, 0).

To find intersection point

Multiplying Eq. (ii) by 120 and subtracting Eq. (ii) from Eq. (i), we get

$$\begin{array}{rcr} 180x + 120y & = & 1500 \\ \underset{-}{\ }120x \underset{-}{+} 120y & = & \underset{-}{\ }1200 \\ \hline 60x & = & 300 \end{array}$$

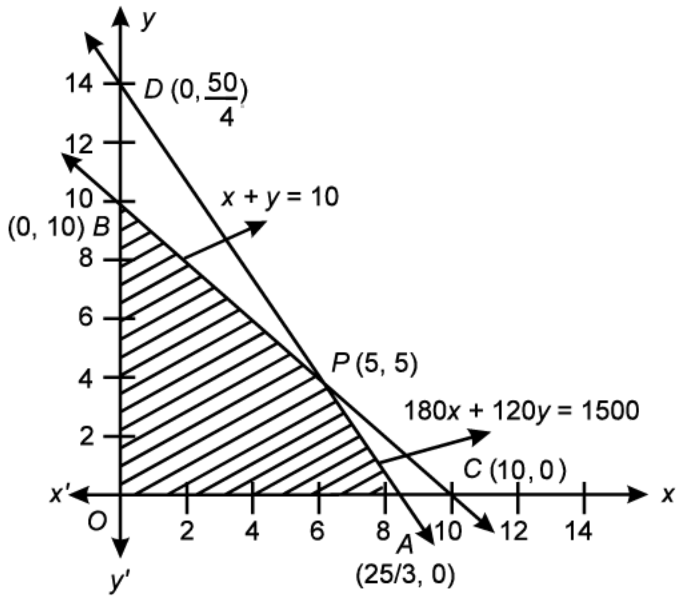
$\Rightarrow$ $x = 5$

Putting $x = 5$ in Eq. (ii), we get

$$5 + y = 10 \Rightarrow y = 5$$

∴ The intersection point is (5, 5).

Now, plot the straight lines on the graph and find the corner points of feasible region.



Hence, *OAPB* is the feasible region.

The corner points of feasible region are $O(0, 0)$, $A\left(\frac{25}{3}, 0\right)$, $P(5, 5)$ and $B(0, 10)$ respectively.

Corner points	$P = 11x + 9y$
$O(0, 0)$	$11(0) + 9(0) = 0$
$A\left(\frac{25}{3}, 0\right)$	$91.3 + 0 = 91.3$
$P(5, 5)$	$55 + 45 = 100$ (Maximum)
$B(0, 10)$	$0 + 90 = 90$

$\therefore$ Maximum profit, P = Rs. 100

Number of rice bags, $x = 5$

Number of wheat bags, $y = 5$

S19. Let x g of wheat and y g of rice be mixed in the daily diet. Let z be the minimum cost of diet.

	Proteins	**Carbohydrates**	**Cost**
1 g wheat	0.1 g	0.25 g	Rs 4/kg
1 g Rice	0.05 g	0.5 g	Rs 6/kg
Minimum requirement	50 g	200 g	

The mathematical formulation of the given L.P.P. is as

$$\text{Minimise } z = \frac{4x}{1000} + \frac{6y}{1000} \quad i.e. \quad z = \frac{x}{250} + \frac{3y}{500}$$

subject to $0.1x + 0.05y \geq 50$ *i.e.* $2x + y \geq 1000$

$$0.25x + 0.5y \geq 200 \quad i.e. \quad x + 2y \geq 800$$

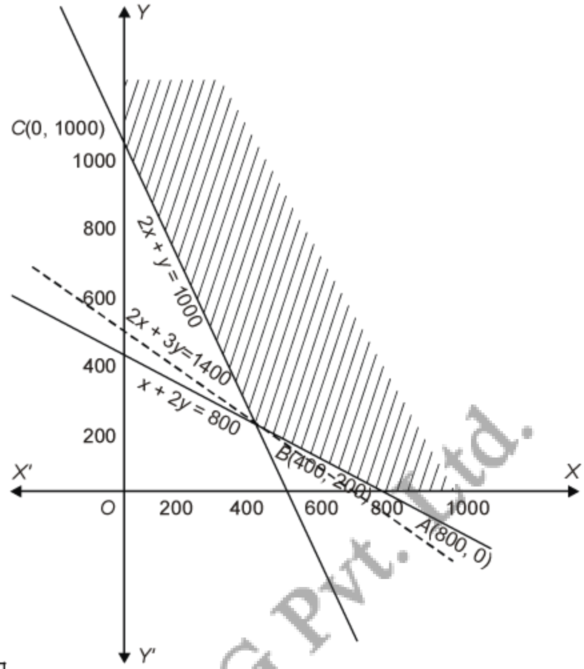
$$x,\ y \geq 0$$

Consider the linear constraint defined by the inequality

$$2x + y \geq 1000$$

First draw the graph of the line $2x + y = 1000$

x	0	500
y	1000	0



Putting (0, 0) in the inequality $2x + y \geq 1000$, we have

$$2 \times 0 + 0 \geq 1000 \quad \Rightarrow \quad 0 \geq 1000, \text{ which is false}$$

So the half plane of $2x + y \geq 1000$ is away from origin

Now consider the linear constraint defined by the inequality

$$x + 2y \geq 800$$

First draw the graph of the line $x + 2y = 800$

x	0	800
y	400	0

Putting (0, 0) in the inequality $x + 2y \geq 800$, we have

$$0 + 2 \times 0 \geq 800 \quad \Rightarrow \quad 0 \geq 800, \text{ which is false}$$

So the half plane of $x + 2y \geq 800$ is away the origin.

Since $\qquad x,\ y \geq 0$

So the feasible region lies in the first quadrant.

The coordinates of the corner points of the feasible region are A (800, 0), B (400, 200) and C (0, 1000). These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $\qquad z = \dfrac{x}{250} + \dfrac{3y}{500}$

At A (800, 0) $\qquad z = \dfrac{800}{250} + \dfrac{3 \times 0}{500} = \dfrac{16}{5} + 0 = 3.2$

At B (400, 200) $\qquad z = \dfrac{400}{250} + \dfrac{3 \times 200}{500} = \dfrac{8}{5} + \dfrac{6}{5} = \dfrac{14}{5} = 2.8$

At C (0, 1000) $\qquad z = \dfrac{0}{250} + \dfrac{3 \times 1000}{500} = 0 + 6 = 6.$

Thus z is minimum at (400, 200) and minimum value = 2.8.

Since the feasible region is unbounded, so we have to determine whether $z = 2.8$ is minimum value or not.

Now consider the linear constraint defined by the inequality

$$\frac{x}{250} + \frac{3y}{500} = 2.8 \quad i.e. \quad 2x + 3y \le 1400$$

First draw the graph of the line $2x + 3y = 1400$

x	700	400
y	0	200

Putting (0, 0) in the inequality $2x + 3y \le 1400$, we have

$$2 \times 0 + 3 \times 0 \le 1400 \quad \Rightarrow \quad 0 \le 1400, \quad \text{which is true}$$

So the half plane of $2x + 3y \le 1400$ is towards the origin.

Now the half plane of $2x + 3y \le 1400$ and the feasible region have no common points.

So the least cost of diet is Rs. 2.80 when 400 g wheat and 200 g rice are mixed.

S20. Let number of cakes of first kind be x and the number of cakes of second kind be y.

The given data can be put in tabular form a follows :

Types of cake	Flour (in g)	Fat (in g)
First kind (x)	300	15
Second kind (y)	150	30
Total amount	7.5 kg or 7500 g	600

The required LPP is

Maximize $Z = x + y$

Subject to the constraints,

$$300x + 150y \le 7500 \quad \ldots \text{(i)}$$
$$300x + 150y = 7500 \quad \text{(eq. form)}$$
$$15x + 30y \le 600 \quad \ldots \text{(ii)}$$
$$15x + 30y = 600 \quad \text{(eq. form)}$$
$$x \ge 0, \; y \ge 0$$

Table for line $300x + 150y = 7500$ is

x	0	25
y	50	0

$\therefore$ It passes through the points (0, 50) and (25, 0).

Table for line $15x + 30y = 600$ is

x	0	40
y	20	0

$\therefore$ It passes through the points (0, 20) and (40, 0).

Multiplying Eq. (ii) by 20 and subtracting Eq. (ii) from Eq. (i), we get

$$\begin{array}{r} 300x + 150y = 7500 \\ \underline{\overset{}{-}300x \overset{}{+} 600y = \overset{}{-}12000} \\ -450y = -4500 \end{array}$$

$\Rightarrow$ $y = 10$

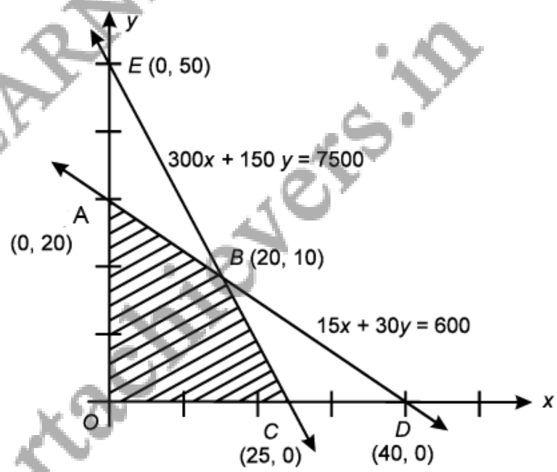
Putting $y = 10$ in Eq. (i), we get

$$300x + 150(10) = 7500$$

$\Rightarrow$ $300x = 6000 \Rightarrow x = 20$

$\therefore$ The point of intersection is B (20, 10).

Now, the graph of the given system of inequalities is as follows



From the figure, we see that $OABC$ is the feasible region.

The corner points of the feasible region are $O(0, 0)$, $A(0, 20)$, $B(20, 10)$ and $C(25, 0)$ respectively.

Now, we evaluate Z at the corner points.

Corner points	Value of $Z = x + y$
$O(0, 0)$	$0 + 0 = 0$
$A(0, 20)$	$0 + 20 = 20$
$B(20, 10)$	$20 + 10 = 30$ (maximum)
$C(25, 0)$	$25 + 0 = 25$

$\therefore$ Maximum number of cakes = 30

One kind of cakes = 20

And other kind of cakes = 10

S21. We can write the given data in tabular form as follows

Cake	Flour	Fat
I kind	200 g	25 g
II kind	100 g	50 g
Total amount	5 kg	1 kg

Suppose the number of cakes of I kind be x and that of II kind be y.

$\therefore$ The required LPP is

Maximum $Z = x + y$

Subject of constraints,

$$200x + 100y \leq 5000 \quad \text{... (i)}$$

$$25x + 50y \leq 1000 \quad \text{... (ii)}$$

$$x \geq 0, y \geq 0$$

$\Rightarrow$ $2x + y \leq 50$ ... (iii)

[$\because$ Dividing both sides of Eq. (i) by 100]

$2x + y = 50$ (in eq. form)

$x + 2y \leq 40$ ... (iv)

[$\because$ Dividing both sides of Eq. (ii) by 25]

$x + 2y = 40$ (in eq. form)

$x \geq 0, y \geq 0$

Table for line $2x + y = 50$ is

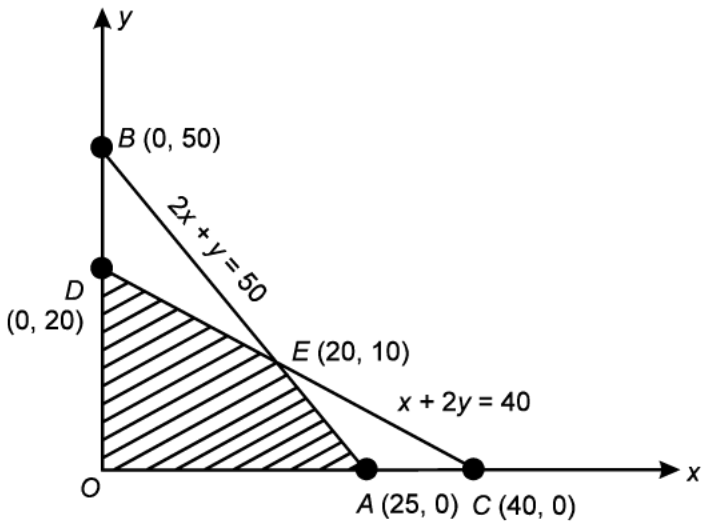
x	25	0
y	0	50

So, it passes through points (25, 0) and (0, 50). Table for line $x + 2y = 40$ is

x	40	0
y	0	20

So, it passes through points (40, 0) and (0, 20).

Graph of above LPP is given as follows



Now, we solve Eqs. (iii) and (iv) to find the point of intersection.

Multiplying Eq. (iv) by 2 and then subtract Eq. (iii) from it, we get

and
$$\begin{array}{rcl} 2x + y &=& 50 \\ \underset{-}{}2x \underset{-}{+} 4y &=& \underset{-}{}80 \\ \hline -3y &=& -30 \end{array}$$

$\Rightarrow$ $y = 10$

Putting $y = 10$ in $x + 2y = 40$, we get

$\therefore$ $x + 20 = 40$

$\Rightarrow$ $x = 20$

The line of intersection is (20, 10).

From the figure, we see that *OAED* is the feasible region.

The corner points of feasible region are $O(0, 0)$, $A(25, 0)$, $D(0, 20)$ and $E(20, 10)$ respectively.

Now, we evaluate Z at the corner points.

Corner points	**Value of $Z = x + y$**
$O(0, 0)$	$0 + 0 = 0$
$A(25, 0)$	$25 + 0 = 25$
$D(0, 20)$	$0 + 25 = 20$
$E(20, 10)$	$20 + 10 = 30$ (maximum)

∴ Maximum number of cakes = 30

∴ 20 cakes of first kind and 10 cakes of second kind be prepared.

S22. Let x kg of food X and y kg of food Y mixes to make the mixture. Let z be the total cost of the mixture.

	Vitamin *A*	**Vitamin *B***	**Vitamin *C***	**Cost**
Food X	1	2	3	Rs 16
Food Y	2	2	1	Rs. 20
Minimum requirement	10	12	8	

Thus the mathematical formulation of the given L.P.P. is as

Minimise $z = 16x + 20y$

subject to $x + 2y \geq 10$, $2x + 2y \geq 12$, $3x + y \geq 8$, $x, y \geq 0$.

Consider the linear constraint defined by the inequality

$$x + 2y \geq 10$$

First draw the graph of the line $x + 2y = 10$

x	0	4
y	5	3

Putting (0, 0) in the inequality $x + 2y \geq 10$, we have

$$0 + 2 \times 0 \geq 10 \quad \Rightarrow \quad 0 \geq 10, \text{ which is false}$$

So the half plane of $x + 2y \geq 10$ is away from origin.

Now consider the linear constraint defined by the inequality

$$2x + 2y \geq 12$$

First draw the graph of the line $2x + 2y = 12$ *i.e.* $x + y = 6$

x	0	2
y	6	4

Putting (0, 0) in the inequality $x + y \geq 6$, we have

$$0 + 0 > 6 \quad \Rightarrow \quad 0 \geq 6, \text{ which is false}$$

So the half plane of $x + y \geq 6$ is away from origin.

Now consider the linear constraint defined by the inequality

$$3x + y \geq 8$$

First draw the graph of the line $3x + y = 8$

x	2	1
y	2	5

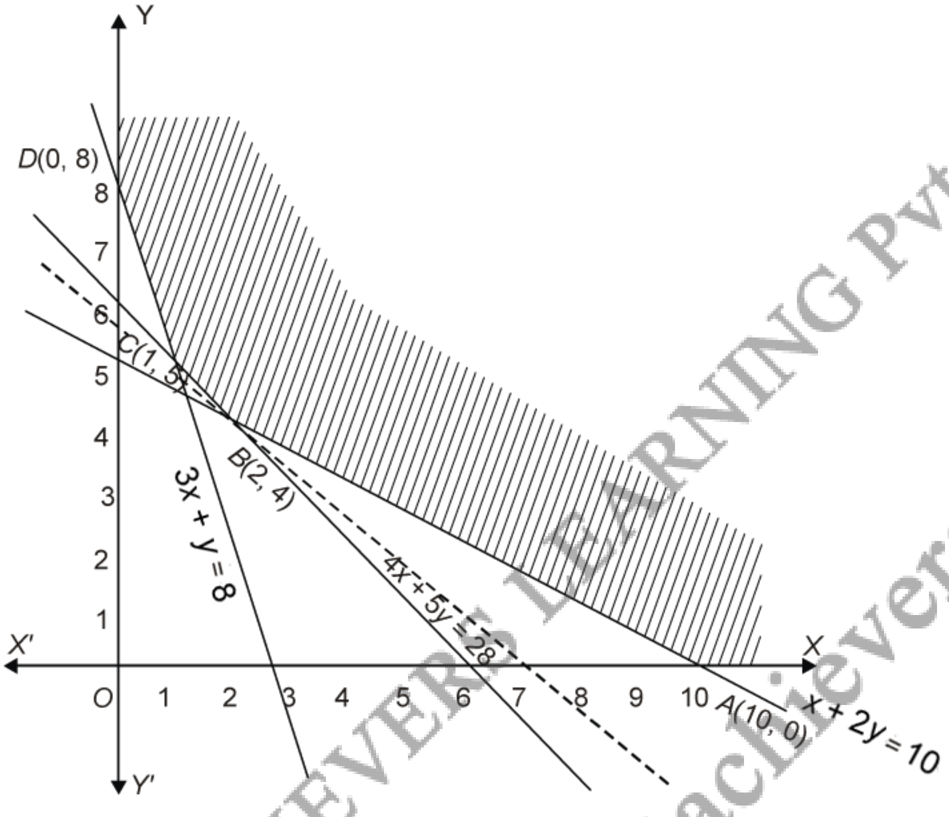
Putting (0, 0) in the inequality $3x + y \geq 8$, we have

$$3 \times 0 + 0 \geq 8 \quad \Rightarrow \quad 0 \geq 8, \text{ which is false}$$

So the half plane of $3x + y \geq 8$ is away from origin.

Since $x, y \geq 0$

So the feasible region lies in the first quadrant.



The coordinates of the corner points of the feasible region are A (10, 0), B (2, 4), C (1, 5) and D(0, 8). These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $z = 16x + 20y$

At A(10, 0) $z = 16 \times 10 + 20 \times 0 = 160 + 0 = 160$

At B(2, 4) $z = 16 \times 2 + 20 \times 4 = 32 + 80 = 112$

At C(1, 5) $z = 16 \times 1 + 20 \times 5 = 16 + 100 = 116$

At D(0, 8) $z = 16 \times 0 + 20 \times 8 = 0 + 160 = 160$

Thus z is minimum at (2, 4) and minimum value = 112

Since the feasible region is unbounded, so we have to determine whether $z = 112$ is the minimum value or not.

Now consider the linear constraint defined by the inequality

$$16x + 20y \le 112$$

First draw the graph of the line $16x + 20y = 112$ *i.e.* $4x + 5y = 28$

x	7	2
y	0	4

Putting (0, 0) in the inequality $4x + 5y \le 28$, we have

$$4 \times 0 + 5 \times 0 \le 28 \quad \Rightarrow \quad 0 < 28, \text{ which is true}$$

So the half plane of $4x + 5y \le 28$ is towards the origin.

Now the half plane of $4x + 5y \le 28$ and feasible region have no common points.

So the least cost of mixture = Rs. 112 when 2 kg of food X and 4 kg of food Y are mixed.

S23. Let x be the number of packets of food P and y be the number of packets of food Q.

Let z be the maximum amount of vitamin A.

Thus the mathematical formulation of the given L.P.P. is as

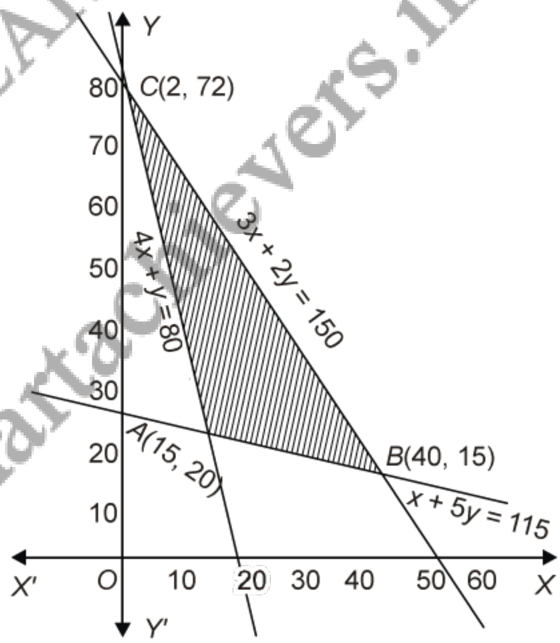
Maximise $z = 6x + 3y$

Subject to $12x + 3y \ge 240$, $4x + 20y \ge 460$, $6x + 4y \le 300$, $x, y \ge 0$.

Consider the linear constraint defined by the inequality

$$12x + 3y \ge 240$$

First draw the graph of the line $12x + 3y = 240$ *i.e.* $4x + y = 80$



x	20	0
y	0	80

Putting (0, 0) in the inequality $4x + y \ge 80$, we have

$$4 \times 0 + 0 \ge 80 \quad \Rightarrow \quad 0 \ge 80, \text{ which is false}$$

So the half plane of $4x + y \ge 80$ is away from origin.

Now consider the linear constraint defined by the inequality

$$4x + 20y \ge 460$$

First draw the graph of the line $4x + 20y = 460$ *i.e.* $x + 5y = 115$

x	0	40
y	23	15

Putting (0, 0) in the inequality $x + 5y \geq 115$, we have

$0 + 5 \times 0 \geq 115 \quad \Rightarrow \quad 0 \geq 115$, which is false

So the half plane of $x + 5y \geq 115$ is away from origin.

Now consider the linear constraint defined by the inequality

$$6x + 4y \leq 300$$

First draw the graph of the line $6x + 4y = 300$ *i.e.* $3x + 2y = 150$

x	50	0
y	0	75

Putting (0, 0) in the inequality $3x + 2y \leq 150$, we have

$$3 \times 0 + 2 \times 0 \leq 150 \quad \Rightarrow \quad 0 \leq 150, \text{ which is true}$$

So the half plane of $3x + 2y \leq 150$ is towards the origin.

Since $\qquad x, y \geq 0$

So the feasible region lies in the first quadrant.

The coordinates of the corner points of the feasible region are $A(15, 20)$, $B(40, 15)$ and $C(2, 72)$.

These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $\qquad z = 6x + 3y$

At $A(15, 20)$ $\qquad z = 6 \times 15 + 3 \times 20 = 90 + 60 = 150$

At $B(40, 15)$ $\qquad z = 6 \times 40 + 3 \times 15 = 240 + 45 = 285$

At $C(2, 72)$ $\qquad z = 6 \times 2 + 3 \times 72 = 12 + 216 = 228$

Thus z is maximum at (40, 15) and maximum value = 285

$\therefore$ Maximum amount of vitamin A = 285 units when 40 packets of food P and 15 packets of food Q be used.

S24. Let Reshma mix x kg food P and y kg of food Q to make the mixture. Let z be the total cost of mixture.

	Food P	*Food Q*	*Minimum requirement*
Vit. A	3	4	8
Vit B	5	2	11
Cost	Rs 60	Rs. 80	

Thus the mathematical formulation of the given L.P.P. is as

Minimise $\qquad z = 60x + 80y$

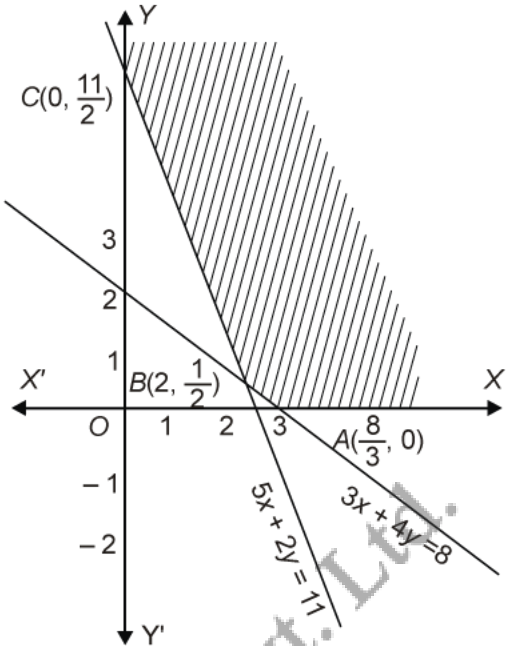
subject to $3x + 4y \geq 8$, $5x + 2y \geq 11$, $x, y \geq 0$.

Consider the linear constraint defined by the inequality

$$3x + 4y \geq 8$$

First draw the graph of the line $3x + 4y = 8$

x	0	4
y	2	–1



Putting (0, 0) in the inequality $3x + 4y \geq 8$, we have

$5 \times 0 + 4 \times 0 \geq 8 \quad \Rightarrow \quad 0 \geq 8$, which is false

So the half plane of $3x + 4y \geq 8$ is away from origin

Now consider the linear constraint defined by the inequality

$$5x + 2y \geq 11$$

First draw the graph of the line $5x + 2y = 11$

x	1	3
y	3	–2

Putting (0, 0) in the inequality $5x + 2y \geq 11$, we have

$$5 \times 0 + 2 \times 0 \geq 11 \quad \Rightarrow \quad 0 \geq 11, \text{ which is false}$$

So the half plane of $5x + 2y \geq 11$ is away from origin.

Since $\qquad x, y \geq 0$

So the feasible region lies in the first quadrant.

The coordinates of the corner points of the feasible region are $A\left(\frac{8}{3}, 0\right)$, $B\left(2, \frac{1}{2}\right)$ and $C\left(0, \frac{11}{2}\right)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $\qquad z = 60x + 80y$

At $A\left(\frac{8}{3}, 0\right)$ $\qquad z = 60 \times \frac{8}{3} + 80 \times 0 = 160 + 0 = 160$

At $B\left(2, \frac{1}{2}\right)$ $\qquad z = 60 \times 2 + 80 \times \frac{1}{2} = 120 + 40 = 160$

At $C\left(0, \frac{11}{2}\right)$ $\qquad z = 60 \times 0 + 80 \times \frac{11}{2} = 0 + 440 = 440$

Thus z is minimum at $\left(\frac{8}{3}, 0\right)$ and $\left(2, \frac{1}{2}\right)$, minimum value = 160

Since the feasible region is unbounded, so we have to determine whether $z = 160$ is the minimum value or not.

Now consider the linear constraint defined by the inequality

$$60x + 80y < 160$$

First draw the graph of the line $60x + 80y = 160$ *i.e.* $3x + 4y = 8$

Putting (0, 0) in the inequality $3x + 4y < 8$, we have

$$3 \times 0 + 4 \times 0 < 8 \quad \Rightarrow \quad 0 < 8, \text{ which is true}$$

So the half plane of $3x + 4y < 8$ is towards the origin.

Now the half plane of $3x + 4y < 8$ and feasible region have no common points.

Thus least cost of mixture is Rs 160 when $\frac{8}{3}$ kg of food *P* and 0 kg of food *Q* or 2 kg of food *P* and $\frac{1}{2}$ kg of food *Q* are mixed.

S25. Let x be the number of packages of screws *A* and y be the number of packages of screws *B* produced in a day. Let z be the total profit of the manufacturer in a day.

	Automatic machine	**Hand operated machine**	**Profit**
Package of screws *A*	4 minutes	6 minutes	Rs. 7
Package of screws *B*	6 minutes	3 minutes	Rs. 10
Time available	4 hours	4 hours	

Thus the mathematical formulation of the given L.P.P. is as

Maximise $\quad z = 7x + 10y$

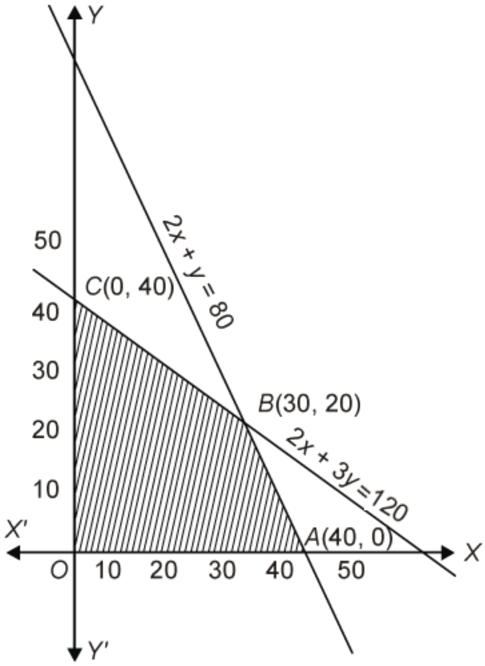
subject to $\quad 4x + 6y \leq 240, \; 6x + 3y \leq 240, \; x, y \geq 0$

Consider the linear constraint defined by the inequality

$$4x + 6y \leq 240$$

First draw the graph of the line $4x + 6y = 240$ *i.e.* $2x + 3y = 120$

x	0	30
y	40	20



Putting (0, 0) in the inequality $4x + 6y \le 240$, we have

$$4 \times 0 + 6 \times 0 \le 240 \quad \Rightarrow \quad 0 \le 240, \text{ which is true}$$

So the half plane of $4x + 6y \le 240$ is towards the origin

Now consider the linear constraint defined by the inequality

$$6x + 3y \le 240$$

First draw the graph of the line $6x + 3y = 240$ *i.e.* $2x + y = 80$

x	40	20
y	0	40

Putting (0, 0) in the inequality $6x + 3y \le 240$, we have

$$6 \times 0 + 3 \times 0 \le 240 \quad \Rightarrow \quad 0 \le 240, \text{ which is true}$$

So the half plane of $6x + 3y \le 240$ is towards the origin.

Since $x, y \ge 0$

So the feasible region lies in the first quadrant.

The coordinates of the corner points of the feasible region are $O(0, 0)$, $A(40, 0)$, $B(30, 20)$ and $C(0, 40)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $z = 7x + 10y$

At $O(0, 0)$ $z = 7 \times 0 + 10 \times 0 = 0$

At $A(40, 0)$ $z = 7 \times 40 + 10 \times 0 = 280 + 0 = 280$

At $B(30, 20)$ $z = 7 \times 30 + 10 \times 20 = 210 + 200 = 410$

At $C(0, 40)$ $z = 7 \times 0 + 10 \times 40 = 400 = 400$

Thus z is maximum at (30, 20) and maximum value = 410

$\therefore$ Maximum profit = Rs. 410 when 30 packages of screws A and 20 package of screws B are produced in a day.

S26. Let the manufacturer produce x trunks of first type and y trunks of second type each day. Let z be the total profit of the manufacturer.

	Trunk of Ist type	Trunk of IInd type	Maximum time available
machine A	3 hours	3 hours	18 hours
machine B	2 hours	3 hours	14 hours
profit	Rs. 30	Rs. 40	

Thus the mathematical formulation of the given L.P.P. is as

Maximise $z = 30x + 40y$

subject to $3x + 3y \le 18,\ 2x + 3y \le 14,\ x, y \ge 0$

Consider the linear constraint defined by the inequality

$$3x + 3y \le 18$$

First draw the graph of the line $3x + 3y = 18$ *i.e.* $x + y = 6$

x	2	3
y	4	3

Putting (0, 0) in the inequality $3x + 3y \le 18$, we have

$$3 \times 0 + 3 \times 0 \le 18 \quad \Rightarrow \quad 0 \le 18, \text{ which is true}$$

So the half plane of $3x + 3y \le 18$ is towards the origin

Now consider the linear constraint defined by the inequality

$$2x + 3y \le 14$$

First draw the graph of the line $2x + 3y = 14$

x	1	4
y	4	2

Putting (0, 0) in the inequality $2x + 3y \le 14$, we have

$$0 \times 0 + 3 \times 0 \le 14 \quad \Rightarrow \quad 0 \le 14, \text{ which is true}$$

So the half plane of $2x + 3y \le 14$ is towards the origin.

Since $x, y \geq 0$

So the feasible region lies in the first quadrant.

The coordinates of the corner points of the feasible region are $O(0, 0)$, $A(6, 0)$, $B(4, 2)$ and $C\left(0, \frac{14}{3}\right)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $z = 30x + 40y$

At $O(0, 0)$ $z = 30 \times 0 + 40 \times 0 = 0$

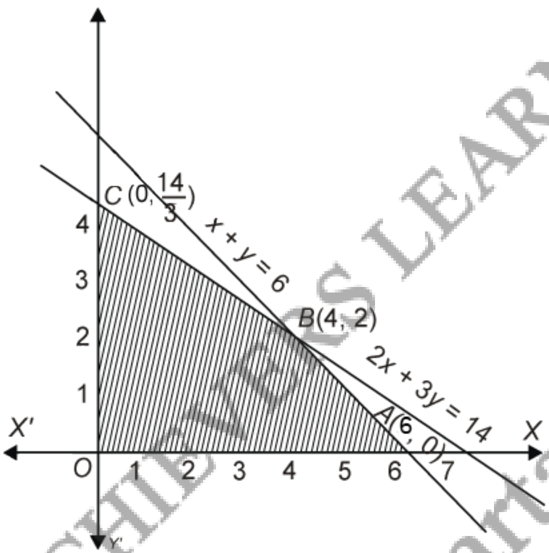
At $A(6, 0)$ $z = 30 \times 6 + 40 \times 0 = 180 + 0 = 180$

At $B(4, 2)$ $z = 30 \times 4 + 40 \times 2 = 120 + 80 = 200$

At $C\left(0, \frac{14}{3}\right)$ $z = 30 \times 0 + 40 \times \frac{14}{3} = 0 + \frac{560}{3} = \frac{560}{3}$

Thus z is maximum at $(4, 2)$ and maximum value = 200

So 4 trunks of first type and 2 trunks of second type should be produced each day in order to make maximum profit.



S27. Let the company manufactures x souvenirs of type A and y souvenirs of type B in a day. Let z be the total profit of the company in a day.

	Cutting	Assembling	Profit
Souvenir of type A	5 minutes	10 minutes	Rs. 5
Souvenir of type B	8 minutes	8 minutes	Rs. 6
Time available	3 hours 20 minutes	4 hours	

Thus the mathematical formulation of the given L.P.P. is as

Maximise $z = 5x + 6y$

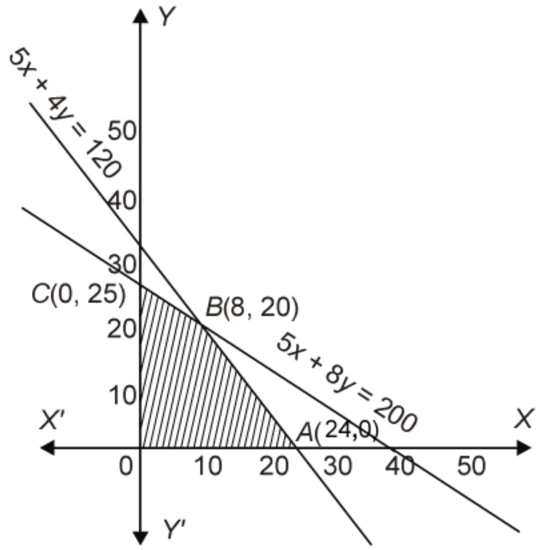
subject to $5x + 8y \leq 200$, $10x + 8y \leq 240$, $x, y \geq 0$

Consider the linear constraint defined by the inequality

$$5x + 8y \le 200$$

First draw the graph of the line $5x + 8y = 200$

x	0	40
y	25	0



Putting (0, 0) in the inequality $5x + 8y \le 200$, we have

$$5 \times 0 + 8 \times 0 \le 200 \quad \Rightarrow \quad 0 \le 200, \text{ which is true}$$

So the half plane of $5x + 4y \le 120$ is towards the origin

Since $\qquad x, y \ge 0$

So the feasible region lies in the first quadrant.

The coordinates of the corner points of the feasible region are $O(0, 0)$, $A(24, 0)$, $B(8, 20)$ and $C(0, 25)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $\qquad z = 5x + 6y$

At $O(0, 0)$ $\qquad z = 5 \times 0 + 6 \times 0 = 0$

At $A(24, 0)$ $\qquad z = 5 \times 24 + 6 \times 0 = 120 + 0 = 120$

At $B(8, 20)$ $\qquad z = 5 \times 8 + 6 \times 20 = 40 + 120 = 160$

At $C(0, 25)$ $\qquad z = 5 \times 0 + 6 \times 25 = 0 + 150 = 150$

Thus z is maximum at (8, 20) and maximum value = 160

$\therefore$ Maximum profit = Rs. 160 when 8 souvenir of type A and 20 souvenir of type B are produced in a day.

S28. Let the manufacturer produces x units of goods A and y units of goods B.

Now, formulate a table for given data.

	A	B	Required Capacity
Workers	2	3	30
Capital	3	1	17
Profit function (z)	100	120	

$\therefore$ The required LPP becomes

Maximise (z) = $100x + 120y$,

Subject to constraints $2x + 3y \le 30$ ... (i)

$3x + y \le 17$ ... (ii)

and $x, y \ge 0$

Now, for solving the above LPP by graphical method firstly we assume all the inequalities in equations.

i.e., $2x + 3y = 30$ ... (iii)

and $3x + y = 17$ ... (iv)

Table for line $2x + 3y = 30$ is

x	0	15
y	10	0

$\therefore$ Eq. (iii) passes through points (0, 10) and (15, 0).

Table for line $3x + y = 17$ is

x	0	$\frac{17}{3}$
y	17	0

$\therefore$ Eq. (iv) passes through points (0, 17) and $\left(\frac{17}{3}, 0\right)$

For determining the intersection point.

Multiplying Eq. (iv) by 3 and subtracting from Eq. (iii), we get

$$\begin{array}{rrl} 2x & +3y = & 30 \\ 9x & +3y = & 51 \\ \hline -7x & = & -21 \end{array}$$

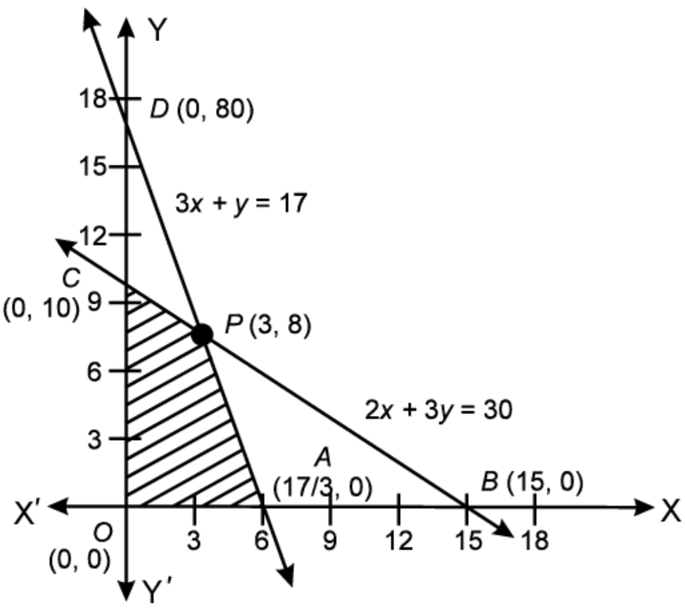
$\Rightarrow$ $x = 3$

and $y = 17 - 3(3) = 17 - 9$

$$y = 8$$

$\therefore$ Intersection point of both lines is $P(3, 8)$.

Now, plotting these points into graph paper [$\because$ Let 1 unit = 1 sq.]



Here, extreme points are O, A, P and C. Which forms a convex polygon $OAPC$.

Now, use corner point method.

Points	Maximize Function $z = 100x + 120y$
$O(0, 0)$	$100 \times 0 + 120 \times 0 = 0 + 0 = 0$
$A\left(\frac{17}{3}, 0\right)$	$100 \times \frac{17}{3} + 120 \times 0 = \frac{1700}{3} = 566.666$
$C(0, 10)$	$100 \times 0 + 120 \times 10 = 1200$
$P(3, 8)$	$100 \times 3 + 120 \times 8 = 300 + 960 = 1260$ (Maximum)

Hence, *A* manufacturer will produce 3 units of goods *A* and 8 unit of goods *B* to maximize the total revenue.

Yes, I agree with this view of the manufacturer because in our Indian constitution according to right of equality clause men and women workers are equally efficient and so should be paid at the same rate.

S29. Let the manufacturer produces x nuts and y bolts.

The given data can be put in tabular form as follows.

Item	Time on machine *A*	Time on machine *B*	Profit (in Rs.)
Nuts (x)	1 h	3 h	17.50
Bolts (y)	3 h	1 h	7.00
	$\leq$ 12 h	$\leq$ 12h	

$\therefore$ The required LPP is

Maximize profit, $P = 17.50x + 7.00y$

Subject to the constraints $x + 3y \leq 12$ ... (i)

$3x + y \leq 12$ ... (ii)

$x, y \geq 0$

Table for line $x + 3y = 12$ is

x	0	12
y	4	0

$\therefore$ Eq. (i) passes through (0, 4) and (12, 0). Table for line $3x + y = 12$ is

x	0	4
y	12	0

$\therefore$ Eq. (ii) passes through (0, 12) and (4, 0).

We need to find intersection point.

Multiplying Eq. (i) a by 3 and subtracting Eq. (ii) from Eq. (i), we get

$$\begin{array}{r} 3x + 9y = 36 \\ \underset{-}{3x} \underset{-}{+} y = \underset{-}{12} \\ \hline 8y = 24 \end{array}$$

$\Rightarrow$ $y = 3$

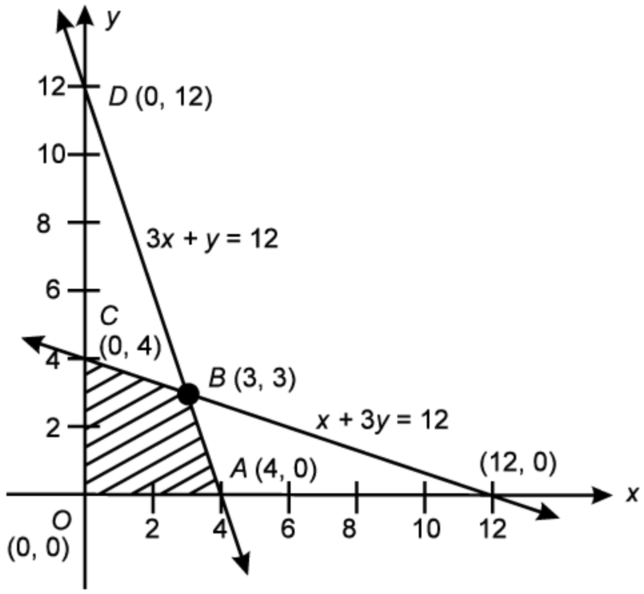
Putting $y = 3$ in Eq. (i), we get

$x + 3(3) = 12$

$\Rightarrow$ $x = 12 - 9 = 3$

$\therefore$ The point of intersection is (3, 3).

Now, the graph of the system of inequalities is given as follows



From the graph, we see that *OABC* is the feasible region.

The corner points of the feasibe region are *O*(0, 0), *C*(0, 4), *B*(3, 3) and *A*(4, 0)

Now, evaluate profit *P* at corner points.

Corner points	**Value of $P = 17.50x + 7.00y$**
O (0, 0)	$17.50(0) + 7.00(0) = 0 + 0 = 0$
A (4, 0)	$17.50(4) + 7.00(0) = 70.00 + 0 = 70.00$
B (3, 3)	$17.50(3) + 7.00(3) = 52.50 + 21.00 = 73.50$ (maximum)
C(0, 4)	$17.50(0) + 7.00(4) = 0 + 28.00 = 28.00$

Hence, the profit is maximum *i.e.*, Rs. 73.50, when he produce 3 nuts and 3 bolts each day.

S30. Let x be the executive class tickets and y be the economy class tickets. Let z be the maximum profit.

Thus the mathematical formulation of the given L.P.P. is as

Maximise $z = 1000x + 600y$

Subject to $x + y \leq 200,\ x \geq 20,\ y \geq 4x,\ x, y \geq 0.$

Consider the linear constraint defined by the inequality

$$x + y \leq 200$$

First draw the graph of the line $x + y = 200$

x	100	80
y	100	120

Putting (0, 0) in the inequality $x + y \le 200$, we have

$$0 + 0 \le 200 \quad \Rightarrow \quad 0 \le 200, \text{ which is true}$$

So the half plane of $x + y \le 200$ is towards the origin.

Now consider the linear constraint defined by the inequality

$$x \ge 20$$

First draw the graph of the line $x = 20$

Putting (0, 0) in the inequality $x \ge 20$, we have

$$0 \ge 20, \text{ which is false}$$

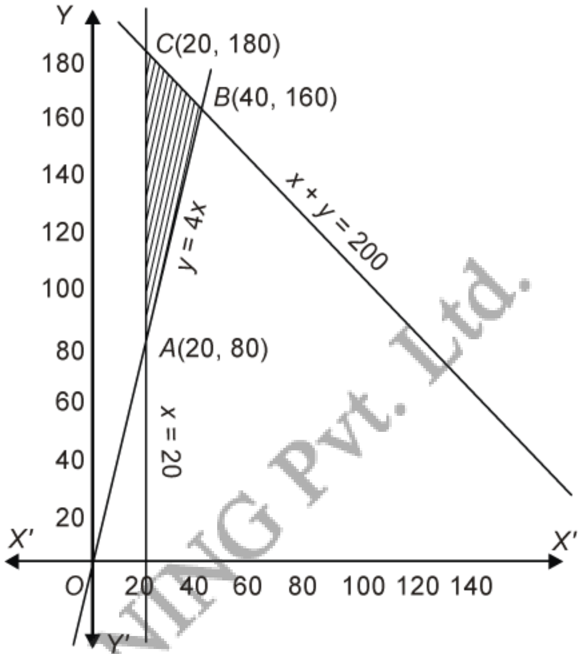
So the half plane of $x \ge 20$ is away from origin.

Now consider the linear constraint defined by the inequality

$$y \ge 4x$$

First draw the graph of the line $y = 4x$

x	10	20
y	40	80



Putting (10, 0) in the inequality $y \ge 4x$, we have

$$0 \ge 4 \times 10 \quad \Rightarrow \quad 0 \ge 40, \text{ which is false}$$

So the half plane of $y \ge 4x$ is away from the point (10, 0).

Since $\qquad x, y \ge 0$

So the feasible region lies in the first quadrant.

The coordinates of the corner points of the feasible region are A (20, 80), B (40, 160) and C (20, 180). These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $\qquad z = 1000x + 600y$

At A (20, 80) $\qquad z = 1000 \times 20 + 600 \times 80 = 20000 + 48000 = 68000$

At B (40, 160) $\qquad z = 1000 \times 40 + 600 \times 160 = 40000 + 96000 = 136000$

At C (20, 180) $\qquad z = 1000 \times 20 + 600 \times 180 = 20000 + 108000 = 128000$

Thus z is maximum at (40, 160) and maximum value = 136000

$\therefore$ Maximum profit = Rs. 136000 when 40 tickets of executive class and 160 tickets of economy class to be sold.

S31. Let x hectares for crop A and y hectares for crop B be allocated.

Now, making a table for given data.

A	B	Extreme Value
x	y	50 hec
10500	9000	Profit
20L/hec	10L/hec	800L atmost

According to the question, We need to maximize profit given by

$z = 10500x + 9000y$, subject to the constraints

$$x + y \leq 50 \quad \text{... (i)}$$

$$20x + 10y \leq 800 \quad \text{... (ii)}$$

and $$x, y \geq 0$$

First, we find all the stationary points of the inequation by converting into equation.

Table for line $x + y = 50$ is

x	0	50
y	50	0

$\therefore$ Eq. (i) passes through points, (0, 50) and (50, 0)

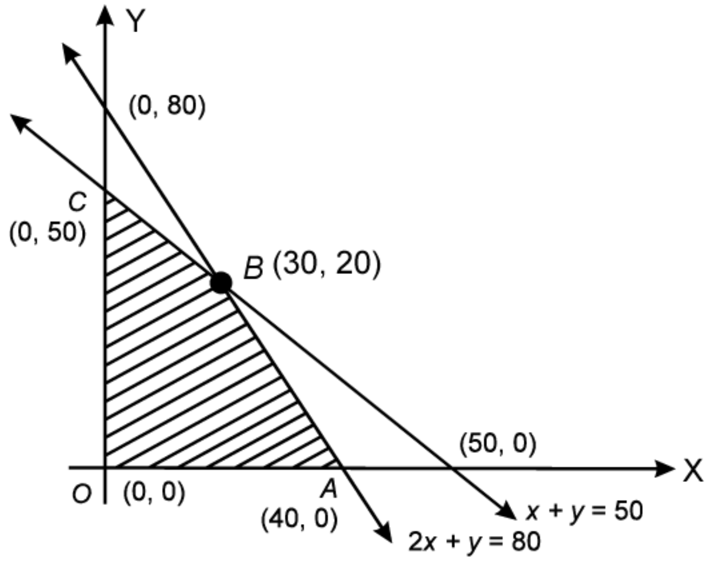
Table for line $20x + 10y = 800$

or $2x + y = 80$ is

x	0	40
y	80	0

$\therefore$ Eq (ii) passes through points (40, 0) and (0, 80) for determining the intersection point solving Eqs. (i) and (ii), we get *B* (30, 20).

Now, plot all the stationary points on graph paper, we get



From figure, we observe that *OABC* is a convex polygon because this region is bounded by extreme points and the points are *O*(0, 0), *A*(40, 0), *C*(0, 50) and *B*(30, 20).

Now we find z at various corner points.

Corner Points	Value of *z*	Nature of *z*
$O(0, 0)$	$z = 0$	
$A(40,0)$	$z = 420000$	
$B(30,20)$	$z = 315000 + 180000 = 495000$	Maximum
$C(0,50)$	$z = 450000$	

For maximize the profit, the land allocated 30 hec for crop *A* and 20 hec for crop *B*.

Yes, I agree with the message that the protection of wildlife is atmost necessary to preserve the balance in environment.

S32. Let two types of books are *x* and *y* respectively. The given data in tabular form as follows

Types of books	Thickness(cm)	Weight (kg)
x	6	1
y	4	$1\frac{1}{2} = \frac{3}{2}$
Less than or equal to	96	21

The required LPP is

Maximize books $Z = x + y$

Subject to the constraints,

$$6x + 4y \le 96 \quad \dots \text{(i)}$$

$$x + \frac{3}{2}y \le 21 \quad \dots \text{(ii)}$$

$$x,\ y \ge 0$$

Table for $6x + 4y = 96$ is

x	0	16
y	24	0

$\therefore$ It passes through the points (0, 24) and (16, 0).

Table for $x + \frac{3}{2}y = 21$

x	0	21
y	14	0

$\therefore$ It passes through points (0, 14) and (21, 0). To find the intersection point from the lines

$$6x + 4y = 96 \quad \text{... (iii)}$$

and

$$2x + 3y = 42 \quad \text{... (iv)}$$

Multiplying Eq. (iv) by 3 and subtracting Eq. (iv) from Eq. (iii), we get

$$\begin{array}{r} 6x + 4y = 96 \\ \underline{\overset{-}{}6x \overset{-}{+} 9y = \overset{-}{}126} \\ -5y = -30 \end{array}$$

$\Rightarrow$ $y = 6$

Putting, $y = 6$ in Eq. (iii), we get

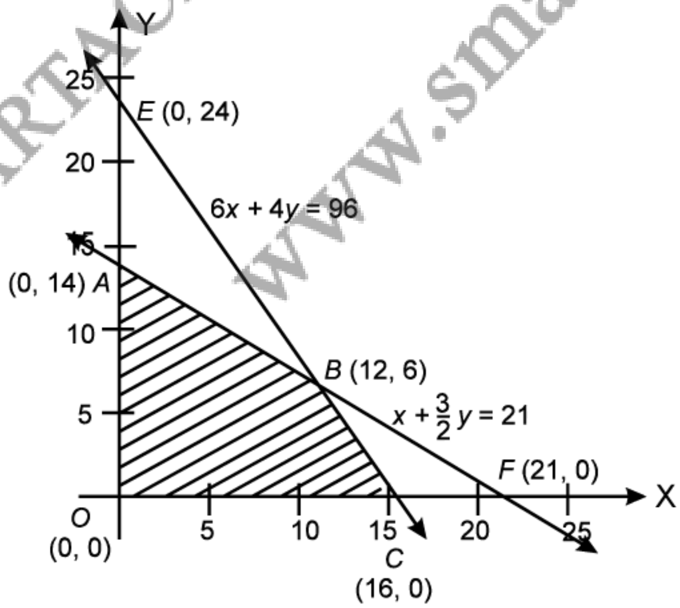
$$6x + 4(6) = 96$$

$\Rightarrow$ $6x = 96 - 24 = 72$

$\Rightarrow$ $x = 12$

$\therefore$ The point of intersection is $B(12, 6)$.

Now, the graph of above LPP is as follows



From the graph, we see that $OABC$ is the feasible region.

Now, the corner points of feasible region are $O(0, 0)$, $A(0, 14)$, $B(12, 6)$ and $C(16, 0)$ respectively.

Now, we find value of Z at various corner points.

Corner points	Value of $Z = x + y$
$O(0, 0)$	$Z = 0 + 0 = 0$
$A(0, 14)$	$Z = 0 + 14 = 14$
$B(12, 6)$	$Z = 12 + 6 = 18$ (Maximum)
$C(16, 0)$	$Z = 16 + 0 = 16$

$\therefore$ The maximum number of books are 18 at $B(12, 6)$.

$\therefore$ Books of I type is 12 and books of II type is 6.

S33. Let number of fans = x

and number of sewing machines = y

The given data can be put in the tabular form as follow

Products	Items	Costs (per item)	Profit cost
Fan (x)	1	Rs. 360	Rs. 22
Sewing machines (y)	1	Rs. 240	Rs. 18
Requirement	20	Rs. 5760	

Required LPP is

Maximize profit $Z = 22x + 18y$

Subject to the constraints, $x + y \leq 20$ [Number of items constraints] ... (i)

$360x + 240y \leq 5760$ [Cost constraints] ... (ii)

$x, y \geq 0$

Table for line $x + y = 20$ is

x	0	20
y	20	0

$\therefore$ It passes through the points (0, 20) and (20, 0). Table for line $360x + 240y = 5760$ is

x	0	16
y	24	0

∴ It passes through the points (0, 24) and (16, 0).

To find point of intersection.

Multiplying Eq. (i) by 360 and subtracting Eq. (ii) from Eq. (i), we get

$$\begin{array}{rcr} 360x + 360y &=& 7200 \\ \underline{-\ 360x \pm 240y} &=& \underline{-\ 5760} \\ 120y &=& 1440 \end{array}$$

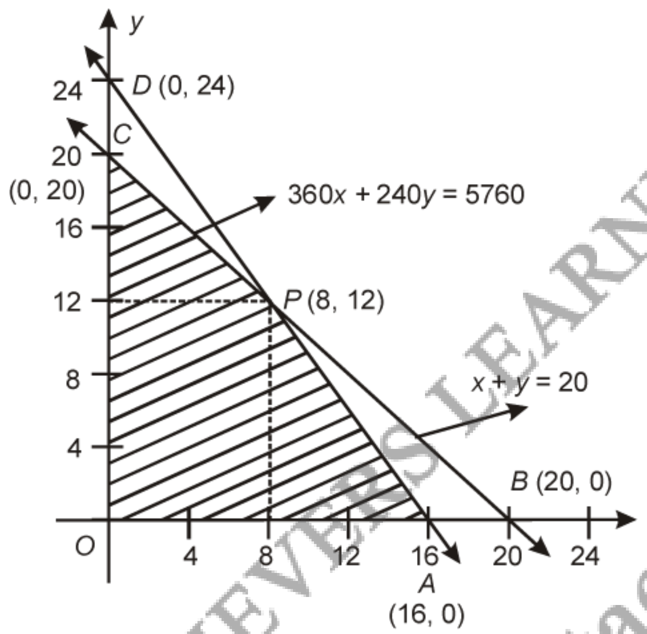
⇒ $y = 12$

Putting $y = 12$ in Eq. (i), we get

$$x + 12 = 20 \Rightarrow x = 8$$

∴ Point of intersection is $P(8, 12)$.

Now, we plot the graph and find the corner points of feasible region.



Hence, $OAPC$ is the feasible region.

∴ The corner points of feasible region are $O(0, 0)$, $A(16, 0)$, $P(8, 12)$ and $C(0, 20)$. Now, we find Z at various corner points.

Corner points	$Z = 22x + 18y$
$O(0, 0)$	$22(0) + 18(0) = 0$
$A(16, 0)$	$352 + 0 = 352$
$P(8, 12)$	$176 + 216 = 392$ (Maximum)
$C(0, 20)$	$0 + 360 = 360$

∴ Maximum profit, Z = Rs. 392 at $x = 8$, $y = 12$.

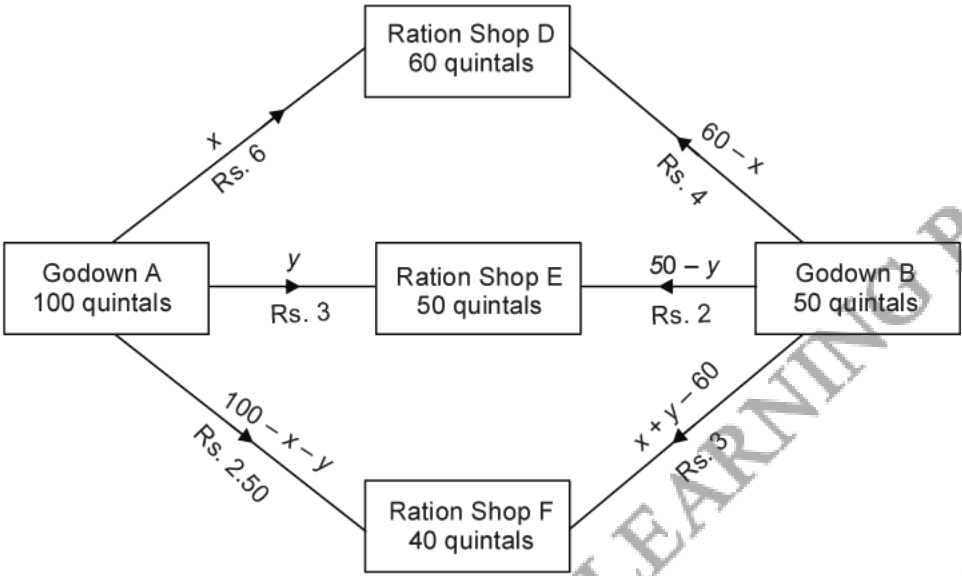
S34. Let the godown A transports x quintals of grain to ration shop D and y quintals of grain to ration shop E.

Since the total capacity of godown A is 100 quintals, so the remaining $(100 - x - y)$ quintals of grain can be transported to ration shop F.

Now the requirement of ration shop D is 60 quintals, out of which x quintals are transported from godown A. The remaining $(60 - x)$ quintals will be transported from godown B.

Also the requirement of ration shop E is 50 quintals, out of which y quintals are transported from godown A. The remaining $(50 - y)$ quintals will be transported from godown B.

Since the total capacity of godown B is 50 quintals, so the remaining $50 - (60 - x + 50 - y) = (x + y - 60)$ quintals can be transported to ration shop F.



Let z be the total cast of transportation, then

$$z = 6x + 3y + 2.50(100 - x - y) + 4(60 - x) + 2(50 - y) + 3(x + y - 60)$$
$$= 6x + 3y + 250 - 2.50x - 2.50y + 240 - 4x + 100 - 2y + 3x + 3y - 180$$
$$= 2.50x + 1.50y + 410.$$

Thus the mathematical formulation of the given L.P.P. is as

Minimise $z = 2.50x + 1.50y + 410$

subject to $x + y \le 100$, $x + y \ge 60$, $x \le 60$, $y \le 50$, $x, y \ge 0$.

Consider the linear constraint defined by the inequality

$$x + y \le 100$$

First draw the graph of the line $x + y = 100$

x	50	40
y	50	60

Putting (0, 0) in the inequality $x + y \le 100$, we have

$0 + 0 \leq 100 \quad \Rightarrow \quad 0 \leq 100$, which is true

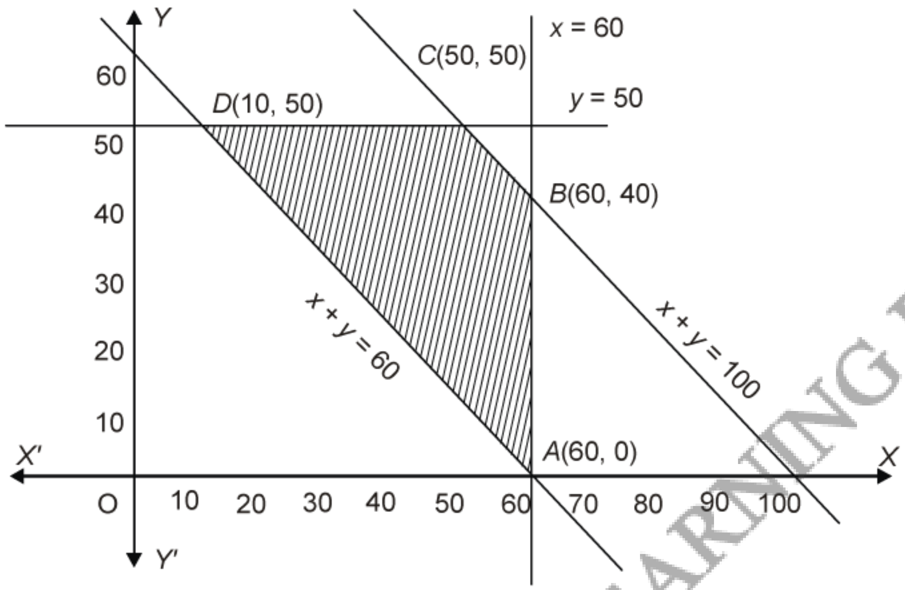
So the half plane of $x + y \leq 100$ is towards the origin

Now consider the linear constraint defined by the inequality

$$x + y \geq 60$$

First draw the graph of the line $x + y = 60$

x	20	30
y	40	30



Putting (0, 0) in the inequality $x + y \geq 60$, we have

$$0 + 0 \geq 60 \quad \Rightarrow \quad 0 \geq 60, \text{ which is false}$$

So the half plane of $x + y \geq 60$ is away from origin.

Now consider the linear constraint defined by the inequality

$$x \leq 60$$

First draw the graph of the line $x = 60$

Putting (0, 0) in the inequality $x \leq 60$, we have

$$0 \leq 60, \text{ which is true}$$

So the half plane of $x \leq 60$ is towards the origin.

Now consider the linear constraint defined by the inequality

$$y \leq 50$$

First draw the graph of the line $y = 50$

Putting (0, 0) in the inequality $y \leq 50$, we have

$$0 \leq 50, \text{ which is true}$$

So the half plane of $x \leq 50$ is towards the origin.

Now consider the linear constraint defined by the inequality

Since $x,\ y \geq 0$

So the feasible region lies in the first quadrant.

The coordinates of the corner points of the feasible region are $A(60, 0)$, $B(60, 40)$, $C(50, 50)$ and $D(10, 50)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $z = 2.50x + 1.50y + 410$

At $A(60, 0)$ $z = 2.50 \times 60 + 1.50 \times 0 + 410 = 150 + 0 + 410 = 560$

At $B(60, 40)$ $z = 2.50 \times 60 + 1.50 \times 40 + 410 = 150 + 60 + 410 = 620$

At $C(50, 50)$ $z = 2.50 \times 50 + 1.50 \times 50 + 410 = 125 + 75 + 410 = 610$

At $D(10, 50)$ $z = 2.50 \times 10 + 1.50 \times 50 + 410 = 25 + 75 + 410 = 510$

Thus z is minimum at (10, 50) and minimum value = 510.

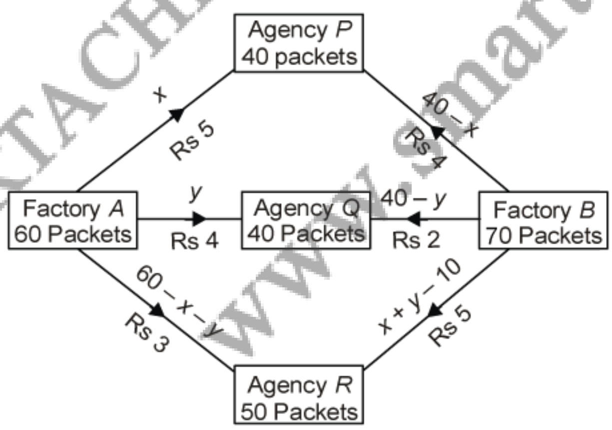
So the minimum transportation cost is Rs. 510 when 10 quintals, 50 quintals, 40 quintals are transported from godown A and 50 quintals, 0 quintal, 0 quintal are transported from godown B to ration shops D, E and F respectively.

S35. Let factory A transport x packets of medicines to agency P and y packets of medicines to agency Q.

Since the total production capacity of factory A is 60 packets, so the remaining $(60 - x - y)$ packets of medicines can be transported to agency R.

Now the requirement of agency P is 40 packets, out of which x packets are transported from factory A. The remaining $(40 - x)$ packets will be transported from factory B.

Also the requirement of agency Q is 40 packets, out of which y packets are transported from factory A. The remaining $(40 - y)$ packets will be transported from factory B. Since the total production capacity of factory B is 70 packets, so the remaining $70 - (40 - x + 40 - y) = (x + y - 10)$ packets can be transported to agency R.



Let z be the total cast of transportation, then

$$z = 5x + 4y + 3\,(60 - x - y) + 4(40 - x) + 2(40 - y) + 5(x + y - 10)$$
$$= 5x + 4y + 180 - 3x - 3y + 160 - 4x + 80 - 2y + 5x + 5y - 50$$
$$= 3x + 4y + 370$$

Thus the mathematical formulation of the given L.P.P. is as

$$\text{Minimise} \quad z = 3x + 4y + 370$$

$$\text{subject to} \quad x + y \leq 60, \quad x + y \geq 10, \quad x \leq 40, \quad y \leq 40, \quad x, y \geq 0.$$

Consider the linear constraint defined by the inequality

$$x + y \leq 60$$

First draw the graph of the line $x + y = 60$

x	40	20
y	20	40

Putting (0, 0) in the inequality $x + y \leq 60$, we have

$$0 + 0 \leq 60 \quad \Rightarrow \quad 0 \leq 60, \text{ which is true}$$

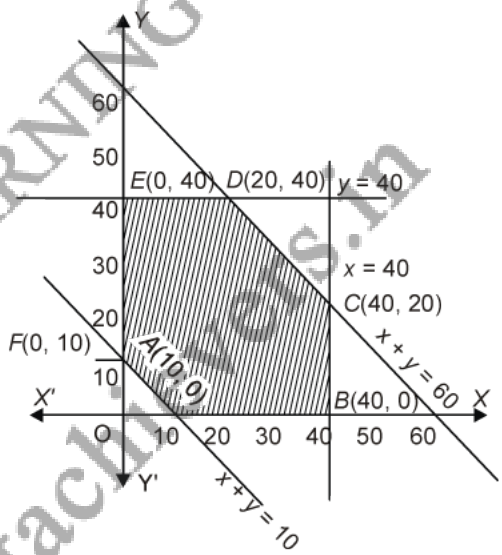
So the half plane of $x + y \leq 60$ is towards the origin

Now consider the linear constraint defined by the inequality

$$x + y \geq 10$$

First draw the graph of the line $x + y = 10$

x	5	6
y	5	4



Putting (0, 0) in the inequality $x + y \geq 10$, we have

$$0 + 0 \geq 10 \quad \Rightarrow \quad 0 \geq 10, \text{ which is false}$$

So the half plane of $x + y \geq 10$ is away from origin.

Now consider the linear constraint defined by the inequality

$$x \leq 40$$

First draw the graph of the line $x = 40$

Putting (0, 0) in the inequality $x \leq 40$, we have

$$0 \leq 40, \text{ which is true}$$

So the half plane of $x \leq 40$ is towards the origin.

Now consider the linear constraint defined by the inequality

$$y \leq 40$$

First draw the graph of the line $y = 40$

Putting (0, 0) in the inequality $y \leq 40$, we have

$$0 \leq 40, \text{ which is true}$$

So the half plane of $y \leq 40$ is towards the origin.

Since $\qquad x, y \geq 0$

So the feasible region lies in the first quadrant.

The coordinates of the corner points of the feasible region are $A(10, 0)$, $B(40, 0)$, $C(40, 20)$, $D(20, 40)$, $E(0, 40)$ and $F(0, 10)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $\quad z = 3x + 4y + 370$

At	$A(10, 0)$	$z = 3 \times 10 + 4 \times 0 + 370 = 30 + 0 + 370 = 400$
At	$B(40, 0)$	$z = 3 \times 40 + 4 \times 0 + 370 = 120 + 0 + 370 = 490$
At	$C(40, 20)$	$z = 3 \times 40 + 4 \times 20 + 370 = 120 + 80 + 370 = 570$
At	$D(20, 40)$	$z = 3 \times 20 + 4 \times 40 + 370 = 60 + 160 + 370 = 590$
At	$E(0, 40)$	$z = 3 \times 0 + 4 \times 40 + 370 = 0 + 160 + 370 = 530$
At	$F(0, 10)$	$z = 3 \times 0 + 4 \times 10 + 370 = 0 + 40 + 370 = 410$

Thus z is minimum at (10, 0) and minimum value = 400.

So the minimum transportation cost is Rs. 400 when 10 packets, 0 packet, 50 packets are transported from factory A and 30 packets, 40 packets, 0 packet are transported from factory B to agencies P, Q and R respectively.

S36. Let investment in first type of bond is Rs. x and investment in second type of bond is Rs. y. Let z be the maximum return.

Thus the mathematical formulation of the given L.P.P. is as

$$\text{Maximize } z = \frac{8}{100}x + \frac{10}{100}y \quad i.e., \quad z = 0.08x + 0.1y$$

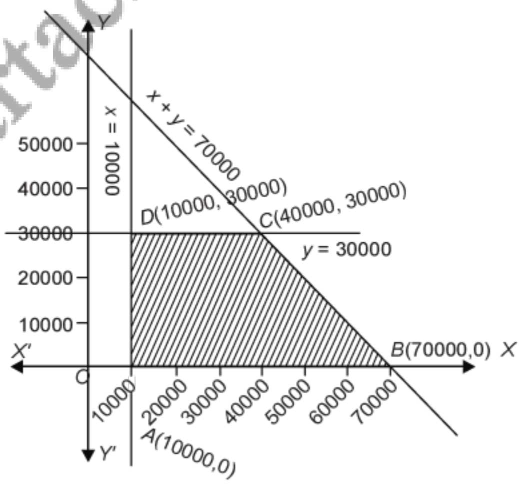
$$\text{subject to } x + y \leq 70000, \quad x \geq 10000, \; y \leq 30000, \quad x, y \geq 0$$

Now consider the linear constraint defined by the inequality

$$x + y \leq 70000$$

First draw the graph of the line $x + y = 70000$

x	20000	30000
y	50000	40000



Putting (0, 0) in the inequality $x + y \leq 70000$, we have

$$0 + 0 \leq 70000 \quad \Rightarrow \quad 0 \leq 70000, \text{ which is true}$$

So the half plane of $x + y \leq 70000$ is towards the origin.

Now consider the linear constraint defined by the inequality

$$x \leq 10000$$

First draw the graph of the line $x = 10000$

Putting (0, 0) in the inequality $x \geq 10000$, we have

$$0 \geq 10000, \text{ which is false}$$

So the half plane of $x \geq 10000$ is away from origin.

Now consider the linear constraint defined by the inequality

$$y \leq 30000$$

First draw the graph of the line $y = 30000$

Putting (0, 0) in the inequality $y \leq 30000$, we have

$$0 \leq 30000, \text{ which is true}$$

So the half plane of $y \leq 30000$ is towards the origin.

Since $\quad x, y \geq 0$

So the feasible region lies in the first quadrant.

The coordinates of the corner points of the feasible region are A (10000, 0), B (70000, 0), C (40000, 30000) and D (10000, 30000). These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $\quad z = 0.08x + 0.1y$

At A (10000, 0) $\quad z = 0.08 \times 10000 + 0.1 \times 0 = 800 + 0 = 800$

At B (70000, 0) $\quad z = 0.08 \times 70000 + 0.1 \times 0 = 5600 + 0 = 5600$

At C (40000, 30000) $\quad z = 0.08 \times 40000 + 0.1 \times 30000 = 3200 + 3000 = 6200$

At D (10000, 30000) $\quad z = 0.08 \times 10000 + 0.1 \times 30000 = 800 + 3000 = 3800$

Thus z is maximum at (40000, 30000) and maximum value = 6200.

$\therefore$ Maximum return = Rs. 6200 when Rs. 40000 invested in first type bond and Rs. 30000 invested in second type bond.

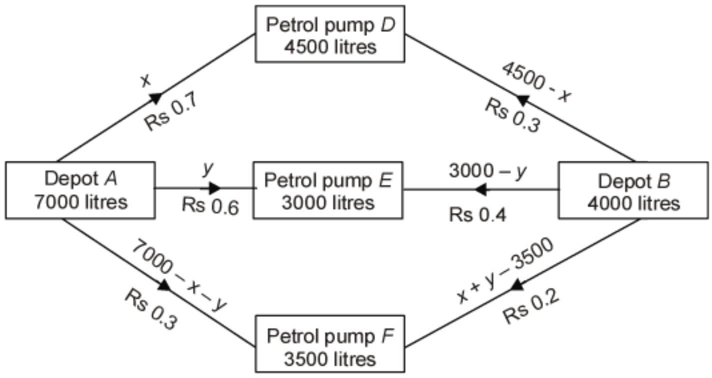
S37. Let the depot A transports x litres of oil to petrol pump D and y litres of oil to petrol pump E.

Since the total capacity of depot A is 7000 litres, so the remaining $(7000 - x - y)$ litres of oil can be transported to petrol pump F.

Now the requirement of petrol pump D is 4500 litres, out of which x litres are transported from depot A. The remaining $(4500 - x)$ litres will be transported from depot B.

Also the requirement of petrol pump E is 3000 litres, out of which y litres are transported from depot A. The remaining $(3000 - y)$ litres will be transported from depot B.

Since the total capacity of depot B is 4000 litres, so the remaining $4000 - (4500 - x + 3000 - y) = x + y - 3500$ litres can be transported to petrol pump F.



Let z be the total cast of transportation, then

$$z = 0.7x + 0.6y + 0.3(7000 - x - y) + 0.3(4500 - x) + 0.4(3000 - y) + 0.2(x + y - 3500)$$

$$= 0.7x + 0.6y + 2100 - 0.3x - 0.3y + 1350 - 0.3x + 1200 - 0.4y + 0.2x + 0.2y - 700$$

$$= 0.3x + 0.1y + 3950.$$

Thus the mathematical formulation of the given L.P.P. is as

Minimise $z = 0.3x + 0.1y + 3950$

subject to $x + y \le 7000$, $x + y \ge 3500$, $x \le 4500$, $y \le 3000$, $x, y \ge 0$.

Consider the linear constraint defined by the inequality

$$x + y \le 7000$$

First draw the graph of the line $x + y = 7000$

x	4000	3000
y	3000	4000

Putting (0, 0) in the inequality $x + y \le 7000$, we have

$0 + 0 \le 7000 \quad \Rightarrow \quad 0 \le 7000$, which is true

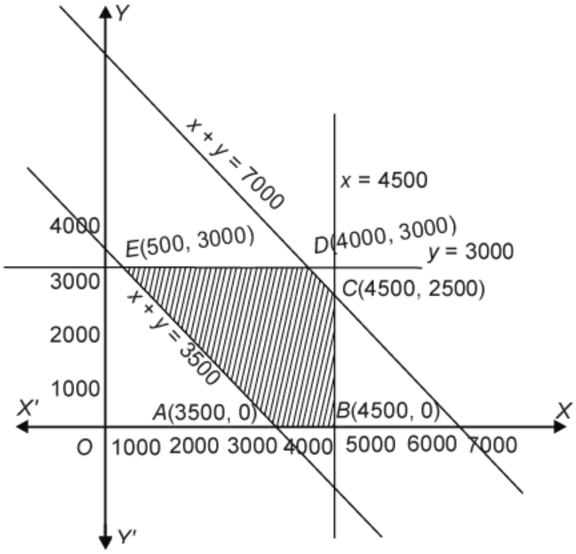
So the half plane of $x + y \le 7000$ is towards the origin

Now consider the linear constraint defined by the inequality

$$x + y \ge 3500$$

First draw the graph of the line $x + y = 3500$

x	1500	2000
y	2000	1500



Putting (0, 0) in the inequality $x + y \geq 3500$, we have

$$0 + 0 \geq 3500 \quad \Rightarrow \quad 0 \geq 3500, \text{ which is false}$$

So the half plane of $x + y \geq 3500$ is away from origin.

Now consider the linear constraint defined by the inequality

$$x \leq 4500$$

First draw the graph of the line $x = 4500$

Putting (0, 0) in the inequality $x \leq 4500$, we have

$$0 \leq 4500, \text{ which is true}$$

So the half plane of $x \leq 4500$ is towards the origin.

Now consider the linear constraint defined by the inequality

$$y \leq 3000$$

First draw the graph of the line $y = 3000$

Putting (0, 0) in the inequality $y \leq 3000$, we have

$$0 \leq 3000, \text{ which is true}$$

So the half plane of $x \leq 3000$ is towards the origin.

Now consider the linear constraint defined by the inequality

Since $x, y \geq 0$

So the feasible region lies in the first quadrant.

The coordinates of the corner points of the feasible region are $A(3500, 0)$, $B(4500, 0)$, $C(4500, 2500)$, $D(4000, 3000)$ and $E(500, 3000)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now $z = 0.3x + 0.1y + 3950$

At $A(3500, 0)$ $z = 0.3 \times 3500 + 0.1 \times 0 + 3950 = 1050 + 0 + 3950 = 5000$

At $B(4500, 0)$ $z = 0.3 \times 4500 + 0.1 \times 0 + 3950 = 1350 + 0 + 3950 = 5300$

At $C(4500, 2000)$ $z = 0.3 \times 4500 + 0.1 \times 2000 + 3950 = 1350 + 200 + 3950 = 5500$

At $D(4000, 3000)$ $z = 0.3 \times 4000 + 0.1 \times 3000 + 3950 = 1200 + 300 + 3950 = 5450$

At $E(500, 3000)$ $z = 0.3 \times 500 + 0.1 \times 3000 + 3950 = 150 + 300 + 3950 = 4400$

Thus z is minimum at (500, 3000) and minimum value = 4400.

So the minimum transportation cost is Rs. 4400 when 500 litres, 3000 litres, 3500 litres are transported from depot *A* and 4000 litres, 0 litre, 0 litre are transported from depot *B* to petrol pumps *D*, *E* and *F* respectively.