Date: 19/10/2021

Q1. Find the unit vector in the direction of sum of vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{j} + \hat{k}$.

Q2. Find the angle between the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} + 4\hat{j} - \hat{k}$.

Q3. Find a unit vector in the direction of $\overrightarrow{PQ}$, where P and Q have cooordinates (5, 0, 8) and (3, 3, 2), respectively.

Q4. If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$, then find the unit vector in the direction of $2\vec{a} - \vec{b}$.

Q5. If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$, then find the unit vector in the direction of $6\vec{b}$.

Q6. If $\vec{a}$ and $\vec{b}$ are the position vectors of $\vec{A}$ and $\vec{B}$ respectively, then find the position vector of a point $\vec{C}$ in $\overrightarrow{BA}$ produced such that $\overrightarrow{BC} = 1.5\overrightarrow{BA}$.

Q7. Prove that in any $\triangle ABC$, $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$, where a, b and c are the magnitudes of the sides opposite to the vertices A, B and C, respectively.

Q8. Find a vector of magnitude 11 in the direction opposite to that of $\overrightarrow{PQ}$, where P and Q are the points (1, 3, 2) and (– 1, 0, 8), respectively.

Q9. Find the position vector of a point R which divides the line joining the two points P and Q with position vectors $\overrightarrow{OP} = 2\vec{a} + \vec{b}$ and $\overrightarrow{OQ} = \vec{a} - 2\vec{b}$, respectively in the ration 1 : 2 (i) internally and (ii) externally.

Q10. If the points (– 1, – 1, 2), (2, m, 5) and (3, 11, 6) are collinear, find the value of m.

Q11. Find all vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane of $\hat{i} + 2\hat{j} + \hat{k}$ and $-\hat{i} + 3\hat{j} + 4\hat{k}$.

Q12. Find a vector $\vec{r}$ of magnitude $3\sqrt{2}$ units which makes an angle of $\dfrac{\pi}{4}$ and $\dfrac{\pi}{2}$ with y and z-axis, respectively.

Q13. Using vectors, prove that the parallelogram on the same base and between the same parallels are equal in area.

Q14. If A, B, C and D are the points with position vectors $\hat{i} + \hat{j} - \hat{k}$, $2\hat{i} - \hat{j} + 3\hat{k}$, $2\hat{i} - 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$, respectively, then find the projection of $\vec{AB}$ along $\vec{CD}$.

Q15. Find the sine of the angle between the vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$.

Q16. Find a vector of magnitude 6, which is perpendicular to both the vectors $2\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} - \hat{j} + 3\hat{k}$.

Q17. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{c} = \hat{i} + 3\hat{j} - \hat{k}$, find λ such that $\vec{a}$ is perpendicular to $\lambda\vec{b} + \vec{c}$.

Q18. Using vectors, find the value of k, such that the points (k, – 10, 3), (1, – 1, 3) and (3, 5, 3) are collinear.

Q19. Prove that in a ΔABC, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$, where a, b, c represent the magnitudes of the sides opposite to vertices A, B, C respectively.

Q20. Using vectors, prove that $\cos(A - B) = \cos A \cos B + \sin A \sin B$.

Q21. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.

Q22. Show that area of the parallelogram whose diagonals are given by $\vec{a}$ and $\vec{b}$ is $\frac{|\vec{a} \times \vec{b}|}{2}$. Also, find the area of the parallelogram, whose diagonals are $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$.

Q23. If a vector $\vec{r}$ has magnitude 14 and direction ratios 2, 3 and -6. Then, find the direction cosines and components of $\vec{r}$, given that $\vec{r}$ makes an acute angle with X-axis.

Q24. A vector $\vec{r}$ is inclined at equal angles to the three axes. If the magnitude of $\vec{r}$ is $2\sqrt{3}$ units, then find the value of $\vec{r}$.

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S1. Let $\vec{c}$ denote the sum $\vec{a}$ and $\vec{b}$.

We have, $$\vec{c} = \vec{a} + \vec{b}$$

$$= 2\hat{i} - \hat{j} + \hat{k} + 2\hat{j} + \hat{k} = 2\hat{i} + \hat{j} + 2\hat{k}$$

$\therefore$ Unit vector in the direction of $\vec{c} = \dfrac{\vec{c}}{|\vec{c}|} = \dfrac{2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{2^2 + 1^2 + 2^2}} = \dfrac{2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{9}}$

$$\vec{c} = \frac{2\hat{i} + \hat{j} + 2\hat{k}}{3}.$$

S2. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 3\hat{i} + 4\hat{j} - \hat{k}$

We know that, angle between two vectors $\vec{a}$ and $\vec{b}$ is given by

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{(2\hat{i} - \hat{j} + \hat{k})(3\hat{i} + 4\hat{j} - \hat{k})}{\sqrt{4+1+1}\sqrt{9+16+1}}$$

$$= \frac{6-4-1}{\sqrt{6}\sqrt{26}} = \frac{1}{2\sqrt{39}}$$

$$\therefore \qquad \theta = \cos^{-1}\left(\frac{1}{2\sqrt{39}}\right).$$

S3. Since, the coordinates of *P* and *Q* are (5, 0, 8) and (3, 3, 2), respectively.

$$\therefore \qquad \overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$= (3\hat{i} + 3\hat{j} + 2\hat{k}) - (5\hat{i} + 0\hat{j} + 8\hat{k})$$

$$= -2\hat{i} + 3\hat{j} - 6\hat{k}$$

$\therefore$ Unit vector in the direction of $\overrightarrow{PQ} = \dfrac{\overrightarrow{PQ}}{\left|\overrightarrow{PQ}\right|} = \dfrac{-2\hat{i} + 3\hat{j} - 6\hat{k}}{\sqrt{2^2 + 3^2 + 6^2}}$

$$= \frac{-2\hat{i}+3\hat{j}-6\hat{k}}{\sqrt{49}} = \frac{-2\hat{i}+3\hat{j}-6\hat{k}}{7}.$$

S4. Here, $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$

Since, $2\vec{a} - \vec{b} = 2(\hat{i} + \hat{j} + 2\hat{k}) - (2\hat{i} + \hat{j} - 2\hat{k})$

$$= 2\hat{i} + 2\hat{j} + 4\hat{k} - 2\hat{i} - \hat{j} + 2\hat{k} = \hat{j} + 6\hat{k}$$

$\therefore$ Unit vector in the direction of $2\vec{a} - \vec{b} = \frac{2\vec{a}-\vec{b}}{|2\vec{a}-\vec{b}|} = \frac{\hat{j}+6\hat{k}}{\sqrt{1+36}} = \frac{1}{\sqrt{37}}(\hat{j}+6\hat{k})$.

S5. Here, $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$

Since, $6\vec{b} = 12\hat{i} + 6\hat{j} - 12\hat{k}$

$\therefore$ Unit vector in the direction of $6\vec{b} = \frac{6\vec{b}}{|6\vec{b}|}$

$$= \frac{12\hat{i}+6\hat{j}-12\hat{k}}{\sqrt{12^2+6^2+12^2}} = \frac{6(2\hat{i}+\hat{j}-2\hat{k})}{\sqrt{324}}$$

$$= \frac{6(2\hat{i}+\hat{j}-2\hat{k})}{18} = \frac{2\hat{i}+\hat{j}-2\hat{k}}{3}.$$

S6. Since, $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OB} = \vec{b}$

$\therefore$ $\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = \vec{a} - \vec{b}$

and $1.5\overrightarrow{BA} = 1.5(\vec{a} - \vec{b})$

Similarly, $\overrightarrow{BC} = 1.5\overrightarrow{BA} = 1.5(\vec{a} - \vec{b})$

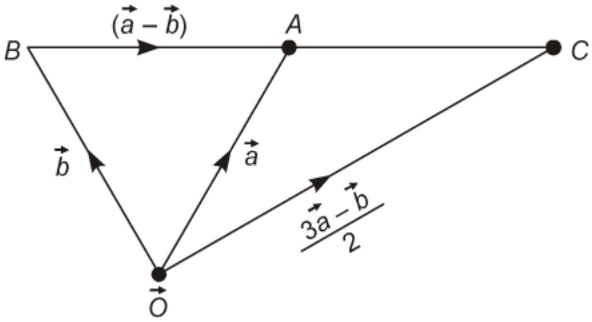
$$\overrightarrow{OC} - \overrightarrow{OB} = 1.5\vec{a} - 1.5\vec{b}$$

$$\overrightarrow{OC} = 1.5\vec{a} - 1.5\vec{b} + \vec{b}$$

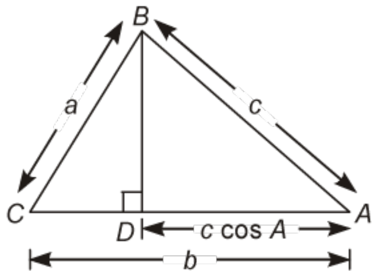
$$= 1.5\vec{a} - 0.5\vec{b}$$

$$= \frac{3\vec{a}-\vec{b}}{2}$$

Graphically, explanation of the above solution is given below:



S7. Here, components of C are $c \cos A$ and $c \sin A$ is drawn.



Since, $\vec{CD} = b - c\cos A$

In ΔBDC, $a^2 = (b - c\cos A)^2 + (c\sin A)^2$

$$a^2 = b^2 + c^2\cos^2 A - 2bc\cos A + c^2\sin^2 A$$

$\Rightarrow$ $2bc\cos A = b^2 - a^2 + c^2(\cos^2 A + \sin^2 A)$

$\Rightarrow$ $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

S8. The vector with initial point $P(1, 3, 2)$ and terminal point $Q(-1, 0, 8)$ is given by

$$\overrightarrow{PQ} = (-1-1)\hat{i} + (0-3)\hat{j} + (8-2)\hat{k}$$

$$= -2\hat{i} - 3\hat{j} + 6\hat{k}$$

Thus, $\overrightarrow{QP} = -\overrightarrow{PQ} = 2\hat{i} + 3\hat{j} - 6\hat{k}$

$=$ $|\overrightarrow{QP}| = \sqrt{2^2 + 3^2 + (-6)^2}$

$$= \sqrt{4 + 9 + 26} = \sqrt{49} = 7$$

Therefore, unit vector in the direction of $\overrightarrow{QP}$ is given

$$\widehat{QP} = \frac{\overrightarrow{QP}}{|\overrightarrow{QP}|} = \frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{7}$$

Hence, the required vector of magnitude 11 in direction of $\overrightarrow{QP}$ is

$$\widehat{QP} = 11\left(\frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{7}\right) = \frac{22}{7}\hat{i} + \frac{33}{7}\hat{j} - \frac{66}{7}\hat{k}.$$

S9. (i) The position vector of the point R dividing the join of P and Q internally in the ratio 1 : 2 is given by

$$\overrightarrow{OR} = \frac{2(2\vec{a}+\vec{b})+1(\vec{a}-2\vec{b})}{1+2} = \frac{5\vec{a}}{3}.$$

(ii) The position vector of the point R' dividing the join of P and Q in the ratio 1 : 2 externally is given by

$$\overrightarrow{OR'} = \frac{2(2\vec{a}+\vec{b})-1(\vec{a}-2\vec{b})}{2-1} = 3\vec{a}+4\vec{b}.$$

S10. Let the given points be $A(-1, -1, 2)$, $B(2, m, 5)$ and $C(3, 11, 6)$. Then

$$\overrightarrow{AB} = (2+1)\hat{i} + (m+1)\hat{j} + (5-2)\hat{k} = 3\hat{i} + (m+1)\hat{j} + 3\hat{k}$$

and $$\overrightarrow{AC} = (3+1)\hat{i} + (11+1)\hat{j} + (6-2)\hat{k} = 4\hat{i} + 12\hat{j} + 4\hat{k}$$

Since, A, B, C are collinear, we have $\overrightarrow{AB} = \lambda\overrightarrow{AC}$, *i.e.*,

$$(3\hat{i} + (m+1)\hat{j} + 3\hat{k}) = \lambda(4\hat{i} + 12\hat{j} + 4\hat{k})$$

$\Rightarrow$ $3 = 4\lambda$ and $m + 1 = 12\lambda$

Therefore, $m = 8$.

S11. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$. Then

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ -1 & 3 & 4 \end{vmatrix}$$

$$= \hat{i}(8-3) - \hat{j}(4+1) + \hat{k}(3+2)$$

$$= 5\hat{i} - 5\hat{j} + 5\hat{k}$$

$\Rightarrow$ $$|\vec{a}\times\vec{b}| = \sqrt{(5)^2+(-5)^2+(5)^2} = \sqrt{3(5)^2} = 5\sqrt{3}.$$

Therefore, unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$ is given by

$$\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{5\hat{i} - 5\hat{j} + 5\hat{k}}{5\sqrt{3}}$$

Hence, vectors of magnitude of $10\sqrt{3}$ that are perpendicular to plane of $\vec{a}$ and $\vec{b}$ are $\pm 10\sqrt{3}\left(\frac{5\hat{i} - 5\hat{j} + 5\hat{k}}{5\sqrt{3}}\right)$, *i.e.*, $\pm 10(\hat{i} - \hat{j} + \hat{k})$

S12. Here, $m = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ and $n = \cos\frac{\pi}{2} = 0$

Since, $l^2 + m^2 + n^2 = 1$

$$l^2 + \frac{1}{2} + 0 = 1$$

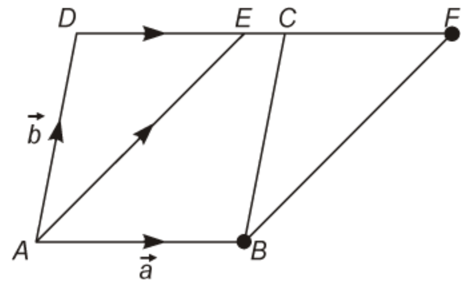
$\Rightarrow$ $$l = \pm\frac{1}{\sqrt{2}}$$

Hence, the required vector $\vec{r} = 3\sqrt{2}\,(l\hat{i} + m\hat{j} + n\hat{k})$ is given by

$$\vec{r} = 3\sqrt{2}\left(\pm\frac{1}{\sqrt{2}}\,\hat{i} + \frac{1}{\sqrt{2}}\,\hat{j} + 0\hat{k}\right) = \vec{r} = \pm 3\hat{i} + 3\hat{j}.$$

S13. Let *ABCD* and *ABFE* are parallelograms on the same base *AB* and between the same parallel lines *AB* and *DF*.

Here, *AB* || *CD* and *AE* || *BF*



Let $\vec{AB} = \vec{a}$ and $\vec{AD} = \vec{b}$

$\therefore$ Area of parallelogram $ABCD = \vec{a} \times \vec{b}$

Now, Area of parallelogram $ABFF = \vec{AB} \times \vec{AE}$

$= \vec{AB} \times (\vec{AD} + \vec{DE})$

$= \vec{AB} \times (\vec{b} + k\vec{a})$ [Let $\vec{DE} = k\vec{a}$, where k is a scalar]

$= \vec{a} \times (\vec{b} + k\vec{a})$

$= (\vec{a} \times \vec{b}) + (\vec{a} \times k\vec{a})$

$= (\vec{a} \times \vec{b}) + k\,(\vec{a} \times \vec{a})$

$= (\vec{a} \times \vec{b})$

= Area of parallelogram *ABCD*. **Hence proved.**

S14. Here, $\vec{OA} = \hat{i} + \hat{j} - \hat{k}$, $\vec{OB} = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{OC} = 2\hat{i} - 3\hat{k}$ and $\vec{OD} = 3\hat{i} - 2\hat{j} + \hat{k}$

$\therefore$ $\vec{AB} = \vec{OB} - \vec{OA}$

$= (2-1)\hat{i} + (-1-1)\hat{j} + (3+1)\hat{k}$

$= \hat{i} - 2\hat{j} + 4\hat{k}$

and $\vec{CD} = \vec{OD} - \vec{OC}$

$= (3-2)\hat{i} + (-2-0)\hat{j} + (1+3)\hat{k}$

$= \hat{i} - 2\hat{j} + 4\hat{k}$

So, the projection of $\vec{AB}$ along $\vec{CD} = \vec{AB} \cdot \frac{\vec{CD}}{|\vec{CD}|}$

$$= \frac{(\hat{i} - 2\hat{j} + 4\hat{k}) \cdot (\hat{i} - 2\hat{j} + 4\hat{k})}{\sqrt{1^2 + 2^2 + 4^2}}$$

$$= \frac{1 + 4 + 16}{\sqrt{21}} = \frac{21}{\sqrt{21}}$$

$$= \sqrt{21} \text{ units.}$$

S15. Here, $a_1 = 3$, $a_2 = 1$, $a_3 = 2$ and $b_1 = 2$, $b_2 = -2$, $b_3 = 4$

We know that,

$$\cos\theta = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}}$$

$$= \frac{3 \times 2 + 1 \times (-2) + 2 \times 4}{\sqrt{3^2 + 1^2 + 2^2}\sqrt{2^2 + (-2)^2 + 4^2}}$$

$$= \frac{6 - 2 + 8}{\sqrt{14}\sqrt{24}} = \frac{12}{2\sqrt{14}\sqrt{6}} = \frac{6}{\sqrt{84}} = \frac{6}{2\sqrt{21}} = \frac{3}{\sqrt{21}}$$

$\therefore$ $$\sin\theta = \sqrt{1 - \cos^2\theta}$$

$$= \sqrt{1 - \frac{9}{21}} = \sqrt{\frac{12}{21}} = \frac{2\sqrt{3}}{\sqrt{3}\sqrt{7}} = \frac{2}{\sqrt{7}}$$

S16. Let $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 4\hat{i} - \hat{j} + 3\hat{k}$

So, any vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$ is given as

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 2 \\ 4 & -1 & 3 \end{vmatrix}$$

$$= \hat{i}(-3 + 2) - \hat{j}(6 - 8) + \hat{k}(-2 + 4)$$

$$= -\hat{i} + 2\hat{j} + 2\hat{k} = \vec{r}$$ [Say]

A vector of magnitude 6 in the direction of $\vec{r}$

$$= \frac{\vec{r}}{|\vec{r}|} \cdot 6 = \frac{-\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{1^2 + 2^2 + 2^2}} \cdot 6$$

$$= \frac{-6}{3}\hat{i} + \frac{12}{3}\hat{j} + \frac{12}{3}\hat{k}$$

$$= -2\hat{i} + 4\hat{j} + 4\hat{k}.$$

S17. We have, $\lambda\vec{b} + \vec{c} = \lambda(\hat{i} + \hat{j} - 2\hat{k}) + (\hat{i} + 3\hat{j} - \hat{k})$

$$= (\lambda + 1)\hat{i} + (\lambda + 3)\hat{j} - (2\lambda + 1)\hat{k}$$

Since, $\vec{a} \perp (\lambda\vec{b} + \vec{c})$, $\vec{a} \cdot (\lambda\vec{b} + \vec{c}) = 0$

$\Rightarrow$ $(2\hat{i} - \hat{j} + \hat{k}) \cdot [(\lambda + 1)\hat{i} + (\lambda + 3)\hat{j} - (2\lambda + 1)\hat{k}] = 0$

$\Rightarrow$ $2(\lambda + 1) - (\lambda + 3) - (2\lambda + 1) = 0$

$\Rightarrow$ $\lambda = -2.$

S18. Let the points are $A(k, -10, 3)$, $B(1, -1, 3)$ and $C(3, 5, 3)$

So, $\vec{AB} = \vec{OB} - \vec{OA}$

$$= (\hat{i} - \hat{j} + 3\hat{k}) - (k\hat{i} - 10\hat{j} + 3\hat{k})$$

$$= (1 - k)\hat{i} + (-1 + 10)\hat{j} + (3 - 3)\hat{k}$$

$$= (1 - k)\hat{i} + 9\hat{j} + 0\hat{k}$$

$\therefore$ $|\vec{AB}| = \sqrt{(1-k)^2 + (9)^2 + 0} = \sqrt{(1-k)^2 + 81}$

Similarly, $\vec{BC} = \vec{OC} - \vec{OB}$

$$= (3\hat{i} + 5\hat{j} + 3\hat{k}) - (\hat{i} - \hat{j} + 3\hat{k})$$

$$= 2\hat{i} + 6\hat{j} + 0\hat{k}$$

$\therefore$ $|\vec{BC}| = \sqrt{2^2 + 6^2 + 0} = 2\sqrt{10}$

and $\vec{AC} = \vec{OC} - \vec{OA}$

$$= (3\hat{i} + 5\hat{j} + 3\hat{k}) - (k\hat{i} - 10\hat{j} + 3\hat{k})$$

$$= (3 - k)\hat{i} + 15\hat{j} + 0\hat{k}$$

$\therefore$ $|\vec{AC}| = \sqrt{(3 - k)^2 + 225}$

If A, B and C are collinear, then sum of modulus of any two vectors will be equal to the modulus of third vectors.

For $|\vec{AB}|+|\vec{BC}|=|\vec{AC}|$

$$\sqrt{(1-k)^2+81}+2\sqrt{10} = \sqrt{(3-k)^2+225}$$

$$\Rightarrow \quad \sqrt{(3-k)^2+225}-\sqrt{(1-k)^2+81} = 2\sqrt{10}$$

$$\Rightarrow \quad \sqrt{9+k^2-6k+225}-\sqrt{1+k^2-2k+81} = 2\sqrt{10}$$

$$\Rightarrow \quad \sqrt{k^2-6k+234}-2\sqrt{10} = \sqrt{k^2-2k+82}$$

$$\Rightarrow k^2 - 6k + 234 + 40 - 2\sqrt{k^2-6k+234}\cdot 2\sqrt{10} = k^2 - 2k + 82$$

$$\Rightarrow \quad k^2 - 6k + 234 + 40 - k^2 + 2k - 82 = 4\sqrt{10}\sqrt{k^2+234-6k}$$

$$\Rightarrow \quad -4k + 192 = 4\sqrt{10}\sqrt{k^2+234-6k}$$

$$\Rightarrow \quad -k + 48 = \sqrt{10}\sqrt{k^2+234-6k}$$

On squaring both sides, we get

$$48 \times 48 + k^2 - 96k = 10\,(k^2 + 234 - 6k)$$

$$\Rightarrow \quad k^2 - 96k - 10k^2 + 60k = -48 \times 48 + 2340$$

$$\Rightarrow \quad -9k^2 - 36k = -48 \times 48 + 2340$$

$$\Rightarrow \quad (k^2 + 4k) = 16 \times 16 - 260 \qquad \text{[Dividing by 9 in both sides]}$$

$$\Rightarrow \quad k^2 + 4k = -4$$

$$k^2 + 4k + 4 = 0$$

$$\Rightarrow \quad (k+2)^2 = 0$$

$$\therefore \quad k = -2.$$

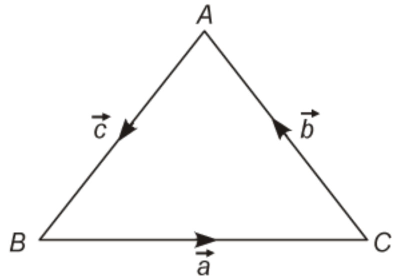
S19. Let the three sides of the triangle *BC*, *CA* and *AB* be represented by $\vec{a}$, $\vec{b}$ and $\vec{c}$, respectively.

We have, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ *i.e.*, $\vec{a}+\vec{b}=-\vec{c}$

which pre cross multiplying by $\vec{a}$, and post cross multiplying by $\vec{b}$, gives

$$\vec{a}\times\vec{b} = \vec{c}\times\vec{a}$$

and $$\vec{a}\times\vec{b} = \vec{b}\times\vec{c}$$



respectively. Therefore,

$$\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$$

$$\Rightarrow \quad |\vec{a}\times\vec{b}| = |\vec{b}\times\vec{c}| = |\vec{c}\times\vec{a}|$$

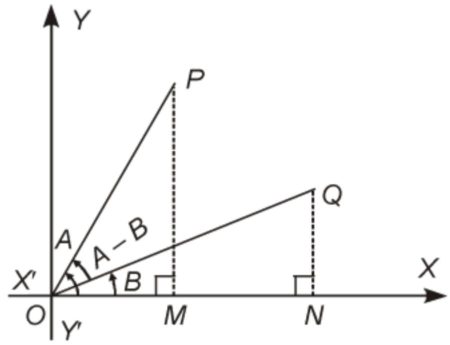
$$\Rightarrow \quad |\vec{a}||\vec{b}|\sin(\pi - C) = |\vec{b}||\vec{c}|\sin(\pi - A) = |\vec{c}||\vec{a}|\sin(\pi - B)$$

$$\Rightarrow \quad ab\sin C = bc\sin A = ca\sin B$$

Dividing by *abc*, we get

$$\frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin B}{b} \qquad i.e., \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

S20. Let $\widehat{OP}$ and $\widehat{OQ}$ be unit vectors making angles A and B, respectively, with positive direction of x-axis. Then $\angle QOP = A - B$.



We know, $\widehat{OP} = \overrightarrow{OM} + \overrightarrow{MP} = \hat{i}\cos A + \hat{j}\sin A$ and $\widehat{OQ} = \overrightarrow{ON} + \overrightarrow{NQ} = \hat{i}\cos B + \hat{j}\sin B$.

By definition $\qquad \widehat{OP} \cdot \widehat{OQ} = |\widehat{OP}|\,|\widehat{OQ}|\cos(A - B)$

$$= \cos(A - B) \qquad [\because\ |\widehat{OP}| = 1 = |\widehat{OQ}|] \quad \dots \text{(i)}$$

In terms of components, we have

$$\widehat{OP} \cdot \widehat{OQ} = (\hat{i}\cos A + \hat{j}\sin A)\cdot(\hat{i}\cos B + \hat{j}\sin B)$$
$$= \cos A\cos B + \sin A\sin B \quad \dots \text{(ii)}$$

From Eq. (i) and (ii), we get

$$\cos(A - B) = \cos A\cos B + \sin A\sin B.$$

S21. Let, $\qquad \vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$

Also, $\qquad \vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$

For $\quad \vec{a} \times \vec{c} = \vec{b}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \hat{j} - \hat{k}$$

$\Rightarrow \ \hat{i}(z - y) - \hat{j}(z - x) + \hat{k}(y - x) = \hat{j} - \hat{k}$

$\therefore$
$$z - y = 0 \quad \dots \text{(i)}$$
$$x - z = 1 \quad \dots \text{(ii)}$$
$$x - y = 1 \quad \dots \text{(iii)}$$

Also, $\qquad \vec{a} \cdot \vec{c} = 3$

$$(\hat{i}+\hat{j}+\hat{k})\cdot(x\hat{i}+y\hat{j}+z\hat{k}) = 3$$

$$\Rightarrow \quad x + y + z = 3 \quad \text{... (iv)}$$

On adding Eqs. (ii) and (iii), we get

$$2x - y - z = 2 \quad \text{... (v)}$$

On solving Eq. (iv) and (v), we get

$$x = \frac{5}{3}$$

$$\therefore \quad y = \frac{5}{3} - 1 = \frac{2}{3} \quad \text{and} \quad z = \frac{2}{3}$$

Now,

$$\vec{c} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$$

$$= \frac{1}{3}(5\hat{i} + 2\hat{j} + 2\hat{k}).$$

S22. Let $ABCD$ be a parallelogram such that

$$\vec{AB} = \vec{p}, \quad \vec{AD} = \vec{q} \quad \Rightarrow \quad \vec{BC} = \vec{q}$$

By triangle law of addition, we get

$$\vec{AC} = \vec{p} + \vec{q} = \vec{a} \quad \text{[Say]} \quad \text{... (i)}$$

Similarly,

$$\vec{BD} = -\vec{p} + \vec{q} = \vec{b} \quad \text{[Say]} \quad \text{... (ii)}$$

On adding Eqs. (i) and (ii), we get

$$\vec{a} + \vec{b} = 2\vec{q} \quad \Rightarrow \quad \vec{q} = \frac{1}{2}(\vec{a} + \vec{b})$$

On subtracting Eq. (ii) from Eq. (i), we get

$$\vec{a} - \vec{b} = 2\vec{p} \quad \Rightarrow \quad \vec{p} = \frac{1}{2}(\vec{a} - \vec{b})$$

Now,

$$\vec{p} \times \vec{q} = \frac{1}{4}(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$$

$$= \frac{1}{4}(\vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{b}]$$

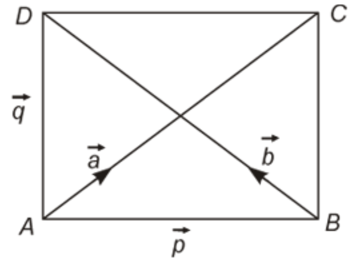
$$= \frac{1}{4}[\vec{a} \times \vec{b} + \vec{a} \times \vec{b}]$$

$$= \frac{1}{2}(\vec{a} \times \vec{b})$$

So,Area of a parallelogram $ABCD$ = $|\vec{p} \times \vec{q}| = \frac{1}{2}|\vec{a} \times \vec{b}|$

Now, area of a parallelogram, whose diagonals are $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$.

$$= \frac{1}{2}|(2\hat{i} - \hat{j} + \hat{k}) \times (\hat{i} + 3\hat{j} - \hat{k})|$$



$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix}$$

$$= \frac{1}{2}|[\hat{i}(1-3) - \hat{j}(-2-1) + \hat{k}(6+1)]|$$

$$= \frac{1}{2}|-2\hat{i} + 3\hat{j} + 7\hat{k}|$$

$$= \frac{1}{2}\sqrt{4+9+49}$$

$$= \frac{1}{2}\sqrt{62} \text{ Sq. units.}$$

S23. Here $|\vec{r}| = 14$, $\vec{a} = 2k$, $\vec{b} = 3k$ and $\vec{c} = -6k$

$\therefore$ Direction cosines l, m and n are

$$l = \frac{\vec{a}}{|\vec{r}|} = \frac{2k}{14} = \frac{k}{7}$$

$$m = \frac{\vec{b}}{|\vec{r}|} = \frac{3k}{14}$$

and $$n = \frac{\vec{c}}{|\vec{r}|} = \frac{-6k}{14} = \frac{-3k}{7}$$

Also, we know that

$$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \quad \frac{k^2}{49} + \frac{9k^2}{196} + \frac{9k^2}{49} = 1$$

$$\Rightarrow \quad \frac{4k^2 + 9k^2 + 36k^2}{196} = 1$$

$$\Rightarrow \quad k^2 = \frac{196}{49} = 4$$

$$\Rightarrow \quad k = \pm 2$$

So, the direction cosines (l, m, n) are $\frac{2}{7}, \frac{3}{7}$ and $\frac{-6}{7}$.

[Since, $\vec{r}$ makes an acute angle with X-axis]

$\because \quad \vec{r} = \hat{r} \cdot |\vec{r}|$

$\therefore \quad \vec{r} = (l\hat{i} + m\hat{j} + n\hat{k})|\vec{r}|$

$$= \left(\frac{+2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) \cdot 14$$

$$= +4\hat{i} + 6\hat{j} - 12\hat{k}.$$

S24. We have, $|\vec{r}| = 2\sqrt{3}$

Since, $\vec{r}$ is equally inclined to the three axes, so direction cosines of the unit vector $\vec{r}$ will be same i.e., $l = m = n$.

We know that, $l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad l^2 + l^2 + l^2 = 1$

$\Rightarrow \quad l^2 = \frac{1}{3}$

$\Rightarrow \quad l = \pm\left(\frac{1}{\sqrt{3}}\right)$

So, $\hat{r} = \pm\frac{1}{\sqrt{3}}\hat{i} \pm \frac{1}{\sqrt{3}}\hat{j} \pm \frac{1}{\sqrt{3}}\hat{k}$

$\therefore \quad \vec{r} = \hat{r}|\vec{r}| \qquad \left[\because \hat{r} = \frac{\vec{r}}{|\vec{r}|}\right]$

$$= \left[\pm\frac{1}{\sqrt{3}}\hat{i} \pm \frac{1}{\sqrt{3}}\hat{j} \pm \frac{1}{\sqrt{3}}\hat{k}\right]2\sqrt{3} \qquad [\because |r| = 2\sqrt{3}]$$

$$= \pm 2\hat{i} \pm 2\hat{j} \pm 2\hat{k} = \pm 2(\hat{i} + \hat{j} + \hat{k})$$