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Q1. If $A = \{1, 2, 3\}$ and f, g are relations corresponding to the subset of $A \times A$ indicated against them, which of f, g is a function? Why?

$$f = \{(1, 3), (2,3), (3, 2)\}$$
$$g = \{(1, 2), (1, 3), (3, 1)\}$$

Q2. If $f = \{(5, 2), (6, 3)\}$ and $g = \{(2, 5), (3, 6)\}$, write the range of f and g.

Q3. Let the function $f : R \to R$ be defined by $f(x) = 4x - 1$, $\forall x \in R$. Then show that f is one-one.

Q4. Let C be the set of complex numbers. Prove that the mapping $f : C \to R$ given by $f(z) = |z|$, $\forall z \in C$, is neither one-one not onto.

Q5. Are the following set of ordered pairs functions? If so examine whether the mapping is injective or surjective.

(i) $\{(x, y) : x$ is a person, y is the mother of $x\}$.

(ii) $\{(a, b) : a$ is a person, b is an ancestor of $a\}$.

Q6. Let D is the domein of the real valued function f defined by $f(x) = \sqrt{25 - x^2}$. Then write D.

Q7. If $f, g : R \to R$ be defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$, $\forall x \in R$, respectively. Then, find $g o f$.

Q8. If the mappings f and g are given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$ write fog.

Q9. If functions $f : A \to B$ and $g : B \to A$ satisfy $gof = I_A$, then show that f is one-one and g is onto.

Q10. If, $f = \{(5, 2), (6, 3)\}$; $g = \{(2, 5), (3, 6)\}$. Write fog.

Q11. Let $f : R \to R$ be the function defined by $f(x) = 2x - 3$, $\forall x = R$. Write f^{-1}.

Q12. If $A = \{a, b, c, d\}$ and the function $f = \{(a, b), (b, d), (c, d), (d, c)\}$, write f^{-1}.

Q13. Using the definition, prove that the function $f : A \to B$ is invertible if and only if f is both one-one and onto.

Q14. Let $f : R \to R$ be the function defined by $f(x) = 4x - 3$, $\forall x = R$. Then write f^{-1}.

Q15. If $A = \{a, b, c, d\}$ and $f = \{(a, b), (b, d\}, (c, a\}, (d, c)\}$. Show that f is one-to-one from A to A. Find f^{-1}.

Q16. Is the binary operation $*$ defined on Z (set of integer) by $m * n = m - n + mn$ $\forall m, n \in Z$ commutative?

Q17. Let $A = \{a, b, c\}$ and the relation R be defined on A as follows:

$$R = \{(a, a), (b, c), (a, b)\}$$

Then write minimum number of ordered pairs to be added in R to make R reflexive and transitive.

Q18. In the set N of natural numbers, define the binary operation $*$ by $m * n = g.c.d(m, n)$, $m, n \in N$. Is the operation $*$ commutative and associative?

Q19. Let $A = \{0, 1, 2, 3\}$ and define a relation R on A as follows:

$$R = \{(0, 0), (0, 1), (0, 3), (1, 0), (1, 1), (2, 2), (3, 0), (3, 3)\}$$

is R reflexive? Symmetric? Transitive.

Q20. Let R be the equivalence relation in set z of integers given by $R = \{(a, b) : 2$ devides $a - b\}$ write the equivalence class [0].

Q21. Is $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ a function? If g is described by $g(x) = \alpha x + \beta$, then what value should be assigned to α and β?

Q22. Let R be the set of real numbers and $f : R \to R$ be the function defined by $f(x) = 4x + 5$. Show that f is invertible and find f^{-1}.

Q23. Give an example of a map:

(i) which is one-one but not onto.

(ii) which is not one-one but onto.

(iii) which is neither one-one nor not onto.

Q24. Given, $A = \{2, 3, 4\}$, $B = \{3, 5, 6, 7\}$. Construct an example of each of the following:

(i) an injective mapping from A to B.

(ii) a mapping from A to B which is not injective.

(iii) a mapping from B to A.

Q25. Let $f = R \to R$ be the function defined by $f(x) = \frac{1}{2 - \cos x}$, $\forall x \in R$. Then, find the range of f.

Q26. Let $X = \{1, 2, 3)$ and $Y = \{4, 5\}$. Find whether the following subsets of $X \times Y$ are functions from X to Y or not.

(i) $f = \{(1, 4), (1, 5), (2, 4), (3, 5)\}$

(bb) $g = \{(1, 4), (2, 4), (3, 4)\}$

(iii) $h = \{(1, 4), (2, 5), (3, 5)\}$

(iv) $k = \{(1, 4), (2, 5)\}$

Q27. Let n be a fixed positive integer. Define a relation R in Z as follows $\forall a, b \in Z$, aRb if and only if $a - b$ is divisible by n. Show that R is an equivalence relation.

Q28. Let R be ralation defined on the set of natural number N as follows, $R = \{(x, y) : x \in N, y \in N, 2x + y = 41\}$. Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive.

Q29. If $A = \{1, 2, 3, 4\}$, define relations on A which have properties of being

(i) reflexive, transitive but not symmetric.

(ii) symmetric but neither reflexive nor transitive.

(iii) reflexive, symmetric and transitive.

Q30. Let $*$ be the binary operation defined on Q. Find which of the following binary operations are commutative

(i) $a * b = a - b$, $\forall a, b \in Q$

(ii) $a * b^2 = a^2 + b^2$, $\forall a, b \in Q$

(iii) $a * b = a + ab$, $\forall a, b \in Q$

(iv) $a * b = (a - b)^2$ $\forall a, b \in Q$

Q31. If $*$ be the binary operation defined on R by $a * b = 1 + ab$, $\forall a, b \in R$. Then prove that operation is commulative but not associative.

Q32. Let $A = [-1, 1]$, then, discuss whether the following functions defined on A are one-one, onto or bijective.

(i) $f(x) = \frac{x}{2}$ (ii) $g(x) = |x|$ (iii) $h(x) = x|x|$ (iv) $k(x) = x^2$

Q33. Each of the following defines a relation of N

(i) x is greater than y, $x, y \in N$ (ii) $x + y = 10$, $x, y \in N$

(iii) xy is square of an integer $x, y \in N$ (iv) $x + 4y = 10$, $x, y \in N$

Determine which of the above relations are reflexive, symmetric and transitive.

Q34. Let $f, g : R \to R$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x$ $\forall x \in R$. Then find fog and gof.

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S1. f is a function since each element of A in the first place in the ordered pairs is related to only one element of A in the second place while g is not a function because 1 is related to more than one element of A, namely, 2 and 3.

S2. The range of $f = \{2, 3\}$ and the range of $g = \{5, 6\}$.

S3. For any two elements $x_1, x_2 \in \mathbf{R}$ such that $f(x_1) = f(x_2)$, we have

$$4x_1 - 1 = 4x_2 - 1$$

$\Rightarrow$ $4x_1 = 4x_2 \quad i.e., \quad x_1 = x_2$

Hence f is one-one.

S4. The mapping $f : C \to R$

Given that, $f(z) = |z|, \ \forall z \in C$

$$f(1) = |1| = 1$$

$$f(-1) = |-1| = 1$$

$$f(1) = f(-1)$$

But $1 \neq -1$.

So, $f(z)$ is not one-one. Also, $f(z)$ is not onto as there is no pre-image for any negative element or R under the mapping $f(z)$.

S5. (i) Given set of ordered pair is $\{(x, y) : x$ is a person, y is the mother of $x\}$.

It represent a function. Here, the image of distinct elements of x under f are not distinct, so it is not a injective but it is a surjective.

(ii) Set of ordered pairs = $\{(a, b) : a$ is a person, b is an ancestor of $a\}$.

Here, each element of domain does not have a unique image. So, it does not represent function.

S6. Since D_f is the set of all real values for which function is decfined.

Given function is $\sqrt{25 - x^2}$.

$\therefore$ For real value of $f(x)$.

$$25 - x^2 \geq 0 \quad \Rightarrow \quad (5 - x)(5 + x) \geq 0$$

$\Rightarrow$ $-5 \leq x \leq 5$

$\therefore$ $D_f = [-5, 5]$.

S7. Given that, $f(x) = 2x + 1$ and $g(x) = x^2 - 2, \ \forall x \in R$

$$gof(x) = g\{f(x)\}$$

$$= (2x + 1)^2 - 2$$

$$= 4x^2 + 4x + 1 - 2$$
$$= 4x^2 + 4x - 1.$$

S8. Given that, $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$

Now, $fog(2) = f\{g(2)\} = f(3) = 5$

$fog(5) = f\{g(5)\} = f(1) = 2$

$fog(1) = f\{g(1)\} = f(3) = 5$

$fog = \{(2, 5), (5, 2), (1, 5)\}$

S9. Given that, $f : A \to B$ and $g : B \to A$ satisfy $gof = I_A$

$\because \quad gof = I_A$

$\Rightarrow \quad gof\{f(x_1)\} = gof\{f(x_2)\}$

$\Rightarrow \quad g(x_1) = g(x_2)$

$\therefore \quad x_1 = x_2$

Hence, f is one-one and g is onto.

S10. $f(g(2)) = f(5) = 2$

$f(g(3)) = f(6) = 3$

$\therefore \quad f(g(x)) = \{(2, 2), (3, 3)\}$

$\therefore \quad fog = \{(2, 2), (3, 3\}.$

S11. Given that, $f(x) = 2x - 3, \ \forall x = R$

Now, let $y = 2x - 3$

$2x = y + 3$

$$x = \frac{y+3}{2}$$

$$\therefore \quad f^{-1}(x) = \frac{x+3}{2}$$

S12. Given that, $A = \{a, b, c, d\}$

and $f = \{(a, b), (b, d), (c, d), (d, c)\}$

Since, f is not bijective hence f^{-1} does not exist.

S13. A function $f : X \to Y$ is defined to be invertible, if there exist a function $f : Y \to X$ such that $gof = I_X$ and $fog = I_Y$. The function is called the inverse of f and is denoted by f^{-1}.

S14. Given that, $f(x) = 4x - 3 = y$ (say), then

$4x = y + 3$

$\Rightarrow \qquad x = \dfrac{y+3}{4}$

Hence $\qquad f^{-1}(y) = \dfrac{y+3}{4} \quad \Rightarrow \quad f^{-1}(x) = \dfrac{x+3}{4}$.

S15. $\because \qquad A = \{a, b, c, d\}$

$$f = \{(a, b), (b, d\}, (c, a\}, (d, c)\}$$

$\because$ f is ont-to-one, because image of each element of set A is assigned to a distinct image of set A. Also f is onto $f(A) = A$. Moreover,

$$f' = \{(b, a), (d, b\}, (a, c\}, (c, d)\}.$$

S16. No. Since for $1, 2 \in \mathbf{Z}$, $1 * 2 = 1 - 2 + 1.2 = 1$ while $2 * 1 = 2 - 1 + 2.1 = 3$, so that $1 * 2 \neq 2 * 1$.

S17. A relation R in A is said to be reflexive if aRa for all $a \in A$, and it is said to be transitive if aRb and $bRc \Rightarrow aRc \ \forall\, a, b, c \in A$.

Given relation, $\qquad R = \{(a, a), (b, c), (a, b)\}$

To make R is reflexive we must add (b, b) and (c, c) to R. Also to make R is transitive we must add (a, c) to R.

So, minimum number of ordered pair is to be added are (b, b), (c, c), (a, c).

S18. The operation is clearly commutative since

$$m * n = g.c.d\,(m, n) = g.c.d\,(n, m) = n * m \quad \forall m, n \in \mathbf{N}.$$

It is also associative because for $l, m, n \in \mathbf{N}$, we have

$$l * (m * n) = g.c.d\,(l, g.c.d\,(m, n))$$

$$= g.c.d\,(g.c.d\,(l, m), n)$$

$$= (l * m) * n.$$

S19. A relation R in A is said to be reflexive if aRA for all $a \in A$, R is symmetric if $aRb \Rightarrow bRA$, $\forall a, b \in A$ and it is said to be transitive if aRb and $bRc \Rightarrow aRc \ \forall\, a, b, c \in A$.

Since R is $\{(0, 0), (0, 1), (0, 3), (1, 0), (1, 1), (2, 2), (3, 0), (3, 3)\}$.

Obviously, R is reflexive and symmetric, but not transitive since for $(1, 0) \in R$ and $(0, 3) \in R$, whereas $(1, 3) \notin R$.

S20. $\because$ equivalence relation devides the set into pairwise disjoint subsets called equivalence classes whose collection is called a partition of set.

Union of all the equivalence classes gives the whole set, hence equivalence class for relation R is

$$[0] = \{0, \pm 2, \pm 4, \pm 6, ...\}.$$

S21. Given that, $\qquad g(x) = \alpha x + \beta$

$$g(1) = \alpha + \beta$$

$$\alpha + \beta = 1 \qquad \dots \text{(i)}$$

$$g(2) = 2\alpha + \beta$$

$$2\alpha + \beta = 3. \qquad \text{... (ii)}$$

From Eqs. (i) and (ii), we get

$$2(1 - \beta) + \beta = 3$$

$\Rightarrow$ $$2 - 2\beta + \beta = 3$$

$\Rightarrow$ $$2 - \beta = 3$$

$$\beta = -1$$

If $\beta = -1$, then $\alpha = 2$

$$\alpha = 2, \quad \beta = -1.$$

S22. Here the function $f : R \to R$ is defined as $f(x) = 4x + 5 = y$ (say). Then

$$4x = y - 5 \quad \text{or} \quad x = \frac{y-5}{4}$$

This leads to a function $g : R \to R$ is defined as:

$$g(y) = \frac{y-5}{4}$$

Therefore, $$(gof)(x) = g(f(x) = g(4x + 5)$$

$$= \frac{4x+5-5}{4} = x$$

or $$gof = I_R$$

Similarly $$(fog)(y) = f(g(y))$$

$$= f\left(\frac{y-5}{4}\right)$$

$$= 4\left(\frac{y-5}{4}\right) + 5 = y$$

or $$fog = I_R$$

Hence f is invertible and $f^{-1} = g$ which is given by

$$f^{-1}(x) = \frac{x-5}{4}.$$

S23. (i) Let $f : N \to N$, be a mapping defined by $f(x) = 2x$ which is one-one.

For $$f(x_1) = f(x_2)$$

$\Rightarrow$ $$2x_1 = 2x_2$$

$$x_1 = x_2$$

Further f is not onto, as for $1 \in N$, there does not exist any x in N such that $f(x) = 2x + 1$.

(ii) Let $f : N \to N$ given $f(1) = f(2) = 1$ and $f(x) = x - 1$ for every $x > 2$ is onto but not one-one. f is not one-one as $f(1) = f(2) = 1$. But f is onto.

(iii) The mapping $f : R \to R$ defined as $f(x) = x^2$, is neither one-one nor onto.

S24. Given that, $A = \{2, 3, 4\}, \quad B = \{3, 5, 6, 7\}$

(i) Let $f : A \to B$ denote a mapping

$$f = \{x, y) : y = x + 3\}$$

i.e., $f = \{(2, 5), (3, 6), (4, 7)\}$, which is an injective mapping.

(ii) Let $g : A \to B$ denote a mapping such that $g = \{(2, 2), (3, 5), (4, 5)\}$, which is not an injective mapping.

(iii) Let $h : B \to A$ denote a mapping such that $h = \{(2, 2), (5, 3), (6, 4), (6, 4)\}$, which is a mapping from B to A.

S25. Given that,

$$f(x) = \frac{1}{2 - \cos x}, \ \forall x \in R$$

Let

$$y = \frac{1}{2 - \cos x}$$

$\Rightarrow$ $2y - y\cos x = 1$

$\Rightarrow$ $y\cos x = 2y - 1$

$\Rightarrow$ $\cos x = \dfrac{2y-1}{y} = 2 - \dfrac{1}{y} \quad \Rightarrow \quad \cos x = 2 - \dfrac{1}{y}$

$\Rightarrow$ $-1 \le \cos x \le 1 \quad \Rightarrow \quad -1 \le 2 - \dfrac{1}{y} \le 1$

$\Rightarrow$ $-3 \le -\dfrac{1}{y} \le -1$

$\Rightarrow$ $\dfrac{1}{3} \le y \le 1$

So, range of y is $\left[\dfrac{1}{3}, 1\right]$.

S26. Given that $X = \{1, 2, 4) \quad \text{and} \quad Y = \{4, 5\}$

$$X \times Y = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$$

(i) $f = \{(1, 4), (1, 5), (2, 4), (3, 5)\}$

f is not a function because f has not unique image.

(ii) $g = \{(1, 4), (2, 4), (3, 4)\}$

Since, g is a function as each element of the domain has unique image.

(iii) $h = \{(1, 4), (2, 5), (3, 5)\}$

It is clear that h is a function.

(iv) $k = \{(1, 4), (2, 5)\}$

k is not a function as 3 has not an image under the mapping.

S27. Given that, $\forall a, b \in Z$, aRb if and only if $a - b$ is divisible by n.

Now,

I. Reflexive

$aRa \Rightarrow (a - a)$ is divisible by n, which is true for any integer a as '0' is divisible by n.

Hence, R is reflexive.

II. Symmetric

aRb

$\Rightarrow$ $a - b$ is divisible by n.

$\Rightarrow$ $-b + a$ is divisible by n.

$\Rightarrow$ $-(b - a)$ is divisible by n.

$\Rightarrow$ $(b - a)$ is divisible by n.

$\Rightarrow$ bRa

Hence, R is symmetric.

III. Transitive

$\Rightarrow$ $(a - b)$ is divisible by n and $(b - c)$ is divisible by n.

$\Rightarrow$ $(a - b) + (b - c)$ is divisible by n.

$\Rightarrow$ $(a - c)$ is divisible by n.

$\Rightarrow$ aRc

Hence, R is transitive.

So, R is an equivalence relation.

S28. Given that, $R = \{(x, y) : x \in N, y \in N, 2x + y = 41\}$

Domain = $\{1, 2, 3, ..., 20\}$

Range = $\{1, 3, 5, 7, ..., 39\}$

$R = \{(1, 39), (2, 37), (3, 35), ..., (19, 3), (20, 1)\}$

R is not reflexive as $(2, 2) \in R$

$2 \times 2 + 2 \neq 41$

So, R is not symmetric.

As $(1, 39) \in R$ but $(39, 1) \notin R$

So, R is not transitive.

As $(11, 19) \in R$ but $(19, 1) \in R$

But $(11, 3) \notin R$

Hence, R is neither reflexive, nor symmetric and nor transitive.

S29. Given that, $A = \{1, 2, 3, 4\}$

(i) Let, $R_1 = \{(1, 1), (1, 2), (2, 3), (2, 2), (1, 3), (3, 3)\}$

R_1 is reflexive, since, (1, 1) (2, 2) (3, 3) lie in R_1.

Now, $(1, 2) \in R_1, \quad (2, 3) \in R_1 \quad \Rightarrow \quad (1, 3) \in R_1$

Hence, R_1 is also transitive but $(1, 2) \in R_1 \quad \Rightarrow \quad (2, 1) \notin R_1$.

So, it is not symmetric.

(ii) Let $R_2 = \{(1, 2), (2, 1)\}$

Now, $(1, 2) \in R_2, \quad (2, 1) \in R_2$

So, it is symmetric.

(iii) Let $R_3 = \{(1, 2), (2, 1), (1, 1), (2, 2), (3, 3), (1, 3), (1, 3), (3, 1), (2, 3)\}$

Hence, R_3 is reflexive, symmetric and transitive.

S30. Given that $*$ be the binary operation defined on Q.

(i) $a * b = a - b, \; \forall a, b \in Q$ and $b * a = b - a$

So, $a * b \neq b * a$

Hence, $*$ is not commutative.

(ii) $a * b = a^2 + b^2$

$b * a = b^2 + a^2$

So, $*$ is commutative. [Since, '+' is on rational is commutative]

(iii) $a * b = a + ab$

$b * a = b + ab$

Clearly, $a + ab \neq b + ab$

So, $*$ is not commutative.

(iv) $a * b = (a - b)^2, \; \forall a, b \in Q$

$b * a = (b - a)^2$

$\because \quad (a - b)^2 = (b - a)^2$

Hence, $*$ is commutative.

S31. Given that, $a * b = 1 + ab, \quad \forall a, b \in R$

$a * b = ab + 1 = b * a$

So, $*$ is a commulative binary operation.

Also, $a * (b * c) = a * (1 + bc) = 1 + a(1 + bc)$ ... (i)

$a * (b * c) = 1 + a + abc$

$(a * b) * c = (1 + ab) * c$

$= 1 + (1 + ab) * c = 1 + c + abc.$

From Eq. (i) and (ii), we get

$$a * (b * c) \neq (a * b) * c.$$

So $*$ is not associative.

Hence, $*$ is commulative but not associative.

S32. Given that, $A = [-1, 1]$

(i) $f(x) = \frac{x}{2}$

Let $f(x_1) = f(x_2)$

$\Rightarrow \quad \frac{x_1}{2} = \frac{x_2}{2} \quad \Rightarrow \quad x_1 = x_2$

So, $f(x)$ is one-one.

Now, let $y = \frac{x}{2}$

$\Rightarrow \quad x = 2y \notin A, \quad \forall\, y \in A$

As for $y = 1 \in A, \quad x = 2 \notin A$

So, $f(x)$ is not onto.

Also, $f(x)$ is not bijective as it is not onto.

(ii) $g(x) = |x|$

Let $g(x_1) = g(x_2)$

$\Rightarrow \quad |x_1| = |x_2| \quad \Rightarrow \quad x_1 = \pm x_2$

So, $g(x)$ is not one-one.

Now, let $y = |x_1| \quad \Rightarrow \quad x = \pm y \notin A, \quad \forall\, y \in A$

So, $g(x)$ is not onto.

Also, $g(x)$ is not bijective.

(iii) $h(x) = x|x|$

Let $h(x_1) = h(x_2)$

$\Rightarrow \quad x_1|x_1| = x_2|x_2| \quad \Rightarrow \quad x_1 = \pm x_2$

So, $h(x)$ is not one-one.

Now, let $y = x|x|$

$y = x^2 \in A, \quad \forall\, x \in A$

So, $h(x)$ is not onto.

Also, $h(x)$ is not bijective.

(iv) $k(x) = x^2$

Let $k(x_1) = k(x_2)$

$\Rightarrow$ $x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

Thus, $k(x)$ is not one-one.

Now, let $y = x^2$

$\Rightarrow$ $x = \sqrt{y} \notin A, \ \forall y \in A$

As for $y = -1$, $\sqrt{-1} \notin A$.

Hence, $k(x)$ is neither one-one nor onto.

S33. (i) x is greater than y, $x, y \in N$

$(x, x) \in R$

For xRx $x > x$ is not true for any $x \in N$.

Therefore, R is not reflexive.

Let $(x, y) \in R \Rightarrow xRy$

$x > y$

but $y > x$ is not true for any $x, y \in N$

Thus, R is not symmetric.

Let xRy and yRz

$x > y$ and $y > z \Rightarrow x > z$

$\Rightarrow$ xRz

So, R is transitive.

(ii) $x + y = 10$, $x, y \in N$

$R = \{(x, y);\ x + y = 10,\ x, y \in N\}$

$R = \{(1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2), (9, 1)\}$ $(1, 1) \notin R$

So, R is not reflexive.

$(x, y) \in R \Rightarrow (y, x) \in R$

Therefore, R is symmetric

$(1, 9) \in R, \quad (9, 1) \in R \Rightarrow (1, 1) \notin R$

Hence, R is not transitive.

(iii) Given xy, is square of an integer $x, y \in N$

$\Rightarrow$ $R = \{(x, y) : xy$ is a square of an integer $x, y \in N\}$

$(x, x) \in R, \quad x \in N$

As x^2 is square of an integer for any $x \in N$.

Hence, R is reflexive.

If $(x, y) \in R \Rightarrow (y, x) \in R$

Therefore, R is symmetric

If $(x, y) \in R, \quad (y, z) \in R$

So, xy is square of an integer and yz is square of an integer.

Let $xy = m^2$ and $yz = n^2$ for some $m, n \in Z$

$x = *$ and $z = *$

$xz = *$, which is square of an integer.

So, R is transitive.

(iv) $x + 4y = 10, \ x, y \in N$

$R = \{(x, y) : x + 4y = 10, x, y \in N\}$

$R = \{(2, 2), (6, 1)\}$

$(1, 1), (3, 3), ,... \notin R$

Thus R is not reflexive.

$(6, 1) \in R$ but $(1, 6) \notin R$

Hence, R is not symmetric.

$(x, y) \in R \Rightarrow x + 4y = 10$ but $(y, z) \in R$

$y + 4z = 10 \Rightarrow (x, z) \in R$

So, R is transitive.

S34. Here, $f(x) = |x| + x$ which can be redefined as

$$f(x) = \begin{cases} 2x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Similarly, the function g defined by $g(x) = |x| - x$ may be redefined as

$$g(x) = \begin{cases} 0 & \text{if } x \geq 0 \\ -2x & \text{if } x < 0 \end{cases}$$

Therefore, gof gets defined as:

For $x \geq 0$, $(gof)(x) = g(f(x)) = g(2x) = 0$

and for $x < 0$, $(gof)(x) = g(f(x)) = g(0) = 0$.

Consequently, we have $(gof)(x)$, $\forall x \in \boldsymbol{R}$.

Similarly, fog gets defined as:

For $x \geq 0$, $(fog)(x) = f(g(x) = f(0) = 0$.

and for $x < 0$, $(fog)(x) = f(g(x)) = f(-2x) = -4x$.

i.e., $$(fog)(x) = \begin{cases} 0, & x > 0 \\ -4x, & x < 0 \end{cases}$$