Chapter Eleven

DUAL NATURE OF RADIATION AND MATTER

11.1 INTRODUCTION

The Maxwell's equations of electromagnetism and Hertz experiments on the generation and detection of electromagnetic waves in 1887 strongly established the wave nature of light. Towards the same period at the end of 19th century, experimental investigations on conduction of electricity (electric discharge) through gases at low pressure in a discharge tube led to many historic discoveries. The discovery of X-rays by Roentgen in 1895, and of electron by J. J. Thomson in 1897, were important milestones in the understanding of atomic structure. It was found that at sufficiently low pressure of about 0.001 mm of mercury column, a discharge took place between the two electrodes on applying the electric field to the gas in the discharge tube. A fluorescent glow appeared on the glass opposite to cathode. The colour of glow of the glass depended on the type of glass, it being yellowish-green for soda glass. The cause of this fluorescence was attributed to the radiation which appeared to be coming from the cathode. These *cathode rays* were discovered, in 1870, by William Crookes who later, in 1879, suggested that these rays consisted of streams of fast moving negatively charged particles. The British physicist J. J. Thomson (1856-1940) confirmed this hypothesis. By applying mutually perpendicular electric and magnetic fields across the discharge tube, J. J. Thomson was the first to determine experimentally the speed

and the specific charge [charge to mass ratio (e/m)] of the cathode ray particles. They were found to travel with speeds ranging from about 0.1 to 0.2 times the speed of light (3×10^8 m/s). The presently accepted value of e/m is 1.76×10^{11} C/kg. Further, the value of e/m was found to be independent of the nature of the material/metal used as the cathode (emitter), or the gas introduced in the discharge tube. This observation suggested the universality of the cathode ray particles.

Around the same time, in 1887, it was found that certain metals, when irradiated by ultraviolet light, emitted negatively charged particles having small speeds. Also, certain metals when heated to a high temperature were found to emit negatively charged particles. The value of e/m of these particles was found to be the same as that for cathode ray particles. These observations thus established that all these particles, although produced under different conditions, were identical in nature. J. J. Thomson, in 1897, named these particles as *electrons*, and suggested that they were fundamental, universal constituents of matter. For his epoch-making discovery of electron, through his theoretical and experimental investigations on conduction of electricity by gasses, he was awarded the Nobel Prize in Physics in 1906. In 1913, the American physicist R. A. Millikan (1868-1953) performed the pioneering oil-drop experiment for the precise measurement of the charge on an electron. He found that the charge on an oil-droplet was always an integral multiple of an elementary charge, 1.602×10^{-19} C. Millikan's experiment established that *electric charge is quantised.* From the values of charge (e) and specific charge (e/m), the mass (m) of the electron could be determined.

11.2 ELECTRON EMISSION

We know that metals have free electrons (negatively charged particles) that are responsible for their conductivity. However, the free electrons cannot normally escape out of the metal surface. If an electron attempts to come out of the metal, the metal surface acquires a positive charge and pulls the electron back to the metal. The free electron is thus held inside the metal surface by the attractive forces of the ions. Consequently, the electron can come out of the metal surface only if it has got sufficient energy to overcome the attractive pull. A certain minimum amount of energy is required to be given to an electron to pull it out from the surface of the metal. This minimum energy required by an electron to escape from the metal surface is called the *work function* of the metal. It is generally denoted by ϕ_0 and measured in eV (electron volt). One electron volt is the energy gained by an electron when it has been accelerated by a potential difference of 1 volt, so that 1 eV $= 1.602 \times 10^{-19}$ J.

This unit of energy is commonly used in atomic and nuclear physics. The work function (ϕ_0) depends on the properties of the metal and the nature of its surface.

The minimum energy required for the electron emission from the metal surface can be supplied to the free electrons by any one of the following physical processes:

(i) *Thermionic emission:* By suitably heating, sufficient thermal energy can be imparted to the free electrons to enable them to come out of the metal.

275

(ii) *Field emission*: By applying a very strong electric field (of the order of 10^8 V m^{-1}) to a metal, electrons can be pulled out of the metal, as in a spark plug.

(iii) *Photoelectric emission*: When light of suitable frequency illuminates a metal surface, electrons are emitted from the metal surface. These photo(light)-generated electrons are called *photoelectrons*.

11.3 PHOTOELECTRIC EFFECT

11.3.1 Hertz's observations

The phenomenon of photoelectric emission was discovered in 1887 by Heinrich Hertz (1857-1894), during his electromagnetic wave experiments. In his experimental investigation on the production of electromagnetic waves by means of a spark discharge, Hertz observed that high voltage sparks across the detector loop were enhanced when the emitter plate was illuminated by ultraviolet light from an arc lamp.

Light shining on the metal surface somehow facilitated the escape of free, charged particles which we now know as electrons. When light falls on a metal surface, some electrons near the surface absorb enough energy from the incident radiation to overcome the attraction of the positive ions in the material of the surface. After gaining sufficient energy from the incident light, the electrons escape from the surface of the metal into the surrounding space.

11.3.2 Hallwachs' and Lenard's observations

Wilhelm Hallwachs and Philipp Lenard investigated the phenomenon of photoelectric emission in detail during 1886-1902.

Lenard (1862-1947) observed that when ultraviolet radiations were allowed to fall on the emitter plate of an evacuated glass tube enclosing two electrodes (metal plates), current flows in the circuit (Fig. 11.1). As soon as the ultraviolet radiations were stopped, the current flow also stopped. These observations indicate that when ultraviolet radiations fall on the emitter plate C, electrons are ejected from it which are attracted towards the positive, collector plate A by the electric field. The electrons flow through the evacuated glass tube, resulting in the current flow. Thus, light falling on the surface of the emitter causes current in the external circuit. Hallwachs and Lenard studied how this photo current varied with collector plate potential, and with frequency and intensity of incident light.

Hallwachs, in 1888, undertook the study further and connected a negatively charged zinc plate to an electroscope. He observed that the zinc plate lost its charge when it was illuminated by ultraviolet light. Further, the uncharged zinc plate became positively charged when it was irradiated by ultraviolet light. Positive charge on a positively charged zinc plate was found to be further enhanced when it was illuminated by ultraviolet light. From these observations he concluded that negatively charged particles were emitted from the zinc plate under the action of ultraviolet light.

After the discovery of the electron in 1897, it became evident that the incident light causes electrons to be emitted from the emitter plate. Due

to negative charge, the emitted electrons are pushed towards the collector plate by the electric field. Hallwachs and Lenard also observed that when ultraviolet light fell on the emitter plate, no electrons were emitted at all when the frequency of the incident light was smaller than a certain minimum value, called the *threshold frequency*. This minimum frequency depends on the nature of the material of the emitter plate.

It was found that certain metals like zinc, cadmium, magnesium, etc., responded only to ultraviolet light, having short wavelength, to cause electron emission from the surface. However, some alkali metals such as lithium, sodium, potassium, caesium and rubidium were sensitive even to visible light. All these *photosensitive substances* emit electrons when they are illuminated by light. After the discovery of electrons, these electrons were termed as *photoelectrons.* The phenomenon is called *photoelectric effect.*

11.4 EXPERIMENTAL STUDY OF PHOTOELECTRIC EFFECT

Figure 11.1 depicts a schematic view of the arrangement used for the experimental study of the photoelectric effect. It consists of an evacuated glass/quartz tube having a thin photosensitive plate C and another metal plate A. Monochromatic light from the source S of sufficiently short wavelength passes through the window W and falls on the photosensitive plate C (emitter). A transparent quartz window is sealed on to the glass tube, which permits ultraviolet radiation to pass through it and irradiate the photosensitive plate C. The electrons are emitted by the plate C and are collected by the plate A (collector), by the electric field created by the battery. The battery maintains the potential difference between the plates C and A, that can be varied. The polarity of the plates C and A can be reversed by a commutator. Thus, the plate A can be maintained at a desired positive or negative potential with respect to emitter C.

When the collector plate A is positive with respect to the emitter plate C, the electrons are attracted to it. The emission of electrons causes flow of electric current in the circuit. The potential difference between the emitter and collector plates is measured by a voltmeter (V) whereas the resulting photo current flowing in the circuit is measured by a microammeter (μA). The photoelectric current can be increased or decreased by varying the potential of collector plate A with respect to the emitter plate C. The intensity and frequency of the incident light can be varied, as can the potential difference *V* between the emitter C and the collector A.

We can use the experimental arrangement of Fig. 11.1 to study the variation of photocurrent with (a) intensity of radiation, (b) frequency of incident radiation, (c) the potential difference between the plates A and C, and (d) the nature of the material of plate C. Light of different frequencies can be used by putting appropriate coloured filter or coloured glass in the path of light falling
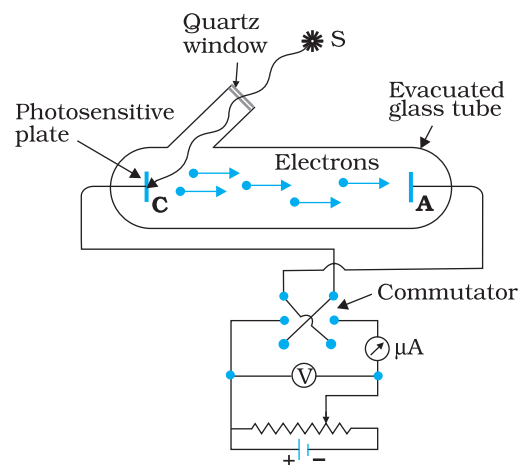

FIGURE 11.1 Experimental arrangement for study of photoelectric effect.
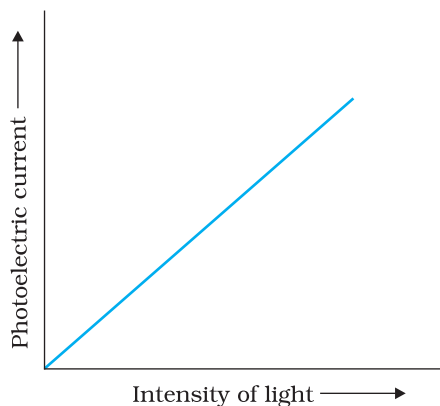
277

FIGURE 11.2 Variation of Photoelectric current with intensity of light.

on the emitter C. The intensity of light is varied by changing the distance of the light source from the emitter.

11.4.1 Effect of intensity of light on photocurrent

The collector A is maintained at a positive potential with respect to emitter C so that electrons ejected from C are attracted towards collector A. Keeping the frequency of the incident radiation and the potential fixed, the intensity of light is varied and the resulting photoelectric current is measured each time. It is found that the photocurrent increases linearly with intensity of incident light as shown graphically in Fig. 11.2. The photocurrent is directly proportional to the number of photoelectrons emitted per second. This implies that *the number of photoelectrons emitted per second is directly proportional to the intensity of incident radiation.*

11.4.2 Effect of potential on photoelectric current

We first keep the plate A at some positive potential with respect to the plate C and illuminate the plate C with light of fixed frequency v and fixed intensity I_1. We next vary the positive potential of plate A gradually and measure the resulting photocurrent each time. It is found that the photoelectric current increases with increase in positive (accelerating) potential. At some stage, for a certain positive potential of plate A, all the emitted electrons are collected by the plate A and the photoelectric current becomes maximum or saturates. If we increase the accelerating potential of plate A further, the photocurrent does not increase. This maximum value of the photoelectric current is called *saturation current.* Saturation current corresponds to the case when all the photoelectrons emitted by the emitter plate C reach the collector plate A.
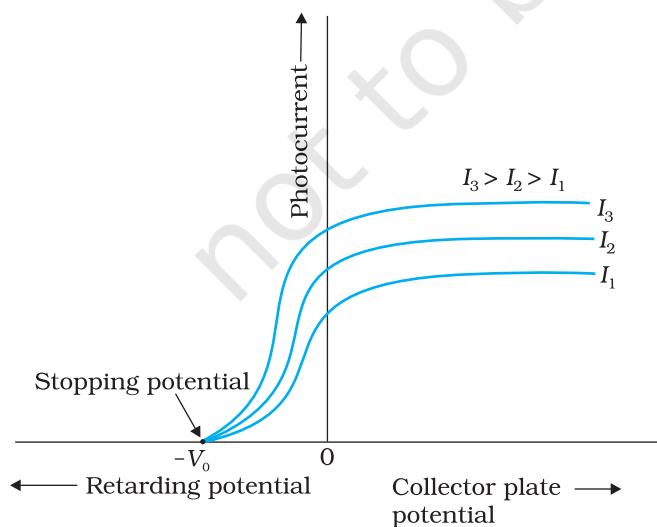

FIGURE 11.3 Variation of photocurrent with collector plate potential for different intensity of incident radiation.

We now apply a negative (retarding) potential to the plate A with respect to the plate C and make it increasingly negative gradually. When the polarity is reversed, the electrons are repelled and only the sufficiently energetic electrons are able to reach the collector A. The photocurrent is found to decrease rapidly until it drops to zero at a certain sharply defined, critical value of the negative potential V_0 on the plate A. For a particular frequency of incident radiation, *the minimum negative (retarding) potential V_0 given to the plate A for which the photocurrent stops or becomes zero is called the cut-off or stopping potential.*

The interpretation of the observation in terms of photoelectrons is straightforward. All the photoelectrons emitted from the metal do not have the

278

same energy. Photoelectric current is zero when the stopping potential is sufficient to repel even the most energetic photoelectrons, with the maximum kinetic energy (K_{max}), so that

$K_{max} = e\, V_0$ (11.1)

We can now repeat this experiment with incident radiation of the same frequency but of higher intensity I_2 and I_3 ($I_3 > I_2 > I_1$). We note that the saturation currents are now found to be at higher values. This shows that more electrons are being emitted per second, proportional to the intensity of incident radiation. But the stopping potential remains the same as that for the incident radiation of intensity I_1, as shown graphically in Fig. 11.3. Thus, *for a given frequency of the incident radiation, the stopping potential is independent of its intensity.* In other words, the maximum kinetic energy of photoelectrons depends on the light source and the emitter plate material, but is independent of intensity of incident radiation.

11.4.3 Effect of frequency of incident radiation on stopping potential

We now study the relation between the frequency ν of the incident radiation and the stopping potential V_0. We suitably adjust the same intensity of light radiation at various frequencies and study the variation of photocurrent with collector plate potential. The resulting variation is shown in Fig. 11.4. We obtain different values of stopping potential but the same value of the saturation current for incident radiation of different frequencies. The energy of the emitted electrons depends on the frequency of the incident radiations. The stopping potential is more negative for higher frequencies of incident radiation. Note from Fig. 11.4 that the stopping potentials are in the order $V_{03} > V_{02} > V_{01}$ if the frequencies are in the order $\nu_3 > \nu_2 > \nu_1$. This implies that greater the frequency of incident light, greater is the maximum kinetic energy of the photoelectrons. Consequently, we need greater retarding potential to stop them completely. If we plot a graph between the frequency of incident radiation and the corresponding stopping potential for different metals we get a straight line, as shown in Fig. 11.5.

The graph shows that

(i) the stopping potential V_0 varies linearly with the frequency of incident radiation for a given photosensitive material.

(ii) there exists a certain minimum cut-off frequency ν_0 for which the stopping potential is zero.
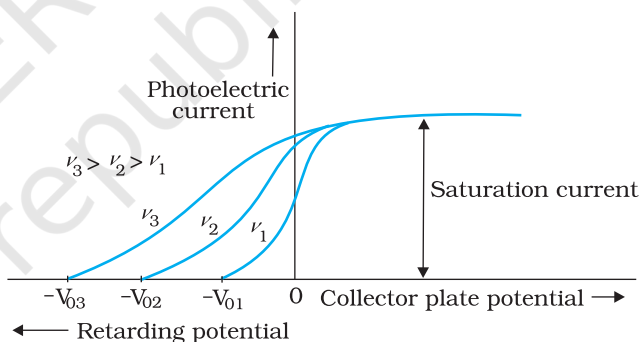

FIGURE 11.4 Variation of photoelectric current with collector plate potential for different frequencies of incident radiation.
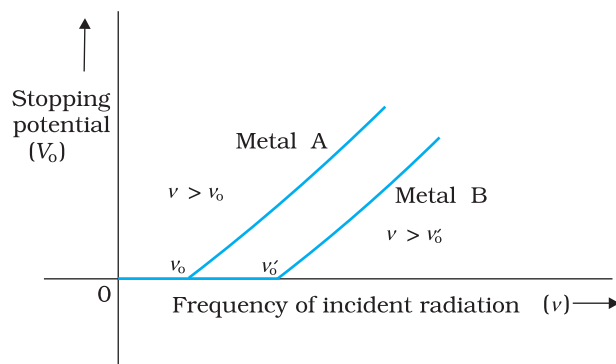

FIGURE 11.5 Variation of stopping potential V_0 with frequency ν of incident radiation for a given photosensitive material.

279

These observations have two implications:

(i) *The maximum kinetic energy of the photoelectrons varies linearly with the frequency of incident radiation, but is independent of its intensity.*

(ii) *For a frequency v of incident radiation, lower than the cut-off frequency v_0, no photoelectric emission is possible even if the intensity is large.*

This minimum, cut-off frequency v_0, is called the *threshold frequency.* It is different for different metals.

Different photosensitive materials respond differently to light. Selenium is more sensitive than zinc or copper. The same photosensitive substance gives different response to light of different wavelengths. For example, ultraviolet light gives rise to photoelectric effect in copper while green or red light does not.

Note that in all the above experiments, it is found that, if frequency of the incident radiation exceeds the threshold frequency, the photoelectric emission starts instantaneously without any apparent time lag, even if the incident radiation is very dim. It is now known that emission starts in a time of the order of 10^{-9} s or less.

We now summarise the experimental features and observations described in this section.

(i) For a given photosensitive material and frequency of incident radiation (above the threshold frequency), the photoelectric current is directly proportional to the intensity of incident light (Fig. 11.2).

(ii) For a given photosensitive material and frequency of incident radiation, saturation current is found to be proportional to the intensity of incident radiation whereas the stopping potential is independent of its intensity (Fig. 11.3).

(iii) For a given photosensitive material, there exists a certain minimum cut-off frequency of the incident radiation, called the *threshold frequency*, below which no emission of photoelectrons takes place, no matter how intense the incident light is. Above the threshold frequency, the stopping potential or equivalently the maximum kinetic energy of the emitted photoelectrons increases linearly with the frequency of the incident radiation, but is independent of its intensity (Fig. 11.5).

(iv) The photoelectric emission is an instantaneous process without any apparent time lag ($\sim 10^{-9}$s or less), even when the incident radiation is made exceedingly dim.

11.5 PHOTOELECTRIC EFFECT AND WAVE THEORY OF LIGHT

The wave nature of light was well established by the end of the nineteenth century. The phenomena of interference, diffraction and polarisation were explained in a natural and satisfactory way by the wave picture of light. According to this picture, light is an electromagnetic wave consisting of electric and magnetic fields with continuous distribution of energy over the region of space over which the wave is extended. Let us now see if this

wave picture of light can explain the observations on photoelectric emission given in the previous section.

According to the wave picture of light, the free electrons at the surface of the metal (over which the beam of radiation falls) absorb the radiant energy continuously. The greater the intensity of radiation, the greater are the amplitude of electric and magnetic fields. Consequently, the greater the intensity, the greater should be the energy absorbed by each electron. In this picture, the maximum kinetic energy of the photoelectrons on the surface is then expected to increase with increase in intensity. Also, no matter what the frequency of radiation is, a sufficiently intense beam of radiation (over sufficient time) should be able to impart enough energy to the electrons, so that they exceed the minimum energy needed to escape from the metal surface . A threshold frequency, therefore, should not exist. These expectations of the wave theory directly contradict observations (i), (ii) and (iii) given at the end of sub-section 11.4.3.

Further, we should note that in the wave picture, the absorption of energy by electron takes place continuously over the entire wavefront of the radiation. Since a large number of electrons absorb energy, the energy absorbed per electron per unit time turns out to be small. Explicit calculations estimate that it can take hours or more for a single electron to pick up sufficient energy to overcome the work function and come out of the metal. This conclusion is again in striking contrast to observation (iv) that the photoelectric emission is instantaneous. In short, the wave picture is unable to explain the most basic features of photoelectric emission.

11.6 EINSTEIN'S PHOTOELECTRIC EQUATION: ENERGY QUANTUM OF RADIATION

In 1905, Albert Einstein (1879-1955) proposed a radically new picture of electromagnetic radiation to explain photoelectric effect. In this picture, photoelectric emission does not take place by continuous absorption of energy from radiation. Radiation energy is built up of discrete units – the so called *quanta of energy of radiation*. Each quantum of radiant energy has energy $h\nu$, where h is Planck's constant and ν the frequency of light. In photoelectric effect, an electron absorbs a quantum of energy ($h\nu$) of radiation. If this quantum of energy absorbed exceeds the minimum energy needed for the electron to escape from the metal surface (work function ϕ_0), the electron is emitted with maximum kinetic energy

$$K_{max} = h\nu - \phi_0 \tag{11.2}$$

More tightly bound electrons will emerge with kinetic energies less than the maximum value. Note that the intensity of light of a given frequency is determined by the number of photons incident per second. Increasing the intensity will increase the number of emitted electrons per second. However, the maximum kinetic energy of the emitted photoelectrons is determined by the energy of each photon.

Equation (11.2) is known as *Einstein's photoelectric equation*. We now see how this equation accounts in a simple and elegant manner all the observations on photoelectric effect given at the end of sub-section 11.4.3.
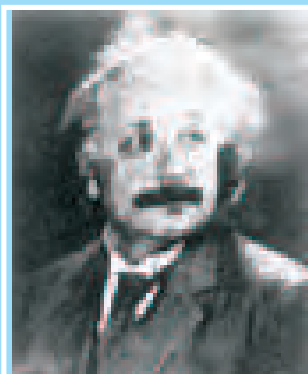
Albert Einstein (1879 – 1955) Einstein, one of the greatest physicists of all time, was born in Ulm, Germany. In 1905, he published three path-breaking papers. In the first paper, he introduced the notion of light quanta (now called photons) and used it to explain the features of photoelectric effect. In the second paper, he developed a theory of Brownian motion, confirmed experimentally a few years later and provided a convincing evidence of the atomic picture of matter. The third paper gave birth to the special theory of relativity. In 1916, he published the general theory of relativity. Some of Einstein's most significant later contributions are: the notion of stimulated emission introduced in an alternative derivation of Planck's blackbody radiation law, static model of the universe which started modern cosmology, quantum statistics of a gas of massive bosons, and a critical analysis of the foundations of quantum mechanics. In 1921, he was awarded the Nobel Prize in physics for his contribution to theoretical physics and the photoelectric effect.

- According to Eq. (11.2), K_{max} depends linearly on v, and is independent of intensity of radiation, in agreement with observation. This has happened because in Einstein's picture, photoelectric effect arises from the absorption of a single quantum of radiation by a single electron. The intensity of radiation (that is proportional to the number of energy quanta per unit area per unit time) is irrelevant to this basic process.

- Since K_{max} must be non-negative, Eq. (11.2) implies that photoelectric emission is possible only if
  $h v > \phi_0$
  or $v > v_0$, where

$$v_0 = \frac{\phi_0}{h} \qquad (11.3)$$

Equation (11.3) shows that the greater the work function ϕ_0, the higher the minimum or threshold frequency v_0 needed to emit photoelectrons. Thus, there exists a threshold frequency v_0 (= ϕ_0/h) for the metal surface, below which no photoelectric emission is possible, no matter how intense the incident radiation may be or how long it falls on the surface.

- In this picture, intensity of radiation as noted above, is proportional to the number of energy quanta per unit area per unit time. The greater the number of energy quanta available, the greater is the number of electrons absorbing the energy quanta and greater, therefore, is the number of electrons coming out of the metal (for $v > v_0$). This explains why, for $v > v_0$, photoelectric current is proportional to intensity.

- In Einstein's picture, the basic elementary process involved in photoelectric effect is the absorption of a light quantum by an electron. This process is instantaneous. Thus, whatever may be the intensity i.e., the number of quanta of radiation per unit area per unit time, photoelectric emission is instantaneous. Low intensity does not mean delay in emission, since the basic elementary process is the same. Intensity only determines how many electrons are able to participate in the elementary process (absorption of a light quantum by a single electron) and, therefore, the photoelectric current.

Using Eq. (11.1), the photoelectric equation, Eq. (11.2), can be written as

$$e V_0 = h v - \phi_0; \text{ for } v \geq v_0$$

$$\text{or } V_0 = \frac{h}{e} v - \frac{\phi_0}{e} \qquad (11.4)$$

This is an important result. It predicts that the V_0 versus v curve is a straight line with slope = (h/e),

independent of the nature of the material. During 1906-1916, Millikan performed a series of experiments on photoelectric effect, aimed at disproving Einstein's photoelectric equation. He measured the slope of the straight line obtained for sodium, similar to that shown in Fig. 11.5. Using the known value of e, he determined the value of Planck's constant h. This value was close to the value of Planck's contant (= 6.626×10^{-34} J s) determined in an entirely different context. In this way, in 1916, Millikan proved the validity of Einstein's photoelectric equation, instead of disproving it.

The successful explanation of photoelectric effect using the hypothesis of light quanta and the experimental determination of values of h and ϕ_0, in agreement with values obtained from other experiments, led to the acceptance of Einstein's picture of photoelectric effect. Millikan verified photoelectric equation with great precision, for a number of alkali metals over a wide range of radiation frequencies.

11.7 PARTICLE NATURE OF LIGHT: THE PHOTON

Photoelectric effect thus gave evidence to the strange fact that light in interaction with matter behaved as if it was made of quanta or packets of energy, each of energy $h\nu$.

Is the light quantum of energy to be associated with a particle? Einstein arrived at the important result, that the light quantum can also be associated with momentum ($h\nu/c$). A definite value of energy as well as momentum is a strong sign that the light quantum can be associated with a particle. This particle was later named *photon*. The particle-like behaviour of light was further confirmed, in 1924, by the experiment of A.H. Compton (1892-1962) on scattering of X-rays from electrons. In 1921, Einstein was awarded the Nobel Prize in Physics for his contribution to theoretical physics and the photoelectric effect. In 1923, Millikan was awarded the Nobel Prize in physics for his work on the elementary charge of electricity and on the photoelectric effect.

We can summarise the photon picture of electromagnetic radiation as follows:

(i) In interaction of radiation with matter, radiation behaves as if it is made up of particles called photons.

(ii) Each photon has energy E (=$h\nu$) and momentum p (= $h\nu/c$), and speed c, the speed of light.

(iii) All photons of light of a particular frequency ν, or wavelength λ, have the same energy E (=$h\nu = hc/\lambda$) and momentum p (= $h\nu/c$= h/λ), whatever the intensity of radiation may be. By increasing the intensity of light of given wavelength, there is only an increase in the number of photons per second crossing a given area, with each photon having the same energy. Thus, photon energy is independent of intensity of radiation.

(iv) Photons are electrically neutral and are not deflected by electric and magnetic fields.

(v) In a photon-particle collision (such as photon-electron collision), the total energy and total momentum are conserved. However, the number of photons may not be conserved in a collision. The photon may be absorbed or a new photon may be created.

283

EXAMPLE 11.1

Example 11.1 Monochromatic light of frequency 6.0 ×10^{14} Hz is produced by a laser. The power emitted is 2.0 ×10^{-3} W. (a) What is the energy of a photon in the light beam? (b) How many photons per second, on an average, are emitted by the source?

Solution

(a) Each photon has an energy
$E = h\,\nu = (\,6.63 \times 10^{-34}\,J\,s) (6.0 \times 10^{14}\,Hz)$
$= 3.98 \times 10^{-19}\,J$

(b) If N is the number of photons emitted by the source per second, the power P transmitted in the beam equals N times the energy per photon E, so that $P = N\,E$. Then

$$N = \frac{P}{E} = \frac{2.0 \times 10^{-3}\,W}{3.98 \times 10^{-19}\,J}$$

$= 5.0 \times 10^{15}$ photons per second.

EXAMPLE 11.2

Example 11.2 The work function of caesium is 2.14 eV. Find (a) the threshold frequency for caesium, and (b) the wavelength of the incident light if the photocurrent is brought to zero by a stopping potential of 0.60 V.

Solution

(a) For the cut-off or threshold frequency, the energy $h\,\nu_0$ of the incident radiation must be equal to work function ϕ_0, so that

$$\nu_0 = \frac{\phi_0}{h} = \frac{2.14\,eV}{6.63 \times 10^{-34}\,J\,s}$$

$$= \frac{2.14 \times 1.6 \times 10^{-19}\,J}{6.63 \times 10^{-34}\,J\,s} = 5.16 \times 10^{14}\ Hz$$

Thus, for frequencies less than this threshold frequency, no photoelectrons are ejected.

(b) Photocurrent reduces to zero, when maximum kinetic energy of the emitted photoelectrons equals the potential energy $e\,V_0$ by the retarding potential V_0. Einstein's Photoelectric equation is

$$eV_0 = h\nu - \phi_0 = \frac{hc}{\lambda} - \phi_0$$

or, $\lambda = hc/(eV_0 + \phi_0)$

$$= \frac{(6.63 \times 10^{-34}\,J\,s) \times (3 \times 10^{8}\,m/s)}{(0.60\,eV + 2.14\,eV)}$$

$$= \frac{19.89 \times 10^{-26}\,J\,m}{(2.74\,eV)}$$

$$\lambda = \frac{19.89 \times 10^{-26}\,J\,m}{2.74 \times 1.6 \times 10^{-19}\,J} = 454\ nm$$

11.8 WAVE NATURE OF MATTER

The dual (wave-particle) nature of light (electromagnetic radiation, in general) comes out clearly from what we have learnt in this and the preceding chapters. The wave nature of light shows up in the phenomena of interference, diffraction and polarisation. On the other hand, in

284

photoelectric effect and Compton effect which involve energy and momentum transfer, radiation behaves as if it is made up of a bunch of particles – the photons. Whether a particle or wave description is best suited for understanding an experiment depends on the nature of the experiment. For example, in the familiar phenomenon of seeing an object by our eye, both descriptions are important. The gathering and focussing mechanism of light by the eye-lens is well described in the wave picture. But its absorption by the rods and cones (of the retina) requires the photon picture of light.

A natural question arises: If radiation has a dual (wave-particle) nature, might not the particles of nature (the electrons, protons, etc.) also exhibit wave-like character? In 1924, the French physicist Louis Victor de Broglie (pronounced as de Broy) (1892-1987) put forward the bold hypothesis that moving particles of matter should display wave-like properties under suitable conditions. He reasoned that nature was symmetrical and that the two basic physical entities – matter and energy, must have symmetrical character. If radiation shows dual aspects, so should matter. De Broglie proposed that the wave length λ associated with a particle of momentum p is given as

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad (11.5)$$

LOUIS VICTOR DE BROGLIE (1892 – 1987)

Louis Victor de Broglie (1892 – 1987) French physicist who put forth revolutionary idea of wave nature of matter. This idea was developed by Erwin Schrödinger into a full-fledged theory of quantum mechanics commonly known as wave mechanics. In 1929, he was awarded the Nobel Prize in Physics for his discovery of the wave nature of electrons.

where m is the mass of the particle and v its speed. Equation (11.5) is known as the *de Broglie relation* and the wavelength λ of the *matter wave* is called *de Broglie wavelength*. The dual aspect of matter is evident in the de Broglie relation. On the left hand side of Eq. (11.5), λ is the attribute of a wave while on the right hand side the momentum p is a typical attribute of a particle. Planck's constant h relates the two attributes.

Equation (11.5) for a material particle is basically a hypothesis whose validity can be tested only by experiment. However, it is interesting to see that it is satisfied also by a photon. For a photon, as we have seen,

$$p = hv /c \qquad (11.6)$$

Therefore,

$$\frac{h}{p} = \frac{c}{v} = \lambda \qquad (11.7)$$

That is, the de Broglie wavelength of a photon given by Eq. (11.5) equals the wavelength of electromagnetic radiation of which the photon is a quantum of energy and momentum.

Clearly, from Eq. (11.5), λ is smaller for a heavier particle (large m) or more energetic particle (large v). For example, the de Broglie wavelength of a ball of mass 0.12 kg moving with a speed of 20 m s^{-1} is easily calculated:

$p = m\,v = 0.12 \text{ kg} \times 20 \text{ m s}^{-1} = 2.40 \text{ kg m s}^{-1}$

$\lambda = \dfrac{h}{p} = \dfrac{6.63 \times 10^{-34} \text{J s}}{2.40 \text{ kg m s}^{-1}} = 2.76 \times 10^{-34} \text{ m}$

This wavelength is so small that it is beyond any measurement. This is the reason why macroscopic objects in our daily life do not show wave-like properties. On the other hand, in the sub-atomic domain, the wave character of particles is significant and measurable.

Example 11.3 What is the de Broglie wavelength associated with (a) an electron moving with a speed of 5.4×10^6 m/s, and (b) a ball of mass 150 g travelling at 30.0 m/s?

Solution

(a) For the electron:
Mass $m = 9.11 \times 10^{-31}$ kg, speed $v = 5.4 \times 10^6$ m/s. Then, momentum
$p = m\,v = 9.11 \times 10^{-31}$ (kg) $\times 5.4 \times 10^6$ (m/s)
$p = 4.92 \times 10^{-24}$ kg m/s
de Broglie wavelength, $\lambda = h/p$

$\quad = \dfrac{6.63 \times 10^{-34} \text{ J s}}{4.92 \times 10^{-24} \text{ kg m/s}}$

$\lambda = 0.135$ nm

(b) For the ball:
Mass $m' = 0.150$ kg, speed $v' = 30.0$ m/s.
Then momentum $p' = m'\,v' = 0.150$ (kg) $\times 30.0$ (m/s)
$p' = 4.50$ kg m/s
de Broglie wavelength $\lambda' = h/p'$.

$\quad = \dfrac{6.63 \times 10^{-34} \text{ J s}}{4.50 \times \text{ kg m/s}}$

$\lambda' = 1.47 \times 10^{-34}$ m

The de Broglie wavelength of electron is comparable with X-ray wavelengths. However, for the ball it is about 10^{-19} times the size of the proton, quite beyond experimental measurement.

EXAMPLE 11.3

SUMMARY

1. The minimum energy needed by an electron to come out from a metal surface is called the work function of the metal. Energy (greater than the work function (ϕ_o) required for electron emission from the metal surface can be supplied by suitably heating or applying strong electric field or irradiating it by light of suitable frequency.

2. Photoelectric effect is the phenomenon of emission of electrons by metals when illuminated by light of suitable frequency. Certain metals respond to ultraviolet light while others are sensitive even to the visible light. Photoelectric effect involves conversion of light energy into electrical energy. It follows the law of conservation of energy. The photoelectric emission is an instantaneous process and possesses certain special features.

286

3. Photoelectric current depends on (i) the intensity of incident light, (ii) the potential difference applied between the two electrodes, and (iii) the nature of the emitter material.

4. The stopping potential (V_o) depends on (i) the frequency of incident light, and (ii) the nature of the emitter material. For a given frequency of incident light, it is independent of its intensity. The stopping potential is directly related to the maximum kinetic energy of electrons emitted: $e\,V_0 = (1/2)\,m\,v_{max}^2 = K_{max}$.

5. Below a certain frequency (threshold frequency) v_0, characteristic of the metal, no photoelectric emission takes place, no matter how large the intensity may be.

6. The classical wave theory could not explain the main features of photoelectric effect. Its picture of continuous absorption of energy from radiation could not explain the independence of K_{max} on intensity, the existence of v_o and the instantaneous nature of the process. Einstein explained these features on the basis of photon picture of light. According to this, light is composed of discrete packets of energy called quanta or photons. Each photon carries an energy E (= $h\,v$) and momentum p (= h/λ), which depend on the frequency (v) of incident light and not on its intensity. Photoelectric emission from the metal surface occurs due to absorption of a photon by an electron.

7. Einstein's photoelectric equation is in accordance with the energy conservation law as applied to the photon absorption by an electron in the metal. The maximum kinetic energy $(1/2)m\,v_{max}^2$ is equal to the photon energy (hv) minus the work function ϕ_0 (= hv_0) of the target metal:

$$\frac{1}{2}\,m\,v_{max}^2 = V_0\,e = hv - \phi_0 = h\,(v - v_0)$$

This photoelectric equation explains all the features of the photoelectric effect. Millikan's first precise measurements confirmed the Einstein's photoelectric equation and obtained an accurate value of Planck's constant h. This led to the acceptance of particle or photon description (nature) of electromagnetic radiation, introduced by Einstein.

8. Radiation has dual nature: wave and particle. The nature of experiment determines whether a wave or particle description is best suited for understanding the experimental result. Reasoning that radiation and matter should be symmetrical in nature, Louis Victor de Broglie attributed a wave-like character to matter (material particles). The waves associated with the moving material particles are called matter waves or de Broglie waves.

9. The de Broglie wavelength (λ) associated with a moving particle is related to its momentum p as: $\lambda = h/p$. The dualism of matter is inherent in the de Broglie relation which contains a wave concept (λ) and a particle concept (p). The de Broglie wavelength is independent of the charge and nature of the material particle. It is significantly measurable (of the order of the atomic-planes spacing in crystals) only in case of sub-atomic particles like electrons, protons, etc. (due to smallness of their masses and hence, momenta). However, it is indeed very small, quite beyond measurement, in case of macroscopic objects, commonly encountered in everyday life.

Physical Quantity	Symbol	Dimensions	Unit	Remarks
Planck's constant	h	$[ML^2T^{-1}]$	J s	$E = h\nu$
Stopping potential	V_0	$[ML^2T^{-3}A^{-1}]$	V	$eV_0 = K_{max}$
Work function	ϕ_0	$[ML^2T^{-2}]$	J; eV	$K_{max} = E - \phi_0$
Threshold frequency	ν_0	$[T^{-1}]$	Hz	$\nu_0 = \phi_0/h$
de Broglie wavelength	λ	$[L]$	m	$\lambda = h/p$

POINTS TO PONDER

1. Free electrons in a metal are free in the sense that they move inside the metal in a constant potential (This is only an approximation). They are not free to move out of the metal. They need additional energy to get out of the metal.
2. Free electrons in a metal do not all have the same energy. Like molecules in a gas jar, the electrons have a certain energy distribution at a given temperature. This distribution is different from the usual Maxwell's distribution that you have learnt in the study of kinetic theory of gases. You will learn about it in later courses, but the difference has to do with the fact that electrons obey Pauli's exclusion principle.
3. Because of the energy distribution of free electrons in a metal, the energy required by an electron to come out of the metal is different for different electrons. Electrons with higher energy require less additional energy to come out of the metal than those with lower energies. Work function is the least energy required by an electron to come out of the metal.
4. Observations on photoelectric effect imply that in the event of matter-light interaction, *absorption of energy takes place in discrete units of hν*. This is not quite the same as saying that light consists of particles, each of energy $h\nu$.
5. Observations on the stopping potential (its independence of intensity and dependence on frequency) are the crucial discriminator between the wave-picture and photon-picture of photoelectric effect.
6. The wavelength of a matter wave given by $\lambda = \dfrac{h}{p}$ has physical significance; its phase velocity v_p has no physical significance. However, the group velocity of the matter wave is physically meaningful and equals the velocity of the particle.

EXERCISES

11.1 Find the
(a) maximum frequency, and
(b) minimum wavelength of X-rays produced by 30 kV electrons.

11.2 The work function of caesium metal is 2.14 eV. When light of frequency 6×10^{14}Hz is incident on the metal surface, photoemission of electrons occurs. What is the

(a) maximum kinetic energy of the emitted electrons,

(b) Stopping potential, and

(c) maximum speed of the emitted photoelectrons?

11.3 The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?

11.4 Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW.

(a) Find the energy and momentum of each photon in the light beam,

(b) How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section which is less than the target area), and

(c) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?

11.5 In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be 4.12×10^{-15} V s. Calculate the value of Planck's constant.

11.6 The threshold frequency for a certain metal is 3.3×10^{14} Hz. If light of frequency 8.2×10^{14} Hz is incident on the metal, predict the cut-off voltage for the photoelectric emission.

11.7 The work function for a certain metal is 4.2 eV. Will this metal give hotoelectric emission for incident radiation of wavelength 330 nm?

11.8 Light of frequency 7.21×10^{14} Hz is incident on a metal surface. Electrons with a maximum speed of 6.0×10^5 m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons?

11.9 Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.

11.10 What is the de Broglie wavelength of

(a) a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s,

(b) a ball of mass 0.060 kg moving at a speed of 1.0 m/s, and

(c) a dust particle of mass 1.0×10^{-9} kg drifting with a speed of 2.2 m/s?

11.11 Show that the wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum (photon).

289