PHYSICS

The following questions given below consist of an "Assertion" (A) and "Reason" (R) Type questions. Use the following Key to choose the appropriate answer.

(A) If both (A) and (R) are true, and (R) is the correct explanation of (A).

(B) If both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) If (A) is true but (R) is false.

(D) If (A) is false but (R) is true.

Q.1 **Assertion :** A rocket launched vertically upward explodes at the highest point it reaches. The explosion produces three fragments with non-zero initial velocity. Then the initial velocity vectors of all the three fragments are in one plane.

Reason : For sum of momentum of three particles to be zero all the three momentum vectors must be coplanar. **[A]**

Sol. For sum of three non null vectors to be zero, three must be coplanar. Hence Reason is a correct explanation for Assertion.

Q.2 **Assertion :** The work done by all forces on a system equals to the change in kinetic energy of that system. This statement is true even if nonconservative forces act on the system.

Reason : The total work done by internal forces may be positive. **[B]**

Sol. Both the Assertion & Reason are true. The work done by all forces on a system is equal to change in its kinetic energy, irrespective of fact whether work done by internal forces is positive, is zero or is negative.

Q.3 **Assertion :** When a spring is elongated work done by spring is negative but when it compressed work done by spring is positive.

Reason : Work done by spring is path independent.

[D]

Q.4 **Assertion :** If in some case work done by a force is path independent then it must be conservative.

Reason : Work done by conservative forces in a round trip must be zero. **[D]**

Q.5 **Assertion :** When a perfectly elastic ball is dropped on floor from some small height then its motion is SHM, if air resistance is neglected.

Reason : The mechanical energy of ball is conserved if air resistance is neglected.

Sol. **[D]**

$$T = 2\sqrt{\frac{2h}{g}}$$

Q.6 **Assertion :** In the reference frame of centre of mass net force acting on system is always zero.

Reason : A pseudo force given by $\vec{P}_S = -m\,\vec{a}_{cm}$ (where m mass of system and $\vec{a}_{cm}$ = acceleration of centre of mass) acts on system which balances all the external forces.

[A]

Q.7 **Assertion :** Two identical block are connected with spring (spring constant = k). The spring is stretched by a distance 'x_0' and released. Work done by spring on any block when it comes to natural length to $\frac{1}{4}kx_0^2$.

Reason : Work done on block can be calculated as $W = \int_0^{(x_0/2)} kx\,dx$ **[C]**

Q.8 **Assertion** : A uniform sphere is placed on a smooth horizontal surface and horizontal force F is applied on it at a distance h above the surface. The acceleration of centre is independent of h.

Reason : Acceleration depends only on force and mass. **[B]**

Q.9 **Assertion** : A body of mass m_1 head on elastically collides with another stationary body of mass m_2. After the collision velocity of mass m_2 is maximum, when $m_1 << m_2$.

Reason : Velocity of second body is always maximum, when its mass m_2 is greater than mass of the hitting body. **[A]**

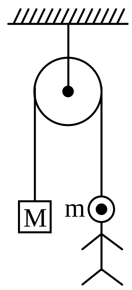
Q.10 **Assertion** : Two blocks of masses m_1 and m_2 are at rest. They are moving towards each other under the mutual internal force. The velocity of centre of mass is zero.

Reason : If no external force act on the system, then velocity of centre of mass unchanged but can never be zero. **[C]**

Q.11 **Assertion** : When a massive projectile collides with a lighter stationary target then maximum speed of target is twice that of projectile.

Reason : It is explained by the momentum and energy conservation. **[A]**

Q.12 **Assertion** : The centre of mass shift upwards, as man moves upwards.



Reason : The centre of mass accelerates under the action of net force. **[A]**

Q.13 **Assertion** : In an elastic collision between two bodies,

the relative speed of the bodies after collision is equal to the relative speed before the collision.

Reason : In a elastic collision, the linear momentum of the system is conserved.

[IIT-2007]

[D]

Q.14 **Assertion :** Acceleration of centre of mass doesn't depend upon internal forces.

Reason : In the reference frame of centre of mass $\vec{a}_{CM} = \frac{\Sigma \vec{F}_{ext}}{M}$ where $\Sigma \vec{F}_{ext}$ = Sum of all real external forces and M = mass of system.

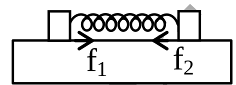
[C]

Sol. In reference frame of centre of mass $\vec{a}_{CM}$ is always zero.

Q.15 **Assertion :** A platform is kept on frictionless horizontal surface. Two block of mass m_1 and m_2 are connected with spring and kept on platform. The spring is stretched and released. If coefficient of friction between blocks and platform is 'μ', then platform will not move only if $m_1 = m_2$.

Reason : Internal force cannot change position of centre of mass. **[B]**

Sol.



Acceleration of platform is zero only when net force on platform in horizontal direction is zero.

i.e. $f_1 = f_2$ where 'f_1' friction force on platform due to 1.

$\Rightarrow m_1 = m_2$

Q.16 **Assertion :** When a spring is elongated work done by spring is negative but when it is compressed work done by spring is positive.

Reason : Work done by spring force is path independent. **[D]**

Q.17 **Assertion :** The centre of mass of an electron and proton when released moves faster towards proton.

Reason : Proton is heavier than electron. **[D]**

Q.18 **Assertion** : A rocket moves forward by pushing the surrounding air backwards.

Reason : It derives the necessary thrust to move forward, according to Newton's third law of motion.

(A) Both Assertion and Reason are true and Reason is a correct explanation of the Assertion

(B) Both Assertion and Reason are true but Reason is not a correct explanation of the Assertion

(C) Both Assertion and Reason are false

(D) Assertion is false but the Reason is true **[D]**

Q.19 **Assertion:** In an elastic collision of two billiards balls, the kinetic energy is not conserved during the short interval of time of collision between the balls.

Reason : Energy spent against friction does not follow the law of conservation of energy. **[C]**

Q.20 **Assertion :** Work done in moving a body in non-uniform circular motion is zero.

Reason : The centripetal force always acts along the radius of the circle. **[D]**

Q.1 A single conservative force acts on a body of mass 1 kg that moves along the x-axis. The potential energy U(x) is given by $U(x) = 20 + (x–2)^2$ where x is in meters. At x = 5.0 m the particle has a kinetic energy of 20 J then –

Column-I	**Column-II**
(A) minimum value of x in meters	(P) 29
(B) maximum value of x in meters	(Q) 7.38
(C) maximum potential energy in joules	(R) 49
(D) maximum kinetic Energy in joules	(S) – 3.38

(A) $\rightarrow$ S (B) $\rightarrow$ Q (C) $\rightarrow$ R (D) $\rightarrow$ P

Q.2 A particle of mass m, kinetic energy K and linear momentum p collides head on elastically with another particle of mass 2m at rest. Match the following (after collision) –

(A) momentum of first particle	(P) $\frac{3}{4}p$
(B) momentum of second particle	(Q) $\frac{8K}{9}$
(C) kinetic energy of first particle	(R) $-\frac{p}{3}$
(D) kinetic energy of second particle	(S) none

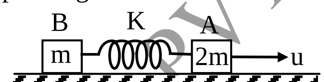
Sol. **A $\rightarrow$ R; B $\rightarrow$ S ; C $\rightarrow$ S ; D $\rightarrow$ Q**

$mv_1 + 2mv_2 = p$

also $\frac{1}{2}mv_1^2 + \frac{1}{2} \times 2m \times v_2^2 = K$

solving $v_1 = -\frac{p}{3m}$ and $v_2 = \frac{2p}{3m}$

Q.3 Two blocks A and B of mass m and 2m respectively are connected by a massless spring of spring constant K. This system lies over a smooth horizontal surface. At t = 0 the block A has velocity u towards right as shown while the speed of block B is zero, and the length of spring is equal to its natural length at that instant. In each situation of column-I, certain statements are given and corresponding results are given in column-II, Match the statements in column-I to the corresponding results in column-II :



smooth horizontal surface

Column I	**Column II**
(A) The velocity of block A	(P) Can never be zero
(B) The velocity of block B	(Q) may be zero at certain instants of time
(C) The kinetic energy of system of two blocks	(R) is minimum at maximum compression of spring
(D) The potential energy of spring	(S) is minimum at maximum extension of spring

Sol. **A $\rightarrow$ P ; B $\rightarrow$ Q ; C $\rightarrow$ P,R ; D $\rightarrow$ Q,S**

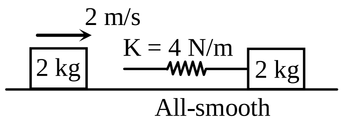
(A) If velocity of block A is zero, from conservation of momentum, speed of block B is 2u. The K.E. of block B = $\frac{1}{2}$ m $(2u)^2 = 2mu^2$ is greater than net mechanical energy of system. Since this is not possible, velocity of a never be zero.

(B) Since initial velocity of B is zero, it shall be zero for many other instants of time.

(C) Since momentum of system is non-zero, K.E. of system cannot be zero. Also K.E of system is minimum at maximum extension of spring.

(D) The potential energy of spring shall be zero whenever it comes to natural length. Also P.E. of spring is maximum at maximum extension of spring.

Q.4 In the arrangement shown in figure match the following :



Column-I	Column-II
(A) Velocity of center of mass	(P) 2 SI unit
(B) Velocity of combined mass when compression in the spring is maximum	(Q) 1 SI unit
(C) Maximum compression in the spring	(R) 4 SI unit
(D) Maximum potential energy stored in the spring	(S) 0.5 SI unit

Sol. **A → Q ; B → Q ; C → Q ; D → P**

$v_{cm} = \dfrac{m_1v_1 + m_2v_2}{m_1 + m_2} = 1$ m/s

During maximum compression also, velocity of combined mass is 1 m/s

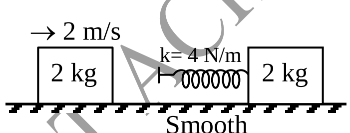
Now, $U_{max} = K_i - K_f$

$= \dfrac{1}{2} \times 2 \times (2)^2 - \dfrac{1}{2} \times 4 \times (1)^2 = 2J$

From $\dfrac{1}{2} K\ X^2_{max} = 2J$

we have, $X_{max} = 1$ m

Q.5 A block of mass 2 kg is moving with a speed of 2 m/s towards other block of same mass as shown. The spring connected to second mass has spring constant 4 N/m. Then, match the following columns



	Column-I		Column-II
(A)	Velocity of centre of mass.	**(P)**	2 SI units
(B)	Velocity at the maximum compression of spring.	**(Q)**	1 SI units
(C)	Maximum compression in spring is	**(R)**	4 SI units
(D)	Maximum potential energy stored in spring.	**(S)**	0.5 SI units

Sol. **A → Q ; B → Q ; C → Q ; D → P**

(A) $v_{cm} = \dfrac{m_1v_1 + m_2v_2}{m_1 + v_2} = 1$ m/s

(B) $v_0 = v_{cm}{}^2 = \dfrac{m_1v_1 + m_2v_2}{m_1 + m_2}$

(C) $\dfrac{1}{2}Kx^2m + \dfrac{1}{2}(m_1 + m_2)\,v_0^2 = \dfrac{1}{2}m_1v_1^2$

where $v_0 = \dfrac{m_1v_1 + m_2v_2}{m_1 + m_2}$

(D) $\dfrac{1}{2}Kx_m^2$

Q.6

Column - I	Column - II
(A) Uniform circular motion	(P) Constant kinetic energy
(B) Elastic collision	(Q) Constant linear momentum
(C) Inelastic collision	(R) Path independent work done
(D) Conservative forces	(S) Zero work done

Sol. **(A) → P, S; (B) → P, Q; (C) → Q; (D) → R**

Q.7 If net external force on a system of particles is zero. Then match the following:

Column - I	Column - II
(A) Velocity of centre of mass	(P) May remain constant
(B) Acceleration of centre of mass	(Q) Must remain constant
(C) Mechanical energy of the system	(R) May remain zero
(D) Kinetic energy of centre of the system	(S) Must remain zero

Sol. **(A) → Q, R; (B) → S; (C) → P, R; (D) → P, R**

Q.8

Column - I	Column - II
(A) Work done by friction	(P) Positive
(B) Work done by conservative force	(Q) Negative
(C) Work done by spring force	(R) Zero
(D) Work done by magnetic force	(S) Not defined

Sol. **(A) → P, Q, R;** **(B) → P, Q, R;**
(C) → P, Q, R; **(D) → R**

Q.9 The potential energy for a conservative system is given by $U = ax^2 - bx$

Column-I	**Column-II**
(A) The net force acting on the system	(P) $b/2a$
(B) The equilibrium position	(Q) $b - 2ax$
(C) The potential energy at the Equilibrium position	(R) $-b^2/4a$
(D) The equilibrium	(S) stable

Sol. **(A) → (Q), (B) → (P), (C) → (R), (D) → (S)**

$F = -\dfrac{dU}{dx} = -[2ax - b] = b - 2ax$

So (A) → (Q)

At equilibrium position

$F = 0$

$\therefore \quad x = b/2a$

SO (B) → (P)

Equilibrium potential energy

$U = \dfrac{a.b^2}{4a^2} - \dfrac{b.b}{2a} = \dfrac{-b^2}{4a}$

So (C) → (R)

$\dfrac{d^2U}{dx^2} = 2a$ hence $\dfrac{d^2U}{dx^2} = +ve$

i.e. system is at stable equilibrium.

So (D) → (S)

Q.10 Let h_0 be the initial height of ball with respect to the earth. The coefficient of restitution is e.

Column-I	**Column-II**
(A) Total distance travelled by the ball before coming to rest	(P) $e^{2n} h_0$
(B) Height attained after n impacts	(Q) $h_0\left(\dfrac{1+e^2}{1-e^2}\right)$
(C) Average force exerted by ball	(R) $P\left(\dfrac{1+e}{1-e}\right)$
(D) Total momentum transferred to the earth	(S) mg

Sol. **(A) → (Q), (B) → (P), (C) → (S), (D) → (R)**

From definition of coefficient of restitution

$e = \dfrac{v_2 - v_1}{u_1 - u_2} = \dfrac{v_1}{u} = \dfrac{\sqrt{2gh_1}}{\sqrt{2gh_0}}$

(h_1 = height attained after impact, h_0 = Drop height)

$h_1 = e^2 h_0$

After n impact / collision

$h_n = e^{2n} h_0$

So (B) → (P)

Total distance travelled by the ball before coming to rest.

$h = h_0 + 2h_1 + 2h_2 + \ldots$

$h = h_0 + 2e^2h_0 + 2e^4h_0 + \ldots$

$= h_0 (1 + e^2) (1 - e^2)^{-1}$

$h = h_0\left(\dfrac{1+e^2}{1-e^2}\right)$

So (A) → (Q)

$\Delta P_1 = P - (-eP) = P(1 + e)$

$\Delta P_2 = eP(1 + e)$

Total momentum transfer is

$\Delta P = P(1 + e) + eP(1 + e) + \ldots$

$= P\left(\dfrac{1+e}{1-e}\right)$

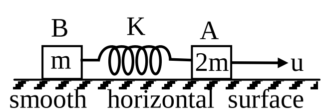
So (D) → (R)

Average force $F = \dfrac{dP}{dt}$

$= \dfrac{P(1+e)(1-e)^{-1}}{\sqrt{\dfrac{2h_0}{g}}\left(\dfrac{1+e}{1-e}\right)} = mg$

So (C) → (S)

Q.11 Two blocks A and B of mass m and 2m respectively are connected by a massless spring of spring constant K. This system lies over a smooth horizontal surface. At t = 0 the block A has velocity u towards right as shown while the speed of block B is zero, and the length of spring is equal to its natural length at that instant. In each situation of column-I, certain statements are given and corresponding results are given in column-II, Match the statements in column-I to the corresponding results in column-II :



Column I	**Column II**
(A) The velocity of	(P) Can never be zero

block A	
(B) The velocity of block B	(Q) may be zero at certain instants of time
(C) The kinetic energy of system of two blocks	(R) is minimum at maximum compression of spring
(D) The potential energy of spring	(S) is minimum at maximum extension of spring

Ans. **A $\rightarrow$ P ; B $\rightarrow$ Q ; C $\rightarrow$ P, R ; D $\rightarrow$ Q, S**

Q.12 If the net force acting on a system is represented by $\vec{F}$ and its momentum is $\vec{p}$, then match the entries of column I with the entries of column II.

Column I	**Column II**
(A) If $\vec{F}$ is constant	(P) $\vec{p}$ may change its direction
(B) If $\vec{F}$ is changing in magnitude	(Q) $\vec{p}$ must change its magnitude
(C) If $\vec{F}$ is changing in direction	(R) $\vec{p}$ may not change its direction
(D) If $\vec{F}$ is zero	(S) $\vec{p}$ must not change its direction

Ans. **A $\rightarrow$ P, Q, R ; B $\rightarrow$ P, Q, R ; C $\rightarrow$ P, Q ; D $\rightarrow$ S**

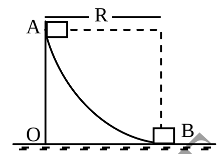
Q.13 Match the following :

(p = momentum of particle, K = kinetic energy of particle)

Column I	**Column II**
(A) p is increased by 200 %, corresponding change in K	(P) 800 %
(B) K is increased by 300 %, corresponding change in p	(Q) 200%
(C) p is increased by 1 %, corresponding change in K	(R) 0.5%
(D) K is increased by 1%, corresponding change in p	(S) 2%
	(T) None

Ans. **A $\rightarrow$ P ; B $\rightarrow$ T ; C $\rightarrow$ S ; D $\rightarrow$ R**

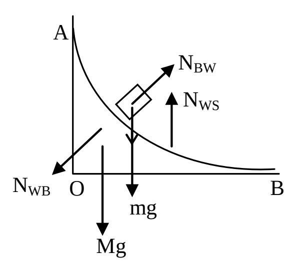
Q.14 A wedge is kept over a frictionless horizontal surface. A block is placed at top 'A' and slips to bottom 'B' of the wedge all surface are frictionless. Then during the motion of block from 'A' to 'B'



Column I	**Column II**
(A) Magnitude of momentum of block + wedge system in horizontal direction	(P) Conserved
(B) Mechanical energy of block	(Q) Not conserved
(C) Magnitude of momentum of block + wedge system	(R) Increases
(D) Angular momentum of block about 'O'	(S) Decreases

Ans. **A $\rightarrow$ P; B $\rightarrow$ Q; C $\rightarrow$ Q ; D $\rightarrow$ P**

Free body diagram of block + wedge system



Net external force in horizontal direction = 0

Net external force in vertical direction $\neq$ 0

Angular momentum of block about O at A and B both are zero.

Q.15 **Column I** **Column II**

Column I	Column II
(A) The average of the mean kinetic energies of the random part of molecular motion	(P) Potential energy
(B) Equal amounts of lead and aluminium subjected to input of equal amounts of heat energy, undergo different temperature changes	(Q) Kinetic energy
(C) Increases as the string in an archer's bow is drawn back	(R) Heat capacity
(D) Ability to do work in virtue of motion	(S) Temperature

Sol. **A $\rightarrow$ S; B $\rightarrow$ R; C $\rightarrow$ P; D $\rightarrow$ Q**

(A) $K.E_{av} = \frac{3}{2} K_B T \Rightarrow S$

(B) $Q = mc\Delta T$ = Constant

$C\Delta T$ = constant

$C_1 \Delta T_1 = C_2 \Delta T_2 \Rightarrow R$

(C) As the elastic string stretches its Potential energy increases $\Rightarrow$ P

(D) Work done = Change in kinetic energy $\Rightarrow$ Q

Q.16 Match the following:

Column-I	Column-II
(A) Work done by all the forces	(P) Change in potential energy
(B) Work done by conservative forces	(Q) Change in kinetic energy
(C) Work done by external forces	(R) Change in mechanical energy
	(S) None

Ans. A $\rightarrow$ Q; B $\rightarrow$ S; C $\rightarrow$ R

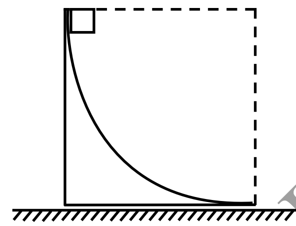
Q.17 Considering 'force' to real as well as pseudo for match the following -

Column-I	Column-II
(A) Laws of motion	(P) Universally applicable
(B) Work energy theorem	(Q) Applicable only in inertial reference frame
(C) Conservation of mechanical energy	(R) No force is present
(D) Conservation of momentum	(S) Non conservative force must do zero work

Sol. **A $\rightarrow$ P ;** **B $\rightarrow$ P ;**

C $\rightarrow$ P,R,S ; **D $\rightarrow$ P,Q,R,S**

Q.18 A block is kept on the top of wedge kept on a horizontal surface. Coefficient of friction between block & wedge is μ' & that between wedge & horizontal surface is μ. As block moves from top to bottom.



Column-I	Column-II
(A) $\mu = 0$, $\mu' = 0$ then mechanical energy of block	(P) conserved
(B) $\mu = 0$, $\mu' = 0$ then magnitude of momentum of block + wedge system	(Q) Increases
(C) $\mu \neq 0$, $\mu' \neq 0$ then magnitude of momentum of wedge in vertical direction	(R) Decrease
(D) $\mu \neq 0$, $\mu' \neq 0$, the magnitude of angular momentum of block about point 'A'	(S) Increases then decreases

Sol. **A $\rightarrow$ R ; B $\rightarrow$ Q ; C $\rightarrow$ P; D $\rightarrow$ P, S**

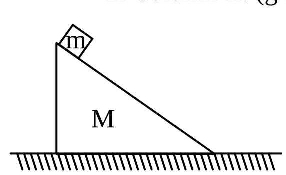
A $\rightarrow$ Work done on block by normal force is –ve.

B $\rightarrow$ External force (gravitational force) acts on System

C $\rightarrow$ Law of momentum conservation

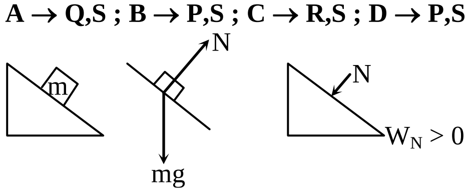
D $\rightarrow$ Law of angular momentum conservation

Q.19 A block of mass m lies on a wedge of mass M. The wedge in turn lies on a smooth horizontal surface. Friction is absent everywhere. The wedge block system is released from rest. All situations given in Column I are to be estimated in duration the block undergoes a vertical displacement 'h' starting from rest (assume the block to be still on the wedge). Match the statements in Column I with the results in Column II. (g is acceleration due to gravity)



Column -I	*Column-II*
(A) Work done by normal reaction acting on the block is	*(P) positive*
(B) Work done by normal reaction (exerted by block) acting on the wedge is	*(Q) negative*
(C) The sum of work done by normal reaction on wedge and Mg	*(R) zero*
(D) Net work done by all forces on the block is	*(S) less than mgh in magnitude*

Sol. **A → Q,S ; B → P,S ; C → R,S ; D → P,S**



angle between N and v of block is abtuse

W_N on block < 0

W_{mg} = mgh (on block)

$\frac{1}{2}mv^2 = W_N + W_{mg}$ or $\frac{1}{2}mv^2 = W_N + mgh$

$\frac{1}{2}mv^2 - mgh = W_N < 0$ or $\frac{1}{2}mv^2 < mgh$

KE of system increases.

Q.20 A particle of mass m, kinetic energy K and momentum p collides head on elastically with another particle of mass 2m at rest. Match the following after collision.

Column-I	Column-II
(A) Momentum of first particle	(P) $\frac{3}{4}p$
(B) Momentum of second particle	(Q) $-\frac{K}{9}$
(C) Kinetic energy of first particle	(R) $-\frac{p}{3}$
(D) Kinetic energy of second particle	(S) $\frac{8K}{9}$
	(T) None

Sol. A → R ; B → T ; C → T ; D → S

Q.1 A thin uniform rod of mass m and length ℓ is free to rotate about its upper end. When it is at rest, it receives an impulse J at its lowest point, normal to its length. Then immediately after impact -

(A) the angular momentum of the rod is $J\ell$.

(B) the angular velocity of the rod is $\frac{3J}{m\ell}$

(C) the kinetic energy of the rod is $\frac{3J^2}{2m}$

(D) the linear velocity of the midpoint of rod is $\frac{3J}{2m}$

Sol. **[A,B,C]**

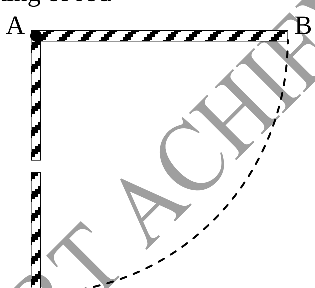
Angular impulse $= J\ell$ = angular moment

$\therefore$ Angular velocity = $\frac{J\ell}{I}$

K.E. = $\frac{1}{2} I\omega^2$

and velocity of mid-point = $\frac{\ell}{2}\omega$

Q.2 A uniform rod AB of length ℓ is free to rotate about a horizontal axis passing through A. The rod is released from rest from horizontal position. If the rod gets broken at mid-point when it becomes vertical then just after breaking of rod –



(A) angular velocity of upper part starts to decrease while that of lower part remains constant

(B) angular velocity of upper part starts to decrease which that of lower part starts to increase

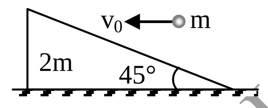
(C) angular velocity of both the parts is identical.

(D) angular velocity of lower parts becomes zero

Sol. **[A]**

Angular momentum is conserned.

Q.3 A ball of mass m hits a wedge of mass '2m' with velocity 'v_0' in horizontal direction and moves in vertically upward direction with velocity '$v_0/2$'. There is no friction between wedge and the surface -



(A) Coefficient of restitution between ball and wedge is 1

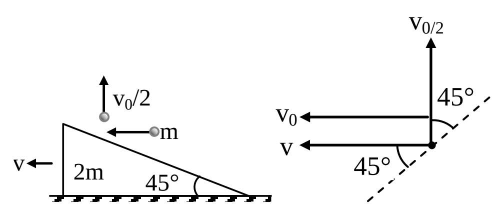
(B) Coefficient of restitution between ball and wedge is $\frac{3}{4}$

(C) Impulse on wedge due to ball is $\frac{\sqrt{5}}{2} mv_0$

(D) Impulse on wedge due to surface is $\frac{mv_0}{2}$

[A,C,D]

Sol.



Momentum of ball + wedge system is conserved in horizontal direction

$\Rightarrow$ $v = v_0/2$

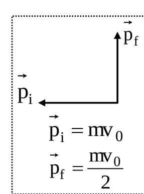
v = velocity of wedge after collision.

$\therefore$ e = $\frac{v_0/2.\cos 45° + v_0/2 \sin 45°}{v_0 \sin 45°} = 1$

Impulse on wedge due to ball = Impulse on ball due to wedge = change in momentum of ball.

$= \sqrt{m^2 v_0^2 + \frac{m^2 v_0^2}{4}}$

$= \frac{\sqrt{5}}{2} mv_0$

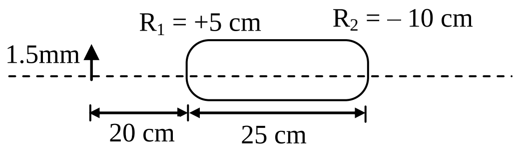


Impulse on wedge due to surface

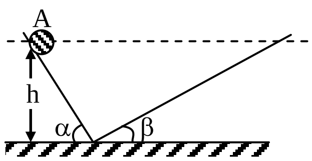
= Change in momentum of ball + wedge system

$= \frac{mv_0}{2}$

Sol.



Q.4 A ball of mass m slips down the frictionless inclined planes as shown.



(A) The period of oscillation of the ball is greater than $\sqrt{\frac{128h}{g}}\left(\frac{1}{\sin\alpha+\sin\beta}\right)$

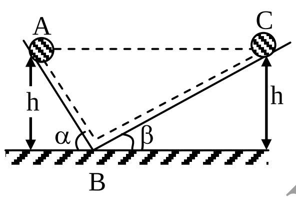
(B) Height attained by ball on inclined plane β is h

(C) After two oscillations ball comes to stop

(D) Ball performs simple harmonic oscillations

[A,B]

Sol.



Body will attained same height 'h' on inclined plane β, because mechanical energy is conserved.

Acceleration along AB = g sin α

h/sinα = AB = $\frac{1}{2}(g\sin\alpha)t_{AB}^2$

$\Rightarrow t_{AB} = \sqrt{\frac{2h}{g\sin^2\alpha}}$

Similarly,

$t_{BC} = \sqrt{\frac{2h}{g\sin^2\beta}}$

Time period of oscillation is

$T = 2t_{AB} + 2t_{BC}$

$$T = \frac{2}{\sin\alpha}\sqrt{\frac{2h}{g}} + \frac{2}{\sin\beta}\sqrt{\frac{2h}{g}}$$

$$T = 2\sqrt{\frac{2h}{g}}\left[\frac{1}{\sin\alpha}+\frac{1}{\sin\beta}\right]$$

$$T = \sqrt{\frac{8h}{g}}\left[\frac{\sin\alpha+\sin\beta}{\sin\alpha\ \sin\beta}\right]$$

$$T = 2\sqrt{\frac{8h}{g}}\left[\frac{\sin\alpha+\sin\beta}{2\sin\alpha\sin\beta}\right]$$

$$T > 2\sqrt{\frac{8h}{g}}\left[\frac{2}{\sin\alpha+\sin\beta}\right]$$

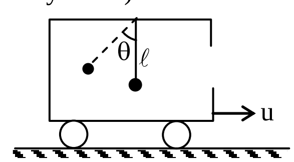
{Applying AM > HM}

$$T > \sqrt{\frac{128h}{g}}\left[\frac{1}{\sin\alpha+\sin\beta}\right]$$

Since acceleration is constant and always directed down along the inclined plane, motion is not simple harmonic but only oscillatory.

Since friction is absent, ball never comes to stop.

Q.5 A pendulum of length ℓ is suspended on a flat car that is moving with a velocity u on the horizontal road. If the car is suddenly stopped, then : (Assume bob of pendulum does not collide anywhere)



(A) the maximum angle θ with the initial vertical line through which the pendulum swing is $\sin^{-1}\left[\frac{u}{2\sqrt{g\ell}}\right]$

(B) the maximum angle θ with the initial vertical line through which the pendulum swing is $2\sin^{-1}\left[\frac{u}{2\sqrt{g\ell}}\right]$

(C) if maximum angle is 60°, ℓ = 5 m and g = 9.8 m/s^2, then the initial speed of car u is 7 m/s

(D) if maximum angle θ is 30°, ℓ = 5 m and g = 9.8 m/s^2 , then the initial speed of car u is 6 m/s

[B,C]

Sol. Using conservation of energy

$$\frac{1}{2}mu^2 + 0 = 0 + mg\ell(1 - \cos\theta)$$

$$u^2 = 2g\ell \times 2\sin^2\frac{\theta}{2}$$

or $\frac{\theta}{2} = \sin^{-1}\left[\frac{u}{2\sqrt{g\ell}}\right]$

$$\Rightarrow \theta = 2\sin^{-1}\left[\frac{u}{2\sqrt{g\ell}}\right]$$

Q.6 A body of mass m was hauled up the hill with constant speed v by a force F which at each point was directed along tangent to the path. If length of base is ℓ and height of hill is h, then which of the following are correct ?

(A) Work done by gravity is – mgh

(B) Work done by friction is – $\mu mg\ell$

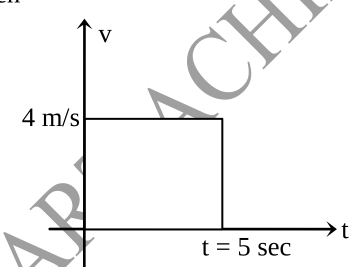
(C) Work done by gravity is path independent

(D) Work done by friction is path independent

[A,C]

Sol. Gravity is a conservative force and here frictional force depends on radius of curvature.

Q.7 Two blocks of masses 2 kg each are moving in opposite direction with equal speed collides at t = 5 sec. The magnitude of relative velocity (v) is plotted against time 't'. The loss in kinetic energy is K and coefficient of restitution in e, then –



(A) K = 8J (B) K = 16J

(C) e = 0.5 (D) e = 0 **[A,D]**

Sol. As relative velocity after collision (t = 5 sec) becomes zero, therefore e = 0

Now, $\Delta K_{loss} = \frac{1}{2} \times \frac{m_1 m_2}{m_1 + m_2} \times (u_1 - u_2)^4(1 - e^2)$

$$= \frac{1}{2} \times \frac{2 \times 2}{2 + 2} \times (4)^2 \times (1 - 0) = 8\text{ J}$$

Q.8 A particle strikes a horizontal smooth floor with a velocity is making an angle θ with the floor and rebounds with velocity v making an angle ϕ with the floor. If the coefficient of restitution between the particle and the floor is e, then -

(A) The impulse delivered by the floor to the body is $mu(1 + e)\sin\theta$

(B) $\tan\phi = e\tan\theta$

(C) $v = u\sqrt{1 - (1 - e^2)\sin^2\theta}$

(D) the ratio of the final kinetic energy to the initial kinetic energy is $(\cos^2\theta + e^2\sin^2\theta)$

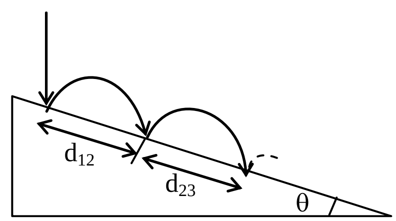
Sol.[A,B,C,D]

$J = \Delta p = m(v\sin\phi + mu\sin\theta)$

$= mv_{app}(1 + e)$

$v = u\sqrt{1 - (1 - e^2)\sin^2\theta}$

Q.9 An elastic ball is dropped on a long inclined plane. If bounces, hits the plane again, bounces and so on. let us Label the distance between the point of the first and second hit d_{12} and the distance between the points of second and the third hit is d_{23}. find the ratio of d_{12}/d_{23} .



(A) $\frac{2}{1}$ (B) $\frac{1}{2}$

(C) $\frac{4}{1}$ (D) $\frac{1}{4}$

Sol. **[B]**

If we rotate the coordinate system 80 that the ramp is horizontal then the free fall acceleration will have two components one downward $a_y = -g\cos\theta$ and one horizontal $a_x = g\sin\theta$. In this frame, the initial velocity will have components given by $V_y = V_0\cos\theta$ & $V_x = V_0\sin\theta$ time for each bounce is given by $t_b = \frac{2V_y}{-ay} = \frac{2V_0}{g}$

The horizontal displacement $\Delta x = V_x t + 0.5\, a_x t^2$ given these equation

$$d_{12} = V_0\sin\theta\left(\frac{2V_0}{g}\right) + 0.5g\sin\theta\left(\frac{2V_0}{g}\right)^2$$

$$= \frac{4V_0^2 \sin\theta}{g}$$

$$d_{13} = V_0\sin\theta\left(\frac{4V_0}{g}\right) + 0.5\ g\ \sin\theta\ \left(\frac{4V_0}{g}\right)^2$$

$$= \frac{12V_0^2\ \sin\theta}{g}$$

Since $d_{23} = d_{13} - d_{12} = \dfrac{8V_0^2\ \sin\theta}{g}$

$$\frac{d_{12}}{d_{23}} = \frac{1}{2}$$

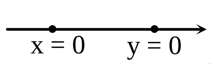
Q.10 Two blocks of mass m_1 and m_2, resting on a frictionless table, are connected by a stretched spring and then released -

(A) ratio of speed of blocks is m_2/m_1
(B) ratio of kinetic energy of blocks is m_2/m_1
(C) centre of mass will move towards heavier block
(D) mechanical energy will remain conserve in this process

Sol. **[A,B,D]** $m_1v_1 = m_2v_2$

$$\frac{v_1}{v_2} = \frac{m_2}{m_1},\ K = \frac{p^2}{2m} \text{ or } \frac{K_1}{K_2} = \frac{m_2}{m_1}$$

Q.11 A particle of mass m = 1 kg, lying on x-axis, experiences a force given by law F = x (3x – 2) N, where x is the x-coordinate of the particle in meters

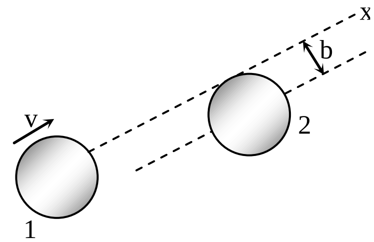


The points on x-axis where the particle is in equilibrium are -

(A) x = 0 (B) $x = \frac{1}{3}$
(C) $x = \frac{2}{3}$ (D) x = 1

Sol. **[A,C]**
The particle is at equilibrium (F = 0) at x = 0 and x = 2/3

Q.12 A ball of radius r moving with a speed v collides elastically with another identical stationary ball. The impact parameter for the collision is b (see figure) –



(A) After collision ball 1 will come to rest and ball 2 will move with velocity making an angle $\sin^{-1}\left(\frac{b}{2r}\right)$ below the x-axis

(B) After collision ball 1 will move with some finite velocity making an angle $\cos^{-1}\left(\frac{b}{2r}\right)$ above x-axis and ball 2 will move with some different making an angle $\sin^{-1}\left(\frac{b}{2r}\right)$ below the x-axis

(C) For perfectly elastic head on collision b = 0, and for perfectly elastic oblique collision 0 < b < 2r

(D) The balls must scatter at right angles

[B,C,D]

Q.13 Two equal sphere of mass m are in contact on a smooth horizontal table. A third identical sphere impinges symmetrically on them and reduces to rest. Then –

(A) Coefficient of restitution is e = 2/3

(B) Loss of kinetic energy $\frac{1}{6}$ mu^2 where u is velocity before impact

(C) After the collision, velocity of equal mass sphere is $\frac{u}{\sqrt{3}}$

(D) Loss of kinetic energy $\frac{1}{3}$ mu^2 **[A,B,C]**

Q.14 A ball moving with a velocity v hits a massive wall moving towards the ball with a velocity u. An elastic impact lasts for a time Δt. Then-

(A) The average elastic force acting on the ball is $\frac{m(u+v)}{\Delta t}$

(B) The average elastic force acting on the ball is $\frac{2m(u+v)}{\Delta t}$

(C) The kinetic energy of the ball increases by $2mu(u + v)$

(D) The kinetic energy of the ball remains the same after the collision **[B,C]**

Q.15 A particle strikes a horizontal smooth floor with a velocity u making an angle θ with the floor and rebounds with velocity v making an angle ϕ with the floor. The coefficient of restitution between the particle and the floor is e. Then-

(A) The impulse delivered by the floor to the body is $mu(1 + e) \sin\theta$

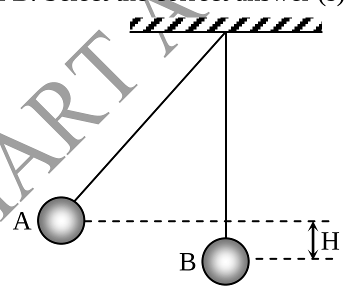
(B) $\tan \phi = e \tan \theta$

(C) $v = u \sqrt{1-(1-e^2)\sin^2\theta}$

(D) The ratio of the final kinetic energy to the initial kinetic energy is $\cos^2\theta + e^2\sin^2\theta$

[All]

Q.16 Two small balls A and B of mass M and 3M hang from the ceiling by strings of equal length. The ball A is drawn aside so that it is raised to a height H. It is then released and collides with ball B. Select the correct answer (s).



(A) If collision is perfectly elastic, ball B will rise to a height H/4

(B) If the collision is perfectly elastic ball A will rise to a height H/4

(C) If the collision is perfectly inelastic, the combined mass will rise to a height H/16

(D) If the collision is perfectly inelastic, the combined mass will rise to a height H/4

[A,B,C]

Q.17 A neutron collides head–on with a stationary hydrogen atom in ground state. Which of the following statement is/are correct ? (Take mass of H–atom equal to mass of neutron)

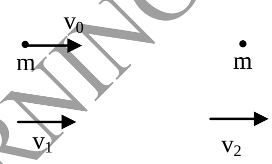
(A) If kinetic energy of the neutron is less than 13.6 eV, collision must be elastic

(B) If kinetic energy of the neutron is less than 13.6 eV, collision may be inelastic

(C) Inelastic collision may take place only when initial kinetic energy of neutron is greater than 20.4 eV

(D) Perfectly inelastic collision cannot take place **[A,C]**

Sol.



$mv_0 = mv_1 + mv_2$

$\Rightarrow v_1 + v_2 = v_0$ .... (1)

$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + E$

$v_0^2 = v_1^2 + v_2^2 + 2\frac{E}{m}$ .... (2)

$\Rightarrow v_1v_2 = \frac{E}{m}$

$(v_1 - v_2)^2 = v_1^2 + v_2^2 - 2v_1v_2 \geq 0$

$\Rightarrow (v_1 + v_2)^2 - 4v_1v_2 \geq 0$

$v_0^2 \geq \frac{4E}{m}$

$\frac{1}{2}mv_0^2 \geq 2E$

E = Minimum Excitation energy

= 10.2 eV

$\therefore KE_{initial} \geq 20.4$ eV

Q.18 A ball hits the floor and rebounds after an inelastic collision. In this case- **[IIT-1986]**

(A) The momentum of the ball just after the collision is the same as that just before the collision

(B) The mechanical energy of the ball remains the same in the collision

(C) The total momentum of the ball and the earth is conserved

(D) The total energy of the ball and the earth is conserved **[C,D]**

Q.19 Which of the following is/are conservative force (s) -

(A) $\vec{F} = 2r^3\ \vec{r}$ (B) $\vec{F} = -\frac{5}{r}\ \vec{r}$

(C) $\vec{F} = \frac{3(X\hat{i} + Y\hat{j})}{(X^2 + Y^2)^{3/2}}$ (D) $\vec{F} = \frac{3(Y\hat{i} + X\hat{j})}{(X^2 + Y^2)^{3/2}}$

[A,B,C]

Sol. Since W = $\int \vec{F}.\vec{dr}$

Clearly for forces (A) and (B) the integration do not require any information of the path taken.

For (C) : $W_C = \int \frac{3(X\hat{i} + Y\hat{j})}{(X^2 + Y^2)^{3/2}}\ (dX\,\hat{i}\ + dY\,\hat{j})$

$= 3\int \frac{XdX + YdY}{(X^2 + Y^2)^{3/2}}$

Taking : $X^2 + Y^2 = t$

$2XdX + 2Y\,dY = dt$

$\Rightarrow XdX + YdY = \frac{dt}{2}$

$\Rightarrow W_C = 3\int \frac{dt/2}{t^{3/2}} = \frac{3}{2}\int \frac{dt}{t^{3/2}}$

which is solvable

Hence (A), (B) and (C) are conservative forces. But (D) requires some more information on path. Hence non conservative.

Q.20 A small ball of mass m is released from rest at a height h, above ground at time t = 0. At time $t = t_0$, the ball comes to rest at a height h_2 above ground. Consider the ground to be perfectly rigid and neglect air friction. In the time interval from

$t = 0$ to $t = t_0$, pick up the correct statements -

(A) work done by gravity on ball is $mg(h_1 - h_2)$

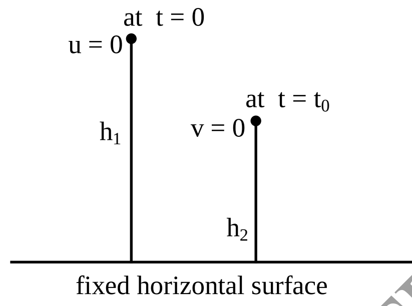
(B) work done by ground on ball for duration of contact is $mg\,(h_2 - h_1)$

(C) Average acceleration of the ball is zero

(D) Net work done on the ball by all forces except gravity is $mg(h_2 - h_1)$ **[A,C,D]**

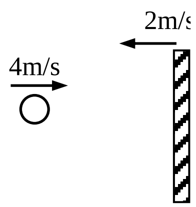
Sol. From the figure work done by gravity from $t = 0$ to $t = t_0$ is

$W = mg\,(h_1 - h_2)$



Since initial and final velocity of ball is zero its average acceleration will be zero. Since net work done is zero from time interval $t = 0$ to $t = t_0$. Hence work done by forces except gravity is $mg\,(h_2 - h_1)$.

Q.1 A ball of mass 2kg moving with speed 4m/s hits a wall normally moving with speed 2 m/s in opposite direction. Coefficient of restitution between wall & ball is e = $\frac{1}{2}$. If ball remains in contact with wall till wall has moved 3m then average force (in N) on ball by wall is -



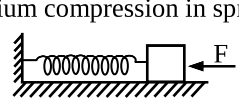
Sol. **[3]**

Velocity of ball after collision = e (V_1 + V_2) + V_2

V_1 = 4m/s, V_2 = 2m/s

$$\langle F\rangle = \frac{K_2 - K_1}{d} = \frac{\frac{1}{2}\times 2(5^2 - 4^2)}{3} = 3N$$

Q.2 A block is attached with spring and is kept on frictionless surface as shown. It is pushed by a force. Ratio of maximum compression and equilibrium compression in spring is -



Sol. **[2]**

Max. compression = $\frac{2F}{k}$

Equilibrium compression = $\frac{F}{k}$

Q.3 A single conservative force acts on a body of mass 1 kg that moves along the x-axis. The potential energy U(x) is given by U (x) = 20 + $(x - 2)^2$, where x is in meters. At x = 5.0 m the particle has a kinetic energy of 20 J, then the maximum kinetic energy of body in J is . **[0029]**

Sol. At x = 5 m, U = 29 J and K = 20 J

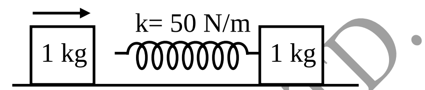
$\therefore$ E = 49 J

Now, E = U_{min} + K_{max}

49 = 20 + K_{max}

K_{max} = 29 J

Q.4 Each of the blocks shown in figure has mass 1 kg. The rear block moves with a speed of 2m/s towards the front block kept at rest. The spring attached to the front block is light and has a spring constant 50 N/m. The maximum compression of the spring is given by $\frac{X}{10}$ m, then find X.



Sol. For maximum compression, velocity of each block is same.

By momentum conservation.

1 × 2 = 1 × v + 1 × v

v = 1 m/s

By energy conservation.

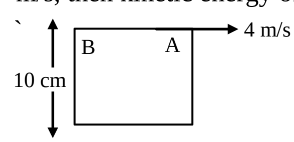
$\frac{1}{2}$ × 1 × 2^2 = $\frac{1}{2}$ × 1 × $(1)^2$ + $\frac{1}{2}$ × 1 × $(1)^2$ + $\frac{1}{2}$ kx^2

$\Rightarrow$ x = 0.2 m

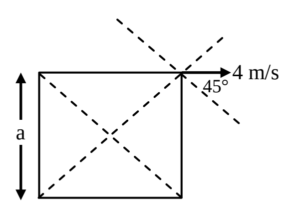
= 2/10 m

X = 2m

Q.5 A cube of mass 3 kg is kept on a frictionless horizontal surface. The block is given an impulse so that point 'A' acquires velocity 4 m/s in the direction shown. If speed of point B is $4\sqrt{2}$ m/s, then kinetic energy of block is –



Sol.



Let angular velocity about centre of mass be 'ω' and velocity centre of mass be v_{CM}

$$\omega = \frac{v_A \cos 45°}{(a/\sqrt{2})} = 40 \text{ rad/sec } (a = 10 \text{ cm})$$

v = v_A sin 45° = $2\sqrt{2}$ m/s

$\therefore$ K.E = $\frac{1}{2}mv_{CM^2} + \frac{1}{2}I\omega^2$ = 16J

Q.6 Machine gun mounted on car is firing 30 bullets per minute onto the truck moving with speed 90 km/hr. The car is chasing truck with speed 180 km/hr. Numbers of bullet hitting the truck per min is : (Speed f bullet with respect to ground = 300 m/s) **[0033]**

Sol. Time between firing of two bullets = 2 sec

$\therefore$ Distance between two consecutive bullets

= 500 m

$\therefore$ Time taken by one bullet getting into truck just after a bullet hits the truck

$$t = \frac{500}{275} \text{ sec}$$

$\therefore$ Bullet hitting the truck per sec

$$= \frac{275}{500} \times 60$$

= 33

Q.7 A body starts moving from origin at t = 0 with a velocity of $5\hat{i}$ in x-y plane under the action of force producing an acceleration of $(3\hat{i} + 2\hat{j})$ m/s^2, then y-co-ordinate in meters of body when x-co-ordinate is 84 m is. **[0036]**

Sol. $x = u_xt + \frac{1}{2}a_xt^2$

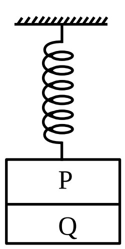
$\Rightarrow$ $84 = 5t + \frac{1}{2} \times 3t^2$ $\Rightarrow$ t = 6 sec

$\therefore$ $y = u_yt + \frac{1}{2}a_yt^2$

Hence, $y = \frac{1}{2} \times 2 \times 36 = 36$ m

Q.8 Two blocks P and Q of masses 0.3 kg and 0.4 kg respectively are stuck to each other by some weak glue as shown in the figure. They hang together at the end of a spring with a spring constant

k = 200 N/m. The block Q suddenly falls free due to failure of glue, then find maximum kinetic energy of the block P during subsequent motion in mJ.



Sol. $A = \frac{0.4g}{200}$

= 2 cm motion starts from lower extreme

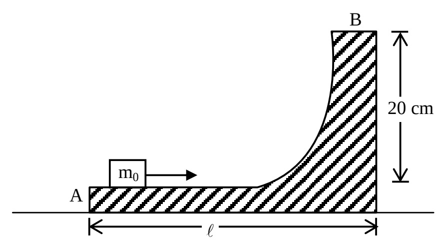
$KE_{max} = \frac{1}{2}m\omega^2A^2 = \frac{1}{2} \times 0.3 \times \frac{k}{m}A^2$

$= \frac{1}{2} \times 0.3 \times \frac{200}{0.3} \times \left(\frac{2}{100}\right)^2$

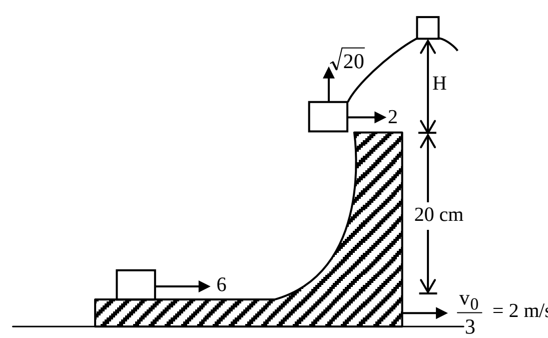
= 0.04 J

= 40 mJ

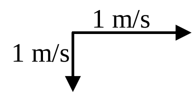
Q.9 A wedge of mass M = 2 m_0 rests on a smooth horizontal plane. A small block of mass m_0 rests over it at left end A as shown in figure. A sharp impulse is applied on the block, due to which it starts moving to the right with velocity v_0 = 6 m/s. At highest point of its trajectory, the block collides with a particle of same mass m_0 moving vertically downwards with velocity v = 2 m/s and gets stuck with it. If the combined mass lands at the end point A of the body of mass M, calculate length ℓ in cm. Neglect friction, take g = 10 m/s^2. **[0040]**



Sol.



At highest point combined mass velocity is



$H = \frac{20}{2g} = 1$m

$$1.2 = 1.T + \frac{1}{2}\ g\, T^2$$

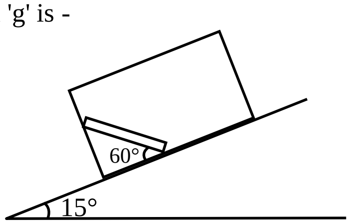
$T = 0.4$ sec

$u_{rel} = 1$ m/s

$\Rightarrow \ell = u_{rel}\, T = 0.4$ m

$= 40$ cm

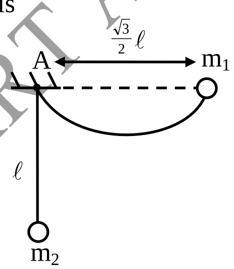
Q.10 A rod is kept inclined inside a box as shown in figure. The box is kept on a inclined plane as shown. All surfaces are frictionless. Let acceleration of box on inclined plane so that rod doesn't slip inside box be 'a'. Then ratio of 'a' and 'g' is -



Sol. **[1]**
Obvious from figure that angle between g_{eff} and vertical
is 60° – 15° = 45°

$\therefore\ \frac{a}{g} = \tan 45° = 1$

Q.11 Two balls of mass $m_1 = 100$ gm and $m_2 = 300$ gm are suspended from point A by two equal inextensible threads, each of length $\ell = \frac{32}{35}$ m. Ball of mass m_1 is drawn aside and held at the same level as A but a distance $\frac{\sqrt{3}}{2}\ell$ from A as shown. When ball m_1 is released, it collides elastically with stationary ball m_2. Then velocity in m/s with which the ball m_1 collides with the ball m_2 is

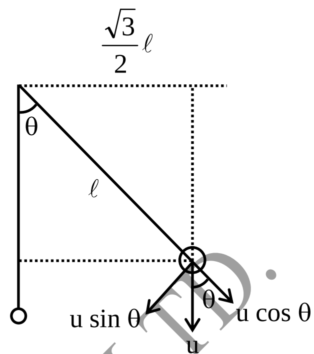


Sol. [4] When string become taut

$\sin\theta = \frac{\sqrt{3}}{2}$

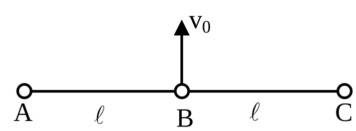
$\Rightarrow \theta = 60°$ and

$u = \sqrt{2g.\frac{\ell}{2}} = \sqrt{g\ell}$



$$\therefore\ \frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1\left(\sqrt{g\ell}\,\sin\theta\right)^2 + m_1g\ell\,(1-\cos\theta)$$

Q.12 Three identical balls each of mass m = 0.5 kg are connected with ideal string as shown in figure and this system rests on a smooth horizontal table. At moment t = 0 ball B is imparted a velocity $v_0 = 9$ m/s as shown. Then the velocity of A in m/s when it collides with ball C.



Sol. [6] Let velocity of A and C in direction perpendicular to motion of B at the time of collision is v_1 and in the direction of motion of B is v_2 then,

by conservation of momentum

$3mv_2 = mv_0 \Rightarrow v_2 = 3$ m/s

by conservation of energy

$$2\times\left[\frac{1}{2}m(v_2)^2 + \frac{1}{2}mv_1^2\right] + \frac{1}{2}\ mv_2^2 = \frac{1}{2}\ mv_0^2$$

$\therefore\ v_1 = 3\sqrt{3}$ m/s $\Rightarrow v_{net} = \sqrt{v_1^2 + v_2^2}$

Q.13 A steel ball of radius R = 20 cm and m = 2 kg is rotating about a horizontal diameter with angular speed $\omega_0 = 50$ rad/sec. This rotating ball is dropped on a rough horizontal floor and fall freely through a height h = 1.25 m. The coefficient of restitution is e = 1 and coefficient of friction between ball and floor is μ = 0.3. Then the distance in m between the point of first and second impact of the ball and floor is

Sol. [3] Ball collides with speed $V_y = \sqrt{2gh}$ = 5 m/s and rebound with same speed as e = 1

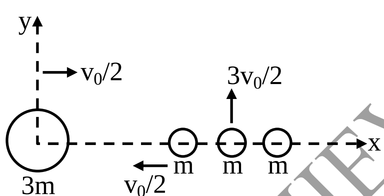
$\therefore$ Impulse of normal reaction = $2mV_y$ = 20 kg s^{-1}

$\therefore$ Impulse of friction = 0.3 × 20 = mV_x

or V_x = 3 m/s

Q.14 A projectile is fixed with velocity v_0 = 2m/s at angle 60º with horizontal. At top of its trajectory it explodes into three fragment of equal mass. First fragment retraces the path, second moves vertically upward with speed $\frac{3V_0}{2}$. The speed of third fragment (in m/s) is –

Sol. **[5]**



Let velocity of third fragment be $\vec{v}$

$\vec{P}_i = \vec{P}_f$

$\Rightarrow 3m.\frac{v_0}{2}\hat{i} = -m\frac{v_0}{2}\hat{i} + m.\frac{3v_0}{2}\hat{j} + m\vec{v}$

$\Rightarrow \vec{v} = 4\frac{v_0}{2}\hat{i} - \frac{3v_0}{2}\hat{j}$

$\Rightarrow v = \frac{5v_0}{2}$

Q.15 A block is hanged from spring in a cage. Elongation in spring is $x_1 = 4\sqrt{2}$ mm and $x_2 = 3\sqrt{2}$ mm when cage moves up and down respectively with same acceleration. The expansion (in mm) in spring when the cage move horizontally with same acceleration.

Sol. **[5]**

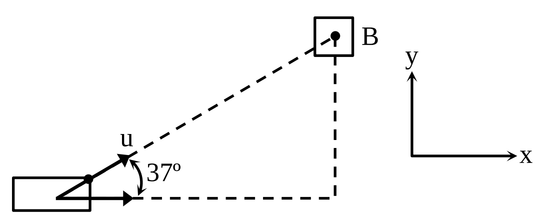
$$\left.\begin{aligned} x_1 &= \frac{m}{k}(g+a) \\ x_2 &= \frac{m}{k}(g-a) \\ x_3 &= \frac{m}{k}\sqrt{g^2+a^2} \end{aligned}\right\} \begin{aligned} &m = \text{mass of pendulum} \\ &k = \text{spring constant} \end{aligned}$$

$\therefore x_1^2 + x_2^2 = \frac{m^2}{k^2}.2(g^2 + a^2)$

$= 2\ x_3^2$

$\Rightarrow x_3 = \sqrt{\frac{x_1^2 + x_2^2}{2}}$

Q.16 A man moving on plank 'A' throws a ball of mass 'm' with horizontal component of velocity $u = 3\sqrt{34}$ m/s w.r.t. ground towards a stationary plank 'B' as shown in figure, while man of plank B catches the ball. The combined mass of plank 'A' with man as well as plank B with man is '2m'. Initially plank 'A' is moving with speed $u\hat{i}$ w.r.t. ground. There is no friction between plank and ground. The relative velocity of B (in m/s) w.r.t. A after man on 'B' catches the ball minus 10 m/s is -



Horizontal plane

Sol. **[7]**

By momentum conservation

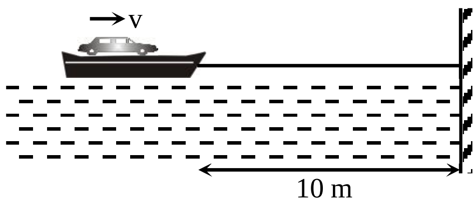
$3mu\hat{i} = m(u\cos 37^o\hat{i} + u\sin 37^o\hat{j}) + 2m\vec{v}_A$

$\Rightarrow \vec{v}_A = \frac{11u}{10}\hat{i} - \frac{3u}{10}\hat{j}$

Similarly $\vec{v}_B = \frac{4u}{15}\hat{i} + \frac{3u}{10}\hat{j}$

$\therefore |v_{BA}|$ = 17m/s.

Q.17 A car of mass 200 kg initially at rest on a boat of mass 1000 kg tied to the wall of dock, through a massless inextensible string. The car accelerates from rest to velocity 10 m/s in 2 sec. At t = 2 sec car applies brake and comes to rest relative to boat in negligible time. Neglecting friction between boat and water, the time at which boat will strike the wall is -



Sol. **[8]**

Velocity of boat after comes to rest : $\frac{200\times10}{200+1000}$

$\therefore$ time taken by boat in hitting the wall

$= 2 + \frac{10(200+1000)}{200\times10} = 8$ sec.

Q.18 A ball of mass m moving with K.E. 3J undergoes head on collision with another stationary ball of mass 2m. During impact maximum potential energy of system can be –

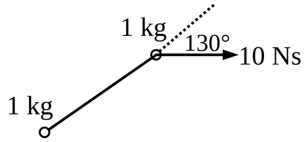
Sol. **[2]**

Maximum potential energy

$= \frac{1}{2}mv^2 - \frac{1}{2} \times 3m\left(\frac{v}{3}\right)^2$

$= \frac{2}{3}\left(\frac{1}{2}mv^2\right)$

Q.19 Two balls of mass 1 kg each are connected by inextensible massless string. The system is resting on smooth horizontal surface. An impulse of $10\sqrt{3}$ Ns is applied to one of the balls at an angle 30° with the line joining two balls in horizontal direction as shown in figure. Assuming that string remains taut after the impulse, the magnitude of impulse (in Ns) minus 10 Ns is -

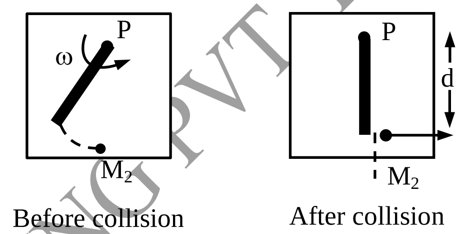


Sol. **[5]**

$10\sqrt{3}\cos 30° - I = I \Rightarrow I = 15$ Ns

($\therefore$ mass will more with same velocity along string)

Q.20 A system contains ball of mass M_2 and uniform thin rod of mass M_1 and length 'd'. The rod is attached to frictionless horizontal table by a pivot at point 'P' and initially rotates at an angular speed ω as shown. As a result, rod stops and ball moves in the direction shown. If collision is elastic. The ratio M_1/M_2 is -



Sol. **[3]**

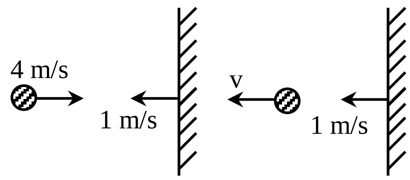
$\frac{M_1 d^2}{3}\omega = M_2 v d$

$\Rightarrow v = d\omega \Rightarrow \frac{M_1}{M_2} = \frac{3}{1}$

Q. 1 A ball of mass m approaches a wall of mass M (>> m) with speed 4 m/s along the normal to the wall. The speed of wall is 1m/s towards the ball. The speed of the ball after an elastic collision with the wall is -

(A) 5 m/s away from the wall
(B) 9 m/s away from the wall
(C) 3 m/s away from the wall
(D) 6 m/s away from the wall **[D]**

Sol.



Before collision After collision

Let v be the velocity of ball after collision, collision is elastic

$\therefore$ e =1

or

relative velocity of separation = relative velocity of approach

$\therefore$ v – 1 = 4 + 1

or v = 6 m/s (away from the wall)

Q.2 One end of a spring of force constant k is fixed to a vertical wall and the other to body of mass m resting on a smooth horizontal surface. There is another wall at a distance x_0 from the body. The spring is then compressed by $2x_0$ and released. The time taken to strike the wall first time is -

(A) $\frac{\pi}{6}\sqrt{\frac{m}{k}}$ (B) $\sqrt{\frac{m}{k}}$

(C) $\frac{2\pi}{3}\sqrt{\frac{m}{k}}$ (D) $\frac{\pi}{4}\sqrt{\frac{m}{k}}$

Sol. **[C]**

The total time from A to C

$t_{AC} = t_{AB} + t_{BC} = \frac{T}{4} + t_{BC}$

Where T = Time period of oscillation of spring-mass system t_{BC} can be given by

$BC = AB \sin\left(\frac{2\pi}{T}\right)t_{BC}$

Putting $\frac{BC}{AB} = \frac{1}{2}$, we get

$t_{BC} = \frac{T}{12}$

$\therefore t_{BC} = \frac{2\pi}{3}\sqrt{\frac{m}{k}}$

Q. 3 A block of mass 2 kg is kept at origin at t = 0 and is having velocity $4\sqrt{5}$ m/s in positive x-direction. The only force acting on it is a conservative and its potential energy is defind as

$U = -x^3 + 6x^2 + 15$ (SI units). Its velocity when its acceleration is minimum after t = 0 is-

(A) 8 m/s (B) 4 m/s

(C) $10\sqrt{24}$ m/s (D) 20 m/s

Sol. **[A]**

$F = -\frac{dU}{dx} \Rightarrow F = 3x^2 - 12x$

Now F = min. $\Rightarrow \frac{dF}{dx} = 0 \Rightarrow x = 2$ m

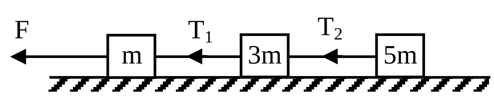
$U_i + K_i = U_f + K_f$

$$15 + \frac{1}{2} \times 2 \times 80 = [-(2)^3 + 6 \times (2)^2 + 15] + \frac{1}{2} \times 2 \times v^2$$

$v^2 = 64$

v = 8m/sec

Q.4 Figure shows a system of three masses being pulled with a force F. The masses are connected to each other by strings. The horizontal surface is frictionless. The tension T_1 in the first string is 16 N. The acceleration of the system is -



(A) $\frac{1}{m}$ (B) $\frac{2}{m}$

(C) $\frac{3}{m}$ (D) $\frac{4}{m}$ **[B]**

Sol. $T_1 = 8$ ma

$16 = 8\,ma$

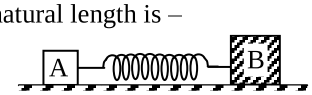
$a = \dfrac{2}{m}$

Q. 5 As observed in the laboratory system, a 6 MeV proton is incident on a stationary ^{12}C target. Velocity of center of mass of the system is-
(Take mass of proton to be 1 amu)

(A) 2.6×10^6 m/s (B) 6.2×10^6 m/s

(C) 10×10^6 m/s (D) 10 m/s **[A]**

Q. 6 Two blocks A and B of mass m and 2m are connected by a massless spring of force constant k. They are placed on a smooth horizontal plane. Spring is stretched by an amount x and then released. The relative velocity of the blocks when the spring comes to its natural length is –



(A) $\left(\sqrt{\dfrac{3k}{2m}}\right)x$ (B) $\left(\sqrt{\dfrac{2k}{3m}}\right)x$

(C) $\sqrt{\dfrac{2kx}{m}}$ (D) $\sqrt{\dfrac{3km}{2x}}$

Sol. **[A]**

From conservation of mechanical energy

$$\frac{1}{2}kx^2 = \frac{1}{2}\mu v_r^2 \qquad \text{... (1)}$$

Here, μ = reduced mass of the blocks

$$= \frac{(m)(2m)}{m+2m} = \frac{2}{3}m$$

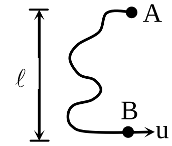
and v_r = relative velocity of the two.

Substituting in Equation (1), we get

$$kx^2 = mv_r^2$$

$$\therefore v_r = \left(\sqrt{\frac{3k}{2m}}\right)x$$

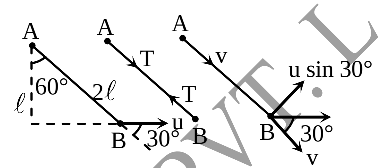
Q.7 Two particles A and B each of mass m are attached by a light inextensible string of length 2ℓ. The whole system lies on a smooth horizontal table with B initially at a distance ℓ from A. The particle at end B is projected across the table with speed u perpendicular to AB. Velocity of ball A just after the string is taut, is –



(A) $\dfrac{u\sqrt{3}}{4}$ (B) $u\sqrt{3}$

(C) $\dfrac{u\sqrt{3}}{2}$ (D) $\dfrac{u}{2}$

Sol. **[A]**



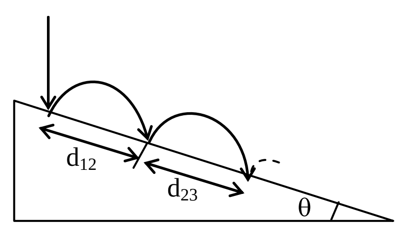
When the string jerk tight both particles begin to move with velocity components v in the direction AB. Using conservation of momentum in the direction AB

$mu \cos 30° = mv + mv$

or $v = \dfrac{u\sqrt{3}}{4}$

Hence, the velocity of ball A just after the jerk is $\dfrac{u\sqrt{3}}{4}$

Q. 8 An elastic ball is dropped on a long inclined plane. If bounces, hits the plane again, bounces and so on. let us Label the distance between the point of the first and second hit d_{12} and the distance between the points of second and the third hit is d_{23}. find the ratio of d_{12}/d_{23} .



(A) $\dfrac{2}{1}$ (B) $\dfrac{1}{2}$

(C) $\dfrac{4}{1}$ (D) $\dfrac{1}{4}$

Sol. **[B]**

If we rotate the coordinate system 80 that the ramp is horizontal then the free fall acceleration will have two components one downward $a_y = -g \cos\theta$ and one horizontal $a_x = g \sin\theta$. In this frame, the initial velocity will have components given by $V_y = V_0\cos\theta$ & $V_x = V_0 \sin\theta$ time for each bounce is given by $t_b = \frac{2V_y}{-ay} = \frac{2V_0}{g}$

The horizontal displacement

$\Delta x = V_x t + 0.5\, a_x t^2$

given these equation

$$d_{12} = V_0 \sin\theta \left(\frac{2v_0}{g}\right) + 0.5g \sin\theta \left(\frac{2V_0}{g}\right)^2$$

$$= \frac{4V_0^2 su\theta}{g}$$

$$d_{13} = V_0\sin\theta\left(\frac{4V_0}{g}\right) + 0.5\, g \sin\theta \left(\frac{4V_0}{g}\right)^2$$

$$= \frac{12V_0^2 \sin\theta}{g}$$

Since $d_{23} = d_{13} - d_{12} = \frac{8V_0^2 \sin\theta}{g}$

$$\frac{d_{12}}{d_{23}} = \frac{1}{2}$$

Q.9 In a free space, a rifle of mass M shoots a bullet of mass m at a stationary block of mass M distance D away from it. When the bullet has moved through a distance d towards the block, the centre of mass of the bullet-block system is at a distance of -

(A) $\frac{m(D-d)}{M+m}$ from the block

(B) $\frac{(m+M)d}{M}$ from rifle

(C) $\frac{M(D+d)}{M+m}$ from bullet

(D) None of these

Sol. **[C]**

If x is distance moved by rifle when bullet has traveled through a distance d, then –

$$Mx = md \Rightarrow x = \frac{md}{M}$$

So, distance of bullet from block D – d and distance between block and rifle = D + x

$\therefore$ Distance of C.M from block in

$$r_1 = \frac{Mx(0)+m(D-d)}{m+M} = \frac{(D-d)m}{m+M}$$

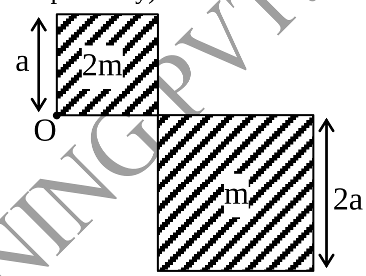
Distance of C.M from rifle

$$= \frac{m(x+d)+M(D+x)}{m+M}$$

Also distance of C.M from bullet

$$= \frac{m\times o+M(D-d)}{M+m}$$

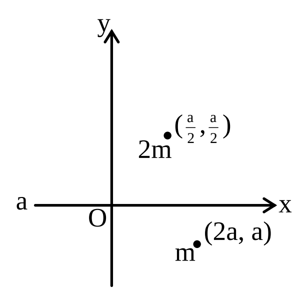
Q.10 The distance of centres of mass of two square plates system a shown from point O. If masses of plates are 2m and m is (their edges are 'a' and '2a' respectively)-



(A) $\frac{a}{2}$ (B) a

(C) $\frac{3a}{2}$ (D) $\frac{2a}{3}$

Sol. **[B]**



$$x_{cm} = \frac{2m\times\frac{a}{2}+m\times 2a}{3m} = a$$

$$y_{cm} = \frac{2m\times\frac{a}{2}-ma}{3m} = 0$$

Q.11 A solid cube of edge 'a' is molten and moulded in eight identical small solid cubes and are placed on one other on a straight line with the edge of bottom cube on the same horizontal plane on which big cube was placed, then the vertical shift in centre of mass is-

(A) $\frac{3a}{2}$ (B) 2a

(C) $\frac{5a}{2}$ (D) 3a

Sol. **[A]**

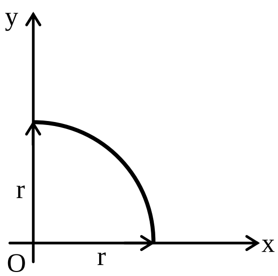
Final hight of centre of mass

$$= 4\ell \begin{pmatrix} \text{where } 8\ell^3 = a^3 \\ \Rightarrow \quad \ell = \dfrac{a}{2} \end{pmatrix}$$

Initial height of centre of mass $= \dfrac{a}{2}$

$\therefore \quad \Delta h = 4\ell - \dfrac{a}{2} = 2a - \dfrac{a}{2} = \dfrac{3a}{2}$

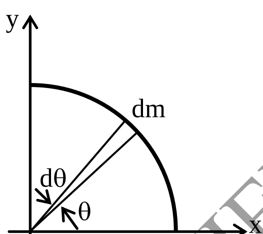
Q.12 The coordinates of centre of mass of the following quarter circular arc is -



(A) $\left(\dfrac{r}{2}, \dfrac{r}{2}\right)$ (B) $\left(\dfrac{2r}{3}, \dfrac{2r}{3}\right)$

(C) $\left(\dfrac{2r}{\pi}, \dfrac{2r}{\pi}\right)$ (D) $\left(\dfrac{4r}{\pi}, \dfrac{4r}{\pi}\right)$

Sol. **[C]**



$dm = \lambda R d\theta = \dfrac{2m}{\pi} d\theta$

$\therefore \quad x_{cm} = \dfrac{1}{m}\int x\,dm$

$= \dfrac{1}{m}\int_0^{\pi/2} R\cos\theta \dfrac{2m}{\pi} d\theta$

Similarly $y_{cm} = \dfrac{1}{m}\int_0^{\pi/2} R\sin.\theta \dfrac{2m}{\pi} d\theta$

Q.13 A nucleus moving with velocity $\vec{v}$ emits an α-particle. If the velocities of α-particle and the remaining nucleus be $\vec{v}_1$ and $\vec{v}_2$ and their masses be m_1 and m_2, then -

(A) All velocity vectors $\vec{v}$, $\vec{v}_1$ and $\vec{v}_2$ must be parallel

(B) $\vec{v}$ must be parallel to $(\vec{v}_1 + \vec{v}_2)$

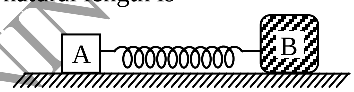
(C) $\vec{v}$ must be parallel to $(m_1\vec{v}_1 + m_2\vec{v}_2)$

(D) None of above

Sol. **[C]**

Conceptual

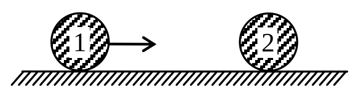
Q.14 Two blocks A and B of mass m and 2m are connected by a massless spring of force constant k. They are placed on a smooth horizontal plane. Spring is stretched by an amount x and then released. The relative velocity of the blocks when the spring comes to its natural length is -



(A) $\left(\sqrt{\dfrac{3k}{2m}}\right)x$ (B) $\left(\sqrt{\dfrac{2k}{3m}}\right)x$

(C) $\sqrt{\dfrac{2kx}{m}}$ (D) $\sqrt{\dfrac{3km}{2x}}$ **[A]**

Q.15 Ball 1 collides with an another identical ball 2 at rest as shown in figure. For what value of coefficient of restitution e, the velocity of second ball becomes two times that of 1 after collision ?



(A) 1/3 (B) 1/2

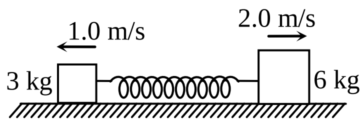
(C) 1/4 (D)1/6 **[A]**

Q.16 After perfectly inelastic collision between two identical particles moving with same speed in different directions, the speed of the particles become half the initial speed. The angle between the velocities of the two before collision is -

(A) 60º (B) 45º

(C) 120º (D) 30º **[C]**

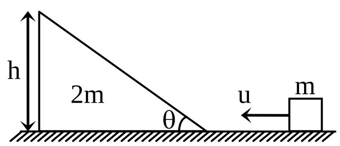
Q.17 Two blocks of mass 3 kg and 6 kg respectively are placed on a smooth horizontal surface. They are connected by a light spring of force constant k = 200 N/m. Initially the spring is unstretched. The indicated velocities are imparted to the blocks. The maximum extension of the spring will be –



(A) 30 cm (B) 25 cm

(C) 20 cm (D) 15 cm **[A]**

Q.18 A block of mass m is pushed towards a movable wedge of mass 2m and height h with a velocity u. All surfaces are smooth. The minimum value of u for which the block will reach the top of the wedge is –



(A) $2\sqrt{gh}$ (B) $\sqrt{3gh}$

(C) $\sqrt{6gh}$ (D) $\sqrt{\frac{3}{2}gh}$ **[B]**

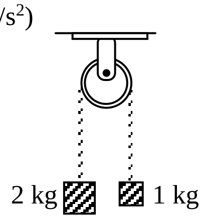
Q.19 A projectile of mass 3 m explodes at highest point of its path. It breaks into three equal parts. One part retraces its path, the second one comes to rest. The range of the projectile was 100 m if no explosion would have taken place. The distance of the third part from the point of projection when it finally lands on the ground is

(A) 100 m (B) 150 m

(C) 250 m (D) 300 m **[C]**

Q.20 Two blocks of masses 2 kg and 1 kg respectively are tied to the ends of a string which passes over a light frictionless pulley. The masses are held at rest at the same horizontal level and then released. The distance traversed by centre of mass in 2 s is –

($g = 10\ m/s^2$)



(A) 1.42 m (B) 2.22 m

(C) 3.12 m (D) 3.33 m

Sol. **[B]**

Here acceleration of system = 10/9 m/sec^2

$S_{CM} = \frac{1}{2} a_{CM} t^2 \Rightarrow 2.22$ m

Q.21 In one dimensional collision of two particles velocities are interchanged when

(i) collision is elastic and mass are equal

(ii) collision is inelastic but masses are unequal

Select the correct alternative

(A) only (i) is correct

(B) only (ii) is correct

(C) both (i) and (ii) are correct

(D) both (i) and (ii) are wrong **[A]**

Q.22 A heavy elastic ball falls freely from point A at a height H_0 onto the smooth horizontal surface of an elastic plate. As the ball strikes the plate another such ball is dropped from the same point A. At what time t, after the second ball is dropped, and at what height will the balls meet ?

(A) $\sqrt{\frac{H_0}{2g}}$; $\frac{3H_0}{4}$ (B) $\sqrt{\frac{2H_0}{g}}$; $\frac{3H_0}{4}$

(C) $\sqrt{\frac{H_0}{2g}}$; $\frac{H_0}{4}$ (D) $\sqrt{\frac{H_0}{g}}$, $\frac{H_0}{4}$ **[A]**

Sol. $t = \sqrt{\frac{H_0}{2g}}$; $h_1 = \frac{3}{4} H_0$

The velocity of the first ball at the moment it strikes the plate will be $v_0 = \sqrt{2gH_0}$. Since the impact is elastic, the ball will begin to rise after the impact with a velocity of the same magnitude v_0. During the time t the first ball will rise to a height

$$h_1 = v_0 t - \frac{gt^2}{2}$$

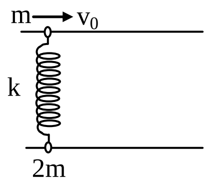
During this time the second ball will move down from a point A a distance

$$h_2 = \frac{gt^2}{2}$$

At the moment the balls meet, $h_1 + h_2 = H_0$. Hence,

$$t = \frac{H_0}{v_0} = \sqrt{\frac{H_0}{2g}}.$$

Q.23 Two ring of mass m and 2 m are connected with a mass less spring and can slips over two frictionless parallel horizontal rails as shown in figure. Ring of mass m is given velocity 'v_0' in the direction shown. Maximum stretch in spring will be -



(A) $\sqrt{\frac{m}{k}}v_0$ (B) $\sqrt{\frac{3m}{k}}v_0$

(C) $\sqrt{\frac{2m}{3k}}v_0$ (D) $\sqrt{\frac{2m}{k}}v_0$ **[C]**

Sol. Maximum expansion in spring is given by

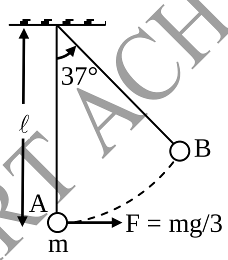
$$\frac{1}{2}kx_{max}^2 = \frac{1}{2}\mu v_0^2$$

[μ = Reduced mass]

$$\Rightarrow x_{max} = \sqrt{\frac{\mu}{k}}\ .\ v_0$$

$$= \sqrt{\frac{2m}{3k}}\ v_0$$

Q.24 A pendulum of mass 'm' is pulled from position 'A' by applying a constant horizontal force F = mg/3. Velocity at point 'B' shown in figure –



(A) $\sqrt{\frac{2\ell g}{3}}$ (B) $\sqrt{\frac{3\ell g}{5}}$

(C) $\sqrt{\frac{4}{5}\ell g}$ (D) Zero **[D]**

Sol. $W_{net} = \Delta K$

$$\Rightarrow F \sin\theta.\ell - mg\ \ell\ (1 - \cos\theta) = \frac{1}{2}mv^2$$

($\because$ $\theta = 37°$ and $F = \frac{mg}{3}$)

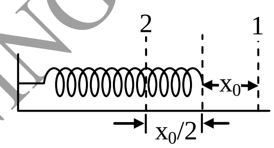
$$\Rightarrow v = \left\{\frac{2\ell}{5m}(3F - mg)\right\}^{1/2} = 0$$

Q.25 Assuming that potential energy of spring is zero when it is stretched by 'x_0', its potential energy when it is compressed by '$x_0/2$' is -

(A) $\frac{3}{8}kx_0^2$ (B) $-\frac{3}{4}kx_0^2$

(C) $-\frac{3}{8}kx_0^2$ (D) $\frac{1}{8}kx_0^2$ **[C]**

Sol. Change in potential energy is independent of reference



$$\therefore\ U_2 - U_1 = \frac{1}{2}k\left(\frac{x_0}{2}\right)^2 - \frac{1}{2}\ kx_0^2$$

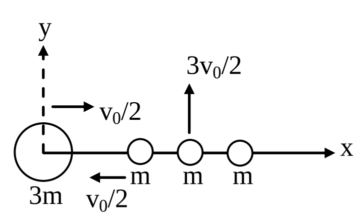
$$= -\frac{3}{8}kx_0^2$$

Q.26 A projectile is fixed with velocity v_0 at an angle 60° with horizontal. At top of its trajectory it explodes into three fragment of equal mass. First fragment retraces the path, second moves vertically upward with speed $\frac{'3v_0'}{2}$. The speed of third fragment –

(A) $\frac{3v_0}{2}$ (B) $\frac{5v_0}{2}$

(C) v_0 (D) $2v_0$ **[B]**

Sol.



Let velocity of third fragment be $\vec{v}$

$$\vec{P}_i = \vec{P}_f$$

$$\Rightarrow 3m\ .\ \frac{v_0}{2}\hat{i} = -\ m\frac{v_0}{2}\hat{i}\ + m\ .\ \frac{3v_0}{2}\hat{j}\ +\ m\vec{v}$$

$$\Rightarrow v = \frac{4v_0}{2}\hat{i} - \frac{3v_0}{2}\hat{j}$$

$$\Rightarrow v = \frac{5v_0}{2}$$

Q.27 A block is hanged from spring in a cage. Elongation in spring is 'x_1' and 'x_2' when cage moves up and down respectively with same acceleration. The expansion in spring when the cage move horizontally with same acceleration -

(A) $\frac{x_1 + x_2}{2}$ (B) $\sqrt{\frac{x_1^2 - x_2^2}{2}}$

(C) $\sqrt{\frac{x_1^2 + x_2^2}{2}}$ (D) $\sqrt{x_1 x_2}$ **[C]**

Sol. $x_1 = \frac{m}{k}(g + a)$

$x_2 = \frac{m}{k}(g - a)$

$x_3 = \frac{m}{k}\sqrt{g^2 + a^2}$

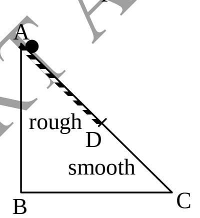
[m = mass of pendulum
k = spring constant]

$$\therefore\ x_1^2 + x_2^2 = \frac{m^2}{k^2}.2(g^2 + a^2)$$

$$= 2x_3^2$$

$$\Rightarrow x_3 = \sqrt{\frac{x_1^2 + x_2^2}{2}}$$

Q.28 ABC is a fixed incline plane with D mid point of AC. Part AD of incline plane is rough such that when a sphere released from A starts rolling, while the part DC is smooth. The sphere reaches the bottom point C, then –



(A) It is in pure rolling in the part DC

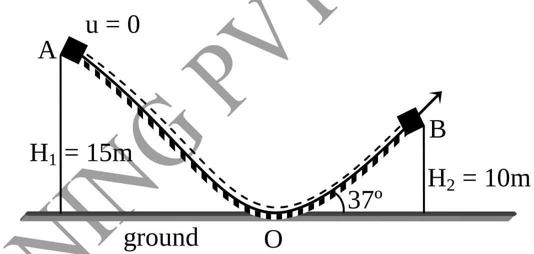
(B) Work done by friction on the sphere is negative when it moves from A to D

(C) Mechanical energy of sphere remains constant for its motion from A to C

(D) All of the above **[C]**

Sol As surface DC is smooth it will slip but as in path AD it is in pure rolling therefore work done by friction is zero. Hence mechanical energy is conserved.

Q. 29 A body starts slipping on a smooth track from point A and leaves the track from point B as shown. The part OB of track is straight at angle 37° with horizontal. The maximum height of body from ground when it is in air is : (g = 10 m/s^2)



(A) 16.8 m (B) 13.6 m

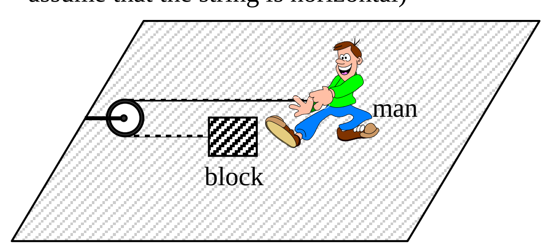
(C) 11.8 m (D) None of these

[C]

Sol. $v_B^2 = 2g\,(H_1 - H_2) = 100\ m^2/s^2$

Now, required height = $H_2 + \frac{v_B^2 \sin^2\theta}{2g} = 11.8$ m

Q. 30 The acceleration of man of mass 40 kg plus block of 10 kg system shown in figure, if the man applies a force 30 N on the string (the plane in fixed, smooth and horizontal, also assume that the string is horizontal)



(A) 0.6 m/s^2 (B) Zero

(C) 1.2 m/s^2 (D) 2.4 m/s^2 **[C]**

Sol. $a_{CM} = \frac{F_{ext}}{\text{Mass of system}} = \frac{60}{50} = 1.2\ m/s^2$

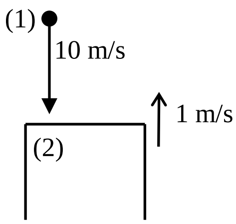
Q.31 A ball falls from a height of 5m and strikes a lift which is moving in the upward direction with a velocity of 1m/s, then the velocity with which the ball rebounds after collision will be -

(A) 11 m/s downwards
(B) 12 m/s upwards
(C) 13 m/s upwards
(D) 12 m/s downwards **[B]**

Sol. Velocity after a fall of 5m = $\sqrt{2\times10\times5}$ = 10 m/s

$\bar{v}_1 - \bar{v}_2 = e\,(\bar{u}_2 - \bar{u}_1)$

$v_1 - 1 = 1 - (-10) = 12$ m/s



Q.32 A rocket is fired with a speed u = 3$\sqrt{gR}$ from the earth surface. What will be its speed at interstellar space ?

(A) zero (B) $\sqrt{2gR}$
(C) $\sqrt{7gR}$ (D) $\sqrt{3gR}$ **[C]**

Sol. From conservation of mechanical energy

$(K.E. + P.E.)_{surface} = (K.E. + P.E.)_{infinity}$

$$\frac{1}{2}m\left(3\sqrt{gR}\right)^2 + \left(-\frac{GMm}{R}\right) = \frac{1}{2}m\,v_\infty^2 + 0$$

or $v_\infty = \sqrt{7gR}$

Q.33 A pulley fixed with celling carries a string with blocks of mass 'm' and '3m' attached to its ends. The masses of string and pulley are negligible. When the system is released, its centre of mass moves with acceleration –

(A) g (B) g/s
(C) g/4 (D) zero **[C]**

Sol. Acceleration of each mass w.r.t. pulley

$= \dfrac{3mg - mg}{(3m+m)} = g/2$

Acceleration of centre of mass

$= \dfrac{(3m)\,g/2 - m\,g/2}{(3m+m)} = g/4$

Q.34 A ball moving with a velocity v hits a massive wall moving towards the ball with a velocity u. An elastic impact lasts for a time Δt.

(A) The average elastic force acting on the ball is $\dfrac{m(u+v)}{\Delta t}$

(B) The average elastic force acting on the ball is $\dfrac{2m(u+v)}{\Delta t}$

(C) The K.E. of the ball increases by mu (u + v)

(D) The K.E. of the ball remains the same after the collision. **[B]**

Sol. $\bar{F} = \dfrac{m\Delta\bar{v}}{\Delta t} = \dfrac{m[\bar{v}_f - \bar{v}_i]}{\Delta t}$

$F = \dfrac{m[(v+2u) - (-u)]}{\Delta t} = \dfrac{2m(u+v)}{\Delta t}$

Q.35 Two masses of 1g and 4g are moving with equal kinetic energies. The ratio of the magnitudes of their momenta is -

(A) 4 : 1 (B) $\sqrt{2}$: 1
(C) 1 : 2 (D) 1 : 16 **[C]**

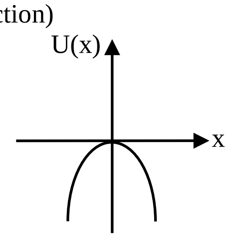
Sol. P = $\sqrt{2Km}$

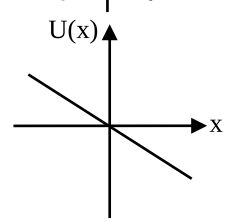
or P $\propto \sqrt{m}$

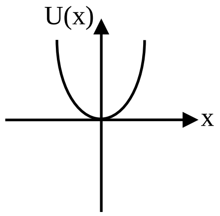
$\dfrac{m_1}{m_2} = \dfrac{1}{4}$

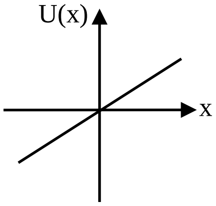
$\therefore \dfrac{P_1}{P_2} = \dfrac{1}{2}$

Q.36 A particle is placed at the origin and a force F = kx is acting on it (where k is a positive constant). If U (0) = 0, the graph of U (x) versus x will be: (where U is the potential energy function)

(A)


(B)


(C) 

(D) 

[A]

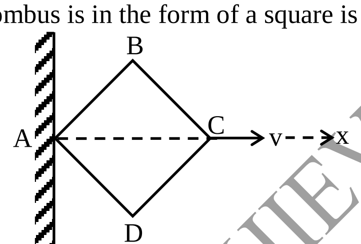
Sol. From $F = -\dfrac{dU}{dx}$

$$\int_0^{U(x)} dU = -\int_0^x F dx = -\int_0^x (kx)\, dx$$

$$\therefore\ U(x) = -\frac{kx^2}{2}$$

as U (0) = 0

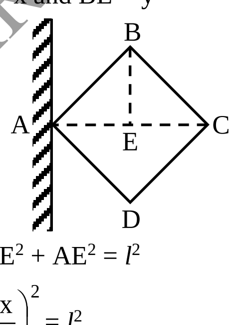
Q.37 Four rods each of length l have been hinged to form a rhombus. Vertex A is fixed to rigid support, vertex C is being moved along the x-axis with a constant velocity v as shown in the figure. The rate at which vertex B is approaching the x-axis at the moment the rhombus is in the form of a square is –



(A) $\dfrac{v}{4}$ (B) $\dfrac{v}{3}$

(C) $\dfrac{v}{2}$ (D) $\dfrac{v}{\sqrt{2}}$ **[C]**

Sol. Let AC = x and BE = y



Then, $BE^2 + AE^2 = l^2$

or $y^2 + \left(\dfrac{x}{2}\right)^2 = l^2$

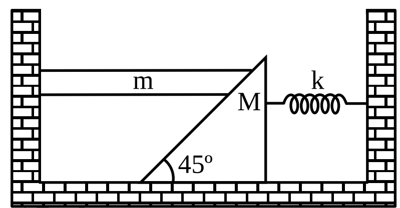
$$\therefore\ 2y\left(\frac{dy}{dt}\right) + \frac{x}{2}\cdot\frac{dx}{dt} = 0$$

$$\therefore\ \left(-\frac{dy}{dt}\right) = \frac{1}{2}\left(\frac{x}{2y}\right)\cdot\frac{dx}{dt}$$

x = 2y, when the rhombus is a square.

Hence, $v_B = \dfrac{1}{2}\ v_c = \dfrac{v}{2}$

Q.38 All surfaces shown in figure are smooth. System is released with the spring unstretched. In equilibrium, compression in the spring wiil be –



(A) $\dfrac{mg}{\sqrt{2}k}$ (B) $\dfrac{2mg}{k}$

(C) $\dfrac{(M+m)g}{\sqrt{2}k}$ (D) $\dfrac{mg}{k}$ **[D]**

Sol. Let N be the normal reaction between m and M,

Equilibrium of M

N sin 45º = kx … (i)

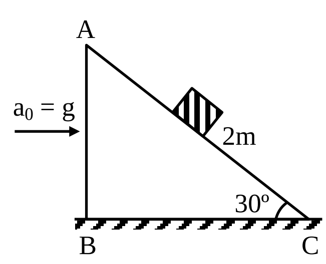
Equilibrium of m in vertical direction gives

N cos 45º = mg … (ii)

From Eqs. (i) and (ii), we get

$$x = \frac{mg}{k}$$

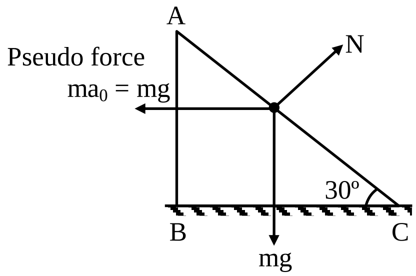
Q.39 A block is placed on an inclined plane moving towards right horizontally with an acceleration a_0 = g. The length of the plane AC = 1 m. Friction is absent everywhere. The time taken by the block to reach from C to A is : (g = 10 m/s^2)



(A) 1.2 s (B) 0.74 s

(C) 2.56 s (D) 0.42 s **[B]**

Sol. Drawing free body diagram of block with respect to plane.



Acceleration of the block up the plane is

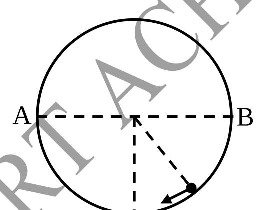
$$a = \frac{mg\cos 30º - mg\sin 30º}{m}$$

$$= \left(\frac{\sqrt{3}-1}{2}\right)g = 3.66 \text{ m/s}^2$$

applying $s = \frac{1}{2}at^2$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times 1}{3.66}} = 0.74 \text{ s}$$

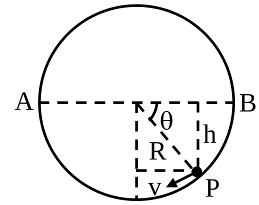
Q.40 A particle of mass m oscillates along the horizontal diameter AB inside a smooth spherical shell of radius R. At any instant the kinetic energy of the particle is K. Then the force applied by particle on the shell at this instant is –



(A) $\frac{K}{R}$ (B) $\frac{2K}{R}$

(C) $\frac{3K}{R}$ (D) $\frac{K}{2R}$ **[C]**

Sol. Let velocity of particle at point P is v.
From conservation of mechanical energy



$$\frac{1}{2}mv^2 = K = mgh$$

Let N be the normal reaction between the particle and the shell at this instant. Then

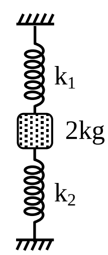
$$N - mg\sin\theta = \frac{mv^2}{R} \quad \left(\frac{mv^2}{R} = \frac{2K}{R}\right)$$

or $N = mg\left(\frac{h}{R}\right) + \frac{2K}{R} = \frac{K}{R} + \frac{2K}{R}$

(mgh = K)

$\therefore$ $N = \frac{3K}{R}$ = force on shell

Q.41 A 2 kg block is connected with two springs of force constants $k_1 = 100$ N/m and $k_2 = 300$ N/m as shown in figure. The block is released from rest with the springs unstretched. The acceleration of the block in its lowest position is : ($g = 10$ m/s^2) –



(A) zero (B) 10 m/s^2 upwards
(C) 10 m/s^2 downwards (D) 5 m/s^2 uuwards

[B]

Sol. Let x be the maximum displacement of block downwards. Then from conservation of mechanical energy:
decrease in potential energy of 2 kg block = increase in elastic potential energy of both the springs

$\therefore$ $mg\,x = \frac{1}{2}(k_1 + k_2)\,x^2$

or $x = \frac{2mg}{k_1 + k_2} = \frac{(2)(2)(10)}{100+300} = 0.1$ m

Acceleration of block in this position is –

$$a = \frac{(k_1 + k_2)x - mg}{m} \qquad \text{(upwards)}$$

$$= \frac{(400)(0.1) - (2)(10)}{2}$$

$$= 10 \text{ m/s}^2 \qquad \text{(upwards)}$$

Q.42 A U^{238} nucleus initially at rest emits an α-particle and is converted into a Th^{234} nucleus. If the KE of the α-particle is 4.1 MeV what is the recoil energy of the Th nucleus ?

(A) 1 MeV (B) 0.60 MeV
(C) 0.07 MeV (D) 0.005 MeV **[C]**

Q.43 A monkey of mass 20kg rides on a 40kg trolley moving with constant speed of 8m/s along a horizontal track. If the monkey jumps vertically to grab the overhanging branch of a tree, the speed of the trolley after the monkey has jumped off is -

(A) 8 m/s (B) 1 m/s
(C) 4 m/s (D) 12 m/s **[A]**

Q.44 A system of two particle move under the influence of mutual gravitational attraction. If $\vec{p}$ represent the linear momentum and $\vec{J}$ the angular momentum, then the correct formulae are-

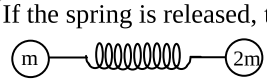
(A) $\Delta \vec{J_1} + \Delta \vec{J_2} = 0$ and $\Delta \vec{p_1} + \Delta \vec{p_2} = 0$

(B) $\Delta \vec{J_1} + \Delta \vec{J_2} \neq 0$ and $\Delta \vec{p_1} + \Delta \vec{p_2} \neq 0$

(C) $\Delta \vec{J_1} + \Delta \vec{J_2} \neq 0$ and $\vec{p_1} + \Delta \vec{p_2} = 0$

(D) $\Delta \vec{J_1} + \Delta \vec{J_2} = 0$ and $\vec{p_1} + \Delta \vec{p_2} \neq 0$ **[A]**

Q.45 Two masses m and 2m are attached to two ends of an ideal spring and the spring is in the compressed state. The energy of spring is 60 joule. If the spring is released, then-



(A) the energy of both bodies will be same
(B) energy of smaller body will be 10J
(C) energy of smaller body will be 20J
(D) energy of smaller body will be 40 J **[D]**

Q.46 A nucleus of mass M emits a X-ray photon of frequency ν, What is the loss of internal energy by nucleus ?

(A) $h\nu$

(B) $\frac{h^2\nu^2}{2MC^2}$

(C) $h\nu\left(1 - \frac{h\nu}{2MC^2}\right)$

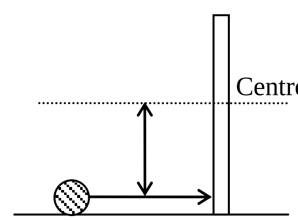
(D) $h\nu\left(1 + \frac{h\nu}{2MC^2}\right)$ **[D]**

Q.47 A nucleus of mass number A originally at rest emits α-particle with speed v. The recoil speed of daughter nucleus is-

(A) $\frac{4v}{A-4}$ (B) $\frac{4v}{A+4}$

(C) $\frac{v}{A-4}$ (D) $\frac{v}{A+4}$ **[A]**

Q.48 A meter scale lying horizontally on a frictionless table is struck by a ball as shown in the diagram. Which of the following quantities is conserved for the ball-scale system-



(A) Linear momentum
(B) Kinetic energy of translation
(C) Angular momentum
(D) Linear momentum and angular momentum both **[D]**

Q.49 Two elastic bodies P and Q having equal masses are moving along the same line with velocities of 16 m/s and 10m/s respectively. Their velocities after the elastic collision will be in m/s -

(A) 0 and 25 (B) 5 and 20
(C) 10 and 16 (D) 20 and 5 **[C]**

Q.50 Two solid balls of rubber A and B whose masses are 200gm and 400gm respectively, are moving in mutually opposite directions. If the velocity A is 0.3 m/s and both the balls come to rest after collision, then the velocity of ball B is-

(A) 0.15 ms^{-1} (B) –0.15 ms^{-1}

(C) 1.5 ms^{-1} (D) none of these **[B]**

Q.1

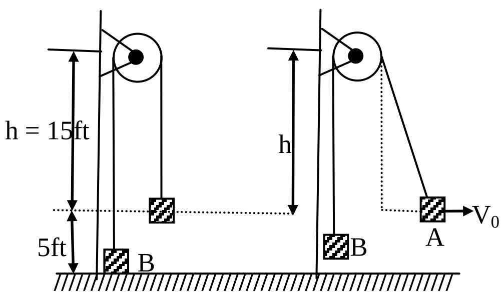
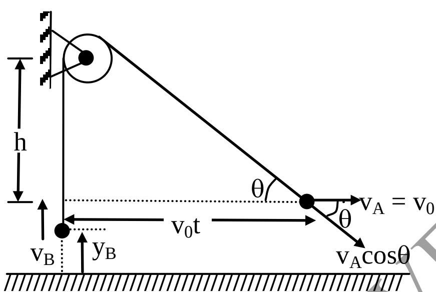


Fig.1 Fig.2

Initial position of blocks A & B are shown in figure 1. Block A is pulled horizontally with constant velocity v_0 = 10 m/s. Find the time taken by the block B to reach the pulley.

Sol.



$v_B = v_A \cos\theta$, $v_B = v_o\cos\theta$: Since the length of string to be constant velocity of both ends of string along the length of string must be equal.

$$\frac{dy_B}{dt} = v_B = \frac{v_0(v_0t)}{\sqrt{h^2+(v_0t)^2}}$$

$$\int_0^{20} dy_B = \int_0^t \frac{v_0^2 t dt}{\sqrt{h^2+(v_0t)^2}}$$

$$20 = v_0^2 \int_0^t \frac{t\,dt}{\sqrt{h^2+(v_0t)^2}} \quad \ldots\ldots\ldots\ldots(i)$$

= let $\quad h^2 + (vot)^2 = x^2$

$2\, v_0^2 tdt = 2xdx$

$$\text{R.H.S.} = \int \frac{v_0^2 t\,dt}{\sqrt{h^2+(v_0t)^2}} = \int \frac{xdx}{x}$$

$$= \int dx = x = \sqrt{h^2+(v_0t)^2}$$

From (i)

$$20 = \left.\sqrt{h^2+(v_0t)^2}\right]_0^t$$

$20 = h^2 + (v_0t)^2 - h$

$(20 + h)^2 = h^2 + (v_0t)^2$

h = 15 m.

$(35)^2 - (15)^2 = (v_0t)^2$

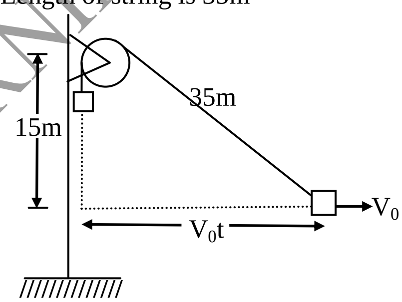
$1225 - 225 = (v_0t)^2$

$$t = \sqrt{\frac{1000}{v_0^2}}$$

$t = \sqrt{10}$ = **3.162 sec.**

Alternate Solution of Q.9:

Length of string is 35m

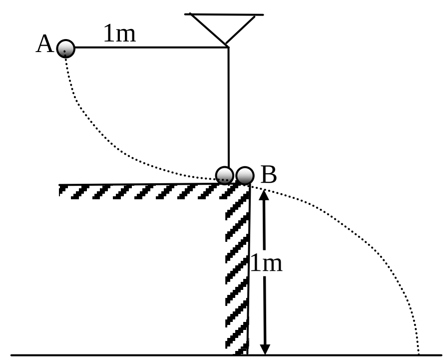


$(v_0t)^2 = 35^2 - 15^2 = 1000$

$$T = \sqrt{\frac{1000}{v_0^2}} = \sqrt{10}$$

T = 3.162 sec.

Q.2 A small steel ball B is at rest on the edge of a table of height 1m. Another steel ball A, used as the bob of a meter-long simple pendulum, is released from rest with the pendulum suspension horizontal & swings against B as shown in figure. The masses of the balls are identical & collision is elastic.



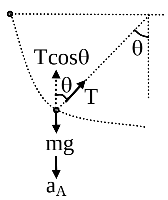
Considering motion of B only up until the moment it first hits the ground.

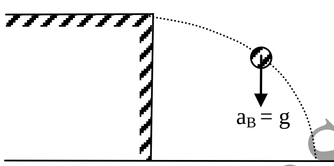
(i) Which ball is in motion for longer time.

(ii) Which ball covers greater distance.

Sol. (i) The vertical acceleration of ball B falling from the table is always g, which makes it possible to determine the time (approximately half a second) it takes to fall 1m. The motion of the bob of the simple pendulum is rather complicated as no small amplitude approximation is possible, and therefore the time during which it it is in motion is not easy to determine. What can stated with certainty is that, since the thread exerts an upwards force on it, its vertical acceleration is always less than g. Therefore the vertical motion of ball A takes a longer time than the vertical free fall of ball B. Ball A stays in motion for longer. (see figure below)

For ball A: **For ball B :**





$mg - T\cos\theta = ma_A$

$a_B = g$

$a_A = g - \frac{T}{m}\cos\theta$

since $a_B > a_A$; ball B remains in motion for smaller time.

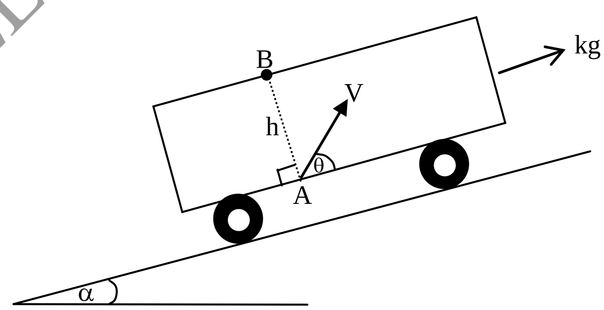
(ii) The bob of the pendulum describes one-quarter of a circle (a path of approximately 1.5m). The other ball, B follows a parabolic path, the length of which cannot be determined by elementary method. However , it is easy to see that it hits the ground at a distance of vt = $\sqrt{2\times g\times 1}\sqrt{2\times 1/g}$ = 2m from the edge of the table. The length of its path is therefore not less than the shortest distance between the beginning and end points of its motion, namely $\sqrt{5}\,m \approx$ 2.2m.

In summary: ball B moves on a longer path, but in a shorter time, than the bob of the pendulum.

Q.3 A vehicle is moving with constant acceleration **kg** up a slope of inclination α, the floor of the vehicle being parallel to the slope. Show that if a particle is falling freely inside the vehicle its acceleration relative to the vehicle makes an angle β with the floor, where

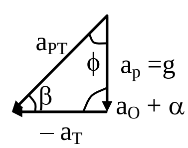
cot β = k sec α + tan α

A particle is projected inside the vehicle from a point A on the floor, its initial velocity relative to the vehicle being V at an angle θ (> β) with the floor, as shown in the diagram. It strikes the ceiling at B, where AB is perpendicular the floor and AB = h. By considering motion parallel to the slope, show that the time of flight is $\frac{2V\cos\theta}{g\cos\alpha\cot\beta}$ and deduce that h = $\frac{2V^2\cos\theta\tan\beta\sin(\theta-\beta)}{g\cos\alpha\cos\beta}$.



Sol. Acceleration of particle with respect to trolley

$\overrightarrow{a_{PT}} = \overrightarrow{a_P} - \overrightarrow{a_T}$



$\phi = 180 - [90 + \alpha + \beta]$

$= 90 - (\alpha + \beta)$

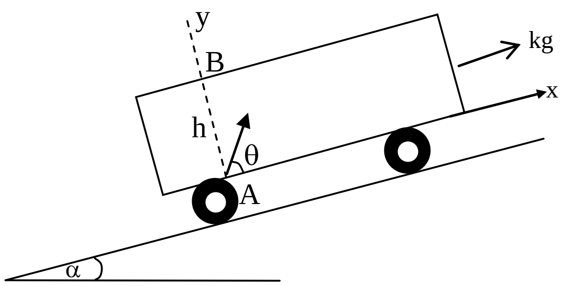
$$\frac{\sin\beta}{a_P} = \frac{\sin\phi}{a_T} = \frac{\sin(90+\alpha)}{a_{PT}}$$

$$\frac{\sin\beta}{g} = \frac{\sin[90-(\alpha+\beta)]}{kg}$$

$k \sin\beta = \cos(\alpha + \beta)$

$k \sin\beta = \cos\alpha \cos\beta - \sin\alpha \sin\beta$

$k \cos\alpha + \tan\alpha = \cot\beta$ ….. (i)



$(a_x)_{PT} = -[g \sin\alpha + kg]$

with respect to observer in trolley (vehicle)

$x_{PT} = (u_x)_{Pt} t + \frac{1}{2}(a_x)_{Pt} t^2$

$0 = v\cos\theta\, T - \frac{1}{2}(g\sin\alpha + kg)T^2$

$T = \frac{2v\cos\theta}{g[k+\sin\alpha]}$ …………(ii)

From (i)

$k + \sin\alpha = \cot\beta \cos\alpha$

Putting in (ii)

$T = \frac{2v\cos\theta}{g\cos\alpha\cot\beta}$

In y–direction :

$Y_{PT} = (u_y)_{PT} t + \frac{1}{2}(a_y)_{PT} t^2$

$h = v \sin q\, T - \frac{1}{2} g\cos\alpha\, T^2$

$h = T\left[v\sin\theta - \frac{g\cos\alpha}{2}T\right]$

$h = \frac{2v\cos\theta}{g\cos\alpha\cot\beta}\left[v\sin\theta - \frac{g\cos\alpha}{2}\frac{2v\cos\theta}{g\cos\alpha\cot\beta}\right]$

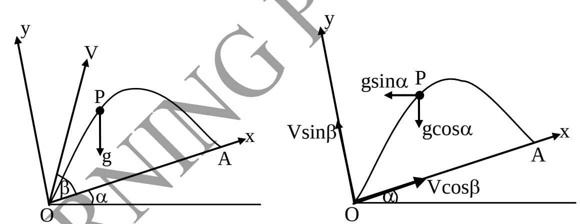
$h = \frac{2v^2\cos\theta}{g\cos\alpha\cot\beta}\left[\sin\theta - \frac{\cos\theta.\sin\beta}{\cos\beta}\right]$

$h = \frac{2v^2\cos\theta}{g\cos\alpha\cot\beta}\frac{[\sin(\theta-\beta)]}{\cos\beta}$

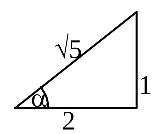
$\mathbf{h} = \frac{2V^2\cos\theta\tan\beta\sin(\theta-\beta)}{g\cos\alpha\cos\beta}$

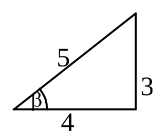
Q.4 A particle is projected from a point O on a smooth inclined plane inclined to the horizontal at arc tan $\frac{1}{2}$. The particle is projected at arc tan $\frac{3}{4}$ to the plane, hits the plane at a higher point A and rebounds. OA is a line of greatest slope and 'e' is the coefficient of restitution between P and the plane. If P continues to move up the plane after the impact find the possible values of 'e'.

Sol.



In the diagrams, $\tan\alpha = \frac{1}{2} \Rightarrow$





and $\tan\beta = \frac{3}{4} \Rightarrow$

Taking inclined axes as shown,

$\ddot{x} = -\frac{1}{5}g\sqrt{5}$ $\quad \ddot{y} = -\frac{2}{5}g\sqrt{5}$

$\dot{x} = \frac{4}{5}V - \frac{1}{5}gt\sqrt{5}$ $\quad \dot{y} = \frac{3}{5}V - \frac{2}{5}gt\sqrt{5}$

$x = \frac{4}{5}Vt - \frac{1}{10}gt^2\sqrt{5}$ $\quad y = \frac{3}{5}Vt - \frac{1}{5}gt^2\sqrt{5}$

When the particle hits the plane at A, y = 0,

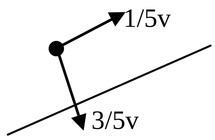
i.e., $\frac{3}{5}Vt - \frac{1}{5}g\sqrt{5} = 0 \Rightarrow t = \left(\frac{3V\sqrt{5}}{5g}\right)$

Therefore, at A, $\dot{x} = \frac{4}{5}V - \frac{1}{5}g\sqrt{5}\,(3V\sqrt{5}/5g)$

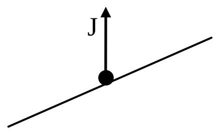
$= \frac{1}{5}V$

$$\dot{y} = \frac{3}{5}V - \frac{2}{5}g\sqrt{5}\,(3V\sqrt{5}/g) = -\frac{3}{5}V$$

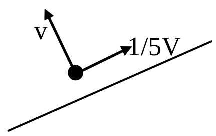
We can now investigate the effect of the impact.



Just before impact



At impact



At impact

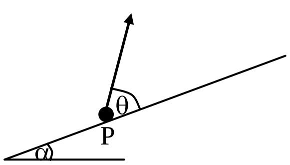
Using the law of restitution perpendicular to the plane gives

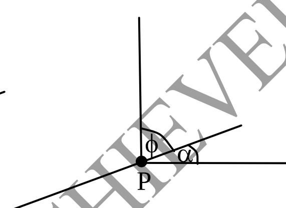
$$v = e\left(\frac{3}{5}V\right)$$

Parallel to the plane the velocity component is unchanged.

Just after impact, the resultant velocity of P is inclined to the plane at an angle θ, where

$$\tan\theta = v \div \left(\frac{1}{5}V\right) = \left(\frac{3}{5}eV\right) \div \left(\frac{1}{5}V\right) = 3e$$





If P continues to move up the plane then

$\theta < \phi \Rightarrow \tan\theta < \tan\phi$

Now $\phi = \frac{1}{2}\pi - \alpha$ $\quad\Rightarrow \tan\theta = \cot\alpha$

Therefore $\quad \tan\theta < \cot\alpha$

i.e., $\quad 3e < 2$

$\Rightarrow \quad e < \frac{2}{3}$

We also know that $e \geq 0$, therefore the range of value sof e for which P continues to move up the plane after impact is

$$\mathbf{0 \leq e < \frac{2}{3}}.$$

Q.5 An object of mass m is at rest in equilibrium at origin. At t = 0, a new force $\vec{F}(t)$ is applied that has components. $F_x(t) = K_1 + K_2y$; $F_y(t) = K_3t$

Where K_1, K_2 & K_3 are constant. Calculate the position $\vec{r}(t)$ & velocity $\vec{v}(t)$ vectors as a functions of time.

Sol. $F_x(t) = K_1 + K_2y$

$F_y(t) = K_3t$

$$\frac{dv_x}{dt} = a_x = \frac{F_x}{m} = \frac{K_1}{m} + \frac{K_2}{m}y \quad \text{…………..(i)}$$

$$\frac{dv_y}{dt} = a_y = \frac{F_y}{m} = \frac{k_3}{m}t \quad \text{………….(ii)}$$

$$v_y = \frac{K_3}{m}\frac{t^2}{2} + C$$

At t = 0, $v_y = 0 \Rightarrow C = 0$

$$\frac{dy}{dt} = v_y = \frac{K_3}{m}\frac{t^2}{2} + C$$

$$y = \frac{K_3}{6m}t^3 + C'$$

at t = 0, $y = 0 \Rightarrow C' = 0$

$$y = \frac{K_3}{6m}t^3 \quad \text{…………..(iii)}$$

Putting (iii) in (i)

$$\frac{dv_x}{dt} = \frac{K_1}{m} + \frac{K_2}{m}\cdot\frac{K_3}{6m}t^3$$

$$\frac{dx}{dt} = v_x = \frac{K_1}{m}t + \frac{K_2K_3}{6m^2}\frac{t^4}{4} + C''$$

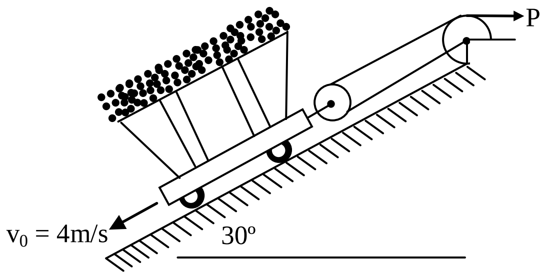
at t = 0, $v_x = 0 \Rightarrow C'' = 0$

$$\frac{dx}{dt} = v_x = \frac{K_1}{m}t + \frac{K_2K_3}{24m^2}t^4$$

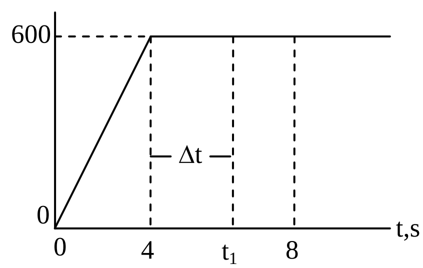
$$x = \frac{K_1}{m}\frac{t^2}{2} + \frac{K_2K_3}{120m^2}t^5$$

Therefore velocity $\vec{v} = v_x\hat{i} + v_y\hat{j}$

Q.6 The loaded 150 kg skip is rolling down the incline at 4m/s when a force P is applied to the cable as shown at time t = 0. The force P is increased uniformly with the time until it reaches 600N at t = 4s, after which time it remains constant at this value. Calculate (a) The time t_1 at which the skip reverse its direction and (b) The velocity v of the skip at t =8s. Treat the skip as a particle.



Sol.

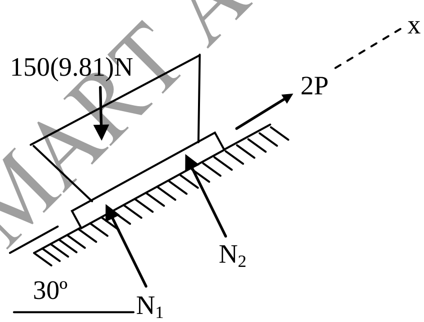


(i) The skip reverses direction when its velocity becomes zero. We will assume that this condition occurs at t = 4 + Δt s. The impulse-momentum equation applied consistently in the positive x-direction gives $\left[\int \Sigma F_x dt = m\Delta v_x\right]$

$\frac{1}{2}$(4) (2) (600) + 2 (600) Δt – 150(9.81) sin 30º

(4 + Δt) = 150 (0 – [–4])

46Δt = 1143 Δt = 2.46s t = 4 + 2.46 = **6.46** Ans.



(ii) Applying the impulse-momentum equation to the entire interval gives $\left[\int \Sigma F_x dt = m\Delta v_x\right]$.

$\frac{1}{2}$(4) (2) (600) + 4(2) (600) – 150(9.81) sin 30º

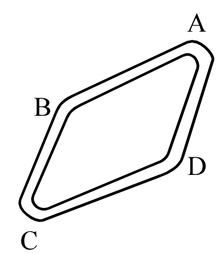
(8) = 150 (v – 1 [–4])

150v = 714 **v = 4.76 m/s**

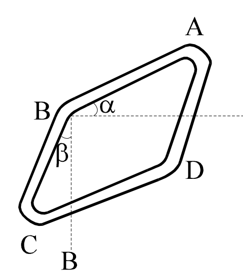
The same result is obtained by analyzing the interval from t_1 to 8s.

Helpful Hint : The free-body diagram keeps us from making the error of using the impulse of P rather than 2P or of forgetting the impulse of the component of the weight. The first term in the equation is the triangular area of the P-t relation for the first 4s, doubled for the force 2P.

Q.7 A tube in the shape of a rhombus with rounded corners is placed in a vertical plane as shown in fig. A ball is allowed to roll inside the tube along sides AB and BC, and then allowed to roll along sides AD and DC. In which will it roll faster ? The length of the rhombus's side is A.



Sol.



When sides AB and DC of the rhombus (fig.) are almost horizontal, it is atonce obvious that the ball will roll faster down sides AD and DC (the second case in the problem). This can be seen from the fact that the ball will travel along DC at a high average velocity, acquired from it's motion along side AD. But in the first the case ball will travel along AB with a very small average velocity (since its acceleration is small).

The result found for this particular case remains true for the general case, as may be verified from the following calculation. Let sides AB and DC from an angle of α with the horizontal and sides BC and AD an angle of β with the vertical. If the ball rolls along sides AB and BC it spends time $t_1 + t_2$ on this, where t_1 is spent on traveling along AB and t_2 on traveling along BC. The acceleration during motion along AB equals $g\sin\alpha$. Therefore to calculate t_1 we have the equation

$$A = \frac{g\sin\alpha t_1^2}{2}$$

The acceleration during motion along BC equals $g \cos \beta$, and this motion takes place with an initial velocity of $\sqrt{2Ag\sin\alpha}$, so we can find t_2 by solving the equation

$$A = \sqrt{2Ag\sin\alpha}\, t_2 + g\cos\beta \frac{t_2^2}{2}$$

For the second case we shall have the same expressions with the only difference that the acceleration $g\sin\alpha$ and and $g\cos\beta$ must change places. So the sum of the two times for the first case will be

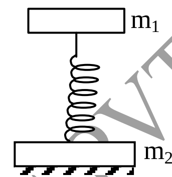
$$\sqrt{\frac{2A}{g\sin\alpha}} + \frac{-\sqrt{2Ag\sin\alpha} + \sqrt{2Ag\sin\alpha + 2Ag\cos\beta}}{g\cos\beta},$$

And for the second case the sum of the two times will be

$$\sqrt{\frac{2A}{g\cos\beta}} + \frac{-\sqrt{2Ag\cos\beta} + \sqrt{2Ag\cos\beta + 2Ag\sin\alpha}}{g\sin\alpha}$$

Since it is clear that $g \sin \alpha$ is less than $g \cos \beta$, we find that the sum of the two times in the first case is greater than in the second.

Q.8 Two laminas of mass m_1 and m_2 are joined by a spring (fig.) With what force should the upper lamina be pressed downwards so that when the force is removed the upper lamina should spring back and raise the lower lamina a little too ? The coefficient of elasticity of the spring is k. Assume that Hooke's law is applicable throughout. Ignore the mass of the spring.



Sol. The spring is compressed initially (by comparison with its normal length) by the weight of the upper lamina by an amount

$$x_1 = \frac{m_1 g}{k}$$

For the spring to raise the lower lamina on expansion, it must be extended, by comparison with its normal length, by an amount greater than

$$x_2 = \frac{m_2 g}{k}$$

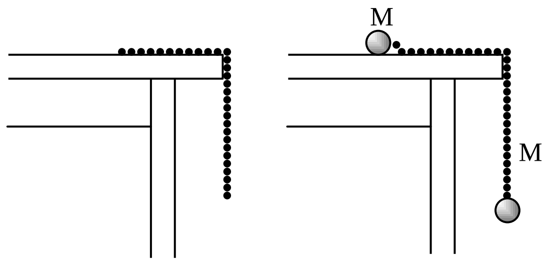
Consequently, the upper lamina must be depressed with such a force that when it is released it should jump up through a height greater than

$$x_1 + x_2 = \frac{(m_1 + m_2)g}{k},$$

reckoning form the position which the upper lamina occupy before it was depressed. But the upper lamina will jump up release through the same distance above the position of equilibrium as it lies below this position when depressed. Thus, if upper lamina is to carry the lower one up with it a little, it must be depressed through a distance greater than $x_1 + x_2$. For upper lamina must be depressed with a force greater than

$\mathbf{k (x_1 + x_2) = (m_1 + m_2) g.}$

Q.9 A chain is lying on an absolutely smooth table, half of it hanging over the edge of the table (fig.a). How will the time it takes to slip off the table be affected if two weights of equal mass be attached, one to each end (fig.b) ?



(a) (b)

Sol. Let the mass of the whole chain be **m**, and the mass of unit length of the chain be $m_0 = m/\ell$. Let a part of the chain of length x be hanging down from the table at any moment; to begin with, $x = \ell/2$, where ℓ is the length of the whole chain. Then the force which causes the chain to move will be proportional to the length of chain hanging down, i.e. m_0xg. The acceleration of the chain will equal m_0xg/m. To begin with, when $m_0x = m/2$ the acceleration equals g/2, but then it increases so that the chain's motion is not uniformly accelerated.

If identical masses M be attached to the ends of the chain, then at the moment when a part of the chain of length x is hanging from the table, the value of the moving force will be $m_0xg + Mg = (m_0x + M)g$, and the chain's acceleration at the moment will be $(m_0x + M)x/(m + 2M)$. To solve the problem as to which case cause the chain to slip off faster, we must find in which case will the acceleration increase at the greater rate. For this we must compare the two expressions :

$$\frac{m_0 xg}{m} \text{ and } \frac{(m_0 x + M)g}{m + 2M}$$

To compare these fractions, we shall put them over a common denominator and their numerator. The numerator of the first fraction will be $m_0xgm + 2m_0xgM$, and that of the second fraction will be $m_0xgm + Mgm$.

It is clear that these numerators are equal at the beginning when $m_0x = m/2$. In the subsequent moments, the acceleration will be greater in the first case than in the second.

So the chain will slip of more quickly when there are no masses attached to its ends.

This result can also be obtained from the law of conservation of energy, taking into account that at the moment when the end of the chain slips off the table, its centre of gravity is at a distance of $\ell/2$ below the edge of the table. Then if v_1 be the velocity at that moment and if there are no masses attached to the end,

$$mg\,\frac{\ell}{2} = \frac{mv_1^2}{2}$$

(Potential Energy) = (Kinetic Energy)

or $g\ell = v_1^2$

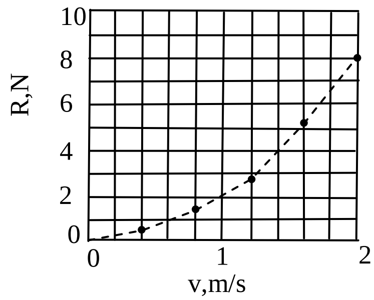
while if masses are attached to the ends

$$(m + M)\,g\,\frac{\ell}{2} = \frac{m + 2M}{2}\,v_2^2$$

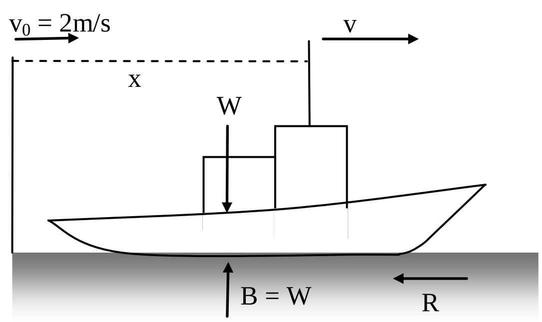
$$\text{or } g\ell = \frac{(m + 2M)v_2^2}{m + M}$$

i.e. the time of slipping for the chain must be less when there are no masses attached to the ends.

Q.10 The design model for a new ship has a mass of 10 kg and is tested in an experimental towing tank to determine its resistance to motion through her water at various speeds. The test results are plotted on the accompanying graph, and the resistance R may be closely approximated by the dashed parabolic curve shown. If the model is released when it has speed of 2m/s, determine the time **t** required for it to reduce its speed to 1m/s and the corresponding travel distance x.



Sol.



We approximate the resitance-velocity by $R = kv^2$ and find k by substituting R = 8N and v = 2 m/s into the equation, which gives $k = 8/2^2 = 2N\text{-}s^2/m^2$. Thus $R = 2v^2$.

The only horizontal force on the model is R, so that's

$[\Sigma F_x = ma_x]$ $\quad -R = ma_x$ or $\quad -2v^2 = 10\dfrac{dv}{dt}$

We separate the variables and integrate to obtain

$$\int_0^t dt = -5\int_0^v \frac{dv}{v^2} \quad t = 5\left(\frac{1}{v} - \frac{1}{2}\right)\text{sec}$$

Thus, when $v = v_0/2 = 1$ m/s the time is $t = 5\left(\frac{1}{1} - \frac{1}{2}\right) = 2.5$s

The distance traveled during the 2.5 second is obtained by integrating v = dx/dt. Thus, v = 10/(5 + 2t) so that

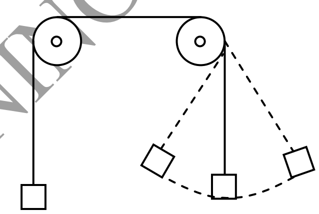
$$\int_0^x dx = \int_0^{2.5} \frac{10}{5+2t}\,dt$$

$x = \dfrac{10}{2}\ \text{In}\ (5 + 2t)\Big|_0^{2.5}$ = **3.47m** **Ans.**

Helpful Hints :

1. Be careful to observe the minus sign for R.
2. Suggestion: Express the distance x after release in terms of the velocity v and see if you agree with the resulting relation $x = 5\ \ln(v_0/v)$

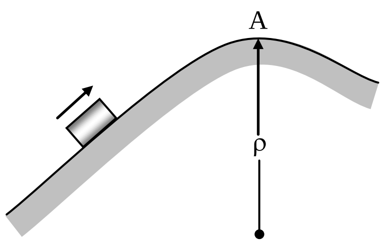
Q.11 Two identical weightless pulleys are mounted with parallel axes at the same height. A non-elastic weightless rope is passed through both pulleys with identical weights on the ends (fig.)



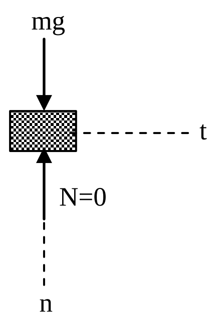
The system is in equilibrium. One of the weights is pulled to one side and released. Is equilibrium broken ?

Sol. Equilibrium will be broke since the tension in the rope from which the swinging load is suspended cannot remain constant and equal to the weight of its load. At the limits of the swing, where the load's velocity is zero, the tension in the rope will be equal to the component of the load's weight acting in the line of the rope, i.e., it will be less than the load's weight. At the centre point of the swing the tension in the rope must not only balance the load's weight, but must also impart to it the necessary centripetal force upwards, i.e. its tension must be greater than the load's weight. Therefore if the right-hand load swings, the left-hand load can not remain at rest and the resulting motion is of a more complicated character than that of a pendulum's oscillations, suspended from a fixed point. Experiment and calculation show that the swinging load will outpull the other.

Q.12 Determine the maximum speed **v** which the sliding block may have as it passes point A without losing contact with the surface.



Sol.



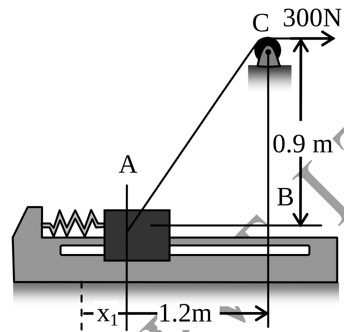
The condition for loss of contact is that the normal force N which the surface exerts on the block goes to zero. Summing forces in the normal direction gives

$[\Sigma F_n = ma_n]$ $\quad mg = \dfrac{v^2}{\rho}$ $\quad \mathbf{v} = \sqrt{g\rho}$

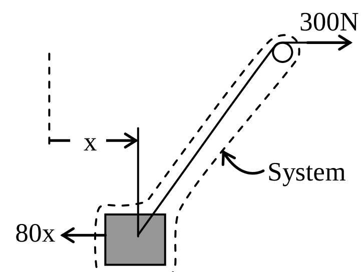
Ans.

If the speed at A were less than $\sqrt{g\rho}$, then an upward normal force exerted by the surface on the block would exist. In order for the block to have a speed at A which is greater than $\sqrt{g\rho}$, some type of constraint, such as a second curved surface above the block, would have to be introduced to provide additional downward force.

Q.13 The 50 kg block at **A** is mounted on rollers so that it moves along the fixed horizontal rail with negligible friction under the action of the constant 300-N force in the cable. The block is released from rest at A, with the spring to which it is attached extended an initial amount x_1 = 0.233m. The spring has a stiffness K = 80 N/m. Calculate the velocity of the block as it reaches position **B**.



Sol.



It will be assumed initially that the stiffness of the spring is small enough to allow the block to reach position B. The active-force diagram for the system composed of both block and cable is shown for a general position. The spring force 80x and the 300N. tension are the only forces external to this system which do work on the system. The force exerted on the block by the rail, the weight of the block and the reaction of the small pulley on the cable do no work on the system and are not included on the active-force diagram.

$[\, U = \int F dx \,]$

$$U_{1-2} = -\int_{0.233}^{1.433} 80x\, dx = -40x^2 \Big|_{0.233}^{1.433} = -80.0 \text{ J}$$

The work done on the system by the constant 300-N force in the cable is the force times the net horizontal movement of the cable over pulley C, which is $\sqrt{(1.2)^2 + (0.9)^2}$ – 0.9 = 0.6 m. Thus, the work done is 300 (0.6) = 180 J. We now apply the work-energy equation to be system and get

$$[U_{1-2} = \Delta T] \quad -80.0 + 180 = \frac{1}{2}(50)(v^2 - 0)$$

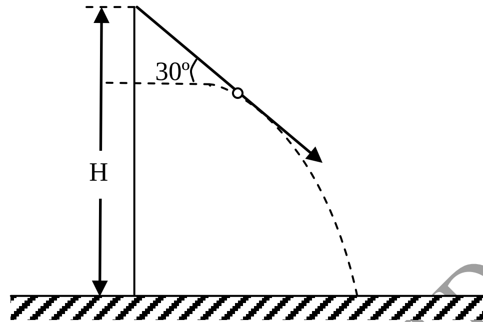
v = 2.0 m/s **Ans.**

We take special note of the advantage to our choice of system. If the block alone had constituted the system, the horizontal component of the 300-N cable tension acting on the block would have to be integrated over the 1.2-m displacement. This step would require considerably more effort then was needed in the solution as presented. If there had been appreciable friction between the block and its guiding rail, we would have found it necessary to insolate the block alone in order to compute the variable normal force and, hence, the variable friction force. Integration of the friction force over the displacement would then be required to evaluate the negative work which it would do.

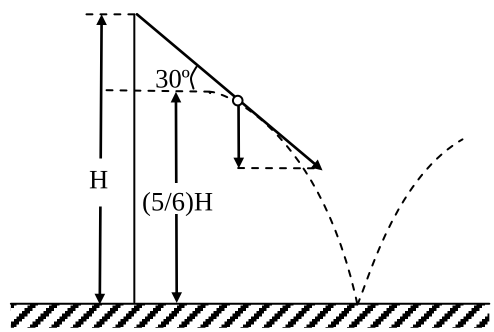
Helpful Hint :

If the variable x had been measured from the starting position A, the spring force would be 80(0.233 + x), and the limits of integration would be 0 and 1.2m.

Q.14 Starting from a height H, a ball slips without friction, down a smooth plane inclined at an angle of 30º to the horizontal (fig.) The length of the plane is H/3. The ball then falls on to a horizontal surface with an impact that way be taken as perfectly elastic. How high does the ball rise after striking the horizontal plane ?



Sol. The ball slips from the plane **(see Fig.)** with a velocity of $\sqrt{gH/3}$ at an inclination of 30º to the horizontal. Then the ball describes a parabola and falls on to the horizontal plane with a velocity inclined at some unknown angle to the horizontal. But the height to which the ball will rise after an absolutely elastic impact on the plane depends on only the vertical component of this velocity. The value of this component can be found by calculating the speed with which the ball will fall from a height of $\mathbf{\frac{5}{6}H}$ with an initial velocity of $\mathbf{\frac{1}{2}\sqrt{gH/3}}$.



From the equation

$$\frac{5}{6}H = \frac{1}{2}\sqrt{\frac{gH}{3}}\,t + \frac{gt^2}{2}$$

We find that the time of fall of the ball

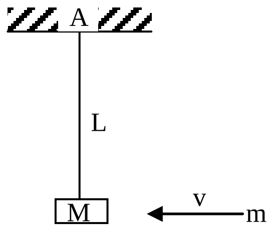
$$t = \frac{\sqrt{21}-1}{2}\sqrt{\frac{H}{3g}}.$$

Therefore its velocity at the end of the fall will be

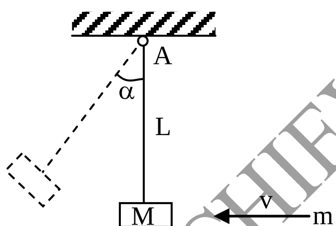
$$v = v_0 + gt = \frac{\sqrt{21}}{2}\sqrt{\frac{gH}{3}}.$$

Therefore the height to which the ball will rise after its elastic impact on the plane will equal $\mathbf{\frac{v^2}{2g} = \frac{7H}{8}}$.

Q.15 A bullet of mass **m** hits a wooden block of mass **M**, which is suspended from a thread of length ℓ (a ballistic pendulum), and is embedded in it. Find through what angle the block will swing if the bullet's velocity is **v** (fig.).



Sol. A bullet of mass m, traveling with a velocity v, has momentum **mv**. After the bullet embeds itself in the block, the block plus the bullet will have exactly the same momentum (the impact is completely inelastic). Therefore the velocity $\mathbf{v_1}$, which the block acquires immediately upon the bullet's hitting it, will be determined from the law of conservation of momentum : mv = (M + m) v_1. Also the kinetic energy of block and bullet will be



$$\frac{(M+m)v_1^2}{2} = \frac{m}{M+m}\cdot\frac{mv^2}{2}.$$

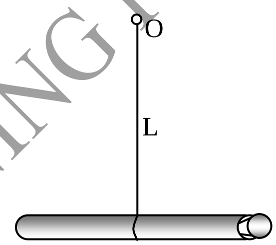
Then the block will rise, and this kinetic energy will be changed into potential energy. Since the whole mass (M + m) is virtually at a distance of ℓ from the point of suspension A (Fig.), its centre of gravity will rise, in consequence of a swing through an angle α on the part of the pendulum, through height $\Delta h = \ell\,(1 - \cos\theta)$. At the farthest point of its swing, through an angle α_0, the potential energy must equal the initial kinetic energy, i.e.

$$\frac{m}{M+M}\cdot\frac{mv^2}{2} = (M + m)\, g\ell\, (1 - \cos \alpha_0).$$

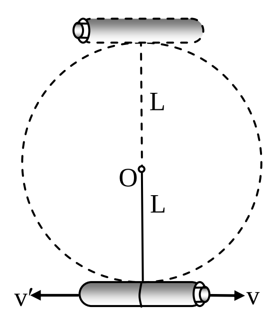
Hence the angle through which the pendulum swings is given by the relationship

$$\sin^2\frac{\alpha_2}{2} = \frac{m^2v^2}{4(M+m)^2 g\ell}.$$

Q.16 A test-tube of mass **M**, closed with a cork of mass **m**, contains a drop of ether. When the test-tube is heated, the cork flies out under the pressure of the ether-gases. The test-tube is suspended by a weightless, rigid bar of length **L** (Fig.) What is the least initial velocity which will cause the test-tube to describe a full circle about the pivot **O** ?



Sol. Let the minimum velocity with which the cork must fly out of the test-tube so that the test-tube should describe a full circle about O be **v (Fig.)** Then the cork will acquire momentum of **mv**, and the test-tube will accordingly acquire momentum of **Mv′** and the law of conservation of momentum tells us that then v′ = mv/M. At the initial moment of its movement the test-tube will have a kinetic energy of $Mv'^2/2$. This energy must go towards raising a mass M through a height of 2L.



From the law of the conservation of energy we shall obtain the equation $\frac{Mv'^2}{2}$ = Mg2L.

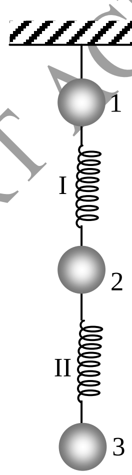
Hence, substituting for v′, we get :

$$\mathbf{v = \frac{2M\sqrt{gL}}{m}}.$$

Q.17 Three balls of equal mass are suspended from a thread and two springs of the same elasticity so that the distances between the first and second ball and the second and third are the same **(fig.).** Thus the centre of gravity of the whole system coincides with the centre of the second ball. If the thread supporting the top ball be cut, the system will fall and the acceleration of the system's centre of gravity will be

$$\frac{mg+mg+mg}{3m} = g$$

(according to Newton's second law, the acceleration of the centre of gravity of a system equals the sum of the forces acting on the system from outside divided by the system's total mass). But spring I will pull the second ball upwards with greater force than spring II will pull is downwards (the force of spring I at the initial moment f_{10} = 2mg, while the force of spring II at the initial moment f_{20} = mg) and therefore the centre of the second ball will have, at the initial moment, an acceleration of less than g. And yet the centre of gravity of the whole system must move with an acceleration of **g** the whole time. Explain the contradiction.



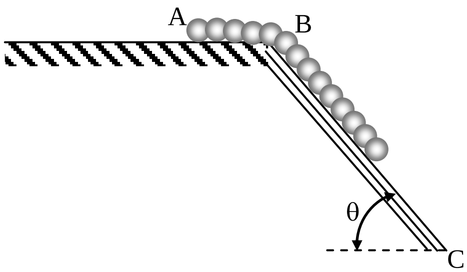
Sol. Springs I and II having equal elasticity, act with different forces of 2mg and mg at the initial moment, and yet they have the same length. Thus, when they are not under stress, they must have different lengths. In a free fall, both springs must cease to be deformation will disappear) and since this normal length is not the same for the two springs, the distance between the centre of the first and second ball and the second and third ball will no longer be the same. Thus the centre of the second ball will cease to be the centre of gravity of the system of three balls after the beginning of the fall.

Q.18 The principle of the automatic weapon is based on the utilization of the phenomenon of recoil which takes place on firing: the breech-block moves backwards after firing and compresses the spring, which in turn brings the reloading mechanism into action. Find what must be the velocity of the bullet for the breech-block to move back a distance **a**, if the mass of the bullet be **m**, the mass of the breech-block to move back a distance **a**, if the mass of the bullet be **m**, the mass of breech-block **M** and the coefficient of elasticity of the spring **k**. Neglect the mass of the charge.

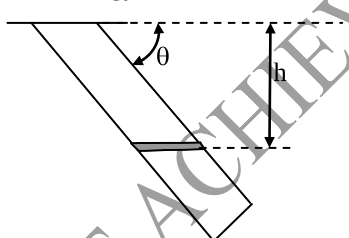
Sol. For the breech-block to move back a distance **a**, it is necessary that the work done in overcoming the elastic force of the spring should be $\mathbf{ka^2/2}$. This work will be done at the expense of the kinetic energy which the breech-block acquires from the recoil. If the breech-block has initial velocity u, its kinetic energy $Mu^2/2 = ka^2/2$, hence $u = a\sqrt{k/M}$. On the other hand, the absolute momentum **Mu** of the breech-block on firing must equal the momentum of the bullet, **mv** (since they are opposite in direction and must give a sum of zero). Therefore

$$v = \frac{Mu}{m} = \frac{\mathbf{a}}{\mathbf{m}}\sqrt{\mathbf{kM}}.$$

Q.19 A chain of length **L** and mass **m** is placed upon a smooth surface (see. Figure). The length of $\overline{BA}$ is **L–b**. Calculate the velocity of the chain when its end reaches B.



Sol. To solve this problem, we use the principle of conservation of energy. Let us denote by y = 0 the plane $\overline{AB}$. It will be the reference plane of the potential energy. In changing its position from A to B, the chain's potential energy changes, and as a result, its velocity changes. The potential energy is calculated by integration over the length of the chain. The mass of the chain per unit length is $\ell = \frac{m}{L}$. The contribution of a piece of length **dr** to the potential energy is :



$du = -gh\,dm = -\lambda hgdr$ …..(i)

Where $h = r \sin\theta$ (see figure). The initial potential energy is, therefore,

$$u_i = \int_0^b (-\lambda g \sin\theta) r\, dr = -\frac{1}{2}\lambda g b^2 \sin\theta$$

…..(ii)

Similarly, the final potential energy is :

$$u_f = \int_0^L (-\lambda g \sin\theta) r\, dr = -\frac{1}{2}\lambda g L^2 \sin\theta$$

…..(iii)

From the principle of conservation of energy, we know that :

$E_{k(i)} + u_i = E_{k(f)} + u_f$ …..(iv)

Where $E_{k(i)} = 0$ Hence,

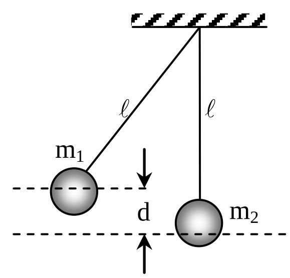
$$E_{k(f)} = u_i - u_f = \frac{1}{2}\frac{m}{L} g\sin\theta\,(L^2 - b^2)\ \frac{1}{2} mv^2$$

…..(v)

From which follows

$$v = \sqrt{\frac{g\sin\theta}{L}(L^2 - b^2)}$$ …..(iv)

Q.20 Two pendulums each of length l are initially situated as in figure. The first pendulum is released and strikes the second. Assume that the collision in completely inelastic and neglect the mass of the strings and any frictional effects. How high does the center of mass rise after the collision ?



Ans. $d\left[\frac{m_1}{m_1 + m_2}\right]^2$