Vectors

Single Correct Answer Type

1. P(i – j + 3k)&Q(3i + 3j + 3k)are two points in space. Equation of a plane is $\bar{r}\cdot(5i+2j-7k)+9=0$, then the points P & Q

a) Lie on same side and equidistant from the plane.

b) Lie on either side and equidistant from the plane.

c) Lie on same side of a plane & at unequal distances from the plane

d) Lie on opposite side & at unequal distances from the plane

Key. B

Sol. $\bar{r}\cdot\bar{m}=d,\ A(\bar{a})$

distance from $A(\bar{a})$ to the plane $\bar{r}\cdot\bar{m}=d$ is $\frac{\bar{d}-\bar{a}\cdot\bar{m}}{|\bar{m}|}$

If $A(\bar{a})=i-j+3k\ then\ d=\frac{9}{\sqrt{78}}$

If $A(\bar{a})=3i+3j+3k$ then d = $\frac{-9}{\sqrt{78}}$

2. The length of the perpendicular from the origin to the plane passing through the point $\bar{a}$ & containing the line $\bar{r}=\bar{b}+\lambda\bar{c}$ is

a) $\frac{\left[\bar{a}\ \bar{b}\ \bar{c}\right]}{\left|\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}\right|}$

b) $\frac{\left[\bar{a}\ \bar{b}\ \bar{c}\right]}{\left|\bar{a}\times\bar{b}+\bar{b}\times\bar{c}\right|}$

c) $\frac{\left[\bar{a}\ \bar{b}\ \bar{c}\right]}{\left|\bar{b}\times\bar{c}+\bar{c}\times\bar{a}\right|}$

d) $\frac{\left[\bar{a}\ \bar{b}\ \bar{c}\right]}{\left|\bar{c}\times\bar{a}+\bar{a}\times\bar{b}\right|}$

Key. C

Sol. Given plane passes through $\bar{a}$ & $\vec{b}$ containing the line is $\left[\overline{AP}\ \overline{AB}\ \ \bar{c}\right]=0$

$\Rightarrow(\bar{r}-\bar{a})\cdot\left((\bar{b}-\bar{a})\times\bar{c}\right)=0$

$\Rightarrow\bar{r}\cdot\left(\bar{b}\times\bar{c}+\bar{c}\times\bar{a}\right)=\left[\bar{a}\ \bar{b}\ \bar{c}\right]$

length of $\perp^{r}$ from the origin $=\frac{\left[\bar{a}\ \bar{b}\ \bar{c}\right]}{\left|\bar{b}\times\bar{c}+\bar{c}\times\bar{a}\right|}$

3. Equation of the plane through $(3,4,-1)$ which is parallel to the plane

$\bar{r}.(2\bar{i}-3\bar{j}+5\bar{k})+7=0$ is

1. $\bar{r}.(2\bar{i}-3\bar{j}+5\bar{k})+11=0$ 2. $\bar{r}.(3\bar{i}+4\bar{j}-\bar{k})+11=0$

3. $\bar{r}.(3\bar{i}+4\bar{j}-\bar{k})+7=0$ 4. $\bar{r}.(2\bar{i}-3\bar{j}+5\bar{k})-7=0$

Key. 1

Sol. Equation of any plane parallel to the given plane is $r.(2i-3j+5k)+\lambda=0.$

If $r=xi+yi+zk,$ we get $2x-3y+5k+\lambda=0$

This plane passes through the point $(3,4,-1)$ if $2\times3-3\times4+5(-1)+\lambda=0$ or it $x=11$ and hence the equation of the required plane is $r.(2i-3j+5k)+11=0$

4. Let $\bar{a}=\bar{i}+\bar{j}+\bar{k},\bar{b}=\bar{i}-\bar{j}+2\bar{k}$ and $\bar{c}=x\bar{i}+(x-2)\bar{j}-\bar{k}.$ If the vector $\bar{c}$ lies in the plane of $\bar{a}$ and $\bar{b}$, then x equals

1. 0 2. 1 3. -4 4. -2

Key. 4

Sol. Since the three vectors are coplanar $\begin{vmatrix}1 & 1 & 1\\ 1 & -1 & 2\\ x & x-2 & -1\end{vmatrix}=0$

$$\Rightarrow\begin{vmatrix}1 & 0 & 0\\ 1 & -2 & 1\\ x & -2 & -1-x\end{vmatrix}=0$$

$$\Rightarrow -2(-1-x)+2=0 \qquad \Rightarrow \qquad x=-2$$

5. Equation of the plane containing the lines $\bar{r}=\bar{i}+2\bar{j}-\bar{k}+\lambda(\bar{i}+2\bar{j}-\bar{k})$ and $\bar{r}=\bar{i}+2\bar{j}-\bar{k}+\mu(\bar{i}+\bar{j}+3\bar{k})$ is

1. $\bar{r}.(7\bar{i}-4\bar{j}-\bar{k})=0$ 2. $7(x-1)-4(y-1)-(z+3)=0$

3. $\bar{r}.(\bar{i}+2\bar{j}-\bar{k})=0$ 4. $\bar{r}.(\bar{i}+\bar{j}+3\bar{k})=0$

Key. 1

Sol. Since both the given lines pass through the point with position vector $i+2j-k,$ the required plane also passes through $i+2j-k$ and normal to the plane is perpendicular to the vectors $i+2j-k$ and $i+j+3k.$ If $d=ai+bj+ck$ is normal to the required plane, then $a+2b-c=0$ and $a+b+3c=0$

$$\Rightarrow \frac{a}{7}=\frac{b}{-4}=\frac{c}{-1}\Rightarrow d=7i-4j-k.$$

So the required plane passes through $i+2j-k$ and the normal to plane is $7i-4j-k,$ hence required equation is $[r-(i+2j-k)].(7i-4j-k)=0$

$$r.(7i-4j-k)=1\times 7+2(-4)+(-1)(-1)=0$$

Also since the required plane passes through $i+2j-k, i.e.$ the point $-(1,2,-1)$ and the direction ratios of the normal to the plane are 7,-4,-1, the equation of the plane in Cartesian form can be written as $7(x-1)-4(y-2)-(z+1)=0$

Use the result number 11 given in vectorial equations.

6. The Cartesian equation of the plane passing through the line of intersection of the planes $r.(2i-3j+4k)=1$ and $r.(i-j)+4=0$ and perpendicular to the plane $r.(2i-j+k)+8=0$ is

1. $3x-4y+4z=5$
2. $x-2y+4z=3$
3. $5x-2y-12z+47=0$
4. $2x+3y+4=0$

Key. 3

Sol. Equation of any plane passing through the intersection of the planes $r.(2i-3j+4k)=1$ and $r.(i-j)+4=0$ is $2x-3y+4z-1+\lambda(x-y+4)=0$ or $(2+\lambda)x-(3+\lambda)y+4z+4\lambda-1=0$

The plane is perpendicular to the plane $r.(2i-j+k)+8=0$ if

$$\Rightarrow 2(2+\lambda)+(3+\lambda)4=0.$$

$\Rightarrow 11+3\lambda=0\Rightarrow\lambda=-11/3.$ and the required equation of the plane is

$$3(2x-3y+4z-1)-11(x-y+4)=0\Rightarrow 5x-2y-12z+47=0$$

7. If the vector $2i-3j+7k$ is inclined at angles α,β,γ with the coordinate axes, then

1. $3\cos\alpha=2/\sqrt{62}$
2. $2\cos\beta=-3/\sqrt{62}$
3. $\cos\gamma=7/\sqrt{62}$
4. $2\cos\alpha=-3\cos\beta=7\cos\gamma$

Key. 3

Sol. $\cos\alpha=2/\sqrt{62},\cos\beta=-3\sqrt{62},\cos\gamma=7/\sqrt{62}.$

8. If $\bar{r}.\bar{n}=q$ is the equation of a plane normal to the vector $\bar{n}$ then the length of the perpendicular from the origin on the plane is

1. $|q|$ 2. $|\bar{n}|$ 3. $|q||\bar{n}|$ 4. $\frac{|q|}{|\bar{n}|}$

Key. 4

Sol. Equation of the plane is $r.\frac{n}{|x|}=\frac{q}{|n|}$ $i.e., r.n=\frac{q}{|n|}$. So the required length $=q/|n|$.

9. If $\alpha(\bar{a}\times\bar{b})+\beta(\bar{b}\times\bar{c})+\gamma(\bar{c}\times\bar{a})=\bar{0}$, Then

(A) $\bar{a}, \bar{b}, \bar{c}$ are coplanar only if none of a, b, g is zero

(B) $\bar{a}, \bar{b}, \bar{c}$ are coplanar if atleast one of a, b, g is non zero

(C) $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar for any α, β, γ (D) none of these

Key. B

Sol. We have

$$\alpha(\bar{a}\times\bar{b})+\beta(\bar{b}\times\bar{c})+\gamma(\bar{c}\times\bar{a})=\bar{0}$$

Taking dot product with c, we have

$$\alpha[\bar{a}\,\bar{b}\,\bar{c}]+\beta[\bar{b}\,\bar{c}\,\bar{c}]+\gamma[\bar{c}\,\bar{a}\,\bar{c}]=0$$

i.e. $\alpha[\bar{a}\,\bar{b}\,\bar{c}]+0+0=0$

i.e., $\alpha[\bar{a}\,\bar{b}\,\bar{c}]=0$

Similarly, taking dot product with b and c, we have

Now, even if one of $\alpha, \beta, \gamma\neq 0$, then we have $[a\ b\ c]=0$

$\Rightarrow$ a, b, c are coplanar

10. If $\bar{a}$ *and* $\bar{b}$ are unit vectors and $\bar{c}$ is a vector such that $\bar{c}=\bar{a}\times\bar{c}+\bar{b}$ then

(A) $[\bar{a}\ \bar{b}\ \bar{c}]=\bar{b}.\bar{c}-(\bar{a}.\bar{b})^2$ (B) $[\bar{a}\ \bar{b}\ \bar{c}]=0$

(C) Maximum value of $[\bar{a}\ \bar{b}\ \bar{c}]=\frac{1}{2}$ (D) Minimum value of $[\bar{a}\ \bar{b}\ \bar{c}]$ is $\frac{1}{2}$

Key. A,C

Sol. $\bar{c}.\bar{a}=((\bar{a}\times\bar{c})+\bar{b}).\bar{a}=\bar{b}.\bar{a}$

$\bar{b}\times\bar{c}=(\bar{b}.\bar{c})+\bar{a}-(\bar{a}-\bar{b}).\bar{c}$

$\therefore [\bar{a}\,\bar{b}\,\bar{c}]=\bar{b}.\bar{c}-(\bar{a}-\bar{b}).(\bar{a}.\bar{c})$

Also $\bar{c}.\bar{b}=1-[\bar{a}\,\bar{b}\,\bar{c}]$

$\therefore 2\ [\bar{a}\,\bar{b}\,\bar{c}]=1-(\bar{a}.\bar{b})^2\leq 1$

$\therefore \left[\bar{a}\,\bar{b}\,\bar{c}\right] \le \frac{1}{2}$

11. If the four faces of a tetrahedron are represented by the equations $\bar{r}.\left(\alpha\bar{i}+\beta\bar{j}\right)=0, \bar{r}.\left(\beta\bar{j}+\gamma\bar{k}\right)=0, \bar{r}.\left(\gamma\bar{k}+\alpha\bar{i}\right)=0$ *and* $\bar{r}.\left(\alpha\bar{i}+\beta\bar{j}+\gamma\bar{k}\right)=P$

then volume of the tetrahedron (in cubic units) is

a) $\left|\frac{P^3}{6\alpha\beta\gamma}\right|$ b) $\left|\frac{4P^3}{6\alpha\beta\gamma}\right|$ c) $\left|\frac{3P^3}{6\alpha\beta\gamma}\right|$ d) none of these

Key. B

Sol. Conceptual

12. A non - zero vector $\vec{a}$ is parallel to the line of intersection of the plane P_1 determined by $\hat{i}+\hat{j}$ and $\hat{i}+2\hat{j}$ and plane P_2 determined by vector $2\hat{i}-\hat{j}$ and $3\hat{i}+2\hat{k}$, then angle between $\vec{a}$ and $\hat{i}-2\hat{j}+2\hat{k}$ vector is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) none of these

Key. D

Sol. Conceptual

13. If $\vec{a}'=\hat{i}+\hat{j}, \vec{b}'=\hat{i}+\hat{j}+2\hat{k}$ & $\vec{c}'=2\hat{i}+\hat{j}-\hat{k}$. Then altitude of the parallelpiped formed by the vectors
$\vec{a},\vec{b},\vec{c}$ having base formed by $\vec{b}$ & $\vec{c}$ is ($\vec{a},\vec{b},\vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vectors)

(A) 1 (B) $\frac{3\sqrt{2}}{2}$ (C) $\frac{1}{\sqrt{6}}$ (D) $\frac{1}{\sqrt{2}}$

Key. D

Sol. Volume of the parallelepiped formed by $\vec{a}',\vec{b}',\vec{c}'$ is 4

$\therefore$ Volume of the parallelepiped formed by $\vec{a},\vec{b},\vec{c}$ is $\frac{1}{4}$

$$\vec{b}\times\vec{c}=\frac{\left(\vec{c}'\times\vec{a}'\right)\times\vec{c}}{4}=\frac{1}{4}\vec{a}'$$

$$\therefore \left|\vec{b}\times\vec{c}\right|=\frac{\sqrt{2}}{4}=\frac{1}{2\sqrt{2}}$$

$\therefore$ length of altitude $=\frac{1}{4}\times 2\sqrt{2}=\frac{1}{\sqrt{2}}$.

14. A unit vector $\vec{a}$ in the plane of $\vec{b}=2\hat{i}+\hat{j}$ & $\vec{c}=\hat{i}-\hat{j}+\hat{k}$ is such that $\vec{a}\wedge\vec{b}$ = $\vec{a}\wedge\vec{d}$ where $\vec{d}=\hat{j}+2\hat{k}$ is

(A) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$ (B) $\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$ (C) $\frac{2\hat{i}+\hat{j}}{\sqrt{5}}$ (D) $\frac{2\hat{i}-\hat{j}}{\sqrt{5}}$

Key. B

Sol. Let $\vec{a}=\lambda\vec{b}+\mu\vec{c}$, then $\frac{\vec{a}.\vec{b}}{ab}=\frac{\vec{a}.\vec{d}}{ad}$

i.e. $\frac{\left(\lambda\vec{b}+\mu\vec{c}\right).\vec{b}}{b}=\frac{\left(\lambda\vec{b}+\mu\vec{c}\right).\vec{d}}{d}$

i.e. $\frac{\left[\lambda\left(2\hat{i}+j\right)+\mu\left(\hat{i}-j+k\right)\right].\left(2\hat{i}+j\right)}{\sqrt{5}}=\frac{\left[\lambda\left(2\hat{i}+j\right)+\mu\left(\hat{i}-j+k\right)\right].\left(\hat{i}+2k\right)}{\sqrt{5}}$

i.e. $\lambda(4+1)+\mu(2-1)=\lambda(1)+\mu(-1+2)$ *i.e.* $41=0$ *i.e.* $1=0$

$\therefore \vec{a}=\frac{\hat{i}-j+k}{\sqrt{3}}$

15. Let $\vec{r},\vec{a},\vec{b}\&\vec{c}$ be four non-zero vector such that $\vec{r}.\vec{a}=0,|\vec{r}\times\vec{b}|=|\vec{r}||\vec{b}|,|\vec{r}\times\vec{c}|=|\vec{r}||\vec{c}|$, then [abc] =

(A) |a| |b| |c| (B) - |a| |b| |c| (C) 0 (D) none of these

Key. C

Sol. $\vec{r}.\vec{a}=0,\left|\vec{r}\times\vec{b}\right|=|\vec{r}|\left|\vec{b}\right|\ \&\left|\vec{r}\times\vec{c}\right|=|\vec{r}||\vec{c}|$

$\Rightarrow \vec{r}\perp\vec{a},\vec{b},\vec{c}$

$\therefore \vec{a},\vec{b},\vec{c}$ are coplaner

$\therefore\left[\vec{a}\,\vec{b}\,\vec{c}\right]=0$

16. If $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$, then $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is equal to

(A) $6(\vec{b}\times\vec{c})$ (B) $6(\vec{c}\times\vec{a})$ (C) $6(\vec{a}\times\vec{b})$ (D) none of these

Key. A

Sol.

$\vec{a}+2\vec{b}+3\vec{c}=\vec{0}\Rightarrow\vec{a}\times\vec{b}+3\vec{c}\times\vec{b}=\vec{0}$ *i.e.* $\vec{a}\times\vec{b}=3\vec{b}\times\vec{c},\vec{a}\times\vec{c}+2\vec{b}\times\vec{c}=\vec{0}$ *i.e.* $2\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

$\therefore\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=3\vec{b}\times\vec{c}+\vec{b}\times\vec{c}+2\vec{b}\times\vec{c}=6\vec{b}\times\vec{c}$

17. If $\left(\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)\right)\cdot\left(\vec{a}\times\vec{d}\right)=0,$ then which of the following is always true

(A) $\vec{a},b,\vec{c},\vec{d}$ are necessarily coplaner

(B) either $\vec{a}$ or $\vec{d}$ must lie in the plane of $\vec{b}$ or $\vec{c}$

(C) either $\vec{b}$ or $\vec{c}$ must lie in place of $\vec{a}$ and $\vec{d}$

(D) either $\vec{a}$ or $\vec{b}$ must lie in plane of $\vec{c}$ and $\vec{d}$

Key. C

Sol. $\left(\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)\right)\cdot\left(\vec{a}\times\vec{d}\right)=0,$

$\Rightarrow\left(\left[\vec{a}\vec{c}\vec{d}\right]\vec{b}-\left[\vec{b}\vec{c}\vec{d}\right]\vec{a}\right)\cdot\left(\vec{a}\times\vec{d}\right)=0$

$\Rightarrow\left[\vec{a}\vec{c}\vec{d}\right]\left[\vec{b}\vec{a}\vec{d}\right]=0$

$\Rightarrow$ either $\vec{c}$ and $\vec{b}$ must lie in the plane of $\vec{a}$ and $\vec{d}$.

18. Let $\vec{r} = (\vec{a} \times \vec{b}) \sin x + (\vec{b} \times \vec{c}) \cos y + 2(\vec{c} \times \vec{a})$ where $\vec{a}\ \vec{b}\ \vec{c}$ are three noncoplanar vectors. If $\vec{r}$ is perpendicular to $\vec{a} + \vec{b} + \vec{c}$, then minimum value of $x^2 + y^2$ is

(A) π^2 (B) $\frac{\pi^2}{4}$ (C) $\frac{5\pi^2}{4}$ (D) none of these

Key. C

Sol. $\vec{r} = (\vec{a} \times \vec{b}) \sin x + (\vec{b} \times \vec{c}) \cos y + 2(\vec{c} \times \vec{a})$

$\vec{r}\,.\,(\vec{a} + \vec{b} + \vec{c}) = 0$

$\Rightarrow \quad [\vec{a}\,\vec{b}\,\vec{c}]\,(\sin x + \cos y + 2) = 0$

$[\vec{a}\,\vec{b}\,\vec{c}] \neq 0 \quad \Rightarrow \quad \sin x + \cos y = -2$

this is possible only when sin x = – 1 and cos y = –1

for $x^2 + y^2$ to be minimum $x = -\frac{\pi}{2}$ and $y = \pi$

$\Rightarrow$ minimum value of $(x^2 + y^2)$ is $= \frac{\pi^2}{4} + \pi^2 = \frac{5\pi^2}{4}$

19. The position vector of the centre of the circle $|\vec{r}| = 5$, $\vec{r}\,.\,(\hat{i} + \hat{j} + \hat{k}) = 3\sqrt{3}$

(A) $\hat{i} + \hat{j} + \hat{k}$ (B) $3(\hat{i} + \hat{j} + \hat{k})$

(C) $(\sqrt{3} + \hat{i} + \hat{k})$ (D) None of these

Key. C

Sol. Centre of the circle is the foot of perpendicular drawn from origin to the plane $\vec{r}\,.\left(\hat{i}+\hat{j}+\hat{k}\right) = 3\sqrt{3}$

equation of perpendicular is $\vec{r} = \lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$

Let $\lambda\left(\hat{i}+\hat{j}+\hat{k}\right)$ lie on the plane $\vec{r}\,.\left(\hat{i}+\hat{j}+\hat{k}\right) = 3\sqrt{3}$

$\therefore \quad 3\lambda = 3\sqrt{3} \quad$ i.e. $\quad \lambda = \sqrt{3}$

$\therefore \quad$ the centre is $\sqrt{3}\left(\hat{i}+\hat{j}+\hat{k}\right)$

20. The locus represented by $xy + yz = 0$ is

(A) A pair of perpendicular lines

(B) a pair of parallel lines

(C) A pair of parallel planes

(D) a pair of perpendicular planes

Key. D

Sol. $xy + yz = 0$

$y(x + z) = 0$

i.e. $y = 0$ or $x + z = 0$ which is a pair of perpendicular planes.

Vectors

Integer Answer Type

1. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar vectors and

$$\left[\left(\vec{a}+\vec{b}\right)\times(\vec{b}-\vec{c})\ \ \left(\vec{b}+\vec{c}\right)\times\left(\vec{c}+\vec{a}\right)\ \ (\vec{c}-\vec{a})\times\left(\vec{a}+\vec{b}\right)\right]=K\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$$ then value of K is ?

Key. 4

Sol. $[(\vec{a}+\vec{b})\times(\vec{b}-\vec{c})\ \ (\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\ \ (\vec{c}-\vec{a})\times(\vec{a}+\vec{b})]$

$=[\vec{a}\times\vec{b}-\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\quad -\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\quad -\vec{a}\times\vec{b}-\vec{b}\times\vec{c}+\vec{c}\times\vec{a}]$

$=[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]\begin{vmatrix}1 & -1 & 1\\ -1 & 1 & 1\\ -1 & -1 & 1\end{vmatrix}$

$=4[\vec{a}\,\vec{b}\,\vec{c}]^2$

2. OABC is regular tetrahedron of unit edge length with volume V then $12\sqrt{2}V =$

Key. 2

Sol. $\left[\bar{a}\ \bar{b}\ \bar{c}\right]^2=\begin{vmatrix}\bar{a}\cdot\bar{a} & \bar{a}\cdot\bar{b} & \bar{a}\cdot\bar{c}\\ \bar{b}\cdot\bar{a} & \bar{b}\cdot\bar{b} & \bar{b}\cdot\bar{c}\\ \bar{c}\cdot\bar{a} & \bar{c}\cdot\bar{b} & \bar{c}\cdot\bar{c}\end{vmatrix}=\begin{vmatrix}1 & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & 1 & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2} & 1\end{vmatrix}=\frac{1}{2}$

$\Rightarrow\left[\bar{a}\ \bar{b}\ \bar{c}\right]=\frac{1}{\sqrt{2}}$ volume $=\frac{1}{6}\left[\bar{a}\ \bar{b}\ \bar{c}\right]=\frac{1}{6\sqrt{2}}$

$12\sqrt{2}V=2$

3. Two points P and Q are given in the rectangular cartesian co-ordinate system on the curve $y = 2^{x+2}$, such that $\overrightarrow{OP}.\hat{i}=-1$ and $\overrightarrow{OQ}.\hat{i}=2$. The magnitude of the vector $\overrightarrow{OQ}-4\overrightarrow{OP}$ is $10l$ where l =__ (where O is origin)

Key. 1

Sol. Let $P(x_1,y_1)$ and $Q(x_2,y_2)$ then $y_1=2^{x_1+2}$ and $y_2=2^{x_2+2}$ and $\overrightarrow{OP}.\hat{i}=-1$

$\Rightarrow\ (x_1\hat{i}+y_1\hat{i}).\hat{i}=-1 \Rightarrow\ x_1=-1$

and correspondingly $y_1=2^{-1+2}$, ie. $y_1 = 2$.

4. ABC is any triangle and O is any point in the plane of the same. If AO, BO and CO meet the sides BC, CA and AB in D,E,F respectively, then $\frac{OD}{AD}+\frac{OE}{BE}+\frac{OF}{CF}$ =____.

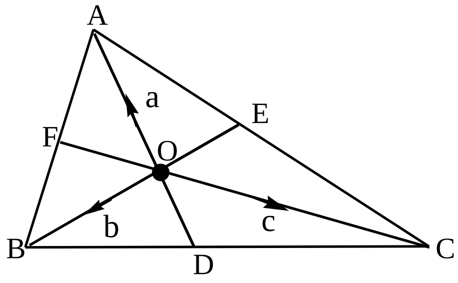
Key. 1

Sol. $\overrightarrow{OD}=x\overrightarrow{OA}\Rightarrow\ \vec{r}=-x\vec{a}$

$\because \bar{a}, \bar{b}, \bar{c}$ are coplanor

$l\vec{x} + m\vec{b} + n\vec{c} = 0$

$\because \vec{r}, \vec{b}, \vec{c}$ are collinear $\frac{-l}{x} + m + x = 0 \Rightarrow x = \frac{l}{m+n}$



$\overrightarrow{AD} = -x\vec{a} - \vec{a} = -(x+1)\vec{a}$

$\therefore \frac{OD}{AD} = \frac{+x}{x+1} = \frac{l}{l+m+n}$. Similary $\frac{OE}{BE} = \frac{m}{l+m+n}, \frac{OF}{CF} = \frac{n}{l+m+n}$

$\therefore \frac{OD}{AD} + \frac{OE}{BE} + \frac{OC}{CA} = 1$

5. The vectors $\bar{a}, \bar{b}$ & $\bar{c}$ each two of which are non–collinear. If $\bar{a}+\bar{b}$ is collinear with $\bar{c}, \bar{b}+\bar{c}$ is collinear with $\bar{a}$ & $|\bar{a}| = |\bar{b}| = |\bar{c}| = \sqrt{2}$. Then the value of $|\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a}| =$

Key. 3

Sol. $\bar{a}+\bar{b} = \lambda\bar{c}, \bar{b}+\bar{c} = m\bar{a}$

$\Rightarrow \bar{a}+\bar{b}+\bar{c} = \bar{0}$

$$\Rightarrow |\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a}| = \left| -\frac{\left(|\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2\right)}{2} \right| = 3$$

6. The equation of conic section can also be given by two dimensional vectors. The vector equation of conic must be a relation satisfied by position vectors of all the points on the conic. The position vector of a general point may be taken as $\vec{r}$ .The eccentricity of the conic $|\vec{r} - \hat{i} - \hat{j}| + |\vec{r} + \hat{i} + \hat{j}| = 3$ is “e” then $\left[\sqrt{2}e^{-1}\right]$ where [.] denotes greatest integer function

Key. 1

Sol. e = $2\sqrt{2}/3$

7. Find the distance of the point $\hat{i}+2\hat{j}+3k$ from the plane $\vec{r}.\left(\hat{i}+\hat{j}+k\right)=5$ measured parallel to the vector $2\hat{i}+3\hat{j}-6k$.

Key. 7

Sol. The distance of the point 'a' from the plane $\vec{r}.\vec{n}=q$ measured in the direction of the unit vector b is $=\frac{q-\vec{a}.\vec{n}}{b.\vec{n}}$

Here $\vec{a}=\hat{i}+2\hat{j}+3k, \vec{n}=\hat{i}+\hat{j}+k$ and q = 5

Also $b=\frac{2\hat{i}+3\hat{j}-6k}{\sqrt{(2)^2+(3)^2+(-6)^2}}=\frac{2\hat{i}+3\hat{j}-6k}{7}$

$\therefore$ The required distance

$$=\frac{5-\left(\hat{i}+2\hat{j}+3k\right).\left(\hat{i}+\hat{j}+k\right)}{\frac{1}{7}\left(2\hat{i}+3\hat{j}-6k\right).\left(\hat{i}+\hat{j}+k\right)}$$

$$\frac{5-(1+2+3)}{\frac{1}{7}(2+3-6)}=7$$

8. If $\vec{a},\vec{b},\vec{c}$ be non-coplanar unit vectors equally inclined to one another at an acute angle θ, and if $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$ then $p-r=$__ $(p,q,r \in R)$

ans: 0.

Sol. taking dot product with $\vec{a}=\left[\bar{a}\bar{b}\bar{c}\right]=p+q\cos\theta+r\cos\theta---(1)$

taking dot product with $\vec{c}=\left[\bar{a}\bar{b}\bar{c}\right]=p\cos\theta+q\cos\theta+r---(2)$

From (1) and (2) $p=r$.

9. Let A be a point on the line $\bar{r}=\left(-3\hat{i}+6j+3k\right)+t\left(2\hat{i}+3j-2k\right)$ and B be a point on the line $\bar{r}=6j+s\left(2\hat{i}+2j-k\right)$. The least value of the distance AB is

ANS : 5

HINT $\quad Let\ A_o=(-3,6,3), B_o=(0,6,0); \bar{c}=(2,3,-2)\ \&\ \bar{d}=(2,2,-1)$

Then $AB_{min}=\left|proj\ of\ \overrightarrow{A_OB_o}\ on\ \vec{c}\times\vec{d}\right|=\frac{\left|(3,0,-3).(1,-2,-2)\right|}{3}=3$

10. If $\bar{a},\bar{b},\bar{c}$ are unit vectors such that $\bar{a}$ is perpendicular to plane of $\bar{b}$ and $\bar{c}$ and the angle between $\bar{b}\ \&\ \bar{c}$ is $\frac{\pi}{3}$ the $\left|\bar{a}+\bar{b}+\bar{c}\right|$ is

KEY : 2

SOL : $|\bar{a}| = |\bar{b}| = |\bar{c}| = 1 \& \bar{a}.\bar{b} = 0 \& \bar{a}.\bar{c} = 0$

$$\bar{b}.\bar{c} = |\bar{b}||\bar{c}|\cos\frac{\pi}{3} = \frac{1}{2}.$$

$$\therefore \left|\bar{a} + \bar{b} + \bar{c}\right|^2 = 3 + 2.0 + 2.0 + 1 = 4$$

$$\therefore \left|\bar{a} + \bar{b} + \bar{c}\right| = 2$$

11. Find the distance of the point $\hat{i} + 2\hat{j} + 3k$ from the plane $\vec{r}.\left(\hat{i} + \hat{j} + k\right) = 5$ measured parallel to the vector $2\hat{i} + 3\hat{j} - 6k$.

Key. 7

Sol. The distance of the point 'a' from the plane $\vec{r}.\vec{n} = q$ measured in the direction of the unit vector b is $= \frac{q - \vec{a}.\vec{n}}{b.\vec{n}}$

Here $\vec{a} = \hat{i} + 2\hat{j} + 3k, \vec{n} = \hat{i} + \hat{j} + k$ and q = 5

Also $b = \frac{2\hat{i} + 3\hat{j} - 6k}{\sqrt{(2)^2 + (3)^2 + (-6)^2}} = \frac{2\hat{i} + 3\hat{j} - 6k}{7}$

$\therefore$ The required distance

$$= \frac{5 - \left(\hat{i} + 2\hat{j} + 3k\right).\left(\hat{i} + \hat{j} + k\right)}{\frac{1}{7}\left(2\hat{i} + 3\hat{j} - 6k\right).\left(\hat{i} + \hat{j} + k\right)}$$

$$\frac{5 - (1 + 2 + 3)}{\frac{1}{7}(2 + 3 - 6)} = 7$$

12. The projection length of a variable vector $x\hat{i} + y\hat{j} + z\hat{k}$ on the vector $\vec{p} = \hat{i} + 2\hat{j} + 3\hat{k}$ is 6. Let ℓ be the minimum projection length of the vector $x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ on the vector $\vec{p}$, then the value of $\sqrt[3]{l^2 + 15^2}$ is

Key. 9

Sol. Projection length = $|\vec{a}.\vec{p}|$

So, $\frac{|x + 2y + 3z|}{\sqrt{14}} = 6$

$\Rightarrow |x + 2y + 3z| = 6\sqrt{14}$

$\Rightarrow |(x\hat{i} + \sqrt{2}y\hat{j} + \sqrt{3}z\hat{k}).(\hat{i} + \sqrt{2}\hat{j} + \sqrt{3}\hat{k})| = 6\sqrt{14}$

$\Rightarrow (x^2 + 2y^2 + 3z^2)(1 + 2 + 3)\cos^2\theta = \left(6\sqrt{14}\right)^2$

$\Rightarrow \frac{x^2 + 2y^2 + 3z^2}{\sqrt{14}} \geq 6\sqrt{14} \Rightarrow l = 6\sqrt{14}$

So, $(l^2 + 15^2)^{1/3} = (504 + 225)^{1/3} = (729)^{1/3} = 9.$

13. Non-zero vectors $\vec{a}, \vec{b}, \vec{c}$ satisfy $\vec{a}.\vec{b} = 0$, $(\vec{b}-\vec{a}).(\vec{b}+\vec{c}) = 0$ and $2|\vec{b}+\vec{c}| = |\vec{b}-\vec{a}|$. If $\vec{a} = \mu\vec{b}+4\vec{c}$ then the value of μ is

Key. 0

Sol. $\vec{c} = \dfrac{\vec{a}-\mu\vec{b}}{4}$ and $\vec{a}.\vec{b} = 0$

Now, $(\vec{b}-\vec{a}).(\vec{b}+\vec{c}) = 0 \Rightarrow (\vec{b}-\vec{a}).\left(\vec{b}+\dfrac{\vec{a}-\mu\vec{b}}{4}\right) = 0$

$\Rightarrow (4-\mu)\, b^2 = a^2$ ($\therefore$ $\mu < 4$) … (i)

Again $4|\vec{b}+\vec{c}|^2 = |\vec{b}-\vec{a}|^2 \Rightarrow 4\left|\dfrac{(4-\mu)\vec{b}+\vec{a}}{4}\right|^2 = |\vec{b}-\vec{a}|^2$

$\Rightarrow 4\left(\dfrac{4-\mu}{4}\right)^2 b^2 + \dfrac{a^2}{4} = b^2 + a^2 \Rightarrow ((4-\mu)^2 - 4)b^2 = 3a^2$ … (ii)

(i) & (ii) we get $\dfrac{(4-\mu)^2-4}{4-\mu} = \dfrac{3}{1} \Rightarrow \mu^2 - 5\mu = 0$

$\Rightarrow \mu = 0$ or 5 but as $\mu < 4$, so, $\mu = 0$.

14. Angle θ is made by line of intersection of planes $\vec{r}\cdot(\hat{i}+2j+3k) = 0$ and $\vec{r}\cdot(3\hat{i}+3j+k) = 0$ with j, where $\cos\theta = \sqrt{\dfrac{\lambda}{3}}$, then λ is

Ans. 2

Sol. Conceptual