Trigonometry

Single Correct Answer Type

1. $\sec h^{-1}(\sin\theta) =$

1) $\log\tan\frac{\theta}{2}$ 2) $\log\sin\frac{\theta}{2}$ 3) $\log\cos\frac{\theta}{2}$ 4) $\log\cot\frac{\theta}{2}$

Key. 4

Sol. $\log_e\left[\frac{1+\sqrt{\cos^2\theta}}{\sin\theta}\right]$

$= \log_e \cot\theta/2$

2. The value of the expression $\sec h^2\left(\operatorname{Tan}h^{-1}(1/2)\right) + \operatorname{cos}ech^2\left(\cot h^{-1}3\right)$ is

A) $\frac{35}{9}$ B) $\frac{43}{4}$ C) $\frac{35}{4}$ D) $\frac{43}{9}$

Key. 3

Sol. Conceptual

3. If $x = \log\left[\cot\left(\frac{\pi}{4}+\theta\right)\right]$ *then* $\sinh x =$

1) $\tan 2\theta$ 2) $\cot 2\theta$ 3) $-\tan 2\theta$ 4) $-\cot 2\theta$

Key. 3

Sol. $x = \log\left[\cot(\pi/4+\theta)\right]$

$= \log\left[\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right] \Rightarrow e^x = \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$

$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{1}{2}\left[\frac{(\cos\theta-\sin\theta)^2 - (\cos\theta+\sin\theta)^2}{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}\right]$

$= \frac{1}{2}\left[\frac{-4\cos\theta\sin\theta}{\cos^2\theta-\sin^2\theta}\right] = \frac{-\sin 2\theta}{\cos 2\theta} = -\tan 2\theta$

4. If $Sinh^{-1}\, 2x = 2Cosh^{-1}y, then$

1) $x^2+y^2=x^4$ 2) $x^2+y^2=4$

3) $x^2+y^2=y^4$ 4) $x^2=y^2$

Key. 3

Sol. $\sinh^{-1}2x = 2\cosh^{-1}y$

$2x = \sinh\left(2\cosh^{-1}y\right) = 2\sinh\left(\cosh^{-1}y\right)\cosh\left(\cosh^{-1}y\right)$

$= 2\sinh\left(\sinh^{-1}\left(\sqrt{y^2-1}\times y\right)\right)$

$2x = 2y\sqrt{y^2-1}$

$\Rightarrow x^2 + y^2 = y^4$

5. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60^0. He moves away from the pole along the line BC to a point D such that CD =7 m. From D the angle of elevation of the point A is 45^0. Then the height of the pole is

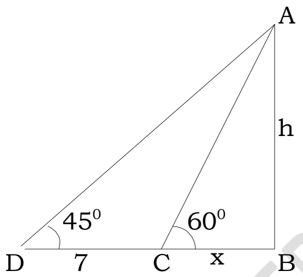
1) $\frac{7\sqrt{3}}{2}\left(\sqrt{3}+1\right)m$ 2) $\frac{7\sqrt{3}}{2}\left(\sqrt{3}-1\right)m$ 3) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}+1}m$ 4) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}-1}m$

Key. 1

Sol. $x = h\cot 60^0 = h/\sqrt{3}$

$x + 7 = h\cot 45^0 \Rightarrow h = h - h/\sqrt{3} = 7$

$\Rightarrow h = \frac{7\sqrt{3}}{\sqrt{3}-1}$



6. The angle of elevation of an object from a point P on the level ground is α. Moving d metres on the ground towards the object, the angle of elevation is found to be β. Then the height (in metres) of the object is

1) d tan α 2) d cot β

3) $\frac{d}{\cot\alpha + \cot\beta}$ 4) $\frac{d}{\cot\alpha - \cot\beta}$

Key. 4

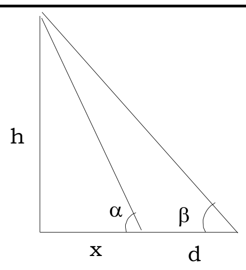
Sol. $\text{Tan}\alpha = \frac{h}{x+d}$

$\Rightarrow x + d = h\cot\alpha$

$\text{Tan}\beta = \frac{h}{x} \Rightarrow x = h\cot\beta$

$x + d - x = h[\cot\alpha - \cot\beta]$

$h = \frac{d}{\cot\alpha - \cot\beta}$



7. The angle of elevation of a cloud from a point h mt above the surface of a lake is θ and the angle of depression of its reflection in the lake is φ. The height of the cloud is

1) $\frac{h\sin(\varphi+\theta)}{\sin(\varphi-\theta)}$ 2) $\frac{h\sin(\varphi-\theta)}{\sin(\varphi+\theta)}$ 3) $\frac{h\sin(\theta+\varphi)}{\sin(\theta-\varphi)}$ 4) $\frac{h\sin(\theta-\varphi)}{\sin(\theta+\varphi)}$

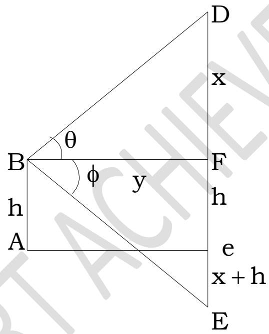
Key. 1

Sol. $\text{Tan}\theta = \frac{x}{y}$

$\text{Tan}\phi = \frac{2h+x}{y}$

$\Rightarrow x = \frac{2h}{\cot\theta.\tan\phi - 1}$

$CD = h + x = \frac{h\sin(\phi+\theta)}{\sin(\phi-\theta)}$



8. If $\tan\left(\frac{\pi}{4}+\frac{y}{2}\right) = \tan^3\left(\frac{\pi}{4}+\frac{x}{2}\right)$, then $\sin x\left(\frac{3+\sin^2 x}{1+3\sin^2 x}\right)$ equals

(A) cos y (B) sin y
(C) sin 2y (D) 0

Key. B

Sol. $$\frac{1+\tan\frac{y}{2}}{1-\tan\frac{y}{2}} = \left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)^3$$

Square both sides, we get

$$\frac{1+\sin y}{1-\sin y}=\frac{(1+\sin x)^3}{(1-\sin x)^3}$$

Using componendo and dividendo

$$\frac{2\sin y}{2}=\frac{(3+\sin^2 x)}{1+3\sin^2 x}\sin x$$

9. If $x=\sin\frac{2\pi}{7}+\sin\frac{4\pi}{7}+\sin\frac{8\pi}{7}$ and $y=\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{8\pi}{7}$ then $x^2+y^2=$

A. 1 B. 2 C. 3 D. 4

KEY. B

SOL. $x^2+y^2=3+2\left(\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}\right)=2$

10. If $0<A<B<\pi,\sin A-\sin B=\frac{1}{\sqrt{2}},\cos A-\cos B=\sqrt{\frac{3}{2}}$ then A+B=

A. $\frac{2\pi}{3}$ B. $\frac{5\pi}{6}$ C. π D. $\frac{4\pi}{3}$

KEY. D

SOL. $(\sin A-\sin B)^2+(\cos A-\cos B)^2=2\Rightarrow B=A+\frac{\pi}{2}\, and\; A=\frac{5\pi}{12}$

11. $\cot\frac{7\pi}{6}+2\cot\frac{3\pi}{8}+\cot\frac{15\pi}{16}=$

A. -4 B. 4 C. 1 D. 0

KEY. A

SOL. $\tan\frac{\pi}{16}-\cot\frac{\pi}{16}+2\cot\left(\frac{3\pi}{8}\right)=-2\cot\frac{\pi}{8}+2\tan\frac{\pi}{6}=-4$

12. Tan $\frac{4\pi}{5}-Tan\frac{2\pi}{15}+\sqrt{3}Tan\frac{4\pi}{5}Tan\frac{2\pi}{15}=$

A. $\sqrt{3}$ B. $\frac{1}{\sqrt{3}}$ C. $-\sqrt{3}$ D. $-\frac{1}{\sqrt{3}}$

KEY. C

SOL. $TanA-TanB-TanATanBTan(A-B)=Tan(A-B)$

13. If $x_1, x_2, x_3, ...x_n$ are in A.P. Whose common difference is α, then the value of $\sin\alpha[\sec x_1 \sec x_2 + \sec x_2 \sec x_3 + ... + \sec x_{n-1} \sec x_n] =$

A. $\frac{\sin n\alpha}{\cos x_n \cos x_1}$ B. $\frac{\sin(n-1)\alpha}{\cos x_n \cos x_1}$ C. $\frac{\sin(n+1)\alpha}{\cos x_n \cos x_1}$ D. $\frac{\cos(n-1)\alpha}{\cos x_n \cos x_1}$

KEY. B

SOL. $= \frac{\sin(x_2 - x_1)}{\cos x_1 \cos x_2} + \frac{\sin(x_3 - x_2)}{\cos x_2 \cos x_3} + ... + \frac{\sin(x_n - x_{n-1})}{\cos x_{n-1} \cos x_n}$

$= \tan x_2 - tax_1 + \tan x_3 - \tan x_2 + ... + \tan x_n - \tan x_{n-1}$

$= \tan x_n - \tan x_1 = \frac{\sin(x_n - x_1)}{\cos x_n \cos x_1} = \frac{\sin(n-1)\alpha}{\cos x_n \cos x_1}$

14. If $a\sin^2 x + b\cos^2 x = c, b\sin^2 y + a\cos^2 y = d$ and atanx = btany then $\frac{a^2}{b^2} =$

A. $\frac{(a-d)(c-a)}{(b-c)(d-b)}$ B. $\frac{(a+d)(c+a)}{(b+c)(d+b)}$ C. $\frac{(a-d)(b-a)}{(a-c)(c-b)}$ D. $\frac{(d-a)(c-a)}{(b-c)(d-b)}$

KEY. A

SOL. $a\tan^2 x + b = c(1+\tan^2 x)$

$$\Rightarrow \tan^2 x = \left(\frac{c-b}{a-c}\right), \tan^2 y = \left(\frac{d-a}{b-d}\right)$$

$$\frac{a^2}{b^2} = \frac{\tan^2 y}{\tan^2 x} = \frac{(a-d)(c-a)}{(b-c)(d-b)}$$

15. If $\cos^3 x \sin 2x = \sum_{r=0}^{n} a_r \sin(rx), \forall x \in R$ then

A. $n=5, a_1 = \frac{1}{2}$ B. $n=5, a_1 = \frac{1}{4}$ C. $n=5, a_2 = \frac{1}{8}$ D. $n=5, a_2 = \frac{1}{4}$

KEY. B

SOL. $\cos^3 x \sin 2x = \cos^2 x . \cos x \sin 2x$

$$= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{2\sin 2x \cos x}{2}\right) = \frac{1}{4}(1-\cos 2x)(\sin 3x + \sin x)$$

$$= \frac{1}{4}[\sin 3x + \sin x - \frac{1}{2}(2\sin 3x \cos 2x) - \frac{1}{2}(2\cos 2x \sin x)]$$

$$= \frac{1}{4}[\sin 3x + \sin x - \frac{1}{2}(\sin 5x + \sin x) - \frac{1}{2}(\sin 3x - \sin x)] = \frac{1}{4}[\sin x + \frac{1}{2}\sin 3x - \frac{1}{2}\sin 5x]$$

$a_1 = \frac{1}{4}; a_3 = \frac{1}{8}; n = 5$

16. If , $\cos\theta = \dfrac{a\cos\phi + b}{a + b\cos\phi}$ then $\tan\theta/2$ is equal to

A. $\sqrt{\left(\dfrac{a-b}{a+b}\right)}\tan(\phi/2)$ B. $\sqrt{\left(\dfrac{a+b}{a-b}\right)}\cos(\phi/2)$ C. $\sqrt{\left(\dfrac{a-b}{a+b}\right)}\sin(\phi/2)$ D. none of these

Key. A

Sol. $\tan\theta/2 = \sqrt{\left(\dfrac{1-\cos\theta}{1+\cos\theta}\right)}$

$$= \sqrt{\frac{1-\left(\dfrac{a\cos\phi + b}{a + b\cos\phi}\right)}{1+\left(\dfrac{a\cos\phi + b}{a + b\cos\phi}\right)}}$$

$$= \sqrt{\frac{(a-b)(1-\cos\phi)}{(a+b)(1+\cos\phi)}}$$

$$= \sqrt{\frac{(a-b)}{(a+b)}}\tan(\phi/2)$$

17. If in a triangle ABC, $\cos 3A + \cos 3B + \cos 3C = 1,$ then one angle must be exactly equal to

A. $\dfrac{\pi}{3}$ B. $\dfrac{2\pi}{3}$ C. π D. $\dfrac{\pi}{6}$

Key. B

Sol. $\therefore \cos 3A + \cos 3B + \cos 3C = 1$

$$\Rightarrow \cos 3A + \cos 3B + \cos 3C - 1 = 0$$

$$\Rightarrow \cos 3A + \cos 3B + \cos 3C + \cos 3\pi = 0$$

$$\Rightarrow 2\cos\left(\frac{3A+3B}{2}\right)\cos\left(\frac{3A-3B}{2}\right) + 2\cos\left(\frac{3\pi+3C}{2}\right)\cos\left(\frac{3\pi-3C}{2}\right) = 0$$

$$\Rightarrow 2\cos\left(\frac{3\pi-3C}{2}\right)\left\{\cos\left(\frac{3A-3B}{2}\right) + \cos\left(\frac{3\pi+3C}{2}\right)\right\} = 0$$

$$\Rightarrow 2\cos\left(\frac{3\pi}{2} - \frac{3C}{2}\right)2\cos\left(\frac{3\pi+3C+3A-3B}{4}\right).\cos\left(\frac{3\pi+3C-3A+3B}{4}\right) = 0$$

$$\Rightarrow 2\cos\left(\frac{3\pi}{2}-\frac{3C}{2}\right)2\cos\left(\frac{3\pi}{2}-\frac{3B}{2}\right).\cos\left(\frac{3\pi}{2}-\frac{3A}{2}\right)=0$$

$$\Rightarrow \quad -4\sin\left(\frac{3A}{2}\right)\sin\left(\frac{3B}{2}\right)\sin\left(\frac{3C}{2}\right)=0$$

$$\Rightarrow \quad \sin\left(\frac{3A}{2}\right)\sin\left(\frac{3B}{2}\right)\sin\left(\frac{3C}{2}\right)=0$$

$$\therefore \frac{3A}{2}=\pi \text{ or } \frac{3B}{2}=\pi \text{ or } \frac{3C}{2}=\pi$$

$$\therefore A=\frac{2\pi}{3} \text{ or } B=\frac{2\pi}{3} \text{ or } C=\frac{2\pi}{3}$$

18. The value of $\sum_{r=0}^{10}\cos^3\frac{\pi r}{3}$ is equal to

(A) $\frac{-9}{2}$ (B) $\frac{-7}{2}$

(C) $\frac{-9}{8}$ (D) $\frac{-1}{8}$

Key. D

Sol. $I=\sum_{r=0}^{10}\frac{1}{4}\left(\cos 3\frac{\pi r}{3}+3\cos\frac{\pi r}{3}\right)$

$=\sum_{r=0}^{10}\frac{1}{4}\left(\cos\pi r+3\cos\frac{\pi r}{3}\right)$

$=\frac{1}{4}(I_1+I_2)$

$\therefore \quad I_1=\sum_{r=0}^{10}\cos\pi r=1-1+1-1+.....-1+1=1$

$$I_2=3\sum_{r=0}^{10}\cos\frac{\pi r}{3}=\frac{3\cos\left(\frac{10}{2}\frac{\pi}{3}\right)\sin\frac{11\pi}{3}}{\sin\frac{\pi}{6}}=-\frac{1\times 3}{2}=-\frac{3}{2}$$

$$\Rightarrow \quad I=\frac{1}{4}\left(1-\frac{3}{2}\right)=-\frac{1}{8}$$

19. The number of distinct real roots of the equation $\tan x=mx, m>1$ in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ is

A) 1 B) 2 C) 3 D) 0

Key. C

Sol. Conceptual

20. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points in the XY-Plane whose co-ordinates satisfy the equation $\cot^2(x+y)+\tan^2(x+y)+y^2+2y-1=0$. The minimum distance between P and Q is

A) $\pi/4$ B) $\pi/2$ C) $3\pi/4$ D) π

Key. B

Sol. $[\cot(x+y)-\tan(x+y)]^2+(y+1)^2=0$

$\therefore \tan^2(x+y)=1$ and $y=-1$

21. If α is the angle which each side of a regular polygon of n sides subtends at its centre then $1+\cos\alpha+\cos 2\alpha+\cos 3\alpha+....+\cos(n-1)\alpha$ is equal to

(a) n (b) 0 (c) 1 (d) n – 1

Key. B

Sol. $\cos\alpha+\cos(\alpha+\beta)+.....+\cos(\alpha+(n-1)\beta)=\dfrac{\sin\frac{nB}{2}}{\sin\frac{B}{2}}\sin\left(\alpha+\dfrac{(n-1)\beta}{2}\right)$

22. If $\lfloor C=90^0$ in ΔABC, then $\tan^{-1}\left(\dfrac{a}{b+c}\right)+\tan^{-1}\left(\dfrac{b}{c+a}\right)$ is equal to

a) $\dfrac{\pi}{2}$ b) $\dfrac{\pi}{4}$ c) $\dfrac{\pi}{3}$ d) π

Ans. b

Sol. $\tan^{-1}\left(\dfrac{\frac{a}{b+c}+\frac{b}{c+a}}{1-\frac{a}{b+c}\cdot\frac{b}{c+a}}\right)$ as $\dfrac{ab}{(b+c)(c+a)}<1$

But in right angled ΔABC

$c^2 = a^2 + b^2$

$\therefore \tan^{-1}(1)=\dfrac{\pi}{4}$

23. In a ΔABC, $\dfrac{a^2+b^2+c^2}{\Delta}$ is always

a) $\geq 6\sqrt{3}$ b) $\geq 4\sqrt{3}$ c) $\geq 8\sqrt{3}$ d) $\geq 12\sqrt{3}$

Ans. b

Sol. $\dfrac{a^2+b^2+c^2}{\Delta}\geq 4\sqrt{3}$: use the fact that $\Delta \leq \dfrac{(a+b+c)^2}{12\sqrt{3}}$

24. In triangle ABC, the value of the expression $\sum_{r=0}^{n} {}^{n}C_r\, a^r b^{n-r} \cos\left(rB-(n-r)A\right)$ is equal to

a) C^n b) Zero c) a^n d) b^n

Ans. a

Sol. It is the expansion of $\left(a\cos B+b\cos A\right)^n = C^n$

25. Total number of solution of $2^{\cos x} = |\sin x|$ in $[-2\pi, 5\pi]$ is equal to

a) 12 b) 14 c) 16 d) 15

Ans. b

Sol. Drawn the graphs of both. Total intersection points are 14.

26. If $2\sec 2\alpha = \tan\beta + \cot\beta$, then one positive value of $\alpha+\beta$ is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) 0

Ans. b

Sol. $2\sec 2\alpha = \left(\frac{1}{\sin\beta\cos\beta}\right)$

$\Rightarrow 2\alpha = \frac{\pi}{2} - 2\beta \Rightarrow \alpha+\beta = \frac{\pi}{4}$

27. If in a triangle $\frac{s-a}{11} = \frac{s-b}{12} = \frac{s-c}{13}$ and $\lambda \tan^2(A/2) = 455$, then λ must be

a) 1155 b) 1551 c) 5511 d) 1515

Ans. a

Sol. $\frac{s-a}{11} = \frac{s-b}{12} = \frac{s-c}{13} = \frac{s}{36}$ calculate $\tan^2(A/2) = \frac{13}{33}$

$\lambda = 1155$

28. The value of $\sin^3 10^0 + \sin^3 50^0 - \sin^3 70^0$ is equal to

a) $-\frac{3}{2}$ b) $\frac{3}{4}$ c) $-\frac{3}{4}$ d) $-\frac{3}{8}$

Ans. d

Sol. We have $\sin^3 10^0 + \sin^3 50^0 - \sin^3 70^0$

$= \frac{1}{4}\left[\left(3\sin 10^0 - \sin 30^0\right) + \left(3\sin 50^0 - \sin 150^0\right) - \left(3\sin 70^0 - \sin 120^0\right)\right]$

$= \frac{1}{4}\left[3\left(\sin 10^0 + \sin 50^0 - \sin 70^0\right) - \frac{3}{2}\right]$

$= \frac{1}{4}\left[3\left(\sin 10^0 - 2\cos 60^0 \cdot \sin 10^0\right) - \frac{3}{2}\right] = -\frac{3}{8}$

29. If $\tan(\alpha-\beta)=\dfrac{\sin 2\beta}{3-\cos 2\beta}$, then

a) $\tan\alpha = 2\tan\beta$ b) $\tan\beta = 2\tan\alpha$ c) $2\tan\alpha = 3\tan\beta$ d) $3\tan\alpha = 2\tan\beta$

Ans. a

Sol. We have $\dfrac{\sin 2\beta}{3-\cos 2\beta}=\dfrac{2\sin\beta\cdot\cos\beta}{2-2\cos 2\beta+1+\cos 2\beta}$

$=\dfrac{2\sin\beta\cdot\cos\beta}{4\sin^2\beta+2\cos^2\beta}=\dfrac{\tan\beta}{1+2\tan^2\beta}=\dfrac{2\tan\beta-\tan\beta}{1+2\tan^2\beta}$

$=\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\cdot\tan\beta}=\dfrac{2\tan\beta-\tan\beta}{1+2\tan^2\beta}$

$\therefore \tan\alpha = 2\tan\beta$

30. In a triangle ABC, if angle C is obtuse and angles A and B are given by roots of the equation $\tan^2 x + p\tan x + q = 0$, then the value of q is

a) greater than 1 b) less than 1 c) equal to 1 d) 0

Ans. b

Sol. We have A + B = π – C

$=\tan(A+B)=-\tan C$

$=\dfrac{\tan A+\tan B}{1-\tan A\cdot\tan B}>0$ $[\because \tan A>0, \tan B>0, \tan C<0]$

= $\tan A\cdot\tan B<1 \Rightarrow q<1$

31. If 2sinx – cos2x = 1, then $\cos^2 x + \cos^4 x$ is equal to

a) 1 b) – 1 c) $-\sqrt{5}$ d) $\sqrt{5}$

Ans. a

Sol. Given $2\sin x + 2\sin^2 x - 1 = 1$

Or, $\sin^2 x + \sin x - 1 = 0$

$\therefore \sin x=\dfrac{-1\pm\sqrt{1+4}}{2}=\dfrac{-1+\sqrt{5}}{2}$

$\Rightarrow \sin^2 x=\dfrac{3-\sqrt{5}}{2}\Rightarrow \cos^2 x=\dfrac{\sqrt{5}-1}{2}$

$\therefore \cos^2 x\left(1+\cos^2 x\right)=\dfrac{\sqrt{5}-1}{2}\times\dfrac{\sqrt{5}+1}{2}=1$

32. If ABCD is a cyclic quadrilateral such that 13cosA + 12 = 0 and 3tanB – 4 = 0, then the quadratic equation whose roots are tanC and cosD is

a) $15x^2 + 60x - 11 = 0$ b) $60x^2 + 11x - 15 = 0$

c) $11x^2 + 60x - 15 = 0$ d) none of these

Ans. b

Sol. In a cyclic quadrilateral, no angle is greater than 180^0

Here $\cos A=-\dfrac{12}{13}\Rightarrow\dfrac{\pi}{2}<A<\pi$ and 0 < C < π/2 (since A + C = 180^0)

$\therefore \tan A=-\dfrac{5}{12}\Rightarrow \tan C=\dfrac{5}{12}$

Also $\tan B = \frac{4}{3} \Rightarrow 0 < B < \frac{\pi}{2}$ and $\frac{\pi}{2} < D < \pi$ (since B + D = 180^0)

$\therefore \cos B = \frac{3}{5} \Rightarrow \cos D = -\frac{3}{5}$

Now, the required equation is

$$x^2 - \left(\frac{5}{12} - \frac{3}{5}\right)x + \left(\frac{5}{12}\right)\left(-\frac{3}{5}\right) = 0$$

$$\Rightarrow 60x^2 + 11x - 15 = 0$$

33. If A, B, C are the angles of a triangle such that $\cot\frac{A}{2} = 3\tan\frac{C}{2}$, then sinA, sinB, sinC are in

a) A.P b) G.P c) H.P d) none of these

Ans. a

Sol. Given $\cot\frac{A}{2}\cdot\cot\frac{C}{2} = 3$

$$\Rightarrow \frac{\cos\frac{A}{2}\cdot\cos\frac{C}{2}}{\sin\frac{A}{2}\cdot\sin\frac{C}{2}} = 3 \Rightarrow \frac{\cos\frac{A-C}{2}}{\cos\frac{A+C}{2}} = 2$$ (using componendo and dividendo)

$$\Rightarrow \frac{2\sin\frac{A+C}{2}\cos\frac{A-C}{2}}{2\sin\frac{A+C}{2}\cdot\cos\frac{A+C}{2}} = 2$$

= 2sinB = sinA + sinC

34. If $\frac{2\tan\alpha}{1+\sec\alpha+\tan\alpha} = \lambda$, then $\frac{2\tan\alpha/2}{1+\tan\alpha/2}$ is equal to

a) $\frac{1}{\lambda}$ b) λ c) $1-\lambda$ d) $1+\lambda$

Ans. b

Sol. We have $\frac{2\tan\alpha}{1+\sec\alpha+\tan\alpha} = \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}$

$$= 2\frac{2\tan\alpha/2}{\left(1+\tan^2\alpha/2\right)+\left(1-\tan^2\alpha/2\right)+2\tan\alpha/2} = \frac{2\tan\alpha/2}{1+\tan\alpha/2}$$

35. In ΔABC, if $b^2 + c^2 = 2a^2$, then the value of $\frac{\cot A}{\cot B + \cot C}$ is

a) 1/2 b) 3/2 c) 5/2 d) 5/3

Ans. a

Sol. $\dfrac{\cot A}{\cot B+\cot C}=\dfrac{\dfrac{R\left(b^2+c^2-a^2\right)}{abc}}{\dfrac{R\left(a^2+c^2-b^2\right)}{abc}+\dfrac{R\left(a^2+b^2-c^2\right)}{abc}}=1/2$

36. If $0\le A,B,C\le\pi$ and A + B + C = π, then the minimum value of sin 3A + sin 3B + sin 3C is

a) -2 b) $-\dfrac{3\sqrt{3}}{2}$ c) 0 d) none of these

Ans. a

Sol. Since A + B + C = π

$\Rightarrow$ all of sin3A, sin3B, sin3C can't be negative

Let us take sin3A = - 1 $\Rightarrow$ A = $\pi/2$

$\Rightarrow$ sin3A = - 1, sin3B = - 1 and sin 3C = 0

So minimum value is – 2 1. Let $\theta\in\left(0,\pi/4\right)$ and $t_1=\left(\tan\theta\right)^{\tan\theta}$, $t_2=\left(\tan\theta\right)^{\cot\theta}$, $t_3=\left(\cot\theta\right)^{\tan\theta}$, $t_4=\left(\cot\theta\right)^{\cot\theta}$ then

a) $t_1>t_2>t_3>t_4$ b) $t_4>t_3>t_1>t_2$

c) $t_3>t_1>t_2>t_4$ d) $t_2>t_3>t_1>t_4$

Key. B

Sol. $\theta\in\left(0,\dfrac{\pi}{4}\right)$

therefore, $\tan\theta<\cot\theta$

since $\tan\theta<1\,\&\cot\theta>1$

therefore, $\left(\tan\theta\right)^{\cot\theta}<1$ and $\left(\cot\theta\right)^{\tan\theta}>1$

therefore, $t_4>t_1$

37. If $\theta=\dfrac{2\pi}{7}$ then the value of $\tan\theta\tan2\theta+\tan2\theta\tan4\theta+\tan4\theta\tan\theta$ is

a) –1 b) 0 c) $\dfrac{1}{8}$ d) - 7

Key. D

Sol. $7\theta=2\pi$

$\theta+2\theta+4\theta=2\pi$

$\cos\left(\theta+2\theta+4\theta\right)=1$

Expanding and dividing with $\cos\theta\cos2\theta\cos4\theta$ we have

$$\tan\theta\tan2\theta+\tan2\theta\tan4\theta+\tan4\theta\tan\theta=1-\frac{1}{\cos\theta\cos2\theta\cos4\theta}=1-\frac{1}{\left(\frac{1}{8}\right)}=-7$$

$$\left(\because\cos\theta\cos2\theta\cos4\theta=\frac{\sin8\theta}{8\sin\theta}=\frac{1}{8}\right)$$

38. If $k_1 = \tan 27\theta - \tan\theta$ and $k_2 = \frac{\sin\theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin 9\theta}{\cos 27\theta}$ then

a) $k_1 = 2k_2$ b) $k_1 = k_2$ c) $k_1 = -k_2$ d) $2k_1 = k_2$

Key. A

Sol. $\tan 3\theta - \tan\theta = \frac{\sin 2\theta}{\cos 3\theta \cos\theta} = \frac{2\sin\theta}{\cos 3\theta}$ (1)

$\tan 9\theta - \tan 3\theta = \frac{2\sin 3\theta}{\cos 9\theta}$ (2)

$\tan 27\theta - \tan 9\theta = \frac{2\sin 9\theta}{\cos 27\theta}$ (3)

Adding (1), (2), (3) $k_1 = 2k_2$

39. If $\frac{\cos x}{a} = \frac{\cos(x+\theta)}{b} = \frac{\cos(x+2\theta)}{c} = \frac{\cos(x+3\theta)}{d}$ then $\frac{a+c}{b+d}$ is equal to

a) $\frac{a}{d}$ b) $\frac{c}{b}$

c) $\frac{b}{c}$ d) $\frac{d}{a}$

Key. C

Sol. For each of the ratio be k

$\frac{a+c}{b+d} = \frac{k\cos x + k\cos(x+2\theta)}{k\cos(x+\theta) + k\cos(x+3\theta)} = \frac{2\cos(x+\theta)\cos\theta}{2\cos(x+2\theta)\cos\theta}$

$= \frac{\cos(x+\theta)}{\cos(x+2\theta)} = \frac{k\cos(x+\theta)}{k\cos(x+2\theta)} = \frac{b}{c}$

$\Rightarrow$ (c) is correct.

40. If $\cos\alpha = \frac{2\cos\beta - 1}{2 - \cos\beta}(0 < \alpha, \beta < \pi)$, then $\frac{\tan\frac{\alpha}{2}}{\tan\frac{\beta}{2}}$ is equal to

a) 1 b) $\sqrt{2}$

c) $\sqrt{3}$ d) $\frac{1}{\sqrt{3}}$

Key. C

Sol. Take $\beta = 120^o$, then

$\cos\alpha = \frac{2\left(-\frac{1}{2}\right) - 1}{2 - \left(-\frac{1}{2}\right)} = \frac{2}{5/2} = \frac{4}{5}$

$\cos\alpha = -\frac{4}{5} \Rightarrow \tan\alpha = \frac{3}{4}$

If $\tan\alpha = -\frac{3}{4}$, then we get $\frac{\tan\alpha/2}{\tan\beta/2} = \sqrt{3}$ and $\tan\frac{\alpha}{2} = 3$

$$\Rightarrow \frac{3}{4} = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} \Rightarrow \tan\frac{\alpha}{2} = 3 \Rightarrow \frac{\tan\frac{\alpha}{2}}{\tan\frac{\beta}{2}} = \frac{\frac{1}{3}}{\frac{1}{\sqrt{2}}} = \sqrt{3}$$

41. Cot 16^0 Cot 44^0 + Cot44^0 Cot 76^0 – Cot 76^0 Cot 16^0 =

a) 1 b) 2 c) 3 d) 4

Key. C

Sol.

$$\frac{\cos 16 \cos 44}{\sin 16.\sin 44} - 1 + \frac{\cos 44.\cos 76}{\sin 44 \sin 76} - 1 - \frac{\cos 76 \cos 16}{\sin 76.\sin 16} - 1 + 3$$

$$= \frac{\cos 60}{\sin 16.\sin 44} + \frac{\cos 120}{\sin 44 \sin 76} - \frac{\cos 60}{\sin 76.\sin 16} + 3 = \frac{1}{2}\left(\frac{\sin 76 - \sin 16}{\sin 16.\sin 44.\sin 76}\right) - \frac{1}{2\sin 76.\sin 16} + 3 = 3.$$

37. The value of x which satisfies equation $2\tan^{-1} 2x = \sin^{-1}\frac{4x}{1+4x^2}$ is

a) $\left[\frac{1}{2}, \infty\right)$ b) $\left(-\infty, -\frac{1}{2}\right]$ c) $[-1, 1]$ d) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Ans. d

Sol. $-\frac{\pi}{2} \le 2\tan^{-1} 2x \le \frac{\pi}{2}$

$\Rightarrow -\frac{1}{2} \le x \le \frac{1}{2}$

38. Number of integral solutions of the equation $3\tan^{-1} x + \cos^{-1}\left(\frac{1-3x^2}{\left(1+x^2\right)^{3/2}}\right) = 0$ is

a) 1 b) 2 c) 0 d) infinite

Ans. b

Sol. Let $\tan^{-1} x = \theta, \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$3\theta + \cos^{-1}(\cos 3\theta) = 0$

$\cos^{-1}(\cos 3\theta) = -3\theta \Rightarrow -\pi \le 3\theta \le 0$

$\Rightarrow -\frac{\pi}{3} \le \theta \le 0$

$\Rightarrow x \in \left[-\sqrt{3}, 0\right]$, so number of integral solutions is 2.

39. In a triangle ABC, with $A = \frac{\pi}{7}, B = \frac{2\pi}{7}, C = \frac{4\pi}{7}$, then $a^2 + b^2 + c^2$ is (R = circumradius of ΔABC)

a) $4R^2$ b) $6R^2$ c) $7R^2$ d) $8R^2$

Ans. c

Sol. $a^2 + b^2 + c^2 = 4R^2\left(\sin^2 A + \sin^2 B + \sin^2 C\right)$

$$= 2R^2\left(1-\cos 2A+1-\cos 2B+1-\cos 2C\right) = 2R^2\left[3-\left(\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{8\pi}{7}\right)\right]$$

$$= 2R^2\left[3-\left(\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}\right)\right]$$

$$= 2R^2\left[3-\left(2\sin\frac{\pi}{7}\cdot\cos\frac{2\pi}{7}+2\sin\frac{\pi}{7}\cdot\cos\frac{4\pi}{7}+2\sin\frac{\pi}{7}\cdot\cos\frac{6\pi}{7}\right)\frac{1}{2\sin\frac{\pi}{7}}\right]$$

$$= 2R^2\left[3-\frac{1}{2\sin\frac{\pi}{7}}\left(\sin\frac{3\pi}{7}-\sin\frac{\pi}{7}+\sin\frac{5\pi}{7}-\sin\frac{3\pi}{7}+\sin\frac{7\pi}{7}-\sin\frac{5\pi}{7}\right)\right]$$

$$= 2R^2\left[3+\frac{1}{2}\right] = 7R^2$$

40. For which value of x, $\sin(\cot^{-1}(x + 1)) = \cos(\tan^{-1} x)$

a) $\frac{1}{2}$ b) 0 c) 1 d) $-\frac{1}{2}$

Ans. d

Sol. $\sin\left(\cot^{-1}\left(x+1\right)\right) = \sin\left(\sin^{-1}\left(\frac{1}{\sqrt{x^2+2x+2}}\right)\right)$

$$\Rightarrow \sin\left(\cot^{-1}\left(x+1\right)\right) = \frac{1}{\sqrt{x^2+2x+2}}$$

$$\cos\left(\tan^{-1}x\right) = \cos\left(\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right) = \frac{1}{\sqrt{1+x^2}}$$

$$\Rightarrow \frac{1}{x^2+2x+2} = \frac{1}{1+x^2}$$

41. If the equation $x^2 + 12 + 3\sin(a + bx) + 6x = 0$ has atleast one real solution where $a,b \in [0, 2\pi]$, then value of $\cos\theta$ where θ is least positive value of a + bx is

a) π b) 2π c) 0 d) $\frac{\pi}{2}$

Ans. c

Sol. $\left(x+3\right)^2+3+3\sin\left(a+bx\right)=0$

x = - 3, sin(a + bx) = - 1

$$\Rightarrow \sin\left(a-3b\right) = -1$$

$$a-3b = \left(4n-1\right)\frac{\pi}{2},\ n\in 1$$

n = 1

a – 3b = $3\pi/2$

cos(a – 3b) = 0

42. In any ΔABC, which is not right angled, $\sum \cos A \operatorname{cosec} B \operatorname{cosec} C$ is

a) constant b) less than 1 c) greater than 2 d) none of these

Ans. a

Sol. $$\sum \frac{\cos A}{\sin B \sin C} = \frac{-\sum \cos(B+C)}{\sin B \sin C} = \sum(1 - \cot B \cot C) = 3 - \sum \cot A \cot B = 2$$

43. Range of $f(x) = \sin^6 x + \cos^6 x$ is

(A) [0, 1] (B) $[0, \sqrt{2}]$

(C) $\left[\frac{1}{\sqrt{2}}, \frac{3}{4}\right]$ (D) $\left[\frac{1}{4}, 1\right]$

Key. D

Sol. $$f(x) = (\sin^2 x + \cos^2 x)^3 - 3\sin^2 x \cos^2 x(\sin^2 x + \cos^2 x) = 1 - \frac{3}{4}\sin^2 2x$$

Range of $\sin^2 2x$ is $[0,1]$

Range of $f(x)$ is $\left[\frac{1}{4}, 1\right]$.

Note: Certain questions are better done by avoiding derivatives. Derivatives is one of the tools to determine extrema.

44. The maximum value of $4\sin^2 x + 3\cos^2 x + \sin\frac{x}{2} + \cos\frac{x}{2}$ is

a) $4+\sqrt{2}$ b) $3+\sqrt{2}$ c) 9 d) 4

Key. A

Sol. Maximum value of $4\sin^2 x + 3\cos^2 x$ i.e., $\sin^2 x + 3$ is 4 and that of $\sin\frac{x}{2} + \cos\frac{x}{2}$ is $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$, both attained at $x = \pi/2$. Hence the given function has maximum value of $4+\sqrt{2}$

45. If $\sin\theta + \sin 2\theta + \sin 3\theta = \sin\alpha$ and $\cos\theta + \cos 2\theta + \cos 3\theta = \cos\alpha$, then θ is equal to

a) $\frac{\alpha}{2}$ b) α c) 2α d) $\frac{\alpha}{6}$

Key. A

Sol. $\sin\theta + \sin 3\theta + \sin 2\theta = \sin\alpha$

$\Rightarrow$ $2\sin 2\theta \cos\theta + \sin 2\theta = \sin\alpha$

$\Rightarrow$ $\sin 2\theta(2\cos\theta + 1) = \sin\alpha$ ....(1)

Now $\cos\theta + \cos 3\theta + \cos 2\theta = \cos\alpha$

$\cos 2\theta(2\cos\theta + 1) = \cos\alpha$ ....(2)

From (1) and (2),

$$\tan 2\theta = \tan\alpha \Rightarrow 2\theta = \alpha \Rightarrow \theta = \frac{\alpha}{2}$$

46. If $\pi < 2\theta < \frac{3\pi}{2}$, then $\sqrt{2+\sqrt{2+2\cos 4\theta}}$ equals to

a) $-2\cos\theta$ b) $-2\sin\theta$ c) $2\cos\theta$ d) $2\sin\theta$

Key. D

Sol. $\sqrt{2+2(1+\cos 4\theta)} = \sqrt{2+2|\cos 2\theta|}$

$= \sqrt{2(1-\cos 2\theta)}$

$= 2|\sin\theta| = 2\sin\theta$ as $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$

47. $\cos^2\alpha + \cos^2(\alpha+120^0) + \cos^2(\alpha-120^0)$ is equal to

a) $\frac{3}{2}$ b) 1 c) $\frac{1}{2}$ d) 0

Key. A

Sol. $\cos^2\alpha + \cos^2(\alpha+120^0) + \cos^2(\alpha-120^0)$

$= \cos^2\alpha + \{\cos(\alpha+120^0) + \cos(\alpha-120^0)\}^2 - 2\cos(\alpha+120^0)\cos(\alpha-120^0)$

$= \cos^2\alpha + \{2\cos\alpha\cos 120^0\}^2 - 2\{\cos^2\alpha - \sin^2 120^0\}$

$= \cos^2\alpha + \cos^2\alpha - 2\cos^2\alpha + 2\sin^2 120^0$

$= 2\sin^2 120^0 = 2\times\frac{3}{4} = \frac{3}{2}$

Trigonometry
Integer Answer Type

1. Let $f(x)=0$ be an equation of degree six, having integer coefficients and whose one root is $2\cos\frac{\pi}{18}$. Then, the sum of all the roots of $f^1(x)=0$, is

Key. 0

Sol. Let $\theta=\frac{\pi}{18}\Rightarrow 6\theta=\frac{\pi}{3}\Rightarrow \cos 6\theta=\frac{1}{2}$

$\Rightarrow 4\cos^3 2\theta-3\cos 2\theta=\frac{1}{2}\Rightarrow 8\left(2\cos^2\theta-1\right)^3-6\left(2\cos^2\theta-1\right)=1$ let $2\cos\theta=x$

$\Rightarrow 8\left(2.\frac{x^2}{4}-1\right)^3-6\left(2.\frac{x^2}{4}-1\right)=1$

$\Rightarrow \left(x^2-2\right)^3-3\left(x^2-2\right)=1$

$\Rightarrow x^6-6x^4+9x^2-3=0$

$f^1(x)=6x\left(x^4-4x^2+3\right)$

$f^1(x)=0\Rightarrow x=0,\pm1,\pm\sqrt{3}$

2. If $\cos\theta+\cos^2\theta+\cos^3\theta=1$ and $\sin^6\theta=a+b\sin^2\theta+c\sin^4\theta$ then a + b + c =

KEY. 0

SOL. $\cos\theta(1+\cos^2\theta)=\sin^2\theta$

$(1-\sin^2\theta)[2-\sin^2\theta]^2=\sin^4\theta$

$\sin^6\theta=4-8\sin^2\theta+4\sin^4\theta$

a = 4, b = -8, c =4

a + b + c = 0

3. If $\frac{\sin\theta}{\cos 3\theta}+\frac{\sin 3\theta}{\cos 9\theta}+\frac{\sin 9\theta}{\cos 27\theta}=\frac{1}{A}[\tan B\theta-\tan C\theta]$ then (27A – B – 27C) =

KEY. 0

SOL. $\frac{\sin\theta}{\cos 3\theta}=\frac{2\sin\theta\cos\theta}{2\cos 3\theta\cos\theta}=\frac{\sin 2\theta}{2\cos 3\theta\cos\theta}=\frac{\sin(3\theta-\theta)}{2\cos 3\theta\cos\theta}$

$\therefore \frac{\sin\theta}{\cos 3\theta}=\frac{1}{2}[\tan 3\theta-\tan\theta]\rightarrow(1)$

$\frac{\sin 3\theta}{\cos 9\theta}=\frac{1}{2}[\tan 9\theta-\tan 3\theta]\rightarrow(2)$

$$\frac{\sin 9\theta}{\cos 27\theta}=\frac{1}{2}[\tan 27\theta-\tan 9\theta]\to(3)$$

$$\therefore (1)+(2)+(3)\Rightarrow\frac{1}{2}[\tan 27\theta-\tan\theta]$$

A = 2, B = 27, C = 1

27A – B – 27 C = 0

4. If $\frac{\tan x}{2}=\frac{\tan y}{3}=\frac{\tan z}{5}$ and x + y + z = π, $\tan^2 x+\tan^2 y+\tan^2 z=\frac{38}{K}$ then K =

KEY. 3

SOL. tanx = 2t, tany = 3t, tanz = 5t

$$\sum\tan x=\pi(\tan x)\Rightarrow t^2=\frac{1}{3}$$

$$\tan^2 x+\tan^2 y+\tan^2 z=t^2(4+9+25)=38t^2, K=3$$

5. If $\tan\alpha$ is an integral solution of $4x^2-16x+15<0$ and $\cos\beta$ is the slope of the bisector of the angle in the first quadrant between the x and y axis. Then $\sin(\alpha+\beta):\sin(\alpha-\beta)=$

KEY. 1

SOL. $4x^2-16x+15<0$

$$4x^2-10x-6x+15<0$$

$$2x(2x-5)-3(2x-5)<0$$

$$\frac{3}{2}<x<\frac{5}{2}\Rightarrow x=2$$

$$\tan\alpha=2;\cos\beta=1$$

$$\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}=\frac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}=\frac{2+0}{2-0}=1$$

6. If $\sin\theta+\sin^2\theta+\sin^3\theta=1$, then the value of $\cos^6\theta-4\cos^4\theta+8\cos^2\theta$ must be

Key. 4

Sol. We have, $\sin\theta(1+\sin^2\theta)=1-\sin^2\theta$

$$\Rightarrow \quad \sin\theta(2-\cos^2\theta)=\cos^2\theta$$

Squaring both sides, we get

$$\sin^2\theta(2-\cos^2\theta)^2 = \cos^4\theta$$

$$\Rightarrow \quad (1-\cos^2\theta)(4-4\cos^2\theta+\cos^4\theta) = \cos^4\theta$$

$$\Rightarrow \quad -\cos^6\theta+5\cos^4\theta-8\cos^2\theta+4=\cos^4\theta$$

$$\therefore \quad \cos^6\theta-4\cos^4\theta+8\cos^2\theta+4=\cos^4\theta$$

7. If $\alpha = \frac{\pi}{14}$, then the value of (tan α tan 2α + tan 2α tan 4α + tan 4α tan α) is

Key. 1

Sol. $\alpha + 2\alpha + 4\alpha = 7\alpha = \frac{\pi}{2}$

$$\tan\frac{\pi}{2} = \tan(\alpha+2\alpha+4\alpha)$$

$$= \frac{\tan\alpha+\tan 2\alpha+\tan 4\alpha-\tan\alpha.\tan 2\alpha.\tan 4\alpha}{1-\tan\alpha.\tan 2\alpha-\tan\alpha.\tan 4\alpha-\tan 2\alpha.\tan 4\alpha}$$

tan α.tan 2α + tan2α. tan4α + tanα.tan4α = 1

8. In a □ABC, a = 5, b = 4 and $\cos(A-B) = \frac{31}{32}$, then c must be

Ans. 6

Sol. $\cos(A-B) = \dfrac{1-\tan^2\left(\dfrac{A-B}{2}\right)}{1+\tan^2\left(\dfrac{A-B}{2}\right)} = \tan\left(\dfrac{A-B}{2}\right) = \dfrac{1}{\sqrt{63}}$

Use Napier's analogy, we will get $\cot\frac{C}{2} = \frac{9}{\sqrt{63}}$

Then $\cos C = \frac{1}{8},\ c = 6$

9. In a triangle ABC, if r_1, r_2, r_3 are the ex-radius then $\frac{bc}{r_1}+\frac{ac}{r_2}+\frac{ab}{r_3} = k\frac{abc}{2\Delta}\left[\frac{s}{a}+\frac{s}{b}+\frac{s}{c}-3\right]$ then k is equal to

Ans. 2

Sol. $r_1 = \frac{\Delta}{s-a},\ r_2 = \frac{\Delta}{s-b},\ r_3 = \frac{\Delta}{s-c}$ substitute this value and take abc common

L.H.S = $\frac{abc}{\Delta}\left[\frac{s-a}{a}+\frac{s-b}{b}+\frac{s-c}{c}\right] = \frac{abc}{\Delta}\left[\sum\frac{s}{a}-3\right] \Rightarrow k = 2$

10. In □ABC, if I is incentre then AI + BI + CI ≥ □r then find □□

Ans. 6

Sol. $AI = r\operatorname{cosec}\left(\frac{A}{2}\right)$

$AI + BI + CI = r\left(\operatorname{cosec}\left(\frac{A}{2}\right) + \operatorname{cosec}\left(\frac{B}{2}\right) + \operatorname{cosec}\left(\frac{C}{2}\right)\right)$

$A.M \ge G.M$

$\operatorname{cosec}\left(\frac{A}{2}\right) + \operatorname{cosec}\left(\frac{B}{2}\right) + \operatorname{cosec}\left(\frac{C}{2}\right) \ge \left(\operatorname{cosec}\left(\frac{A}{2}\right) + \operatorname{cosec}\left(\frac{B}{2}\right) + \operatorname{cosec}\left(\frac{C}{2}\right)\right)^{1/3}$

$\ge 3(8)^{1/3} \ge 6$

11. If $\alpha+\beta+\gamma=\pi$ and $\tan\left[\frac{\alpha+\beta-\gamma}{4}\right]\tan\left[\frac{\gamma+\alpha-\beta}{4}\right]\tan\left[\frac{\gamma+\beta-\alpha}{4}\right] = 1$ then the value of $1+\cos\alpha+\cos\beta+\cos\gamma$ is K - 1 where K is

Key. 1

Sol. $A=\frac{\beta+\gamma-\alpha}{4}$ $B=\frac{\gamma+\alpha-\beta}{4}$ $C=\frac{\alpha+\beta-\gamma}{4}$

$\Rightarrow \tan A\tan B\tan C = 1$

$\frac{\sin A\sin B}{\cos A\cos B} = \frac{1}{\tan C} \Rightarrow \frac{\sin A\sin B - \cos A\cos B}{\sin A\sin B + \cos A\cos B} = \frac{1-\tan C}{1+\tan C}$

$\Rightarrow \frac{-\cos(A+B)}{\cos(A-B)} = \frac{\sin\left(\frac{\pi}{4}-C\right)}{\cos\left(\frac{\pi}{4}-C\right)}$

$\Rightarrow 2\sin\left(\frac{\pi}{4}-C\right)\cos(A-B) + 2\cos\left(\frac{\pi}{4}-C\right)\cos(A+B) = 0$

$\Rightarrow \sin\left(\frac{\pi}{4}-C+A-B\right) + \sin\left(\frac{\pi}{4}-C-A+B\right) + \cos\left(\frac{\pi}{4}-C+A+B\right) + \cos\left(\frac{\pi}{4}-C-A-B\right) = 0.....(1)$

$A-B-C=\frac{\pi}{4}-\alpha$ $B-A-C=\frac{\pi}{4}-\beta$ $C-A-B=\frac{\pi}{4}-\gamma$ $A+B+C=\frac{\pi}{4}$

$(1) \Rightarrow \cos\alpha+\cos\beta+\cos\gamma+1=0$

12. The value of $-2\left(\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}\right)$ is

Key. 1

Sol. $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7} = \frac{2\sin\frac{\pi}{7}\cos\frac{2\pi}{7} + 2\sin\frac{\pi}{7}\cos\frac{4\pi}{7} + 2\sin\frac{\pi}{7}\cos\frac{6\pi}{7}}{2\sin\frac{\pi}{7}}$

$= \frac{\left(\sin\frac{3\pi}{7}-\sin\frac{\pi}{7}\right)+\left(\sin\frac{5\pi}{7}-\sin\frac{3\pi}{7}\right)+\left(\sin\pi-\sin\frac{5\pi}{7}\right)}{2\sin\frac{\pi}{7}} = \frac{\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}} = -\frac{1}{2}$

13. If $A+B+C=180^0$, $\frac{\sin 2A+\sin 2B+\sin 2C}{\sin A+\sin B+\sin C}=k\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ then the value of k is

Key. 8

Sol. From conditional identities we have

$$\frac{\sin 2A+\sin 2B+\sin 2C}{\sin A+\sin B+\sin C}=\frac{4\sin A\sin B\sin C}{4\cos(A/2)\cos(B/2)\cos(C/2)}$$

$$=8\sin(A/2)\sin(B/2)\sin(C/2)$$

$\Rightarrow$ k = 8

14. If A, B and C are the angles of a triangle, then minimum value of

$$\left(\tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}\right)$$

Key. 1

Sol. We have $\frac{A}{2}+\frac{B}{2}+\frac{C}{2}=\frac{\pi}{2}$, so that

$$\tan\left(\frac{A}{2}+\frac{B}{2}\right)=\tan\left(\frac{\pi}{2}-\frac{C}{2}\right)\Rightarrow\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}=\frac{1}{\tan\frac{C}{2}}$$

$$\Rightarrow\tan\frac{A}{2}\tan\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{C}{2}\tan\frac{A}{2}=1$$

$$\Rightarrow\tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}-1=\frac{1}{2}\left[\sum 2\tan^2\frac{A}{2}-\sum 2\tan\frac{A}{2}\tan\frac{B}{2}\right]$$

$$=\frac{1}{2}\left[\left(\tan\frac{A}{2}-\tan\frac{B}{2}\right)^2+\left(\tan\frac{B}{2}-\tan\frac{C}{2}\right)^2+\left(\tan\frac{C}{2}-\tan\frac{A}{2}\right)^2\right]\geq 0$$

15. The value of $\sqrt{3}\operatorname{cosec}20^0-\sec 20^0$ is

Key. 4

Sol. $L.H.S=\frac{\sqrt{3}}{\sin 20^0}-\frac{1}{\cos 20^0}=\frac{\sqrt{3}\cos 20^0-\sin 20^0}{\sin 20^0\cos 20^0}$

$$=\frac{4.\left(\frac{\sqrt{3}}{2}\cos 20^0-\frac{1}{2}\sin 20^0\right)}{2\sin 20^0\cos 20^0}$$

$$=4.\frac{\sin 60^0\cos 20^0-\cos 60^0\sin 20^0}{\sin 40^0}=4\frac{\sin 40^0}{\sin 40^0}=4$$

16. $\sin 12^0.\sin 48^0.\sin 54^0=\frac{1}{a}$ then the value of a is

Key. 8

Sol. Let $\theta=12^0$

$$L.H.S=\frac{1}{\sin 72^0}\sin 12^0\sin 48^0\sin 72^0\sin 54^0$$

$$=\frac{1}{4}\frac{\sin 3\left(12^0\right)\sin 54^0}{\sin 72^0}=\frac{\sin 36^0 \sin 54^0}{8\sin 36^0 \cos 36^0}=\frac{\cos 36^0}{8\cos 36^0}=\frac{1}{8}$$

17. If $\alpha+\beta+\gamma=\pi$ and $\tan\left(\frac{\beta+\gamma-\alpha}{4}\right)\tan\left(\frac{\gamma+\alpha-\beta}{4}\right)\tan\left(\frac{\alpha+\beta-\gamma}{4}\right)=1$ then the value of $1+\cos\alpha+\cos\beta+\cos\gamma$ is

Key. 0

Sol. Let $A=\frac{\beta+\gamma-\alpha}{4};\ B=\frac{\gamma+\alpha-\beta}{4};\ C=\frac{\alpha+\beta-\gamma}{4}$

tan A tan B tan C = 1

or $\frac{\sin A\sin B}{\cos A\cos B}=\frac{1}{\tan C}$ or $\frac{\sin A\sin B-\cos A\cos B}{\sin A\sin B+\cos A\cos B}=\frac{1-\tan C}{1+\tan A}$

or, $-\frac{\cos(A+B)}{\cos(A-B)}=\frac{\sin\left(\frac{\pi}{4}-C\right)}{\cos\left(\frac{\pi}{4}-C\right)}$

or $2\sin\left(\frac{\pi}{4}-C\right)\cos(A-B)+2\cos\left(\frac{\pi}{4}-C\right)\cos(A+B)=0$

or $\sin\left(\frac{\pi}{4}\text{-C+A-B}\right)+\sin\left(\frac{\pi}{4}\text{-C-A+B}\right)+\cos\left(\frac{\pi}{4}\text{-C+A+B}\right)+\cos\left(\frac{\pi}{4}\text{-C-A-B}\right)=0$ ....(1)

$$A-B-C=\frac{\beta+\gamma-\alpha-\gamma-\alpha+\beta-\alpha-\beta+\gamma}{4}=\frac{\beta+\gamma-3\alpha}{4}=\frac{\pi-4\alpha}{4}=\frac{\pi}{4}-\alpha$$

Similarly $B-A-C=\frac{\pi}{4}-\beta$ and $C-A-B=\frac{\pi}{4}-\gamma$

and $C+A+B=\frac{\alpha+\beta+\gamma}{4}=\frac{\pi}{4}$

$\therefore$ Equation (1) reduces to,

$$\sin\left\{\frac{\pi}{4}+(\text{A-B-C})\right\}+\sin\left\{\frac{\pi}{4}+(\text{B-C-A})\right\}+\cos\left\{\frac{\pi}{4}-(\text{C-A-B})\right\}+\cos\left\{\frac{\pi}{4}-(\text{C+A+B})\right\}=0$$

or $\sin\left(\frac{\pi}{4}+\frac{\pi}{4}-\alpha\right)+\sin\left(\frac{\pi}{4}+\frac{\pi}{4}-\beta\right)+\cos\left(\frac{\pi}{4}-\frac{\pi}{4}+\gamma\right)+\cos\left(\frac{\pi}{4}-\frac{\pi}{4}\right)=0$

or $\cos\alpha+\cos\beta+\cos\gamma+1=0$.

18. In an acute angled triangle ABC, minimum value of $\sum\tan A\tan B$ is

Key. 9

Sol. $(\tan A\tan B-1)+(\tan B\tan C-1)+(\tan C\tan A-1)$

$$\Rightarrow\frac{\tan A+\tan B}{\tan C}+\frac{\tan B+\tan C}{\tan A}+\frac{\tan C+\tan A}{\tan B}$$

$$\Rightarrow\left(\frac{\tan A}{\tan C}+\frac{\tan C}{\tan A}\right)+\left(\frac{\tan A}{\tan B}+\frac{\tan B}{\tan A}\right)+\left(\frac{\tan B}{\tan C}+\frac{\tan C}{\tan B}\right)\ge 6$$

$$\therefore\sum\tan A\tan B\ge 9$$

19. The value of $\sum_{r=0}^{10}\cos^3\frac{r\pi}{3}$ is equal to $\frac{-a}{b}$ then the value of b is (where g. c. d of (a, b)is 1)

Key. 8

Sol. $\sum_{r=0}^{10}\cos^3\frac{r\pi}{3}=\frac{1}{4}\sum_{r=0}^{10}\left(3\cos\frac{r\pi}{3}+\cos r\pi\right)$

$$=\frac{1}{4}\left[3\left(\cos 0+\cos\frac{\pi}{3}+.....+\cos\frac{10\pi}{3}+\left(1-1+.....-1=1\right)\right)\right]$$

$$=\frac{\frac{3}{4}\left[\cos\left(\frac{10\pi}{6}\right)\sin\left(\frac{11\pi}{6}\right)\right]}{\sin\frac{\pi}{6}}+\frac{3}{4}=-\frac{1}{8.}$$