# Trigonometric Equations

## *Single Correct Answer Type*

1. If $2\tan^2 x-5\sec x=1$ for exactly 7 distinct values of $x\in\left[0,\frac{n\pi}{2}\right],n\in N$ then the greatest value of n is

A. 13 B. 17 C. 19 D. 15

KEY. D

SOL. $\sec x=3\Rightarrow\cos x=\frac{1}{3}$

Which gives two values of x in each of $[0,2\pi],(2\pi,4\pi],(4\pi,6\pi]$ and one value in $6\pi+\frac{3\pi}{2}=15\frac{\pi}{2}$

$\therefore$ greatest value of n = 15

2. Let $S=\{a\in N,a\le 100\}$. If the equation $[Tan^2 x]-Tanx-a=0$ has real roots then number of elements in S is (where [ ] is step function).

A. 10 B. 8 C. 9 D. 0

KEY. C

SOL. Given equation is true only when Tan x is an integer $Tanx=\frac{1\pm\sqrt{4a+1}}{2}\Rightarrow 4a+1$ is perfect square and $4a+1\le 401$

3. If $\sin^2(\theta-\alpha)\cos\alpha=\cos^2(\theta-\alpha)\sin\alpha=m\sin\alpha\cos\alpha$ then

A. $|m|\le\frac{1}{\sqrt{2}}$ B. $|m|\ge\frac{1}{\sqrt{2}}$ C. $|m|\ge 1$ D. $|m|\le 1$

KEY. B

SOL. $\frac{\sin^2(\theta-\alpha)}{\sin\alpha}=\frac{\cos^2(\theta-\alpha)}{\cos\alpha}=m=\frac{1}{\sin\alpha+\cos\alpha}\Rightarrow|m|\ge\frac{1}{\sqrt{2}}$

4. The number of values of y in $[-2\pi,2\pi]$ satisfying the equation $|\sin 2x|+|\cos 2x|=|\sin y|$ is

A. 1 B. 2 C. 3 D. 4

KEY. D

SOL. $1\le|\sin 2x|+|\cos 2x|\le\sqrt{2}$ and $|\sin y|\le 1\Rightarrow\sin y=\pm 1\Rightarrow y=\pm\frac{\pi}{2},\pm\frac{3\pi}{2}$

5. Let $\theta\in[0,4\pi]$ satisfying the equation $(\sin\theta+2)(\sin\theta+3)(\sin\theta+4)=6$. If the sum of all values of $\theta$ is $K\pi$ then value of K is

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A. 6 B. 5 C. 4 D. 2

KEY. B

SOL. $\sin\theta = -1 \Rightarrow \theta = \frac{3\pi}{2}, \frac{7\pi}{2}$

$\therefore K = 5$

6. If $\tan\left(\frac{\pi}{4}+\frac{y}{2}\right) = \tan^3\left(\frac{\pi}{4}+\frac{x}{2}\right)$, then $\sin x\left(\frac{3+\sin^2 x}{1+3\sin^2 x}\right)$ equals

(A) cos y (B) sin y

(C) sin 2y (D) 0

Key. B

Sol. $$\frac{1+\tan\frac{y}{2}}{1-\tan\frac{y}{2}} = \left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)^3$$

Square both sides, we get

$$\frac{1+\sin y}{1-\sin y} = \frac{(1+\sin x)^3}{(1-\sin x)^3}$$

Using componendo and dividendo

$$\frac{2\sin y}{2} = \frac{(3+\sin^2 x)}{1+3\sin^2 x}\sin x$$

7. The number of solutions of the equation $16\left(\sin^5 x+\cos^5 x\right) = 11\left(\sin x+\cos x\right)$ in the interval $\left[0, 2\pi\right]$ is

(A) 6 (B) 7
(C) 8
(D) 9

KEY : A

HINT: $16\ (\sin^5 x+\cos^5 x) - 11(\sin x+\cos x) = 0$

$\Rightarrow (\sin + \cos x)\{16(\sin^4 x - \sin^3 x\cos\ x + \sin^2 x\cos^2 x - \sin\ x\cos^3 x + \cos^4 x) - 11\} = 0$

$\Rightarrow (\sin x+\cos x)\{16(1-\sin^2 x\cos^2 x - \sin\ x\cos x) - 11\} =$

$\Rightarrow (\sin x+\cos x)\ (4\sin x\cos x - 1)\ (4\sin x\cos x + 5) = 0$

$As\ \ 4\sin x\cos x + 5 \neq 0$, WE HAVE

$\sin x + \cos x = 0, 4\sin x\cos x - 1 = 0$

THE REQUIRED VALUES ARE $\pi/12, 5\pi/12, 9\pi/12, 13\pi/12, 17\pi/12, 21\pi/12, -$ THEY ARE 6 SOLUTIONS ON $\left[0, 2\pi\right]$

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8. Sum of integral values of n such that sin x $\left(2\sin x+\cos x\right)=n,$ has at least one real solution is

(A) 3 (B) 1 (C) 2 (D) 0

Key : A

Hint : $2\sin^2 x+\frac{2\sin x\cos x}{2}=n$

$\sin 2x-2\cos x=2n-2$

$-\sqrt{5}\le 2n-2\le\sqrt{5}$

$\Rightarrow\ 1-\frac{\sqrt{5}}{2}\le n\le 1+\frac{\sqrt{5}}{2}$

$\therefore$ (A)

9. The equation 2x = (2n + 1) $\pi$ $\left(1-\cos x\right)$, (where n is a positive integer)

(A) has infinitely many real roots (B) has exactly one real root

(C) has exactly 2n + 2 real roots (D) has exactly 2n + 3 real roots

Key : C

Hint : $\sin^2\left(\frac{x}{2}\right)=\frac{x}{\left(2n+1\right)\pi}$

the graph of $\sin^2\left(\frac{x}{2}\right)$ will be above the x–axis and will be meeting the x-axis at 0, 2$\pi$, 4$\pi$, . . . etc. It will attain maximum values at odd multiples of $\pi$ lie. $\pi$, 3$\pi$, . . . (2n + 1)$\pi$. The last point after which graph of $y=\frac{x}{\left(2n+1\right)\pi}$ will stop cutting will be $\left(2n+1\right)\pi$.

Total intersection = $2\left(n+1\right)$

10. If sin A = sin B and cos A = cos B, A > B, then

(a) sin (1/2) (A – B) = 0 (b) sin (1/2) (A + B) = 0

(c) cos (1/2) (A – B) = 0 (d) cos (1/2) (A + B) = 0

Key: a

Hint: sin A = sin B $\Rightarrow$ sin A – sin B = 0

$\Rightarrow\ 2\sin\frac{A-B}{2}\cos\frac{A+B}{2}=0$ (1)

and cos A = cos B $\Rightarrow$ cos B – Cos A = 0

$\Rightarrow\ 2\sin\frac{A+B}{2}\sin\frac{A-B}{2}=0$ (2)

Equations (1) and (2) are simultaneously true if sin (1/2) (A – B) = 0, while the other factors sin (1/2) (A+ B) and cos (1/2) (A + B) cannot both be zero simultaneously.

11. Number of solutions of the equation

tan x + sec x = 2 cos x

lying in the interval [0, 2$\pi$] is

(a) 0 (b) 1 (c) 2 (d) 3

Key: c

Hint: The given equation can be written as

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$\frac{1+\sin x}{\cos x} = 2\cos x$

$\Rightarrow$ 1 + sin x = 2 $\cos^2$x = 2(1 – $\sin^2$x)

$\Rightarrow$ 2 $\sin^2$x + sin x – 1 = 0

$\Rightarrow$ (1 + sin x)(2 sin x – 1) = 0

$\Rightarrow$ sin x = –1 or 1/2

Now sin x = –1 $\Rightarrow$ tan x and sec x not defined. Sin x = 1/2 $\Rightarrow$ x= π/6 or 5π/6.

$\therefore$ The required number of solution is 2.

12. The number of solutions of the pair of equations $2\sin^2\theta - \cos 2\theta = 0, 2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is

A. 0 B. 1 C. 2 D. 4

$\therefore$ KEY.C

$\therefore$ SOL. $2\sin^2\theta - \cos 2\theta = 0 \Rightarrow 1 - \cos 2\theta = 0 \Rightarrow \cos 2\theta = \frac{1}{2}$

$\therefore$ $2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$\therefore$ $2\cos^2\theta - 3\sin\theta = 0$

$\therefore$ $(2\sin\theta - 1)(\sin\theta + 2) = 0 \Rightarrow \sin\theta = \frac{1}{2}$

$\therefore$ $\theta = \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow No.of\ solutions = 2$

13. $\left|x^2 \sin x + \cos^2 x\, e^x + \ln^2 x\right| < x^2 |\sin x| + \cos^2 x\, e^x + \ln^2 x$ true for $x \in$

(A) $(-\pi, 0)$ (B) $\left(0, \frac{\pi}{2}\right)$

(C) $\left(\frac{\pi}{2}, \pi\right)$ (D) $(2n\pi, (2n+1)\pi)\, n \in N$

Key. A

Sol. $|a+b+c| < |a| + |b| + |c|$

If a, b, c do not have same sign.

So $x^2$sinx < 0

If $x \in (-\pi, 0)$

14. The number of solutions of $\sin^2$x$\cos^2$x = 1 + $\cos^2$x $\sin^4$x in the interval [0, 2π] is

(A) 0 (B) 1

(C) 2 (D) 3

Key. A

Sol. $\sin^2$x$\cos^2$x (1 – $\sin^2$x) = 1

$\sin^2$x$\cos^4$x = 1

No values of x for L.H.S. = R.H.S.

15. If $\log_{0.5}$ sinx = 1 – $\log_{0.5}$ cosx then number of values of $x \in [-2\pi, 2\pi]$ is

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(A) 1 (B) 2
(C) 3 (D) 4

Key. D

Sol. $\log_{0.5} \sin x = 1 - \log_{0.5} \cos x$

$x \in [-2\pi, 2\pi]$

$\sin x > 0$ and $\cos x > 0$

$\sin x \cos x = \frac{1}{2}$

$\sin 2x = 1 \quad 2x \in [-4\pi, 4\pi]$

$\Rightarrow$ 4 solutions

16. If $f(x) = \frac{\sin 3x}{\sin x}, x \neq n\pi$ then the range of values of f(x) for real values of x is

(A) [–1, 3] (B) (–∞, –1)
(C) (3, ∞) (D) [–1, 3)

Key. D

Sol. $3 - 4\sin^2 x = y$

$\therefore \quad \sin^2 x = \frac{3-y}{4}$. But $0 < \sin^2 x \le 1$ $(\because \sin x = 0 \Rightarrow x = n\pi)$

$\therefore$ $0 < \frac{3-y}{4} \le 1$ or $0 < 3 - y \le 4$.

17. If $2^{\sqrt{\sin^2 x - 2\sin x + 5}} \frac{1}{4^{\sin^2 y}} \le 1$, then the ordered pair (x, y) is equal to (m, n ∈ I)

(A) $x = (4n+1)\frac{\pi}{2}, y = (2m+1)\frac{\pi}{2}$ (B) $x = 2n\pi, y = 2m\pi$

(C) $x = (2n+1)\frac{\pi}{2}, y = (2m+1)\frac{\pi}{2}$ (D) $x = n\pi, y = m\pi$

Key. A

Sol. $\sin^2 x - 2\sin x + 5 = (\sin x - 1)^2 + 4 \ge 4$

$\therefore \quad 2^{\sqrt{\sin^2 x - 2\sin x + 5}} \ge 2^2 = 4$

and $\sin^2 y \le 1 \Rightarrow \frac{1}{4^{\sin^2 y}} \ge \frac{1}{4}$

$\therefore$ LHS ≥ 1 and according to question LHS ≤ 1, so therefore, LHS = 1
for which

$\therefore \sin^2 x - 2\sin x + 5 = 4$

$(\sin x - 1)^2 = 0$

$\sin x = 1 \Rightarrow x = (2n+1)\frac{\pi}{2}$

and $\operatorname{cosec}^2 y = 1$, $\sin^2 y = 1$ or $\cos y = 0$

$y = (2m+1)\frac{\pi}{2}$

18. The number of solutions of the equation $\left|\sin\frac{\pi}{2}(1-x) + \cos\frac{\pi}{2}(1-x)\right| = \sqrt{\left|\log_e |x|^3 + 1\right|}$ is /are.

a) 4 b) 6 c) 8 d)10

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Key. B

Sol. Simplify, $\sin \pi \ x = \left|\left(\log|x|\right)^3\right|$ By graph, we get 6 solutions.

19. $\sin^2x \cos^2x = 1 + \cos^2x \sin^4x$

Number of solutions in the interval $[0, 2\pi]$ is

(A) 0 (B) 1

(C) 2 (D) 3

Key. A

Sol. $\sin^2x \cos^2x(1 - \sin^2x) = 1$

$\sin^2x \cos^4x = 1$

No values of x possible.

20. If $0 < x < 1000$ and $\left[\frac{x}{2}\right]+\left[\frac{x}{3}\right]+\left[\frac{x}{5}\right]=\frac{31}{30}x$ where [.] GIF; the number of possible values of *x* is

A) 34 B) 33 C) 32 D) 35

Key. B

Sol. LHS is integer

$\therefore$ RHS must be integer for which *x* is multiple of 30

$x = 30, 60, 90, ....., 990$

No. of possible values = 33

21. A set of values of x, satisfying the equation $\cos^2\left(\frac{1}{2}px\right)+\cos^2\left(\frac{1}{2}qx\right)=1$ form an Arithmetic progression with common difference

a) $\frac{2}{p+q}$ b) $\frac{2}{p-q}$ c) $\frac{\pi}{p+q}$ d) none

Key. D

Sol. 1 + cos px + 1 + cos qx = 2

$$\Rightarrow \cos\left(\frac{p+q}{2}\right)x \cos\left(\frac{p-q}{2}\right)x = 0$$

$$\Rightarrow x = \frac{(2n+1)\pi}{p+q} \text{ or } \frac{(2n+1)\pi}{p-q}$$

for n = 0, $\pm1, \pm2, .........$

forms an AP with common difference $\frac{2\pi}{p+q}$ or $\frac{2\pi}{p-q}$

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22. If $\tan\beta = 2\sin\alpha.\sin\gamma.\operatorname{cosec}(\alpha+\gamma)$, then $\cot\alpha, \cot\beta, \cot\gamma$ are in

A) A.P. B) G.P. C) H.P. D) none of these

Key. A

Sol. $\tan\beta = 2\sin\alpha.\sin\gamma.\operatorname{cosec}(\alpha+\gamma) = \dfrac{2\sin\alpha\sin\gamma}{\sin(\alpha+\gamma)}$

$\cot\beta = \dfrac{\sin(\alpha+\gamma)}{2\sin\alpha\sin\gamma}$

i.e. $2\cot\beta = \dfrac{\sin\alpha\cos\gamma + \cos\alpha\sin\gamma}{\sin\alpha\sin\gamma} = \cot\alpha + \cot\gamma$

23. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50° =$

A) $\frac{4}{5}$ B) $\frac{1}{3}$ C) $\frac{3}{4}$ D) 3

Key. C

Sol. $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50° =$

$= \frac{1}{2}\left[1 + \cos 20° - (\cos 60° + \cos 40°) + (1 + \cos 100°)\right]$

$= \frac{1}{2}\left[1 + \cos 20° - \frac{1}{2} - \cos 40° + 1 - \cos 80°\right] = \frac{1}{2}\left[\frac{3}{2} + \cos 20° - (2\cos 60° \cos 20°)\right] = \frac{3}{4}$

24. If $\cos^6\alpha + \sin^6\alpha + k\sin^2 2\alpha = 1 \ \forall \alpha \in (0, \pi/2)$, then k is

A) $\frac{3}{4}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{1}{8}$

Key. A

Sol. The given condition can be written

$(\cos^2\alpha + \sin^2\alpha)^3 - 3\sin^2\alpha\cos^2\alpha(\cos^2\alpha + \sin^2\alpha) + k\sin^2 2\alpha = 1$

$\Rightarrow \quad \left(-\frac{3}{4}\right)\sin^2 2\alpha + k\sin^2 2\alpha = 0$,

Showing that $k = \frac{3}{4}$.

25. The most general solution of the equations $\tan\theta = -1, \cos\theta = \frac{1}{\sqrt{2}}$ is

A) $n\pi + 7\pi/4$ B) $n\pi + (-1)^n \frac{7\pi}{4}$ C) $2n\pi + \frac{7\pi}{4}$ D) none of these

Key. C

Sol. We have $\tan\theta = -1$ and $\cos\theta = \frac{1}{\sqrt{2}}$

The value of $\theta$ lying between $\frac{3\pi}{2}$ and $2\pi$ and satisfying these two is $\frac{7\pi}{4}$. Therefore the most general solution is $\theta = 2n\pi + 7\pi/4$ where $n \in Z$

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26. $\dfrac{\sin^3\theta-\cos^3\theta}{\sin\theta-\cos\theta}-\dfrac{\cos\theta}{\sqrt{1+\cot^2\theta}}-2\tan\theta\cot\theta=-1$ if:

A) $\theta\in\left(0,\dfrac{\pi}{2}\right)$ B) $\theta\in\left(\dfrac{\pi}{2},\pi\right)$ C) $\theta\in\left(\pi,\dfrac{3\pi}{2}\right)$ D) $\theta\in\left(\dfrac{3\pi}{2},2\pi\right)$

Key. B

Sol. note: $\sin\theta\neq\cos\theta$

$\Rightarrow \quad \theta\notin\left(0,\dfrac{\pi}{2}\right)\cup\left(\pi,\dfrac{3\pi}{2}\right);\ \theta\neq\dfrac{\pi}{2},\dfrac{3\pi}{2},\ 0,\pi,\ 2\pi$

And equality holds if $\theta\in\left(\dfrac{\pi}{2},\pi\right)$

27. The least positive values of x satisfy the equation $8^{1+|\cos x|+\cos^2 x+|\cos^3 x|+\ldots\infty=4^3}$ will be (where |cos x| < 1)

A) $\dfrac{\pi}{3}$ B) $\dfrac{2\pi}{3}$ C) $\dfrac{\pi}{4}$ D) none of these

Key. A

Sol. $1+|\cos x|+\cos^2 x$..........

$=\dfrac{1}{1-|\cos x|} \quad \Rightarrow \quad \dfrac{1}{8^{1-|\cos x|}=4^3}$

$\Rightarrow \quad 2^{\frac{3}{1-|\cos x|}}=2^6 \quad \Rightarrow \quad \dfrac{3}{1-|\cos x|}=6 \quad \Rightarrow \quad 1-|\cos x|=\dfrac{1}{2}$

$|\cos x|=\dfrac{1}{2} \quad \Rightarrow \quad \cos x=\pm\dfrac{1}{2}$

$\cos x=\dfrac{1}{2}$ will give least positive value of x

$x=\dfrac{\pi}{3}$ Ans.

28. Value of $\dfrac{3+\cot 80^\circ\cot 20^\circ}{\cot 80^\circ+\cot 20^\circ}$ is equal to

A) $\sqrt{2}\ \cos x$ B) $-\sqrt{2}\ \cos x$ C) $\sqrt{2}\ \sin x$ D) $-\sqrt{2}\ \sin x$

Key. B

Sol. $\dfrac{3+\dfrac{\cos 80^\circ\cos 20^\circ}{\sin 80^\circ\sin 20^\circ}}{\dfrac{\cos 80^\circ}{\sin 80^\circ}+\dfrac{\cos 20^\circ}{\sin 20^\circ}}=\dfrac{2\sin 80^\circ\sin 20^\circ+\cos 80^\circ\cos 20^\circ+\sin 80^\circ\sin 20^\circ}{\sin 20^\circ\cos 80^\circ+\cos 20^\circ\sin 80^\circ}$

$=\dfrac{\cos 60^\circ-\cos 100^\circ+\cos 60^\circ}{\sin 100^\circ}=\dfrac{1-\cos 100^\circ}{\sin 100}=\tan 50^\circ$

29. If $\sin x+\cos x=\sqrt{2}\cos x$, then cosx – sin x is equal to

A) $\sqrt{2}\cos x$ B) $-\sqrt{2}\cos x$ C) $\sqrt{2}\sin x$ D) $-\sqrt{2}\sin x$

Key. C

Sol. $\cos x+\sin x=\sqrt{2}\cos x$

$\sin x=\left(\sqrt{2}-1\right)\cos x$

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$\cos x = \frac{1}{\left(\sqrt{2}-1\right)}\sin x$

$\cos x = \left(\sqrt{2}+1\right)\sin x$

$\cos x - \sin x = \sqrt{2}\sin x$

30. If $\frac{\sin x}{a} = \frac{\cos x}{b} = \frac{\tan x}{c} = k$, then $bc + \frac{1}{ck} + \frac{ak}{1+bk}$ is equal to

A) $k\left(a+\frac{1}{a}\right)$ B) $\frac{1}{k}\left(a+\frac{1}{a}\right)$ C) $\frac{1}{k^2}$ D) $\frac{a}{k}$

Key. B

Sol. $\frac{\cos x \tan x}{k^2} + \frac{1}{\tan x} + \frac{\sin x}{1+\cos x} = \frac{\sin x}{k^2} + \frac{\cos x\left(1+\cos x\right)+\sin^2 x}{\sin x\left(1+\cos x\right)} = \frac{a}{k} + \frac{1}{\sin x} = \frac{a}{k} + \frac{1}{ak}$

31. In a triangle ABC, if $\sin A \cos B = \frac{1}{4}$ and 3 tan A = tan B, then $\cot^2 A$

A) 2 B) 3 C) 4 D) 5

Key. B

Sol. 3 sin A cos B = sin B cos A

$\cos A \sin B = \frac{3}{4}$

$\sin\left(A+B\right) = 1 \Rightarrow C = \frac{\pi}{2}, B = \frac{\pi}{2} - A$

$3 \tan A = \tan\left(\frac{\pi}{2} - A\right)$

$3 = \cot^2 A$

32. If $\Delta = \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 1 & \sin\theta & 1 \end{vmatrix}$, then maximum value of is

A) 1 B) 9 C) 16 D) none of these

Key. D

Sol. $\Delta = \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 1 & \sin\theta & 1 \end{vmatrix}$ applying $R_3 \to R_3 - R_1$

$= \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 1 & \sin\theta - 3\cos\theta & 1 \end{vmatrix} = -\left(\sin\theta - 3\cos\theta\right)\left(3\cos\theta - \sin\theta\right) = \left(3\cos\theta - \sin\theta\right)^2$

Now, $-\sqrt{9+1} \le 3\cos\theta - \sin\theta \le \sqrt{9+1} \Rightarrow \left(3\cos\theta - \sin\theta\right)^2 \le 10$

33. If $\frac{\sin^3\theta - \cos^3\theta}{\sin\theta - \cos\theta} - \frac{\cos\theta}{\sqrt{\left(1+\cot^2\theta\right)}} - 2\tan\theta\cot\theta = -1,\ \theta \in \left[0, 2\pi\right]$, then

A) $\theta \in \left(0, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}\right\}$ B) $\theta \in \left(\frac{\pi}{2}, \pi\right) - \left\{\frac{3\pi}{4}\right\}$ C) $\theta \in \left(\pi, \frac{3\pi}{2}\right) - \left\{\frac{5\pi}{4}\right\}$ D) $\theta \in \left(0, \pi\right) - \left\{\frac{\pi}{4}, \frac{\pi}{2}\right\}$

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Key. D

Sol. Since $\sin\theta-\cos\theta\neq 0$

$\tan\theta\neq 1$

$\therefore \quad \theta\neq\frac{\pi}{4},\frac{5\pi}{4}$

Now $\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta-|\sin\theta|\cos\theta-2\tan\theta\cot\theta=-1$

$\Rightarrow \quad 1+\cos\theta(\sin\theta-|\sin\theta|)-2=-1 \quad\Rightarrow\quad \cos\theta(\sin\theta-|\sin\theta|)=0$

$\therefore \quad \theta\in(0,\pi)\left\{\frac{\pi}{4},\frac{\pi}{2}\right\}$

34. In the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ the equation $\log_{\sin}(\cos 2\theta)=2$ has

A) no solution B) a unique solution C) two solutions D) infinitely many solutions

Key. B

Sol. $\therefore \quad -\frac{\pi}{2}\leq\theta\leq\frac{\pi}{2} \quad\Rightarrow\quad -1\leq\sin\theta\leq 1$

Here $0<\sin\theta<1 \quad\Rightarrow\quad \log_{\sin\theta}\cos 2\theta=2$

$\cos 2\theta=\sin^2\theta \quad\Rightarrow\quad \log_{\sin\theta}\cos 2\theta=2$

$3\sin^2\theta=1 \quad\Rightarrow\quad \sin^2\theta=\frac{1}{3}$

$\therefore \quad \sin\theta=\frac{1}{\sqrt{3}}\{\therefore \quad 0<\sin\theta<1\}$ a unique solution

35. If $\sin 2A=\frac{1}{2}$ and $\sin 2B=-\frac{1}{2}$, then which one of the following is false

A) sin (A + B) may be 0 B) cos (A – B) may be zero

C) sin(A + B) or cos (A – B) is zero D) sin (A + B) = 0

Key. D

Sol. $\sin 2A=\frac{1}{2},\sin 2B=-\frac{1}{2}$

sin2A + sin 2B = 0

2sin(A + B) cos (A – B) = 0

Sin(A + B) = 0 or cos(A – B) = 0

36. If $\sin 2\beta$ is the G.M. between $\sin\alpha$ and $\cos\beta$, then $\cos 4\beta$ is equal to

A) $2\sin^2\left(\frac{\pi}{4}-\alpha\right)$ B) $2\cos^2\left(\frac{\pi}{4}-\alpha\right)$ C) $2\cos^2\left(\frac{\pi}{2}+\alpha\right)$ D) $2\sin^2\left(\frac{\pi}{4}+\alpha\right)$

Key. A

Sol. $\sin 2\beta=\sqrt{\sin\alpha.\cos\alpha}$

$\cos 4\beta=1-2\sin^2 2\beta=1-2\sin\alpha.\cos\alpha=(\sin\alpha-\cos\alpha)^2=2\sin^2\left(\alpha-\frac{\pi}{4}\right)$

Or $=2\sin^2\left(\frac{\pi}{4}-\alpha\right)$

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37. Number of ordered pairs (a, x) satisfying the equation $\sec^2(a+2)x + a^2 - 1 = 0; -\pi < x < \pi$ is

A) a = - 3 and b = 1 B) a = 1 and $b = -\frac{1}{3}$ C) $a = \frac{1}{6}$ and $b = \frac{1}{2}$ D) none of these

Key. C

Sol. Given equation $\sec^2(a+2)x + a^2 - 1 = 0$

$\Rightarrow \quad \tan^2(a+2)x + a^2 = 0 \quad \Rightarrow \quad \tan^2(a+2)x = 0$ and a = 0

$\Rightarrow \quad \tan^2 2x = 0 \quad \Rightarrow \quad \tan^2 2x = 0 \quad \Rightarrow \quad x = 0, \frac{\pi}{2}, \frac{\pi}{2}$

$\therefore \quad (0,0),(0,\pi/2),(0,-\pi/2)$ are ordered pairs satisfying the equation

38. In a parallelogram ABCD $|\vec{AB}| = a, |\vec{AD}| = b$ and $|\vec{AC}| = c$ . Then has the value

A) $\frac{3a^2 + b^2 - c^2}{2}$ B) $\frac{a^2 + 3b^2 - c^2}{2}$ C) $\frac{a^2 - b^2 + 3c^2}{2}$ D) $\frac{a^2 + 3b^2 + c^2}{2}$

Key. A

Sol. $\therefore \quad \vec{DB} = \vec{DA} + \vec{AB}$ or $\vec{DA} = \vec{DB} - \vec{AB}$

$\therefore \quad (\vec{DA})^2 = (\vec{DB})^2 + (\vec{AB})^2 - 2\vec{DB}.\vec{AB}$

In parallelogram $2(a^2 + 2b^2) = c^2 + DB^2$

$\therefore \quad (DB)^2 = 2a^2 + 2b^2 - c^2$

From (i) $\Rightarrow \quad b^2 = 2a^2 + 2b^2 - c^2 + a^2 - 2\vec{AB}, \vec{DB}$

$\therefore \quad \vec{AB}.\vec{DB} = \frac{3a^2 + b^2 - c^2}{2}$

39. Let $\vec{a}(x) = (\sin x)\hat{i} + (\cos x)\hat{j}$ and $\vec{b}(x) = (\cos 2x)\hat{i} + (\sin 2x)\hat{j}$ be two variable vectors $(x \in R)$, then $\vec{a}(x)$ and $\vec{b}(x)$ are

A) collinear for unique value of x B) perpendicular for infinitely many values of x

C) zero vectors for unique value of x D) none of these

Key. B

Sol. If $\vec{a}(x)$ and $\vec{b}(x)$ are perpendicular then $\vec{a}.\vec{b} = 0$

$\Rightarrow$ sin x cos 2x + cos sin 2x = 0

Sin (3x) = 0 sin0

$x = \frac{n\pi}{3}$

For infinitely many value of x.

40. If are two vectors, such that $\vec{a}\,\vec{b}$ and < 0 and $|\vec{a}\,\vec{b}| = |\vec{a} \times \vec{b}|$, then angle between vectors is

A) $\pi$ B) $\frac{7\pi}{4}$ C) $\frac{\pi}{4}$ D) $\frac{3\pi}{4}$

Key. D

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Sol. $\left|\vec{a}.\vec{b}\right| = \left|\vec{a}\times\vec{b}\right|$

$\left|\vec{a}\right| = \left|\vec{b}\right|\cos\theta = \left|\vec{a}\right|\left|\vec{b}\right|\left|\sin\theta\right|$ (where $\theta$ is angle between $\vec{a}$ and $\vec{b}$)

$\Rightarrow \quad \left|\cos\theta\right| \quad \Rightarrow \quad \left|\sin\pi\right|$

$\Rightarrow \quad \theta = \dfrac{\pi}{4}$ or $\dfrac{3\pi}{4}$ $\quad (as \le \theta \le \pi)$

But $\quad \vec{a}.\vec{b} < 0 \quad \theta = \dfrac{3\pi}{4}$

41. If $\hat{a}, \hat{b}$ and $\hat{c}$ are three unit vectors, such that $\hat{a}+\hat{b}+\hat{c}$ is also a unit vector and 1, 2 and 3 are angles between the vectors $\hat{c}\ \hat{a}, \hat{b}; \hat{b}, \hat{c}$ and $\hat{c}, \hat{a}$ respectively, then among

A) all are acute angles

B) all are right angles

C) at least one is obtuse angle

D) none of these

Key. C

Sol. Given condition $\left(\vec{a}+\vec{b}+\vec{c}\right).\left(\vec{a}+\vec{b}+\vec{c}\right)=1$

$\left|a^2\right|+\left|b^2\right|+\left|c^2\right|+2|a||b|\cos\theta+2|b||c|\cos\theta_2+2|c||a|\cos\theta_3=1$

$\Rightarrow \quad \cos\theta_1+\cos\theta_2+\cos\theta_3=-1$

$\Rightarrow$ one of 1, 2 and 3 should be obtuse angle

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# Trigonometric Equations

## *Integer Answer Type*

1. The number of ordered pairs (x,y) where $x, y \in [0,10]$ satisfying $\left(\sqrt{\sin^2 x - \sin x + \frac{1}{2}}\right).2^{\sec^2 y} \le 1$ is 2K then K =

KEY. 8

SOL. $\sqrt{\sin^2 x - \sin x + \frac{1}{2}} = \sqrt{(\sin x - \frac{1}{2})^2 + \frac{1}{4}} \ge \frac{1}{2}$ and

$(\sec^2 y) \ge 1, 2^{\sec^2 y} \ge 2$

It is possible only when $\sin x = \frac{1}{2}, \sec^2 y = 1$

$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

$y = 0, \pi, 2\pi, 3\pi$

No. of ordered pairs = 16

2. The A.M. of the solutions of the equation $4\cos^3 x - 4\cos^2 x - \cos(\pi + x) - 1 = 0$ in the interval $(0, 315)$ is $(17K\pi)$ then K =

KEY. 3

SOL. $(4\cos^2 x + 1)(\cos x - 1) = 0 \Rightarrow \cos x = 1$

$x = 2\pi, 4\pi, 6\pi, ......100\pi$

$A.M. = \frac{2(\pi + 2\pi + .....+ 50\pi)}{50} = 51\pi$

3. The no.of values of $x \in [0, 4\pi]$ satisfying $|\sqrt{3}\cos x - \sin x| \ge 2$ is

KEY. 4

SOL. Since the maximum value of $\sqrt{3}\cos x - \sin x$ is 2

$|\sqrt{3}\cos x - \sin x| = 2$ only $\cos(x + \frac{\pi}{6}) = \pm 1$

$x = \frac{5\pi}{6}, \frac{11\pi}{6}, \frac{17\pi}{6}, \frac{23\pi}{6}$

4. If $\theta \in [0, 5\pi]$ and $r \in R$ such that $2\sin\theta = r^4 - 2r^2 + 3$ then the maximum no.of values of the pair $(r, \theta)$ is ______

KEY. 6

SOL. $2\sin\theta = (r^2 - 1)^2 + 2$

This is possible only $\sin\theta = 1, r^2 = 1, r = \pm 1$

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$\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$

No.of values of the pair = 6.

5. If $[\sin x] + \left[\frac{x}{2\pi}\right] + \left[\frac{2x}{5\pi}\right] = \frac{9x}{10\pi}$ when the number of solutions in the interval (30, 40) is (where [.] is GIF)

Key. 1

Sol. $[\sin x] = \frac{x}{2\pi} - \left[\frac{x}{2\pi}\right] + \frac{2x}{5\pi} - \left[\frac{2x}{5\pi}\right]$ No. of solutions = 1.

6. Let '$k$' be sum of all '$x$' in the interval $[0, 2\pi]$ such that $3\cot^2 x + 8\cot x + 3 = 0$. Then the value of $\frac{k}{\pi}$ is

Key. 5

Sol. $\cot x = u : 3u^2 + 8u + 3 = 0$

Both roots are real and product of roots $= 1$

But $\cot x$ bijection in $(0, \pi)$. Let $x_1, x_2$ are roots such that $0 < x_1\, x_2 < \pi$. But $\cot x_1, \cot x_2$ are both negative.

$\therefore \frac{\pi}{2} < x_1, x_2 < \pi$

But $\pi < x_1 + x_2 < 2\pi$

$\cot x_1 . \cot x_2 = 1$

$\cot x_1 . \cot\left(\frac{3\pi}{2} - x_1\right) = 1$

$\therefore x_1 + x_2 = \frac{3\pi}{2}$ similarly $x_3 + x_4 = \frac{7\pi}{2}$

$\therefore k = 5\pi$

7. The number of solutions of equation $8[x^2 - x] + 4[x] = 13 + 12[\sin x]$ is (here [.] represents greatest integer less than or equal to '$x$')

Key. 0

Sol. $\because R.H.S$ is always odd.

While $L.H.S$ is always even.

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8. Let x be in radians with $0 < x < \frac{\pi}{2}$. If $\sin(2\sin x) = \cos(2\cos x)$; then $\tan x + \cot x$ can be written as $\frac{a}{\pi^c - b}$ where $a, b, c \in N$. Then the value of $\left(\frac{a+b+c}{25}\right)$ is

Key: 2

Hint: $\sin(2\sin x) = \sin\left(\frac{\pi}{2} - 2\cos x\right)$

$\sin x + \cos x = \frac{\pi}{4}$

s.o.b.s

$1 + \sin 2x = \frac{\pi^2}{16}$

$\sin 2x = \frac{\pi^2 - 16}{16}$

$\therefore \tan x + \cot x = \frac{2}{\sin 2x} = \frac{2 \times 16}{\pi^2 - 16} = \frac{32}{\pi^2 - 16}$

$\therefore a = 32, b = 16, c = 2$

$\frac{a+b+c}{25} = 2$

9. If a is irrational then number of solutions of the equation $1 + \sin^2 ax = \cos x$

1) 0 2) 2 3) 1 4) infinite

KEY : 3

HINT: CONCEPUAL

10. Find the number of pairs (x, y) satisfying the equation $\sin x + \sin y = \sin(x+y)$ and $|x| + |y| = 1$.

Key. 6

Sol. The first equation can be written as

$2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x+y}{2}\right)$

or $2\sin\left(\frac{x+y}{2}\right)\left\{\cos\left(\frac{x-y}{2}\right) - \cos\left(\frac{x+y}{2}\right)\right\} = 0$

or $2\sin\left(\frac{x+y}{2}\right) 2\sin\left(\frac{x}{2}\right)\sin\left(\frac{y}{2}\right) = 0$

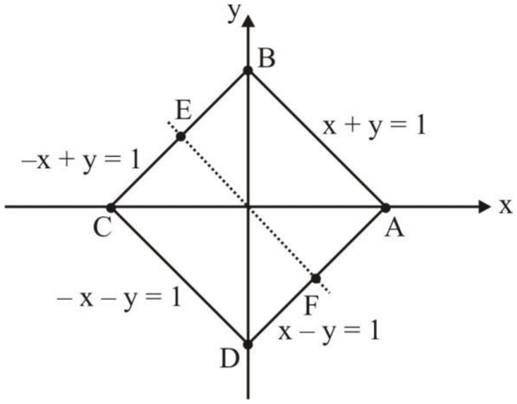
Either $\frac{x+y}{2} = n\pi$

or $\frac{x}{2} = n\pi$ or $\frac{y}{2} = n\pi$

Either x + y = 2nπ or x = 2nπ

Or $y = 2n\pi$

$\therefore$ $|x| + |y| = 1$




$\therefore$ $|x| \le 1$

and $|y| \le 1$

Hence, $x + y = 0$

or $x = 0$ or $y = 0$ clearly $y = 0$ cuts the curve $|x|+|y|=1$ at A, C, $x = 0$, cuts the curve $|x|+|y|=1$ at B, D and $x + y = 0$ cuts the curve at E, F, hence 6 solutions are possible

11. The positive integer value of n > 3 satisfying the equation $\frac{1}{\sin\frac{\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}} + \frac{1}{\sin\frac{3\pi}{n}}$ is

______________

KEY. 7

SOL. $\frac{1}{\sin\theta} = \frac{1}{\sin 2\theta} + \frac{1}{\sin 3\theta}$

$\frac{1}{\sin\theta} = \frac{\sin 3\theta + \sin 2\theta}{\sin 2\theta \sin 3\theta}$

$\sin 2\theta \sin 3\theta = \sin\theta(\sin 3\theta + \sin 2\theta)$

$2\cancel{\sin\theta}\cos\theta \sin 3\theta = \cancel{\sin\theta}(\sin 3\theta + \sin 2\theta)$

$\sin 4\theta + \cancel{\sin 2\theta} = \sin 3\theta + \cancel{\sin 2\theta}$

$4\theta = \pi - 3\theta$

$7\theta = \pi$

$7.\frac{\pi}{n} = \pi$

$n = 7$

12. Set $a, b \in [-\pi, \pi]$ such that $\cos(a-b) = 1$ and $\cos(a+b) = \frac{1}{e}$. The number of pairs (a, b) satisfying the above system of equation is

Key. 4

Sol. Q $\cos(a-b) = \cos 0$

$\therefore$ $a - b = 2n\pi, n \in I$

$a - b = -2\pi, 0, 2\pi$

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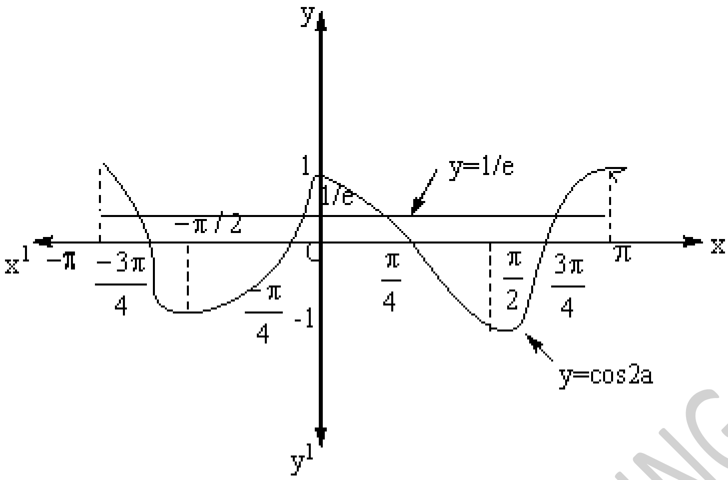
$\therefore \quad a+b=2\pi+2a, 2a, 2a-2\pi$

$\therefore \quad \cos(a+b)=\cos 2a, \cos 2a, \cos 2a=\frac{1}{e}$

$y=\cos 2a=\frac{1}{e}$




Hence, number of solutions is 4.

13. $2\cot^2 x-5\operatorname{cosec} x$ is equal to 1 for exactly 7 distinct values of $x\in[0,n\pi]$, then the greatest value of *n* is ............

Key. 7

Sol. $2\cot^2 x-5\operatorname{cosec} x=1$

$\Rightarrow \quad 2\operatorname{cosec}^2 x-5\operatorname{cosec} x-3=0$

$\Rightarrow \quad 2(\operatorname{cosec} x+1)(\operatorname{cosec} x-3)=0$

$\operatorname{cosec} x=-\frac{1}{2}, \operatorname{cosec} x=3$

$\operatorname{cosec} x=3$ gives the solution in 1st and 2nd quadrant, while $\operatorname{cosec} x=-\frac{1}{2}$ gives no solution. So, in $[0,2\pi]$, we get only two solutions. In $[0,6\pi]$, we get 6 solution and between $6\pi$ and $7\pi$, we get the seventh solution. Hence, n =7

14. The number of solutions of the equation $\sin^5 x-\cos^5 x=\frac{1}{\cos x}-\frac{1}{\sin x}(\sin x\neq\cos x)$ is

Key. 0

Sol. Given that

$\sin^5 x-\cos^5 x=\frac{\sin x-\cos x}{\sin x\cos x}$

$$\sin x\cos x\left[\frac{\sin^5 x-\cos^5 x}{\sin x-\cos x}\right]=1$$

$$\sin x\cos x\{\sin^4 x+\sin^3 x\cos x+\sin^2 x\cos^2 x+\sin x\cos x(\sin^2 x+\cos^2 x)\}=1$$

$$\sin x\cos x\{(\sin^2 x+\cos^2 x)^2-\sin^2 x\cos^2 x+\sin x\cos x\}=1$$

$$\Rightarrow\ \frac{1}{2}\sin 2x\left[1-\frac{1}{4}\sin^2 2x+\frac{1}{2}\sin 2x\right]=1$$

$$\sin 2x(\sin^2 2x-2\sin 2x-4)=-8$$

$$\sin^3 2x-2\sin^2 x-4\sin 2x+8=0$$

$$(\sin 2x-2)^2(\sin 2x+2)=0$$

$\Rightarrow \quad \sin 2x=\pm 2$ which is impossible

15. The number of solutions of x, which satisfy the equation $l\log_{|\sin x|}\left(1+\cos x\right)=2$ . when x $\in$ [0, 2π] is

Key. 0

Sol. $\log_{|\sin x|}(1 + \cos x) = 2 \Rightarrow 1 + \cos x = |\sin x|^2$

$\Rightarrow \quad 1 + \cos x = 1 - \cos^2 x \Rightarrow \cos x(1 + \cos x) = 0$

But $(1 + \cos x) \neq 0 \Rightarrow \cos x = 0, \Rightarrow \sin x = 1$.

But sinx = 1 is not possible because the base of log can not be 1. Hence no solution.

16. If x, y $\in$ [0, 10], then the number of solutions (x, y) of the inequation $3^{\sec^2 x-1}\sqrt{9y^2-6y+2}\le 1$ is

Key. 4

Sol. $3^{\tan^2 x}.\sqrt{(3y-1)^2+1}\le 1$

$3^{\tan^2 x}\ge 1$ and $\sqrt{(3y-1)^2+1}\ge 1$ $\tan^2 x = 0$, y = 1/3 x = 0, π, 2π, 3π

17. If a triangle ABC, prove that sin 10A + sin 10B + sin 10C = 4sin 5A sin 5B sin 5C.

Sol. LHS = 2 sin 5 (A + B) cos 5(A – B) + 2 sin 5C cos 5C.

$=2\sin\left(5\pi-5C\right)$ cos 5(A – B) + 2sin 5C cos 5C

= 2 sin 5C [cos (5A – 5B) + cos 5C]

= 2 sin 5C [cos (5A – 5B) + cos 5 $\left(\pi-\overline{AB}\right)$ ]

= 2 sin 5c [cos (5A – 5B) – cos (5A + 5B)]

= 4 sin 5A sin 5B sin 5C = RHS.

18. If $\tan\beta=\frac{\tan\alpha+\tan\gamma}{1+\tan\alpha\tan\gamma}$, prove that $\sin 2\beta=\frac{\sin 2\alpha+\sin 2\gamma}{1+\sin 2\alpha\sin 2\gamma}$

Sol. $\frac{\tan\alpha+\tan\gamma}{1+\tan\alpha\tan\gamma}=\frac{\sin\left(\alpha+\gamma\right)}{\cos\left(\alpha-\gamma\right)}$

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$$\therefore \qquad \sin 2\beta = \frac{2\tan\beta}{1+\tan^2\beta} = \frac{2\dfrac{\sin(\alpha+\gamma)}{\cos(\alpha-\gamma)}}{1+\dfrac{\sin^2(\alpha+\gamma)}{\cos^2(\alpha-\gamma)}}$$

$$= \frac{2\sin(\alpha+\gamma)\cos(\alpha-\gamma)}{\cos^2(\alpha-\gamma)+\sin^2(\alpha+\gamma)}$$

$$= \frac{\sin 2\alpha + \sin 2\gamma}{\dfrac{1+\cos 2(\alpha-\gamma)}{2}+\dfrac{1-\cos 2(\alpha+\gamma)}{2}}$$

$$= \frac{\sin 2\alpha + \sin 2\gamma}{1+\dfrac{1}{2}\left(\cos 2(\alpha-\gamma)-\cos 2(\alpha+\gamma)\right)}$$

$$= \frac{\sin^2\alpha + \sin\gamma}{1+\sin 2\alpha \sin 2\gamma}$$

= RHS

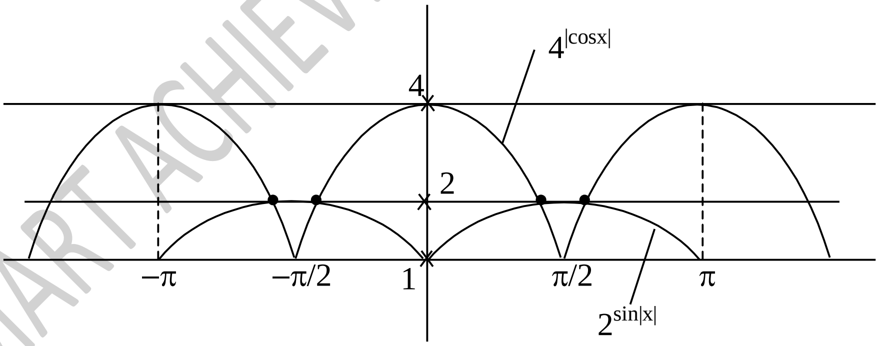
19. Solve the equation $\cos^{n+1}x - \sin^{n+1}x = 1$, where n is an odd natural number.

Sol. The given equation $\cos^{n+1}x - \sin^{n+1}x = 1$, where n + 1 an even integer.
Since LHS $\le$ 1 and RHS $\ge$ 1
$\Rightarrow \quad \cos^{n+1}x = 1 + \sin^{n+1}x = 1 \quad \Rightarrow \quad \sin x = 0 \quad \Rightarrow \quad x = n\pi$

20. Number of solutions of $2^{\sin|x|} = 4^{|\cos x|}$ in $[-\pi, \pi]$ is equal to

Key. 4

Sol. Number of solution of the equation is the number of intersection points of graphs $2^{\sin|x|}$ and $4^{|\cos x|}$ in $[-\pi,\pi]$
-




There are 4 intersection points in $[-\pi,\pi]$.