Sets & Relations, Mathematical Induction, Mathematical Reasoning

Single Correct Answer Type

1. If p, q, r are three propositions then the negation of $p \rightarrow (q \wedge r)$ is logically equivalent to

(a) $(p \vee \sim q) \wedge (p \vee \sim r)$ (b) $(p \wedge \sim q) \vee (p \wedge \sim r)$

(c) $(\sim p \vee q) \wedge (\sim p \vee r)$ (d) $(\sim p \wedge q) \vee (\sim p \wedge r)$

Key. B

Sol. $\sim \{p \rightarrow (q \wedge r)\} \equiv \sim \{\sim p \vee (q \wedge r)\}$

$\equiv p \wedge (\sim q \vee \sim r) \equiv (p \wedge \sim q) \vee (p \wedge \sim r)$

2. If the inverse of the conditional $p \rightarrow (\sim q \wedge \sim r)$ is false, then the truth values of the proportions p,q,r are respectively

(a) T,T,T (b) T,F,F (c) F,T,T (d) F,F,F

Key. D

Sol. The inverse of given conditional is $\sim p \rightarrow \sim(\sim q \wedge \sim r) \equiv \sim p \rightarrow (q \vee r)$. This is false implies that ~p is true and $(q \vee r)$ is false $\therefore$ p is false and each of q,r is false.

3. S – I : $\sim(\sim p \leftrightarrow \sim r) \equiv p \leftrightarrow q$

S – II : $\sim p \leftrightarrow \sim q \equiv (p \vee \sim q) \wedge (q \vee \sim p)$

Which of the following is true about above two statements S – I and S – II.

(a) Both S – I, S – II are true and S – II is a correct explanation of S – I.

(b) Both S – I, S – II are true, but S – II is not a correct explanation of S – I.

(c) S – I is true and S – II is false (d) S – I is false and S – II is true

Key. D

Sol. S – I : $\sim(\sim p \leftrightarrow \sim q) \equiv \sim\{(\sim p \rightarrow \sim q) \wedge (\sim q \rightarrow \sim p)\}$

$\equiv \sim\{(p \vee \sim q) \wedge (q \vee \sim p)\}$

$\equiv (\sim p \wedge q) \vee (\sim q \wedge p)$

But $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p) \equiv (\sim p \vee q) \wedge (\sim q \vee p)$

$\therefore$ S – I is false

S – II : $\sim p \leftrightarrow \sim q \equiv (\sim p \rightarrow \sim q) \wedge (\sim q \rightarrow \sim p) \equiv (p \vee \sim q) \wedge (q \vee \sim p)$

$\therefore$ S – I is true

4. $((p \rightarrow q) \wedge (\sim p \rightarrow q)) \rightarrow q$ is logically equivalent to

(a) a tautology (b) a contradiction (c) $(\sim p \vee p) \rightarrow q$ (d) $(p \wedge \sim p) \rightarrow q$

Key. A

Sol. $\left((p \to q) \wedge (\sim p \to q)\right) \to q$

$\equiv (\sim p \vee q) \wedge (p \vee q) \to q$

$\equiv (\sim p \wedge q) \vee q \to \equiv c \vee q \to q \equiv q \to q \equiv \sim q \vee q \equiv t$

5. Which of the following is a contradiction?

(a) $p \to (q \to p)$ (b) $p \to (p \vee q)$

(c) $(p \vee q) \to (\sim p \wedge \sim q)$ (d) $(p \vee \sim p) \to (q \wedge \sim q)$

Key. D

Sol. a) $p \to (q \to p) \equiv p \to (\sim q \vee p) \equiv \sim p \vee (\sim q \vee p) \equiv (\sim p \vee p) \vee \sim q \equiv t \vee \sim q \equiv t$

b) $p \to (p \vee q) \equiv \sim p \vee (p \vee q) \equiv t \vee q \equiv t$

c) $p \vee q \to (\sim p \wedge \sim q) \equiv \sim (p \vee q) \vee (\sim (p \vee q)) \equiv \sim (p \vee q)$

d) $(p \vee \sim p) \to (q \wedge \sim q) \equiv t \to c \equiv (\sim t \vee c) \equiv c \vee c \equiv c$

6. Let 'A' be a non-empty sub-set of R. Let 'P' be the statement "There is a rational number $x \in A$ such that $2x - 1 \geq 0$". Which of the following statements is the negation of the statement P?

(a) There is a rational number $x \in A$ such that $x < \frac{1}{2}$

(b) There is no rational number $x \in A$ such that $x < \frac{1}{2}$

(c) $x \in A$ and $x \leq \frac{1}{2} \Rightarrow$ x is not rational

(d) Every rational number $x \in A$ satisfies $x < \frac{1}{2}$

Key. D

Sol. Negation of P : There does not exists a rational number $x \in A$ such that $2x - 1 \geq 0$

ie ; for every rational number $x \in A, 2x - 1 \ngeq 0$

ie ; for every rational number $x \in A, 2x - 1 < 0$

7. The dual of converse of the conditional $(p \vee q) \to \sim q$ is logically equivalent to

(a) a tautology (b) a contradiction (c) $(p \wedge q)$ (d) $(\sim p \vee \sim q)$

Key. C

Sol. Converse of $\{(p \vee q) \to \sim q\}$ is $\{\sim q \to (p \vee q)\}$

Which is logically equivalent to $q \vee (p \vee q) \equiv (p \vee q)$

Dual of $(p \vee q)$ is $(p \wedge q)$

8. Which of the following are mathematically acceptable statements ?

(i) All prime numbers are odd numbers (ii) Every set is a finite set

(iii) $\sqrt{2}$ is a rational number or an irrational number

(a) Only (i), (ii) (b) Only (ii), (iii) (c) Only (i), (iii) (d) All (i), (ii), (iii)

Key. D

Sol. (i), (ii), (iii) are mathematically acceptable statements with truth values F, F, T respectively.

9. Which of the following is not a negation of the statement. “There exists a rational number x such that x^2 = 2”.

(i) There does not exists a rational number x such that $x^2 = 2$

(ii) For all rational numbers x, $x^2 \neq 2$ (iii) For no rational number x, $x^2 \neq 2$

(a) Only (i) (b) Only (ii) (c) Only (iii) (d) (ii), (iii)

Key. C

Sol. Negation of P is “For no rational number x, $x^2 = 2$”. Hence (iii) is not a negation of P.

10. Let R, S are two symmetric relations and SoR, RoS are their composite relations. Then which of the following is true?

(a) RoS and SoR are equal

(b) RoS and SoR are symmetric relations

(c) RoS and SoR are symmetric only when R = S

(d) RoS and SoR are symmetric if f RoS = SoR

Key. D

Sol. RoS is symmetric if $(RoS)^{-1} = RoS$

But $(RoS)^{-1} = S^{-1}oR^{-1} = SoR$ $\therefore$ RoS is symmetric iff RoS = SoR.

Similarly SoR is symmetric iff SoR = RoS

11. R = {(1, 2), (2, 3), (3 4)} be a relation on the set of natural numbers. Then the least number of elements that must be included in R to get a new relation S where S is an equivalence relation, is

(a) 5 (b) 7 (c) 9 (d) 11

Key. D

Sol. (1, 1), (2, 2), (3, 3), (4, 4) are to be included so that S is reflexive.

(2, 1), (3, 2), (4, 3) are to be included so that S is symmetric.

(1, 3), (2, 4) are to be included so that S is transitive.

Then (3, 1), (4, 2) are to be included so that S is symmetric.

12. Let R = {(3, 3), (6, 6), (9, 9), (6, 12), (3, 9), (3, 12), (3, 6)} be a relation on the set A = {3, 6, 9, 12}Then the relation R^{-1} is

(a) not reflexive (b) not symmetric (c) transitive (d) all the above

Key. D

Sol. R^{-1} = {(3, 3), (6, 6), (9, 9), (12, 6), (9, 3), (12, 3), (6, 3)}.

(12, 12) $\in R^{-1} \Rightarrow R^{-1}$ is not reflexive

(12, 6) $\in R^{-1}$ but (6, 12) $\notin R^{-1} \Rightarrow R^{-1}$ is not symmetric

The condition (x, y), (y, z) $\in R^{-1} \Rightarrow (x,z) \in R^{-1}$ is satisfied by R^{-1}

$\therefore$ R^{-1} is transitive.

13. Let R be the real line. Consider the following subsets of the plane R x R.

S = {(x, y) : y = 2x – 1 and – 1 < x < 1 }

T = {(x, y) : xy is a rational number}

Then which of the above two relations is an equivalence relation?

(a) Only S (b) Only T (c) Both S,T (d) Neither S nor T

Key. D

Sol. $x = 0 \in (-1,1)$ But (0, 0) $\notin S$

$\therefore$ S is not reflexive and hence it is not an equivalence relation.

$x = \sqrt[3]{2} \in R$, but (x, x) $\notin T$ when $x = \sqrt[3]{2}$

$\therefore$ R is not reflexive and hence it is not an equivalence relation.

14. In a town of 10,000 families it was found that 40% families buy news paper A, 20% families buy news paper B, and 10% families buy news paper C. Also 5% families buy A and B, 3% buy B and C, 4% buy A and C, and 2% buy all the three news papers. Then the number of families which buy exactly one of A, B, C is

(a) 4800 (b) 5200 (c) 5400 (d) 6400

Key. B

Sol. Number of families which buy exactly one of A, B, C

= n(A) + n(B) + n(C) – 2 $\left[n(A\cap B) + n(B\cap C) + n(C\cap A)\right] + 3\left[n(A\cap B\cap C)\right]$

15. Let H be the set of all houses in a city where each house is faced in one of the directions East, West, North, South.

Let R = {(x, y) : (x, y) $\in$ HXH and x,y are faced in same direction} Then the relation R is

(a) Not reflexive, symmetric and transitive

(b) Reflexive, symmetric, not transitive

(c) Symmetric, not reflexive, not transitive

(d) An equivalence relation

Key. D

Sol. Clearly R is reflexive, symmetric & transitive.

16. Let A, B are two sets such that n(A) = 4 and n(B) = 6. Then the least possible number of elements in the power set of $(A \cup B)$ is

(a) 16 (b) 64 (c) 256 (d) 1024

Key. B

Sol. $\text{Min}\left[n(A\cup B)\right] = \text{Max}\{n(A), n(B)\} = 6$

$\text{If } n(A\cup B) = 6$ then n $\left[P(A\cup B)\right] = 2^6 = 64$

17. Let R be a relation defined by R = {(4, 5), (1, 4), (4, 6), (7, 6), (3, 7)} on N. Then RoR^{-1} is

(a) symmetric, reflexive, but not transitive

(b) symmetric, transitive, but not reflexive

(c) reflexive, anti symmetric, and not transitive

(d) a partial order relation

Key. B

Sol. $R = \{(4,5),(1,4),(4,6),(7,6),(3,7)\}$

$R^{-1} = \{(5,4),(4,1),(6,4),(6,7),(7,3)\}$

$ROR^{-1} = \{(5,5),(5,6),(4,4),(6,5),(6,6),(7,7)\}$

Clearly ROR^{-1} is not relative, but it is symmetric and transitive

18. Let R be a relation defined on the set of real numbers by aRb U 1 + ab > 0. Then R is

(a) reflexive, symmetric, but not transitive

(b) symmetric, transitive, but not reflexive

(c) symmetric, not reflexive, not transitive

(d) reflexive, anti symmetric, not transitive

Key. A

$a\ R\ a\ \forall$ real number 'a' $[\therefore 1 + a^2 > 0] \Rightarrow$ R is reflexive

Sol. $a\ R\ b \Rightarrow 1 + ab > 0 \Rightarrow 1 + ba > 0 \Rightarrow b\ R\ a \Rightarrow$ R is symmetric

If $a = \frac{1}{2}, b = \frac{-2}{3}, c = -3$ then a R b and b R c. But $(a,c) \notin R \therefore R$ is not transitive.

19. Let R_1, R_2 are relations defined on Z such that aR_1b U (a – b) is divisible by 3 and

a R_2 b U (a – b) is divisible by 4. Then which of the two relations (R_1 Ė R_2), (R_1 Ç R_2) is an equivalence relation?

(a) (R_1 È R_2) only

(b) (R_1 Ç R_2) only

(c) Both (R_1 È R_2),(R_1 Ç R_2)

(d) Neither (R_1 È R_2) nor (R_1 Ç R_2)

Key. B

Sol. Clearly R_1,R_2 are equivalence relations Þ both R_1 È R_2 and R_1 Ç R_2 are also equivalence relations

20. Consider the following relations :

R = {(x, y) / x, y are real numbers and x = wy for some rational number w}

S = $\left\{ \frac{m}{n}, \frac{p}{q} / m, n, p \text{ and } q \text{ are integers such that } n, q \ ^1\ 0 \text{ and } qm = pn \right\}$. Then

(a) Both R, S are equivalence relations

(b) R is an equivalence relation, but not S

(c) S is an equivalence relation, but not R

(d) Neither R nor S is an equivalence relation

Key. C

Sol. $x\,R\,x\ \forall$ *real number* $x\ [\because x = 1x] \Rightarrow R$ *is reflexive*

If x = 0, y = 2 then xRy but (y, x) $\notin R \Rightarrow R$ is not symmetric

$\therefore$ R is not an equivalence relation.

Clearly $\left(\frac{a}{b}, \frac{a}{b}\right) \in S\ \forall a, b \in Z,\ b \neq 0$

Clearly $\left(\frac{a}{b}, \frac{c}{d}\right) \in S \Rightarrow \left(\frac{c}{d}, \frac{a}{b}\right) \in S\ \forall\ a, b, c, \in Z,$ and $b, d \neq 0$

Now $\left(\frac{a}{b}, \frac{c}{d}\right) \in S$ and $\left(\frac{c}{d}, \frac{e}{f}\right) \in S \Rightarrow \left(\frac{a}{b}, \frac{e}{f}\right) \in S\ \forall\ a, b, c, d, e, f \in Z$ and $b, d, f \neq 0$

because ad = bc, cf = de $\Longrightarrow$ af = be

$\therefore$ S is an equivalence relation.

21. Let * and o defined by a*b = 2^{ab} and aob = a^b for a,b $\in$ R^+ where R^+ is the set of all positive real numbers. Then which of the above two binary operations are associative?

(a) Only '*' (b) Only 'o' (c) Both '*' and 'o' (d) Neither '*' nor 'o'

Key. D

Sol. $(a*b)*c = (2^{ab})*c = (2)^{(2^{ab}c)}$ and

$a*(b*c) = a*(2^{bc}) = 2^{(a2^{bc})}$ which are not equal

$(aob)oc = (a^b)oc = (a^b)^c$ and $ao(boc) = ao(b^c) = (a)^{bc}$ which are not equal

22. For all $n \in N$ $\{3(5^{2n+1}) + 2^{3n+1}\}$ is divisible by k, where k is prime. Then the least prime greater than k is

(a) 13 (b) 17 (c) 19 (d) 29

Key. C

Sol. It can be verified that given expression is divisible by 17 for n = 1, 2

$\therefore$ K = 17 and least prime greater than k is 19

23. Assertion A : For all $n \in N$, of $2.1^2 + 3.2^2 + 4.3^2 +$ ------- to n terms = $\frac{1}{12}$ (n) (n +1) (n+2) (3n+1)

Reason R : If $n \in N$ then $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$ and $\sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2$

(a) Both A, R are true and R is the correct explanation of A.

(b) Both A, R are true, but R is not the correct explanation of A

(c) A is true, R is false (d) A is false, R is true

Key. A

Sol. $T_n = (n+1)n^2 = n^3 + n^2$

$$S_n = \Sigma T_n = \Sigma n^2 = \frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{1}{12}(n)(n+1)(n+2)(3n+1)$$ (or) Give value for n and find k.

24. Which of the following two statements is / are true?

S_1 = The sum of the cubes of three successive natural numbers is always divisible by 9

S_2 = The sum of the squares of three successive even natural numbers is always divisible by 8.

(a) Only S_1 (b) Only S_2 (c) Both S_1, S_2 (d) Neither S_1 nor S_2

Key. A

Sol. By verification S_1 is true & S_2 is false.

25. The value of $\sum_{r=1}^{n} \left[\frac{1^2 + 2^2 + 3^2 + - - - + r^2}{2r+1}\right]$ = k(n) (n + 1) (n + 2) where k = ___

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$ (c) $\frac{1}{18}$ (d) $\frac{1}{6}$

Key. C

Sol. $$\sum_{r=1}^{n} \frac{r(r+1)(2r+1)}{6(2r+1)} = \frac{1}{6}\left[\Sigma (r^2 + r)\right] = \frac{1}{6}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{1}{18}(n)(n+1)(n+2)$$

$\therefore$ K = 18

(or) Give value for n and find K

26. Let s(k) : 1 + 3 + 5 + ----- + (2k – 1) = 2 + k^2. Then which of the following is true?

(a) s(3) is true (b) s(k) $\Rightarrow$ s(k + 1)

(c) s(k) $\not\Rightarrow$ s(k + 1)

(d) Principle of mathematical induction can be used to prove the formula

Key. B

Sol. S(3) is the statement : 1 + 3 + 5 = 3 + 3^2 which is false. If S(k) is true then by adding (2k + 1) we get 1+3+5+ ----- + (2k – 1) + (2k + 1)

= 3 + $(k + 1)^2$

$\therefore\ s(k) \Rightarrow s(k+1)$

27. Two relations R and S are defined on set A = {1, 2, 3, 4, 5} as following.

R = {(x, y) : $|x^2 - y^2| < 16$}

S = {(x, y) ; $x \le y$}

Then which of the above two relations is an equivalence relation?

(a) Only R (b) Only S (c) Both R, S (d) Neither R nor S

Key. D

Sol. R is not transitive. Taking x = 2, y = 4, z = 5,

Both (x, y), (y, z) $\in R$. But (x, z) $\notin R$

S is anti symmetric, reflexive and transitive.

$\therefore$ Neither R, nor S is an equivalence relation