Quadratic Equations & Theory of Equations

Integer Answer Type

1. If λ is the minimum value of the expression $|x-p|+|x-15|+|x-p-15|$ for x in the range $p \le x \le 15$ where 0 < *p* < 15. Then $\frac{\lambda}{5}$ =

Key. 3

Sol. $|x-p| = x-p \quad (Since\, x \ge p)$

$|x-15| = 15-x \; (Since\, x \le 15)$

$|x-(p+15)| = (p+15)-x \; (as\; 15+p > x)$

$\therefore$ expression reduces to

$E = x-p+15-x+p+15-x$

$E = 30-x$

$\therefore E_{\min}$ occurs when $x=15$

$\therefore \lambda = 15$

2. Let $P(x) = x^2+bx+c$, where b and c are integer. If $P(x)$ is a factor of both x^4+6x^2+25 and $3x^4+4x^2+28x+5$, find the value of P(1).

Key. 4

Sol. Since $P(x)$ divides into both of them

Hence P(x) also divides

$(3x^4+4x^2+28x+5)-3(x^4+6x^2+25)$

$= -14x^2+28x-70 = -14(x^2-2x+5)$

Which is a quadratic, Hence $P(x) = x^2-2x+5$

$\therefore P(1) = 4$

3. Largest integral value of m for which the quadratic expression $y = x^2+(2m+6)x+4m+12$ is always positive, $\forall x \in R$, is

Key. 0

Sol. $D < 0 \Rightarrow -3 < m < 1 \Rightarrow m = 0$

4. The number of solution of the equation $e^{2x}+e^{x}+e^{-2x}+e^{-x}=3(e^{-2x}+e^{x})$ is

Key. 1

Sol. $x=ln2$

5. Let a,b,c be the three roots of the equation $x^3+x^2-333x-1002=0$. If P= $a^3+b^3+c^3$ then the value of $\frac{P}{2006}$ =__

Key. 1

Sol. Let α be the root of the given cubic where α can take values a, b, c

Hence $\alpha^3+\alpha^2-333\alpha-1002=0$ or $\alpha^3=1002+333\alpha-\alpha^2$

$\therefore \Sigma\alpha^3=\Sigma 1002+333\Sigma\alpha-\Sigma\alpha^2=3006+333\Sigma\alpha-\left[(\Sigma\alpha)^2-2\Sigma\alpha_1\alpha_2\right]$

But $\Sigma\alpha=-1;\Sigma\alpha_1\alpha_2=-333$

$\therefore a^3+b^3+c^3=3006-333-[1+666]=3006-333-667=3006-100=2006$ = P

6. The number of the distinct real roots of the equation $(x+1)^5=2(x^5+1)$ is

Key. 3

Sol. $(x+1)^5=2(x^5+1)$

Let $f(x)=\frac{(x+1)^5}{(x^5+1)}$ $(x\neq -1)$

$\Rightarrow$ $f'(x)=\frac{5(x+1)^4(1-x^4)}{(x^5+1)^2}$

$\Rightarrow$ $x=1$ is maximum

As, $f(0)=1$ and $f(1)=16$

And $\lim\limits_{x\to\pm\infty} f(x)=1 \Rightarrow f(x)=2$ has two solutions but given equation has three solutions.

because x = -1 included.

7. The equation $2(\log_3 x)^2-|\log_3 x|+a=0$ has exactly four real solutions if $a\in\left(0,\frac{1}{K}\right)$, then the value of K is __

Key. 8

Sol. on putting $\log_3 x=t,$ we get

$2t^2-|t|+a=0$ ...(i)

If $t>0,$ then $2t^2-t+a=0$ ...(ii)

If $t<0,$ then $2t^2+t+a=0$ ...(iii)

If Eq. (i) has four roots then Eq. (ii) must have both roots positive and Eq. (iii) has both roots negative. Now, Eq. (ii) has both roots positive, if $D>0$

$a/2>0$

$\Rightarrow \quad 1-8a>0, a>0$

$\Rightarrow \quad a\in\left(0,\frac{1}{8}\right)$ on taking intersection.

Again, Eq. (iii) has both roots negative, if $D>0, a/2>0.$

We again get $a\in\left(0,\frac{1}{8}\right)\Rightarrow K=80$

8. Let α,β be the roots of $x^2-x+p=0$ and λ,δ be the roots of $x^2-4x+q=0$ such that $\alpha,\beta,\gamma,\delta$ are in G.P and $p\ge 2.$ If $a,b,c\in\{1,2,3,4,5\}$, let the number of equation of the form $ax^2+bx+c=0$ which have real roots be *r,* then the minimum value of $\frac{pqr}{1536}=$

Key. 1

Sol. $(\alpha+\beta)=1, \alpha\beta=p, \gamma+\delta=4, \gamma\delta=q$

Since $\alpha,\beta,\gamma,\delta$ are in G.P

$$\therefore \frac{\beta}{\alpha}=\frac{\delta}{\gamma}\Rightarrow\frac{\beta+\alpha}{\beta-\alpha}=\frac{\delta+\gamma}{\delta-\gamma}\Rightarrow\frac{(\beta+\alpha)^2}{(\beta+\alpha)^2-4\alpha\beta}=\frac{(\delta+\gamma)^2}{(\delta+\gamma)^2-4\delta\gamma}$$

$$\Rightarrow\frac{1}{1-4p}=\frac{16}{16-4q}=\frac{4}{4-q}$$

$$\Rightarrow 4-q=4-16p$$

Now, $p\ge 2 \therefore q\ge 32$

For the given equation $ax^2+bx+c=0$ to have real roots $b^2-4ac\ge 0$

$$\therefore ac\le\frac{b^2}{4}$$

b	$\frac{b^2}{4}$	Possible values of ac such that $ac\le\frac{b^2}{4}$	No. of possible pairs (a,c)		Value of ac	Possible pairs (a,c)
					1	(1,1)
2	1	1	1		2	(1,2), (2,1)
3	2.25	1,2	3		3	(1,3). (3,1)
4	4	1,2,3,4	8		4	(1,4),(4,1),(2,3)
5	6.25	1,2,3,4,5,6	12		5	(1,5),(5,1)
		Total	24		6	(2,3),(3,2)

Hence number of quadratic equation with real roots, r = 24

Now from (i) and (ii) the minimum value of $pqr=2.32.24=1536$

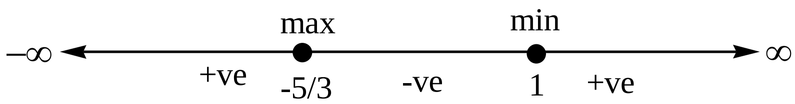
9. Let α, β and γ be the roots of equation $f(x)=0,$ where $f(x)=x^3+x^2-5x-1.$ Then the value of $\left|[\alpha]+[\beta]+[\gamma]\right|,$ where $[.]$ denotes the greatest integer function, is equal to

Key. 3

Sol. Given $f(x)=x^3+x^2-5x-1$

$\therefore f'(x)=3x^2+2x-5$. The roots of $f'(x)=0$ are $-\frac{5}{3}$ and 1

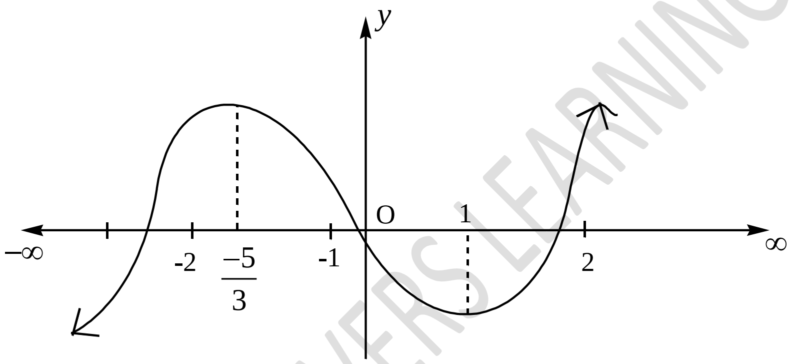
Writing the sign scheme for $f'(x)$,



Also , $f(-\infty)=-\infty<0, f(\infty)=\infty>0$

$f(1)=-4, f\left(-\frac{5}{3}\right)=\frac{148}{27}$

Now, graph of $y=f(x)$ is as follows



$f(-3)=-27+9+15-1=-4<0$

$f(-2)=-8+4+10-1>0$

$f(-1)=4>0, f(0)=-1<0$

$f(2)=1>0$

$\therefore -3<\alpha<-2, -1<\beta<0, 1<\gamma<2$

$|[\alpha]+[\beta]+[\gamma]|=|-3-1+1|=3$

10. The set of real parameter 'a' for which the equation $x^4-2ax^2+x+a^2-a=0$ has all real solutions, is given by $\left[\frac{m}{n},\infty\right)$ where *m* and *n* are relatively prime positive integers, then the value of $(m+n)$ is

Key. 7

Sol. We have $a^2-(2x^2+1)a+x^4+x=0$

$$\therefore a=\frac{(2x^2+1)\pm\sqrt{(2x^2+1)^2-4(x^4+x)}}{2}$$

$$2a=(2x^2+1)\pm(2x-1)$$

On solving +ve & -ve sign we got

$a \geq \frac{3}{4}$

$\therefore m+n=7$

11. Number of positive integer n for which n^2+96 is a perfect square is

Key. 4

Sol. Suppose m is positive integer such that $n^2+96=m^2$ then

$(m-n)(m+n)=96$

As $m-n<m+n$ and $m-n, m+n$ both must be even

So, the only possibilities are

$m-n=2, m+n=48 : m-n=4, m+n=24$

$m-n=6, m+n=16 : m-n=8, m+n=12$

So, the solutions of (m,n) are $(25,23),(14,10),(11,5),(10,2)$

12. If α, β be the roots of $x^2+px-q=0$ and γ, δ are the roots of $x^2+px+r=0$, $q+r \neq 0$, then $\frac{(\alpha-\gamma)(\alpha-\delta)}{(\beta-\gamma)(\beta-\delta)}$ is equal to

Key. 1

Sol. Here, $\alpha+\beta=-p=\gamma+\delta$

$(\alpha-\gamma)(\alpha-\delta)=\alpha^2-\alpha(\gamma+\delta)+\gamma\delta=\alpha^2-\alpha(\alpha+\beta)+r$

$=-\alpha\beta+r=q+r$

Similarly $(\beta-\gamma)(\beta-\delta)=q+r$

So, ratio is 1

13. Number of real roots of $2x^{99}+3x^{98}+2x^{97}+3x^{96}+.......+2x+3=0$ is

Key. 1

Sol. Given equation can be written as $(2x+3)(x^{98}+x^{96}+......+1)=(2x+3)\frac{(x^{100}-1)}{x^2-1}$

So, the real roots are $x=\pm 1, \frac{-3}{2}$, out of which ± 1 are not roots of given equation.

14. If λ is the minimum value of the expression $|x-p|+|x-15|+|x-p-15|$ for x in the range $p \leq x \leq 15$ where 0 < *p* < 15. Then $\frac{\lambda}{5}$ =

Key. 3

Sol. $|x-p|=x-p \quad (Since\, x \geq p)$

$|x-15|=15-x \ (Since\, x \leq 15)$

$|x-(p+15)| = (p+15)-x \; (as\; 15+p > x)$

$\therefore \exp ression\; reduces\;\; to$

$E = x - p + 15 - x + p + 15 - x$

$E = 30 - x$

$\therefore E_{\min}\; occurs\; when\; x = 15$

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15. Let $P(x) = x^2 + bx + c$, where b and c are integer. If $P(x)$ is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, find the value of P(1).

Key. 4

Sol. Since $P(x)$ divides into both of them

Hence P(x) also divides

$(3x^4 + 4x^2 + 28x + 5) - 3(x^4 + 6x^2 + 25)$

$= -14x^2 + 28x - 70 = -14(x^2 - 2x + 5)$

Which is a quadratic, Hence $P(x) = x^2 - 2x + 5$

$\therefore P(1) = 4$

16. Largest integral value of m for which the quadratic expression $y = x^2 + (2m+6)x + 4m + 12$ is always positive, $\forall x \in R$, is

Key. 0

Sol. $D < 0 \Rightarrow -3 < m < 1 \Rightarrow m = 0$

17. For a twice differentiable function $f(x)$, $g(x)$ is defined as $g(x) = f'(x)^2 + f''(x)f(x)$ on $[a,e]$. If for $a < b < c < d < e$, $f(a) = 0$, $f(b) = 2$, $f(c) = -1$, $f(d) = 2$, $f(e) = 0$ then find the minimum number of zeros of g(x).

Key. 6

Sol. $g(x) = f'(x)^2 + f''(x)f(x) = \frac{d}{dx}\left(f(x)f'(x)\right)$

Let $h(x) = f(x)f'(x)$

Then, $f(x) = 0$ has four roots namely a, α, β, e

where $b < \alpha < c$ and $c < \beta < d$.

And $f'(x) = 0$ at three points k_1, k_2, k_3 where

$a < k_1 < \alpha, \alpha < k_2 < \beta, \beta < k_3 < e$

[$\because$ Between any two roots of a polynomial function $f(x) = 0$ there lies atleast one root of $f'(x) = 0$]

There are atleast 7 roots of $f(x).f'(x) = 0$

$\therefore$ There are atleast 6 roots of $\frac{d}{dx}\left(f(x)f'(x)\right) = 0$ i.e. of $g(x) = 0$

18. $f(x)$ is a polynomial of 6th degree and $f(x) = f(2-x)\ \forall\ x \in R$. If $f(x) = 0$ has 4 distinct real roots and two real and equal roots then sum of roots of $f(x) = 0$

Key. 6

Sol. $f(\alpha) = f(2-\alpha) = 0$ sum of roots $= 4$

When $\alpha \neq 2-\alpha$

Where $\alpha = 2-\alpha$ i.e., $\alpha = 1$ sum of roots $= 2$

$\therefore$ Total sum $= 6$

19. $(1+x)(1+x+x^2)(1+x+x^2+x^3)\ldots\ldots.(1+x+x^2+.....+x^{100})$

When written in the ascending power of x then (the highest exponent of x) $-$ 5045 is

Key. 5

Sol. Highest exponent of x = 1 + 2 + 3 + ….. + 100 $= \frac{100(101)}{2} : 5050$

20. If the roots of the equation $x^3 - ax^2 + 14x - 8 = 0$ are all real and positive, then the minimum value of [a] (where [a] is the greatest integer of a) is

Key. 6

Sol. $f(x) = x^3 - ax^2 + 14x - 8 = 0$

$\frac{\alpha+\beta+\gamma}{3} \geq (\alpha.\beta\gamma)^{1/3}$

$\frac{a}{3} \geq (8)^{1/3}$

$a \geq 6$

Quadratic Equations & Theory of Equations

Single Correct Answer Type

1. Let α and β be the roots of $x^2-6x-2=0$ with $\alpha>\beta$ if $a_n=\alpha^n-\beta^n$ for $n\geq 1$ then the value of $\frac{a_{10}-2a_8}{3a_9}=$

1) 1 2) 2
3) 3 4) 4

Key. 2

Sol. $\alpha^2-6\alpha-2=0$ $\beta^2-6\beta-2=0$

$\Rightarrow \alpha^{10}-6\alpha^9-2\alpha^8=0$ ....(1)

$\Rightarrow \beta^{10}-6\beta^9-2\beta^8=0$ ....(2)

subtract (2) from (1)

2. If a,b,c are positive real numbers such that $a+b+c=1$ then the least value of $\frac{(1+a)(1+b)(1+c)}{(1-a)(1-b)(1-c)}$ is

1) 16 2) 8
3) 4 4) 5

Key. 2

Sol. $a=1-b-c$

$\Rightarrow 1+a=(1-b)+(1-c)\geq 2\sqrt{(1-b)(1-c)}$

$\therefore (1+a)(1+b)(1+c)\geq 8(1-a)(1-b)(1-c)$

3. The range of values of $'a'$ for which all the roots of the equation $(a-1)(1+x+x^2)^2=(a+1)(1+x^2+x^4)$ are imaginary is

1) $(-\infty,-2]$ 2) $(2,\infty)$
3) $(-2,2)$ 4) $[2,\infty)$

Key. 3

Sol. The given equation can be written as $(x^2+x+1)(x^2-ax+1)=0$

4. If α,β are the roots of the equation $ax^2+bx+c=0$ and $S_n=\alpha^n+\beta^n$ then $aS_{n+1}+bS_n+cS_{n-1}=$ $(n\geq 2)$

1) 0 2) $a+b+c$
3) $(a+b+c)n$ 4) $n^2\ abc$

Key. 1

Sol. $S_{n+1}=\alpha^{n+1}+\beta^{n+1}$

$=(\alpha+\beta)(\alpha^n+\beta^n)-\alpha\beta(\alpha^{n-1}+\beta^{n-1})$

$= -\frac{b}{a}.S_n - \frac{c}{a}.S_{n-1}$

5. A group of students decided to buy a Alarm Clock priced between Rs. 170 to Rs 195. But at the last moment, two students backed out of the decision so that the remaining students had to pay 1 Rupee more than they had planned. If the students paid equal shares, the price of the Alarm Clock is

1) 190 2) 196
3) 180 4) 171

Key. 3

Sol. Let cost of clock = x
number of students $= n$
then $\frac{x}{n-2} = \frac{x}{n} + 1 \Rightarrow x = \frac{n^2 - 2n}{2}$
$\Rightarrow 170 \le \frac{n^2 - 2n}{2} \le 195$

6. If $\tan A, \tan B$ are the roots of $x^2 - Px + Q = 0$ the value of $\sin^2(A+B) =$ $(where\ P, Q \in R)$

1) $\frac{P^2}{P^2 + (1-Q)^2}$ 2) $\frac{P^2}{P^2 + Q^2}$
3) $\frac{Q^2}{P^2 + (1-Q)^2}$ 4) $\frac{P^2}{(P+Q)^2}$

Key. 1

Sol. $\tan(A+B) = \frac{P}{1-Q}$ then $\sin^2(A+B) = \frac{\tan^2(A+B)}{1+\tan^2(A+B)}$

7. The number of solutions of $|[x] - 2x| = 4$ where $[x]$ is the greatest integer $\le x$ is

1) 2 2) 4
3) 1 4) Infinite

Key. 2

Sol. If $x = n \in Z$, $|n - 2n| = 4 \Rightarrow n = \pm 4$
If $x = n + K$ where $0 < K < 1$ then $|n - 2(n+k)| = 4$, it is possible if $K = \frac{1}{2}$
$\Rightarrow |-n-1| = 4$
$\therefore n = 3, -5$

8. Let a, b and c be real numbers such that $a + 2b + c = 4$ then the maximum value of $ab + bc + ca$ is

1) 1 2) 2 3) 3 4) 4

Key. 4

Sol. Let $ab + bc + ca = x$

$\Rightarrow 2b^2+2(c-2)b-4c+c^2+x=0$

Since $b\in R,$

$\therefore c^2-4c+2x-4\le 0$

Since $c\in R$

$\therefore x\le 4$

9. For the equation $3x^2+px+3=0, p>0,$ if one root is the square of the other then value of P is

1) $\frac{1}{3}$ 2) 1

3) 3 4) $\frac{2}{3}$

Key. 3

Sol. $\alpha+\alpha^2=-\frac{p}{3}$

$\alpha^3=1$

10. If the equations $2x^2+kx-5=0$ and $x^2-3x-4=0$ have a common root, then the value of k is

1) -2 2) -3

3) $\frac{27}{4}$ 4) $-\frac{1}{4}$

Key. 2

Sol. If 'α' is the common root then $2\alpha^2+k\alpha-5=0,\ \alpha^2-3\alpha-4=0$ solve the equations.

11. If α and β are the roots of the equation $x^2-x+1=0$ then $\alpha^{2009}+\beta^{2009}=$

1) 1 2) 2

3) -1 4) -2

Key. 1

Sol. $x=\frac{1\pm i\sqrt{3}}{2}$

$\therefore \alpha=-\omega,\ \beta=-\omega^2$

12. If $P(Q-r)x^2+Q(r-P)x+r(P-Q)=0$ has equal roots then $\frac{2}{Q}=$

$(where\ P,Q,r\in R)$

1) $\frac{1}{P}+\frac{1}{r}$ 2) $\frac{1}{P}-\frac{1}{r}$

3) $P+r$ 4) Pr

Key. 1

Sol. Product of the roots $=1$

13. If $(1+K)\tan^2 x-4\tan x-1+K=0$ has real roots $\tan x_1$ and $\tan x_2$ then

1) $k^2\le 5$
2) $k^2\ge 6$
3) $k=3$
4) $k>10$

Key. 1

Sol. Discriminate ≥ 0

14. α,β are the roots of $ax^2+bx+c=0$ and γ,δ are the roots of $px^2+qx+r=0$ and D_1,D_2 be the respective discriminants of these equations. If α,β,γ and δ are in A.P. then $D_1:D_2=$ (where $\alpha,\beta,\gamma,\delta\in R$ & $a,b,c,p,q,r\in R$)

1) $a^2:p^2$
2) $a^2:b^2$
3) $c^2:r^2$
4) $a^2:r^2$

Key. 1

Sol. $\beta=\alpha+d,\ \gamma=\alpha+2d,\ \delta=\alpha+3d$

$d^2=\dfrac{D_1}{a^2}=\dfrac{D_2}{p^2}$

15. If $x^2+4y^2-8x+12=0$ is satisfied by real values of x and y then $'y'\in$

1) $[2,6]$
2) $[2,5]$
3) $[-1,1]$
4) $[-2,-1]$

Key. 3

Sol. $x^2-8x+(4y^2+12)=0$ is a quadratic in $'x'$, $'x'$ is real then discriminate ≥ 0

16. For $x>0, 0\le t\le 2\pi, K>\dfrac{3}{2}+\sqrt{2},$ K being a fixed real number the minimum value of $x^2+\dfrac{K^2}{x^2}-2\left\{(1+\cos t)x+\dfrac{K(1+\sin t)}{x}\right\}+3+2\cos t+2\sin t$ is

a) $\left\{\sqrt{K}-\left(1+\dfrac{1}{\sqrt{2}}\right)\right\}^2$
b) $\dfrac{1}{2}\left\{\sqrt{K}-\left(1+\dfrac{1}{\sqrt{2}}\right)\right\}^2$
c) $3\left\{\sqrt{K}-\left(1+\dfrac{1}{\sqrt{2}}\right)\right\}^2$
d) $2\left\{\sqrt{K}-\left(1+\dfrac{1}{\sqrt{2}}\right)\right\}^2$

Key. D

Sol. Given expansion = $\{x-(1+\cos t)\}^2+\left\{\dfrac{K}{x}-(1+\sin t)\right\}^2$

17. Let $\phi(x)=\dfrac{(x-b)(x-c)}{(a-b)(a-c)}f(a)+\dfrac{(x-c)(x-a)}{(b-c)(b-a)}f(b)+\dfrac{(x-a)(x-b)}{(c-a)(c-b)}f(c)-f(x)$

Where $a<c<b$ and $f^{11}(x)$ exists at all points in (a,b). Then, there exists a real number $\mu, a<\mu<b$ such that

$\dfrac{f(a)}{(a-b)(a-c)}+\dfrac{f(b)}{(b-c)(b-a)}+\dfrac{f(c)}{(c-a)(c-b)}=$

a) $f^{11}(\mu)$ b) $2f^{11}(\mu)$ c) $\frac{1}{2}f^{11}(\mu)$ d) $\frac{1}{3}f^{111}(\mu)$

Key. C

Sol. Apply RT's, twice

18. If α,β,γ are the roots of the equation $x^3 + px + q = 0$, then the value of the determinant $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is

(A) 4 (B)2 (C)0 (D) –2

Key. C

Sol. Since α,β,γ are the roots of $x^3 + px + q = 0$

$\therefore \quad \alpha+\beta+\gamma = 0$

Applying $C_1 \to C_1 + C_2 + C_3$, then

$$\begin{vmatrix} \alpha+\beta+\gamma & \beta & \gamma \\ \alpha+\beta+\gamma & \gamma & \alpha \\ \alpha+\beta+\gamma & \alpha & \beta \end{vmatrix} = \begin{vmatrix} 0 & \beta & \gamma \\ 0 & \gamma & \alpha \\ 0 & \alpha & \beta \end{vmatrix} = 0$$

19. The number of points (p, q) such that $p,q \in \{1,2,3,4\}$ and the equation $px^2+qx+1=0$ has real roots is

A. 7 B. 8 C. 9 D. None of these

Key. A

Sol. $px^2+qx+1=0$ has real roots if $q^2-4p \ge 0$ or $q^2 \ge 4p$

Since $p,q \in \{1,2,3,4\}$

The required points are(1,2), (1,3),(1, 4), (2,3),(2,4),(3,4),(4,4)

So the required number is 7

20. The value of *b* and *c* for which the identity $f(x+1)-f(x)=8x+3$ is satisfied, where $f(x)=bx^2+cx+d$ are

(A) b = 2, c = 1 (B) b = 4, c = –1

(C) b = – 1, c = 4 (D) b = – 1, c = 1

Key. B

Sol. $\because f(x+1)-f(x)=8x+3$

$\Rightarrow \quad \{b(x+1)^2+c(x+1)+d\}-\{bx^2+cx+d\}=8x+3$

$\Rightarrow \quad b\{(x+1)^2-x^2\}+c=8x+3$

$\Rightarrow \quad b(2x+1)+c=8x+3$ on comparing

2b = 8 and b + c = 3

Then, b = 4 and c = –1

21. Let $f\left(x\right)=ax^2+bx+c$, $g\left(x\right)=ax^2+px+q$ where a, b, c, q, p, $\in$ R and $b\neq p$. If their discriminants are equal and f(x) = g(x) has a root α, then
1) α will be A.M. of the roots of f(x) = 0, g(x) = 0
2) α will be G.M of all the roots of f(x) = 0, g(x) = 0
3) α will be A.M of the roots of f(x) = 0 or g(x) = 0
4) α will be G.M of the roots of f(x) = 0 or g(x) = 0

Key. 1

Sol. $a\alpha^2+b\alpha+c=a\alpha^2+p\alpha+q\Rightarrow\alpha=\dfrac{q-c}{b-p}\rightarrow(i)$

And $b^2-4ac=p^2-4aq$

$\Rightarrow b^2-p^2=4a(c-q)$

$\Rightarrow b+p=\dfrac{4a(c-q)}{b-p}=-4a\alpha \qquad (\text{from}(i))$

$\alpha=\dfrac{-(b+p)}{4a}=\dfrac{\frac{-b}{a}-\frac{p}{a}}{4}$ which is A.M of all the roots of f(x) = 0 and g(x) = 0

22. If the equations $x^2+2\lambda x+\lambda^2+1=0,\ \lambda\in R$ and $ax^2+bx+c=0$ where a, b, c are lengths of sides of triangle have a common root, then the possible range of values of λ is
1) (0, 2) 2) $\left(\sqrt{3},3\right)$ 3) $\left(2\sqrt{2},3\sqrt{2}\right)$ 4) $\left(0,\infty\right)$

Key. 1

Sol. $(x+\lambda)^2+1=0$ has clearly imaginary roots

So, both roots of the equations are common

$\therefore\dfrac{a}{1}=\dfrac{b}{2\lambda}=\dfrac{c}{\lambda^2+1}=k(say)$

Then a = k, b = $2\lambda k$, c = $\left(\lambda^2+1\right)$ k

As a, b, c are sides of triangle

$a+b>c\ \Rightarrow 2\lambda+1>\lambda^2+1\Rightarrow\lambda^2-2\lambda<0$

$\Rightarrow\lambda\in(0,2)$

The other conditions also imply same relation.

23. The number of real or complex solutions of $x^2-6|x|+8=0$ is
1) 6 2) 7 3) 8 4) 9

Key. 1

Sol. If x is real, $x^2-6|x|+8=0\ \Rightarrow |x|^2-6|x|+8=0\Rightarrow |x|=2,4\Rightarrow x=\pm2,\pm4$

If x is non – real, say $x=\alpha+i\beta$, then

$(\alpha+i\beta)^2-6\sqrt{\alpha^2+\beta^2}+8=0 \qquad \left(|\alpha+i\beta|=\sqrt{\alpha^2+\beta^2}\right)$

$\left(\alpha^2-\beta^2+8-6\sqrt{\alpha^2+\beta^2}\right)+2i\alpha\beta=0$

Comparing real and imaginary parts,

$\alpha\beta=0\ \Rightarrow\alpha=0$ (if $\beta=0$ then x is real.)

& $-\beta^2+8-6\sqrt{\beta^2}=0$

$\beta^2 \pm 6\beta - 8 = 0 \Rightarrow \beta = \frac{\mp 6 \pm \sqrt{68}}{2}$

ie., $\beta = \pm\left(3-\sqrt{17}\right)$

Hence $\pm\left(3-\sqrt{17}\right)i$ are non-real roots.

24. If $x_1, x_2\left(x_1 > x_2\right)$ are abscissae of points P, Q lying on $y = 2x^2 - 4x - 5$ such that the tangents drawn at these points pass through the point (0, -7), then $3x_1 - 2x_2$ equals to

1) 4 2) 5 3) 6 4) 7

Key. 2

Sol. Let (α, β) be point on the curve such that the tangent drawn at (α, β) passes through (0, 7)

$y^1 = 4x - 4 \Rightarrow y^1_{(\alpha,\beta)} = 4\alpha - 4$

Tangent at (α, β) is $y - \beta = (4\alpha - 4)(x - \alpha)$ pass through (0, -7) $\Rightarrow -7 - \beta = (4\alpha - 4)(0 - \alpha)$

But $\beta = 2\alpha^2 - 4\alpha - 5$ $\therefore$ It follows that α^2 = 1

$\Rightarrow \alpha = \pm 1$

So, $x_1 = 1,\ x_2 = -1$

So, $3x_1 - 2x_2 = 5$.

25. Let $f(x) = x^2 + 5x + 6$, then the number of real roots of $\left(f(x)\right)^2 + 5f(x) + 6 - x = 0$ is

1) 1 2) 2 3) 3 4) 0

Key. 4

Sol. Use "f(x) = x has non real roots $\Rightarrow$ f(f(x)) = x also has non-real roots"

26. Sum of the roots of the equation is $4^x - 3\left(2^{x+3}\right) + 128 = 0$

1) 5 2) 6 3) 7 4) 8

Key. 3

Sol. Put $2^x = y.$ Equation becomes

$y^2 - 3(8y) + 128 = 0 \Rightarrow y^2 - 24y + 128 = 0$

$\Rightarrow (y-8)(y-16) = 0 \Rightarrow y = 16, 8$

$\Rightarrow 2^x = 16, 8 \Rightarrow x = 4, 3$

$\therefore$ Sum of the roots is 7.

27. The number of solutions of $\sqrt{3x^2 + x + 5} = x - 3$ is

1) 0 2) 1 3) 2 4) 4

Key. 1

Sol. Note that we must have $3x^2 + x + 5 \geq 0$ and $x - 3 \geq 0$ or $x \geq 3.$

$\sqrt{3x^2 + x + 5} = x - 3$.... (1)

Squaring both sides of (1), we get

$3x^2+x+5=x^2-6x+9$

$\Rightarrow 2x^2+7x-4=0\Rightarrow(2x-1)(x+4)=0$

$\Rightarrow x=1/2,-4$

None of these satisfy the inequality $x\geq 3.$ Thus, (1) has no solution.

28. The value of a for which one root of the quadratic equation.
$\left(a^2-5a+3\right)x^2+(3a-1)x+2=0$ is twice as large as other, is

1) $-2/3$ 2) $1/3$ 3) $-1/3$ 4) $2/3$

Key. 4

Sol. $\left(a^2-5a+3a\right)x^2+(3a-1)x+2=0$ ...... (1)

Let α and 2α be the roots of (1), then

$\left(a^2-5a+3\right)\alpha^2+(3a-1)\alpha+2=0$ .... (2)

and $\left(a^2-5a+3\right)\left(4\alpha^2\right)+(3a-1)(2\alpha)+2=0$ ..... (3)

Multiplying (2) by 4 and subtracting it form (3) we get $(3a-1)(2\alpha)+6=0$

Clearly $a\neq 1/3.$ Therefore, $\alpha=-3/(3a-1)$

Putting this value in (2) we get

$\left(a^2-5a+3\right)(9)-(3a-1)^2(3)+2(3a-1)^2=0$

$\Rightarrow 9a^2-45a+27-\left(9a^2-6a+1\right)=0\Rightarrow -39a+26=0$

$\Rightarrow a=2/3.$

For $x=2/3,$ the equation becomes $x^2+9x+18=0,$ whose roots are $-3,-6.$

29. If $f(x)=x^2+2bx+2c^2$ and $g(x)=-x^2-2cx+b^2$ are such that
$\min f(x)>\max g(x),$ then relation between b and c, is

1) no relation 2) $0<c<b/2$ 3) $|c|<\dfrac{|b|}{\sqrt{2}}$ 4) $|c|>\sqrt{2}|b|$

Key. 4

Sol. $f(x)=(x+b)^2+2c^2-b^2$

$\Rightarrow \min f(x)=2c^2-b^2$

Also $g(x)=-x^2-2cx+b^2=b^2+c^2-(x+c)^2$

$\Rightarrow \max g(x)=b^2+c^2$

As $\min f(x)>\max g(x),$ we get $2c^2-b^2>b^2+c^2$

$\Rightarrow c^2>2b^2\Rightarrow |c|>\sqrt{2}|b|$

30. The equation $(\cos p-1)x^2+(\cos p)x+\sin p=0$ in variable x has real roots, if p belongs to the interval

1) $(0,2\pi)$ 2) $(-\pi,0)$ 3) $(-\pi/2,\pi/2)$ 4) $(0,\pi)$

Key. 4

Sol. $(\cos p-1)x^2+(\cos p)x+\sin p=0$ ...... (1)

Discriminant of (1) is given by

$D=\cos^2 p-4(\cos p-1)\sin p=\cos^2 p+4(1-\cos p)\sin p$

Note that $\cos^2 p\ge 0, 1-\cos p\ge 0$. Thus, $D\ge 0$ if $\sin p\ge 0$ *i.e.* if $p\in(0,\pi)$.

31. If $x^2+2ax+10-3a>0$ for each $x\in R$, then

1) $a<-5$ 2) $-5<a<2$ 3) $a>5$ 4) $2<a<5$

Key. 2

Sol. $x^2+2ax+10-3a>0\forall x\in R$

$\Rightarrow(x+a)^2-(a^2+10-3a)>0\forall x\in R$

$\Rightarrow a^2+3a-10<0$

$\Rightarrow(a+5)(a-2)<0$

$\Rightarrow -5<a<2$

32. Sum of all the values of x satisfying the equation $\log_{17}\log_{11}\left(\sqrt{x+11}+\sqrt{x}\right)=0$ is

1) 25 2) 36 3) 171 4) 0

Key. 1

Sol. $\log_{17}\log_{11}\left(\sqrt{x+1}+\sqrt{x}\right)=0$ .... (1)

Equation (1) is defined if $x\ge 0$.

We can rewrite (1) as $\log_{11}\left(\sqrt{x+11}+\sqrt{x}\right)=17^0=1$

$\Rightarrow\sqrt{x+11}+\sqrt{x}=11^1=11$

$\Rightarrow\sqrt{x+11}=11-\sqrt{x}$

Squaring both sides we get $x+11=121-22\sqrt{x}+x$

$\Rightarrow 22\sqrt{x}=110\Rightarrow\sqrt{x}=5$ or $x=25$

This clearly satisfies (1). Thus, sum of all the values satisfying (1) is 25.

33. The number of solutions of the equations of the equation $x^2+[x]-4x+3=0$ is Where [ ] denotes G.I.F.

1) 0 2) 1 3) 2 4) 3

Key. 1

Sol. Given equation can be written as $(x^2-3x+3)-f=O$ where $f=x-[x]$ *and* $O\le f<1$

$\therefore O\le x^2-3x+3<1$

solving $x^2-3x+3=O$; roots are Imaginary

$\therefore x^2-3x+3\ge O\forall x\in R$

solving $x^2-3x+3<1\Rightarrow 1<x<2$

if $1<x<2; [x]=1$.

putting $[x]=1$ in the given equation and solving we get $x=2$. But $1<x<2$ $\therefore$ the given equation has no solution.

34. The number of values of $'a'$ for which the equation $(x-1)^2=|x-a|$ has exactly three solutions is

1) 1 2) 2 3) 3 4) 4

Key. 3

Sol. $|x-a|=(x-1)^2$ Iff $a=x\pm(x-1)^2$

No of solutions = no of intersection its between

$y=a;\ f(x)=x^2-x+1$ and $g(x)=-x^2+3x-1$. clearly the graphs of $f(x),g(x)$ are tangents to each other at $A(1,1)$. The line $y=a$ intersects the two graphs at three points Iff it passes through one of the three pts A,B, C. Here $B=\left(\frac{1}{2},\frac{3}{4}\right)$ vertex of f and $C=\left(\frac{3}{2},\frac{5}{4}\right)$ vertex of 'g' i.e if $a\in\left\{\frac{3}{4},\frac{5}{4},1\right\}$

35. If a, b, c are positive numbers such that a>b>c and the equation $(a+b-2c)x^2+(b+c-2a)x+(c+a-2b)=0$ has a root in the interval $(-1,0)$, then

A) b cannot be the G.M. of a, c B) b may be the G.M. of a, c

C) b is the G.M. of a, c D) none of these

Key. A

Sol. Let $f(x)=(a+b-2c)x^2+(b+c-2a)x+(c+a-2b)$

According to the given condition, we have

$$f(0)f(-1)<0$$

i.e. $(c+a-2b)(2a-b-c)<0$

i.e. $(c+a-2b)(a-b+a-c)<0$

i.e. $c+a-2b<0$ $\quad [a>b>c,\ given \Rightarrow a-b>0, a-c>0]$

i.e. $b>\frac{a+c}{2}$

$\Rightarrow$ *b* cannot be the G.M. of *a, c,* since G.M < A.M. always.

36. Let α,β (a < b) be the roots of the equation $ax^2+bx+c=0$. If $\lim_{x\to m}\frac{|ax^2+bx+c|}{ax^2+bx+c}=1$, then

A) $\frac{|a|}{a}=-1, m<\alpha$ B) $a>0,\alpha<m<\beta$ C) $\frac{|a|}{a}=1, m>\beta$ D) $a<0, m>\beta$

Key. C

Sol. According to the given condition, we have

$$|am^2+bm+c|=am^2+bm+c$$

i.e. $am^2+bm+c>0$

$\Rightarrow$ if *a< 0,* the *m* lies in (α,β)

and if *a>0,* then *m* does not lies in (α,β)

Hence, option (c) is correct, since

$$\frac{|a|}{a}=1\Rightarrow a>0$$

And in that case *m* does not lie in (α,β).

37. Let $f(x)$ be a function such that $f(x)=x-[x]$, where $[x]$ is the greatest integer less than or equal to x. Then the number of solutions of the equation $f(x)+f\left(\frac{1}{x}\right)=1$ is (are)

A) 0 B) 1 C) 2 D) infinite

Key. D

Sol. Given, $f(x)=x-[x],\ x\in R-\{0\}$

Now $f(x)+f\left(\frac{1}{x}\right)=1$ $\therefore$ $x-[x]+\frac{1}{x}-\left[\frac{1}{x}\right]=1$

$\Rightarrow\left(x+\frac{1}{x}\right)-\left([x]+\left[\frac{1}{x}\right]\right)=1$ $\Rightarrow\left(x+\frac{1}{x}\right)=[x]+\left[\frac{1}{x}\right]+1$ ...(i)

Clearly ,R.H.S is an integer $\therefore$ L. H. S. is also an integer

Let $x+\frac{1}{x}=k$ an integer $\Rightarrow x^2-kx+1=0$

$\therefore x=\frac{k\pm\sqrt{k^2-4}}{2}$

For real values of x, $k^2-4\ge 0\Rightarrow k\ge 2$ or $k\le -2$

We also observe that *k=2* and -2 does not satisfy equation *(i)*

$\therefore$ The equation *(i)* will have solutions if $k>2$ or $k<-2$, where $k\in z$.

Hence equation *(i)* has infinite number of solutions.

38. If both the roots of $(2a-4)9^x-(2a-3)3^x+1=0$ are non-negative, then

A) $0<a<2$ B) $2<a<\frac{5}{2}$ C) $a<\frac{5}{4}$ D) $a>3$

Key. B

Sol. Putting $3^x=y$, we have

$$(2a-4)y^2-(2a-3)y+1=0$$

This equation must have real solution

$\Rightarrow$ $(2a-3)^2-4(2a-4)\ge 0$

$\Rightarrow$ $4a^2-20a+25\ge 0$

$\Rightarrow$ $(2a-5)^2\ge 0.$ This is true.

$y=1$ satisfies the equation

Since 3^x is positive and $3^x\ge 3^0,\ y\ge 1$

Product of the roots $=1\times y>1$

$\Rightarrow$ $\frac{1}{2a-4}>1$

$\Rightarrow$ $2a-4<1\Rightarrow\ a<\frac{5}{2}$

Sum of the roots $=\frac{2a-3}{2a-4}>1$

$\Rightarrow$ $\frac{(2a-3)-(2a-4)}{2a-4}>0$

$\Rightarrow \quad \frac{1}{2a-4} > 0 \Rightarrow a > 2$

$\Rightarrow \quad 2 < a < \frac{5}{2}$

39. If the equation $x^2 + 9y^2 - 4x + 3 = 0$ is satisfied for real values of x and y then

A) $x \in [1,3], y \in [1,3]$ B) $x \in [1,3], y \in \left[\frac{-1}{3}, \frac{1}{3}\right]$

C) $x \in \left[\frac{-1}{3}, \frac{1}{3}\right], y \in [1,3]$ D) $x \in \left[\frac{-1}{3}, \frac{1}{3}\right], y \in \left[\frac{-1}{3}, \frac{1}{3}\right]$

Key. B

Sol. Given equation is $x^2 + 9y^2 - 4x + 3 = 0$ …(i)

Or, $x^2 - 4x + 9y^2 + 3 = 0.$

Since x is real $\therefore \ (-4)^2 - 4(9y^2 + 3) \ge 0$

Or, $16 - 4(9y^2 + 3) \ge 0$ or, $4 - 9y^2 - 3 \ge 0$

Or, $9y^2 - 1 \le 0$ or, $9y^2 \le 1$ or, $y^2 \le \frac{1}{9}$

Now $y^2 \le \frac{1}{9} \Leftrightarrow -\frac{1}{3} \le y \le \frac{1}{3}$ …(ii)

Equation (i) can also be written as

$9y^2 + 0y + x^2 - 4x + 3 = 0$ …(iii)

Since y is real $\therefore 0^2 - 4.9(x^2 - 4x + 3) \ge 0$

Or, $x^2 - 4x + 3 \le 0$

$\Rightarrow x \in [1,3]$

40. The equation $a_8x^8 + a_7x^7 + a_6x^6 + ... + a_0 = 0$ has all its roots positive and real $\left(where\ a_8 = 1, a_7 = -4, a_0 = 1/2^8\right)$, then

A) $a_1 = \frac{1}{2^8}$ B) $a_1 = -\frac{1}{2^4}$ C) $a_2 = \frac{7}{2^5}$ D) $a_2 = \frac{7}{2^8}$

Key. B

Sol. Let the roots be $\alpha_1, \alpha_2, ..., \alpha_8$

$\Rightarrow \quad \alpha_1 + \alpha_2 + ... + \alpha_8 = 4$

$\alpha_1\alpha_2.....\alpha_8 = \frac{1}{2^8}$

$\Rightarrow \quad (\alpha_1\alpha_2.....\alpha_8)^{1/8} = \frac{1}{2} = \frac{\alpha_1 + \alpha_2 + ... + \alpha_8}{8}$

$\Rightarrow$ AM=GM $\Rightarrow$ all the roots are equal to $\frac{1}{2}$.

$\Rightarrow \qquad a_1 = -{}^8C_7\left(\frac{1}{2}\right)^7 = -\frac{1}{2^4}$

$a_2 = {}^8C_6\left(\frac{1}{2}\right)^6 = -\frac{7}{2^4}$

$a_3 = -{}^8C_5\left(\frac{1}{2}\right)^5$

41. If every root of a polynomial equation (of degree 'n') $f(x)=0$ with leading coefficient "1" is real and distinct, then the equation $f''(x)f(x)-\{f'(x)\}^2=0$ has.

(A) at least one real root (B) no real root
(C) at most one real root (D) exactly two real roots

Key. B

Sol. Let $f(x)=(x-a_1)(x-a_2)............(x-an)$ where $a_1,a_2.........a_{n\in R}$ take log both sides and differentiate. Then

$$\frac{f'(x)}{f(x)}=\frac{1}{x-a_1}+\frac{1}{x-a_2}+.........+\frac{1}{x-a_n}$$

Again diff w.r.t. 'x'

$$\frac{f\ f''-(f')^2}{f^2}=-\left[\frac{1}{(x-a_1)^2}+\frac{1}{(x-a_2)^2}+......\frac{1}{(x-a_n)^2}\right]$$

$<0\forall x\in R$

$\Rightarrow\ f\ f''-(f')^2=0$ has no real root

42. If $f(x)$ is a polynomial of least degree such that $f(r)=\frac{1}{r}$, $r=1,2,3,$___9, then $f(10)=$ ___

A. 1 B. $\frac{1}{2}$ C. $\frac{1}{10}$ D. $\frac{1}{5}$

Key. D

Sol. $xf(x)-1=0$ has roots 1,2,3 ____9

$xf(x)-1=A(x-1)(x-2)$ _____x-9

Put $x=0\Rightarrow A=\frac{1}{9!}$

Put $x=10\Rightarrow 10f(10)-1=1\Rightarrow f(10)=\frac{1}{5}$

43. The number of ordered pairs of integers (x, y) satisfying the equation $x^2+6x+y^2=4$ is

A. 2 B. 8 C. 6 D. 10

Key. B

Sol. $(x+3)^2+y^2=13$

$x+3=\pm 2, y=\pm 3 \, or \, x+3=\pm 3, y=\pm 2$

44. The number of non-negative integer solutions of $x+y+2z=20$ is

A. 76 B. 84 C. 112 D. 121

Key. D

Sol. $x+y=20-2Z,\ Z=0,1,2,...10$

The number of solutions (non –ve) is $\sum_{Z=0}^{10}(20-2Z+1)_{C_1}=121$

45 If $a+b+c=0$ for $a,b,c\in R$, then the equation $3ax^2+2bx+c=0$ has

A. Atleast one root in $[0,1]$

B. One root in $[2,3]$ and another root in $[-2,-1]$

C. Imaginary roots

D. Atleast one root in $[1,2]$

Key. A

Sol. Let $f(x)=ax^3+bx^2+cx$. Then f is continuous and differentiable in $[0,1]$, $f(0)=f(1)=0$. Hence by Rolle's theorem there exists $k\in(0,1)$ such that $3ak^2+2bk+c=0$

46. If a,b,c be the sides of a triangle ABC and if roots of the equation a(b – c)x² + b(c – a) x + c(a – b) = 0 are equal, then $\sin^2\left(\frac{A}{2}\right),\sin^2\left(\frac{B}{2}\right),\sin^2\left(\frac{C}{2}\right)$ are in

(A) AP (B)GP (C) HP (D) AGP

Key. C

Sol. $\because$ a(b – c) + b(c – a) + c (a – b) = 0

$\therefore$ x = 1 is a root of the equation

a(b – c)x² + b(c – a)x + c(a – b) =0

Then, other root = 1 ($\because$ roots are equal)

$\therefore$ $\alpha\times\beta=\frac{c(a-b)}{a(b-c)}$

$\Rightarrow$ ab – ac = ca – bc

$\therefore$ $b=\frac{2ac}{a+c}$

$\therefore$ a, b, c are in HP

Then, $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in AP.

$\Rightarrow \frac{s}{a},\frac{s}{b},\frac{s}{c}$ are in AP

$\Rightarrow \frac{s}{a}-1,\frac{s}{b}-1,\frac{s}{c}-1$ are in AP.

$\Rightarrow \frac{(s-a)}{a},\frac{(s-b)}{b},\frac{(s-c)}{c}$ are in AP.

Multiplying in each by $\frac{abc}{(s-a)(s-b)(s-c)}$

Then $\frac{bc}{(s-b)(s-c)}, \frac{ca}{(s-c)(s-a)}, \frac{ab}{(s-a)(s-b)}$ are in AP.

$\Rightarrow$ $\frac{(s-b)(s-c)}{bc}, \frac{(s-c)(s-a)}{ca}, \frac{(s-a)(s-b)}{ab}$ are in HP.

Or $\sin^2\left(\frac{A}{2}\right), \sin^2\left(\frac{B}{2}\right), \sin^2\left(\frac{C}{2}\right)$ are in HP

47. If α, β, γ are the roots of the equation $x^3 + px + q = 0$, then the value of the determinant $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is

(A) 4 (B)2 (C)0 (D) –2

Key. C

Sol. Since α, β, γ are the roots of $x^3 + px + q = 0$

$\therefore$ $\alpha + \beta + \gamma = 0$

Applying $C_1 \to C_1 + C_2 + C_3$, then

$$\begin{vmatrix} \alpha+\beta+\gamma & \beta & \gamma \\ \alpha+\beta+\gamma & \gamma & \alpha \\ \alpha+\beta+\gamma & \alpha & \beta \end{vmatrix} = \begin{vmatrix} 0 & \beta & \gamma \\ 0 & \gamma & \alpha \\ 0 & \alpha & \beta \end{vmatrix} = 0$$

48. The value of *b* and *c* for which the identity $f(x+1) - f(x) = 8x + 3$ is satisfied, where $f(x) = bx^2 + cx + d$ are

(A) b = 2, c = 1 (B) b = 4, c = –1

(C) b = – 1, c = 4 (D) b = – 1, c = 1

Key. B

Sol. $\because f(x+1) - f(x) = 8x + 3$

$\Rightarrow$ $\{b(x+1)^2 + c(x+1) + d\} - \{bx^2 + cx + d\} = 8x + 3$

$\Rightarrow$ $b\{(x+1)^2 - x^2\} + c = 8x + 3$

$\Rightarrow$ $b(2x+1) + c = 8x + 3$ on comparing

2b = 8 and b + c = 3

Then, b = 4 and c = –1

49. If *a*, *b*, *c* are positive numbers such that a>b>c and the equation $(a+b-2c)x^2 + (b+c-2a)x + (c+a-2b) = 0$ has a root in the interval $(-1, 0)$, then

A) b cannot be the G.M. of a, c B) b may be the G.M. of a, c

C) b is the G.M. of a, c D) none of these

Key. A

Sol. Let $f(x) = (a+b-2c)x^2 + (b+c-2a)x + (c+a-2b)$

According to the given condition, we have

$f(0)\, f(-1) < 0$

i.e. $(c+a-2b)(2a-b-c) < 0$

i.e. $(c+a-2b)(a-b+a-c) < 0$

i.e. $c+a-2b<0$ $\quad [a>b>c,\ given \Rightarrow a-b>0, a-c>0]$

i.e. $b>\dfrac{a+c}{2}$

$\Rightarrow$ *b* cannot be the G.M. of *a, c,* since G.M < A.M. always.

50. The values of 'a' for which the quadratic expression $ax^2+(a-2)x-2$ is negative for exactly two integral values of x, belongs to

(A) $[-1,1]$ (B) $[1,2)$

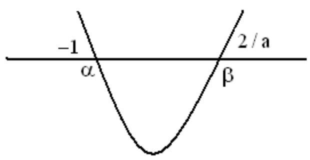
(C) $[3,4]$ (D) $[-2,-1)$

Key. B

Sol. Let $f(x)=ax^2+(a-2)x-2$

$f(x)$ is negative for two integral values of x, so graph should be vertically upward parabola i.e., $a>0$

Let two roots of $f(x)=0$ are α and β then $\alpha,\beta=\dfrac{-(a-2)\pm(a+2)}{2a}$



$\Rightarrow \alpha=-1, \beta=\dfrac{2}{a} \Rightarrow 1<\beta\le 2 \Rightarrow 1<\dfrac{2}{a}\le 2 \Rightarrow a\in[1,2]$