Probability

Single Correct Answer Type

1. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is

1)37/256 2)28/256 3)128/256 4)219/256

Key. 2

Sol. np=6

npq=2

$\Rightarrow q=\frac{1}{2}, p=\frac{1}{2}, n=8$

2. For a binomial distribution $\bar{x}=4, \sigma=\sqrt{3}$. Then P(X=r)=

1) $^{16}C_r\left(1/4\right)^r\left(3/4\right)^{16-r}$ 2) $^{12}C_r\left(1/4\right)^r\left(3/4\right)^{12-r}$

3) $^{16}C_r\left(3/4\right)^r\left(1/4\right)^{16-r}$ 4) $^{12}C_r\left(3/4\right)^r\left(1/4\right)^{12-r}$

Key. 1

Sol. np=4

npq=3

$\Rightarrow q=\frac{3}{4}, p=\frac{1}{4}, n=16$

3. If X is Poisson variate with $P\left(X=0\right)=P\left(X=1\right), then\ P\left(X=2\right)=$

1)e/2 2)e/6 3)1/(6e) 4)1/(2e)

Key. 4

Sol. $P\left(x=r\right)=\frac{e^{-\lambda}\lambda^r}{r!}$

4. In a Poisson distribution the variance is m. The sum of the terms in odd places in this distribution is

1) e^{-m} 2) $e^{-m}\cosh m$ 3) $e^{-m}\sinh m$ 4) $e^{-m}\coth m$

Key. 2

Sol. Conceptual

5. Two natural numbers *a* and *b* are selected at random. The probability that a^2+b^2 is divisible by 7 is

(a) 3/8 (b) 1/7 (c) 3/49 (d) 1/49

Key. D

Sol. $a_1 b$ are is of then form

$a_1 b \in \{7m, 7m+1, 7m+2, 7m+3, 7m+4, 7m+6\}$

$a_1^2 b^2 \in \{7m_1, 7m_1+1, 7m_1+4, 7m_1+2, 7m_1+2, 7m_1+4, 7m_1+1\}$

$\therefore a^2, b^2$ must be of the form 7m.

Probability = $\frac{1}{49}$

6. If a and b are chosen randomly from the set consisting of numbers 1, 2, 3, 4, 5, 6 with replacement. Then probability that $\underset{x\to 0}{Lt}\left(\frac{a^x+b^x}{2}\right)^{2/x}=6$ is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{9}$ (d) $\frac{2}{9}$

Key. C

Sol. $\underset{x\to 0}{Lt}\left(\frac{a^x+b^x}{2}\right)^{2/x}=6$

$= e^{\underset{x\to 0}{Lt}\left(\frac{a^x-1}{x}\right)+\left(\frac{b^x-1}{x}\right)}=6$

$= e^{\log a+\log b}=6$

ab = 6

$(a,b)=(1,6),(6,1),(2,3),(3,2)$

Required probability = $\frac{4}{6\times 6}=\frac{1}{9}$

7. Thirty-two players ranked 1 to 32 are playing in a knockout tournament. Assume that in every match between any two players, the better-ranked player wins, the probability that ranked 1 and ranked 2 players are winner and runner up respectively, is

(A) 16/31 (B) 1/2 (C) 17/31 (D) None of these

Key. A

Sol. For ranked 1 and 2 players to be winners and runners up res., they should not be paired with each other in any rounded. Therefore, the required probability $\frac{30}{31}\times\frac{14}{15}\times\frac{6}{7}\times\frac{2}{3}=\frac{16}{31}$

8. A coin is tossed 7 times. Then the probability that at least 4 consecutive heads appear is

(A) 3/16 (B) 5/32 (C) 5/16 (D) 1/8

Key. B

Sol. Let H denote the head,
T the tail.
* Any of the head or tail

P(H) = $\frac{1}{2}$, P(T) = $\frac{1}{2}$ P(*) = 1

HHHH*** $=\left(\frac{1}{2}\right)^4\times 1=\frac{1}{16}$

THHHH** $= \left(\frac{1}{2}\right)^5 \times 1 = \frac{1}{32}$

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$\frac{5}{32}$

9. A fair coin is tossed 5 times then probability that two heads do not occur consecutively (No two heads come together)

1. $\frac{1}{16}$ 2. $\frac{15}{32}$ 3. $\frac{13}{32}$ 4. $\frac{7}{16}$

Key. 3

Sol. $p\left(\frac{E}{no\,heads}\right) + p\left(\frac{E}{1(head)}\right) + p\left(\frac{E}{2-heads}\right) + p\left(\frac{E}{3-heads}\right)$

Where $E \rightarrow$gtg n two consecutive heads.

$= \frac{1}{32} + \frac{5}{32} + \frac{6}{32} + \frac{1}{32} = \frac{14}{32} = \frac{7}{16}$

10. A man throws a die until he gets a number bigger than 3. The probability that he gets 5 in the last throw

1. $\frac{1}{3}$ 2. $\frac{1}{4}$ 3. $\frac{1}{6}$ 4. $\frac{1}{36}$

Key. 1

Sol. P(gtg a number bigger than 3) = $\frac{1}{2}$

P(gtg 5 in throw)= $\frac{1}{6}$

$E \rightarrow$gtg 5 in last throw when he gets a number bigger than 3

$P(E) = \frac{1}{6} + \frac{1}{2}.\frac{1}{6} + \frac{1}{2}.\frac{1}{2}.\frac{1}{6} + ...\infty$

$= \frac{1}{6} \times \frac{1}{1-\frac{1}{2}} = \frac{1}{3}$

11. A bag contains 4-balls two balls are drawn from the bag and are found to be white then probability that all balls in the bag are white

1. $\frac{1}{5}$ 2. $\frac{2}{5}$ 3. $\frac{3}{5}$ 4. $\frac{4}{5}$

Key. 3

Sol. $P(E)=\dfrac{\frac{1}{3}\cdot\frac{4_{C_2}}{4_{C_2}}}{\frac{1}{3}\left\{\frac{2_{C_2}}{4_{C_2}}+\frac{3_{C_2}}{4_{C_2}}+\frac{4_{C_2}}{4_{C_2}}\right\}}$

$=\dfrac{1}{\frac{1+3+6}{6}}=\frac{6}{10}=\frac{3}{5}$

12. A randomly selected year is containing 53 Mondays then probability that it is a leap year

1. $\frac{2}{5}$ 2. $\frac{3}{5}$ 3. $\frac{4}{5}$ 4. $\frac{1}{5}$

Key. 1

Sol. Selected year may non leap year with a probability $\frac{3}{4}$

Selected year may leap year with a probability $\frac{1}{4}$

$E\rightarrow$Even that randomly selected year contains 53 Mondays

$P(E)=\frac{3}{4}\times\frac{1}{7}+\frac{1}{4}\times\frac{2}{7}=\frac{5}{28}$

$P\left(\frac{leapyear}{E}\right)=\dfrac{\frac{2}{28}}{\frac{5}{28}}=\frac{2}{5}$

13. When 5-boys and 5-girls sit around a table the probability that no two girls come together

1. $\frac{1}{120}$ 2. $\frac{1}{126}$ 3. $\frac{3}{47}$ 4. $\frac{4}{7}$

Key. 2

Sol. $E\rightarrow$first boys can be arranged in $\lfloor 4$ ways, then there are 5-gaps between boys in 5-gaps, 5-girls can be arranged in $\lfloor 5$ ways

$P(E)=\dfrac{\lfloor 5\lfloor 4}{\lfloor 9}=\dfrac{5\times4\times3\times2}{5\times6\times7\times8\times9}=\dfrac{1}{126}$

14. There are m-stations on a railway line. A train has to stop at 3 intermediate stations then probability that no two stopping stations are adjacent

1. $\frac{1}{m\,c_3}$ 2. $\frac{3}{mc_3}$ 3. $\frac{m-2_{C_3}}{mc_3}$ 4. $\frac{mc_2}{m\,c_3}$

Key. 3

Sol. Let 3-stopping stations be S_1, S_2, S_3 then are m-3 stations remaining. Between these m-3 stations there are m-2 places select any 3 for S_1, S_2, S_3, then there are no two stopping stations are adjacent

$$P(E) = \frac{m-2_{C_3}}{m_{C_3}}$$

15. The probability that randomly selected positive integer is relatively prime to 6

1. $\frac{1}{2}$ 2. $\frac{1}{3}$ 3. $\frac{1}{6}$ 4. $\frac{5}{6}$

Key. 2

Sol. Among every 6-consecutive integers one divisible by 6 and other integers leaves remainders 1,2,3,4,5 when divided by 6

The numbers which leave the remainder 1 and 5 are relatively prime to 6

Required probability $\frac{2}{6} = \frac{1}{3}$

16. A and B are events such that P(A)=0.3 $P(A \cup B) = 0.8$. If A and B are independent then $P(B)$ =

1. $\frac{1}{7}$ 2. $\frac{3}{7}$ 3. $\frac{5}{7}$ 4. $\frac{6}{7}$

Key. 3

Sol. $P(A \cap B) = P(A).P(B)$

$P(A \cup B) = P(A) + P(B) - P(A)P(B)$

$0.8 = 0.3 + P(B)(1-0.3)$

$0.5 = P(B)(0.7) \Rightarrow P(B) = \frac{5}{7}$

17. In a 3×3 matrix the entries a_{ij} are randomly selected from the digits {0,1,2 __9} with replacement. The probability that the numbers of the form xyz where x,y,z are the elements in each row will be divisible by 11 is $\frac{7^{K_1}.13^{K_2}}{10^9}$ then $K_1 + K_2 =$ ____

Key. 6

Sol. The number of multiples of 11 from 000 to 999 is 91.

The required probability $=(\frac{91}{1000})^3=\frac{7^3\times13^3}{10^9}$

18. Triangles are formed with vertices of a regular polygon of 20 sides. The probability that no side of the polygon is a side of the triangle is $\frac{\lambda}{57}$ Then $\frac{\lambda}{40}$ is ____

Key. 1

Sol. The total number of triangles $=20_{C_3}=1140$ there are 20 triangles with two sides of polygon there are 20×16 triangles with are side of polygon $\therefore$ required probability $=\frac{1140-20-320}{1140}=\frac{800}{1140}=\frac{40}{57}$

19. The probability that $\sin^{-1}(\sin x)+\cos^{-1}(\cos y)$ is an integer $x, y\in\{1,2,3,4\}$ is

A. $\frac{1}{16}$ B. $\frac{3}{16}$ C. $\frac{15}{16}$ D. $\frac{1}{17}$

Key. B

Sol. $Sin^{-1}(\sin x)+\cos^{-1}(\cos y)$ to be integer $x\in[-\frac{\pi}{2}\frac{\pi}{2}]$ and

$y\in[0\,\pi]\Rightarrow x=1 and\ y=1,2,3.$ required probability $=\frac{3}{16}$.

20. Seven coupons are selected at random one at a time with replacement from 15 coupons numbered 1 to 15. The probability that the largest number appearing on a selected coupon is 9, is

A) $\left(\frac{9}{16}\right)^6$ B) $\left(\frac{8}{15}\right)^7$ C) $\left(\frac{3}{5}\right)^7$ D) None of these

Key. D

Sol. Each coupon can be selected in 15 ways. The total number of ways of choosing 7 copouns is 15^7. If largest number is 9, then the selected numbers have to be from 1 to 9 excluding those consisting of only 1 to 8.

Probability desired is $\frac{9^7-8^7}{15^7}$

$$=\left(\frac{3}{5}\right)^7-\left(\frac{8}{15}\right)^7$$

21. If the letters of the word MATHEMATICS are arranged arbitrarily, the probability that C comes before E, E before H,H before I and I before S is

A) $\frac{1}{75}$ B) $\frac{1}{24}$ C) $\frac{1}{120}$ D) $\frac{1}{720}$

Key. C

Sol. The total numbers of arrangements is $\frac{11!}{2!2!2!}=\frac{11!}{8}$

The number of arrangements in which C, E, H, I, S appear in that order

$=\binom{11}{5}\frac{6!}{2!2!2!}=\frac{1}{8.5!}$

Probability $=\frac{11!}{8.5!}\div\frac{11!}{8!}=\frac{1}{5!}=\frac{1}{120}$

22. A signal which can be green or red with probability $\left(\frac{4}{5}\right)$ and $\left(\frac{1}{5}\right)$ respectively is received by the station A and Transmitted to B. The probability each station receive signal correctly $=\left(\frac{3}{4}\right)$. If signal in received in B is green. The probability original signal was green.

(A) $\frac{3}{5}$ (B) $\frac{6}{7}$ (C) $\frac{20}{23}$ (D) $\frac{9}{20}$

Key. C

Sol. G = Original signal green. A = A receive correct signal, B = B receive signal correct. E is signal received by B is green.

$P\left(\frac{G}{E}\right)=\frac{P(G\cap E)}{P(E)}$

$P(E)=P(GAB)+P(G\bar{A}\bar{B})+P(\bar{G}A\bar{B})+P(\bar{G}\bar{A}B)$

$=\frac{4}{5}.\frac{3}{4}.\frac{3}{4}+\frac{4}{5}.\frac{1}{4}.\frac{1}{4}+\frac{1}{5}.\frac{3}{4}.\frac{1}{4}+\frac{1}{5}.\frac{1}{4}.\frac{3}{4}=\frac{46}{80}$

$P(G\cap E)=P(GAB)+P(G\bar{A}\bar{B})=\frac{40}{80}$

23. Three fair and unbiased dice and rolled at a time. The probability that the numbers shown are totally different.

(A) $\frac{5}{9}$ (B) $\frac{27}{216}$ (C) $\frac{4}{9}$ (D) $\frac{2}{3}$

Key. A

Sol. $n(s)=6^3$

$n(E)=$ Available 6 different numbers or 3 places in 6_{P_3} Hence $P(E)\frac{5}{9}$

24. A bag contains 7 black and 4 white balls two balls are drawn at a time from the bag. The probability at least one white ball is selected is

(A) $\frac{7}{11}$ (B) $\frac{5}{11}$ (C) $\frac{28}{55}$ (D) $\frac{34}{55}$

Key. D

Sol. $1-\left[\frac{7_{C_2}}{11_{C_2}}\right]$

25. There are ten pairs of shoes in a cup board out or which 4 are picked up at random one after the other. The probability that there is at least one pair is

(A) $\frac{4}{11}$ (B) $\frac{3}{11}$ (C) $\frac{33}{107}$ (D) $\frac{99}{323}$

Key. D

Sol. Out of 20 shoes 4 be taken in 20_{P_4}.

Ways of getting no. pair = 20 x 18 x 16 x 14

Probability of no.pair $=\frac{224}{323}$

at least one pair = $1-\frac{224}{323}=\frac{99}{323}$

26. Out of 10 persons sitting at a round table. Three persons are selected at random one after the other. The chance that no two of the selected are together.

(A) $\frac{1}{2}$ (B) $\frac{2}{5}$ (C) $\frac{5}{9}$ (D) $\frac{5}{12}$

Key. D

Sol. Out of 10 persons 3 can be selected in 10_{P_3} ways = 10.9.8 = 720. First person in 10 ways.

Other 2 in 7_{C_2} ways in which 2 are together in 6 ways. Hence

$$P(E)=\frac{10\times\left(7_{C_2}-6\right)\times 2}{720}=\frac{5}{12}.$$

27. A bag contains n white and n red balls. Pairs of balls are drawn with out replacement until the bag is empty. The probability that each pair consists of one white and one red ball is

(A) $\frac{\lfloor n.\lfloor n}{\lfloor 2n}.2^{n-1}$ (B) $\frac{(\lfloor n).2^n}{\lfloor 2n}$

(C) $\frac{(\lfloor n)^2.2^n}{\lfloor 2n}$ (D) $\frac{(\lfloor n).2^{n-1}}{\lfloor 2n}$

Key. C

Sol. $n(s)=2n_{C_2}.2n-2_{C_2}......2_{C_2}$

$n(E)=n^2(n-1)^2(n-2)^2...1^2=(\lfloor n)^2$

28. Let w be complex cube root of unity with $w\neq 1$. A fair die is thrown 3 times. If r_1, r_2, r_3 be the numbers obtained on the die the probability that $w^{r_1}+w^{r_2}+w^{r_3}=0$ is

(A) $\frac{1}{18}$ (B) $\frac{1}{9}$ (C) $\frac{2}{9}$ (D) $\frac{1}{36}$

Key. C

Sol. r_1, r_2, r_3 must be of the four 3n, 3n+1, 3n+2 $P(E) = \frac{\lfloor 3 . {}^2C_1 . {}^2C_1 . {}^2C_1}{6^3} = \frac{2}{9}$

29. Four persons are selected at random out of 3 men, 2 women and 4 children. The probability that there are exactly 2 children in the selection is

A) 11/21 B) 9/21 C) 10/21 D) 8/21

Key. C

Sol. Req. = 2 childrens and 2 others $= \frac{{}^4C_2 \times {}^5C_2}{{}^9C_4} = \frac{10}{21}$

30. 10 different books and 2 different pens are given to 3 boys so that each gets equal number of things. The probability that the same boy does not receive both the pens is

A) $\frac{5}{11}$ B) $\frac{7}{11}$ C) $\frac{10}{11}$ D) $\frac{6}{11}$

Key. C

Sol. n (S) = ${}^{12}C_4$ x ${}^8C_4$ x ${}^4C_4$ x 3!

n (E) = the number of ways in which one boy gets both the pens

= ${}^{10}C_2$ x ${}^8C_4$ x ${}^4C_4$ x (3!)

$\therefore P(E) = 1 - \frac{{}^{10}C_2 \times {}^8C_4 \times {}^4C_4 \times (3!)}{{}^{12}C_4 \times {}^8C_4 \times {}^4C_4 3!} = 1 - \frac{1}{11} = \frac{10}{11}$

31. From a bag containing 10 distinct balls, 6 balls are drawn simultaneously and replaced. Then 4 balls are drawn. The probability that exactly 3 balls are common to the drawings is

A) 8/21 B) 6/19 C) 5/24 D) 9/22

Key. A

Sol. Let S be the sample space of the composite experiment of drawing 6 in the first draw and then four in second draw then $n(S) = {}^{10}C_6 \times {}^{10}C_4$

$\therefore$ Required Probability $= \frac{{}^{10}C_6 \times {}^6C_3 \times {}^4C_1}{{}^{10}C_6 \times {}^{10}C_4} = \frac{80 \times 24}{10 \times 9 \times 8 \times 7} = \frac{8}{21}$

32. If the letters of the word MATHEMATICS are arranged arbitrarily, the probability that C comes before E, E before H,H before I and I before S is

A) $\frac{1}{75}$ B) $\frac{1}{24}$ C) $\frac{1}{120}$ D) $\frac{1}{720}$

Key. C

Sol. The total numbers of arrangements is $\frac{11!}{2!2!2!}=\frac{11!}{8}$

The number of arrangements in which C, E, H, I, S appear in that order

$=\binom{11}{5}\frac{6!}{2!2!2!}=\frac{1}{8.5!}$

Probability $=\frac{11!}{8.5!}\div\frac{11!}{8!}=\frac{1}{5!}=\frac{1}{120}$

33. There are 12 pairs of shoes in a box. Then the possible number of ways of picking 7 shoes so that there are exactly two pairs of shoes are

A) 63360 B) 63300 C) 63260 D) 63060

Key. A

Sol. Total number of ways of picking up 7 shoes with 2 pairs is ${}^{12}C_2\times{}^{10}C_3\times2^3$

34. A pair of unbiased dice are rolled together till a sum of either 5 or 7 is obtained. Probability that 5 comes before 7 is

A) $\frac{1}{5}$ B) $\frac{2}{5}$ C) $\frac{3}{5}$ D) $\frac{4}{5}$

Key. B

Sol. A-event that sum 5 occurs, B-sum 7 occurs

$P(A)=\frac{1}{9},\ P(B)=\frac{1}{6}$,probability that neither a sum 5 or 7 occur $P=\frac{13}{18}$

$P=(A\,occurs\,before\,B)=\frac{1}{9}+\left(\frac{13}{18}\right)\left(\frac{1}{9}\right)+........\infty=\frac{2}{5}$

35. Team A plays with 5 other teams exactly once. Assuming that for each match the probabilities of a win, draw and loss are equal, then

A) the probability that A wins and loses equal number of matches is $\frac{34}{81}$

B) the probability that A wins and loses equal number of matches is $\frac{17}{81}$

C) the probability that A wins more number of matches than it loses is $\frac{17}{81}$

D) the probability that A loses more number of matches than it wins is $\frac{16}{81}$

Key. B

Sol. Probability of equal number of W and L is

$(0)W, (0)L + (1)W, (1)L + (2)W, (2)L$

$$= \left(\frac{1}{3}\right)^5 + {}^5C_1 . {}^4C_1 \left(\frac{1}{3}\right)^5 + {}^5C_2 . {}^3C_2 \left(\frac{1}{3}\right)^5 = \frac{17}{81}$$

36. A box contains 24 identical balls of which 12 are white and 12 black. The balls are drawn at random from the box one at a time with replacement. The probability that a white ball is drawn for the 4th time on the 7th draw is

A) $\frac{5}{64}$ B) $\frac{27}{32}$ C) $\frac{5}{32}$ D) $\frac{1}{2}$

Key. C

Sol. In any trail, P(getting white ball) $= \frac{1}{2}$

P(getting black ball) $= \frac{1}{2}$

Now, required event will occur if in the first six trails 3 white balls are drawn in any one of the

3 trails from six. The remaining 3 trails must be kept reserved for black balls. This can happen in ${}^6C_3 \times {}^3C_3 = 20$ ways.

So, required probability $= 20 \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^3 \times \frac{1}{2} = \frac{5}{32}$

37. Four identical dice are rolled once. Probability that atleast 3 different numbers appear on them is

A) $\frac{13}{42}$ B) $\frac{17}{42}$ C) $\frac{23}{42}$ D) $\frac{25}{42}$

Key. D

Sol. '$aaaa$' can appear in ${}^6C_1$ ways

'$aaab$' can appear in $2\left({}^{6}C_{2}\right)=30$

'$aabb$' can appear in ${}^{6}C_{2}=15$

'$aabc$' can appear in $3\left({}^{6}C_{3}\right)=60$

'$abcd$' can appear in ${}^{6}C_{4}=15$

Probability = $\frac{60+15}{6+30+15+60+15}=\frac{25}{42}$

38. If $x,y,z\in R$ and $x+y+z=5,\ xy+yz+zx=3$, probability for x to be positive only is

A) $\frac{3}{16}$ B) $\frac{5}{16}$ C) $\frac{13}{16}$ D) $\frac{15}{16}$

Key. C

Sol. Range of x is $\left[-1,\frac{13}{3}\right]$

Since probability of x to be

Positive is $\frac{\int_{0}^{\frac{13}{3}}dx}{\int_{-1}^{\frac{13}{3}}dx}=\frac{\frac{13}{3}}{\frac{13}{3}+1}=\frac{13}{16}$

39. If F is the set of all onto functions from a set of vowels to set having 3 elements and $f\in F$ is chosen randomly ,then the probability that $f^{-1}(x)$ is a singleton is

A) $\frac{7}{15}$ B) $\frac{8}{15}$ C) $\frac{9}{15}$ D) $\frac{10}{15}$

Key. A

Sol. No.of onto functions from A having 5 elements to set B having 3 elements is 150 we shall now count onto function which satisfy $f^{-1}(x)$ is singleton. We can choose a singleton in ${}^{5}C_{1}$ ways . The remain 4 elements can be mapped to remaining 2 elements in $2^{4}-2=14$ ways

$\therefore$ desired prob $=\frac{5(14)}{150}=\frac{7}{15}$

40. Probability that a random chosen three digit number has exactly 3 factors is

A) $\frac{2}{225}$ B) $\frac{7}{900}$ C) $\frac{1}{300}$ D) $\frac{4}{900}$

Key. B

Sol. A number has exactly 3 factors if the number is square of a prime number ,

squares of 11,13,17,19,23,29,31 are 3 digit numbers, required probability $=\frac{7}{900}$

41. If a, b are chosen randomly from the numbers present on a unbiased die with replacement. Probability that $\lim_{x\to 0}\left(\frac{a^x+b^x}{2}\right)^{\frac{2}{x}}=6$ is

A) $\frac{1}{3}$ B) $\frac{1}{4}$ C) $\frac{1}{9}$ D) $\frac{2}{9}$

Key. C

Sol. $\lim_{x\to 0}\left(\frac{a^x+b^x}{2}\right)^{\frac{2}{x}}=ab=6$

Total number of possible ways in which a, b can take values = 36

Possible ways are $\{(1,6),(6,1),(3,2),(2,3)\}$

Prob $=\frac{1}{9}$

42. Six persons stand at random in a queue for buying cinema tickets. Individually three of them have only a fifty rupee note each while each of the other three have a hundred rupee note only. The booking clerk has an empty cash box, probability that six persons get tickets without waiting for change is ----, (cost of one ticket is Rs.50/- and each person gets one ticket only)

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$

Key. C

Sol. Here random experiment is arranging 6 persons in a line $n(S)=6!=720$

Let $'a'$ denote person having Rs.50/-, $'b'$ denote person having Rs.100/- note each since all the six person, should get ticket first place should be occupied by a person having Rs.50/- and sixth place should be occupied by person having Rs.100/- possible cases are

a	aa	bb	b
a	ab	ab	b
a	ab	ba	b
a	ba	ab	b
a	ba	ba	b

'a's can arrange among themselves and

'b's can arrange among themselves in

$n(E) = 3!3! + 3!3! + 3!3! + 3!3! + 3!3! = 180$

Probability $= \frac{180}{720} = \frac{1}{4}$

43. A number is chosen at random from the set of all 4-digit numbers each of which contains not more than 2 different digits ,probability that it does not contain the digit zero is

A) $\frac{7}{64}$ B) $\frac{37}{64}$ C) $\frac{47}{64}$ D) $\frac{57}{64}$

Key. D

Sol. If $a \neq 0$ the numbers with O & a are

$aooo, aooa, aoao, aaoo, aaao, aaoa$ & $aoaa$. These are $9 \times 7 = 63\ (a \neq b, ab \neq 0)$

Now, $aaab, aaba, abaa, baaa$ are $4 \times 9 \times 8 = 288$ & $aabb, abab, abba$ are $3 \times 9 \times 8 = 216$ & $aaaa$ are 9

$\therefore$ prob $= \frac{288+216+9}{63+288+216+9} = \frac{513}{576} = \frac{57}{64}$

44. There are 3 bags .Bag 1 contain 2 red and $a^2 - 4a + 8$ black balls, bag 2 contains 1 red and $a^2 - 4a + 9$ black balls and bag 3 contains 3 red and $a^2 - 4a + 7$ black balls .A ball is draw at random from at random chosen bag. Then maximum value of probability that it is a red ball is

A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) $\frac{2}{9}$ D) $\frac{4}{9}$

Key. A

Sol. Req. prob $= \frac{1}{3}\left(\frac{6}{a^2 - 4a + 10}\right)$

$(P(A))_{max} = \frac{1}{3} \times \frac{6}{6} = \frac{1}{3}$

45. Let p, q be chosen one by one from the set $\{1, \sqrt{2}, \sqrt{3}, 2, e, \pi\}$ with replacement. Now a circle is drawn taking (p, q) as its centre then the probability that atmost two rational points exist on circle is (Rational points are those points whose both co-ordinates are rational)

A) $\frac{3}{4}$ B) $\frac{5}{6}$ C) $\frac{7}{8}$ D) $\frac{8}{9}$

Key. D

Sol. Suppose there exist three rational points or more on the circle

$x^2 + y^2 + 2gx + 2fy + c = 0$

If $(x_1, y_1)\,(x_2, y_2)\,(x_3, y_3)$ be those three points $S_{11} = 0,\ S_{22} = 0,\ S_{33} = 0$

on solving we get g, f, c as rational

Possible values of p are 1,2

q are 1,2

(p, q) be chosen as 4 ways

(p, q) can be chosen without restriction in $6 \times 6 = 36$

Prob $= 1 - \frac{4}{36} = 1 - \frac{1}{9} = \frac{8}{9}$

46. A, B are two independent events such that $P(A) > \frac{1}{2}$ $P(B) > \frac{1}{2}$. If $P(A \cap \bar{B}) = \frac{3}{25}$ and $P(\bar{A} \cap B) = \frac{8}{25}$ then $P(A \cap B) =$

A) 3/4 B) 2/3 C) 12/25 D) 18/25

Key. C

Sol. Let $P(A) = x$ and $P(B) = y$. $x > \frac{1}{2}, y > \frac{1}{2}$

$P(A - B) = x - xy = \frac{3}{25}$ and $P(B - A) = y - xy = \frac{8}{25}$

47. Two persons A and B are throwing 3 dice taking turns. If A throws 8 then the probability that B throws a higher number is

A) 5/27 B) 9/17 C) 8/27 D) 20/27

Key. D

Sol. Let E be the event that B throws a number more than 8. Then $P(E) = 1 - P(\overline{E})$

$|\overline{E}|$ = Number of positive integral solutions of $x + y + z \le 8$

$\therefore |\overline{E}| = {}^8c_3 = 56$ and $|S| = 216$ $\therefore P(\overline{E}) = \frac{56}{216} = \frac{7}{27}$

48. Three groups of children contain respectively 3 girls and 1 boy; 2 girls and 2 boys; 1 girl and 3 boys. One child is selected at random from each group. The probability that the three selected consists of 1 girl and 2 boys is

A) 5/9 B) 13/32 C) 12/19 D) 25/64

Key. B

Sol. $\frac{3}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{2}{4}\times\frac{1}{4}=\frac{26}{64}$

49. From a bag containing 10 distinct balls, 6 balls are drawn simultaneously and replaced. Then 4 balls are drawn. The probability that exactly 3 balls are common to the drawings is

A) 8/21 B) 6/19 C) 5/24 D) 9/22

Key. A

Sol. Let S be the sample space of the composite experiment of drawing 6 in the first draw and then four in second draw then $|S|={}^{10}C_6\times{}^{10}C_4$

$\therefore$ Required Probability = $\frac{{}^{10}C_6\times{}^{6}C_3\times{}^{4}C_1}{{}^{10}C_6\times{}^{6}C_4}=\frac{80\times24}{10\times9\times8\times7}=\frac{8}{21}$

50. Two persons each make a single throw with a pair of dice. The probability that their scores are equal is

A) 65/648 B) 69/648 C) 73/648 D) 91/648

Key. C

Sol. Required Probability = $=\frac{1^2+2^2+3^2+4^2+5^2+6^2+5^2+4^2+3^2+2^2+1^2}{36^2}$

51. A bag contains 4 red and 3 blue balls. Two drawings of two balls are made. The probability that the first drawing gives 2 red balls and the second drawing gives two blue balls if the balls are not returned to the bag after the first draw is

A) 2/49 B) 3/35 C) 3/10 D) 1/4

Key. B

Sol. $\frac{4c_2}{7c_2}\times\frac{3c_2}{5c_2}=\frac{4\times3}{7\times6}\times\frac{3}{10}=\frac{3}{35}$

52. A team has probability 2/3 of winning a game whenever it plays. If the team plays 4 games then the probability that it wins more than half of the games is

A) 17/25 B) 15/19 C) 16/27 D) 13/20

Key. C

Sol. Let p be the probability that the team wins a game. Let $q=1-p$ .Then the random variable "number of wins" follows the binomial distribution $P(X=K)={}^{4}C_kq^{4-k}p^k, k=0,1,2,3,4$.

Required probability = ${}^4C_3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 + {}^4C_4\left(\frac{2}{3}\right)^4 = \frac{16}{27}$.

53. Four Identical oranges and six distinct apples (each a different variety) are distributed randomly into five distinct boxes. The probability that each box gets a total of two objects is

A) $\frac{813}{109375}$ B) $\frac{162}{21875}$ C) $\frac{323}{43750}$ D) $\frac{151}{21875}$

Key. B

Sol. The total number of ways to put four identical oranges and six distinct apples into five distinct boxes is

$\left({}^{5+4-1}C_4\right).5^6 = 70\times 5^6$

To satisfy the criteria that each box contains two object we make three cases

(1) Two oranges in each of the two boxes and no oranges in the other three boxes

Number of ways $= {}^5C_2 \times \frac{\lfloor 6}{(2!)^3} = 900$

(2) Two oranges in one box, one orange in each of the two other boxes =

$5\times\left({}^4C_2\right)\times\frac{\lfloor 6}{(2!)^2} = 5400$

(3) One orange in each of the four boxes $5.\frac{\lfloor 6}{2!} = 1800$

The total number of ways $= 900 + 5400 + 1800 = 8100$

Probability $= \frac{8100}{70\times 5^6} = \frac{162}{21875}$

54. A bag contains two red balls and two green balls. A person randomly pulls out a ball, replacing it with a red ball regardless of the colour.What is the probability that all the balls are red after three such replacement ?

A) 3/8 B) 7/16 C) 5/32 D) 9/32

Key. D

Sol. In order that all balls are red after 3 replacements, two of the three balls selected must have green.

There could be three cases.

I : Red, Green, Red.

II : Green, red, Green

III : Green,Green,Red.

The probabilities is

In case I = $\frac{1}{2}.\frac{1}{2}.\frac{1}{4} = \frac{1}{16}$

In case II = $\frac{1}{2}.\frac{3}{4}.\frac{1}{4} = \frac{3}{32}$

In case III = $\frac{1}{2}.\frac{1}{4}.1 = \frac{1}{8}$

The required probability = $\frac{1}{16}+\frac{3}{32}+\frac{1}{8}=\frac{9}{32}$

55. Each face of a cubical die is numbered with a distinct number from among the first six odd numbers; such that the sum of the two numbers on any pair of opposite face is 12, if ten such dies are thrown simultaneously, then find the probability that the sum of the numbers that turn up is exactly 53

A) $\frac{53}{6^{10}}$ B) $\frac{153}{6^{10}}$ C) $\frac{3}{6^{10}}$ D) 0

Key. D

Sol. With 10 dice, the number on each face being odd, we can never get an odd number as their sum

56. Three fair coins are tossed simultaneously .Let E be the event of getting three heads or three tails,F be the event of at least two heads and G be the event of at most two heads then which of the following is true.

A) $P(E\cap F)=P(E).P(F)$ B) $P(E\cap G)=P(E).P(G)$

C) $P(F\cap G)=P(F).P(G)$ D) None.

Key. A

Sol. $P(E)=\frac{2}{8}=\frac{1}{4},P(F)=\frac{4}{8}=\frac{1}{2},P(G)=\frac{7}{8},$

$P(E\cap F)=\frac{1}{8},P(F\cap G)=\frac{3}{8},P(E\cap G)=\frac{1}{8}$

57. If E and F are two independent events such that $P(E\cap F)=\frac{1}{6},P(\overline{E}\cap\overline{F})=\frac{1}{3}$ and $P((E)-P(F))(1-P(F))>0$, Then

A) $P(E)=\frac{1}{2},P(F)=\frac{1}{3}$ B) $P(E)=\frac{1}{3},P(F)=\frac{1}{2}$

C) $P(E)=\frac{1}{4},P(F)=\frac{2}{3}$ D) $P(E)=\frac{2}{3},P(F)=\frac{1}{4}$

Key. A

Sol. $P(E)+P(F)=\frac{5}{6}$

$P(E)-P(F)=\frac{1}{6}$

58. Consider all the 3digit numbers abc (where $a\neq 0$) if a number is selected at random then the probability that the number is such that $a+b+c=6$ is

A) $\frac{2}{15}$ B) $\frac{7}{75}$ C) $\frac{7}{600}$ D) $\frac{7}{300}$

Key. D

Sol. Since $a+b+c=6$, the possible digit selections are
$(1,2,3),(1,1,4),(2,2,2),(0,1,5),(0,2,4),(0,3,3),(0,0,6)$

The required number of ways $6+3+1+4+4+2+1=21$

Required probability = $\dfrac{21}{9\times10\times10}=\dfrac{7}{300}$

59. The Probability that in a family of 5 members, exactly two members have birthday on Sunday is

A) $\dfrac{12\times5^3}{7^5}$ B) $\dfrac{10\times6^3}{5^7}$ C) $\dfrac{12\times6^2}{5^7}$ D) $\dfrac{10\times6^3}{7^5}$

Key. D

Sol. Required Probability = $\dfrac{{}^5C_2\times6\times6\times6}{7\times7\times7\times7\times7}$

60. If three numbers are chosen randomly from the set $\{1,3,3^2,.......,3^n\}$ without replacement then the probability that they form an increasing geometric progression is

A) $\dfrac{3}{2n}$ if n is odd B) $\dfrac{3}{2n}$ if n is even

C) $\dfrac{3n}{n^2-1}$ if n is even D) $\dfrac{3n}{2(n^2-1)}$ if n is odd

Key. A

Sol. Number of triplets $\left(3^r,3^{r+1},3^{r+2}\right)(0\le r\le n)$ is n-1

Number of triplets $\left(3^r,3^{r+2},3^{r+4}\right)(0\le r\le n)$ is n-3

Number of triplets $\left(3^r,3^{r+\frac{n-1}{2}},3^{r+n-1}\right)(n \text{ odd})$ is 2

And Number of triplets $\left(3^r,3^{r+\frac{n}{2}},3^{r+n}\right)(n \text{ even})$ is 1.

$\therefore$ If n is odd, the number of favorable out comes $=(n-1)+(n-3)+...+4+2=\dfrac{n^2-1}{4}$

And if n is even, the number of favorable out comes

$=(n-1)+(n-3)+...+3+1=\dfrac{n}{2}\times\dfrac{n}{2}=\dfrac{n^2}{4}$

Probability = $\dfrac{=(n^2-1)/4}{{}^{(n+1)}C_3}=\dfrac{3}{2n}$ if n is odd

$=\dfrac{n^2/4}{{}^{(n+1)}C_3}=\dfrac{3n}{2(n^2-1)}$ if n is even.

61. A fair coin is tossed until one of the two sides occurs twice in a row, The Probability that the number of tosses required is even is

A) 1/3 B) 2/3 C) 1/4 D) 3/4

Key. B

Sol. A= $\{HH, HTHH, HTHTHH, ...\}$

And B= $\{TT, THTT, THTHTT, ...\}$ P(A) = $\frac{1}{2^2}+\frac{1}{2^4}+.....=\frac{1}{3}$

$P(B)=\frac{1}{3}$

Required Probability $=\frac{1}{3}+\frac{1}{3}$

62. A determinant is chosen at random from the set of all determinants of order 2 with elements 1 and 0 only. The probability that the value of the determinant is positive is

A) $\frac{1}{8}$ B) $\frac{3}{16}$ C) $\frac{5}{16}$ D) $\frac{1}{16}$

Key. B

Sol. no. of determinants formed $=n(s)=2^4=16$

The determinants $\begin{vmatrix}1 & 0\\0 & 1\end{vmatrix}\begin{vmatrix}1 & 0\\1 & 1\end{vmatrix}\begin{vmatrix}1 & 1\\0 & 1\end{vmatrix}$ are positive $\Rightarrow n(E)=3$

$\therefore P(E)=\frac{3}{16}$

63. Two numbers x and y are chosen at random from the set $\{1,2,3,----3n\}$. The probability that $x^2 - y^2$ is divisible by 3 is

A) $\frac{5n-3}{3(3n-1)}$ B) $\frac{2n}{3(3n-1)}$

C) $\frac{5n}{3(3n-1)}$ D) $\frac{n}{3n-1}$

Key. A

Sol.

G_1 = 1	4	7	----	3n–1
G_2 = 2	5	8	----	32–2
G_3 = 3	6	9	----	3n

We have $n(s)={}^{3n}C_2$ for n(E) both numbers can be selected from same group or one number from G_2 and one from G_3

$\therefore n(E)={}^{3n}C_2+nc_1\times nc_1$

$\therefore P(E)=\frac{3.{}^nC_2+n^2}{{}^{3n}C_2}$

64. In shuffling a pack of cards 3 are accidentally dropped. The chance that the missing cards are of different suits is.

A) $\frac{169}{425}$ B) $\frac{169}{1700}$

C) $\frac{1}{4}$ D) $\frac{169}{2550}$

Key. A

Sol. $n(S)={}^{52}C_3$ $n(E)={}^{4}C_3\times{}^{13}C_1\times{}^{13}C_1\times{}^{13}C_1$

65. Two small squares on a chess board are selected at random. The probability that they have a common side is.

A) $\frac{1}{36}$ B) $\frac{1}{9}$ C) $\frac{1}{3}$ D) $\frac{1}{18}$

Key. D

Sol. $n(S)={}^{64}C_2$

n(E) = selecting two consecutive squares from a row or column

$=7\times8+7\times8=112$

66. In a convex hexagon two diagnols are drawn at random. The probability that the diagnols intersect at an interior point of the hexagon is

A) $\frac{5}{12}$ B) $\frac{7}{12}$ C) $\frac{2}{5}$ D) $\frac{1}{3}$

Key. A

Sol. $n(S)={}^{9}C_2$ $n(E)={}^{6}C_4$

67. If 6 articles are distributed at random among 6 persons the probability that at least one person does not get any article is

A) $\frac{319}{324}$ B) $\frac{317}{324}$ C) $\frac{313}{324}$ D) $\frac{79}{162}$

Key. A

Sol. $n(S)=6^6$ $n(E)=6^6-6!$

68. A car is parked by an owner amongst 25 cars in a row not at either end. In his return he finds that exactly 15 places are still occupied. The probability that both the neighbouring places are empty is

A) $\frac{91}{276}$ B) $\frac{15}{184}$ C) $\frac{15}{92}$ D) none

Key. C

Sol. It is given that 15 places are occupied. 14 other cars are parked no. of ways of selecting 14 places from 24 in ${}^{24}C_{14}$ ways. Excluding the neighbouring places there are 22 places in where 14 cars can be parked in ${}^{22}C_{14}$ ways.

$$\therefore P(E) = \frac{{}^{22}C_{14}}{{}^{24}C_{14}}$$

69. Suppose $f(x) = x^3 + ax^2 + bx + c$, where a, b, c are chosen respectively by throwing a die three times. The probability that f(x) is an increasing function is.

A) $\frac{4}{9}$ B) $\frac{3}{8}$ C) $\frac{2}{5}$ D) $\frac{8}{17}$

Key. A

Sol. $f'(x) = 3x^2 + 2ax + b$ f(x) is increasing $f'(x) \geq 0 \forall x$ *and for* $f'(x) = 0$ should not form an interval

$\therefore a^2 - 3b \leq 0$

This is true for exactly 16 ordered pairs (a,b) $1 \leq a, b \leq 6$ (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (2,2) (2,3) (2,4) (2,5) (2,6) (3,3) (3,4) (3,5) (3,6) (4,6)

$$\therefore P(E) = \frac{16}{36} = \frac{4}{9}$$

70. A and B throw alternatively with a pair of dice .A wins if he throws a sum 6 before B throws 7 and B wins if he throws a 7 before A throws sum 6.If A starts the game ,his chance of winning is

a) $\frac{30}{61}$ b) $\frac{31}{61}$ c) $\frac{15}{61}$ d) $\frac{60}{61}$

Key. A

Sol. A's chance of winning in a throw = $\frac{5}{36}$, B's chance of winning in a throw = $\frac{1}{6}$

A's chance of losing in a throw = $\frac{31}{36}$, B's chance of losing in a throw = $\frac{5}{6}$

A can winning the game = $\frac{5}{36} + \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36} + \frac{31}{36} \times \frac{5}{6} \times \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36} + ----$

$$= \frac{5}{36}\left[1 + \frac{155}{216} + \left(\frac{155}{216}\right)^2 + ----\right] = \frac{30}{61}$$

71. From a group of n persons arranged in a circle, 3 persons are selected at random .If probability that no two adjacent persons are selected is $\frac{2}{7}$ then n=

a) 6 b) 7 c) 8 d) 9

Key. C

Sol. P(no two adjacent persons are selected) = $\frac{n(n-4)(n-5)/6}{n(n-1)(n-2)/6}=\frac{(n-4)(n-5)}{(n-1)(n-2)}=\frac{2}{7}$

$$\Rightarrow 5n^2-57n+136=0 \Rightarrow n=8$$

72. A box contains 5 pairs of shoes. If 6 shoes are selected at random, the probability that exactly one pair of shoes is obtained is

a) $\frac{4}{21}$ b) $\frac{2}{21}$ c) $\frac{8}{21}$ d) $\frac{16}{21}$

Key. C

Sol. From 10 shoes, 6 can be selected in 10_{C_6} ways and if there is to be one pair among them if can be selected in 5_{C_1} ways. Other 4 shoes can be selected in $4_{C_4}(2^4)$ =16 ways .

Hence probability = $\frac{5\times 16}{10_{C_6}}=\frac{8}{21}$

73. An unbiased coin is tossed 12 times .The probability that at least 7 consecutive heads show up is

a) $\frac{1}{128}$ b) $\frac{1}{64}$ c) $\frac{9}{256}$ d) $\frac{7}{256}$

Key. D

Sol. The sequence of consecutive heads may starts with 1st toss or 2nd toss or 3rd toss ---- or at 6th toss. In any case ,if it starts with r th throw , the first (r-2) throws may be head or tail but (r-1)st throw must be tail, after which again a head or tail can show up:

HHH------H x x x x x or x x x --------- x T H HH-------H x x------

7 times (r-2) (r-1)st

$\therefore$ Probability = $\frac{1}{2^7}+\frac{1}{2}\cdot\frac{1}{2^7}+\frac{1}{2}\cdot\frac{1}{2^7}+\text{---------}+\frac{1}{2}\cdot\frac{1}{2^7}=\frac{1}{2^7}\left[1+\frac{5}{2}\right]=\frac{7}{2^8}$

5 times

74. Six fair dice are thrown independently .The probability that there are exactly 2 different pairs (A pair is an ordered combination like 2, 2, 1, 3, 5, 6) is

a) $\frac{5}{72}$ b) $\frac{25}{72}$ c) $\frac{125}{144}$ d) $\frac{5}{36}$

Key. B

Sol. Total no. of outcomes = 6^6 .number of ways of choosing 4 other different numbers is 6_{C_2} and choosing 2 out of remaining 4 can be lone in 4_{C_2} ways. Also number of ways of arranging 6 numbers of which 2 are alike and 2 are alike is $\frac{6}{2!2!}$.

$\therefore$ Required probability= $\frac{6_{C_2}\times 4_{C_2}\times \frac{6}{2!2!}}{6^6}=\frac{25}{72}$.

75. Two integers x and y are chosen from the set {0, 1, 2, 3, -----, 2n}, with replacement, the probability that $|x-y|\le n(n\in N)$ is

a) $\frac{3n^2+3n+1}{(2n+1)^2}$ b) $\frac{3n^2+3n}{(2n+1)^2}$ c) $\frac{3n^2+1}{(2n+1)^2}$ d) $\frac{n^2+n+1}{(2n+1)^2}$

Key. A

Sol. x and y can be any one of (2n+1) numbers given . $|x-y|\le n \Rightarrow x-n\le y\le x+n$

Hence number of possibilities of y, for x=0,1,2,3,----n-1,n,n+1----2n are n+1,n+2,n+3n------2n,2,+1,2n,2,-1,-----n+1 respectively.

$\therefore$ Probability = $\frac{2\left(\overline{n+1}+\overline{n+2}+------+\overline{2n}\right)+2n+1}{(2n+1)^2}=\frac{3n^2+3n+1}{(2n+1)^2}$

76. A die is rolled three times, the probability of getting large number than the previous number is

A) 1/54 B) 5/54 C) 5/108 D) 13/108

Key. B

Sol. If the 2nd number is $i(i>1)$ the no.of favourable ways = $(i-1)\times(6-i)$

$n(E)=$ total no.of favourable ways = $\sum_{i=1}^{6}(i-1)\times(6-i)=1\times4+2\times3+3\times2+4\times1=20$

Required probability = $\frac{20}{216}=\frac{5}{54}$

77. 10 apples are distributed at random among 6 persons. The probability that at least one of them will receive none is

A) $\frac{6}{143}$ B) $\frac{{}^{14}C_4}{{}^{15}C_5}$ C) $\frac{137}{143}$ D) $\frac{143}{137}$

Key. C

Sol. The required probability = 1 – probability of each receiving at least one = $1-\frac{n(E)}{n(S)}$.

Now, the number of integral solutions of $x_1+x_2+x_3+x_4+x_5+x_6=10$

Such that $x_1\ge1, x_2\ge1,...., x_6\ge1$ gives $n(E)$ and the number of integral solutions of $x_1+x_2+x_3+x_4+x_5+x_6=10$ such that $x_1\ge0, x_2\ge0,...., x_6\ge0$ gives $n(S)$

$\therefore$ The required probability = $1-\frac{^{10-1}C_{6-1}}{^{10+6-1}C_{6-1}}=1-\frac{^{9}C_{5}}{^{15}C_{5}}=\frac{137}{143}$.

78. A, B are two independent events such that $P(A)>\frac{1}{2}$ $P(B)>\frac{1}{2}$. If $P(A\cap\bar{B})=\frac{3}{25}$ and $P(\bar{A}\cap B)=\frac{8}{25}$ then $P(A\cap B)=$

A) 3/4 B) 2/3 C) 12/25 D) 18/25

Key. C

Sol. Let $P(A)=x$ and $P(B)=y$. $x>1/2, y>1/2$

$P(A-B)=x-xy=3/25$ and $P(B-A)=y-xy=8/25$

79. Two persons A and B are throwing 3 dice taking turns. If A throws 8 then the probability that B throws a higher number is

A) 5/27 B) 9/17 C) 8/27 D) 7/27

Key. D

Sol. Let E be the event that B throws a number more than 8. Then $P(E)=1-P(\bar{E})$

$|\bar{E}|$ = Number of positive integral solutions of $x+y+z\le 8$

$\therefore |\bar{E}|={}^{8}c_{3}=56$ and $|S|=216$ $\therefore P(\bar{E})=\frac{56}{216}=\frac{7}{27}$

80. Three groups of children contain respectively 3 girls and 1 boy; 2 girls and 2 boys; 1 girl and 3 boys. One child is selected at random from each group. The probability that the three selected consists of 1 girl and 2 boys is

A) 5/9 B) 13/32 C) 12/19 D) 25/64

Key. B

Sol. $\frac{3}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{2}{4}\times\frac{1}{4}=\frac{26}{64}$

81. A certain type of missile hits its target with probability 0.3. The number of missiles that should be fired so that there is atleast an 80% probability of hitting a target is

A) 3 B) 4 C) 5 D) 6

Key. C

Sol. Let n be the required number.

$\therefore$ The probability that 'n' missiles miss the target is $(0.7)^n$. We require $1-(0.7)^n>0.8$

i.e., $(0.7)^n<0.2$. The least value of 'n' satisfying this inequality is 5.

82. A team has probability 2/3 of winning a game whenever it plays. If the team plays 4 games then the probability that it wins more than half of the games is

A) 17/25 B) 15/19 C) 16/27 D) 13/20

Key. C

Sol. Let p be the probability that the team wins a game. Let $q=1-p$.Then the random variable "number of wins" follows the binomial distribution $P(X=K)={}^4C_k q^{4-k}p^k, k=0,1,2,3,4$.

Required probability = ${}^4C_3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3+{}^4C_4\left(\frac{2}{3}\right)^4=\frac{16}{27}$.

83. From a bag containing 10 distinct balls, 6 balls are drawn simultaneously and replaced. Then 4 balls are drawn. The probability that exactly 3 balls are common to the drawings is

A) 8/21 B) 6/19 C) 5/24 D) 9/22

Key. A

Sol. Let S be the sample space of the composite experiment of drawing 6 in the first draw and then four in second draw then $|S|={}^{10}C_6\times{}^{10}C_4$

$\therefore$ Required Probability = $\frac{{}^{10}C_6\times{}^6C_3\times{}^4C_1}{{}^{10}C_6\times{}^6C_4}=\frac{80\times24}{10\times9\times8\times7}=\frac{8}{21}$

84. Two persons each make a single throw with a pair of dice. The probability that their scores are equal is

A) 65/648 B) 69/648 C) 73/648 D) 91/648

Key. C

Sol. Required Probability = $=\frac{1^2+2^2+3^2+4^2+5^2+6^2+5^2+4^2+3^2+2^2+1^2}{36^2}$

85. $'A'$ is a 3×3 matrix with entries from the set $\{-1,0,1\}$. The probability that $'A'$ is neither symmetric nor skew symmetric is

A) $\frac{3^9-3^6-3^3+1}{3^9}$ B) $\frac{3^9-3^6-3^3}{3^9}$ C) $\frac{3^9-1}{3^{10}}$ D) $\frac{3^9-3^3+1}{3^9}$

Key. A

Sol. Conceptual

86. The probability that in a family of 5 members, exactly two members have birthday on Sunday is

(A) $\frac{12\times5^3}{7^5}$ (B) $\frac{10\times6^3}{5^7}$

(C) $\frac{12\times6^2}{5^7}$ (D) $\frac{10\times6^3}{7^5}$

Key: D

Hint: Required probability $=\frac{{}^5C_2\times6\times6\times6}{7\times7\times7\times7\times7}=\frac{10\times6^3}{7^5}$

87. If a is an integer lying in the closed interval [- 5 ,30] , then the probability that the graph of $y = x^2 + 2(a+4)x - 5a + 64$ is strictly above the x-axis is

a) 2/9 b) 1/6 c) 1/2 d) 5/9

Key: C

Hint $(a+4)^2 + 5a - 64 < 0 \Rightarrow -16 < a < 3$

$\therefore probability = \frac{18}{36} = 2/9$

88. If three numbers are chosen randomly from the set $\{1, 3, 3^2, ..., 3^n\}$ without replacement, then the probability that they form an increasing geometric progression is

a) $\frac{3}{2n}$ *if n is odd* b) $\frac{3}{2n}$ *if n is even*

c) $\frac{3n}{2(n^2-1)}$ *if n is even* d) $\frac{3n}{2(n^2-1)}$ *if n is odd*

Key: A,C

Hint: Number of triplets $(3^r, 3^{r+1}, 3^{r+2})(0 \le r \le n)$ is $n-1$

Number of triplets $(3^r, 3^{r+2}, 3^{r+4})(0 \le r \le n)$ is $n-3$

Number of triplets $\left(3^r, 3^{r+\frac{n-1}{2}}, 3^{r+n-1}\right)(n\ odd)$ is 2

and no of triplets $\left(3^r, 3^{r+\frac{n}{2}}, 3^{r+n}\right)(n\ even)$ is 1

$\therefore$ If n is odd, the number of favourable outcomes

$= (n-1) + (n-3) + ... + 4 + 2 = \frac{n^2-1}{4}$

and if n is even, the number of favourable outcomes

$= (n-1) + (n-3) + ... + 3 + 1 = \frac{n}{2} \times \frac{n}{2} = n^2/4$

$\therefore$ Prob = $\frac{(n^2-1)/4}{(n+1)C_3} = 3/2n$ if n is odd

$= \frac{n^2/4}{(n+1)C_3} = \frac{3n}{2(n^2-1)}$ if n is even

89. The probabilities of A, B, C solving a problem independently are respectively $\frac{1}{4},\frac{1}{5},\frac{1}{6}$. If 21 such problems are given to A, B, C then the probability that atleast 11 problems can be solved by them is

a) ${}^{21}C_{11}\left(\frac{1}{2}\right)^{11}$ b) $\frac{1}{2}$ c) $\left(\frac{1}{2}\right)^{11}$ d) ${}^{21}C_{11}\frac{2^{11}}{3^{21}}$

Key: B

Hint: No. of trials n = 21

Success is solving the problem

$$\therefore p = P(A\cup B\cup C) = 1 - P\left(\overline{A}\right)P\left(\overline{B}\right)P\left(\overline{C}\right)$$

$$= 1 - \frac{3}{4}\times\frac{4}{5}\times\frac{5}{6}$$

$$= \frac{1}{2}$$

$$q = \frac{1}{2}$$

Find $P\left(X \ge 11\right)$

90. Four identical oranges and six distinct apples (each a different variety) are distributed randomly into five distinct boxes. The probability that each box gets a total of two objects is

(A) $\frac{813}{109375}$ (B) $\frac{162}{21875}$ (C) $\frac{323}{43750}$ (D) $\frac{151}{21875}$

Key: B

Hint; The total number of ways to put four identical oranges and six distinct apples into five distinct boxes is

$$\left({}^{5+4-1}C_4\right).\, 5^6 = 70\times 5^6$$

to satisfy the criteria that each box contains two objects we make three cases(based on number of oranges to go into a box)

1. two oranges in each of the two boxes and no oranges in the other three boxes. number of ways $= {}^5C_2 \times \frac{\lfloor 6}{\lfloor 2\lfloor 2\lfloor 2} = 900$

2. two oranges in one box, one orange in each of the two other boxes

$$(5)\times\left({}^4C_2\right)\times\frac{\lfloor 6}{\lfloor 2\lfloor 2\lfloor 1\lfloor 1} = 5.6.180$$

$$= 5400$$

3. one orange in each of the four boxes

$$= 5 \cdot \frac{\lfloor 6}{\lfloor 2\lfloor 1\lfloor 1\lfloor 1} = 5\times 360 = 1800$$

the total number of ways $= 900 + 5400 + 1800 = 8100$

probability $= \frac{8100}{70\times 5^6} = \frac{162}{21875}$

91. A bag contains two red balls and two green balls. A person randomly pulls out a ball, replacing it with a red ball regardless of the colour. What is the probability that all the balls are red after three such replacements?

(A) $\frac{3}{8}$ (B) $\frac{7}{16}$

C) $\frac{5}{32}$ D) $\frac{9}{32}$

Key: D

Hint In order that all balls are red after 3 replacements , two of the three balls selected must have been green. There could be three cases

I : red , green , red

ii : green , red , green

iii : green , green , red (since they are now all red)

the probability is

case (i) is $\frac{1}{2}.\frac{1}{2}.\frac{1}{4}=\frac{1}{16}$

case (ii) is $\frac{1}{2}.\frac{3}{4}.\frac{1}{4}=\frac{3}{32}$

case (iii) is $\frac{1}{2}.\frac{1}{4}.1=\frac{1}{8}$

the required probability $=\frac{1}{16}+\frac{3}{32}+\frac{1}{8}=\frac{9}{32}$

92. In a test student either guesses or copies or knows the answer to a multiple choice questions with four choices in which exactly one choice is correct. The probability that he makes a guess is $\frac{1}{3}$; The probability that he copies the answer is $\frac{1}{6}$.The Probability that his answer is correct given that he copied it is $\frac{1}{8}$.Find the probability that he knew the answer to the question given that he correctly answered it is

(A) $\frac{29}{35}$ (B) $\frac{24}{29}$ (C) $\frac{1}{7}$ (D) $\frac{1}{9}$

Key: B

Hint: Let ‘A’ be the event of guessing the correct answer.

‘B’ be the event of copying the correct answer.

‘C’ be the event of knowing the correct answer.

‘D’ be the event that his answer is correct

$$P(A)=\frac{1}{3}$$

$$\left.\begin{aligned} P(B) &= \frac{1}{6} \\ P(C) &= \frac{1}{2} \\ P(\frac{D}{B}) &= \frac{1}{8} \\ P(\frac{D}{A}) &= \frac{1}{4} \\ P(\frac{D}{C}) &= 1 \end{aligned}\right\} \qquad P\left(\frac{C}{D}\right) = \frac{P(C).P\left(\frac{D}{C}\right)}{P(D\cap A)+P(D\cap B)+P(D\cap C)} = \frac{24}{29}$$

93. Consider the system of equations $ax+by=0$ and $cx+dy=0$ where $a,b,c,d \in \{1,2\}$. The probability that the system of equations has a unique solution is

A) 3/8 B) 5/16

C) 9/16 D) 5/8

Key: D

Hint: 1) $ad=1, bc=4 \Rightarrow (a=1, d=1, b=2, c=2)$

2) $ad=1, bc=2 \Rightarrow (a=1, d=1, b=1, c=2), (a=1, d=1, b=2, c=1)$

3) $ad=2, bc=1 \Rightarrow (a=1, d=2, b=1, c=1), (a=2, d=1, b=1, c=1)$

4) $ad=2, bc=4 \Rightarrow (a=1, d=2, b=2, c=2), (a=2, d=1, b=2, c=2)$

5) $ad=4, bc=1, \Rightarrow (a=2, d=2, b=1, c=1)$

6) $ad=4, bc=2 \Rightarrow (a=2, d=2, b=1, c=2), (a=2, d=2, b=2, c=1)$

$\Rightarrow$ required probability = $\frac{10}{16} = \frac{5}{8}$

94. Let A, B, C be any three events in a sample space of a random experiment. Let the events $E_1 =$ exactly one of A, B occurs $E_2 =$ exactly one of B, C occurs, $E_3 =$ exactly one of C, A occurs, $E_4 =$ all of A, B, C occurs, $E_5 =$ atleast one of A, B, C occurs. $P(E_1)=P(E_2)=P(E_3)=1/3,\ P(E_4)=\frac{1}{9}$ then $P(E_5)=$

A) $\frac{1}{9}$ B) $\frac{7}{9}$ C) $\frac{5}{18}$ D) $\frac{11}{18}$

Key: D

Hint: $E_1=(A\cap\bar{B})\cup(\bar{A}\cap B),\ E_2=(B\cap\bar{C})\cup(\bar{B}\cap C), E_3=(C\cap\bar{A})\cup(\bar{C}\cap A)$

$E_4 = A\cap B\cap C \quad E_5 = A\cup B\cup C$.

95. The numbers *a,b* are chosen from the set {1,2,3,4,---,10} such that $a \le b$ with replacement. The probability that *a* divides *b* is

(A) $\frac{5}{11}$ (B) $\frac{29}{55}$ (C) $\frac{27}{55}$ (D) None

Key: C

Hint: Number of ways of choosing *a&b* s.t $a \le b = 10+9+8+---+1=55$

" " " *a&b* s.t *a* divides *b* =10+5+3+2+2+5=27

$\therefore$ Required probability= $\frac{27}{55}$

96. An electric component manufactured by a company is tested for its defectiveness by a sophisticated device. Let 'A' denote the event " the device is defective " and 'B' the event "the testing device reveals the component to be defective" . Suppose $P(A)=\alpha$ and $P(B/A)=P(\bar{B}/\bar{A})=1-\alpha$. Where $0<\alpha<1$. If it is given that the testing device revels it to be defective , then the probability that the component is not defective is

A) $\frac{1}{4}$ B) $\frac{3}{4}$ C) 0.7 D) 0.5

Key: D

Hint:
$$P\left(\frac{\bar{A}}{B}\right)=\frac{P(\bar{A}).P(B/\bar{A})}{P(A).P(B/A)+P(\bar{A}).P(B/\bar{A})}$$
$$=\frac{(1-\alpha)\alpha}{\alpha(1-\alpha)+(1-\alpha)\alpha}=\frac{1}{2}$$

97. P (B) =

a) $\frac{6^3}{7^3}$ b) $\frac{5^3}{7^3}$

c) $\frac{(2^6-2)^3}{2^{18}}$ d) $\frac{(2^6-1)^3}{2^{18}}$

Key: C

Hint: no. of ways of selecting atleast one but not al red balls for bag B_1 when considered them differently = ${}^6C_1+{}^6C_2+......+{}^6C_5=2^6-2$.

Similarly for black and white, no. of ways of selecting atleast one but not all for bag B_1 =

$2^6-2,\ Hence\ n(B)=(2^6-2)^3 \Rightarrow P(B)=\frac{(2^6-2)^3}{2^{18}}$

(Though no. of different ways of giving atleast one but not all Red (Identical balls) ball to bag B_1 is (4+1) = 5 i.e., no. of different ways of giving at least one ball of each colour in (4+1) (4+1) (4+1) = 5^3. but these are not equally likely so we can not use this.)

98. P (C/B) =

a) $\frac{{}^{12}C_6-1}{(2^6-1)^2}$ b) $\frac{({}^{12}C_6-2)}{(2^6-2)^2}$ c) $\frac{1}{5}$ d) $\frac{7}{25}$

Key: B

Hint: $P(C/B)=\frac{P(B\cap C)}{P(B)}=\frac{n(B\cap C)}{n(B)}$

$(B\cap C)$ event is same as selecting (1W, 1R), (2W, 2R), (3W, 3R) (4W, 4R) (5W, 5R) , and at leat one but not all black balls when considering all balls different

$$n(B\cap C)=\left({}^6C_1{}^6C_1+{}^6C_2{}^6C_2+......+{}^6C_5.{}^6C_5\right).\left({}^6C_1+{}^6C_2+......+{}^6C_5\right)$$

$$=\left({}^{12}C_6-2\right)\left(2^6-2\right)\Rightarrow P(C/B)=\frac{{}^{12}C_6-2}{\left(2^6-2\right)^2}$$

(Though no. of different ways of event $B\cap C$ is 5^2. when balls are identical. i.e., selecting (1W, 1R), (2W, 2R) .....(5W, 5R) → 5 ways and selecting atleast one but not all black balls → 5 ways.)

99. If E and F are two independent events, such that $P(E\cap F)=\frac{1}{6}, P\left(E^C\cap F^C\right)=\frac{1}{3}$ and $\left(P(E)-P(F)\right)\left(1-P(F)\right)>0$, then

(A) $P(E)=\frac{1}{2}$ (B) $P(E)=\frac{1}{4}$ (C) $P(F)=\frac{1}{3}$ (D) $P(F)=\frac{2}{3}$

Key: A, C

Hint: $P(E\cap F)=P(E).P(F)=\frac{1}{6}$ ......(i)

$P\left(E^c\cap F^c\right)=\left(1-P(E)\right)\left(1-P(F)\right)=\frac{1}{3}$

$\Rightarrow P(E)+P(F)=\frac{5}{6}$ ....(ii)

$\Rightarrow \left|P(E)-P(F)\right|=\frac{1}{6}$

As $\left(P(E)-P(F)\right)\left(1-P(F)\right)>0$

$\Rightarrow P(E)>P(F)\Rightarrow P(E)-P(F)=\frac{1}{6}......(iii)$

Solving (ii) and (iii) $\Rightarrow P(E)=\frac{1}{2}, P(F)=\frac{1}{3}$

100. Mr. A makes a bet with Mr. B that in a single throw with two dice he will throw a total of seven before B throws four. Each of them has a pair of dice and they throw simultaneously until one of them wins equal throws being disregarded. Probability that B wins, is

(A) $\frac{1}{3}$ (B) $\frac{4}{11}$ (C) $\frac{5}{16}$ (D) $\frac{6}{17}$

Key: A

Hint: We have $P(A)=P(7)=\frac{6}{36}m, P(B)=P(4)=\frac{3}{36}$

Since equal throws are disregarded,

Hence in each throw A is twice as likely to win as B.

Let $P(B)=p, P(A)=2p$

$$3p = 1 \Rightarrow P = \frac{1}{3}$$

101. Each face of a cubical die is numbered with a distinct number from among the first six odd numbers, such that the sum of the two numbers on any pair of opposite face is 12. if ten such dices are thrown simultaneously, then find probability that the sum of the numbers that turn up is exactly 53.

(A) $\frac{53}{6^{10}}$ (B) $\frac{153}{6^{10}}$

(C) $\frac{3}{6^{10}}$ (D) 0\

Key : D

Sol : With 10 dice, the number on each face being odd, we can never get an odd number, since the sum of 10 odd numbers can never be an odd. Hence required probability = 0.

102. Two cards are selected at randomly from a pack of ordinary playing cards. If there found to be of different colours (Red & Black), then conditional probability that both are face cards is

(A) $\frac{36}{325}$ (B) $\frac{18}{169}$

(C) $\frac{9}{169}$ (D) none of these

Key : C (final key)

Sol : Let A $\rightarrow$ they are face cards, B $\rightarrow$ they are of different colours

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{{}^{12}C_2 - 2 \times {}^{6}C_2}{13 \times 26} = \frac{18}{169}.$$

103. Sum of the series S= $\sum_{r=1}^{\infty} P_r$ is

a) 1 b) $\frac{1}{6}$

c) $\frac{1}{5}$ d) $\frac{2}{3}$

key : A (final key)

sol : $S = \frac{1}{5}\left[1 + \sum_{r=7}^{\infty}\left(\frac{1}{6}\right)^{r-6} - \sum_{r=2}^{\infty}\left(\frac{1}{6}\right)^{r-1}\right]$

$$S = \frac{1}{5}$$

104. In a knockout tournament 16 equally skilled players namely P_1, P_2, -------- P_{16} are participating. In each round players are divided in pairs at random and winner from each pair moves in the next round. If P_2 reaches the semifinal, then the probability that P_1 will win the tournament is.

a) $\frac{3}{64}$ b) $\frac{1}{16}$ c) $\frac{1}{20}$ d) $\frac{1}{15}$

Key: C

Hint: Let E_1 = P_1 win the tournament, E_2 = P_2 reaches the semifinal since all players are equally skilled and there are 4 persons in the semifinal $P(E_2)=\frac{{}^{15}C_3}{{}^{16}C_4}=\frac{4}{16}=\frac{1}{4}$

$E_1 \cap E_2$ = P_1 & P_2 both are in semifinal and P_1 wins in semifinal and final

$$P(E_1 \cap E_2)=\frac{{}^{16-2}C_2}{{}^{16}C_4}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{3}{16.15}=\frac{1}{80}$$

$$P(E_1/E_2)=\frac{P(E_1\cap E_2)}{P(E_2)}=\frac{1/80}{1/4}=\frac{1}{20}$$

105. $'A'$ is a 3×3 matrix with entries from the set $\{-1,0,1\}$. The probability that $'A'$ is neither symmetric nor skew symmetric is

A) $\frac{3^9-3^6-3^3+1}{3^9}$ B) $\frac{3^9-3^6-3^3}{3^9}$ C) $\frac{3^9-1}{3^{10}}$ D) $\frac{3^9-3^3+1}{3^9}$

Key. A

Sol. Total number of matrices that can be formed is 3^9.

Let $A=[a_{ij}]_{3\times 3}$ where $a_{ij}\in\{-1,0,1\}$

If $'A'$ is symmetric then $a_{ij}=a_{ij}\ \forall\ i,j$

If $'A'$ is skew symmetric then $a_{ij}=-a_{ij}\ \forall\ i,j$

106. The digits of a nine – digit number are 1,2,3,4,5,6,7,8,9 written in random order, then the probability that the number is divisible by 11 is

a) $\frac{11}{126}$ b) $\frac{13}{126}$ c) $\frac{11}{104}$ d) $\frac{5}{63}$

Key. A

Sol. Total number of numbers having 9 digits = 9!

A number is divisible by 11 is the difference between the sum of the digits in odd places and the sum of the digits in even places is it self divisible by 11

As the sum digits is 45, the only possibility of the numbers being divisible by 11 is when the sum of the digits in odd places is 28 and the sum of the digits in even places in 17.

$\therefore$ Number of favorable cases = $11\times 5!\times 4!$

$\therefore$ Required probability $= \dfrac{11\times 5!\times 4!}{9!} = \dfrac{11}{126}$

107. That probability that a randomly chosen 3 digit in number has exactly 3 factors is

a) 2/225 b) 7/900 c) 7/300 d) 3/500

Key. B

Sol. A number has exactly 3 factors if the number is squares of a prime number. Squares of 11,13,17,19,23,29,31 are 3- digit numbers. Hence, the required probability is 7/900

108. If a is a positive integer and $a \in$ [1,10], then the probability that the graph of the function $f(x) = x^2 - 2(4a-1)x + 15a^2 - 2a - 7$ is strictly above the x – axis is

a) $\frac{3}{10}$ b) $\frac{1}{10}$ c) $\frac{1}{5}$ d) $\frac{1}{4}$

Key. B

Sol. $f(x) = x^2 - 2(4a-1)x + 15a^2 - 2a - 7 > 0 \; \forall x \in R$ if (i) coefficient of $x^2 > 0$ (ii) $\Delta < 0$

$$\therefore D = 4(4a-1)^2 - 4(15a^2 - 2a - 7) < 0 \Rightarrow a^2 - 6a + 8 < 0$$

$$\Rightarrow a \in (2,4)$$

$$\Rightarrow a = 3 \text{ only}$$

$$Hence, probability = \frac{1}{10}$$

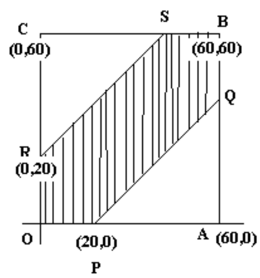
109. Two persons A and B agree to meet at a place between 10a.m to 11 a.m. The first one to arrive waits for 20 minutes and then leave. If the time of their arrival be independent and at random, what is the probability that A and B meet?

a) $\frac{1}{3}$ b) $\frac{4}{9}$ c) $\frac{5}{9}$ d) $\frac{2}{3}$

Key. C

Sol. Let A and B arrive at the place of their meeting x minutes and y minutes after 10 a.m.

Then they will meet if $|x-y| \le 20$



Then the area representing the favourable cases

= Area OPQBRS = 2000sq.units.

Total area = 3600sq. units

$\therefore$ Required probability = $\frac{5}{9}$

110. A man takes a step forward with probability 0.4 and one step backward with probability 0.6. Then the probability that at the end of eleven steps he is one step away from the starting point is

(a) ${}^{11}C_5 \times (0.48)^5$ (b) ${}^{11}C_6 \times (0.24)^5$

(c) ${}^{11}C_5 \times (0.12)^5$ (d) ${}^{11}C_6 \times (0.72)^6$

Key. B

Sol. It is possible if he moves (i) 6 steps forward 5 steps backward or (ii) 6 steps backward 5 steps forward

Required probability $= {}^{11}C_6\left[(0.4)^6(0.6)^5 + (0.4)^5(0.6)^6\right] = {}^{11}C_6 \times (0.24)^5$

111. A drawer contains 6 black socks and r red socks $(r \geq 2)$. For the probability of drawing 2 red socks at random from the drawer is to be at least $\frac{1}{2}$, minimum number of socks in the drawer must be

a) 15 b) 16 c) 21 d) 22

Key. C

Sol. Given $\frac{{}^{r}C_2}{{}^{(r+6)}C_2} \geq \frac{1}{2} \Rightarrow (r-15)(r+2) \geq 0 \Rightarrow r \geq 15$

$\therefore$ minimum number of socks = 21

112. A fair coin is tossed 6 times. The probability of getting at least 4 consecutive heads is

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{1}{8}$ d) $\frac{1}{16}$

Key. C

Sol. P(at least 4 consecutive heads) = P (4 consecutive heads)

+ P(5 consecutive

$$= \left(2\left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^6\right) = \left(2\left(\frac{1}{2}\right)^6\right) + \left(\frac{1}{2}\right)^6 = 4\left(\frac{1}{2}\right)^6 + 2\left(\frac{1}{2}\right)^5 = \frac{1}{8}$$

113. A letter is known to have come from CHENNAI, JAIPUR, NAINITAL, MUMBAI. On the postmark only the two consecutive letters AI are legible. The probability that it came from MUMBAI is

(a) $\frac{39}{190}$ (b) $\frac{42}{149}$ (c) $\frac{39}{191}$ (d) $\frac{38}{149}$

Key. B

Sol. A_1 : Selecting a pair of consecutive letters from the word CHENNAI

A_2 : Selecting a pair of consecutive letters from the word JAIPUR

A_3 : Selecting a pair of consecutive letters from the word NAINITAL

A_4 : Selecting a pair of consecutive letters from the word MUMBAI

E: Selecting a pair of consecutive letters AI

Required probability $= P\left(\frac{A_1}{E}\right) = \frac{\frac{1}{5}}{\frac{1}{6}+\frac{1}{5}+\frac{1}{7}+\frac{1}{5}} = \frac{42}{149}$

114. X follows a binomial distribution with parameters n and p and Y follows a binomial distribution with parameters m and p. If X and Y are independent then $P\left(\frac{X=r}{X+Y=r+s}\right) =$ ------

(a) $\frac{{}^nC_r.{}^mC_s}{{}^{(m+n)}C_{(r+s)}}$ (b) $\frac{3\,{}^mC_{r+s}}{{}^{(m+n)}C_{(r+s)}}$

(c) $\frac{2\left({}^mC_r\right)\left({}^nC_s\right)}{{}^{(m+n)}C_{(r+s)}}$ (d) $\frac{\left({}^mC_r\right)\left({}^nC_r\right)}{{}^{(m+n)}C_{(r+s)}}$

Key. A

Sol. $P\left(\frac{X=r}{X+Y=r+s}\right) = \frac{P\left[(X=r)\cap(X+Y=r+s)\right]}{P(X+Y=r+s)} = \frac{P(X=r)P(Y=s)}{P(X+Y=r+s)}$

$\therefore P\left(\frac{X=r}{X+Y=r+s}\right) = \frac{\left({}^nC_r.q^{n-r}p^r\right)\left({}^mC_s.q^{m-s}p^s\right)}{{}^{(m+n)}C_{(r+s)}p^{r+s}.q^{m+n-r-s}} = \frac{{}^nC_r.{}^mC_s}{{}^{(m+n)}C_{(r+s)}}$

115. If the cube of a natural number ends with a prime digit then the probability of its fourth power ending not with a prime digit, is

(A) 3/10 (B) 9/10

(C) 4/9 (D) 3/4

Key. D

Sol. Total cases are with numbers ending with 3, 5, 7 or 8.

Favourable cases are with numbers ending with 3, 7 or 8.

So, the required probability = 3/4

116. Consider the following three words (written in capital letters): 'PRANAM', 'SALAAM' and 'HELLO'. One of the three words is chosen at random and a letter from it is drawn. The letter is found to be 'A' or 'L' then the probability that it has come from the word 'PRANAM', is

(A) 0 (B) 1/3

(C) 2/5 (D) 5/21

Key. D

Sol. Let Q $\rightarrow$ event that 'PRANAM' is selected. S $\rightarrow$ event that 'SALAAM' is selected

H $\rightarrow$ event that 'HELLO' is selected. E $\rightarrow$ event that the letter chosen is A or L.

P (Q/E) = $\dfrac{P(Q)P(E/Q)}{P(Q)P(E/Q)+P(S)P(E/S)+P(H)P(E/H)}$ = $\dfrac{\frac{1}{3}\times\frac{2}{6}}{\frac{1}{3}\times\frac{2}{6}+\frac{1}{3}\times\frac{4}{6}+\frac{1}{3}\times\frac{2}{5}}=\dfrac{5}{21}$

117. A fair coin is tossed 10 times and the outcomes are listed. Let H_i be the event that the i^{th} outcome is a head and A_m be the event that the list contains exactly m heads, then

(A) H_3 and A_4 are independent (B) A_1 and A_9 are independent

(C) H_2 and A_5 are independent (D) H_4 and H_8 are not independent

Key. C

Sol. $P(H_i) = \dfrac{1}{2}$, $P(A_m) = \dfrac{{}^{10}C_m}{2^{10}}$

$P(H_i \cap A_m) = \dfrac{{}^{9}C_{m-1}}{2^{10}}$

For H_i & A_m to be independent,

$\dfrac{{}^{9}C_{m-1}}{2^{10}} = \dfrac{1}{2}\times\dfrac{{}^{10}C_m}{2^{10}} \Rightarrow 1 = \dfrac{1}{2}\times\dfrac{10}{m} \Rightarrow m = 5.$

118. 64 players play in a knockout tournament. Assuming that all the players are of equal strength, the probability that P_1 loses to P_2 and P_2 becomes the eventual winner is

a) $\dfrac{1}{612}$ b) $\dfrac{1}{672}$ c) $\dfrac{1}{512}$ d) $\dfrac{1}{63.2^6}$

Key. B

Sol. $\dfrac{{}^{62}c_5}{{}^{63}c_6}\cdot\dfrac{1}{64}=\dfrac{1}{672}$

119. Team A plays with 5 other teams exactly once. Assuming that for each match the probabilities of a win, draw and loss are equal, then

a) The probability that A wins and loses equal number of matches is $\dfrac{34}{81}$

b) The probability that A wins and loses equal number of matches is $\dfrac{17}{81}$

c) The probability that A wins more number of matches than it loses is $\dfrac{17}{81}$

d) The probability that A loses more number of matches than it wins is $\dfrac{16}{81}$

Key. B

Sol. Prob.of equal no. of W and L = 0 wins, 0 losses + 1W, 1L + 2W,

2L = $\left(\dfrac{1}{3}\right)^5 + {}^5c_1.{}^4c_1.\left(\dfrac{1}{3}\right)^5 + {}^5c_2.{}^3c_2.\left(\dfrac{1}{3}\right)^5 = \dfrac{17}{81}$

120. The probability that the fourth powers of a number ends in 1 is

a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{10}$

Key. B

Sol. The fourth power of a number ends with 1 if the last digit is 1, 3, 7, 9

$\therefore$ required probability = 4/10 = 2/5

121. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{5}$ d) $\frac{1}{5}$

Key. C

Sol. $\frac{{}^4c_1}{{}^4c_2 + {}^4c_1} = \frac{4}{10} = \frac{2}{5}$

122. Suppose f(x) =x^3 +ax^2 + bx + c where a, b, c are chosen respectively by throwing a die three times, then the probability that f(x) is an increasing function is

a) $\frac{4}{9}$ b) $\frac{3}{8}$ c) $\frac{2}{5}$ d) $\frac{16}{34}$

Key. A

Sol. $f'(x) = 3x^2 + 2ax + b$

$f'(x) \geq 0, \forall x$ discriminiant, $(2a)^2$– 4 x3 x $(2a)^2 - 4\times3\times b \leq 0 \Rightarrow a^2 - 3b \leq 0$

This is true for exactly 16 ordered pairs (a, b) namely (1, 1), (1, 2), (1, 3), (1,4), (1, 5) (1, 6), (2, 2) (2, 3), (2, 4, (2, 5), (2, 6), (3, 3) (3, 4), (3,5) , (3, 6) and (4, 6)

Thus, required probability = $\frac{16}{36} = \frac{4}{9}$

123. A bag initially contains one red ball and two blue balls. An experiment consisting of selecting a ball at random, noting its colour and replacing it together with an additional ball of the same colour. If three such trials are made, then

a) Probability that atleast one blue ball is drawn is 0.9

b) Probability that exactly one blue ball is drawn is 0.2

c) Probability that all the drawn balls are red given that all the drawn balls are of the same colour is 0.2

d) Probability that atleast one red ball is drawn is 0.6

Key. D

Sol. Prob. That atleast one blue ball is drawn

= 1- prob that all the balls drawn are red.

= $1 - \frac{1}{3}.\frac{1}{2}.\frac{3}{5} = 1 - \frac{1}{10} = 0.9$

Prob. That exactly one blue ball is drawn

= $\frac{1}{3}.\frac{1}{2}.\frac{2}{5} + \frac{2}{3}.\frac{1}{4}.\frac{2}{5} = 0.2$

Prob. that all drawn balls are red given that all the drawn balls of the same colour

$$= \frac{\frac{1}{10}}{\frac{1}{10}+\frac{4}{10}} = \frac{1}{5} = 0.2$$

Prob.that atleast one red ball is drawn $= 1 - \left(\frac{2}{3}.\frac{3}{4}.\frac{4}{5}\right)$ = 0.6

124. A has 3-shares in a lottery containing 3 prizes and 9-blanks, B has 2-Shares in a lottery containing 2 prizes and 6 – blanks then ratio of their success is

1. 952:715 2. 715:259 3. 265:233 4. 125 : 752

Key. 1

Sol. A has 3-shares $\Rightarrow P\left(A\, gets\, success\right) = 1 - \frac{9_{C_3}}{12_{C_3}} = \frac{34}{55}$

$$P\left(B\, gets\, success\right) = 1 - \frac{6_{C_2}}{8_{C_2}} = 1 - \frac{15}{28} = \frac{13}{28}$$

$$P(A):P(B) = \frac{34}{55}:\frac{13}{28} = 952:715$$

125. If a is an integer lying in [-5,30] then probability that graph of $y = x^2 + 2(a+4)x - 5a + 64$ is strictly above the x-axis

1. $\frac{1}{6}$ 2. $\frac{2}{9}$ 3. $\frac{3}{5}$ 4. $\frac{1}{5}$

Key. 2

Sol. n(s) =36

$y = x^2 + 2(a+4)x - 5x + 64$ lies above the X-axis is

If $4(a+4)^2 - 4(1)(-5a+64) < 0$

$$\Rightarrow -16 < a < 3$$

$$\Rightarrow a = -5, -4, -3, -2, -1, 0, 1, 2$$

n(E) =8

$$\therefore P(E) = \frac{8}{36} = \frac{2}{9}$$

126. There are 4-machines and it is known that exactly two of them are faulty they are tested one by one in a random order till both faulty machines are Identified. The probability that only two tests are needed

1. $\frac{1}{3}$ 2. $\frac{1}{6}$ 3. $\frac{1}{4}$ 4. $\frac{3}{4}$

Key. 2

Sol. $P(E)=\frac{2}{4}\times\frac{1}{3}=\frac{1}{6}$

127. Two numbers x and y are chosen such that $x\in[0,4], y\in[0,4]$ then probability that $y^2\le x$

1. $\frac{1}{3}$
2. $\frac{2}{3}$
3. $\frac{1}{4}$
4. $\frac{3}{4}$

Key. 1

Sol. n(S)=16

n(E)= $\int_0^4 \sqrt{x}dx=\frac{16}{3}$

$P(E)=\frac{\frac{16}{3}}{16}=\frac{1}{3}$

128. A fair coin is tossed 5 times then probability that two heads do not occur consecutively (No two heads come together)

1. $\frac{1}{16}$
2. $\frac{15}{32}$
3. $\frac{13}{32}$
4. $\frac{7}{16}$

Key. 3

Sol. $p\left(\frac{E}{no\,heads}\right)+p\left(\frac{E}{1(head)}\right)+p\left(\frac{E}{2-heads}\right)+p\left(\frac{E}{3-heads}\right)$

Where $E\rightarrow$gtg n two consecutive heads.

$=\frac{1}{32}+\frac{5}{32}+\frac{6}{32}+\frac{1}{32}=\frac{14}{32}=\frac{7}{16}$

129. A man throws a die until he gets a number bigger than 3. The probability that he gets 5 in the last throw

1. $\frac{1}{3}$
2. $\frac{1}{4}$
3. $\frac{1}{6}$
4. $\frac{1}{36}$

Key. 1

Sol. P(gtg a number bigger than 3) = $\frac{1}{2}$

P(gtg 5 in throw)= $\frac{1}{6}$

$E\rightarrow$gtg 5 in last throw when he gets a number bigger than 3

$$P(E)=\frac{1}{6}+\frac{1}{2}.\frac{1}{6}+\frac{1}{2}.\frac{1}{2}.\frac{1}{6}+....\infty$$

$$=\frac{1}{6}\times\frac{1}{1-\frac{1}{2}}=\frac{1}{3}$$

130. A bag contains 4-balls two balls are drawn from the bag and are found to be white then probability that all balls in the bag are white

1. $\frac{1}{5}$ 2. $\frac{2}{5}$ 3. $\frac{3}{5}$ 4. $\frac{4}{5}$

Key. 3

Sol. $$P(E)=\frac{\frac{1}{3}.\frac{4_{C_2}}{4_{C_2}}}{\frac{1}{3}\left\{\frac{2_{C_2}}{4_{C_2}}+\frac{3_{C_2}}{4_{C_2}}+\frac{4_{C_2}}{4_{C_2}}\right\}}$$

$$=\frac{1}{\frac{1+3+6}{6}}=\frac{6}{10}=\frac{3}{5}$$

131. A randomly selected year is containing 53 Mondays then probability that it is a leap year

1. $\frac{2}{5}$ 2. $\frac{3}{5}$ 3. $\frac{4}{5}$ 4. $\frac{1}{5}$

Key. 1

Sol. Selected year may non leap year with a probability $\frac{3}{4}$

Selected year may leap year with a probability $\frac{1}{4}$

$E\rightarrow$ Even that randomly selected year contains 53 Mondays

$$P(E)=\frac{3}{4}\times\frac{1}{7}+\frac{1}{4}\times\frac{2}{7}=\frac{5}{28}$$

$$P\left(\frac{leapyear}{E}\right)=\frac{\frac{2}{28}}{\frac{5}{28}}=\frac{2}{5}$$

132. When 5-boys and 5-girls sit around a table the probability that no two girls come together

1. $\frac{1}{120}$ 2. $\frac{1}{126}$ 3. $\frac{3}{47}$ 4. $\frac{4}{7}$

Key. 2

Sol. $E\rightarrow$ first boys can be arranged in $\lfloor 4$ ways, then there are 5-gaps between boys in 5-gaps, 5-girls can be arranged in $\lfloor 5$ ways

$$P(E)=\frac{\lfloor 5 \lfloor 4}{\lfloor 9}=\frac{5\times4\times3\times2}{5\times6\times7\times8\times9}=\frac{1}{126}$$

133. There are m-stations on a railway line. A train has to stop at 3 intermediate stations then probability that no two stopping stations are adjacent

1. $\frac{1}{m\,c_3}$ 2. $\frac{3}{mc_3}$ 3. $\frac{m-2_{C_3}}{mc_3}$ 4. $\frac{mc_2}{m\,c_3}$

Key. 3

Sol. Let 3-stopping stations be S_1, S_2, S_3 then are m-3 stations remaining. Between these m-3 stations there are m-2 places select any 3 for S_1, S_2, S_3, then there are no two stopping stations are adjacent

$$P(E)=\frac{m-2_{C_3}}{m_{C_3}}$$

134. A has 3-shares in a lottery containing 3 prizes and 9-blanks, B has 2-Shares in a lottery containing 2 prizes and 6 – blanks then ratio of their success is

1. 952:715 2. 715:259 3. 265:233 4. 125 : 752

Key. 1

Sol. A has 3-shares $\Rightarrow$ $P(A\,gets\,success)=1-\frac{9_{C_3}}{12_{C_3}}=\frac{34}{55}$

$$P(B\,gets\,success)=1-\frac{6_{C_2}}{8_{C_2}}=1-\frac{15}{28}=\frac{13}{28}$$

$$P(A):P(B)=\frac{34}{55}:\frac{13}{28}=952:715$$

135. If a is an integer lying in [-5,30] then probability that graph of $y=x^2+2(a+4)x-5a+64$ is strictly above the x-axis

1. $\frac{1}{6}$ 2. $\frac{2}{9}$ 3. $\frac{3}{5}$ 4. $\frac{1}{5}$

Key. 2

Sol. n(s) =36

$y=x^2+2(a+4)x-5x+64$ lies above the X-axis is

If $4(a+4)^2-4(1)(-5a+64)<0$

$\Rightarrow -16<a<3$

$\Rightarrow a=-5,-4,-3,-2,-1,0,1,2$

n(E) =8

$\therefore P(E)=\frac{8}{36}=\frac{2}{9}$

136. There are 4-machines and it is known that exactly two of them are faulty they are tested one by one in a random order till both faulty machines are Identified. The probability that only two tests are needed

1. $\frac{1}{3}$ 2. $\frac{1}{6}$ 3. $\frac{1}{4}$ 4. $\frac{3}{4}$

Key. 2

Sol. $P(E)=\frac{2}{4}\times\frac{1}{3}=\frac{1}{6}$

137. Two numbers x and y are chosen such that $x\in[0,4], y\in[0,4]$ then probability that $y^2\le x$

1. $\frac{1}{3}$ 2. $\frac{2}{3}$ 3. $\frac{1}{4}$ 4. $\frac{3}{4}$

Key. 1

Sol. n(S)=16

n(E)= $\int_0^4 \sqrt{x}dx=\frac{16}{3}$

$$P(E)=\frac{\frac{16}{3}}{16}=\frac{1}{3}$$

138. The probability that randomly selected positive integer is relatively prime to 6

1. $\frac{1}{2}$ 2. $\frac{1}{3}$ 3. $\frac{1}{6}$ 4. $\frac{5}{6}$

Key. 2

Sol. Among every 6-consecutive integers one divisible by 6 and other integers leaves remainders 1,2,3,4,5 when divided by 6

The numbers which leave the remainder 1 and 5 are relatively prime to 6

Required probability $\frac{2}{6}=\frac{1}{3}$

139. A and B are events such that P(A)=0.3 $P(A\cup B)=0.8$. If A and B are independent then $P(B)$ =

1. $\frac{1}{7}$ 2. $\frac{3}{7}$ 3. $\frac{5}{7}$ 4. $\frac{6}{7}$

Key. 3

Sol. $P(A\cap B)=P(A).P(B)$

$$P(A\cup B)=P(A)+P(B)-P(A)P(B)$$

$0.8 = 0.3 + P(B)(1-0.3)$

$0.5 = P(B)(0.7) \Rightarrow P(B) = \frac{5}{7}$

140. Thirty-two players ranked 1 to 32 are playing in a knockout tournament. Assume that in every match between any two players, the better-ranked player wins, the probability that ranked 1 and ranked 2 players are winner and runner up respectively, is

(A) 16/31 (B) 1/2 (C) 17/31 (D) None of these

Key. A

Sol. For ranked 1 and 2 players to be winners and runners up res., they should not be paired with each other in any rounded. Therefore, the required probability $\frac{30}{31} \times \frac{14}{15} \times \frac{6}{7} \times \frac{2}{3} = \frac{16}{31}$

141. A coin is tossed 7 times. Then the probability that at least 4 consecutive heads appear is

(A) 3/16 (B) 5/32 (C) 5/16 (D) 1/8

Key. B

Sol. Let H denote the head,
T the tail.
* Any of the head or tail

P(H) = $\frac{1}{2}$, P(T) = $\frac{1}{2}$ P(*) = 1

HHHH*** = $\left(\frac{1}{2}\right)^4 \times 1 = \frac{1}{16}$

THHHH** = $\left(\frac{1}{2}\right)^5 \times 1 = \frac{1}{32}$

THHHH = $\left(\frac{1}{2}\right)^5 \times 1 = \frac{1}{32}$

**THHHH = $\left(\frac{1}{2}\right)^5 \times 1 = \frac{1}{32}$

$\frac{5}{32}$

142. Two natural numbers *a* and *b* are selected at random. The probability that $a^2 + b^2$ is divisible by 7 is

(a) 3/8 (b) 1/7 (c) 3/49 (d) 1/49

Key. D

Sol. a_1b are is of then form

$a_1b \in \{7m, 7m+1, 7m+2, 7m+3, 7m+4, 7m+6\}$

$a_1^2b^2 \in \{7m_1, 7m_1+1, 7m_1+4, 7m_1+2, 7m_1+2, 7m_1+4, 7m_1+1\}$

$\therefore a^2, b^2$ must be of the form 7m.

Probability = $\frac{1}{49}$

143. If a and b are chosen randomly from the set consisting of numbers 1, 2, 3, 4, 5, 6 with replacement. Then probability that $\underset{x\to 0}{Lt}\left(\frac{a^x+b^x}{2}\right)^{2/x}=6$ is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{9}$ (d) $\frac{2}{9}$

Key. C

Sol. $\underset{x\to 0}{Lt}\left(\frac{a^x+b^x}{2}\right)^{2/x}=6$

$$=e^{\underset{x\to 0}{Lt}\left(\frac{a^x-1}{x}\right)+\left(\frac{b^x-1}{x}\right)}=6$$

$$=e^{\log a+\log b}=6$$

ab = 6

$(a,b)=(1,6),(6,1),(2,3),(3,2)$

Required probability = $\frac{4}{6\times 6}=\frac{1}{9}$

144. An urn contains five balls. Two balls are drawn and found to be white. Probability that all balls are white, is

(A) $\frac{1}{3}$ (B) $\frac{2}{9}$

(C) $\frac{1}{2}$ (D) $\frac{3}{4}$

Key. C

Sol. Event A_1 = urn contains 5 white balls
Event A_2 = urn contains 4 white balls
Event A_3 = urn contains 3 white balls
Event A_4 = urn contains 2 white balls

Event A = Drawing two white balls when two balls are drawn from five balls

$$P\left(\frac{A_1}{A}\right)=\frac{P(A_1)P\left(\frac{A}{A_1}\right)}{P(A_1)P\left(\frac{A}{A_1}\right)+P(A_2)P\left(\frac{A}{A_2}\right)+P(A_3)P\left(\frac{A}{A_3}\right)+P(A_4)P\left(\frac{A}{A_4}\right)}$$

$$P(A_1)=P(A_2)=P(A_3)=P(A_4)=\frac{1}{4}$$

$$P\left(\frac{A}{A_1}\right)=1,\ P\left(\frac{A}{A_2}\right)=\frac{{}^4C_2}{{}^5C_2},\ P\left(\frac{A}{A_3}\right)=\frac{{}^3C_2}{{}^5C_2}$$

$$P\left(\frac{A}{A_4}\right)=\frac{{}^2C_2}{{}^5C_2}$$

145. Three numbers a, b, c are choosen randomly from the set of natural numbers. The probability that '$a^2+b^2+c^2$' is divisible by 7 is

(A) 1/3 (B) 1/4

(C) 1/5 (D) 1/7

Key. D

Sol. Numbers are of the form: 7k, 7k + 1, 7k + 2, 7k + 3, 7k + 4, 7k + 5, 7k + 6 their squares: 7k, 7k + 1, 7k + 4, 7k + 2, 7k + 2, 7k + 1, 7k + 1.

So, for $a^2 + b^2 + c^2$ to be a multiple of 7, either all the three squares should be of the form 7k or they belong to the catagories 7k + 1, 7k + 2, 7k + 4 separately.

So, required prob., = $\left(\frac{1}{7}\right)^3 + 3!\left(\frac{2}{7}\right)\left(\frac{2}{7}\right)\left(\frac{2}{7}\right) = \frac{1}{7}$

146. If two events A and B are such that $P(\bar{A}) = 0.3$, $P(B) = 0.4$, $P(A \cap \bar{B}) = 0.5$, then the value of $P(B/(A \cup \bar{B}))$ is

(A) 1/2 (B) 1/4

(C) 3/4 (D) 4/5

Key. B

Sol. $P(B)\ (A \cup \bar{B})) = \frac{P(B \cap (A \cup \bar{B})}{P(A \cup \bar{B})} = \frac{P(A) - P(A \cap \bar{B})}{P(A) + P(\bar{B}) - P(A \cap \bar{B})}$

$= \frac{0.7 - 0.5}{0.7 + 0.6 - 0.5} = \frac{0.2}{0.8} = \frac{1}{4}$

147. If the third power of a natural no. ends with a prime digit then the probability of its fourth power ending not with a prime digit, is

(A) 3/10 (B) 9/10

(C) 4/9 (D) 3/4

Key. D

Sol. Total cases are when numbers ending with 3, 5, 7, or 8

Favourable cases are when numbers are ending with 3, 7, or 8

So, the required probability = 3/4

148. A fair coin is tossed 10 times and the outcomes are listed. Let H_i be the event that the ith out–come is a head and A_m be the event that the list contains exactly m heads, then

(A) H_3 and A_4 are independent (B) A_1 and A_9 are independent

(C) H_2 and A_5 are independent (D H_4 and H_8 are not independent

Key. C

Sol. $P(H_i) = \frac{1}{2}$, $P(A_m) = \frac{{}^{10}C_m}{2^{10}}$

$P(H_i \cap A_m) = \frac{{}^{9}C_{m-1}}{2^{10}}$

For H_i and A_m to be independent

$\frac{{}^{9}C_{m-1}}{2^{10}} = \frac{1}{2} \times \frac{{}^{10}C_m}{2^{10}} \Rightarrow 1 = \frac{1}{2} \times \frac{10}{m} \Rightarrow m = 5$

149. A fair coin is tossed 9 times. Heads are coming 7 times. The probability that among these heads atleast 6 are occurring consecutively, is

(A) 1/8 (B) 1/5

(C) 1/4 (D) 1/3

Key. C

Sol. Out of 7 heads exactly six consecutive heads occure in 6 ways and all seven heads consecutively can occur in 3 ways so the required probability = $\frac{6+3}{{}^{9}C_7} = \frac{9}{36} = \frac{1}{4}$.

150. One ticket is selected at random from 100 tickets numbered 00, 01, 02, ……. , 99. Suppose *X* and *Y* are the sum and product of the digits found on the ticket, then $P(X=7/Y=0)$ is given by

A) 2/3 B) 2/19 C) 1/50 D) None of these

Key. B

Sol. We have $(X=7)=\{07,16,25,34,43,52,61,70\}$

And $(Y=0)=\{00,01,02,...,09,10,20,30,.....,90\}$

Thus, $(X=7)\cap(Y=0)=\{07,70\}$

$$\therefore P(X=7/Y=0)=\frac{P\{(X=7)\cap(Y=0)\}}{P(Y=0)}=\frac{2}{19}.$$

151. If the mean and variance of a binomial variate *X* are 7/3 and 14/9 respectively, then the probability that *X* takes value 6 or 7 is equal to

A) $\frac{1}{729}$ B) $\frac{5}{729}$ C) $\frac{7}{729}$ D) $\frac{13}{729}$

Key. B

Sol. We have $np=\frac{7}{3}, npq=\frac{14}{9}$, therefore

$$\frac{7}{3}q=\frac{14}{9}\Rightarrow q=\frac{2}{3}\Rightarrow p=\frac{1}{3}.$$

Thus, $n(1/3)=7/3\Rightarrow n=7$

Now, $P(X=6 \, or \, 7)=P(X=6)+P(X=7)$

$$={}^7C_6p^6q^1+{}^7C_7p^7q^0$$

$$=7\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)+\left(\frac{1}{3}\right)^7=\frac{5}{729}.$$

152. If *A* and *B* are events of the same random experiment with $P(A)=0.2, P(B)=0.5$, then maximum value of $P(\overline{A}\cap B)$ is

A) 1/4 B) 1/2 C) 1/8 D) 1/16

Key. B

Sol. $P(\overline{A}\cap B)\le P(B)$ if $P(A)+P(B)\le 1$

$\therefore$ maximum value = 0.5

153. A pair of fair dice is rolled together till a sum of either 5 or 7 is obtained. The probability that 5 comes before 7 is

A) 0.2 B) 0.3 C) 0.4 D) 0.5

Key. C

Sol. Let A denote the event that a sum of 5 occurs, B the event that a sum of 7 occurs and C the event that neither a sum of 5 nor a sum of 7 occurs we have

$$P(A)=\frac{4}{36}=\frac{1}{9}, P(B)=\frac{6}{36}=\frac{1}{6} \text{ and } P(C)=\frac{26}{36}=\frac{13}{18}.$$

Thus, P (A occurs before B)

$$=P(A)+P(C)P(A)+P(C)P(C)P(A)+.......$$

$$= \frac{P(A)}{1-P(C)} = \frac{\frac{1}{9}}{1-\frac{13}{18}} = \frac{2}{5}.$$

154. Sixteen players $P_1, P_2, ..., P_{16}$ play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair. Assuming that all the players are of equal strength, the probability that exactly one of the two players P_1 and P_2 is among the eight winners is

A) 4/15 B) 7/15 C) 8/15 D) 17/30

Key. C

Sol. Let $E_1(E_2)$ denote the event that P_1 and P_2 are paired (not paired) together and let A denote the event that one of two players P_1 and P_2 is among the winners.

Since, P_1 can be paired with any of the remaining 15 players.

We have, $P(E_1) = \frac{1}{15}$

And $P(E_2) = 1 - P(E_1) = 1 - \frac{1}{15} = \frac{14}{15}$

In case E_1 occurs, it is certain that one of P_1 and P_2 will be among the winners. In case E_2 occurs, the probability that exactly one of P_1 and P_2 is among the winners is

$$P\left\{\left(P_1 \cap \overline{P_2}\right) \cup \left(\overline{P_1} \cap P_2\right)\right\} = P\left(P_1 \cap \overline{P_2}\right) + P\left(\overline{P_1} \cap P_2\right)$$

$$= \left(\frac{1}{2}\right)\left(1-\frac{1}{2}\right) + \left(1-\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

i.e, $P(A/E_1) = 1$ and $P(A/E_2) = \frac{1}{2}$

By the total probability Rule,

$$P(A) = P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right)$$

$$= \frac{1}{15}(1) + \frac{14}{15}\left(\frac{1}{2}\right) = \frac{8}{15}.$$

155. If four cards are drawn at random from a pack of 52 playing cards, then the probability of getting at least two kings is

(a) $\frac{6961}{52_{C_4}}$ (b) $\frac{11}{52_{C_4}}$

(c) $\frac{6768}{52_{C_4}}$ (d) $\frac{24}{52_{C_4}}$

Key. A

Sol. Probability at least two king $= \frac{4_{C_2}48_{C_2} + 4_{C_3}48_{C_1} + 4_{C_4}}{52_{C_4}} = \frac{6961}{52_{C_4}}$

156. Two 8 faced dice (numbered from 1 to 8) are tossed. The probability that the product of two counts is a square number, is

(A) 1/8 (B) 7/32

(C) 3/16 (D) 3/8

Key. C

Sol. n(s) = 8 × 8 = 64

Square values that product can take are 1, 4, 9, 16, 25, 36, 49, 64

4 : (1, 4), (2, 2), (4, 1)

16 : (2, 8) (4, 4), (8, 2)

For other values, there is only one way of getting the product

$\therefore$ n(E) = 2 × 3 + 6 × 1 = 12

$\therefore$ P(E) = $\frac{12}{64} = \frac{3}{16}$.

157. A die is thrown a fixed number of times. If probability of getting even number 3 times is same as the probability of getting even number four times, then probability of getting even number exactly once is

(A) 1/4 (B) 3/128

(C) 5/64 (D) 7/128

Key. D

Sol. $^nC_3 \left(\frac{1}{2}\right)^n = {^nC_4} \left(\frac{1}{2}\right)^n$

Where n = number of times die is thrown

$\Rightarrow {^nC_3} = {^nC_4} \Rightarrow n = 7$

$\therefore$ Required prob. = $^7C_1 \left(\frac{1}{2}\right)^7 = \frac{7}{128}$

158. The sum of two natural numbers is 30 .The probability that their product is less than 150 is

a) $\frac{1}{5}$ b) $\frac{3}{29}$ c) $\frac{1}{6}$ d) $\frac{6}{29}$

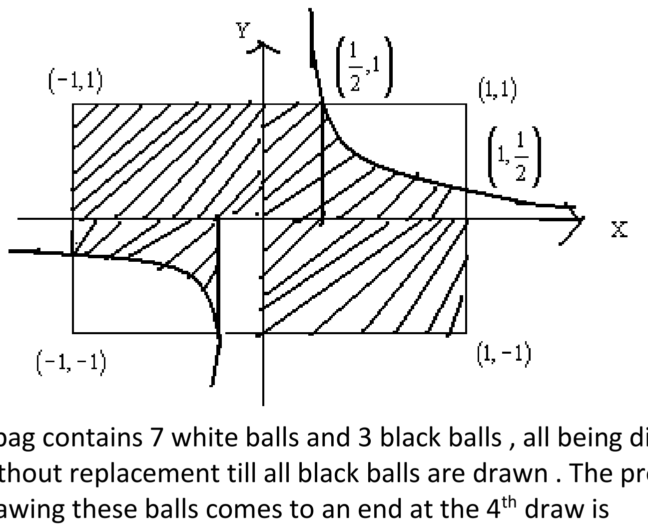
Key. D

Sol. Let the numbers be *x,y* .Given *x+y*=30, $x, y \in N \Rightarrow x = 1, 2, 3, ...29$

$$P(xy < 150) = P\left((30-x)x < 150\right)$$

$$= P(x > 15 + \sqrt{75}) = P(x = 24, 25, 26, 27, 28, 29)$$

$$= \frac{5}{29}$$



159. A bag contains 7 white balls and 3 black balls , all being distinct . Balls are drawn one by one without replacement till all black balls are drawn . The probability that the procedure of drawing these balls comes to an end at the 4th draw is

a) $\frac{1}{40}$ b) $\frac{1}{20}$ c) $\frac{1}{10}$ d) $\frac{1}{80}$

Key. A

Sol. The procedure comes to an end at 4 th draw if in the first 3 draws , 2 black balls drawn and in the 4 th drawn remaining black ball is drawn

$\therefore$ Required probability = $\frac{(3_{C_2})(7_{C_1})}{10_{C_3}}.\frac{1}{7}=\frac{1}{40}$

160. If a and b are selected at random from the range of y (a, b are distinct positive integers). Then the probability of selecting distinct ordered pairs (a, b) of prime numbers from the range of y, where $y=\frac{147}{x+\frac{1}{x}+5}\quad \forall\ x>0$

a) $\frac{3}{32}$ b) $\frac{2}{15}$ c) $\frac{5}{32}$ d) $\frac{2}{21}$

Key. B

Sol. $\left(x+\frac{1}{x}+5\right)\geq 2+5=7\quad y\leq\frac{147}{7}=21\Rightarrow 0<y\leq 21$ Required prob

$=\frac{^{8}C_2.2!}{^{21}C_2.2!}=\frac{2}{15}$

161. 64 players play in a knockout tournament. Assuming that all the players are of equal strength, the probability that P_1 loses to P_2 and P_2 becomes the eventual winner is

a) $\frac{1}{612}$ b) $\frac{1}{672}$ c) $\frac{1}{512}$ d) $\frac{1}{63.2^6}$

Key. B

Sol. $\frac{^{62}c_5}{^{63}c_6}.\frac{1}{64}=\frac{1}{672}$

162. Team A plays with 5 other teams exactly once. Assuming that for each match the probabilities of a win, draw and loss are equal, then

a) The probability that A wins and loses equal number of matches is $\frac{34}{81}$

b) The probability that A wins and loses equal number of matches is $\frac{17}{81}$

c) The probability that A wins more number of matches than it loses is $\frac{17}{81}$

d) The probability that A loses more number of matches than it wins is $\frac{16}{81}$

Key. B

Sol. Prob.of equal no. of W and L = 0 wins, 0 losses + 1W, 1L + 2W,

2L = $\left(\frac{1}{3}\right)^5+{}^5c_1.{}^4c_1.\left(\frac{1}{3}\right)^5+{}^5c_2.{}^3c_2.\left(\frac{1}{3}\right)^5=\frac{17}{81}$

163. The probability that the fourth powers of a number ends in 1 is

a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{10}$

Key. B

Sol. The fourth power of a number ends with 1 if the last digit is 1, 3, 7, 9

$\therefore$ required probability = 4/10 = 2/5

164. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{5}$ d) $\frac{1}{5}$

Key. C

Sol. $\frac{{}^4c_1}{{}^4c_2 + {}^4c_1} = \frac{4}{10} = \frac{2}{5}$

165. A bag contains 4 red and 3 blue balls. Two drawings of two balls are made. The probability that the first drawing gives 2 red balls and the second drawing gives two blue balls if the balls are not returned to the bag after the first draw is

A) 2/49 B) 3/35 C) 3/10 D) ¼

Key. B

Sol. $\frac{4c_2}{7c_2} \times \frac{3c_2}{5c_2} = \frac{4\times3}{7\times6} \times \frac{3}{10} = \frac{3}{35}$

166. A, B are two independent events such that $P(A) > \frac{1}{2}$ $P(B) > \frac{1}{2}$. If $P(A \cap \bar{B}) = \frac{3}{25}$ and $P(\bar{A} \cap B) = \frac{8}{25}$ then $P(A \cap B) =$

A) 3/4 B) 2/3 C) 12/25 D) 18/25

Key. C

Sol. Let $P(A) = x$ and $P(B) = y$. $x > 1/2, y > 1/2$

$P(A-B) = x - xy = 3/25$ and $P(B-A) = y - xy = 8/25$

167. Three groups of children contain respectively 3 girls and 1 boy; 2 girls and 2 boys; 1 girl and 3 boys. One child is selected at random from each group. The probability that the three selected consists of 1 girl and 2 boys is

A) 5/9 B) 13/32 C) 12/19 D) 25/64

Key. B

Sol. $\frac{3}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{2}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{2}{4}\times\frac{1}{4}=\frac{26}{64}$

168. A certain type of missile hits its target with probability 0.3. The number of missiles that should be fired so that there is atleast an 80% probability of hitting a target is

A) 3 B) 4 C) 5 D) 6

Key. C

Sol. Let n be the required number.

$\therefore$ The probability that 'n' missiles miss the target is $(0.7)^n$. We require $1-(0.7)^n > 0.8$

i.e., $(0.7)^n < 0.2$. The least value of 'n' satisfying this inequality is 5.

169. A team has probability 2/3 of winning a game whenever it plays. If the team plays 4 games then the probability that it wins more than half of the games is

A) 17/25 B) 15/19 C) 16/27 D) 13/20

Key. C

Sol. Let p be the probability that the team wins a game. Let $q = 1 - p$. Then the random variable "number of wins" follows the binomial distribution $P(X = K) = {}^4C_k q^{4-k} p^k, k = 0, 1, 2, 3, 4$.

Required probability = ${}^4C_3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 + {}^4C_4\left(\frac{2}{3}\right)^4 = \frac{16}{27}$.

170. From a bag containing 10 distinct balls, 6 balls are drawn simultaneously and replaced. Then 4 balls are drawn. The probability that exactly 3 balls are common to the drawings is

A) 8/21　B) 6/19　C) 5/24　D) 9/22

Key. A

Sol. Let S be the sample space of the composite experiment of drawing 6 in the first draw and then four in second draw then $|S| = {}^{10}C_6 \times {}^{10}C_4$

$\therefore$ Required Probability = $\frac{{}^{10}C_6 \times {}^6C_3 \times {}^4C_1}{{}^{10}C_6 \times {}^6C_4} = \frac{80 \times 24}{10 \times 9 \times 8 \times 7} = \frac{8}{21}$

171. Two persons each make a single throw with a pair of dice. The probability that their scores are equal is

A) 65/648　B) 69/648　C) 73/648　D) 91/648

Key. C

Sol. Required Probability = $= \frac{1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2}{36^2}$

172. Two numbers x and y are chosen without replacement from the set of {0, 1, 2, ….., 10}. The probability that $|x - y| \le 5$ is

(A) $\frac{2}{11}$　(B) $\frac{3}{11}$

(C) $\frac{15}{22}$　(D) $\frac{8}{11}$

Key. D

Sol. $|x - y| \ge 6$ ordered pair (x, y) are

(0, 6), (0, 7) ….. (0, 10
(1, 7) (1, 8) ….. (1, 10
(2, 8) (2, 9) (2, 10)
(3, 9) (3, 10)
(4, 10) = 15 ways

required probability $1 - \frac{15}{{}^{11}C_2} = 1 - \frac{3}{11} = \frac{8}{11}$

173. Four identical dice are rolled once. The probability that atleast three different numbers appears on them is

A) $\frac{15}{22}$　B) $\frac{25}{42}$　C) $\frac{32}{45}$　D) $\frac{17}{52}$

Key. B

Sol. All identical digits - ${}^6C_1 = 6$

Only two different digits - $3\times {}^6C_2 = 45$

Three distinct digits - $3\times {}^6C_3 = 60$

Four different digits - ${}^6C_4 = 15$

Total possible outcomes = 126

Favourable outcomes = 75.

174. A is a 3×3 matrix with entries from the set $\{-1,0,1\}$. The probability that A is neither symmetric nor skew symmetric is

A) $\frac{3^9-3^6-3^3+1}{3^9}$ B) $\frac{3^9-3^6-3^3}{3^9}$ C) $\frac{3^9-1}{3^{10}}$ D) $\frac{3^9-3^3+1}{3^9}$

Key. A

Sol. Total number of matrices that can be formed is 3^9.

Let $A=\left[a_{ij}\right]_{3\times 3}$ where $a_{ij}\in\{-1,0,1\}$

If A is symmetric then $a_{ij}=a_{ji}\forall i, j$

If A is skew-symmetric then $a_{ij}=-a_{ji}\forall i, j$

175. If the cube of a natural number ends with a prime digit then the probability of its fourth power ending not with a prime digit, is

(A) 3/10 (B) 9/10

(C) 4/9 (D) 3/4

Key. D

Sol. Total cases are with numbers ending with 3, 5, 7 or 8.

Favourable cases are with numbers ending with 3, 7 or 8.

So, the required probability = 3/4

176. Consider the following three words (written in capital letters): 'PRANAM', 'SALAAM' and 'HELLO'. One of the three words is chosen at random and a letter from it is drawn. The letter is found to be 'A' or 'L' then the probability that it has come from the word 'PRANAM', is

(A) 0 (B) 1/3

(C) 2/5 (D) 5/21

Key. D

Sol. Let Q $\rightarrow$ event that 'PRANAM' is selected. S $\rightarrow$ event that 'SALAAM' is selected

H $\rightarrow$ event that 'HELLO' is selected. E $\rightarrow$ event that the letter chosen is A or L.

$$P(Q/E) = \frac{P(Q)P(E/Q)}{P(Q)P(E/Q)+P(S)P(E/S)+P(H)P(E/H)} = \frac{\frac{1}{3}\times\frac{2}{6}}{\frac{1}{3}\times\frac{2}{6}+\frac{1}{3}\times\frac{4}{6}+\frac{1}{3}\times\frac{2}{5}} = \frac{5}{21}$$

Probability

Integer Answer Type

1. In a multiple-choice question, there are five alternative answers, of which one or more than one are correct. A candidate will get marks on the question if he ticks all the correct answers. So he decides to tick answers at random, if the least number of chances ,he should be allowed so that the probability of his getting marks on the question exceeds 1/8 is K, then K =

(the student always attempt the question)

Key. 4

Sol. The probability that he get marks $=\frac{1}{31}$

The probability that he get marks in second trial is $\frac{30}{31}\times\frac{1}{30}=\frac{1}{31}$

The probability that he get marks in third trial is $\frac{1}{31}$

Continuing this process the probability from *r* trial is $\frac{r}{31}>\frac{1}{8}$

$\Rightarrow r>\frac{31}{8}$

$r=4$

2. If n(X) = (K + 1), then the probability of selecting 2 subsets A and B of the set 'X' such that B = A^C is equal to $\frac{1}{2^{m-1}}$ where m - k is equal to

Key. 2

Sol. $n(X)=k+1$

No. of ways to construct $A=2^{k+1}$

No. of ways to construct $B=2^{k+1}$

$\therefore$ Total ways to construct A and B = $2^{k+1}\times 2^{k+1}$

Favourable ways to construct A = 2^{k+1}

Favourable ways to construct B such that B = A^C is = 1

$\therefore$ Favourable ways = $2^{k+1}\times 1$

Required Probability = $\frac{2^{k+1}}{\left(2^{k+1}\right)^2}=\frac{1}{2^{k+1}}$

$\Rightarrow m-1=k+1$

$\Rightarrow m-k=2$

3. A bag contains 10 different balls. Five balls are drawn simultaneously and then replaced and then seven balls are drawn. The probability that exactly three balls are common to the two drawn is *p*, then the value of 12*p* is

Key. 5

Sol. The no. of ways of drawing 7 balls = ${}^{10}C_7$
For each set of 7 balls of the second draw, 3 must be common to the set of 5 balls of the draw, i.e., 2 other balls can be drawn in ${}^{3}C_2$ ways thus, for each set of 7 balls of the second draw, there are ${}^{7}C_3 \times {}^{3}C_2$ ways of making the first draw so that there are 3 balls common.
Hence, the probability of having three balls in common $\frac{{}^{7}C_3 \times {}^{3}C_2}{{}^{10}C_7} = \frac{5}{12}$.

4. The number of ways of arranging the letters of the word NALGONDA, such that the letters of the word GOD occur in that order (G before and O and O before D), is P then $\frac{P}{420} =$

Key. 4

Sol. No. of ways $= \frac{\lfloor 8}{\lfloor 2 \lfloor 2 \lfloor 3} = 1680$

$\Rightarrow \quad \frac{P}{420} = 4$

5. In a group of people, if 4 are selected at a random, the probability that the any two of the four do not have same month of birth is p then $\frac{96p}{11}$ is equal to

Key. 5

Sol. Required probability $= \frac{{}^{12}C_4 \lfloor 4}{12^4} = \frac{55}{96}$

6. Two numbers are selected at random from set of the first 100 natural numbers. The probability that the product obtained is divisible by 3 is k then $\frac{150k}{83}$ is equal to

Key. 1

Sol. Required probability $= \frac{{}^{33}C_2 + {}^{33}C_1 \, {}^{67}C_1}{{}^{100}C_2}$

$= \frac{83}{150}$

7. Functions are formed form A = {1, 2, 3,} to set B = {1, 2, 3, 4, 5} and one function is elected at random. If P the probability that function satisfying f(i) ≤ f(j) whenever i < j then value of 25 p is equal to

Key. 7

Sol. Total number of function = $5^3 = 125$

Number of function satisfying $f(i) \le f(j)$ if $i < j$

$= {}^5C_3 + {}^5C_2\,(1 + 1) + {}^5C_1 = 35$

Required probability $= \dfrac{35}{125} = \dfrac{7}{25}$

8. If the sides of triangle are decided by throwing a die thrice, the probability that the triangle is isosceles or equilateral is $\dfrac{1}{k}$ then k =

Key. 8

Sol. Let the sides be a, b, c

$a = b = 1, c = 1$

$a = b = 2, c = 1, 2, 3$

$a = b = 3, c = 1, 2, 3, 4, 5$

$a = b = 4, c = 1, 2, 3, 4, 5, 6$

$a = b = 5, c = 1, 2, 3, 4, 5, 6$

$a = b = 6, c = 1, 2, 3, 4, 5, 6$

The number of these triangles is $1 + 3 + 5 + 3 \times 6 = 27$

Probability $= \dfrac{27}{6^3} = \dfrac{1}{8}$

9. Four identical dice are rolled once the probability that all the members on them are primes is $\dfrac{L}{8L + 2}$ then L =

Key. 5

Sol. The total number of outcomes:

$aaaa$ appear in $\binom{6}{1} = 6$ ways

$aaab$ appear in $2\binom{6}{2} = 30$ ways

$aabb$ appear in $\binom{6}{2} = 15$ ways

$aabc$ appear in $3\binom{6}{3} = 60$ ways

$abcd$ appear in $\binom{6}{4} = 15$ ways

Total $= 6 + 30 + 15 + 60 + 15 = 126$

The number of ways of primes appearing

$aaaa$ appear in $\binom{3}{1}=3$ ways

$aaab$ appear in $2\binom{3}{2}=6$ ways

$aabb$ appear in $\binom{3}{2}=3$ ways

$aabc$ appear in $3\binom{3}{3}=3$ ways

Total $=3+6+3+3=15$

Probability $=\frac{15}{126}=\frac{5}{42}$

10. If $\{x,y\}$ is a subset of the first 30 natural numbers, then the probability, that x^3+y^3 is divisible by 3, is $\frac{S}{9}$ then S =

Key. 3

Sol. x^3+y^3 is divisible by $3 \Rightarrow x+y$ is divisible by $3 \Rightarrow x,y$ are multiples of 3 or one leaves remainder 1 and the other 2 when divided by 3.

$3,6,9.....30$ are multiple of $3;1,4,7,......,28$ leave remainder 1

$2,5,8,...,29$ leave remainder 2

Probability $=\frac{\binom{10}{1}\binom{10}{1}+\binom{10}{2}}{\binom{30}{2}}$

$=\frac{145}{15\times 29}=\frac{1}{3}$

11. If p, q are chosen randomly with replacement from the set $\{1,2,3,......10\}$, the probability, that the roots of the equation $x^2+px+q=0$ are real, is $\frac{k^2+6}{50}$ then k =

Key. 5

Sol.

p	q
2	1
3	1, 2

4	1 to 4
5	1 to 6
6	1 to 9
7, 8, 9, 10	1 to 10

$p^2 \geq 4q \Rightarrow$

The total number of pairs $(p,q) is 1+2+4+6+9+40=62$

Probability $=\dfrac{62}{10.10}=\dfrac{31}{50}$

12. Total number of divisors of $3^5.5^7.7^9$ which are of the form $4\lambda+1,\ \lambda \geq 0$, is $(60)l$ then l is

Key. 2

Sol. Any positive integral power of 5 is of the form $4\lambda+1$. Even power of 3 and 7 are of the form $4\lambda+1$ and odd powers of 3 and 7 are of the form $4\lambda-1$. The required number $=8(3\times5+3\times5)$

13. If $f(x)=ax^3+bx^2+cx+d$, (a,b,c,d are rational) and roots of $f(x)=0$ are eccentricities of a parabola and a rectangular hyperbola then $a+b+c+d$ equals

Key. 0

Sol. Roots of $f(x)$ are $1,\sqrt{2},-\sqrt{2}$

$$ax^3+bx^2+cx+d=(x-1)(x-\sqrt{2})(x+\sqrt{2})=(x-1)(x^2-2)=x^3-x^2-2x+2$$

$$a=1,b=-1,c=-2,d=2 \Rightarrow a+b+c+d=0$$

14. An unbiased coin is tossed 12 times .The probability that at least 7 consecutive heads show up is $\dfrac{K}{256}$ then K =

Key. 7

Sol. The sequence of consecutive heads may starts with 1st toss or 2nd toss or 3rd toss ---- or at 6th toss. In any case ,if it starts with r th throw , the first (r-2) throws may be head or tail but (r-1)st throw must be tail, after which again a head or tail can show up:

HHH------H x x x x x or x x x -------- x T H HH-------H x x------

7 times (r-2) (r-1)st

$\therefore$ Probability = $\frac{1}{2^7}+\frac{1}{2}\cdot\frac{1}{2^7}+\frac{1}{2}\cdot\frac{1}{2^7}$ +---------+ $\frac{1}{2}\cdot\frac{1}{2^7}=\frac{1}{2^7}\left[1+\frac{5}{2}\right]=\frac{7}{2^8}$

5 times

15. An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the number obtained by adding the numbers on the two faces is noted. If the result is a tail, a card from a well shuffled pack of eleven cards numbered 2, 3, 3, 4, ... 12 is picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is $\frac{P}{792}$ then the digit in tens place of P is

Key. 9

Sol. Let E_1 = the toss result in a head

E_2 = the toss result in a tail

A = noted number is 7 or 8

$\therefore$ P(A) = $P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)$

$= \frac{1}{2}\times\frac{11}{36}+\frac{1}{2}\times\frac{2}{11}=\frac{193}{792}$

$\therefore \frac{P}{792}=\frac{193}{792}$

$\therefore$ P = 193.

16. If the number of ways can the letters of the word INSURANCE be arranged, so that the vowels are never separated is $(1080)n$ then the value of 'n' is

Key. 8

Sol. The word INSURANCE has nine different letters, combining the vowels into one bracket as (IUAE) and treating them as one letter we have six letters viz.

(IUAE), N, S, R, N, C and these can be arranged among themselves in $\frac{6!}{2!}$ ways and four vowels within the bracket can be arranged themselves in $4!$ ways.

Required number of words $= \frac{6!}{2!}\times 4! = 8640$

17. In a class of 10 students, there are 3 girls. If the number of different ways that all the students be arranged in a row such that no two of the three girls are consecutive is $(564480)k$ then the value of 'k' is

Key. 3

Sol. Number of girls = 3, number of boys = 7. Since there is no restriction on boys, therefore first of all arrange the 7 boys in $^7P_7 = 7!$ ways.

B B B B B B B

If the girls are arranged at the places (including the two ends) indicated by crosses, no two of three girls will be consecutive.

Now there are 8 places for 3 girls

3 girls can be arranged in $^8P_3$ ways

Required number $= {^8P_3}7! = \frac{8!}{5!} \times 7! = 336 \times 5040 = 3 \times (564480)$

18. If the number of 3 digit odd numbers divisible by 3, which can be formed using the digits 3, 4, 5, 6 when repetition of digits within the number is allowed is $2k+5$ then the value of 'k' is

Key. 3

Sol. Three digits odd numbers using only 3 and only 5 are 2.

Three digit odd numbers using 3, 4 and 5, are 4.

Three digit odd numbers using 4, 5 and 6 are 2.

Three digit odd numbers using two 6 and one 3 are 1.

Three digit odd numbers using two 3 and one 6 are 2.

So, total three digit numbers $= 11$

19. The number of ways of arranging the letters of the word NALGONDA, such that the letters of the word GOD occur in that order (G before and O and O before D), is P then $\frac{P}{420} =$

Key. 4

Sol. No. of ways $= \frac{\lfloor 8}{\lfloor 2 \lfloor 2 \lfloor 3} = 1680$

$\Rightarrow \frac{P}{420} = 4$

20. $1, \alpha_1, \alpha_2, \alpha_3, ..., \alpha_k$ are divisions of number $N = 2^{n-1}(2^n - 1)$ where $2^n - 1$ is a prime number and $1 < \alpha_1 < \alpha_2 < \alpha_3 < ...... < \alpha_k$ then value of $\left(1 + \frac{1}{\alpha_1} + \frac{1}{\alpha_2} + ... + \frac{1}{\alpha_k}\right)$ is

Key. 2

Sol. Divisors of $N=2^{n-1}\left(2^n-1\right)$ are

$1,2,2^2,.....,2^{n-1},2^n-1,2\left(2^n-1\right),2^2\left(2^n-1\right),.....,2^{n-1}\left(2^n-1\right)$

$$1+\frac{1}{\alpha_1}+\frac{1}{\alpha_2}+......+\frac{1}{\alpha_k}=1+\frac{1}{2}+.....+\frac{1}{2^{n-1}}+\frac{1}{2^n-1}+\frac{1}{2\left(2^n-1\right)}+.....+\frac{1}{2^{n-1}\left(2^n-1\right)}$$

$$=\left(1+\frac{1}{2}+....+\frac{1}{2^{n-1}}\right)+\frac{1}{2^n-1}\left(1+\frac{1}{2}+.....+\frac{1}{2^{n-1}}\right)$$

$$=\left(1+\frac{1}{2}+....+\frac{1}{2^{n-1}}\right)\left(1+\frac{1}{2^n-1}\right)$$

$$=\frac{1.\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}}\left(\frac{2^n}{2^n-1}\right)=\frac{2^n}{2^{n-1}}=2$$

21. If the number of ordered triplets (x,y,z) such that $L.C.M(x,y)=3375,\ L.C.M(y,z)=1125,\ L.C.M(z,x)=3375$ is equal to $'k'$, then $k-47$ is equal to

Key. 3

Sol. $3375=5^3.3^3,\ 1125=5^3.3^2$

Clearly, 3^3 is a factor of $'x'$ and 3^2 is factor of atleast one of $y\ \&\ z$. This can be done in 5 ways.

Also, 5^3 is a factor of atleast two of the numbers x,y,z which can be done in

${}^3C_2\times 4-2=10$

$\therefore\ k=50$

22. If k be the number of 3 digit natural numbers, having sum of their digits atleast 10, then the value of $\frac{k-35}{100}$ is

Key. 7

Sol. We have to calculate number of solution of $a+b+c>9, 1\le a\le 9,\ 0\le b,c\le 9$

$a+b+c+d=9,\ d\ge 0$

Number of solutions is co-efficient of t^9 in

$\left(t+t^2+....+t^9\right)\left(1+t+.....+t^9\right)^2\left(1+t+t^2+......t^9\right)$

$=$ coefficient of t^8 in $\left(1-t^9\right)\left(1-t^{10}\right)^2\left(1-t\right)^{-4}$

$=$ coefficient of t^8 in $(1-t)^{-4} = {}^{8+4-1}C_{4-1} = {}^{11}C_3 = 165$

So, required number of natural numbers $900-165=735$

$\therefore \dfrac{k-35}{100}=7$

23. Eighteen guests have to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on other side. If the number of ways in which the seating arrangements can be made is $\dfrac{11!\times(9!)^2}{(p!)(p+1)!}$.

Then the value of $'p'$ is

Key. 5

Sol. Out of 18 guests half i.e., 9 to be seated on side A and rest 9 on side B.

Now out of 18 guests, 4 particular guess desire to sit on one particular side say side A and other 3 on other side B. Out of rest $18-4-3=11$ guests we can select 5 more for side A and rest 6 can be done in ${}^{11}C_5$ ways and 9 guests on each side of table can be seated in $9!\,9!$ ways.

Thus there are total ${}^{11}C_5\,9!\,9!$ arrangements.

24. If the number of arrangements of the letters of the word BANANA in which the two $N's$ do not appear adjacently is $5k$ then $'k'$ equals

Key. 8

Sol. Required number of ways $=\dfrac{\lfloor 6}{\lfloor 3\lfloor 2}-\dfrac{\lfloor 5}{\lfloor 3}=40$

25. $'m'$ men and $'n'$ women are to be seated in a row so that no two women sit together. If $m>n,$ then the number of ways in which they can be seated is $\dfrac{m!(m+1)!\times\lambda}{(m-n+1)!}$.

Then the value of $'\lambda'$ is

Key. 1

Sol. $'m'$ men can be seated in $m!$ ways creating $(m+1)$ for ladies to sit.

$'n'$ ladies out of $(m+1)$ places (as $n<m$) can be seated in ${}^{m+1}P_n$ ways

$\therefore$ Total ways $= m!\times {}^{m+1}P_n = m!\,\dfrac{(m+1)!}{(m+1-n)!}$

26. A seven digit number made up of all distinct digits $8,7,6,4,2,x,y$ is divisible by 3. The possible number of ordered pairs (x,y) is

Key. 8

Sol. We know that a number is divisible by 3. If sum of its digits is divisible by 3.

Hence we must have $8+7+6+4+2+(x+y)=3k$

$27+x+y=3k$

$\Rightarrow x+y$ is multiple of 3

Hence required (x,y) order pairs

$=(0,3),(0,9),(1,5),(3,0),(3,9),(5,1),(9,0),(9,3)$

27. The number of integral solutions of the equation $2x+2y+z=20$ where $x\ge 0, y\ge 0$ *and* $z\ge 0$ is $11k$ then $k=$

Key. 6

Sol. $2x+2y=20-z \Rightarrow x+y=10-\dfrac{z}{2}$

Coefficients of $P^{10-\frac{z}{2}}$ in $\left(1+P+P^2 ....\right)^2 = 11-\dfrac{z}{2}$

Patting $z=0,2,4,6,8,10,12,14,16,18,20$

The number of integral solutions $=66$

28. The number of numbers from 1 to 100, which are neither divisible by 3 not by 5 nor by 7 is n. Then n/9 is.

Key. 5

Sol. Conceptual

29. There are four balls of different colors and four boxes of colors, same as those of the balls. The number of ways in which the balls, one each in a box, could be placed
such that a ball does not go to a box of its own color, is

Key. 9

Sol. $\therefore$ Number of ways of putting all the 4 balls into boxes of different colour.

$$=4!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right]=4!\left(\frac{1}{2}-\frac{1}{6}+\frac{1}{24}\right)=24\left(\frac{12-4+1}{24}\right)=9$$

30. The digit in tenth place of $1!+2!+3!+......+49!=$

Key. 1

Sol. $1!+2!+3!+4!=33$ Also $5!=120, 6!=720, 7!=5040, 8!=40320 \ \ and \ \ 9!=326880$

The digit in tenth place $1!+2!+3!+....+9!=1$

Also note that $n!$ is divisible by 100 for $n \geq 10$

so that the digit in tenth place $10!+11!+.....+49!$ is zero.

There fore The digit in tenth place of $1!+2!+3!+.....+49!$ is 1.

31. In a group of people, if 4 are selected at a random, the probability that any two of the four do not have same month of birth is p then $\frac{96\,P}{11}$ is equal to

Key. 5

Sol. Required probability $= \frac{{}^{12}C_4 \lfloor 4}{12^4} = \frac{55}{96}$

32. Two persons X and Y go to a hotel. There are two hotels having three rooms each and one hotel having four rooms each, The probability that X and Y are in the same hotel having four rooms is $\frac{k}{15}$ then k is ---

Key. 2

Sol. $P(E) = \frac{2.\,{}^4C_2}{10.9}$

33. Two numbers are selected at random from set of the first 100 natural numbers. The probability that the product obtained is divisible by 3 is k then $\frac{150\,k}{83}$ is equal to

Key. 1

Sol. Required probability $= \frac{{}^{33}C_2 + {}^{33}C_1\,{}^{67}C_1}{{}^{100}C_2} \qquad = \frac{83}{150}$

34. If A and B throw a die each. The probability that A s throw is not greater than Bs throw is K/12, then K=

Key. 7

Sol. If A gets 1, then Bs chances are 1,2,.6 = 6

If A gets 2, then Bs chances are 2,3,... = 5

Similarly so on up to 6 = $\sum 6$

$\therefore P(E)=\frac{21}{36}$

35. Two squares are chosen at random on a chess board. If the chance that may have a contact at corner is $\frac{\mu}{144}$ then μ should be equal to

Key. 7

Sol. Total cases of choosing two squares on a chess board $=64\times63$

Favourable cases $=4\times1$ (corners) $+24\times2+36\times4$ $=4\times49$

Required probability $=\frac{4\times49}{64\times63}=\frac{7}{144}$

$\Rightarrow \mu=7$

36. The probability that a teacher will give a surprise test in a class is $\frac{1}{5}$. If a student is absent twice, the probability that he will miss atleast one test will be $\frac{k}{25}$, then k must be

Key. 9

Sol. Required probability $=1-\frac{4}{5}\times\frac{4}{5}=\frac{9}{25}$

$\Rightarrow$ k = 9

37. A box contains 24 balls of which 12 are black and 12 are white. The balls are drawn at random from the box one at a time with replacement . The probability that a white ball is drawn for the 4^{th} time on the 7^{th} draw is K/32 , then K=

Key. 5

Sol. Required probability = Probability of drawing of 3 W & 3 B balls in first 6 draws and a white ball in 7^{th} draw.

$={}^{6}C_{3}\cdot\frac{1}{2^{7}}=\frac{5}{32}$

38. Three tangents are drawn at random to a given circle. The odds against the circle being inscribed in the triangle formed by them is K to 1 , then K =

Key. 2

Sol. $P(E) = \frac{2}{6} = \frac{1}{3}$

$P(\overline{E}) = \frac{2}{3}$

$\therefore$ odds against = 2 : 1

39. Two non-negative integers are chosen at random. The probability that the sum of their squares is divisible by 5 is $\frac{K}{25}$ then $k =$

Key. 9

Sol. Let the non-negative integers be $x, y, x = 5a + \alpha, y = 5b + \beta$ where

$0 \le \alpha \le 4, 0 \le \beta \le 4$

$x^2 + y^2 = 25(a^2 + b^2) + 10(a\alpha + b\beta) + \alpha^2 + \beta^2$

$\alpha^2 + \beta^2$ is divisible by 5

$\alpha, \beta \in \{(0,0)(1,2)(2,1)\ (1,3)(3,1)(2,4)(4,2)(3,4)(4,3)\}$

Probability $= \frac{9}{25}$ $\therefore$ $k = 9$

40. If the integers m and n are chosen at random from $\{1, 2, 3, \ldots\ldots, 100\}$ then the probability that a number of the form $7^m + 7^n$ is divisible by 5 is equal to $\frac{1}{k}$.

The numerical value of k is

Key. 4

Sol. Total ways of choosing m and n is $n(S) = 100 \times 100$

Now, $7^1 = 7 = 5k + 2, 7^2 = 49 = 5k + 4, 7^3 = 343 = 5k + 3, 7^4 = 2301 = 5k + 1$

The same sequence will repeat for next four powers

$7^5 = 2k + 2, 7^6 = 5k + 4, 7^7 = 5k + 3, 7^8 = 5k + 1$

$\therefore$ $7^1, 7^5, 7^9, \ldots\ldots 7^{97}$ are of the type $5k + 2$

$7^2, 7^6, 7^{10}, \ldots\ldots 7^{98}$ are of the type $5k + 4$

$7^3, 7^7, 7^{11}, \ldots\ldots 7^{99}$ are of the type $5k + 3$

& $7^4, 7^8, 7^{12}, \ldots\ldots 7^{100}$ are of the type $5k + 1$

There will be only four favourable combinations in which $7^m + 7^n$ will be divisible by 5.

$7^m : 5k + 2, 5k + 4, 5k + 3, 5k + 1$

$7^n : 5k + 3, 5k + 1, 5k + 2, 5k + 4$

$\therefore$ Number of favourable cases is

$n(E) = 25 \times 25 + 25 \times 25 + 25 \times 25 + 25 \times 25 = 4 \times 625$

$\therefore$ Required probability $=\frac{n(E)}{n(S)}=\frac{4\times 625}{100\times 100}=\frac{1}{4}$

41. A man parks his car among n cars standing in a row ,his car not being parked at an end, on his return he finds that exactly m of the n cars are still there, probability that both the cars parked on two sides of his car ,have left is $\frac{(n-m)(n-m-1)}{(n-A)(n-B)}$ then $A+B$ is

Key. 3

Sol. Number of ways in which remaining $'m-1'$ cars can take their places (excluding the car of man) $={}^{n-1}C_{m-1}$

No.of ways in which remaining $(m-1)$ cars can take places keeping the two places on two sides of his car vacant $={}^{n-3}C_{m-1}$

Prob $=\frac{{}^{n-3}C_{m-1}}{{}^{n-1}C_{m-1}}=\frac{(n-m)(n-m-1)}{(n-1)(n-2)}$

$\Rightarrow A=1$

$B=2$

$A+B=3$

42. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. If the probability that Mr. A selected the winning horse is $\frac{P}{5}$ then the value of P is

Key. 2

Sol. Out of 5 horses only one is the winning horse. The probability that Mr. A selected the loss horse $=\frac{4}{5}\times\frac{3}{4}$

$\therefore$ The probability that Mr. A selected the winning horse $=1-\frac{4}{5}.\frac{3}{4}=\frac{2}{5}$

43. You are given a box with 20 cards in it. 10 cards of this have the letter I printed on them. The other ten have the letter T printed on them. If you pick up 3 cards at random and keep them in the same order, the probability of making the word IIT is $\frac{K}{38}$. The numerical value of K is

Key. 5

Sol. $\frac{{}^{10}C_1}{{}^{20}C_1}\times\frac{{}^{9}C_1}{{}^{19}C_1}\times\frac{{}^{10}C_1}{{}^{18}C_1}=\frac{10\times 9\times 10}{20\times 19\times 18}=\frac{5}{38}$

44. Two squares are chosen at random from small squares (one by one) drawn on a chess board and

the chance that two squares chosen have exactly one corner in common is $\frac{k}{144}$ then $k=$

Key. 7

Sol. Total number of ways to select

2 unit squares $= {}^{64}C_2$

No.of ways of selecting squares which have a corner in common $=98$

$\therefore$ probability $= \frac{98}{{}^{64}C_2} = \frac{7}{144} \Rightarrow k=7$

45. A die is rolled three times, the probability of getting a large number than the previous number is $\frac{k}{54}$ then the value of k is

Key. 5

Sol. Let the second number be x (where $1 < x < 6$)

Then first number can be chosen in $(x-1)$ ways and third in $(6-x)$ ways

$\therefore$ Favourable cases $= \sum_{x=2}^{5}(x-1)(6-x) = 20$

46. From a bag containing 10 distinct balls, 6 balls are drawn simultaneously and replaced. Then 4 balls are drawn. The probability that exactly 3 balls are common to the drawings is $\frac{m}{21}$. Then the numerical value of m is

Key. 8

Sol. Let S be the sample space of the composite experiment of drawing 6 in the first draw and then four in second draw then $|S| = {}^{10}C_6 \times {}^{10}C_4$

$\therefore$ Required probability $= \frac{{}^{10}C_6 \times {}^{6}C_3 \times {}^{4}C_1}{{}^{10}C_6 \times {}^{10}C_6}$

$= \frac{80 \times 24}{10 \times 9 \times 8 \times 7} = \frac{8}{21}$

47. If two natural numbers x, y are selected at randomly and probability that $x^2 + y^2$ is multiple of 5 is p, then 25 p is

Ans: 9

Hint: Total number of ways of end digits of x,y is 100

and favourable is $8 \times 4 + 2 \times 2 = 36$

So, p = $\dfrac{36}{100}=\dfrac{9}{25}$

CONDITIONAL PROBABILITY

48. Two cards are selected at randomly from a pack of ordinary playing cards. If there found to be of different colours (Red & Black), then conditional probability that both are face cards is

(A) $\dfrac{36}{325}$ (B) $\dfrac{18}{169}$

(C) $\dfrac{9}{169}$ (D) none of these

Key: B

Hint: Let A $\rightarrow$ they are face cards, B $\rightarrow$ they are of different colours

$$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{{}^{12}C_2-2\times{}^{6}C_2}{13\times 26}=\frac{18}{169}$$

49. The probability of a bomb hitting a bridge is ½ and two direct hits are needed to destroy it. The leastnumber of bombs required so that the probability of the bridge being destroyed is greater than 0.9 is

a) 7 b) 9 c) 8 d) 10

KEY : A

HINT. $P(X\geq 2)\geq 0.9$ X follows B.D with parameter n,p=$\dfrac{1}{2}$

50. A special die is so constructed that the probabilities of throwing 1, 2, 3, 4, 5 and 6 are (1 – k)/6, (1 + 2k)/6, (1 – k)/6, (1 + k)/6, (1 – 2k)/6 and (1 + k)/6 respectively. If two such dice are thrown and the probability of getting a sum equal to 9 lies in $\left[\dfrac{1}{9},\dfrac{2}{9}\right]$. Then find the number of integral solutions of k.

Key. 1

Sol. Let E_1, E_2, E_3 E_4, E_5 and E_6 be the events of occurrence of 1, 2, 3, 4, 5 and 6 on the dice respectively, and let E be the event

$$\therefore P(E_1)=\frac{1-k}{6};\ P(E_2)=\frac{1+2k}{6};P(E_3)=\frac{1-k}{6}$$

$$P(E_4)=\frac{1+k}{6};\ P(E_5)=\frac{1-2k}{6};\ P(E_6)=\frac{1+k}{6}\ \text{and}\ \frac{1}{9}\leq P(E)\leq\frac{2}{9}$$

Then, $E\equiv\{(3,6),(6,3),(4,5),(5,4)\}$

Hence, $P(E)=P(E_3E_6)+P(E_6E_3)+P(E_4E_5)+P(E_5E_4)$

$=P(E_3)P(E_6)+P(E_6)P(E_3)+P(E_4)P(E_5)+P(E_5)P(E_4)$

$=2P(E_3)P(E_6)+2P(E_4)P(E_5)$

{Since E_1, E_2, E_3, E_4, E_5 and E_6 are independent}

$= 2\left(\frac{1-k}{6}\right)\left(\frac{1+k}{6}\right) + 2\left(\frac{1+k}{6}\right)\left(\frac{1-2k}{6}\right)$

$= \frac{1}{18}\left[2 - k - 3k^2\right]$

Since, $\frac{1}{9} \le P(E) \le \frac{2}{9}$

$\therefore$ $-\frac{1}{3} \le k \le 0$

$\therefore$ Set of integral value of k = {0}

$\therefore$ Number of integral solution of k is 1

51. If number of numbers greater than 3000, which can be formed by using the digits 0, 1, 2, 3, 4, 5 without repetition, is n then $\frac{n}{230}$ is equal to

Key. 6

Sol. No. of 4 digit numbers = 3 × 5 × 4 × 3 = 180

No. of 5 digit numbers = 5 × 5 × 4 × 3 × 2 = 600

No. of 6 digit numbers = 5 × 5 × 4 × 3 × 2 = 600

$n = 1380$

$\Rightarrow \quad \frac{n}{230} = 6$

52. Nine hundred distinct n-digit numbers are to be formed using only the 3 digits 2, 5, 7. The smallest value of n for which this is possible is

Key. 7

Sol. $3^n \ge 900 \Rightarrow n \ge 7$

53. Out of 5 apples, 10 mangoes and 15 oranges, the number of ways of distributing 15 fruits each to two persons, is n then $\frac{n}{22}$ is equal to

Key. 3

Sol. $x_1 + x_2 + x_3 = 15$

$0 \le x_1 \le 5,\ 0 \le x_2 \le 10,\ 0 \le x_3 \le 15$

n = co-efficient of $x^{15}(1 - x^6)(1 - x^{11})(1 - x^{16})(1 - x)^{-3}$

n = 66

$\frac{n}{22} = 3$

54. A bag contains 10 different balls. Five balls are drawn simultaneously and then replaced and then seven balls are drawn. The probability that exactly three balls are common to the two drawn is *p*, then the value of 12*p* is

Key. 5

Sol. The no. of ways of drawing 7 balls = ${}^{10}C_7$

For each set of 7 balls of the second draw, 3 must be common to the set of 5 balls of the draw, i.e., 2 other balls can be drawn in ${}^{3}C_2$ ways thus, for each set of 7 balls of the second draw, there are ${}^{7}C_3 \times {}^{3}C_2$ ways of making the first draw so that there are 3 balls common.

Hence, the probability of having three balls in common $\frac{{}^{7}C_3 \times {}^{3}C_2}{{}^{10}C_7} = \frac{5}{12}$.

55. In a multiple-choice question, there are five alternative answers, of which one or more than one are correct. A candidate will get marks on the question if he ticks all the correct answers. So he decides to tick answers at random, if the least number of chances ,he should be allowed so that the probability of his getting marks on the question exceeds 1/8 is K, then K =

(the student always attempt the question)

Key. 4

Sol. The probability that he get marks $= \frac{1}{31}$

The probability that he get marks in second trial is $\frac{30}{31} \times \frac{1}{30} = \frac{1}{31}$

The probability that he get marks in third trial is $\frac{1}{31}$

Continuing this process the probability from *r* trial is $\frac{r}{31} > \frac{1}{8}$

$\Rightarrow r > \frac{31}{8}$

$r = 4$

56. 3 couples have to be seated around a circle. Let p be the probability that no couple is together then the value of 30 p is

Key. 8

Sol. $p = \frac{5! - {}^{3}C_1(4!).(2!) + {}^{3}C_2(3!).(2!).(2!) - {}^{3}C_3(2!).(2!).(2!).(2!)}{5!} = \frac{120 - 144 + 72 - 16}{120} = \frac{4}{15}$

So 30p = 8

57. A coin is tossed m + n times (m > n) , the probability of getting m consecutive heads is $\frac{n+k}{2^{m+2}}$ then k = ___

Key. 3

Sol. If m = 3, n = 2

Coin is tossed 5 times, then 3 consecutive heads can come is 5 cases

Probability = $\frac{5}{2^5} = \frac{2+3}{2^{3+2}}$

58. Three identical dies are rolled, the probability that they will get same number on them. If $\frac{K}{28}$ then *K* = ________

Key. 3

Sol. $n(S)=56$

$P(E)=\frac{31}{50}$

59. Two distinct numbers are chosen at random from set {1, 2, ……. 3n}. The probability that x^2-y^2 is divisible by 3 is $\frac{pn+q}{r(3n-1)}$ then $p+q+r=$

Key. 5

Sol. $n(s)={}^{3n}C_2$

Let $A_0=\{3,6,9..........3n\}$

$A_1=\{1,4,7..........3n-2\}$

$A_2=\{5,8,11..........3n-1\}$

(x^2-y^2) divisible by 3. If both x,y should come from A_0 or A_1 or A_2 or one is from A_1 and other from A_2

$n(E)=3^nC_2+{}^nC_1+{}^nC_1=\frac{n}{2}(5n-3)$

$P(E)=\frac{5n-3}{3(3n-1)}$

60. 3 numbers are chosen at random without replacement from $\{1,2,3.......14\}$ Let

Let A = { min of chosen number is 5}

B = { max of chosen no is 11 }

$P(A\cup B)=\frac{K+11}{91}$ then *K* = _____

Key. 8

Sol. $P(A)=\frac{{}^9C_2}{{}^{14}C_3}=\frac{9}{91}, P(B)=\frac{{}^{10}C_2}{{}^{14}C_3}=\frac{45}{364}$

$P(A\cap B)=\frac{{}^5C_1}{{}^{14}C_3}=\frac{5}{364}$

$P(A\cup B)=\frac{19}{91}$

61. There are n lines in a pane, No two of which are parallel and No three of concurrent Let plane be divided in U_n parts then U_3=

Key. 7

Sol. $U_0=1, U_1=2$

The n[th] line will rise to n additional parts when U_{n-1} parts are already there $U_n=U_{n-1}+n$

62. Two natural numbers x, y are selected at random, probability that x^2+y^2 is divisible by 5 is $\frac{k}{25}$ then k = _______

Key. 9

Sol. $P(E)=\frac{9}{25}$

Sample space $S=\{0,1,2,3,4\}\times\{0,1,2,3,4\}$

$E=\{(0,0)(1,2)(2,1)(1,3)(3,1)(2,4)(4,2)(3,4)(4,3)\}$

63. In a group of people, if 4 are selected at a random, the probability that the any two of the four do not have same month of birth is p then $\frac{96p}{11}$ is equal to

Key. 5

Sol. Required probability $=\frac{{}^{12}C_4\underline{|4}}{12^4}=\frac{55}{96}$

64. Two numbers are selected at random from set of the first 100 natural numbers. The probability that the product obtained is divisible by 3 is k then $\frac{150k}{83}$ is equal to

Key. 1

Sol. Required probability $=\frac{{}^{33}C_2+{}^{33}C_1\,{}^{67}C_1}{{}^{100}C_2}$

$=\frac{83}{150}$

65. Functions are formed form A = {1, 2, 3,} to set B = {1, 2, 3, 4, 5} and one function is elected at random. If P the probability that function satisfying $f(i)\le f(j)$ whenever $i<j$ then value of 25 p is equal to

Key. 7

Sol. Total number of function = 5^3 = 125

Number of function satisfying $f(i)\le f(j)$ if $i<j$

$={}^5C_3+{}^5C_2(1+1)+{}^5C_1=35$

Required probability $=\frac{35}{125}=\frac{7}{25}$

66. In a bag there are 15 balls of either red or green colour. Let G_k be the event that it contains exactly k green balls and its probability is proportional to k^2. Now a ball is drawn at random. Let P(A) be the probability that the ball drawn is green. If P(A) = p/q in lowest form then q - p is _____.

Key. 7

Sol. $P(G_k)\ \alpha\ k^2 \Rightarrow P(G_k)=\lambda k^2$

$\sum_{k=0}^{n} P(G_k)=1$ (as these are mutually exclusive and exhaustive events)

$\Rightarrow \lambda \sum_{k=0}^{n} k^2=1 \Rightarrow \lambda=\frac{6}{n(n+1)(2n+1)}$

$P(A)=\sum_{k=0}^{n} P(G_k)\ P(A/G_k)=\sum_{k=0}^{n} \lambda k^2.\frac{k}{n}=\frac{\lambda}{n}.\frac{n^2(n+1)^2}{4}=\frac{3(n+1)}{2(2n+1)}$.

Take n = 15.

67. The probability that a random chosen 3 digit number has exactly 3 factors is $\frac{p}{900}$ (where $p\in N$) then the value of p is _____.

Key. 7

Sol. A number has exactly 3 factors if the number is squares of a prime number. Squares of 11, 13, 17, 19, 23, 29, 31 are 3–digit numbers

$\therefore$ required probability = $\frac{7}{900}$

68. Die A has four red and two white faces whereas die B has two red and four white faces. A coin is flipped once. If it falls a head, the game continues by throwing die A, if it falls tail then die B is to be used. If the probability that die A used is $\frac{32}{33}$ when it is given that red turns up every time in first n throws, then the value of n is ______.

Key. 5

Sol. Let R be the event that a red face appears in each of the first n throws.

E_1 : Die A is used when head has already fallen

E_2 : Die B is used when tail has already fallen.

$\therefore$ P (R/E_1) = $\left(\frac{2}{3}\right)^n$ and $P\left(\frac{R}{E_2}\right)=\left(\frac{1}{3}\right)^n$

As per the given condition

$$\frac{P(E_1).P(R/E_1)}{P(E_1).P(R/E_1)+P(E_2).P(R/E_2)}=\frac{32}{33} \Rightarrow \frac{1/2.(2/3)^n}{\frac{1}{2}.\left(\frac{2}{3}\right)^n+\frac{1}{2}\left(\frac{1}{3}\right)^n}=\frac{32}{33} \Rightarrow \frac{2^n}{2^n+1}=\frac{32}{33}$$

$\Rightarrow$ n = 5.