Mixed Questions

Single Correct Answer Type

1. Consider the quadratic equation $ax^2 - bx + c = 0,\ a,b,c \in N$. If the given equation has two distinct real roots belonging to $(1,2)$ then

A) $1 < a < 5$ B) $a \geq 5$ C) $a = 4$ D) $a = 3$

Key. B

Sol. $\alpha+\beta=\frac{b}{a},\ \alpha\beta=\frac{c}{a},\ \alpha,\beta\in(1,2)$ $\Rightarrow \alpha-1,\ \beta-1,\ 2-\alpha,\ 2-\beta\in(0,1)$

Apply $AM \geq G.M$

$$\frac{(\alpha-1)+(2-\alpha)}{2} > \sqrt{(\alpha-1)(2-\alpha)}$$

And $\frac{(\beta-1)+(2-\beta)}{2} > \sqrt{(\beta-1)(2-\beta)}$

$\Rightarrow (\alpha-1)(2-\alpha) < \frac{1}{4}$ and $(\beta-1)(2-\beta) < \frac{1}{4}$

$$(\alpha-1)(\beta-1)(2-\alpha)(2-\beta) < \frac{1}{16}$$

But $(\alpha-1),\ (2-\alpha),\ (\beta-1),\ (2-\beta) > 0$

$$\Rightarrow\ 0 < (\alpha-1)(2-\alpha)(\beta-1)(2-\beta) < \frac{1}{16}$$

$$0 < \left(\frac{c}{a}-\frac{b}{a}+1\right)\left(4-\frac{2b}{a}+\frac{c}{a}\right) < \frac{1}{16}$$

$$0 < (a-b+c)(4a-2b+c) < \frac{a^2}{16} \qquad \ldots\ldots\ldots(1)$$

$$(a-b+c)(4a-2b+c) \geq 1\ (\because a,b,c\in N)$$

From (1), $\frac{a^2}{16} > 1 \Rightarrow a \geq 5$

2. Tangents $\overline{PA}$ and $\overline{PB}$ are drawn to $y^2 = 4ax$ from P. If $m_{\overline{PA}}$ & $m_{\overline{PB}}$ are the slopes of the tangents satisfying $(m_{\overline{PA}})^2 + (m_{\overline{PB}})^2 = 4$ then the locus of 'P' is

A) $y^2 = 2x(2x+a)$ B) $y^2 = 2x(2x-a)$ C) $y^2 = x(x-a)$ D) $y^2 = ax$

Key. A

Sol. Let $P \equiv (h,k)$

$y = mx + \frac{a}{m}$

$k = mh + \frac{a}{m} \Rightarrow m^2 h + a - mk = 0 \Rightarrow m_{PA} + m_{PB} = \frac{k}{h}$

$m_{\overline{PA}} \times m_{\overline{PB}} = \frac{a}{h}$

But given that $\left(m_{\overline{PA}}\right)^2 + \left(m_{\overline{PB}}\right)^2 = 4$

$\Rightarrow \frac{k^2}{h^2} - \frac{2a}{h} = 4$

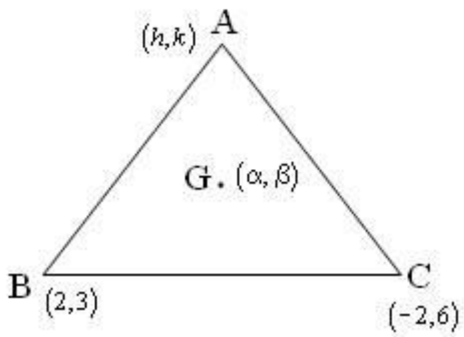
$\Rightarrow k^2 - 2ah = 4h^2$

$\therefore$ Locus of $P(h,k)$ is $y^2 = 2ax + 4x^2 = 2x(2x+a)$

3. In ΔABC, $B(2,3), C(-2,6)$ and perimeter is 14 units then locus of centroid of ΔABC is

A) Circle B) Pair of line C) A line D) Ellipse

Key. D



Sol.

Let $A(h,k)$ be the third vertex and $G(\alpha,\beta)$ be centroid of ΔABC

Perimeter $= AB + AC + BC = 14$

$\Rightarrow \sqrt{(h-2)^2 + (k-3)^2} + \sqrt{(h+2)^2 + (k-6)^2} = 14 - 5 = 9$

$\alpha = \frac{h}{3} \Rightarrow h = 3\alpha$

$\beta = \frac{k+9}{3} \Rightarrow k = 3\beta - 9$

given

$\sqrt{(3\alpha-2)^2 + (3\beta-12)^2} + \sqrt{(3\alpha+2)^2 + (3\beta-15)^2} = 9$

$$\sqrt{(\alpha-\frac{2}{3})^2+(\beta-4)^2}+\sqrt{(\alpha+\frac{2}{3})^2+(\beta-5)^2}=3$$

The distance between $A\left(\frac{2}{3},4\right)$ and $\left(-\frac{2}{3},5\right)$ is $\frac{5}{3}$

As $P(\alpha,\beta)$ is such that $PA+PB=3>AB$

$\Rightarrow$ Locus of $'P'$ is an ellipse.

4. The point of intersection of two tangents of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1,$ the product of whose slopes is c^2, lies on the curve

A) $y^2-b^2=c^2\left(x^2+a^2\right)$ B) $y^2+a^2=c^2\left(x^2-b^2\right)$

C) $y^2+b^2=c^2\left(x^2-a^2\right)$ D) $y^2-a^2=c^2\left(x^2+b^2\right)$

Key. C

Sol. Product of the slopes $=c^2$

$$\Rightarrow \frac{y^2+b^2}{x^2-a^2}=c^2 \Rightarrow y^2+b^2=c^2\left(x^2-a^2\right)$$

5. If four normals can be drawn to a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ from a point $(x_0,0)$ where $x_0>0$ then

A) $x_0>ae^2$ B) $a<x_0<ae^2$ C) $0<x_0<a$ D) $\frac{a}{e}<x_0<ae$

Key. A

Sol. $\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$

$$\frac{ax_o}{\sec\theta}=a^2+b^2 \Rightarrow \sec\theta=\frac{ax_o}{a^2+b^2}$$

$$\Rightarrow \cos\theta=\frac{a^2e^2}{a\,x_0}\ \left(\because a^2+b^2=a^2e^2\right)$$

$$0<\frac{ae^2}{x_0}<1 \Rightarrow x_0>ae^2$$

6. All chords of the curve $x^2+y^2-10x-4y+4=0$, which make a right angle at (8,-2) pass through

a) (2,5) b) (-2,-5) c) (-5,-2) d) (5,2)

Key. D

Sol. (8,-2) lies on the circle $(x-5)^2+(y-2)^2=25$ and a chord making a right angle at (8,-2) must be a diameter of the circle .So they all pass through the centre (5,2)

7. If $0\le \arg(z)\le \frac{\pi}{4}$ then the least value of $\sqrt{2}|2z-4i|$ is

A) 6 B) 1 C) 4 D) 2

Key. C

Sol. The conditions cover the region bounded by X – axis and $y=x$ least value of $|z-2i|$ is the length of perpendicular from (0,2) to y = x which is $\sqrt{2}$

$\therefore \sqrt{2}|2z-4i|$ least = 4

8. If $z_1, z_2 \in C$, $z_1^2+z_2^2\in R$, $z_1\left(z_1^2-3z_2^2\right)=2$ and $z_2\left(3z_1^2-z_2^2\right)=11$, then the value $z_1^2+z_2^2=$

A) 5 B) 6 C) 7 D) 8

Key. A

Sol. $z\bar{z}=1\Rightarrow |z_1z_2+z_2z_3+z_3z_1|=|z_1z_2z_3|\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|$

$=\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|=\left|\overline{z_1+z_2+z_3}\right|=|z_1+z_2+z_3|$

$\Rightarrow k=1$

9. If the equation of one of the tangent to the circle $|z-1-i|=2$ is $z+\bar{z}=-2$ then the equation of tangent perpendicular to it is

A) $z-\bar{z}=2i$ B) $z-\bar{z}=-2i$ C) $z-\bar{z}=3i$ D) $z-\bar{z}=-3i$

Key. B

Sol. Conceptual

10. Let a, b, c be three consecutive terms of an H.P. and all greater than 100. Then the value of $a^{\log_b c}-c^{\log_b a}$ equals

a) 0 b) 2 abc

c) 3 abc d) a + b + c

Key. A

Sol. $a^{\log_b c}=c^{\log_b a}$ always for any defined case

11. Let a, b, c denote the sides of a triangle. Then the quantity $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}$ lies between the limits

a) $\frac{7}{2}$ and 4   b) $\frac{7}{2}$ and $\frac{5}{2}$

c) $\frac{3}{2}$ and 2   d) 4 and 5

Key. C

Sol. observe that $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}$ equals $(a+b+c)\left\{\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}\right\}-3$

Which in turn equals

$\frac{1}{2}\{(a+b)+(b+c)+(c+a)\}\left\{\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}\right\}-3$

But by $AM \geq HM$ are have the above quantity is

$\geq \frac{1}{2}.9-3=\frac{3}{2}$

Suppose a, b, c are arranged such that $a \leq b \leq c$ then

$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b} \leq \frac{a}{a+c}+\frac{c}{c+a}+\frac{c}{a+b}=1+\frac{c}{a+b}$

$<1+1=2$

12. If $x>1, y>1, z>1$ are in G.P., then $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$ are in

a) A.P.   b) G.P.

c) H.P.   d) Not in any progression

Key. C

Sol. $y^2 = xz \Rightarrow 2\ln y = \ln x + \ln z$

$\Rightarrow \ln x, \ln y, \ln z$ are in A.P.

$\Rightarrow 1+\ln x, 1+\ln y, 1+\ln z$ are in A.P.

13. Let $I=\int_0^x \frac{(t-|t|)^2}{1+t^2}dt$. Then

A) $I=0$ if $x>0$ B) $I=4(x-Tan^{-1}x)$ if $x<0$

C) $I=\ln(1+x^2)$ if $x>0$ D) $I=4(x+Tan^{-1}x)$ if $x<0$

Key. A,B

Sol. Conceptual

14. $\int_{-1}^{5}\left[(x+4)\cos(x-2)+e^{|x-2|}\tan(x-2)\right]dx=$

A) $12\sin 3$ B) $8\sin 3$ C) $6\cos 3$ D) 0

Key. A

Sol. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

Applying this to $I = \int_{-1}^{5} e^{|x-2|}\tan(x-2)dx$, we get $I = -I \Rightarrow I = 0$

15. $\int_{-2}^{2} \frac{x^4 \tan^{-1} x + \sin x - 3x^2}{3-|x|} dx =$

A) $48+54\log 3$ B) $48-54\log 3$ C) $36+28\log 2$ D) 0

Key. B

Sol. $\frac{x^4 \tan^{-1} x + \sin x}{3-|x|}$ is odd function

16. Let $f(x) = \begin{cases} x^3 - x^2 + 10x - 5, & x \le 1 \\ -2x + \log_2(b^2-2), & x > 1 \end{cases}$. If $f(x)$ has greatest value at $x = 1$, then $b^2 \in (2, \lambda]$, then λ is

A) 130 B) 103 C) 301 D) 310

Key. A

Sol. $\ln_{x \to 1^+} f(x) \le f(1) \Rightarrow -2 + \log_2(b^2-2) \le 5 \Rightarrow b^2 \le 130$

but $b^2 - 2 > 0 \Rightarrow b^2 > 2$

$\therefore 2 < b^2 \le 130$

$\lambda = 130$

17. Find the area of the smaller region bound by the curves $\sqrt{(x-3)^2 + (y-1)^2} + \sqrt{(x+3)^2 + (y-1)^2} = 6$ & $|x| + |y| = 4$ is

a) 8 square units b) 16 square units c) 9 square units d) 18 square units

Key. C

Sol. First one is a line segment connecting (- 3, 1) to (3, 1)

18. Let A is the number of tangents drawn from a point on the asymptote of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (except origin) to the hyperbola itself. B is the number of normals which can be drawn from centre of $xy = c^2$ to the $xy = c^2$. C is the maximum number of normals which can be drawn from a point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. D is the number of tangent common to both branches of a hyperbola. Then number of normals which can be drawn from the point (ABD, BC) to $y^2 - 48y - 4x + 616 = 0$ is (If A =3, B = 5, C = 4 then ABC = 354)

a) 1 b) 0 c) 2 d) 3

Key. D

Sol. A – 1, B = 2, C = 4, D = 0

From (120, 24) we can draw 3 normals to

$\left(y-24\right)^2 = 4\left(x-10\right)$ since (x – 10) > 2

19. The area of the triangle formed by one of the common tangents of $\frac{x^2}{16}-\frac{y^2}{9}=1$ and $\frac{y^2}{16}-\frac{x^2}{9}=1$ with coordinate axes is

A) 7sq. units B) $\frac{7}{2}$ sq. units C) 6 sq. units D) 9 sq. units

Key. B

Sol. Equations of common tangents are $y=\pm x\pm\sqrt{7}$.

20. One vertex and focus of a hyperbola with eccentricity 3/2 are (3, 0) and (6, 0) respectively. The equation of the hyperbola is

A) $4x^2-5y^2+30y-36=0$ B) $3x^2-7y^2+10x-57=0$

C) $5x^2-4y^2+30x-135=0$ D) $5x^2-3y^2-45=0$

Key. C

Sol. $\frac{SA}{AZ}=\frac{3}{2}$ where $A=$vertex, $S=$focus, $Z=$ foot of the perpendicular from focus on directrix.

21. The radius of the circle passing through the foci of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ and having its centre at (0, 3) is

A) $\sqrt{34}$ B) 5 C) 4 D) 7

Key. A

Sol. Foci $=(\pm5,0)$, radius = distance between (0, 3) and (5, 0) = $\sqrt{34}$

22. If $\frac{\cos x}{\sin ax}$ is a periodic function, then $\underset{m\to\infty}{Lt}\ \underset{n\to\infty}{Lt}\left(1+\cos^{2m} n!\pi a\right)$ is equal to

a) 0 b) 1 c) 2 d) -1

Key. C

Sol. a is rational so $\underset{n\to\infty}{Lt}\ 1+\cos^{2m} n!\pi a = 2$

23. If $f(x)=\begin{cases}[\cos\pi x] & ,x<1\\ |x-2| & ,1\le x<2\end{cases}$ ([.] denotes the greatest integer function) then $f(x)$ is

A) continuous and non-differentiable at $x=-1$ and $x=1$

B) continuous and differentiable at $x=0$

C) discontinuous at $x=1/2$ D) continuous but not differentiable at $x=2$

Key. C

Sol. $f(x)=\begin{cases} -1 & ,\frac{1}{2}<x<1 \\ 0 & ,0<x\le\frac{1}{2} \\ 1 & ,x=0 \\ 0 & ,-\frac{1}{2}\le x<0 \\ -1 & ,-\frac{3}{2}<x<-\frac{1}{2} \\ 2-x & ,1\le x<2 \end{cases}$ clearly discontinuous at $x=\frac{1}{2}$

24. The value of the integral $\int_{-\pi/4}^{\pi/4}\frac{1+\sin x}{\cos x\sqrt{\cos 2x}}dx$ is

a) 0 b) $\pi/4$ c) $\pi/2$ d) π

Key. D

Sol. $I=2\int_0^{\pi/4}\frac{dx}{\cos x\sqrt{\cos 2x}}=2\int_0^{\pi/4}\frac{\sec^2 x}{\sqrt{1-\tan^2 x}}=2\int_0^1\frac{dt}{\sqrt{1-t^2}}=\pi$

25. The area bounded by the curves y = x^2, y = [x+1], $x\le 1$and the y – axis, where[.] denotes the greatest integer not exceeding x, is

a) 2/3 b) 1/3 c) 1 d) 2

Key. A

Sol. Area required = $1-\int_0^1 ydx=1-\int_0^1 x^2dx=\frac{2}{3}$

26. The solution of the initial value problem

$(2\ln x)\frac{dy}{dx}+\frac{y}{x}=\frac{1}{y}\cos x, y>0, x>1 \; and \; y\left(\frac{3\pi}{2}\right)=0$is given by y =

a) $\sqrt{\frac{1-\sin x}{\ln x}}$ b) $\sqrt{\frac{1+\sin x}{\ln x}}$

c) $\sqrt{\frac{1-\cos x}{\ln x}}$ d) $\sqrt{\frac{1+\cos x}{\ln x}}$

Key. B

SOL. $2y\frac{dy}{dx}+y^2\left(\frac{1}{x\ln x}\right)=\frac{\cos x}{\ln x}$

$\Rightarrow\frac{dz}{dx}+\frac{z}{x\ln x}=\frac{\cos x}{\ln x}$ WHERE Z = Y^2

$\Rightarrow y^2\ln x=z\ln x=\sin x+c$ WHERE C = 1

$\Rightarrow y=\sqrt{\frac{1+\sin x}{\ln x}}\;(\because y>0)$

27. The number of ways of forming an arrangement of 4 letters from the letters of the word "IITJEE" is

a) 66 b) 96 c) 102 d) 180

Key. C

Sol. $^4C_4(4!)+\left(^2C_1\right)\frac{4!}{2!}\left(^3C_2\right)+\frac{4!}{(2!)^2}=102$

28. $If\left(1+2x+2x^2\right)^n=\sum_{r=0}^{2n}a_rx^r$, where n is an even positive integer, then the sum $a_0a_{2n}-a_1a_{2n-1}+a_2a_{2n-2}-+....+a_{2n}a_0$ is equal to

a) 2^n

b) 2^{n+1}

c) $^nC_{\frac{n}{2}}2^n$

d) $^nC_{\frac{n}{2}}2^{n+1}$

Key. C

Sol. The given sum is the coefficient of x^{2n} in $\left[\left(1+2x+2x^2\right)\left(1-2x+2x^2\right)\right]^n$ i.e in $\left(1+4x^4\right)^n=n_{C_{\frac{n}{2}}}.2^n$

29. Nine out of 10 urns contain 2 white balls and 2 black balls, while the other urn contains 5 white balls and 1 black ball. A ball drawn from a randomly chosen urn turns out to be white. The probability that the ball is drawn from the urn containing 5 white balls is

a) 1/4

b) 1/8

c) 3/16

d) 5/32

Key. D

Sol. Let A be the event of drawing a ball from one of the 9 urns and B be the event of drawing a ball from the urn containing 5 white and 1 black ball. If W is the event of drawing a white ball, then $P(B/W)=\frac{\left(\frac{1}{10}\right)\left(\frac{5}{6}\right)}{\left(\frac{9}{10}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{10}\right)\left(\frac{5}{6}\right)}=\frac{5}{32}$

30. The number of four digit numbers that can be formed using the digits 1,2,3,4,5,6 that are divisible by 3,when repetition of digits is allowed, is

a) $2^3\times3^2$

b) $2^3\times3^3$

c) $2^3\times3^4$

d) $2^4\times3^3$

Key. D

Sol. The first 3 digits can be filled in 6^3 ways. After filling the first 3 digits, the last digit can be filled using 1, 2, 3, 4, 5, 6 in 6 ways. But the six numbers so formed are consecutive integers out of which only two are divisible by 3. Hence the number of choices = $2\times6^3=2^4\times3^3$

31. The letters of the word "DRAWER" are arranged in alphabetical order. The number of arrangements that precede the word " REWARD" is

a) 241

b) 242

c) 247 d) 248

Key. C

Sol. The letters of the word 'DRAWER' in alphabetical order are A, D, E, R, R and W. The number of words that precede the word "REWARD" is $3\times\frac{5!}{2!}+2\times 4!+3(3!)+1=247$

32. In the expansion of $(1+x)^{2n}\left(\frac{x}{1-x}\right)^{-2n}$,where n is a positive integer, the term independent of x, is

a) $(-1)^{n/2}\ {}^{2n}C_n$ b) $(-1)^n\ {}^{2n}C_n$ c) ${}^{2n}C_n$ d) 1

Key. B

Sol. Term independent of x in $(1+x)^{2n}\left(1-\frac{1}{x}\right)^{2n}$

= coefficient of x^{2n} in $(1-x^2)^{2n}$

= Coefficient of x^{2n} in $\sum_{r=1}^{2n} 2n_{C_r}\left(-x^2\right)^r = (-1)^n\, 2n_{C_n}$

33. When the terms in the binomial expansion of $\left(\sqrt{x}+\frac{1}{2\sqrt[4]{x}}\right)^n$ are arranged in decreasing powers of x, the coefficients of the first three terms are in A.P. The number of terms in the expansion with integer powers of x is

a) 1 b) 2 c) 3 c) 4

Key. C

Sol. $1+\frac{n_{C_2}}{4}=2\left(\frac{n_{C_1}}{2}\right)$ gives n = 8

$\therefore T_{r+1}=8C_r\left(\frac{1}{2}\right)^r x^{(16-3r)/4} \Rightarrow 4/16-3r$ for r = 0, 4 and 8 only

34. $\int\left(\frac{2a+x}{a+x}\right)\sqrt{\frac{a-x}{a+x}}dx=$

a) $\sqrt{a^2-x^2}-2a\sqrt{\frac{a-x}{a+x}}+c$ b) $-\sqrt{a^2-x^2}-2a\sqrt{\frac{a-x}{a+x}}+c$

c) $\frac{1}{a}\tan^{-1}\frac{x}{a}+\ln\left|x+\sqrt{a^2-x^2}\right|+c$ d) $\frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|+\sin^{-1}\frac{x}{a}+c$

Key. A

Sol. Put $\theta=\cos^{-1}\frac{x}{a}(-a<x<a)$. Then $0<\theta<\pi$

and $I = -a\int \frac{(2+\cos\theta)(1-\cos\theta)}{1+\cos\theta} d\theta$

= $-a\int\left\{(1-\cos\theta) + \frac{1-\cos\theta}{1+\cos\theta}\right\} d\theta$

= $-a\left(2\tan\frac{\theta}{2} - \sin\theta\right) + c$

= $\sqrt{a^2 - x^2} - 2a\sqrt{\frac{a-x}{a+x}} + c$

35. $Let\ f(x) = \int_{x-1}^{x+1} \frac{dt}{1+t^8}$ .Then the function f

a) has no extremum

b) has maximum value for some $x \in (0,1)$

c) has minimum value for some $x \in (-1,0)$

d) has maximum value at x = 0

Key. D

Sol. $f(x) = \int_0^{x+1} \frac{dx}{1+x^8} - \int_0^{x-1} \frac{dx}{1+x^8}$

$\Rightarrow f'(x) = \frac{1}{1+(1+x)^8} - \frac{1}{1+(1-x)^8} = \frac{(1-x)^8 - (1+x)^8}{\{1+(1+x)^8\}\{1+(1-x)^8\}}$

$\therefore f'(x) = 0 \Leftrightarrow x = 0$. Also $x < 0 \Rightarrow f'(x) > 0$ and $x > 0 \Rightarrow f'(x) < 0$

$\therefore$ f has maximum value at x = 0

36. A curve passes through the point (0,1)and has the property that the slope of the curve at every point P is twice the y–coordinate of P . If the area bounded by the curve, the axes of coordinates and the line x = -t (t > 0) is A (t), then $\lim_{t\to\infty} A(t)$ =

a) $\frac{1}{2}$

b) $\frac{1}{3}$

c) $\frac{2}{3}$

d) 1

Key. A

Sol. $\frac{dy}{dx} = 2y \Rightarrow \ln|y| = 2x + c \Rightarrow y = ce^{2x}$ where c = 1

$\therefore A(t) = \int_{-t}^{0} e^{2x} dx = \frac{1}{2}\left(1 - e^{-2t}\right) \to \frac{1}{2}$ as $t \to \infty$

37. The solution of the initial value problem

$\left(x^2 + y^2\right)dx = 2xy\ dy, y(1) = 0\ is\ f(x) = x^2 - y^2$ where f(x) is

a) $\frac{1}{x}$

b) x

c) $\frac{1}{x^3}$

d) x^3

Key. B

Sol. The equation can be reduced to $x\frac{dv}{dx}=\frac{1+v^2}{2v}-v=\frac{1-v^2}{2v}\left(v=\frac{y}{x}\right)$

The solution is $ln\left|1-v^2\right|+\ln\left|cx\right|=0$ where c = 1. Therefore $x^2 - y^2 = x$

38. Four boys and four girls are randomly seated in 8 adjacent seats in a conference hall. It is found that all the girls are seated in 4 adjacent seats. The probability that the four boys are also seated in 4 adjacent seats is

a) 1/4

b) 1/2

c) 2/5

d) 3/4

Key. C

Sol. Probability = $\frac{2(4!)(4!)}{4!5!}=\frac{2}{5}$

39. If $\cos x=\tan y$, $\cos y=\tan z$, $\cos z=\tan x$ then the value of $\sin x$ is

(A) $\sin 36^0$

(B) $\cos 36^0$

(C) $2\sin 18^0$

(D) $2\cos 18^0$

Key. C

Sol. $\cos x=\tan y \Rightarrow \cos^2 x=\tan^2 y$

$=\sec^2 y-1=\cot^2 z-1=\operatorname{cosec}^2 z-2=\frac{1}{1-\cos^2 z}-2=\frac{1}{1-\tan^2 x}-2$

$=\frac{2\tan^2 x-1}{1-\tan^2 x}$

$\Rightarrow \cos^2 x=\frac{2\sin^2 x-\cos^2 x}{\cos^2 x-\sin^2 x}\quad \Rightarrow 1-\sin^2 x=\frac{3\sin^2 x-1}{1-2\sin^2 x}$

$\Rightarrow 1-2\sin^2 x-\sin^2 x+2\sin^4 x=3\sin^2 x-1$

$\Rightarrow 2\sin^4 x-6\sin^2 x+2=0$

$\Rightarrow \sin^4 x-3\sin^2 x+1=0$

$\sin x=\frac{\sqrt{5}-1}{2}=2\sin 18^0$

40. The expression $3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi-\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$ is equal to

(A) 0 (B) -1
(C) 1 (D) 3

Key. C

Sol. $3\left[\cos^4\alpha+\sin^4\alpha\right]-2\left[\cos^6\alpha+\sin^6\alpha\right]$

$=3\left[\left(\cos^2\alpha+\sin^2\alpha\right)^2-2\sin^2\alpha.\cos^2\alpha\right]-2\left[\left(\sin^2\alpha+\cos^2\alpha\right)^3-3\sin^2\alpha.\cos^2\alpha.\left(\sin^2\alpha+\cos^2\alpha\right)\right]$

$=3\left[1-2\sin^2\alpha.\cos^2\alpha\right]-2\left[1-3\sin^2\alpha.\cos^2\alpha\right]$

$=3-2=1$

41. The number of pairs (x,y) satisfying the equations $\sin x+\sin y=\sin(x+y)$ and $|x|+|y|=1$ is

(A) 0 (B) 2
(C) 4 (D) 6

Key. D

Sol. $\sin x+\sin y=\sin(x+y)$

$\Rightarrow 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x+y}{2}\right).$

$\Rightarrow 2\sin\left(\frac{x+y}{2}\right)\left[\cos\left(\frac{x-y}{2}\right)-\cos\left(\frac{x+y}{2}\right)\right]=0$

$\Rightarrow 2\sin\left(\frac{x+y}{2}\right)\left[2\sin\frac{x}{2}.\sin\frac{y}{2}\right]=0$

So, $\sin\left(\frac{x+y}{2}\right)=0$ or, $\sin\frac{x}{2}=0$ or, $\sin\frac{y}{2}=0$

Now, $|x|+|y|=1$

So, $|x|\le 1,\ |y|\le 1$

The possibilities are $x=0$ or, $y=0$ or, $x+y=0$

When $x=0$, $y=\pm 1$

When $y=0$, $x=\pm 1$

When $x+y=0$ then $2|x|=1$

$\Rightarrow x=\pm\frac{1}{2}$

So, the solutions are $(0,\pm 1)$, $(\pm 1,0)$, $\left(\frac{1}{2},-\frac{1}{2}\right)$, $\left(-\frac{1}{2},\frac{1}{2}\right)$.

Clearly, there are 6 pairs.

42. Let

$0^0 < \theta < 45^0$, $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$, $t_4 = (\cot\theta)^{\cot\theta}$ then

(A) $t_1 < t_2 < t_3 < t_4$
(B) $t_4 > t_3 > t_1 > t_2$
(C) $t_4 > t_1 > t_2 > t_3$
(D) None of these

Key. B

Sol. $0 < \tan\theta < 1 < \cot\theta$

Since, $\cot\theta > \tan\theta$

$(\cot\theta)^{\tan\theta} > (\tan\theta)^{\tan\theta}$

$\therefore t_3 > t_1$

Again $\cot\theta > 1$

So, $(\cot\theta)^{\cot\theta} > (\cot\theta)^{\tan\theta}$

$\Rightarrow t_4 > t_3$

But $0 < \tan\theta < 1$

So; $(\tan\theta)^{\tan\theta} > (\tan\theta)^{\cot\theta}$

$\Rightarrow t_1 > t_2$

So, $t_4 > t_3 > t_1 > t_2$

43. If $\cos 25^0 + \sin 25^0 = K$, then $\cos 50^0$ is equal to

(A) $K\sqrt{2-K^2}$
(B) $-\sqrt{2-K^2}$
(C) $\sqrt{2-K^2}$
(D) $-\sqrt{K^2-2}$

Key. A

Sol. $\cos 25^0 + \sin 25^0 = K$

Squaring,

$1 + \sin 50^0 = K^2$

$\sin 50^0 = K^2 - 1$

$\cos 50^0 = \sqrt{1-\sin^2 50^0}$

$= \sqrt{1-(K^2-1)^2}$

$= \sqrt{2K^2 - K^4}$

$= K\sqrt{2-K^2}$

44. $\cos 10^0.\cos 20^0.\cos 40^0$ is equal to

(A) $\frac{1}{4}\tan 10^0$
(B) $\frac{1}{8}\cot 10^0$
(C) $\frac{1}{8}\operatorname{cosec} 10^0$
(D) $\frac{1}{8}\sec 10^0$

Key. B

Sol. $\cos 10^0.\cos 20^0.\cos 40^0$

$= \frac{1}{2\sin 10^0}.\sin 20^0.\cos 20^0.\cos 40^0$

$= \frac{1}{4\sin 10^0}.\sin 40^0.\cos 40^0$

$= \frac{1.\sin 80^0}{8.\sin 10^0} = \frac{\sin\left(90^0 - 10^0\right)}{8.\sin 10^0}$

$= \frac{\cos 10^0}{8.\sin 10^0}$

$= \frac{1}{8}\cot 10^0$

45. Which of the following is rational ?

(A) $\sin 18^0.\cos 18^0$ (B) $2\sin^2 15^0$

(C) $\sin 15^0.\cos 75^0$ (D) $\sin 15^0.\sin 75^0$

Key. D

Sol. $\sin 18^0.\cos 18^0 = \frac{1}{2}\sin 36^0 = \frac{1}{8}\sqrt{10 - 2\sqrt{5}}$

$2\sin^2 15^0 = 2\times\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2 = \frac{4-2\sqrt{3}}{4} = \frac{2-\sqrt{3}}{2}$

$\sin 15^0.\cos 75^0 = \sin^2 15^0 = \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2 = \frac{4-2\sqrt{3}}{8} = \frac{2-\sqrt{3}}{4}$

$\sin 15^0.\sin 75^0 = \sin 15^0.\cos 15^0 = \frac{1}{2}\sin 30^0 = \frac{1}{4}$

46. The value of cot $7\frac{1}{2}^0$ is

(A) $\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$ (B) $\sqrt{2}+\sqrt{3}+\sqrt{5}+\sqrt{6}$

(C) $\sqrt{3}+\sqrt{4}+\sqrt{5}+\sqrt{6}$ (D) $\sqrt{2}+\sqrt{3}+\sqrt{6}$

Key. A

Sol. $\cot 7\frac{1}{2}^0 = \frac{\cos 7\frac{1}{2}^0}{\sin 7\frac{1}{2}^0} = \frac{2\cos^2 7\frac{1}{2}^0}{\sin 15^0} = \frac{1+\cos 15^0}{\sin 15^0}$

Put $\sin 15^0 = \frac{\sqrt{3}-1}{2\sqrt{2}}$, $\cos 15^0 = \frac{\sqrt{3}+1}{2\sqrt{2}}$

We get $\cot 7\frac{1}{2}^0 = \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$

47. The value of $\sec\frac{\pi}{7}.\sec\frac{2\pi}{7}.\sec\frac{3\pi}{7}$ is

(A) –8 (B) 8
(C) –4 (D) 4

Key. B

Sol. $\sec\frac{\pi}{7}.\sec\frac{2\pi}{7}.\sec\frac{3\pi}{7} = \frac{1}{-\cos\frac{\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{4\pi}{7}}$

$$= \frac{-2\sin\frac{\pi}{7}}{\sin\frac{2\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{4\pi}{7}}$$

$$= \frac{-4\sin\frac{\pi}{7}}{\sin\frac{4\pi}{7}.\cos\frac{4\pi}{7}}$$

$$= \frac{-8\sin\frac{\pi}{7}}{\sin\left(\pi+\frac{\pi}{7}\right)} = \frac{-8\sin\frac{\pi}{7}}{-\sin\frac{\pi}{7}} = 8$$

48. The number of solutions of the equation $\sec x = \frac{\pi}{2}$ in the interval $\left(-\frac{\pi}{2},\frac{\pi}{4}\right)$ is

(A) 0 (B) 1
(C) 2 (D) 4

Key. B

Sol. $\sec x = \frac{\pi}{2}$

The equation has only one solution in $\left(-\frac{\pi}{2},0\right)$ and no solution in $\left(0,\frac{\pi}{4}\right)$.

Because for $0 < x < \frac{\pi}{4}$

$1 < \sec x < \sqrt{2}$

49. If $\sin x + \operatorname{cosec} x + \tan y + \cot y = 4$ where x and $y \in \left[0,\frac{\pi}{2}\right]$, then $\tan\frac{y}{2}$ is a root of the equation

(A) $\alpha^2 + 2\alpha + 1 = 0$ (B) $\alpha^2 + 2\alpha - 1 = 0$
(C) $2\alpha^2 - 2\alpha - 1 = 0$ (D) $\alpha^2 - \alpha - 1 = 0$

Key. B

Sol. $\sin x + \operatorname{cosec} x \geq 2$ (using $A.M \geq G.M$)

Also $\tan y + \cot y \geq 2$

So, $\sin x + \operatorname{cosec} x + \tan y + \cot y = 4$ is possible when $x = \frac{\pi}{2}$ and $y = \frac{\pi}{4}$

$$\tan y = \tan\frac{\pi}{4} = 1 \quad \Rightarrow \frac{2\tan\frac{y}{2}}{1-\tan^2\frac{y}{2}} = 1 \quad \Rightarrow \tan^2\frac{y}{2} + 2\tan\frac{y}{2} - 1 = 0$$

So, $\tan\frac{y}{2}$ is a root of the equation $\alpha^2 + 2\alpha - 1 = 0$

50. The number of solutions of the equation $|\cos x| = \sin x$, $0 < x < 4\pi$ is

(A) 2 (B) 4

(C)6 (D) 8

Key. B

Sol. The graph of $f(x) = |\cos x|$ and $f(x) = \sin x$ intersect at 4 points i.e $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$

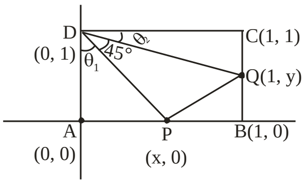
Mixed Questions

Integer Answer Type

1. ABCD is a square of side length 1 unit. P and Q are points on AB and BC such that $\angle PDQ = 45°$. Find the perimeter of ΔPBQ.

Key. 2

Sol.



$\tan\theta_1$ = x and $\tan\theta_2 = 1 - y$

Since, $\theta_1 + \theta_2$ = 45°

$\Rightarrow$ $\frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2}$ = 1

$\Rightarrow$ $\frac{x + (1-y)}{1 - x(1-y)}$ = 1 $\Rightarrow$ $y = \frac{2x}{1+x}$ ...(i)

Now, Perimeter=$1 - x + y + \sqrt{(1-x)^2 + y^2}$

By using (i), we get

Perimeter=2

2. For a twice differentiable function $f(x)$, $g(x)$ is defined as $g(x) = f'(x)^2 + f''(x) f(x)$ on $[a,e]$. If for $a<b<c<d<e$, $f(a)=0$, $f(b)=2$,
$f(c)=-1$, $f(d)=2$, $f(e)=0$ then find the minimum number of zeros of g(x).

Key. 6

Sol. $g\ x \ \square\ f'\ x\ ^2 \ \square\ f''\ x\ f\ x \ \square\ \frac{d}{dx}\ f\ x\ f'\ x$

Let $h\ x \ \square\ f\ x\ f'\ x$

Then, $f\ x\ \square\ 0$ has four roots namely $a, \square, \square, e$

where $b \square \square \square c$ and $c \square \square \square d$.

And $f'\ x\ \square\ 0$ at three points k_1, k_2, k_3 where

$a \square k_1 \square \square, \square \square k_2 \square \square, \square \square k_3 \square e$

[$\because$ Between any two roots of a polynomial function $f\ x\ \square\ 0$ there lies atleast one root of $f'\ x\ \square\ 0$]

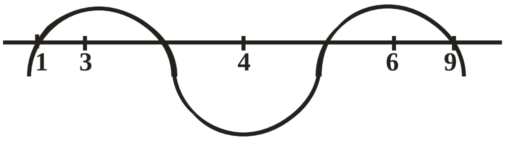
There are atleast 7 roots of $f\ x\ .f'\ x\ \square\ 0$

∴ There are atleast 6 roots of $\frac{d}{dx}\left(f(x)f'(x)\right) = 0$ i.e. of $g(x) = 0$

3. If f(x) is twice differentiable function such that f(1) = 0, f(3) = 2, f(4) = –5, f(6) = 2, f(9) = 0 then the minimum number of zero's of $g'(x) = x^2 f''(x) + 2xf'(x) + f''(x)$ in the interval (1,9) is

Key. 2

Sol. $f'(x) = 0$ has minimum three solution between (1,9)



$f''(x) = 0$ has minimum two solution between (1,9)

Given equations $\frac{d}{dx}\left\{(x^2+1)f'(x)\right\} = 0$

4. The number of integral solution for the equation x + 2y = 2xy is

Key. 2

Sol. $2y = \frac{x}{x-1}$

Since y is an integer 2y is even such that x and x – 1 are consecutive integers and hence the only values of x that satisfy are 2 and 0.

5. If $\int_0^{2\pi} \frac{x\sin^8 x}{\sin^8 x + \cos^8 x}\,dx = k\left(\frac{\pi}{2}\right)^2$, then the value of k is

Key. 4

Sol. $I = \int_0^{2\pi} \frac{x\sin^8 x}{\sin^8 x + \cos^8 x}\,dx = \int_0^{2\pi} \frac{(2\pi - x)\sin^8 x}{\sin^8 x + \cos^8 x}$

$\Rightarrow \quad I = \int_0^{2\pi} \frac{\pi\sin^8 x}{\sin^8 x + \cos^8 x} = 4\int_0^{\frac{\pi}{2}} \frac{\pi\sin^8 x}{\sin^8 x + \cos^8 x}dx = 4\int_0^{\pi/2} \frac{\pi\cos^8 x}{\cos^8 x + \sin^8 x}dx$

$\therefore \quad 2I = 4\pi\int_0^{\pi/2} \frac{\sin^8 x + \cos^8 x}{\cos^8 x + \sin^8 x}dx = 4\pi \times \frac{\pi}{2} = 2\pi^2$

∴ I = π^2. Hence K = 4.

6. The value of the integral $\int_{\frac{-3\pi}{4}}^{\frac{5\pi}{4}} \frac{\cos x + \sin x}{1 + e^{x-\frac{\pi}{4}}}\,dx$ is

Key. 0

Sol. Let $I = \int_{\frac{-3\pi}{4}}^{\frac{5\pi}{4}} \frac{\sin x + \cos x}{e^{x-\frac{\pi}{4}} + 1}dx = \int_{\frac{-3\pi}{4}}^{\frac{5\pi}{4}} \frac{\sqrt{2}\cos\left(x - \frac{\pi}{4}\right)dx}{e^{x-\frac{\pi}{4}} + 1}$. Put $x - \frac{\pi}{4} = t$ and get $I = 0$

7. If $\int \frac{(\sqrt{x})^5}{(\sqrt{x})^7 + x^6} dx = a\,ln\left(\frac{x^K}{x^K+1}\right) + C$, then the value of 2k is

Key. 5

Sol. Let $I = \int \frac{x^{5/2}}{x^{7/2} + x^{12/2}} dx$, put $\sqrt{x} = t$

$\Rightarrow$ I = $2\int \frac{dt}{t^6\left(1+\frac{1}{t^5}\right)}$, put $1+\frac{1}{t^5} = y \Rightarrow I = \frac{2}{5}\ln\left(\frac{x^{5/2}}{x^{5/2}+1}\right) + C$

$\therefore$ a = $\frac{2}{5}$ and k = $\frac{5}{2}$ $\Rightarrow$ 2k = 5.

8. If $\int_0^x \log\sin\theta d\theta + \int_{-x}^x \log(\cos\theta/2 + \sin\theta/2)\, d\theta + \int_{-x}^x \log(\cos\theta + \sin\theta)d\theta$ +

$\int_{-x}^x \log(\cos 2\theta + \sin 2\theta)d\theta + 3x\log 2 = 0$, then $\int_0^x \log\sin 8\theta\, d\theta$ is

Key. 0

Sol. $\int_{-x}^x \log(\cos\theta/2 + \sin\theta/2)\, d\theta = \int_0^x (\log(\cos\theta/2 + \sin\theta/2) + \log(\cos\theta/2 - \sin\theta/2)d\theta$

$= \int_0^x \log\cos\theta d\theta$

So the given definite integral is = $\int_0^x (\log\sin\theta + \log\cos\theta + \log\cos 2\theta + \log\cos 4\theta) + 3x\log 2 =$

0

$\Rightarrow \int_0^x \log\sin 8\theta - 3x\log 2 + 3x\log 2 = 0 \Rightarrow \int_0^x \log\sin 8\theta d\theta = 0$

9. If x = $\int_0^y \frac{dt}{\sqrt{1+9t^2}}$ and $\frac{d^2y}{dx^2}$ = ay, then 'a' is

Key. 9

Sol. x = $\int_0^y \frac{dt}{\sqrt{1+9t^2}}$

$\Rightarrow \frac{dx}{dy} = \frac{1}{\sqrt{1+9y^2}}$

$\Rightarrow \frac{dy}{dx} = \sqrt{1+9y^2}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{18y}{2\sqrt{1+9y^2}}.\frac{dy}{dx}$ = 9y

$\Rightarrow$ a = 9.

10. If $\int x.\frac{\ln(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}dx$ = a $\sqrt{1+x^2}\ln(x+\sqrt{1+x^2})$ + bx + c, then the value of 'a' is

Key. 1

Sol. I = $\int x.\frac{\ln(x+\sqrt{x^2+1})}{\sqrt{x^2+1}}dx$

Put t = $\sqrt{x^2+1} \Rightarrow \frac{dt}{dx} = \frac{x}{\sqrt{x^2+1}}$

$\Rightarrow$ I = $\int \ln(t+\sqrt{t^2-1})\ dt$

I = ln (t + $\sqrt{t^2-1}$) . t - $\int \frac{1+\frac{1}{\sqrt{t^2-1}}}{t+\sqrt{t^2-1}} t\,dt$

I = t.ln (t + $\sqrt{t^2-1}$) - $\frac{1}{2}\int \frac{2t}{\sqrt{t^2-1}}$ dt

I = $\sqrt{1+x^2}$. ln (x + $\sqrt{1+x^2}$) - x + c

$\Rightarrow$ a = 1.

11. If f′(x) = 3x² sin $\frac{1}{x}$ - xcos $\frac{1}{x}$, x ≠ 0, f(0) = 0 then the value of $f\left(\frac{1}{\pi}\right)$ is

Key. 0

Sol. f′(x) = 3x² sin $\frac{1}{x}$ - x cos $\frac{1}{x}$

$\Rightarrow$ f(x) = $\int(3x^2 \sin\frac{1}{x} - x\cos\frac{1}{x})\,dx$

f(x) = sin $\frac{1}{x}.x^3 - \int \cos\frac{1}{x}\left(-\frac{1}{x^2}x^3\right)dx - \int x\cos\frac{1}{x}dx$

f(x) = x³ sin $\frac{1}{x} + c$

f(0) = 0 + c = 0 $\Rightarrow$ c = 0

$\therefore$ f(1/π) = $\frac{1}{\pi^3}$. sinπ + 0 = 0.

12. The value of $\int_{-1}^{3}\{|x-2|+[x]\}dx$ (where [x] stands for greatest integer less than or equal to x), is

Key. 7

Sol. $\int_{-1}^{3}\{|x-2|+[x]\}dx$

$= \int_{-1}^{0}\{|x-2|+[x]\}dx + \int_{0}^{1}\{|x-2|+[x]\}dx + \int_{1}^{2}\{|x-2|+[x]\}dx + \int_{2}^{3}\{|x-2|+[x]\}dx$

$= \int_{-1}^{0}(2-x-1)dx + \int_{0}^{1}(2-x+0)dx + \int_{1}^{2}(2-x+1)dx + \int_{2}^{3}(x-2+2)dx$

$= \left[x-\frac{x^2}{2}\right]_{-1}^{0} + \left[2x-\frac{x^2}{2}\right]_{0}^{1} + \left[3x-\frac{x^2}{2}\right]_{1}^{2} + \left[\frac{x^2}{2}\right]_{2}^{3}$

= - (-1 - $\frac{1}{2}$) + $\left(2-\frac{1}{2}\right)$ + (6 - 2) - (3 - (1/2)) + (9/2) - 2 = 7.

13. If the line $3x-4y-k=0$ touches the circle $x^2+y^2-4x-8y-5=0$ at (a, b) then the positive integral value of $\frac{k+a+b}{5}=$

Key. 4

Sol. $r=5, \pm 5=\frac{(3\times 2)+(-4\times 4)-k}{\sqrt{9+16}} \Rightarrow k=15 \ or \ -35$

Take $k=15$

Now equation of the tangent at (a, b) is $xa+yb-2(x+a)-4(y-b)-5=0$

If ti represents the given line $3x-4y-k=0$

then $\frac{a-2}{3}=\frac{b-4}{-4}=\frac{2a+4b+5}{k}=\lambda$

on simplification $\lambda=1 \Rightarrow a=5, b=0$ and $k+a+b=20$

14. The difference between the radii of the largest and the smallest circles which have their centres on the circumference of the circle $x^2+y^2+2x+4y-4=0$ and passes through the point (a,b) lying outside the given circle, is

Key. 6

Sol. The given circle is $(x+1)^2+(y+2)^2=9$

The points on the circle which are nearest and farthest to the point P(a, b) are Q and R respectively. PQ, PR are normals to the circle. Hence QR = 6.

15. If $p(x)=ax^2+bx \ and \ q(x)=lx^2+mx+n$ with $p(1)=q(1); p(2)-q(2)=1$ and $p(3)-q(3)=4$, then $p(4)-q(4)$ is

Key. 9

Sol. $p(x)=ax^2+bx \ and \ q(x)=lx^2+mx+n$

It is given $p(1)-q(1)=0$

$\Rightarrow (a+b)-(l+m+n)=0$ ....(1)

It is given $p(2)-q(2)=1$

$\Rightarrow (4a+2b)-(4l+2m+n)=1$...........(2)

It is given $p(3)-q(3)=4$

$\Rightarrow (9a+3b)-(9l+3m+n)=4$..........(3)

The value of $p(4)-q(4)=(16a+4b)-(16l+4m+n)-(4)$

$\Rightarrow p(4)-q(4)=9$

16. The number of roots of the equation $2^x + 2^{x-1} + 2^{x-2} = 7^x + 7^{x-1} + 7^{x-2}$ is

Key. 1

Sol. $2^x + 2^{x-1} + 2^{x-2} = 7^x + 7^{x-1} + 7^{x-2}$

$$\Rightarrow 2^x\left(1+\frac{1}{2}+\frac{1}{4}\right)=7^x\left(1+\frac{1}{7}+\frac{1}{7^2}\right)$$

$$\Rightarrow 2^x\left(\frac{7}{4}\right)=7^x\left(\frac{57}{49}\right)$$

$$\Rightarrow \left(\frac{7}{2}\right)^{x-2}=\left(\frac{7}{57}\right)$$

$$\Rightarrow (x-2)\log\left(\frac{7}{2}\right)=\log\left(\frac{7}{57}\right)$$

$$\Rightarrow x = 2+\frac{\log\left(\frac{7}{57}\right)}{\log\left(\frac{7}{2}\right)}$$

Number of roots of the equation is 1.

17. If the lengths of the sides of a right triangle ABC right angled at C are in A.P., find $5(\sin A + \sin B)$.

Key. 7

Sol. here $\angle C = 90^0$, $A + B = 90^0$

$c^2 = a^2 + b^2$ & $2b = a + c$

Since $c = 2b - a$ & $c^2 = a^2 + b^2$

$$\Rightarrow (2b-a)^2 = a^2+b^2 \text{ or } \frac{b}{a}=\frac{4}{3}$$

$$\Rightarrow \frac{\sin B}{\sin A}=\frac{4}{3}\ or\ \frac{\sin B+\sin A}{\sin B-\sin A}=\frac{7}{1} \text{ or } \cot\frac{B-A}{2}=\frac{1}{7}\Rightarrow \cos\frac{A-B}{2}=\frac{7}{5\sqrt{2}}$$

Also $5(\sin A+\sin B)=5\sqrt{2}\cos\frac{A-B}{2}=7$

18. If $\log_x y, \log_z x, \log_y z$ are in G.P., $xyz = 64$ and x^3, y^3, z^3 are in A.P., find $\frac{x+y+z}{3} =$.

Key. 4

Sol. Conceptual

19. If the equation $z^2 - (3+i)z + m + 2i = 0,\ m \in R$ has a real root, then the value of $\frac{9m}{2}$ is

Key. 9

Sol. Conceptual

20. If $1, \alpha, \alpha^2, \alpha^3$ *and* α^4 are the roots of $z^5 = 1$, then the value of $3 + \alpha + \alpha^2 + \frac{1}{\alpha^2} + \frac{1}{\alpha}$ is

Key. 2

Sol. $1 + \alpha + \alpha^2 + \frac{1}{\alpha^2} + \frac{1}{\alpha} = 1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 = 0$

Hence $3 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 = 2$