Matrices & Determents

Integer Answer Type

1. If the integers a, b, c inorder are in A.P., lying between 1 and 9 and a23, b53, and c83 are three-digit numbers, then the value of the determinant $\begin{vmatrix} 2 & 5 & 8 \\ a23 & b53 & c83 \\ a & b & c \end{vmatrix}$ is...

Key. 0

Sol. We have,

$$\begin{vmatrix} 2 & 5 & 8 \\ a23 & b53 & c83 \\ a & b & c \end{vmatrix} = \begin{vmatrix} 2 & 5 & 6 \\ 100a+20+3 & 100b+50+3 & 100c+80+3 \\ a & b & c \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 5 & 8 \\ 100a & 100b & 100c \\ a & b & c \end{vmatrix} + \begin{vmatrix} 2 & 5 & 8 \\ 20 & 50 & 80 \\ a & b & c \end{vmatrix} + \begin{vmatrix} 2 & 5 & 8 \\ 3 & 3 & 3 \\ a & b & c \end{vmatrix}$$

$$= 100\begin{vmatrix} 2 & 5 & 8 \\ a & b & c \\ a & b & c \end{vmatrix} + 10\begin{vmatrix} 2 & 5 & 8 \\ 2 & 5 & 8 \\ a & b & c \end{vmatrix} + 3\begin{vmatrix} 2 & 5 & 8 \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix}$$

$$= 0 + 0 + 3\begin{vmatrix} 2 & 3 & 6 \\ 1 & 0 & 0 \\ a & b-a & c-a \end{vmatrix}$$

(Applying $C_2 \to C_2 - C_1,\ C_3 \to C_3 - C_1$)

$$= -3\left[3(c-a) - 6(b-a)\right] \quad = -9\left[c - a - 2b + 2a\right]$$

$$= -9(a - 2b + c) = 0 \qquad [\because \text{ a, b, c are in A.P., } \therefore 2b = a + c]$$

2. A be set of 3 × 3 matrices formed by entries 0, –1, and 1 only. Also each of 1, -1, 0 occurs exactly three times in each matrix. The number of symmetric matrices with trace (A) = 0 is k, then $\frac{k}{6} = ...$

Key. 6

Sol. For non-diagonal entries, we required even no. of 1, even no. of –1 and even no. of 0, for diagonal three entries are remained, –1, 0, 1. So no. of cases in which trace = 0 are 3! And no. of symmetric matrices for each arrangement of 1, –1, 0 in diagonal = 3!

Total such matrices = 3! × 3! = 36

3. Let A_n, ($n \in N$) be a matrix of order $(2n - 1) \times (2n - 1)$, such that $a_{ij} = 0\ \forall\ i \neq j$ and $a_{ij} = n^2 + i + 1 - 2n\ \forall\ i = j$ where a_{ij} denotes the element of i^{th} row and j^{th} column of A_n.

Let T_n = $(-1)^n \times$ (sum of all the elements of A_n). Find the value of $\left[\dfrac{\sum_{n=1}^{102} T_n}{520200}\right]$, where [.] represents the greatest integer function.

Ans: 2

Hint $a_{ij} = 0 \ \forall \ i \neq j$ and $a_{ij} = (n-1)^2 + i \ \forall \ i = j$

Sum of all the element of $A_n = \sum_{i=1}^{2n-1}[(n-1)^2 + i]$

$= (2n-1)(n-1)^2 + (2n-1)n = 2n^3 - 3n^2 + 3n - 1 = n^3 + (n-1)^3$

So, $T_n = (-1)^n [n^3 + (n-1)^3] = (-1)^n n^3 - (-1)^{n-1}(n-1)^3 = V_n - V_{n-1}$

$$\Rightarrow \sum_{n=1}^{102} T_n = \sum_{n=1}^{102}(V_n - V_{n-1}) = V_{102} - V_0 = (102)^3$$

$$\left[\frac{\sum_{n=1}^{102} T_n}{520200}\right] = 2.$$

4. Find the value of $f\left(\frac{\pi}{6}\right)$, where $f(\theta) = \begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta & -\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$

Key. 1

Sol. Applying $C_1 \to C_1 - \sin\theta C_3$ and $C_2 \to C_2 + \cos\theta C_3$, we get

$$f(\theta) = \begin{vmatrix} 1 & 0 & -\sin\theta \\ 0 & 1 & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$$

Applying $R_3 \to R_3 - \sin\theta R_1 + \cos\theta R_2$, we get

$$f(\theta) = \begin{vmatrix} 1 & 0 & -\sin\theta \\ 0 & 1 & \cos\theta \\ \sin\theta & 0 & \sin^2\theta + \cos^2\theta \end{vmatrix} = 1$$

Thus, $f\left(\frac{\pi}{6}\right) = 1$

5. $\det P = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$, where '$P$' is an orthogonal matrix. Then the value of $|a+b+c|$ is

Key. 1

Sol.
$$PP^T = \begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix}\begin{bmatrix} a & c & b \\ b & a & c \\ c & b & a \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2+b^2+c^2 & ab+bc+ca & ab+bc+ca \\ ab+bc+ca & a^2+b^2+c^2 & ab+bc+ca \\ ab+bc+ca & ab+bc+ca & a^2+b^2+c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

6. If the adjoint of a 3 × 3 matrix P is $\begin{bmatrix} 1 & 2 & 1 \\ 4 & 1 & 1 \\ 4 & 7 & 3 \end{bmatrix}$, then the sum of squares of possible values of determinant of P is

Key. 8

Sol. Adj. P $= \begin{bmatrix} 1 & 2 & 1 \\ 4 & 1 & 1 \\ 4 & 7 & 3 \end{bmatrix}$

$|\text{adj. P}| = 4$

$\Rightarrow |P|^2 = 4 \Rightarrow |P| = \pm 2$

7. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then det($A^{2005}$) equals to

Key. 1

Sol. $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$

$A^3 = A^2A = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 7 & -12 \\ 3 & -5 \end{bmatrix}$

Observing A, A^2, A^3 we can conclude that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

det (A^n) = $\begin{vmatrix} 1+2n & -4n \\ n & 1-2n \end{vmatrix} = 1-4n^2+4n^2 = 1$

$\therefore$ det (A^{2005}) = 1

8. If x, y, z are cube roots of unity and

$D = \begin{vmatrix} x^2+y^2 & z^2 & z^2 \\ x^2 & y^2+z^2 & x^2 \\ y^2 & y^2 & z^2+x^2 \end{vmatrix}$, then the real part of D is

Key. 4

Sol. An applying $R_1 \to R_1 - R_2 - R_3$

$$D = \begin{vmatrix} 0 & -2y^2 & -2x^2 \\ x^2 & y^2+z^2 & x^2 \\ y^2 & y^2 & z^2+x^2 \end{vmatrix}$$

$$= -2\begin{vmatrix} 0 & y^2 & x^2 \\ x^2 & z^2 & 0 \\ y^2 & 0 & z^2 \end{vmatrix}$$

$= 4x^2y^2z^2 = 4\ (1.w.w.^2)^2 = 4$

9. Find the coefficient of x in the determinant $\begin{vmatrix} (1+x)^{a_1b_1} & (1+x)^{a_1b_2} & (1+x)^{a_1b_2} \\ (1+x)^{a_2b_1} & (1+x)^{a_2b_2} & (1+x)^{a_2b_3} \\ (1+x)^{a_3b_1} & (1+x)^{a_3b_2} & (1+x)^{a_3b_3} \end{vmatrix}$, whee $a_1, b_1 \in N$

Ans. $\lambda_1 = 0$

Sol. Let $\begin{vmatrix} (1+x)^{a_2b_1} & (1+x)^{a_1b_2} & (1+x)^{a_1b_2} \\ (1+x)^{a_2b_1} & (1+x)^{a_2b_2} & (1+x)^{a_2b_2} \\ (1+x)^{a_2b_2} & (1+x)^{a_2b_2} & (1+x)^{a_2b_2} \end{vmatrix} = \lambda_0 + \lambda_1 x + \lambda_2 x^2 + \lambda_3 x^3 + ...$

For λ_1 differentiate w.r.t. x and put x = 0

so $\lambda_1 = 0$

10. If if $(x) = \begin{vmatrix} \cos(x+\alpha) & \cos(x+\beta) & \cos(x+\gamma) \\ \sin(x+\alpha) & \sin(x+\beta) & \sin(x+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix}$ and f(2) = 6, then find $\sum_{r=1}^{25} f(r)$

Ans. 150

Sol. Clearly $f'(x) = 0$

$\therefore \quad f(x) = c = 5$

$\therefore \quad \sum_{r=1}^{25} f(r) = \sum_{r=1}^{25} 6 = 150$ Ans.

11. Let $f(x) = \begin{vmatrix} x & 1 & 1 \\ \sin 2\pi x & 2x^2 & 1 \\ x^3 & 3x^4 & 1 \end{vmatrix}$. If f(x) be an odd function and its odd values is equal g(x), then find the value of λ. If $\lambda f(1) g(1) = 4$

Ans. $\lambda = 1$

Sol. f(- x) = - f(x) = g(x)

$\therefore \quad f(x).g(x) = -(f(x))^2$

or $f(1)g(1) = -(f(1))^2 = -\begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 3 & 1 \end{vmatrix} = -4 \Rightarrow \quad \lambda = 1$ Ans.

12. If f(x) satisfies the equation $\begin{vmatrix} f(x+1) & f(x+8) & f(x+1) \\ 1 & 2 & -5 \\ 2 & 3 & \lambda \end{vmatrix} = 0$ for all real x. If f is periodic with period 7, then find the value of $|\lambda|$

Ans. 4

Sol. On solving we get

$(2\lambda+15)f(x+1)-(\lambda+10)f(x+8)-f(x+1)=0$

$(2\lambda+14)f(x+1)=(\lambda+10)f(x+8)$

Since f is periodic with period 7

$\therefore$ f(x + 1) = f(x + 8)

$\Rightarrow \quad 2\lambda+14=\lambda+10 \quad \Rightarrow \quad |\lambda|=4$ Ans.

13. If $A=\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$ and B = (adj A) and C = 5A, then find the value of $\frac{|\text{adj } B|}{|C|}$.

Ans. 1

Sol. $\frac{|\text{adj } B|}{|C|}=\frac{|\text{adj}(\text{adj} A)|}{|5A|}=\frac{|A|^{(3-1)^2}}{5^3|A|}=\frac{|A|^3}{125}$

Now |A| = 5

$\therefore \quad \frac{|\text{adj } B|}{|C|}=1$ Ans.

14. If $A=\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$ and $\phi(x)=(1+x)(1-x)^{-1}$, then prove that $\phi(A)=-A$

Ans. $\phi(A)=(I+A)(I-A)^{-1}=\begin{pmatrix} 2 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -\frac{1}{2} & 0 \end{pmatrix}=\begin{pmatrix} -1 & -2 \\ -1 & -1 \end{pmatrix}=-A$

Sol. $I+A\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}+\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}=\begin{pmatrix} 2 & 2 \\ 1 & 2 \end{pmatrix}$ and $I-A=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}-\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}=\begin{pmatrix} 0 & -2 \\ -1 & 0 \end{pmatrix}$

Now, $|I-A|=\begin{vmatrix} 0 & -2 \\ -1 & 0 \end{vmatrix}=0-2=-2$

adj $(I-A)=\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$

$(I-A)^{-1}=\begin{pmatrix} 0 & -1 \\ -\frac{1}{2} & 0 \end{pmatrix}$

$\therefore \quad \phi(A)=(I+A)(I-A)^{-1}=\begin{pmatrix} 2 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -\frac{1}{2} & 0 \end{pmatrix}=\begin{pmatrix} -1 & -2 \\ -1 & -1 \end{pmatrix}=-A$

15. If $A\begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$, $abc=1$, $A'A=1$, then find the value of $a^3+b^3+c^3$

Ans.

Sol. $A'A = I$

$\therefore \quad |A'A| = |I| \quad \Rightarrow \quad |A| = \pm 1$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \pm 1$$

$\Rightarrow \quad 3abc - a^3 - b^3 - c^3 = \pm 1 \quad \Rightarrow \quad a^3 + b^3 + c^3 = 4$ and 2

16. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $A^2 = 8A + KI_2$, then find the value of $|k|$

Ans. 7

Sol. Here $|A - \lambda| = 0$

$$\begin{vmatrix} 1-\lambda & 0 \\ -1 & 7-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (1-\lambda)(7-\lambda) = 0 \quad \Rightarrow \quad \lambda^2 - 8\lambda + 7 = 0$

$\Rightarrow \quad A^2 - 8A + 7I_2 = 0 \quad \Rightarrow \quad A^2 = 8A - 7I_2$

$\Rightarrow \quad k = -7 \quad \Rightarrow \quad |k| = 7$ Ans.

17. Compute A^{-1} if $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$. Hence solve the system of equations

$$\begin{bmatrix} 3 & 0 & 3 \\ 2 & 1 & 0 \\ 4 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2y \\ z \\ 3y \end{bmatrix}$$

Ans. $\begin{bmatrix} 3 & 0 & 3 \\ 2 & 1 & 0 \\ 4 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2y \\ z \\ 3y \end{bmatrix}$

Sol. Compute A^{-1}. If $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$. Hence solve the system of equations

$$\begin{bmatrix} 3 & 0 & 3 \\ 2 & 1 & 0 \\ 4 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2y \\ z \\ 3y \end{bmatrix}$$

Matrices & Determents

Single Correct Answer Type

1. A and B are two nonsingular matrices so that $A^6 = I$ and $AB^2 = BA(B \neq I)$. A value of K so that $B^K = I$ is

a) 31 b) 32

c) 63 d) 64

Key. C

Sol. $A^5(AB^2) = A^5BA.$

$\Rightarrow B^2 = A^5BA$

$\Rightarrow B^4 = (A^5BA)(A^5BA) = A^5B^2A = A^5(A^5BA)A$

$\Rightarrow B^4 = A^4BA^2$

$\Rightarrow B^8 = (A^4BA^2)(A^4BA^2) = A^4B^2A^2 = A^4(A^5BA)A^2$

$\Rightarrow B^8 = A^3BA^3$

$\Rightarrow B^{16} = (A^3BA^3)(A^3BA^3) = A^3B^2A^3 = A^3(A^5BA)A^3 = A^2BA^4$

$A^{32} = (A^2BA^4)(A^2BA^4) = A^2B^2A^4 = A^2(A^5BA)A^4 = ABA^5$

$A^{64} = (ABA^5)(ABA^5) = AB^2A^5 = A(A^5BA)A^5 = B \Rightarrow A^{63} = I$

2. For each real number x such that – 1 < x < 1, let A (x) be the matrix $(1-x)^{-1}\begin{bmatrix} 1 & -x \\ -x & 1 \end{bmatrix}$ and $z = \frac{x+y}{1+xy}$. Then,

(A) A(z) = A(x) + A(y) (B) A(z) = A(x) $[A(y)]^{-1}$

(C) A(z) = A(x) A(y) (D) A(z) = A(x) – A(y)

Key. C

Sol. $A(z) = A\left(\frac{x+y}{1+xy}\right) = \left[\frac{1+xy}{(1-x)(1-y)}\right]\begin{bmatrix} 1 & -\left(\frac{x+y}{1+xy}\right) \\ -\left(\frac{x+y}{1+xy}\right) & 1 \end{bmatrix}$

$\therefore A(x).A(y) = A(z)$

3. A and B are two non singular matrices so that $A^6 = I$ and $AB^2 = BA(B \neq I)$. A value of K so that $B^K = I$ is

a) 31 b) 32 c) 63 d) 64

Key. C

Sol. $A^5\left(AB^2\right) = A^5BA.$

$\Rightarrow B^2 = A^5BA$

$\Rightarrow B^4 = \left(A^5BA\right)\left(A^5BA\right) = A^5B^2A = A^5\left(A^5BA\right)A$

$\Rightarrow B^4 = A^4BA^2$

$\Rightarrow B^8 = \left(A^4BA^2\right)\left(A^4BA^2\right) = A^4B^2A^2 = A^4\left(A^5BA\right)A^2$

$\Rightarrow B^8 = A^3BA^3$

$\Rightarrow B^{16} = \left(A^3BA^3\right)\left(A^3BA^3\right) = A^3B^2A^3 = A^3\left(A^5BA\right)A^3 = A^2BA^4$

$A^{32} = \left(A^2BA^4\right)\left(A^2BA^4\right) = A^2B^2A^4 = A^2\left(A^5BA\right)A^4 = ABA^5$

$A^{64} = \left(ABA^5\right)\left(ABA^5\right) = AB^2A^5 = A\left(A^5BA\right)A^5 = B \Rightarrow A^{63} = I$

4. If matrix $A = [a_{ij}]_{3\times3}$, matrix $B = [b_{ij}]_{3\times3}$ where $a_{ij} + a_{ji} = 0$ and $b_{ij} - b_{ji} = 0$, then $A^4 . B^3$ is

(A) skew-symmetric matrix (B) singular

(C) symmetric (D) zero matrix

Key. B

Sol. Since matrix A is skew-symmetric,

$\therefore$ | A | = 0

$\therefore$ $| A^4 . B^3 | = 0$

5. If $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, then det (Adj (Adj A)) is

(A) $(14)^4$ (B) $(14)^6$ (C) $(14)^9$ (D) $(14)^2$

Key. A

Sol. | A | = = (1 + 2) – 2 (– 1 – 4) – (1 – 2)

= 3 + 10 + 1 = 14

$\therefore$ det (Adj (Adj A)) = $|\text{Adj } A|^2 = |A|^4 = (14)^4$

6. In the expansion of $\left(\sqrt{\frac{q}{p}} + \sqrt[10]{\frac{p^7}{q^3}}\right)^n$, there is a term similar to pq, then that term is equal to

(A) 210 pq (B) 252 pq (C) 120 pq (D) 45 pq

Key. B

7. Let x, y, z be real numbers such that 3x, 4y and 5z form a geometric progression while x, y, z form an H.P. Then the value of $\frac{x}{z} + \frac{z}{x} = \frac{m}{n}$ where *m* and *n* are relatively prime then, (m + n) is equal to

(A) 29 (B) 39 (C) 49 (D) 59

Key. C

8. If A is a square matrix fo order 3 much that $|A| = 2$ then $\left|\left(adj\, A^{-1}\right)^{-1}\right|$ is

(A) 1 (B) 2 (C) 4 (D) 8

Key. C

9. Let A and B be square matrices of same order satisfying AB = A and BA = B. Then A^2B^2 equals, (O being the zero matrix of the same order as B)

(A) A (B) B (C) I (D) O

Key: A

Hint Conceptual

10. If A and B are square matrices of the same order and A is non-singular, then for a positive integer *n*, $(A^{-1} BA)^n$ is equal to

A) $A^{-n} B^n A^n$ B) $A^n B^n A^{-n}$ C) $A^{-1} B^n A$ D) $n(A^{-1} BA)$

Key: C

Hint: $(A^{-1}BA)^2 = (A^{-1}BA)(A^{-1}BA) = A^{-1}B(AA^{-1})BA = A^{-1}BIBA = A^{-1}B^2A$

$\Rightarrow (A^{-1}BA)^3 = (A^{-1}B^2A)(A^{-1}BA) = A^{-1}B^2(AA^{-1})BA = A^{-1}B^2IBA = A^{-1}B^3A$ and so on

$\Rightarrow (A^{-1}BA)^n = A^{-1}B^nA$

11. If A is a skew–symmetric matrix of order 3, then the matrix A^4 is

(A) skew symmetric (B) symmetric (C) diagonal (D) none of those

Key: B

Hint: We have $A^T = -A$

$(A^4)^T = (A.A.A.A.)^T = A^T A^T A^T A^T$

$\Rightarrow (-A)(-A)(-A)(-A)$

$= (-1)^4 A^4 = A^4$

12. If A and B are symmetric matrices of same order and X= AB + BA and Y = AB – BA, then $(XY)^T$ is equal to

(A) XY (B)YX (C) – YX (D)none of these

Key: C

Hint: X = AB + BA $\Rightarrow X^T = X$

and Y = AB – BA $\Rightarrow Y^T = -Y$

Now, $(XY)^T = Y^T \times X^T = -YX.$

13. If A and B are any two different square matrices of order n with A – B is non-singular $A^3 = B^3$ and A(AB) = B(BA) , then

(A) $A^2 + B^2 = O$ (B) $A^2 + B^2 = I$ (C) $A^2 + B^3 = I$ (D) $A^3 + B^3 = O$

Key: A

Hint: $A^3 = B^3$.......(i)

$A^2B = B^2A$................(ii)

$\left(A^2 + B^2\right)(A - B) = 0$

$\because |A - B| \neq 0$

$A^2 + B^2 = 0$

14. A square matrix A is said to be nilpotent of index m. If $A^m = 0$, now, if for this A $(I - A)^n = I + A + A^2 + ... + A^{m-1}$, then n is equal to

(A) 0 (B) m (C) - m (D) -1

Key: D

Hint: Let $B = I + A + A^2 + ... + A^{m-1}$

$\Rightarrow B(I - A) = (I + A + A^2 + ... + A^{m-1})(I - A)$

$= I - A^m = I$

$\Rightarrow B = (I - A)^{-1} \Rightarrow n = -1$.

15. If $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then A^8 equals

(a) 4B (b)128 B (c) –128 B (d) –64 B

Key: b

Hint: We have A = iB

$\Rightarrow A^2 = (iB)^2 = i^2B^2 = -B^2 = -\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = -2B$

$\Rightarrow A^4 = (-2B)^2 = 4B^2 = 4(2B) = 8B$

$\Rightarrow (A^4)^2 = (8B)^2 \Rightarrow A^8 = 64B^2 = 128B$

16. The number of positive integral solutions of the equation $\begin{vmatrix} y^3+1 & y^2z & y^2x \\ yz^2 & z^3+1 & z^2x \\ yx^2 & x^2z & x^3+1 \end{vmatrix} = 11$ is

(A) 1 (B) 2 (C) 3 (D) 4

Key: C

Hint: Multiply by y,z and x in rows 1,2 and 3 respectively and then take common y, z and x from column 1,2 and 3 respectively, then

$$\begin{vmatrix} y^3+1 & y^3 & y^3 \\ z^3 & z^3+1 & z^3 \\ x^3 & x^3 & x^3+1 \end{vmatrix} = 11$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & y^3 \\ -1 & 1 & z^3 \\ 0 & -1 & x^3+1 \end{vmatrix} = 11 \qquad \left(C_1 \to C_1 - C_2 \text{ and } C_2 \to C_2 - C_3\right)$$

$$\Rightarrow 1\left(x^3+1+z^3\right)+y^3(1)=11 \Rightarrow x^3+y^3+z^3=10$$

So solution are (1,1,2) , (1,2,1) or (2,1,1)

17. If $a - 2b + c = 1$, then the value of $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ is

(A) x (B) $-x$ (C) -1 (D) 1

Key. C

Sol. $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ $\qquad \begin{aligned} a-2b+c&=1 \\ (a-b)+(c-b)&=1 \end{aligned}$

Apply the operation,

$R_1 \to R_1 - 2R_2 + R_3$

$R_3 \to R_3 - R_2$, the determinant reduces to

$$\begin{vmatrix} 0 & 0 & 1 \\ x+2 & x+3 & x+b \\ 1 & 1 & c-b \end{vmatrix} = -1$$

18. If A is involutary matrix, then which of the following is/are correct?

(A) I + A is idempotent (B) I – A is idempotent

(C) (I + A)(I – A) is singular (D) $\dfrac{I+A}{3}$ is idempotent

Key. C

Sol. $A^2 = I$

$(I + A)(I - A) = I - A^2 = I - I = O$

19. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, $C = ABA^T$, then $A^T C^n A$ equals to $\left(n \in I^+\right)$

(A) $\begin{bmatrix} -n & 1 \\ 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -n \\ 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 1 \\ 1 & -n \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$

Key. D

Sol. $A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$

$AA^T = I$ (i)

Now, $C = ABA^T$

$\Rightarrow$ $A^TC = BA^T$ (ii)

Now $A^TC^nA = A^TC.C^{n-1}A$ = $BA^TC^{n-1}A$ (from (ii))

= $BA^TC.C^{n-2}A = B^2A^TC^{n-2}A$ = ……….

= $B^{n-1}A^TCA = B^{n-1}BA^TA$ = $B^n = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$

20. If $p+q+r=0$ and $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$, then K=

1) 0 2) abc 3) pqr 4) a+b+c

Key. 3

Sol. $p+q+r=0 \Rightarrow p^3+q^3+r^3=3pqr$

$\begin{vmatrix} pa & qb & rc \\ qc & rb & pb \\ rb & pc & qa \end{vmatrix} = pqr(a^3+b^3+c^3-3abc)$

$pqr\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} \Rightarrow k = pqr$

21. If $a = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$, then $\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix}$ is

1) purely real 2) purely imaginary

3) a complex number 4) a

Key. 2 or 3

Sol. $a = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = w^2$

$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & w^2 & w \\ 1 & w & w^2 \end{vmatrix} = 3(w-w^2)$ purely imaginary

22. $\begin{vmatrix} {}^xC_r & {}^xC_{r+1} & {}^xC_{r+2} \\ {}^yC_r & {}^yC_{r+1} & {}^yC_{r+2} \\ {}^zC_r & {}^zC_{r+1} & {}^zC_{r+2} \end{vmatrix} - \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$ =

1) 0 2) 2^n 3) ${}^{x+y+z}C_r$ 4) ${}^{x+y+z}C_{r+2}$

Key. 1

Sol. $\begin{vmatrix} {}^xC_r & {}^xC_{r+1} & {}^xC_{r+2} \\ {}^yC_r & {}^yC_{r+1} & {}^yC_{r+2} \\ {}^zC_r & {}^zC_{r+1} & {}^zC_{r+2} \end{vmatrix} = \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+1}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+1}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+1}C_{r+2} \end{vmatrix}$

By applying $C_2 \to C_2 + C_1, C_3 \to C_3 + C_2$

Now apply $C_3 \to C_3 + C_2$, $\begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$

$\therefore$ Ans = 0

23. If A is an orthogonal matrix of order n, then the value of |adj.(adj A)| is
(A) 0 (B) ± 1
(C) n (D) n – 2

Key. B

Sol. $AA' = I$
$\Rightarrow$ $|A| = \pm 1$
$\therefore$ |adj. (adj. A)|
$= |A|^{(n-1)^2} = \pm 1.$

24. If a, b, c, d > 0; x ∈ R and $(a^2 + b^2 + c^2)x^2 - 2(ab + bc + cd)x + b^2 + c^2 + d^2 \le 0$, then
$\begin{vmatrix} 33 & 14 & \log a \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} =$
(A) 1 (B) – 1
(C) 0 (D) none of these

Key. C

Sol. We have
$(a^2 + b^2 + c^2)x^2 - 2(ab + bc + cd)x + b^2 + c^2 + d^2 \le 0$
$\Rightarrow$ $(ax - b)^2 + (bx - c)^2 + (cx - d)^2 \le 0$
$\Rightarrow$ $(ax - b)^2 + (bx - c)^2 + (cx - d)^2 = 0$
$\Rightarrow$ $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = x$
$\Rightarrow$ $b^2 = ac$ or $2\log b = \log a + \log c$,

Now, $\begin{vmatrix} 33 & 14 & \log a \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} = \begin{vmatrix} 130 & 54 & \log a + \log c \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix}$ [Apply $R_1 \to R_1 + R_3$]

$\begin{vmatrix} 0 & 0 & 0 \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} = 0$ [Apply $R_1 \to R_1 - 2R_2$]

25. A square matrix P satisfies $P^2 = I - P$, where I is an identity matrix of order as order of P. if $P^n = 5I - 8P$, then n =
(a) 4 (b) 5
(c) 6 (d) 7

Key. C

SOL. SINCE $P^2 = I - P$ (GIVEN) ——(1)
$P^3 = P(I - P)$
$P^3 = P - P^2 = P - (I - P)$ (USING) —— (II)
$P^3 = 2P - I$
SIMILARLY $P^4 = 2P^2 - P = 2I - 3P$ AND $P^5 = 5P - 3I$
$P^6 = 5P^2 - 3P = 5I - 8P$
$\therefore\ n = 6$

26. If $Y = SX, Z = tX$ all the variables being differentiable functions of x and lower suffices denote the derivative with respect to x and $\begin{vmatrix} X & Y & Z \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix} \div \begin{vmatrix} S_1 & t_1 \\ S_2 & t_2 \end{vmatrix} = X^n$, then $n =$

a) 1 b) 2 c) 3 d) 4

Key. C

Sol. $$\Delta = \begin{vmatrix} X & SX & tX \\ X_1 & SX_1 + S_1X & tX_1 + t_1X \\ X_2 & SX_2 + 2S_1X_1 + S_2X & tX_2 + 2t_1X_1 + t_2X \end{vmatrix}$$

$$\begin{pmatrix} C_2 \leftarrow C_2 - SC_1 \\ C_3 \leftarrow C_3 - C_1 \end{pmatrix}$$

$$= \Delta = \begin{vmatrix} X & 0 & 0 \\ X_1 & S_1X & t_1X \\ X_2 & 2S_1X_1 + S_2X & 2t_1X_1 + t_2X \end{vmatrix}$$

$$= S^2 \begin{vmatrix} S_1 & t_1 \\ 2S_1X_1 + S_2X & 2t_1X_1 + t_2X \end{vmatrix}$$

$$= = X^3 \le \begin{vmatrix} S_1 & t_1 \\ S_2 & t_2 \end{vmatrix} \left(R_2 \leftarrow R_2 - 2X_1R_1\right)$$

$$\therefore n = 3.$$

27. If A and B are two non singular matrices and both are symmetric and commute each other then

a) Both $A^{-1}B$ and $A^{-1}B^{-1}$ are symmetric.
b) $A^{-1}B$ is symmetric but $A^{-1}B^{-1}$ is not symmetric
c) $A^{-1}B^{-1}$ is symmetric but $A^{-1}B$ is not symmetric
d) Neither $A^{-1}B$ nor $A^{-1}B^{-1}$ are symmetric

Key. A

Sol. $AB = BA$

Previous & past multiplying both sides by A^{-1}.

$$A^{-1}(AB)A^{-1} = A^{-1}(BA)A^{-1}$$

$$\left(A^{-1}A\right)\left(BA^{-1}\right) = A^{-1}B\left(AA^{-1}\right)$$

$$\Rightarrow \left(BA^{-1}\right)^1 = \left(A^{-1}B\right)^1 = \left(A^{-1}\right)^1 B^1 \left(\text{reversal laws}\right)$$

$$= A^{-1}B\left(\text{ as } B=B^1\right)$$

$\left(A^{-1}\right)^1 = A^{-1} \Rightarrow A^{-1}B$ is symmetric

Similarly for $A^{-1}B^{-1}$.

28. If $f(x) = ax^2 + bx + c \quad a,b,c \in R$ and the equation $f(x) - x = 0$ has imaginary roots α and β and γ and δ be the roots of $f(f(x)) - x = 0$, then $\begin{vmatrix} 2 & \alpha & \delta \\ \beta & 0 & \alpha \\ \gamma & \beta & 1 \end{vmatrix}$ is

a) 0 b) purely real c) purely imaginary d) none of these

Key. B

Sol. $f(x) - x > 0 \text{ or}, f(x) - x < 0 \forall x \in R$

$f(f(x)) - f(x) > 0 \text{ or } f(f(x)) - f(x) < 0$

Adding, $f(f(x)) - x > 0 \text{ or, f}(f(x)) - x < 0$

$\Rightarrow$ roots of $f(f(x)) - x = 0$ are imaginary.

Let $z = \begin{vmatrix} 2 & \alpha & \delta \\ \beta & 0 & \alpha \\ \gamma & \beta & 1 \end{vmatrix}$

$$\bar{z} = \begin{vmatrix} 2 & \bar{\alpha} & \bar{\delta} \\ \bar{\beta} & 0 & \bar{\alpha} \\ \bar{\gamma} & \bar{\beta} & 1 \end{vmatrix} = \begin{vmatrix} 2 & \beta & \gamma \\ \alpha & 0 & \beta \\ \delta & \alpha & 1 \end{vmatrix} = z$$

29. Suppose a Matrix A satisfies $A^2 - 5A + 7I = 0$ If $A^5 = aA + bI$, then the values of $2a + b$ is.

a) -87 b) -105 c) 1453 d) 1155

Key. A

Sol. $A^3 = AA^2 = A(5A - 7I)$

$= 5A^2 - 7A = 5(5A - 7I) - 7A = 18A - 35I$

$A^4 = A.A^3 = A(18A - 35I) = 18(5A - 7I) - 35A$

$A^5 = 149A - 385I \qquad = 55A - 126I$

$A^5 = 149A - 385I$

$a = 149, b = -385$

30. The digits A, B, C are such that the three digit numbers A88, 6B8, 86C are divisible by 72, then the determinant $\begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 8 & 8 & C \end{vmatrix}$ is divisible by

a) 76 b) 144 c) 216 d) 276

Key. B

Sol. $100A + 80 + 8 = 72\lambda_1$

$600 + 10B + 8 = 72\lambda_2 \quad \lambda_1, \lambda_2, \lambda_3 \in I.$

$800 + 60 + C = 72\lambda_3$

$$\begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 8 & 8 & C \end{vmatrix} \left(R_3 \leftarrow R_3 + 10R_2 + 100R_1\right)$$

$$= \begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 72\lambda_1 & 72\lambda_2 & 72\lambda_3 \end{vmatrix}$$

A88 is div. by 72

$\Rightarrow$ A88 is div. by 9

$\Rightarrow$ A+8+8 is div. by 9

$\therefore\ A = 2$

$6B8$ is div. by 9 $\Rightarrow B = 4.$

31. If the matrix $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ is invertible, then the planes $a_{11}x + a_{12}y + a_{13}z = 0$, $a_{21}x + a_{22}y + a_{23}z = 0$ and $a_{31}x + a_{32}y + a_{33}z = 0$ $(a_{ij} \in R, \forall i, j)$

(A) intersect in a point (B) intersect in a line

(C) have no common point (D) are same

Key. A

Sol. Given matrix A is invertible $\Rightarrow$ det A $\neq 0$

$\Rightarrow$ the given system of equation has only one solution

i.e., (0, 0, 0). Hence option (A) is correct.

32. If A is a skew–symmetric matrix of order 3, then the matrix A^4 is

(A) skew symmetric (B) symmetric

(C) diagonal (D) none of those

Key. B

Sol. We have $A^T = -A$

$(A^4)^T = (A.A.A.A.)^T = A^T A^T A^T A^T$

$\Rightarrow (-A)(-A)(-A)(-A)$

$= (-1)^4 A^4 = A^4$

33. If 'α' is a root of $x^4 = 1$ with negative principal argument, then the principal argument of $\Delta(\alpha)$ where

$$\Delta(\alpha) = \begin{vmatrix} 1 & 1 & 1 \\ \alpha^n & \alpha^{n+1} & \alpha^{n+3} \\ \frac{1}{\alpha^{n+1}} & \frac{1}{\alpha^n} & 0 \end{vmatrix}$$ is

(A) $\frac{5\pi}{14}$ (B) $-\frac{3\pi}{4}$

(C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{4}$

Key. B

Sol. Clearly $\alpha = -i$ where $i^2 = -1$

So $\Delta(\alpha) = \alpha^n \frac{1}{\alpha^n}\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^3 \\ \frac{1}{\alpha} & 1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -i & i \\ i & 1 & 0 \end{vmatrix}$ = 1(–i) + 1(i^2) + (1 + i^2) = – 1 – i

So, principal argument of $\Delta(\alpha)$ is $-\frac{3\pi}{4}$

34. If z is a complex number and $l_1, l_2, l_3, m_1, m_2, m_3$ are all real, then

$$\begin{vmatrix} l_1 z + m_1\bar{z} & m_1 z + l_1\bar{z} & m_1 z + l_1 \\ l_2 z + m_2\bar{z} & m_2 z + l_2\bar{z} & m_2 z + l_2 \\ l_3 z + m_3\bar{z} & m_3 z + l_3\bar{z} & m_3 z + l_3 \end{vmatrix}$$ is equal to

(A) $|z|^2$ (B) 3
(C) $(l_1 l_2 l_3 + m_1 m_2 m_3)^2 |z|^2$ (D) 0

Key. D

Sol. $\begin{vmatrix} l_1 & m_1 & 0 \\ l_2 & m_2 & 0 \\ l_3 & m_3 & 0 \end{vmatrix} \times \begin{vmatrix} z & \bar{z} & 0 \\ \bar{z} & z & 0 \\ 1 & z & 0 \end{vmatrix} = 0$

35. Let A, B be square matrix such that A B = 0 and B is non singular then
(A) |A| must be zero but A may non zero (B) A must be zero matrix
(C) nothing can be said in general about A (D) none of these

Key. B

Sol. A. B = 0 $\Rightarrow$ A. B . B^{-1} = 0.B^{-1}
$\Rightarrow$ A . I = 0
$\Rightarrow$ A = 0

36. The value of $\begin{vmatrix} \sqrt{6} & 2i & 3+\sqrt{6} \\ \sqrt{12} & \sqrt{3}+\sqrt{8}\,i & 3\sqrt{2}+\sqrt{6}\,i \\ \sqrt{18} & \sqrt{2}+\sqrt{12}\,i & \sqrt{27}+2i \end{vmatrix}$ is

a) an integer b) a rational number c) an irrational number d) an imaginary number

Key. A

Sol. Take $\sqrt{6}$ common from C_1 and apply $C_3 \to C_3 - 3C_1$, $C_2 \to C_2 - 2iC_1$

37. If p + q + r = 0 and $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = K\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ then the value of K is

a) p + q –r b) p + q + r c) pqr d) - pqr

Key. D

Sol. $pqr\left(a^3+b^3+c^3\right) - abc\left(p^3+q^3+r^3\right)$

$\Rightarrow pqr\left(a^3+b^3+c^3-3abc\right) - abc\left(p^3+q^3+r^3-3pqr\right)$

$\Rightarrow pqr\left(a^3+b^3+c^3-3abc\right)-abc\left(p+q+r\right)\left(p^2+q^2+r^2-pq-qr-rp\right)$

$= pqr\left(a^3+b^3+c^3-3abc\right)$

38. If f(x), g(x), h(x) are polynomials of degree 4 and $\begin{vmatrix} f(x) & g(x) & h(x) \\ a & b & c \\ p & q & r \end{vmatrix} =$

$mx^4+nx^3+rx^2+5x+t$ be an identity in x, then the value of

$\begin{vmatrix} f'''(0)-f''(0) & g'''(0)-g''(0) & h'''(0)-h''(0) \\ a & b & c \\ p & q & r \end{vmatrix}$ is

a) $(3n-r)$ b) $2(3n-r)$ c) $3(3n-r)$ d) $3n+r$

Key. B

Sol. LHS = $\left(24mx+6n\right)-\left(12mx^2+6nx+2r\right)$

$x=0 \Rightarrow 6n-2r$

$\Rightarrow 2(3n-r)$

39. Let x > 0, y > 0, z > 0 are respectively the 2nd, 3rd, 4th, terms of a G.P and

$\Delta = \begin{vmatrix} x^k & x^{k+1} & x^{k+2} \\ y^k & y^{k+1} & y^{k+2} \\ z^k & z^{k+1} & z^{k+2} \end{vmatrix} = (r-1)^2\left(1-\frac{1}{r^2}\right)$ (where r is the common ratio) then

a) $k=-1$ b) $k=1$ c) $k=0$ d) None of these

Key. A

Sol. $x^k y^k z^k \begin{vmatrix} 1 & ar & a^2r^2 \\ 1 & ar^2 & a^2r^4 \\ 1 & ar^3 & a^3r^6 \end{vmatrix}$

$a^{3(k+1)}.r^{3(2k+1)}\left[(r-1)(r^4-1)-(r^2-1)^2\right] \Rightarrow k=-1$

40. If $f(x) = \begin{vmatrix} x^2-4x+6 & 2x^2+4x+10 & 3x^2-2x+16 \\ x-2 & 2x+2 & 3x-1 \\ 1 & 2 & 3 \end{vmatrix}$. Then the value of

$\int_{-3}^{3} \frac{x^2 \sin x}{1+x^6}.f(x)dx$ is

a) 6 b) 3 c) 0 d) $\frac{\pi}{2}$

Key. C

Sol. $f(x)$ is const.

Hence = 0

41. If A and B are square matrices of order 3 such that $|A| = -1, |B| = 3$ then $|3AB|$ is equals to

A) –9 B) –81 C) –27 D) 81

Key. B

Sol. $|3AB| = 3^3|A||B| = 27 \times -1 \times 3 = -81$

42. A = $[a_{ij}]_{n\times n}$ and a_{ij} = $i^2 - j^2$ then A is necessarily

a) a unit matrix b) symmetric matrix c) skew symmetric matrix d) zero matrix

Key. C

Sol. $a_{ji} = j^2 - i^2 = (i^2 - j^2) = -a_{ij}$

43. If $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$ is an orthogonal matrix then value of x+y is equal to

a) –3 b) 0 c) 1 d) 3

Key. A

Sol. $A.A^T = I \Rightarrow \frac{1}{9}\begin{bmatrix} 9 & 0 & x+4+2y \\ 0 & 9 & 2x+2-2y \\ x+4+2y & 2x+2-2y & x^2+y^2+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

x+4+2y = 0, 2x+2–2y = 0 $\Rightarrow$ x = –2, y = –1

44. If $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ then A^{16} =

a) $\begin{bmatrix} 0 & 256 \\ 256 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$

c) $\begin{bmatrix} -16 & 0 \\ 0 & -16 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 16 \\ 16 & 0 \end{bmatrix}$

Key. B

Sol. $A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$, $A^4 = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$ $A^8 = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$, $A^{16} = \begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$

45. Let a, b, c be positive real numbers. Then the following system of equations in x, y, z

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, -\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ has

a) no solution b) unique solution

c) infinite solution d) finitely many solution

Key. D

Sol. Let $\frac{x^2}{a^2} = X$, $\frac{y^2}{b^2} = Y$, $\frac{z^2}{c^2} = Z$

$X+Y-Z=1$, $X-Y+Z=1$, $-X+Y+Z=1$ on solving $X=Y=Z=1$

$\Rightarrow x=\pm a, y=\pm b, z=\pm c$ $\Rightarrow$ 8 solution

46. The value of determinant $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$ is equal to

a) $(1-a^2-b^2)^3$ b) $(a+b+1)^2(ab+b+a)$ c) $(1+a^2+b^2)^3$ d) $(1-a^2+b^2)^3$

Key. C

Sol. $\Delta=\dfrac{1}{ab}\begin{vmatrix} b(1+a^2-b^2) & 2ab^2 & -2b^2 \\ 2a^2b & a(1-a^2+b^2) & 2a^2 \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$ by $(R_1\times b, R_2\times a)$

$=\begin{vmatrix} 1+a^2-b^2 & 2b^2 & -2b^2 \\ 2a^2 & 1-a^2+b^2 & 2a^2 \\ 2 & -2 & 1-a^2-b^2 \end{vmatrix}$ by $\left(\dfrac{C_1}{b},\dfrac{C_2}{a}\right)$

$\begin{vmatrix} 1+a^2+b^2 & 0 & -2b^2 \\ 1+a^2+b^2 & 1+a^2+b^2 & 2a^2 \\ 0 & -(1+a^2+b^2) & 1-a^2-b^2 \end{vmatrix} (C_1\to C_1+C_2, C_2\to C_2+C_3)$

$=(1+a^2+b^2)^3$

47. If $\begin{vmatrix} a^2+\lambda^2 & ab+c\lambda & ca-b\lambda \\ ab-c\lambda & b^2+\lambda^2 & bc+a\lambda \\ ac+b\lambda & bc-a\lambda & c^2+\lambda^2 \end{vmatrix}\begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}=(1+a^2+b^2+c^2)^3$ then λ is equal to

a) 0 b) 1 c) –1 d) ± 1

Key. B

Sol. If $\Delta=\begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}$ other determinant (say Δ^1) is the cofactor determinant

$\Delta\Delta^1=\Delta^3$ (for 3rd order det)

$\Delta=\lambda(\lambda^2+a^2+b^2+c^2)$ by comparing $\lambda=1$

48. Constant term in f(x) = $\begin{vmatrix} x & (1+\sin x)^3 & \cos x \\ 1 & \ln(1+x) & 2 \\ x^2 & (1+x)^2 & 0 \end{vmatrix}$ when f(x) is expressed polynomial in x, is

a) 0 b) –1 c) 1 d) 2

Key. C

Sol. $f(0)=+1$

49. If A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, B = $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, a, b $\in$ N, then number of matrix 'B' such that AB = BA are

a) 0 b) 1 c) finitely many d) infinite

Key. D

Sol. $AB = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}, BA = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$

AB= BA $\Rightarrow$ a= b

50. If $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then f(α+β) =

a) f(α) + f(β) b) f(α).f(β) c) f(α)–f(β) d) 0

Key. B

Sol. f(α).f(β) = $\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & 0 \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$