Logs & Surds

Single Correct Answer Type

1. Number of ordered triplets of natural number (a, b, c) for which abc ≤ 11 is

(A) 52 (B) 53 (C) 55 (D) 56

Key. D

Sol. abc = 1 in 1 ways
abc = 2, 3, 5, 7, 11 in 15 ways
abc = 4, 9 in 12 ways
abc = 8 in 10 ways
abc = 6, 10 in 18 ways
So, total number of solution is 56

2. The value of $\sqrt{5-\sqrt{10}-\sqrt{15}+\sqrt{6}}$ is

(A) $\frac{\sqrt{5}-\sqrt{3}-\sqrt{2}}{\sqrt{2}}$ (B) $\frac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{\sqrt{2}}$

(C) $\frac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{2}$ (D) $\frac{\sqrt{5}-\sqrt{3}-\sqrt{2}}{2}$

Key. B

Sol. $5-\sqrt{10}-\sqrt{15}+\sqrt{6}$ can be written as

$$\frac{3+2+5-2\sqrt{2}\sqrt{5}-2\sqrt{5}\sqrt{3}+2\sqrt{3}\sqrt{2}}{2}$$

$$=\left(\frac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{\sqrt{2}}\right)^2$$

3. $a>0(a\neq1), b>0(b\neq1)$ such that $a^{\left(\log_a^b\right)^x}=b^{\left(\log_b^a\right)^x}$ then $x=$

(A) 1 (B) -1 (C) $\frac{1}{2}$ (D) 2

Key: C

Hint: Taking $\log_b$ both sides we get

$$\left(\log_a^b\right)^x \log_b^a=\left(\log_b^a\right)^x$$

$$\therefore\left(\log_a^b\right)^x=\left(\log_b^a\right)^{x-1}$$

$$\therefore 1-x=x \Rightarrow x=\frac{1}{2}$$

4. Given that $\log_{10}^{5}=0.70$ and $\log_{10}^{3}=0.48$ then the value of $\log_{30}^{8}$ (correct upto 2 places of decimal) is

(A) 0.56 (B) 0.61 (C) 0.68 (D) 0.73

Key: B

5. The value of $\sqrt{5-\sqrt{10}-\sqrt{15}+\sqrt{6}}$ is

(A) $\dfrac{\sqrt{5}-\sqrt{3}-\sqrt{2}}{\sqrt{2}}$ (B) $\dfrac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{\sqrt{2}}$

(C) $\dfrac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{2}$ (D) $\dfrac{\sqrt{5}-\sqrt{3}-\sqrt{2}}{2}$

Key. B

Sol. $5-\sqrt{10}-\sqrt{15}+\sqrt{6}$ can be written as

$$\frac{3+2+5-2\sqrt{2}\sqrt{5}-2\sqrt{5}\sqrt{3}+2\sqrt{3}\sqrt{2}}{2}$$

$$=\left(\frac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{\sqrt{2}}\right)^2$$

6. The value of $\sqrt{5-\sqrt{10}-\sqrt{15}+\sqrt{6}}$ is

(A) $\dfrac{\sqrt{5}-\sqrt{3}-\sqrt{2}}{\sqrt{2}}$ (B) $\dfrac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{\sqrt{2}}$

(C) $\dfrac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{2}$ (D) $\dfrac{\sqrt{5}-\sqrt{3}-\sqrt{2}}{2}$

Key. B

Sol. $5-\sqrt{10}-\sqrt{15}+\sqrt{6}$ can be written as

$$\frac{3+2+5-2\sqrt{2}\sqrt{5}-2\sqrt{5}\sqrt{3}+2\sqrt{3}\sqrt{2}}{2}$$

$$=\left(\frac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{\sqrt{2}}\right)^2$$

7. There exist positive integers A, B and C with no common factors greater than 1, such that A $\log_{200}5$ + B $\log_{200}2$ = C. The sum A + B + C equals

(A) 5 (B) 6 (C) 7 (D) 8

Key. B

Sol. A $\log_{200}5$ + B $\log_{200}2$ = C

= C

A log 5 + B log 2 = C log 200 = C log($5^2\ 2^3$) = 2C log 5 + 3 C log 2

hence, A = 2C and B = 3C

for no common factor greater than 1, C = 1

$\therefore$ A = 2; B = 3 $\Rightarrow$ A + B + C = 6 Ans.

9. Given real numbers a, b, c > 0 ($\neq$ 1) such that $\log_{\log_c a} e, \log_{(a^{c/2})} e, \log_{(\log_b c)} e$ are in H.P. then c equal to

(a) $\log_a(\log_a b)$ (b) $\log_a(\log_b a)$

(c) $\log_b(\log_b a)$ (d) $\log_b(\log_a b)$

Key. B

SOL. $\mathrm{LOG}_E(\mathrm{LOG}_C A)$, $\mathrm{LOG}_E A^{C/2}$, $\mathrm{LOG}_E(\mathrm{LOG}_B C)$ ARE IN A.P.

$\Rightarrow$ $\mathrm{LOG}_C A$, $A^{C/2}$, $\mathrm{LOG}_B C$ ARE IN G.P.

$\Rightarrow$ $A^C = \mathrm{LOG}_C A \ \mathrm{LOG}_B C$

$\Rightarrow$ $A^C = \mathrm{LOG}_B A$

$\Rightarrow$ $c = \log_a(\log_b a)$