Hyperbola

Single Correct Answer Type

1. A line drawn through the point P (-1, 2) meets the hyperbola $xy = c^2$ at the points A and B. (points A and B lie on same side of P) and Q is a point on AB such that PA, PQ and PB are in H.P then locus of Q is

A. $x - 2y = 2c^2$ B. $2x - y = 2c^2$ C. $2x + y + 2c^2 = 0$ D. $x + 2y = 2c^2$

Key. B
Sol. Locus of Q is $S_1 = 0$

$2x - y = 2c^2$

2. If the asymptote of the hyperbola $(x+y+1)^2 - (x-y-3)^2 = 5$ cut each other at A and the coordinate axis at B and C then radius of circle passing through the points A,B,C is

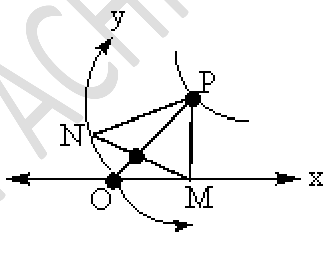
A. 3 B. $\frac{\sqrt{5}}{2}$ C. $\frac{\sqrt{3}}{2}$ D. $\sqrt{3}$

Key. B
Sol. Centre of rectangular hyperbola = (1,-2)
So equation of asymptotes are x = 1, y = -2
So radius of circle $= \frac{\sqrt{5}}{2}$

3. PM and PN are the perpendiculars from any point P on the rectangular hyperbola xy = 8 to the asymptotes. If the locus of the mid point of MN is a conic, then the least distance of (1, 1) to director circle of the conic is

A. $\sqrt{3}$ B. $\sqrt{2}$ C. $2\sqrt{3}$ D. $2\sqrt{5}$

Key. B
Sol. OMPN is rectangle.



$P = (Ct, \frac{c}{t})$

Mid point $= \left(\frac{ct}{2}, \frac{c}{2t}\right) = (x, y)$ $\therefore cy = \frac{c^2}{4} \Rightarrow e = \sqrt{2}$

4. A hyperbola passing through origin has 3x – 4y – 1=0 and 4x – 3y – 6 = 0 as its asymptotes. Then the equations of its transverse and conjugate axes are

A) x – y – 5 = 0 and x + y + 1 = 0 B) x – y = 0 and x + y + 5 = 0
C) x + y – 5 = 0 and x – y – 1 = 0 D) x + y – 1 = 0 and x – y – 5 = 0

Key. C
Sol. Transverse and conjugate axes are the bisectors of the angle between asymptotes.

$$\frac{3x-4y-1}{5} = \pm\left(\frac{4x-3y-6}{5}\right) \text{ etc......}$$

5. If the asymptotes of the hyperbola $(x+y+1)^2-(x-y-3)^2=5$ cuts each other at A and the coordinate axes at B and C, then radius of the circle passing through the points A, B, C is

A) 3 B) $\frac{\sqrt{5}}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $\sqrt{3}$

Key. B

Sol. (B) Centre of rectangular hyperbola (1, –2)

So equation of asymptotes are x = 1, y = –2

So radius of circle = $\frac{\sqrt{5}}{2}$

6. If a chord joining P($a\text{Sec}\theta, a\tan\theta$), Q($a\text{Sec}\alpha, a\tan\alpha$) on the hyperbola $x^2-y^2=a^2$ is the normal at P,then $\text{Tan}\alpha=$

A) $\text{Tan}\theta(4\sec^2\theta+1)$ B) $\text{Tan}\theta(4\sec^2\theta-1)$ C) $\text{Tan}\theta(2\text{Sec}^2\theta-1)$ D) $\text{Tan}\theta(1-2\text{Sec}^2\theta)$

Key. B

Sol. Slope of chord joining P and Q = slope of normal at P

$\frac{\text{Tan}\alpha-\text{Tan}\theta}{\sec\alpha-\sec\theta}=-\frac{\text{Tan}\theta}{\sec\theta}\Rightarrow \text{Tan}\alpha-\text{Tan}\theta=-k\text{Tan}\theta$ and $\sec\alpha-\sec\theta=k\sec\theta$

$\therefore(1-k)Tan\theta=Tan\alpha\rightarrow 1.\ (1+k)\sec\theta=\sec\alpha\rightarrow 2.$

$\left[(1+k)\sec\theta\right]^2-\left[(1-k)\text{Tan}\theta\right]^2 = \sec^2\alpha-\text{Tan}^2\alpha$

$\Rightarrow k=-2\left(\sec^2\theta+\text{Tan}^2\theta\right)=-4\sec^2\theta+2$

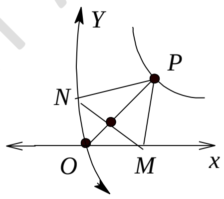
From (1) $Tan\alpha=Tan\theta\ \left(1+4\sec\theta^2-2\right)=\text{Tan}\theta\left(4\sec\theta^2-1\right).$

7. PM and PN are the perpendiculars from any point P on the rectangular hyperbola $xy=c^2$ to the asymptotes. If the locus of the mid point of MN is a conic, then its eccentricity is

A) $\sqrt{3}$ B) $\sqrt{2}$ C) $\frac{1}{\sqrt{3}}$ D) $\frac{1}{\sqrt{2}}$

Key. B

Sol. OMPN is rectangle.



$P=\left(Ct,\frac{c}{t}\right)$

Mid point = $\left(\frac{ct}{2},\frac{c}{2t}\right)=(x,y)$ $\therefore xy=\frac{c^2}{4}\ \Rightarrow e=\sqrt{2}$

8. A variable straight line of slope 4 intersects the hyperbola xy = 1 at two points. The locus of the point which divides the line segment between these two points in the ratio 1 : 2 is

A) $16x^2+10xy+y^2=2$ B) $16x^2-10xy+y^2=2$

C) $16x^2+10xy+y^2=4$ D) $16x^2-10xy+y^2=4$

Key. A

Sol. Let P(h, k)

y - k = 4(x - h) --- (1)

Let it meets xy = 1 ----(2) at A (x_1, y_1) and B (x_2, y_2)

$$x_1+x_2=\frac{4h-k}{4}, x_1x_2=-\frac{1}{4} \text{ Also } \Rightarrow \therefore \frac{2x_1+x_2}{3}=h \Rightarrow x_1=\frac{8h+k}{4}, x_2=\frac{2h+k}{2}$$

$$\Rightarrow 16x^2+10xy+y^2=2$$

9. The length of the transverse axis of the hyperbola $9x^2-16y^2-18x-32y-151=0$ is

1) 8 2) 4 3) 6 4) 2

Key. 1

Sol. Given hyperbola is $\frac{(x-1)^2}{16}-\frac{(y+1)^2}{9}=1$

Length of the transverse axis is 2a=8.

10. The equation of a hyperbola , conjugate to the hyperbola $x^2+3xy+2y^2+2x+3y=0$ is

1) $x^2+3xy+2y^2+2x+3y+1=0$ 2) $x^2+3xy+2y^2+2x+3y+2=0$

3) $x^2+3xy+2y^2+2x+3y+3=0$ 4) $x^2+3xy+2y^2+2x+3y+4=0$

Key. 2

Sol. Let $H=x^2+3xy+2y^2+2x+3y=0$ and C=0 is its conjugate. Then C + H=2A, where A=0 is the combined equation of asymptotes. Equation of asymptotes is $x^2+3xy+2y^2+2x+3y+\lambda=0$, where $\Delta=abc+2fgh-af^2-bg^2-ch^2=0 \Rightarrow \lambda=1$

$\therefore C=2(x^2+3xy+2y^2+2x+3y+1)-(x^2+2y^2+3xy+2x+3y)$

$\Rightarrow$ equation of conjugate hyperbola is $x^2+3xy+2y^2+2x+3y+2=0$

11. If AB is a double ordinate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that ΔOAB is an equilateral triangle O being the origin, then the eccentricity of the hyperbola satisfies

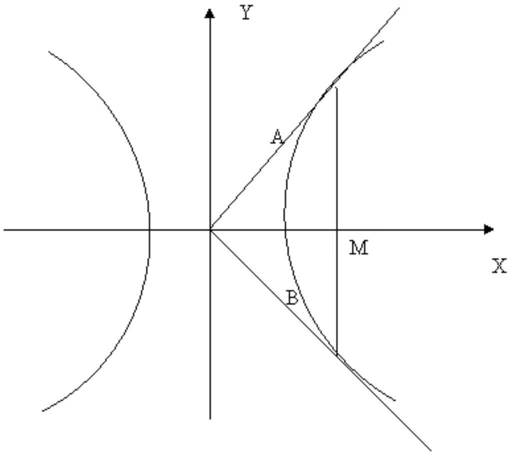
1) $e>\sqrt{3}$ 2) $1<e<\frac{1}{\sqrt{3}}$ 3) $e=\frac{2}{\sqrt{3}}$ 4) $e>\frac{2}{\sqrt{3}}$

Key. 4

Sol. Let the length of the double ordinate be 2ℓ

$\therefore$ AB=2ℓ and AM=BM=ℓ

Clearly ordinate of point A is ℓ.



The abscissa of the point A is given by

$$\frac{x^2}{a^2}-\frac{l^2}{b^2}=1 \Rightarrow x=\frac{a\sqrt{b^2+l^2}}{b}$$

$\therefore$ A is $\left(\frac{a\sqrt{b^2+l^2}}{b}, l\right)$

Since ΔOAB is equilateral triangle, therefore

OA=AB=OB=2l

Also, $OM^2+AM^2=OA^2 \therefore \frac{a\left(b^2+l^2\right)}{b}+l^2=4l^2$

We get $l^2=\frac{a^2b^2}{3b^2-a^2}$

Since $l^2>0 \therefore \frac{a^2b^2}{3b^2-a^2}>0 \Rightarrow 3b^2-a^2>0$

$$\Rightarrow 3a^2\left(e^2-1\right)-a^2>0 \Rightarrow e>\frac{2}{\sqrt{3}}$$

12. If the line $5x+12y-9=0$ is a tangent to the hyperbola $x^2-9y^2=9$, then its point of contact is

1) (-5,4/3) 2) (5,-4/3) 3) $(3,-1/2)$ 4) (5,4/3)

Key. 2

Sol. Common Point

13. Any chord passing through the focus $(ae,0)$ of the hyperbola $x^2-y^2=a^2$ is conjugate to the line

1) $ex-a=0$ 2) $ae+x=0$ 3) $ax+e=0$ 4) $ax-e=0$

Key. 1

Sol. $S_1=0$

14. Number of points from where perpendicular tangents to the curve $\frac{x^2}{16}-\frac{y^2}{25}=1$ can be drawn, is:

1) 1 2) 2 3) 0 4) 3

Key. 3

Sol. Director circle is set of points from where drawn tangents are perpendicular in this case $x^2+y^2=a^2-b^2$ (equation of director circle)i.e., $x^2+y^2=-9$ is not a real circle so there is no points from where tangents are perpendicular.

15. $x^2-y^2+5x+8y-4=0$ represents

1) Rectangular hyperbola 2)Ellipse

3) Hyperbola with centre at (1,1) 4)Pair of lines

Key. 1

Sol. $\Delta\neq 0,\ x^2-ab>0,\ a+b=0$

16.

1) $(2\sqrt{2},2\sqrt{2}),(-2\sqrt{2},-2\sqrt{2})$ 2) $(-3\sqrt{2},-3\sqrt{2}),(3\sqrt{2},3\sqrt{2})$

3) $(2\sqrt{2},-2\sqrt{2}),(-2\sqrt{2},2\sqrt{2})$ 4) (-2. 2)

Key. 1

Sol. foci of $xy=c^2$ is $(\pm c\sqrt{2},\pm c\sqrt{2})$

17. Which of the following is INCORRECT for the hyperbola $x^2-2y^2-2x+8y-1=0$

1) Its eccentricity is $\sqrt{2}$ 2) Length of the transverse axis is $2\sqrt{3}$

3) Length of the conjugate axis is $2\sqrt{6}$ 4) Latus rectum $4\sqrt{3}$

Key. 1

Sol. The equation of the hyperbola is $x^2-2y^2-2x+8y-1=0$

Or $(x-1)^2-2(y-2)^2+6=0$

Or $\frac{(x-1)^2}{-6}+\frac{(y-2)^2}{3}=1;$ or $\frac{(y-2)^2}{3}-\frac{(x-1)^2}{6}=1\rightarrow 1$

Or $\frac{Y^2}{3}-\frac{X^2}{6}=1$, where X = x -1 and Y = y - 2 $\rightarrow 2$

$\therefore$ the centre=(0,0)in the X-Y coordinates.

$\therefore$ the centre=(1,2)in the x-y coordinates .using $\rightarrow 2$

If the transverse axis be of length 2a, then a= $\sqrt{3}$,since in the equation (1) the transverse axis is parallel to the y-axis.

If the conjugate axis is of length 2b,then b = $\sqrt{6}$

But $b^2=a^2\left(e^2-1\right)$

$\therefore$ $6=3\left(e^2-1\right),\therefore e^2=3$ or $e=\sqrt{3}$

The length of the transverse axis = $2\sqrt{3}$

The length of the conjugate axis = $2\sqrt{6}$

Latus rectum $4\sqrt{3}$

18. If the curve $xy=R^2-16$ represents a rectangular hyperbola whose branches lies only in the quadrant in which abscissa and ordinate are opposite in sign but not equal in magnitude, then

1) $|R|<4$ 2) $|R|\geq 4$ 3) $|R|=4$ 4) $|R|=5$

Key. 1

Sol. conceptual

19. If the line ax + by + c=0 is a normal to the curve xy=1,then

1) a > 0, b > 0 2) a < 0,b < 0 3) a < 0,b > 0 4) a=b=1

Key. 3

Sol. Slope of the line $\frac{-a}{b}$ is equal to slope of the normal to the curve.

$\therefore$ either a > 0 & b < 0 (or) a < 0 & b > 0.

20. The equation of normal at $\left(at, \frac{a}{t}\right)$ to the hyperbola $xy = a^2$ is

1) $xt^3 - yt + at^4 - a = 0$
2) $xt^3 - yt - at^4 + a = 0$
3) $xt^3 + yt + at^4 - a = 0$
4) $xt^3 + yt - at^4 - a = 0$

Key. 2

Sol. Equation of tangent is $S_1 = 0$ normal is $\perp^r$ to tangent and passing through

$\left(at, \frac{a}{t}\right)$ is $xt^3 - yt - at^4 + a = 0$

21. The product of perpendiculars from any point P (θ) on the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ to its asymptotes is equal to:

1) $\frac{6}{5}$
2) $\frac{36}{13}$
3) Depending on θ
4) $\frac{5}{6}$

Key. 2

Sol. The product of perpendiculars from any point P (θ) on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to its asymptotes is equal to $\frac{a^2b^2}{a^2+b^2}$

22. The foot of the perpendicular from the focus to an asymptote of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

1) (ae , be)
2) (a/e,b/e)
3) (e/a,e/b)
4) (be,ae)

Key. 2

Sol. Focus S=(ae,0) Equation of one asymptote is bx-ay=0

Let (h,k) be the foot of the perpendicular from s to bx-ay=0

Then $\frac{h-ae}{b} = \frac{k-0}{-a} = \frac{-abe}{a^2+b^2} \Rightarrow \frac{h-ae}{b} = \frac{-abe}{a^2e^2} \;\&\; \frac{k}{-a} = \frac{-abe}{a^2e^2}$

On simplification, we get h=a/e, k=b/e

Foot of the perpendicular is (a/e,b/e)

23. The area of the triangle formed by the asymptotes and any tangent to the hyperbola $x^2 - y^2 = a^2$

1) $4a^2$
2) $3a^2$
3) $2a^2$
4) a^2

Key. 4

Sol. Equation of any tangent to $x^2 - y^2 = a^2$

i.e. $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1 \; is \; \frac{x}{a}\sec\theta - \frac{y}{a}\tan\theta = 1 \rightarrow (1)$

$or \; x\sec\theta - y\tan\theta = a$

equation of other two sides of the triangle are

x-y=0..(2) x + y=0(3)

The two asymptotes of the hyperbola $x^2 - y^2 = a^2$

Are x-y=0 and x + y=0)

Solving (1) (2) and (3) in pairs the coordinates of the vertices of the triangle are (0,0)

$\left(\frac{a}{\sec\theta + \tan\theta}, \frac{a}{\sec\theta + \tan\theta}\right)$

And $\left(\frac{a}{\sec\theta - \tan\theta}, \frac{-a}{\sec\theta - \tan\theta}\right) -$

Area of triangle = $\frac{1}{2}\left|\frac{a^2}{\sec^2\theta - \tan^2\theta} + \frac{a^2}{\sec^2\theta - \tan^2\theta}\right|$

$\frac{1}{2}\left(a^2 + a^2\right) \qquad \because \sec^2\theta - \tan^2\theta = 1$

$= a^2$

24. The foot of the normal $3x + 4y = 7$ to the hyperbola $4x^2 - 3y^2 = 1$ is

1) $(1,1)$ 2) $(1,-1)$ 3) $(-1,1)$ 4) $(-1,-1)$

Key. 1

Sol. Since the point $(1,1)$ lies on the normal and hyperbola it is the foot of the normal

25. Tangent at the point $\left(2\sqrt{2},3\right)$ to the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ meet its asymptotes at A and B, then area of the triangle OAB, O being the origin is

1) 6 sq. units 2) 3 sq. units 3) 12 sq. units 4) 2 sq. units

Key. 1

Sol. Since area of the Δ formed by tangent at any point lying on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and its asymptotes is always constant and is equal to ab. Therefore, required area is 2 X 3=6 square units.

26. Eccentricity of hyperbola $\frac{x^2}{k}+\frac{y^2}{k}=1(k<0)$ is :

1) $\sqrt{1+k}$ 2) $\sqrt{1-k}$ 3) $\sqrt{1+\frac{1}{k^2}}$ 4) $\sqrt{1-\frac{1}{k}}$

Key. 4

Sol. Given equation can be rewritten as $\frac{y^2}{k^2}-\frac{x^2}{(-k)}=1(-k>0)$

$e^2=1+\frac{(-k)}{k^2}=1-\frac{1}{k}\Rightarrow e=\sqrt{1-\frac{1}{k}}$

27. If the circle $x^2+y^2=a^2$ intersect the hyperbola $xy=c^2$ in four points $P(x_1,y_1),Q(x_2,y_2),R(x_3,y_3),S(x_4,y_4)$ then which of the following does not hold

1) $x_1+x_2+x_3+x_4=0$ 2) $x_1x_2x_3x_4=y_1y_2y_3y_4=c^4$

3) $y_1+y_2+y_3+y_4=0$ 4) $x_1+y_2+x_3+y_4=0$

Key. 4

Sol. $x^2+\frac{c^4}{x^2}=a^2$ $\Rightarrow \mathbf{x^4-a^2x^2+c^4=0}$, 4th option does not hold

28. If a normal to the hyperbola x y = c^2 at $\left(ct_1,\frac{c}{t_1}\right)$ meets the curve again at $\left(ct_2,\frac{c}{t_2}\right)$, then:

1) $t_1t_2=-1$ 2) $t_2=-t_1-\frac{2}{t_1}$ 3) $t_2^3t_1=-1$ 4) $t_1^3t_2=-1$

Key. 4

Sol. Equation of normal at $\left(ct_1,\frac{c}{t_1}\right)$ is

$t_1^3x-t_1y-ct_1^4+c=0$

It passes through $\left(ct_2,\frac{c}{t_2}\right)$

Ie., $t_1^3.ct_2-t_1.\frac{c}{t_2}-ct_1^4+c=0$

$\Rightarrow(t_1-t_2)(t_1^3t_2+1)=0$

$\Rightarrow t_1^3t_2=-1$

29. The equation of the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola xy=c^2 is

1) $\frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$ 2) $\frac{x}{x_1-x_2}+\frac{y}{y_1-y_2}=1$ 3) $\frac{y}{x_1+x_2}+\frac{x}{y_1+y_2}=1$ 4) $\frac{x}{y_1-y_2}+\frac{y}{x_1-x_2}=1$

Key. 1

Sol. Mid point of the chord is $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

The equation of the chord in terms of its mid-point is $s_1 = s_{11}$

30. A rectangular hyperbola whose centre is C is cut by any circle of radius r in four points P,Q,R and S. Then $CP^2+CQ^2+CR^2+CS^2=$

1) r^2 2) $2r^2$ 3) $3r^2$ 4) $4r^2$

Key. 4

Sol. $CP=CQ=CR=CS=r$

31. The product of focal distances of the point (4,3) on the hyperbola $x^2-y^2=7$ is

1) 25 2) 12 3) 9 4) 16

Key. 1

Sol. $e=\sqrt{2},\ sp.s'p=(ex_1+a)(ex_1-a)=25$

32. Let $y=4x^2$ & $\frac{x^2}{a^2}-\frac{y^2}{16}=1$ intersect iff

1) $|a|\le\frac{1}{\sqrt{2}}$ 2) $a>\frac{1}{\sqrt{2}}$ 3) $a>-\frac{1}{\sqrt{2}}$ 4) $a>\sqrt{2}$

Key. 1

Sol. $y=4x^2$ & $\frac{1}{4}y=x^2$

Using $\frac{1}{4a^2}y-\frac{y^2}{16}=1$

$\Rightarrow 4y-a^2y^2=16a^2$

$\Rightarrow a^2y^2-4y+16a^2=0$

$\Rightarrow D \geq 0$ for intersection of two curves

$\Rightarrow 16 - 4a^2\left(16a^2\right) \geq 0$

$\Rightarrow 1 - 4a^4 \geq 0$

$\Rightarrow \left(2a^2\right) \leq 1$

$\Rightarrow \left|\sqrt{2}a\right| \leq 1 \Rightarrow -\frac{1}{\sqrt{2}} \leq a \leq \frac{1}{\sqrt{2}}$

33. If angle between the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is 45°, then value of eccentricity e is

1) $\sqrt{4 \pm 2\sqrt{2}}$ 2) $\sqrt{4 + 2\sqrt{2}}$ 3) $\sqrt{4 - 2\sqrt{2}}$ 4) $\sqrt{4 - 3\sqrt{2}}$

Key. 3

Sol. $2\tan^{-1}\frac{b}{a} = 45^\circ \Rightarrow \frac{b}{a} = \tan 22^\circ = \frac{a^2\left(e^2 - 1\right)}{a^2} = \left(\sqrt{2} - 1\right)^2$

$\Rightarrow e^2 - 1 = 3 - 2\sqrt{2} \Rightarrow e = \sqrt{4 - 2\sqrt{2}}$.

34. A hyperbola, having the transverse axis of length $2\sin\theta$, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is

1) $x^2 \operatorname{cosec}^2\theta - y^2 \sec^2\theta = 1$ 2) $x^2 \sec^2\theta - y^2 \operatorname{cosec}^2\theta = 1$

3) $x^2 \sin^2\theta - y^2 \cos^2\theta = 1$ 4) $x^2 \cos^2\theta - y^2 \sin^2\theta = 1$

Key. 1

Sol. Equation of the ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$. Its eccentricity is $e = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

Coordinates of foci are $(\pm 1, 0)$.

Let the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $a = \sin\theta$

Also, $ae_1 = 1 \Rightarrow$ $e_1 = \operatorname{cosec}\theta$

$\therefore b^2 = a^2\left(e_1^2 - 1\right) = 1 - \sin^2\theta = \cos^2\theta$

Equation of the hyperbola is thus $\frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$

35. An ellipse intersects the hyperbola $2x^2-2y^2=1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinates axes, then

1) Equation of ellipse is $x^2+2y^2=1$
2) the foci of ellipse are $(\pm1,0)$
3) equation of ellipse are $x^2+2y^2=4$
4) the foci of ellipse are $(\pm\sqrt{2},0)$

Key. 2

Sol. If two concentric conics intersect orthogonally then they must be confocal, so ellipse and hyperbola will be confocal

$\Rightarrow(\pm ae,0)\equiv(\pm1,0)$

[foci of hyperbola are $(\pm1,0)$]

36. Let $P(6,3)$ be a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal at the point P intersects the x axis at $(9,0)$, then the eccentricity of the hyperbola is:

1) $\sqrt{\frac{5}{2}}$
2) $\sqrt{\frac{3}{2}}$
3) $\sqrt{2}$
4) $\sqrt{3}$

Key. 2

Sol. Normal at (6,3) is

$\frac{a^2x}{6}+\frac{b^2y}{3}=a^2+b^2$,

$\Rightarrow\frac{9a^2}{6}=a^2+b^2\Rightarrow\frac{3}{2}=1+\frac{b^2}{a^2}$

$\therefore\quad \frac{b^2}{a^2}=\frac{1}{2}\Rightarrow e^2-1=\frac{1}{2}\Rightarrow e=\sqrt{\frac{3}{2}}$

37. For hyperbola $\frac{x^2}{\cos^2\alpha}-\frac{y^2}{\sin^2\alpha}=1$, which of the following remains constant with change in 'α'

1) abscissae of vertices
2) abscissae of foci
3) Eccentricity
4) directrix

Key. 2

Sol. Hyperbola is $\frac{x^2}{\cos^2\alpha}-\frac{y^2}{\sin^2\alpha}=1$

Coordinates of vertices are $(\pm\cos\alpha,0)$, eccentricity of the hyperbola is $e=\sqrt{1+\frac{\sin^2\alpha}{\cos^2\alpha}}=|\sec\alpha|$

$\therefore$ Coordinates of foci are thus $(\pm 1, 0)$, which are independent of α.

Directrix is $x = \pm\cos^2\alpha$

38. Equation of a common tangent to the curves $y^2 = 8x$ and $xy = -1$ is

(a) $3y = 9x + 2$ (b) $y = 2x + 1$ (c) $2y = x + 8$ (d) $y = x + 2$

Key. D

Sol. $y^2 = 8k, xy = -1$

Let $P\left(t, \frac{-1}{t}\right)$ be any point on xy = - 1

Equation of the tangent to $xy = -1$ at $P\left(t, \frac{-1}{t}\right)$ is

$$\frac{xy_1 + yx_1}{2} = -1$$

$$\frac{-x}{t} + yt = -2$$

$$y = \frac{x}{t^2} + \left(\frac{-2}{t}\right)................(1)$$

If (1) is tangent to the parabola $y^2 = 8x$ then

$$\frac{-2}{t} = \frac{2}{1/t^2} \Rightarrow t^3 = -1$$

$$t = -1$$

$\therefore$ Common tangent is y = x+2

39. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that OPQ is an equilateral triangle, O being the centre of the hyperbola. Then the eccentricity *e* of the hyperbola, satisfies

(a) $1 < e < 2/\sqrt{3}$ (b) $e = 2/\sqrt{3}$ (c) $e = \sqrt{3}/2$ (d) $e > 2/\sqrt{3}$

Key. D

Sol. If OPQ is equilateral triangle then OP makes 30^0 with x-axis.

$\left(\frac{\sqrt{3}r}{2}, \frac{r}{2}\right)$ ties on hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\Rightarrow r^2 = \frac{16a^2b^2}{12b^2 - 4a^2} > 0$$

$$\Rightarrow 12b^2 - 4a^2 > 0 \Rightarrow \frac{b^2}{a^2} > \frac{4}{12}$$

$$e^2 - 1 > \frac{1}{3}$$

$$e^2 > \frac{4}{3} \Rightarrow e > \frac{2}{\sqrt{3}}$$

40. The locus of a point, from where tangents to the rectangular hyperbola $x^2 - y^2 = a^2$ contain an angle of 45^0, is

(A) $\left(x^2+y^2\right)+a^2\left(x^2-y^2\right)=4a^2$ (B) $2\left(x^2+y^2\right)+4a^2\left(x^2-y^2\right)=4a^2$

(C) $\left(x^2+y^2\right)^2+4a^2\left(x^2-y^2\right)=4a^4$ (D) $\left(x^2+y^2\right)^2+a^2\left(x^2-y^2\right)=a^4$

Key. C

Sol. Equation of tangent to the hyperbola : $y = mx \pm \sqrt{m^2a^2 - a^2}$

$\Rightarrow$ Let $P(x_1, y_1)$ be locus

$\Rightarrow y - mx = \pm\sqrt{m^2a^2 - a^2}$

S.B.S

$\Rightarrow m^2\left(x_1^2 - a^2\right) - 2y_1x_1m + y_1^2 + a^2 = 0$

$$m_1 + m_2 = \frac{2x_1y_1}{x_1^2 - a^2};\ m_1m_2 = \frac{y_1^2 + a^2}{x_1^2 - a^2}$$

$$\tan 45^0 = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$$

$$\Rightarrow (1 + m_1m_2)^2 = (m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1m_2$$

$$\Rightarrow \left(1 + \frac{y_1 + a^2}{x_1^2 - a^2}\right) = \left(\frac{2x_1y_1}{x_1^2 - a^2}\right) - 4\left(\frac{y_1^2 + a^2}{x_1^2 - a^2}\right)$$

41. If a circle cuts the rectangular hyperbola xy=1 in 4 points (x_r, y_r) where r =1,2,3,4. Then ortho centre of triangle with vertices at (x_r, y_r) where r=1,2,3 is

1. (x_4, y_4) 2. $(-x_4, -y_4)$ 3. $(-x_4, +y_4)$ 4. $(+x_4, -y_4)$

Key. 2

Sol. $xy = 1$ cuts the circle in 4-points then $x_1x_2x_3x_4 = 1,\ y_1y_2y_3y_4 = 1$

Ortho centre of triangle with vertices $(x_1, y_1)(x_2, y_2)(x_3, y_3)$

Ie $\left(\dfrac{-1}{x_1x_2x_3}, -(y_1y_2y_3)^{-1}\right)$

$-(-x_4, -y_4)$

42. A hyperbola passing through origin has 3x – 4y – 1=0 and 4x – 3y – 6 = 0 as its asymptotes. Then the equations of its transverse and conjugate axes are

A) x – y – 5 = 0 and x + y + 1 = 0 B) x – y = 0 and x + y + 5 = 0

C) x + y – 5 = 0 and x – y – 1 = 0 D) x + y – 1 = 0 and x – y – 5 = 0

Key. C

Sol. Transverse and conjugate axes are the bisectors of the angle between asymptotes.

$$\frac{3x - 4y - 1}{5} = \pm\left(\frac{4x - 3y - 6}{5}\right) \text{ etc......}$$

43. If the asymptotes of the hyperbola $(x+y+1)^2-(x-y-3)^2=5$ cuts each other at A and the coordinate axes at B and C, then radius of the circle passing through the points A, B, C is

A) 3 B) $\frac{\sqrt{5}}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $\sqrt{3}$

Key. B

Sol. Centre of rectangular hyperbola (1, –2)

So equation of asymptotes are x = 1, y = –2

So radius of circle = $\frac{\sqrt{5}}{2}$

44. If a chord joining P($a\mathrm{Sec}\theta, a\tan\theta$), Q($a\mathrm{Sec}\alpha, a\tan\alpha$) on the hyperbola $x^2-y^2=a^2$ is the normal at P,then $\mathrm{Tan}\alpha=$

A) $\mathrm{Tan}\theta(4\sec^2\theta+1)$ B) $\mathrm{Tan}\theta(4\sec^2\theta-1)$ C) $\mathrm{Tan}\theta(2\mathrm{Sec}^2\theta-1)$ D) $\mathrm{Tan}\theta(1-2\mathrm{Sec}^2\theta)$

Key. B

Sol. Slope of chord joining P and Q = slope of normal at P

$$\frac{\mathrm{Tan}\alpha-\mathrm{Tan}\theta}{\sec\alpha-\sec\theta}=-\frac{\mathrm{Tan}\theta}{\sec\theta}\Rightarrow \mathrm{Tan}\alpha-\mathrm{Tan}\theta=-k\mathrm{Tan}\theta \text{ and } \sec\alpha-\sec\theta=k\sec\theta$$

$$\therefore(1-k)Tan\theta=Tan\alpha\to 1.\ (1+k)\sec\theta=\sec\alpha\to 2.$$

$$\left[(1+k)\sec\theta\right]^2-\left[(1-k)\mathrm{Tan}\theta\right]^2=\sec^2\alpha-\mathrm{Tan}^2\alpha$$

$$\Rightarrow k=-2\left(\sec^2\theta+\mathrm{Tan}^2\theta\right)=-4\sec^2\theta+2$$

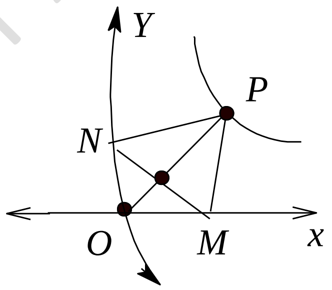
From (1) $Tan\alpha=Tan\theta\left(1+4\sec\theta^2-2\right)=\mathrm{Tan}\theta\left(4\sec\theta^2-1\right).$

45. PM and PN are the perpendiculars from any point P on the rectangular hyperbola $xy=c^2$ to the asymptotes. If the locus of the mid point of MN is a conic, then its eccentricity is

A) $\sqrt{3}$ B) $\sqrt{2}$ C) $\frac{1}{\sqrt{3}}$ D) $\frac{1}{\sqrt{2}}$

Key. B

Sol. OMPN is rectangle.



$$P=\left(Ct,\frac{c}{t}\right)$$

Mid point = $\left(\frac{ct}{2},\frac{c}{2t}\right)=(x,y)$ $\therefore xy=\frac{c^2}{4}\Rightarrow e=\sqrt{2}$

46. A variable straight line of slope 4 intersects the hyperbola xy = 1 at two points. The locus of the point which divides the line segment between these two points in the ratio 1 : 2 is

A) $16x^2+10xy+y^2=2$ B) $16x^2-10xy+y^2=2$

C) $16x^2+10xy+y^2=4$ D) $16x^2-10xy+y^2=4$

Key. A

Sol. Let P(h, k)

y - k = 4(x - h) --- (1)

Let it meets xy = 1 ----(2) at A (x_1, y_1) and B (x_2, y_2)

$$x_1+x_2=\frac{4h-k}{4}, x_1x_2=-\frac{1}{4} \text{ Also } \Rightarrow \therefore \frac{2x_1+x_2}{3}=h \Rightarrow x_1=\frac{8h+k}{4}, x_2=\frac{2h+k}{2}$$

$$\Rightarrow 16x^2+10xy+y^2=2$$

47. From a point *P* on the hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$ straight lines are drawn parallel to the asymptotes of the hyperbola. Then the area of parallelogram formed by the asymptotes and the two lines through P is

A) dependent on coordinates of P B) 4 C) 6 D) $8\sqrt{2}$

Key. B

Sol. Area of parallelogram is $\frac{ab}{2}=\frac{4\times 2}{2}=4$

48. The eccentricity of the conic defined by $\left|\sqrt{(x-1)^2+(y-2)^2}-\sqrt{(x-5)^2+(y-5)^2}\right|=3$

A) 5/2 B) 5/3 C) $\sqrt{2}$ D) $\sqrt{11}/3$

Key. B

Sol. Hyperbola for which (1, 2) and (5, 5) are foci and length of transverse axis 3.

$2ae=5$ and $2a=3$ $\therefore e=5/3$

49. The asymptotes of a hyperbola are $3x-4y+2=0$ and $5x+12y-4=0$. If the hyperbola passes through the point (1, 2) then slope of transverse axis of the hyperbola is

A) 6 B) −7/2 C) −8 D) 1/8

Key. C

Sol. Axes of hyperbola are bisectors of angles between asymptotes.

50. If P is a point on the rectangular hyperbola $x^2 - y^2 = a^2$, C being the center and S, S' are two foci, then $SP.S'P$ =

a) 2 b) $(CP)^2$ c) $(CS)^2$ d) $(SS')^2$

Key. B

Sol. Let P = (a sec θ, a tan θ), $S_1 S^1 = \left(\pm a\sqrt{2}, 0\right)$

SP= $a\left(\sqrt{2}\sec\theta-1\right)$, S^1 P = $a\left(\sqrt{2}\sec\theta+1\right)$

SP-S^1P = a^2 ($\sec^2\theta + \tan^2\theta$) = CP^2

Hyperbola

Integer Answer Type

1. If P (x, y) satisfy $x^2 + y^2 = 1$. Let maximum value of $(x+y)^2$ is λ then number of tangents from $(\lambda, 0)$ to hyperbola $(x-2)^2 - y^2 = 1$ are

Key. 2

Sol. Let P(x, y) = $(\cos\theta, \sin\theta)$

$\therefore \lambda = 2$

No. of tangents from (2, 0) are 0

2. Acute angle between the asymptotes of the hyperbola $x^2 + 2xy - 3y^2 + x + 7y + 9 = 0$ is θ. Then $\tan\theta =$

Key. 2

Sol. Equation of hyperbola is

$$x^2 + 2xy - 3y^2 + x + 7y + 9 = 0$$

The combined equation of asymptotes is $x^2 + 2xy - 3y^2 + x + 7y + K = 0$

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{|a+b|} = \frac{2\sqrt{1+3}}{1-3} = 2$$

3. The equation of Asymptotes of $xy + 2x + 4y + 6 = 0$ is $xy + 2x + 4y + c = 0$, then C = ___

Key. 8

Sol. $xy + 2x + 4y + C = 0$ represents pair of lines $\Rightarrow C = 8$

4. The equation of Asymptotes of $xy + 2x + 4y + 6 = 0$ is $xy + 2x + 4y + c = 0$, then C = ___

Key. 8

Sol. $xy + 2x + 4y + C = 0$ represents pair of lines $\Rightarrow C = 8$

5. Let PN be the ordinate of a point P on the hyperbola $\frac{x^2}{(97)^2} - \frac{y^2}{(79)^2} = 1$ and the tangent at P meets the transverse axis in T, O is the origin. Then $\left[\frac{ON.OT}{2011}\right]$ is equal to (where [.] denotes G.I.F)

Key. 4

Sol. $ON.OT = 97\cos\theta . 97\sec\theta = 97^2$

$$\therefore \left[\frac{ON.OT}{2011}\right] = \left[\frac{97^2}{2011}\right] = 4$$

6. If e is the eccentricity of the hyperbola $(5x - 10)^2 + (5y + 15)^2 = (12x - 5y + 1)^2$ then $\frac{25e}{13}$ is equal to ....

Key. 5

Sol. Equation can be rewritten as $\sqrt{(x-2)^2 + (y+3)^2} = \frac{13}{5}\left|\frac{12x - 5y + 1}{13}\right|$

So, e = $\frac{13}{5}$.

7. If a variable tangent of the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$, cuts the circle $x^2 + y^2 = 4$ at point A, B and locus of mid point of AB is $9x^2 - 4y^2 - \lambda (x^2 + y^2)^2 = 0$ then λ is ....

Key. 1

Sol. Equation of chord of circle with mid point (h, k) is xh + xk = $h^2 + k^2$ or

$y=\left(\frac{-h}{k}\right)x+\frac{h^2+k^2}{k}$, it touches the hyperbola

8. If the angle between the asymptotes of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{\pi}{3}$. Then the eccentricity of conjugate hyperbola is

Key. 2

Sol. $2\tan^{-1}\left(\frac{b}{a}\right)=\frac{\pi}{3}$

$\frac{b}{a}=\frac{1}{\sqrt{3}}$

$e^2=1+\frac{1}{3}=\frac{4}{3}$

$\frac{1}{e'^2}+\frac{1}{e^2}=1$

$\Rightarrow \quad \frac{1}{e'^2}+\frac{3}{4}=1$

$\Rightarrow \quad \frac{1}{e'^2}=\frac{1}{4}\Rightarrow e'=2$

9. If PN be the ordinate of a point P on the hyperbola $\frac{x^2}{(97)^2}-\frac{y^2}{(79)^2}=1$ and the tangent at P meets the transverse axis in T, O is the origin; then $\left[\frac{ON.OT}{7999}\right]$ is...... (where [.] denotes greatest integer function).

Key. 1

10. If the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide, then b =

Key. 4

11. The equation $\frac{x^2}{9-\lambda}+\frac{y^2}{4-\lambda}=1$ represents a hyperbola when $a<\lambda<b$ then $\left[\frac{b+a}{b-a}\right]=$

Where [.] denotes greatest integer function.

Key. 2

Sol. $(9-\lambda)(4-\lambda)<0\Rightarrow 4<\lambda<9\Rightarrow\left[\frac{b+a}{b-a}\right]=\left[\frac{13}{5}\right]=2$

12. If CP, CD are semiconjugate diameters of $5(x-2)^2+4(y-3)^2=20$, then $CP^2+CD^2=$

Key. 9

Sol. $CP^2+CD^2=a^2+b^2$

13. If a hyperbola passes through the focus of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and its transverse and conjugate axes coincides with the major and minor axes of the ellipse, and the product of eccentricities is 1, represented by the equation $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then the value of b^2-a^2 is

Key. 7

Sol. Using the hypothesis, we get equation to hyperbola as $\frac{x^2}{9}-\frac{y^2}{16}=1 \Rightarrow b^2-a^2=7$

14. The product of perpendiculars from any point on the hyperbola $\frac{x^2}{4}-\frac{3y^2}{4}=1$ to its asymptotes is $\frac{1}{K}$, then $K=$

Key. 1

Sol. Product of perpendiculars from any point on the hyperbola to its asymptotes $=\frac{a^2b^2}{a^2+b^2}=\frac{1}{\frac{1}{a^2}+\frac{1}{b^2}}=1$

15. Chords of the hyperbola $x^2-y^2=a^2$ touch the parabola $y^2=4a\,x$. Prove that the locus of their middle points is the curve $y^2(x-a)=x^3$.

Ans. Hence locus is $x^3=y^2(x-a)$.

Sol. Let P(h, k) be midpoint of chords so their equation is $T=S_1$

i.e. $xh-yk=h^2-k^2$ …(i)

Also equation of tangent to the parabola $y^2=4ax$ is

$y=mx+\frac{a}{m}$ …(ii)

$\therefore$ comparing (i) and (ii), we get

$m=\frac{h}{k}$ and $\frac{a}{m}=\frac{k^2-h^2}{k} \Rightarrow \frac{ak}{h}=\frac{k^2-h^2}{k} \Rightarrow \quad h^3=k^2(h-a)$

Hence locus is $x^3=y^2(x-a)$.

16. Prove that chord of a hyperbola, which touches the conjugate hyperbola, is bisects at the point of contact.

Sol. Let $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ …(i) be the hyperbola, then its conjugate hyperbola is

$\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$ …(ii)

Let any point on (ii) be ($(a\tan\theta, b\sec\theta)$, then equation of the tangent to (ii) at this point is

$\frac{y\sec\theta}{b}-\frac{x\tan\theta}{a}=1=\sec^2\theta-\tan^2\theta$

i.e. $\frac{x\tan\theta}{a}-\frac{y\sec\theta}{b}-1=\frac{a^2\tan^2\theta}{a^2}-\frac{b^2\sec^2\theta}{a^2}-1$

which is the equation of the chord of (i) whose mid point is $(a\tan\theta, b\sec\theta)$. Hence the result

17. The asymptotes of a hyperbola are parallel to 2 x + 3y = 0 & 3 x + 2 y = 0. Its centre is (1, 2) & it passes through (5, 3). Find the equation of the hyperbola.

Ans. $(2x+3y-8)(3x+2y-7)-154=0$

Sol. Let the asymptotes be $2x+3y+\lambda=0$ and $3x+2y+\mu=0$. Since asymtotes passes through (1, 2), then $\lambda=-8$ and $\mu=-7$

Thus the equation of asymptotes are

2x + 3y – 8 = 0 and 3x + 2y – 7 = 0

Let the equation of hyperbola be

$$(2x + 3y - 8)(3x + 2y - 7) + v = 0$$

It passes through (5, 3), then

$$(10 + 9 - 8)(15 + 6 - 7) + v = 0$$

$\Rightarrow$ $$11 \times 14 + v = 0$$

$\therefore$ $$v = -154$$

putting the value of v in (1) we obtain

$$(2x + 3y - 8)(3x + 2y - 7) - 154 = 0$$

which is the equation of required hyperbola.