Ellipse

Single Correct Answer Type

1. If a variable tangent of the circle $x^2 + y^2 = 1$ intersect the ellipse $x^2 + 2y^2 = 4$ at P and Q then the locus of the points of intersection of the tangents at P and Q is

A. a circle of radius 2 units

B. a parabola with fouc as (2, 3)

C. an ellipse with eccentricity $\frac{\sqrt{3}}{4}$

D. an ellipse with length of latus rectrum is 2 units

Key. D

Sol. $x^2 + y^2 = 1; x^2 + 2y^2 = 4$

Let $R(x_1, y_1)$ is pt of intersection of tangents drawn at P,Q to ellipse

$\Rightarrow PQ$ is chord of contact of $R(x_1, y_1)$

$\Rightarrow xx_1 + 2yy_1 - 4 = 0$

This touches circle $\Rightarrow r^2(\ell^2 + m^2) = n^2$

$\Rightarrow 1(x_1^2 + 4y_1^2) = 16$

$\Rightarrow x^2 + 4y^2 = 16$ is ellipse $e = \frac{\sqrt{3}}{2}; LL^1 = 2$

2. A circle S = 0 touches a circle $x^2 + y^2 - 4x + 6y - 23 = 0$ internally and the circle $x^2 + y^2 - 4x + 8y + 19 = 0$ externally. The locus of centre of the circle S = 0 is conic whose eccentricity is k then $\left[\frac{1}{k}\right]$ is where [.] denotes G.I.F

A. 7   B. 2   C. 0   D. 3

Key. A

Sol. $c_1(2, -3) r_1 = 6$

$c_2(2, -4) r_2 = 1$

Let C is the center of S = 0

$$\therefore \left.\begin{array}{l} cc_1 = r_1 - r \\ cc_2 = r_1 + r \end{array}\right\} \Rightarrow cc_1 + cc_2 = r_1 + r_2$$

$\therefore$ Locus is an ellipse whose foci are (2, -3) & (2, -4)

$$e=\frac{2ae}{2a}=\frac{c_1c_2}{r_1+r_2}=\frac{1}{7}\Rightarrow k=\frac{1}{7}$$

3. If circum centre of an equilateral triangle inscribed in $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with vertices having eccentric angles α,β,γ respectively is (x_1,y_1) then $\sum\cos\alpha\cos\beta+\sum\sin\alpha\sin\beta=$

A. $\frac{9x_1^2}{a^2}+\frac{9y_1^2}{b^2}+\frac{3}{2}$ B. $9x_1^2-9y_1^2+a^2b^2$ C. $\frac{9x_1^2}{2a^2}+\frac{9y_1^2}{2b^2}-\frac{3}{2}$ D. $\frac{9x_1^2}{a^2}+\frac{9y_1^2}{b^2}+3$

Key. C

Sol. $(x_1,y_1)=\left(\frac{a\sum\cos\alpha}{3},\frac{b\sum\sin\alpha}{3}\right)$

$$\sum\cos\alpha=\frac{3x_1}{a}\text{.............(1)}$$

$$\sum\sin\alpha=\frac{3y_1}{b}\text{.............(2)}$$

Squarding & adding

4. The ratio of the area enclosed by the locus of mid-point of PS and area of the ellipse where P is any point on the ellipse and S is the focus of the ellipse, is

A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{1}{5}$ D. $\frac{1}{4}$

Key. D

Sol. Ellipse equation is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, Area = πab

Let $P=(a\cos\theta,b\sin\theta)$

$S=(ae,0)$

M(h,k) mid point of PS

$$\Rightarrow h=\frac{ae+a\cos\theta}{2};k=\frac{b\sin\theta}{2}$$

$$=\frac{h-\frac{ae}{2}}{a/2}+\frac{k^2}{(b^2/4)}=1\text{, locus of (h,k) is ellipse}$$

Area $= \pi\left(\frac{a}{2}\right)\left(\frac{b}{2}\right) = \frac{1}{4}\pi ab$

5. How many tangents to the circle $x^2 + y^2 = 3$ are there which are normal to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$

A) 3 B) 2 C) 1 D) 0

Key. D

Sol. Equation of normal at $p(3\cos\theta, 2\sin\theta)$ is $3x\sec\theta - 2y\operatorname{cosec}\theta = 5$

$$\frac{5}{\sqrt{9\sec^2\theta + 4\operatorname{cosec}^2\theta}} = \sqrt{3}$$

But Min. of $9\sec^2\theta + 4\operatorname{cosec}^2\theta = 25$

$\therefore$ no such $\overset{-1}{\theta}$ exists.

6. If the ellipse $\frac{x^2}{a^2-3} + \frac{y^2}{a+4} = 1$ is inscribed in a square of side length $a\sqrt{2}$ then a is

A) 4 B) 2 C) 1 D) None of these

Key. D

Sol. Sides of the square will be perpendicular tangents to the ellipse so, vertices of the square will lie on director circle. So diameter of director circle is

$$2\sqrt{(a^2-3)+(a+4)} = \sqrt{2a^2+2a^2}$$

$$2\sqrt{a^2+a+1} = 2a \Rightarrow a = -1$$

But for ellipse $a^2 > 3 \;\&\; a > -4$

So a cannot take the value '-1'

7. Let 'O' be the centre of ellipse for which A,B are end points of major axis and C,D are end points of minor axis, F is focus of the ellipse. If in radius of ΔOCF is '1' then $|AB| \times |CD| =$

A) 65 B) 52 C) 78 D) 47

Key. A

Sol. $r = \frac{\Delta}{S} \Rightarrow \Delta = S$

$$\frac{1}{2}(ae)b = \frac{ae + b + \sqrt{a^2e^2 + b^2}}{2}$$

$$ae = 6 \Rightarrow 6b = 6 + b + \sqrt{36 + b^2} \Rightarrow b = \frac{5}{2}$$

$$\Rightarrow a^2\left(1 - e^2\right) = \frac{25}{4} \Rightarrow a^2 - 36 = \frac{25}{4} \Rightarrow a = \frac{13}{2}$$

8. If the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ meet the ellipse $\frac{x^2}{1} + \frac{y^2}{a^2} = 1$ in four distinct points and $a = b^2 - 10b + 25$, then the value b does not satisfy

1. $(-\infty, 4)$ 2. (4, 6) 3. $(6, \infty)$ 4. [4, 6]

Key. 4

Sol. a > 1

9. The perimeter of a triangle is 20 and the points (-2, -3) and (-2, 3) are two of the vertices of it. Then the locus of third vertex is :

1. $\frac{(x-2)^2}{49} + \frac{y^2}{40} = 1$ 2. $\frac{(x+2)^2}{49} + \frac{y^2}{40} = 1$ 3. $\frac{(x+2)^2}{40} + \frac{y^2}{49} = 1$ 4. $\frac{(x-2)^2}{40} + \frac{y^2}{49} = 1$

Key. 3

Sol. PA + PB + AB = 20 where A & B are foci

10. Tangents are drawn from any point on the circle $x^2+y^2=41$ to the Ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ then the angle between the two tangents is

1. $\frac{\pi}{4}$ 2. $\frac{\pi}{3}$ 3. $\frac{\pi}{6}$ 4. $\frac{\pi}{2}$

Key. 4

Sol. Director circle

11. The area of the parallelogram formed by the tangents at the points whose eccentric angles are $\theta, \theta + \frac{\pi}{2}, \theta + \pi, \theta + \frac{3\pi}{2}$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

1. ab 2. 4ab 3. 3ab 4. 2ab

Key. 2

Sol. Put $\theta = 0^0$

12. A normal to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the axes in L and M. The perpendiculars to the axes through L and M intersect at P. Then the equation to the locus of P is

1. $a^2x^2-b^2y^2=\left(a^2+b^2\right)^2$ 2. $a^2x^2+b^2y^2=\left(a^2+b^2\right)^2$

3. $b^2x^2-a^2y^2=\left(a^2-b^2\right)^2$ 4. $a^2x^2+b^2y^2=\left(a^2-b^2\right)^2$

Key. 4

Sol. $P=\left(x_1,y_1\right),\frac{x}{x_1}+\frac{y}{y_1}=1$ Apply normal condition

13. The points of intersection of the two ellipse $x^2+2y^2-6x-12y+23=0$, $4x^2+2y^2-20x-12y+35=0$

1. Lie on a circle centered at $\left(\frac{8}{3},3\right)$ and of radius $\frac{1}{3}\sqrt{\frac{47}{2}}$

2. Lie on a circle centered at $\left(\frac{8}{3},-3\right)$ and of radius $\frac{1}{3}\sqrt{\frac{47}{3}}$

3. Lie on a circle centered at (8,9) and of radius $\frac{1}{3}\sqrt{\frac{47}{2}}$

4. Are not concyclic

Key. 1

Sol. If $S_1 = 0$ and $S_2 = 0$ are the equations, Then $\lambda S_1+S_2=0$ is a second degree curve passing through the points of intersection of $S_1 = 0$ and $S_2 = 0$

$$\Rightarrow(\lambda+4)x^2+2(\lambda+1)y^2-2(3\lambda+10)x-12(\lambda+1)y+(23\lambda+35)=0$$

For it to be a circle, choose λ such that the coefficients of x^2 and y^2 are equal $\therefore\lambda=2$

This gives the equation of the circle as

$$6\left(x^2+y^2\right)-32x-36y+81=0\{u\sin g(1)\}$$

$$\Rightarrow x^2+y^2-\frac{16}{3}x-6y+\frac{27}{2}=0$$

Its centre is $C\left(\frac{8}{3},3\right)$ and radius is

r = $\sqrt{\frac{64}{9}+9-\frac{27}{2}}=\frac{1}{3}\sqrt{\frac{47}{2}}$

14. In a model, it is shown that an arc of a bridge is semi elliptical with major axis horizontal. If the length of the base is 9m and the highest part of the bridge is 3m from the horizontal; then the height of the arch, 2m from the centre of the base is (in meters)

1. $\frac{8}{3}$ 2. $\frac{\sqrt{65}}{3}$ 3. $\frac{\sqrt{56}}{3}$ 4. $\frac{9}{3}$

Key. 2

Sol. Let the equation of the semi elliptical are be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(y>0)$

Length of the major axis = 2a = 9 $\Rightarrow$ a = 9/2

So the equation of the arc becomes $\frac{4x^2}{81}+\frac{y^2}{9}=1$

If x=2, then $y^2=\frac{65}{9}\Rightarrow y=\frac{1}{3}\sqrt{65}$

15. If a tangent of slope 2 of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is normal to the circle $x^2+y^2+4x+1=0$ then the maximum value of ab is

1. 2 2. 4 3. 6 3. Can n't be found

Key. 2

Sol. A tangent of slope 2 is $y=2x\pm\sqrt{4a^2+b^2}$ this is normal to $x^2+y^2+4x+1=0$ then

$0=-4\pm\sqrt{4a^2+b^2}\Rightarrow 4a^2+b^2=16$ using $Am\geq GM$

$ab\leq 4$

16. The distance between the polars of the foci of the Ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ w.r.to itself is

1. $\frac{25}{2}$ 2. $\frac{25}{9}$ 3. $\frac{25}{8}$ 4. $\frac{25}{3}$

Key. 1

Sol. $\frac{2a}{e}$

17. An ellipse passing through origin has its foci at (5, 12) and (24, 7). Then its eccentricity is

1. $\frac{\sqrt{386}}{38}$ 2. $\frac{\sqrt{386}}{39}$ 3. $\frac{\sqrt{386}}{47}$ 4. $\frac{\sqrt{386}}{51}$

Key. 1

Sol. Conceptual

18. If $e=\frac{\sqrt{3}}{2}$, its length of latusrectum is

1. $\frac{1}{2}$ (length of major axis)
2. $\frac{1}{3}$ (length of major axis)
3. $\frac{1}{4}$ (length of major axis)
4. Length of major axis

Key. 3

Sol. $L.L.R=\frac{2b^2}{a}$

19. Number of normals that can be drawn from the point (0, 0) to $3x^2+2y^2=30$ are

1. 2 2. 4 3. 1 4. 3

Key. 2

Sol. It is centre

20. A tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ cuts the axes in M and N. Then the least length of MN is

1. a + b 2. a – b 3. $a^2 + b^2$ 4. $a^2 – b^2$

Key. 1

Sol. Standard

21. $p(\theta),\ D\left(\theta+\frac{\pi}{2}\right)$ are two points on the Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ Then the locus of point of intersection of the two tangents at P and D to the ellipse is

1. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{4}$ 2. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=4$ 3. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$ 4. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{2}$

Key. 3

Sol. $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ → 1eq

$\frac{x}{a}\cos\left(\frac{\pi}{2}+\theta\right)+\frac{y}{b}\sin\left(\frac{\pi}{2}+\theta\right)=1$ → 2eq

Eliminate θ from 1 and 2

22. The abscissae of the points on the ellipse $9x^2+25y^2-18x-100y-116=0$ lie between

1. 3, -5 2. -4, 6 3. 5, 7 4. 2, 5

Key. 2

Sol. $-5\geq x-1\leq 5$

23. Tangents to the ellipse $b^2x^2+a^2y^2=a^2b^2$ makes angles $\theta_1 \, and \, \theta_2$ with major axis such that $\cot\theta_1+\cot\theta_2=k$. Then the locus of the point of intersection is

1. $xy=2k(y^2+b^2)$ 2. $2xy=k(y^2-b^2)$ 3. $4xy=k(y^2-b^2)$ 4. $8xy=k(y^2-b^2)$

Key. 2

Sol. Apply sum of the slopes = $\frac{2x_1y_1}{x_1^2-a^2}$

24. The equation $\frac{x^2}{10-a}+\frac{y^2}{4-a}=1$ represents an ellipse if

1. a < 4 2. a > 4 3. 4 <a < 10 4. a > 10

Key. 1

Sol. 10 – a > 0, 4 – a > 0

25. The locus of the feet of the perpendiculars drawn from the foci of the ellipse S=0 to any tangent to it is

1. a circle 2. an ellipse 3. a hyperbola 4. not a conic

Key. 1

Sol. Standard

26. If the major axis is “n” (n>1) times the minor axis of the ellipse, then eccentricity is

1. $\frac{\sqrt{n-1}}{n}$ 2. $\frac{\sqrt{n-1}}{n^2}$ 3. $\frac{\sqrt{n^2-1}}{n^2}$ 4. $\frac{\sqrt{n^2-1}}{n}$

Key. 4

Sol. 2a = n(2b)

$\Rightarrow n=\frac{a}{b}$

$\therefore e=\sqrt{\frac{a^2-b^2}{a^2}}=\sqrt{1-\frac{b^2}{a^2}}=$

$\sqrt{1-\frac{1}{n^2}}=\frac{\sqrt{n^2-1}}{n}$

27. If $\left(\sqrt{3}\right)bx+ay=2ab$ is tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then eccentric angle θ is

1. $\frac{\pi}{4}$ 2. $\frac{\pi}{6}$ 3. $\frac{\pi}{2}$ 4. $\frac{\pi}{3}$

Key. 2

Sol. Equation of tangent at a point $\left(a\cos\theta, b\sin\theta\right)$ is $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$

But, it is the same as $\frac{x}{a}\frac{\sqrt{3}}{2}+\frac{y}{b}.\frac{1}{2}=1$

$\therefore\cos\theta=\frac{\sqrt{3}}{2},\sin\theta=\frac{1}{2}\Rightarrow\theta=\frac{\pi}{6}$

28. If PSQ is a focal chord of the ellipse $16x^2+25y^2=400$ such that SP = 8 then the length of SQ =

1. 2 2. $\frac{11}{3}$ 3. 16 4. 25

Key. 1

Sol. $\frac{1}{SP}+\frac{1}{SQ}=\frac{2a}{b^2}$

29. A man running round a race course notes that the sum of the distances of two flag posts from him is 8 meters. The area of the path he encloses in square meters if the distance between flag posts is 4 is

1. $15\sqrt{3}\pi$ 2. $12\sqrt{3}\pi$ 3. $18\sqrt{3}\pi$ 4. $8\sqrt{3}\pi$

Key. 4

Sol. Area = πab

30. The locus of point of intersection of the two tangents to the ellipse $b^2x^2+a^2y^2=a^2b^2$ which makes an angle 60^0 with one another is

1. $4\left(x^2+y^2-a^2-b^2\right)^2=3\left(b^2x^2+a^2y^2-a^2b^2\right)$

2. $3\left(x^2+y^2-a^2-b^2\right)^2=4\left(b^2x^2+a^2y^2-a^2b^2\right)$

3. $3\left(x^2+y^2-a^2-b^2\right)^2=2\left(b^2x^2+a^2y^2-a^2b^2\right)$

4. $3\left(x^2+y^2-a^2-b^2\right)^2=\left(b^2x^2+a^2y^2-a^2b^2\right)$

Key. 2

Sol. $Tan\theta=\frac{2ab\sqrt{S_{11}}}{x_1^{\,2}+y_1^{\,2}-a^2-b^2}$

31. If the equation of the chord joining the points $P(\theta)$ and $D\left(\theta+\frac{\pi}{2}\right)$ on $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $x\cos\alpha+y\sin\alpha=p$ then $a^2\cos^2\alpha+b^2\sin^2\alpha=$

1. $4p^2$ 2. p^2 3. $\frac{p^2}{2}$ 4. $2p^2$

Key. 4

Sol. $\frac{x}{a}\cos\left(\frac{\theta+\theta+\frac{\pi}{2}}{2}\right)+\frac{y}{b}\sin\left(\frac{\theta+\theta+\frac{\pi}{2}}{2}\right)$

$=\cos\left(\frac{\theta-\theta-\frac{\pi}{2}}{2}\right)$ → 1eq

$x\cos\alpha+y\sin\alpha=P$ → 2eq

(1) = (2)

32. The locus of mid point of chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which passes through the foot of the directrix from focus is

1. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{ex}{a^2}$ 2. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{x}{ae}$ 3. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{x}{a^2e}$ 4. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{x}{ae^2}$

Key. 2

Sol. $S_1 = S_{11}$ passes through $\left(\frac{a}{e},0\right)$

33. Consider two points A and B on the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$, circles are drawn having segments of tangents at A and B in between tangents at the two ends of major axis of ellipse as diameter, then the length of common chord of the circles is

A) 8 B) 6 C) 10 D) $4\sqrt{2}$

Key. A

Sol. All such circles pass through foci ∴ The common chord is of the length 2ae $10\times\frac{4}{5}=8$

34. If 'CF' is the perpendicular from the centre C of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ on the tangent at any point P and G is the point where the normal at P meets the major axis, then CF.PG is

A) b^2 B) $2b^2$ C) $\frac{b^2}{2}$ D) $3b^2$

Key. A

Sol. $CF=\frac{ab}{\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}}$ $PG=\frac{b}{a}\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}$

35. The line passing through the extremity A of the major axis and extremity B of the minor axis
of the ellipse $x^2+9y^2=9,$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin 'O' is

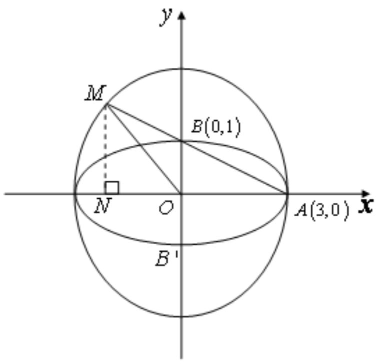
A) $\frac{31}{10}$ B) $\frac{29}{10}$ C) $\frac{21}{10}$ D) $\frac{27}{10}$

Key. D

Sol. Equation of given ellipse is $\frac{x^2}{9}+\frac{y^2}{1}=1$

Equation of auxiliary circle is $x^2+y^2=9.........(1)$

Equation of line AB is $\frac{x}{3}+\frac{y}{1}=1\Rightarrow x=3(1-y)$



Putting this in (1), we get $9(1-y)^2+y^2=9\Rightarrow 10y^2-18y=0\Rightarrow y=0,\frac{9}{5}$

Thus, y coordinate of 'M' is $\frac{9}{5}$

$\Delta OAM=\left(\frac{1}{2}\right)(OA)(MN)=\frac{1}{2}(3)\frac{9}{5}=\frac{27}{10}$

36. The normal at an end of a latus rectum of the ellipse $x^2/a^2 + y^2/b^2 = 1$ passes through an end of the minor axis if

(a) $e^4 + e^2 = 1$ (b) $e^3 + e^2 = 1$ (c) $e^2 + e = 1$ (d) $e^3 + e = 1$

Key. A

Sol. Given ellipse equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Let $P\left(ae, \frac{b^2}{a}\right)$ be one end of latus rectum.

Slope of normal at $P\left(ae, \frac{b^2}{a}\right) = \frac{1}{e}$

Equation of normal is

$$y = \frac{b^2}{a} = \frac{1}{e}(x - ae)$$

It passes through $B^{'}(0, b)$ then

$$b - \frac{b^2}{a} = -a$$

$$a^2 - b^2 = -ab$$

$$a^4e^4 = a^2b^2$$

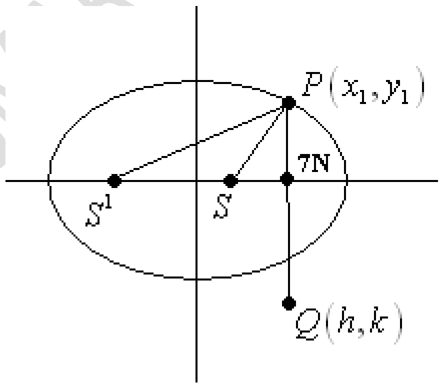
$$e^4 + e^2 = 1$$

37. From any point P lying in first quadrant on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, PN is drawn perpendicular to the major axis such that N lies on major axis. Now PN is produced to the point Q such that NQ equals to PS, where S is a focus. The point Q lies on which of the following lines

(A) $2y - 3x - 25 = 0$ (B) $3x + 5y + 25 = 0$

(C) $2x - 5y - 25 = 0$ (D) $2x - 5y + 25 = 0$

Key. B



Sol.

$$a^2 = 25$$

$$b^2 = 16$$

$e=\sqrt{\frac{25-16}{25}}=\frac{3}{5}$

Let point Q be (h, k), where K < 0

Given that $|K|=a+eh\left(as\ x_1=h\right)$

$-y=a+ex$

$-y=5+\frac{3}{5}x$

$3x+5y+25=0$

38. A circle of radius 'r' is concentric with the Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Then inclination of common tangent with major axis is _______(b<r<a)

1. $\tan^{-1}\left(\frac{b}{a}\right)$ 2. $\tan^{-1}\left(\frac{rb}{a}\right)$ 3. $\tan^{-1}\sqrt{\frac{r^2-b^2}{a^2-r^2}}$ 4. $\frac{\pi}{2}$

Key. 3

Sol. The tangent of Ellipse is $y=mx+\sqrt{a^2m^2+b^2}$, this line touches $x^2+y^2=r^2$

Condition is $\left|\frac{\sqrt{a^2m^2+b^2}}{\sqrt{m^2+1}}\right|=r$

$a^2m^2+b^2=r^2m^2+r^2$

$m^2\left(a^2-r^2\right)=r^2-b^2 \Rightarrow m^2=\frac{r^2-b^2}{a^2-r^2}$

$m=\sqrt{\frac{r^2-b^2}{a^2-r^2}}$

Inclimation is $\tan^{-1}\sqrt{\frac{r^2-b^2}{a^2-r^2}}$

39. A circle cuts the X-axis and Y-axis such that intercept on X-axis is a constant a and intercept on Y-axis is a constant b. Then eccentricity of locus of centre of circle is

1. 1 2. $\frac{1}{2}$ 3. $\sqrt{2}$ 4. $\frac{1}{\sqrt{2}}$

Key. 3

Sol. Locus of centre of circle is a rectangular hyperbola hence its eccentricity is $\sqrt{2}$

40. Consider two points A and B on the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$, circles are drawn having segments of tangents at A and B in between tangents at the two ends of major axis of ellipse as diameter, then the length of common chord of the circles is

A) 8 B) 6 C) 10 D) $4\sqrt{2}$

Key. A

Sol. All such circles pass through foci $\therefore$ The common chord is of the length 2ae

$$10\times\frac{4}{5}=8$$

41. If 'CF' is the perpendicular from the centre C of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ on the tangent at any point P and G is the point where the normal at P meets the major axis, then CF.PG is

A) b^2 B) $2b^2$ C) $\frac{b^2}{2}$ D) $3b^2$

Key. A

Sol. $CF=\frac{ab}{\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}}$ $PG=\frac{b}{a}\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}$

42. The line passing through the extremity A of the major axis and extremity B of the minor axis
of the ellipse $x^2+9y^2=9$, meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin 'O' is

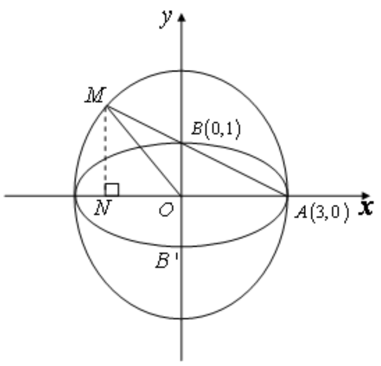
A) $\frac{31}{10}$ B) $\frac{29}{10}$ C) $\frac{21}{10}$ D) $\frac{27}{10}$

Key. D

Sol. Equation of given ellipse is $\frac{x^2}{9}+\frac{y^2}{1}=1$

Equation of auxiliary circle is $x^2+y^2=9.........(1)$

Equation of line AB is $\frac{x}{3}+\frac{y}{1}=1\Rightarrow x=3(1-y)$



Putting this in (1), we get $9(1-y)^2+y^2=9 \Rightarrow 10y^2-18y=0 \Rightarrow y=0,\frac{9}{5}$

Thus, y coordinate of 'M' is $\frac{9}{5}$

$\Delta OAM=\left(\frac{1}{2}\right)(OA)(MN)=\frac{1}{2}(3)\frac{9}{5}=\frac{27}{10}$

43. If $2x^2+y^2-24y+80=0$ then maximum value of x^2+y^2 is

A. 20 B. 40

C. 200 D. 400

Key. D

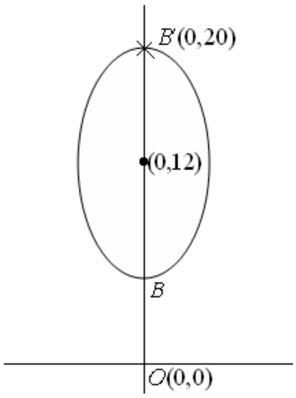
Sol. Given equation is $2x^2+y^2-24y+80=0$

$2x^2+(y-12)^2=64$

$\frac{x^2}{32}+\frac{(y-12)^2}{64}=1$

If is an ellipse with center (0, 12)

If (x, y) is any point on this distance from origin is $\sqrt{x^2+y}$



x^2+y^2 is max If $\sqrt{x^2+y^2}$ is

max

$B^1(1,\infty)$ is at max distance from 0

$\therefore \max(x^2+y^2)=400$

44. An ellipse whose foci $(2,4)$ $(14,9)$ touches x –axis then its eccentricity is

A. $\frac{13}{\sqrt{313}}$ B. $\frac{1}{\sqrt{313}}$

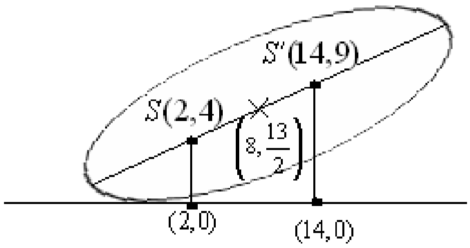
C. $\frac{2}{\sqrt{313}}$ D. $\frac{1}{\sqrt{13}}$

Key. A

Sol. Equation of aurally circle $(x-8)^2+\left(y-\frac{13}{2}\right)^2=a^2$

(2, 0) lies on it

$36+\frac{169}{4}=a^2 \Rightarrow \frac{313}{4}=a^2$



$a=\frac{\sqrt{313}}{2}$

But $SS'=2ae$

$\sqrt{144+25}=2ae$

$13=2ae$

$e=\frac{13}{2a}=\frac{13}{\sqrt{313}}$

45. A circle of radius 2 is concentric with the ellipse $\frac{x^2}{7}+\frac{y^2}{3}=1$ then inclination of common tangent with X-axis

A. $\frac{\pi}{2}$ B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$ D. $\frac{\pi}{6}$

Key. D

Sol. tangent is $y=mx+\sqrt{7m^2+3}$

$\frac{x^2}{7}+\frac{y^2}{3}=1$

$x^2+y^2=4$

It is also touching $x^2+y^2=4$

$\left|\frac{\sqrt{7m^2+3}}{\sqrt{m^2+1}}\right|=2$

$$7m^2+3=4m^2+4$$

$$m^2=\frac{1}{3}\Rightarrow m=\frac{1}{\sqrt{3}}$$

$$\therefore \tan\theta=\frac{1}{\sqrt{3}}$$

$$\theta=\frac{\pi}{6}$$

46. The points of intersection of two ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$ be at the extremeties of conjugate diameters of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then $\frac{a^2}{\alpha^2}+\frac{b^2}{\beta^2}=$

A. 1 B. 2 C. 3 D. 4

Key. B

Sol. Clearly $P(a\cos\theta, b\sin\theta)$ $Q(-a\sin\theta, b\cos\theta)$ are extremities of conjugate diameters of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

P and Q lies m $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$

$$\frac{a^2\cos^2\theta}{\alpha^2}+\frac{b^2\sin^2\theta}{\beta^2}=1$$

$$\frac{a^2\sin^2\theta}{\alpha^2}+\frac{b^2\cos^2\theta}{\beta^2}=1$$

(+) $\frac{a^2}{\alpha^2}+\frac{b^2}{\beta^2}=2$

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47. From the focus $(-5,0)$ of the ellipse $\frac{x^2}{45}+\frac{y^2}{20}=1$ a ray of light is sent which makes angle $\cos^{-1}\left(\frac{-1}{\sqrt{5}}\right)$ with the positive direction of X-axis upon reacting the ellipse the ray is reflected from it. Slope of the reflected ray is

A) $-3/2$ B) $-7/3$ C) $-5/4$ D) $-2/11$

Key. D

Sol. Let $\theta=\cos^{-1}\left(\frac{-1}{\sqrt{5}}\right)\Rightarrow\cos\theta=\frac{-1}{\sqrt{5}}\Rightarrow\tan\theta=-2$

Foci are $(\pm 5,0)$

Equation of line through (-5, 0) with slope –2 is $y=-29x+5)=-2x-10$

This line meets the ellipse above X-axis at $(-6,2)$

$\therefore$ Slope = $\frac{2-0}{-6-5}=-\frac{2}{11}$.

48. If $f(x)$ is a decreasing function for all $x\in R$ and $f(x)>0\ \forall x\in R$ then the range of *K* so that the equation $\frac{x^2}{f(K^2+2K+5)}+\frac{y^2}{f(K+11)}=1$ represents an ellipse whose major axis is the X-axis is

A) $(-2,3)$ B) $(-3,2)$

C) $(-\infty,-3)\cup(2,\infty)$ D) $(-\infty,-2)\cup(3,\infty)$

Key. B

Sol. Conceptual

49. P,Q are points on the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ such that PQ is a chord through the point $R(3,0)$. If $|PR|=2$ then length of chord PQ is

A) 8 B) 6 C) 10 D) 4

Key. C

Sol. Conceptual

50. Let $Q=(3,\sqrt{5}),R=(7,3\sqrt{5})$. A point *P* in the XY-plane varies in such a way that perimeter of ΔPQR is 16. Then the maximum area of ΔPQR is

A) 6 B) 12 C) 18 D) 9

Key. B

Sol. *P* lies on the ellipse for which Q,R are foci and length of major axis is 10 and eccentricity is 3/5.

Ellipse

Integer Answer Type

1. Any ordinate MP of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the auxiliary circle of Q; then locus of the point of intersection of normals of P and Q to the respective curves is a circle of radius ___

Key. 8

Sol. The locus is $x^2+y^2=64$

2. The distance between the directrices of the ellipse $(4x-8)^2+16y^2=\left(x+\sqrt{3}y+10\right)^2$ is K then $\frac{K}{2}$ is

Key. 8

Sol. $(x-2)^2+y^2=\left(\frac{1}{2}\right)^2\frac{\left(x+\sqrt{3}y+10\right)^2}{4}$

$(h, k)=(z, 0), e=½$

Perpendicular distance from $(2, 0)$ to $x+\sqrt{3}y+10=0$ is $\frac{a}{e}-ae$

$2a-\frac{a}{2}=6 \Rightarrow a=4$

Distance between directrics = $\frac{2a}{e}=16=K$

3. A circle concentric to an ellipse $\frac{4x^2}{289}+\frac{4y^2}{\lambda^2}=1\left(\lambda<\frac{17}{2}\right)$ passes through foci F_1 and F_2 cuts the ellipse at 'P' such that area of triangle P F_1 F_2 is 30 sq.units. If $F_1F_2=13K$ where $K\in Z$ then K =

Key. 1

Sol. Since F_1 & F_2 are the ends of the diameter

Area of $\Delta PF_1F_2=\frac{1}{2}(F_1P)(F_2P)=\frac{1}{2}x(17-x)=30 \Rightarrow x=5 \text{ or } 12 \Rightarrow F_1F_2=13$

4. If F_1, F_2 are the feeet of the perpndiculars from foci S_1, S_2 of the ellipse $16x^2+25y^2=400$ on the tangent at any point P on the ellipse then minimum value of $S_1F_1+S_2F_2$ is

Key. 8

Sol. The minimum perpendiculars from two foci upon any tangent is b^2

$$S_1F_1.S_2F_2 = 16$$

$$AM \geq GM \Rightarrow \frac{S_1F_1 + S_2F_2}{2} \geq \sqrt{S_1F_1 \times S_2F_2} \Rightarrow S_1F_1 + S_2F_2 \geq 8$$

5. The equation of an ellipse is given by $5x^2 + 5y^2 - 6xy - 8 = 0$. If r_1, r_2 are distances of points on the ellipse which are at maximum & minimum distance from origin then $r_1 + r_2 =$

Key. 3

Sol. Any point on ellipse at a distance r from origin is $(r\cos\theta, r\sin\theta)$

$\Rightarrow r^2 = \dfrac{8}{5 - 3\sin 2\theta}$ is maximum if $5 - 3\sin 2\theta$ is minimum $\Rightarrow r^2 = 4$

$r^2 m$ in if $(5 - 3\sin 2\theta)$ is maximum = 8 $\Rightarrow r^2 = 1$

$r_1 + r_2 = 2 + 1 = 3$

6. The equation of the curve on reflection of the ellipse $\dfrac{(x-4)^2}{16} + \dfrac{(y-3)^2}{9} = 1$ about the line x–y–2 = 0 is $16x^2 + 9y^2 + ax - 36y + b = 0$ then the value of $a + b - 125 =$

Key. 7

Sol. Let P(4, 0) & Q(0, 3) are two points on given ellipse E_1

P_1 and Q_1 are images of P,Q w.r.to x – y – 2 = 0

$\therefore P_1(2, 2) \quad Q_1(5, -2)$ lies on E_2

$\therefore a = -160,\ b = 292$

7. Number of points on the ellipse $\dfrac{x^2}{50} + \dfrac{y^2}{20} = 1$ from which pair of perpendicular tangents are drawn to the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ is

Key. 4

Sol. Director circle of $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ is $x^2 + y^2 = 25$

The director circle will cut the ellipse $\dfrac{x^2}{50} + \dfrac{y^2}{20} = 1$ at 4 points.

8. If L be the length of common tangent to the ellipse $\frac{x^2}{25}+\frac{y^2}{4}=1$ and the circle $x^2+y^2=16$ intercepted by the coordinate axis then $\frac{\sqrt{3}L}{2}$ is

Key. 7

Sol. The equation of the tangent at $(5\cos\theta, 2\sin\theta)$ is $\frac{x}{5}\cos\theta+\frac{y}{2}\sin\theta=1$

If it is a tangent to the circle then $\frac{1}{\sqrt{\frac{\cos^2\theta}{25}+\frac{\sin^2\theta}{4}}}=4$

$\Rightarrow \cos\theta=\frac{10}{4\sqrt{7}}, \sin\theta=\frac{\sqrt{3}}{2\sqrt{7}}$

Let A and B be the points where the tangent meets the coordinate axis then $A\left(\frac{5}{\cos\theta}, 0\right)$, $B\left(0, \frac{2}{\sin\theta}\right)$

$L=\sqrt{\frac{25}{\cos^2\theta}+\frac{4}{\sin^2\theta}}=\frac{14}{\sqrt{3}}$

9. An ellipse is sliding along the coordinate axes. If the foci of the ellipse are (1, 1) and (3,3) then the area of the director circle of the ellipse is $K\pi$. Then K = __

Key. 7

Sol. Since axes are tangents, $b^2=3$ and $ae=\sqrt{2} \Rightarrow a^2-b^2=2 \therefore a^2=5$

10. Tangents are drawn from points on the line $x-y+2=0$ to the ellipse $x^2+2y^2=2$, then all the chords of contact pass through the point whose distance from $\left(2, \frac{1}{2}\right)$ is

Key. 3

Sol. Consider any point $(t_1, t+2)$, $t \in R$ on the line $x-y+2=0$

The chord of contact of ellipse with respect to this point is $x(t)+2y(t+2)-2=0$

$\Rightarrow (4y-2)+t(x+2y)=0$, $y=\frac{1}{2}, x=-1$

Hence, the point is $\left(-1, \frac{1}{2}\right)$, Where distance from $\left(2, \frac{1}{2}\right)$ is 3.

11. If P and Q are the ends of a pair of conjugate diameters and C is the centre of the ellipse $4x^2+9y^2=36$ then the area of ΔCPQ in square units.

Key. 3

Sol. $\frac{x^2}{9}+\frac{y^2}{4}=1$, so $P=(3\cos\theta, 2\sin\theta)$ and $Q=\left(3\cos\left(\frac{\pi}{2}+\theta\right), 2\sin\left(\frac{\pi}{2}+\theta\right)\right)$

$$\text{Area of } \Delta CPQ = =\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 3\cos\theta & 2\sin\theta & 1 \\ -3\sin\theta & 2\cos\theta & 1 \end{vmatrix}=3.$$

12. The maximum distance from the origin to any normal chord drawn to the ellipse $\frac{x^2}{25}+\frac{y^2}{4}=1$ is

Key. 3

Sol. The other end of the normal drawn at P(t) in $Q\left(t-\frac{2}{t}\right)$

If A is the vertex, slope of AP slope AQ = -1

$$\Rightarrow \frac{2}{t}\frac{(-2)}{\left(t+\frac{2}{t}\right)}=-1 \Rightarrow t^2+2=4 \Rightarrow t^2=2$$

13. The area of the quadrilateral formed by the tangents at the end point of latus rectum to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ is 3K. Then K is equal to

Key. 9

Sol. e = 2/3

Equation of tangent at L is $\frac{2x}{9}+\frac{y}{3}=1$ it meets x-axis at $A\left(\frac{9}{2},0\right)$ & y axis at B(0, 3).

$\therefore$ area = $4\left[\frac{1}{2}.\frac{9}{2}.3\right]=27$

14. If a tangent of slope 2 of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is normal to the circle $x^2+y^2+4x+1=0$, then the maximum value of ab is ______________

Key. 4

Sol. A tangent of slope 2 is $y=2x\pm\sqrt{4a^2+b^2}\rightarrow(1)$

This is normal to the circle $x^2+y^2+4x+1=0$

i.e., (1) passes through $(-2,0)\,4a^2+b^2=16$

Using AM ≥ GM $\Rightarrow \frac{4a^2+b^2}{2}\geq\sqrt{4a^2.b^2}$ $ab\leq 4$

15. If a line through P(a, 2) meets the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ at A and D and meets the axes at B and C, so that PA, PB, PC, PD are in G.P., then the minimum value of $|a|$,is….

Key. 6

Sol. $\frac{x-a}{\cos\theta}=\frac{y-2}{\sin\theta}=r$ $(a+r\cos\theta, 2+r\sin\theta)$ lies on ellipse for A and D.

$$\frac{(a+r\cos\theta)^2}{9}+\frac{(2+r\sin\theta)^2}{4}=1 \Rightarrow r_1r_2=PA.PD$$

PA, PB, PC, PD are in G.P PA. PD = PB. PC. etc.....

16. The number of values of c such that the straight line $y=4x+c$ touches the curve $x^2/4+y^2=1$ is K then K = ………………

Key. 2

Sol. If $y=mx+c$ is tangent to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then

$$C^2=a^2m^2+b^2$$

$$y=4x+c,\quad \frac{x^2}{4}+\frac{y^2}{1}=1$$

$$C=\pm\sqrt{65}$$

17. Tangent is drawn to ellipse $x^2/27+y^2=1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ (where $\theta\in(0,\pi/2)$). Then the value of θ such that sum of intercepts on coordinate axes made by this tangent is least is $\frac{\pi}{K}$ then K =

Key. 6

Sol. $\frac{x^2}{27}+\frac{y^2}{1}=1,\ P(3\sqrt{3}\cos\theta, \sin\theta)$

$$\frac{3\sqrt{3}\cos\theta}{27}+\frac{\sin\theta y}{1}=1$$

$$A\left(\frac{3\sqrt{3}\cos\theta}{27},0\right), B=\left(0,\frac{1}{\sin\theta}\right)$$

$$f(\theta)=3\sqrt{3}\sec\theta+\cos ec\theta$$

$$f^1(\theta)=\frac{3\sqrt{3}\sin\theta}{\cos^2\theta}-\frac{\cos\theta}{\sin^2\theta}=0$$

$$\Rightarrow \tan^3\theta=\frac{1}{3\sqrt{3}}=\left(\frac{1}{\sqrt{3}}\right)^3$$

$$\theta=\frac{\pi}{6}$$

18. The maximum distance from the origin to any normal chord drawn to the ellipse $\frac{x^2}{25}+\frac{y^2}{4}=1$ is

Key. 3

Sol. The other end of the normal drawn at P(t) in $Q\left(t-\frac{2}{t}\right)$

If A is the vertex, slope of AP slope AQ = -1

$$\Rightarrow \frac{2}{t}\frac{(-2)}{\left(t+\frac{2}{t}\right)}=-1 \Rightarrow t^2+2=4 \Rightarrow t^2=2$$

19. The area of the quadrilateral formed by the tangents at the end point of latus rectum to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ is 3K. Then K is equal to

Key. 9

Sol. e = 2/3

Equation of tangent at L is $\frac{2x}{9}+\frac{y}{3}=1$ it meets x-axis at $A\left(\frac{9}{2},0\right)$ & y axis at B(0, 3).

$\therefore$ area = $4\left[\frac{1}{2}.\frac{9}{2}.3\right]=27$

20. If a tangent of slope 2 of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is normal to the circle $x^2+y^2+4x+1=0$, then the maximum value of ab is ______________

Key. 4

Sol. A tangent of slope 2 is $y=2x\pm\sqrt{4a^2+b^2}\rightarrow(1)$

This is normal to the circle $x^2+y^2+4x+1=0$

i.e., (1) passes through $(-2,0)\,4a^2+b^2=16$

Using AM $\geq$ GM $\Rightarrow \frac{4a^2+b^2}{2}\geq\sqrt{4a^2.b^2}$ $ab\leq 4$