Complex Numbers

Single Correct Answer Type

1. If z_1, z_2, z_3 and z_4 be the consecutive vertices of a square, then $z_1^2 + z_2^2 + z_3^2 + z_4^2$ equals

(a) $z_1z_2 + z_2z_3 + z_3z_4 + z_4z_1$

(b) $z_1z_2 + z_1z_3 + z_1z_4 + z_2z_3 + z_2z_4 + z_3z_4$

(c) 0 (d) None of the above

Key. A

Sol. We know that, $\dfrac{z_2 - z_1}{z_4 - z_1} = \dfrac{|z_2 - z_1|}{|z_4 - z_1|}e^{i\pi/2} = i$ (as $|z_2 - z_1| = |z_4 - z_1|$)

$\Rightarrow (z_4 - z_1)^2 + (z_2 - z_1)^2 = 0$

similarly $\dfrac{z_4 - z_3}{z_2 - z_3} = i$

$\Rightarrow (z_4 - z_3)^2 + (z_2 - z_3)^2 = 0$

On adding Eqs (i) and (ii), we get

$2\left[z_1^2 + z_2^2 + z_3^2 + z_4^2 - z_1z_2 - z_4z_1 - z_4z_3 - z_2z_3\right] = 0$

$z_1^2 + z_2^2 + z_3^2 + z_4^2 = z_1z_2 + z_2z_3 + z_3z_4 + z_4z_1$

2. If z_1, z_2 and z_3 are the vertices of an isosceles right angled triangle, right angled at the vertex z_2, then $(z_1 - z_2)^2 + (z_3 - z_2)^2$ equals

(a) 0 (b) $(z_1 - z_3)^2$ (c) $\left(\dfrac{z_1 + z_3}{2}\right)^2$ (d) None of these

Key. A

Sol. we know that $\dfrac{z_3 - z_2}{z_1 - z_2} = \dfrac{|z_3 - z_2|}{|z_1 - z_2|}.e^{-\frac{i\pi}{2}}$

$\Rightarrow z_3 - z_2 = -i(z_1 - z_2)$

$\Rightarrow (z_3 - z_2)^2 + (z_1 - z_2)^2 = 0$

3. Let $C = \cos\dfrac{2\pi}{7} + \cos\dfrac{4\pi}{7} + \cos\dfrac{8\pi}{7}$ and $S = \sin\dfrac{2\pi}{7} + \sin\dfrac{4\pi}{7} + \sin\dfrac{8\pi}{7}$, then

(a) $C = \dfrac{1}{2}$ (b) S= $\dfrac{1}{2}$

(c) $C = \dfrac{\sqrt{7}}{2}$ (d) $S = \dfrac{\sqrt{7}}{2}$

Key. D

Sol. $C + is = e^{i\theta} + e^{i(2\theta)} + e^{i(4\theta)}$, where $\theta = \dfrac{2\pi}{7}$

i.e $C + iS = \alpha + \alpha^2 + \alpha^4$, where $\alpha = e^{i\theta}$ ....(i)

so, $C - iS = \bar{\alpha} + (\bar{\alpha}^2) + (\bar{\alpha}^4) = \alpha^6 + \alpha^5 + \alpha^3$ ....(ii)

$\left(\text{since } \bar{a} = \dfrac{a\bar{a}}{a} = \dfrac{1}{a} = \dfrac{a^7}{a} = a^6 \quad etc\right)$

From Eqs (i) and (ii) , we get

$$2C = a + a^2 + a^3 + a^4 + a^5 + a^6 = \frac{a\left(a^6 - 1\right)}{a - 1}$$

$\Rightarrow \ 2C = \dfrac{1 - a}{a - 1} \left(\because \ a^7 = 1\right)$

$\Rightarrow \ C = -\dfrac{1}{2}$

Again $(C + iS)(C - iS) = \left(a + a^2 + a^4\right)\left(a^6 + a^5 + a^3\right)$

$\Rightarrow \ C^2 + S^2 = 1 + a^6 + a^4 + a + 1 + a^5 + a^3 + a^2 + 1 \qquad (a^7 = 1)$

$\Rightarrow \ \left(-\dfrac{1}{2}\right)^2 + S^2 = 2 + \left(1 + a + a^2 + a^3 + a^4 + a^5 + a^6\right)$

$\Rightarrow \ S^2 = 2 + 0 - \dfrac{1}{4} = \dfrac{7}{4}$

$\Rightarrow \ S = \dfrac{\sqrt{7}}{2}$

4. The point of intersection of the curves $\arg(z - 3i) = \dfrac{3\pi}{4}$ and $\arg(2z + 1 - 2i) = \dfrac{\pi}{4}$ is

(a) $\dfrac{1}{4}(3 + 9i)$ (b) $\dfrac{1}{4}(3 - 9i)$ (c) $\dfrac{1}{2}(3 + 2i)$ (d) None of these.

Key. D

Sol. Clearly the two eqns represent two rays which are not intersecting. Hence no point of intersection.

5. If z_1, z_2, z_3 are non-zero complex numbers representing the points A, B, C such that $\dfrac{2}{z_1} = \dfrac{1}{z_2} + \dfrac{1}{z_3}$. Then

(a) A, B, C are collinear.
(b) Circle passes through points A, B, C has centre at origin O
(c) Circle passes through A, B, C passes through origin.
(d) None of these.

Key. C

Sol. $\dfrac{2}{z_1} = \dfrac{1}{z_2} + \dfrac{1}{z_3} \Rightarrow \ \arg\left(\dfrac{z_2 - z_1}{z_3 - z_1}\right) = \arg\left(-\dfrac{z_2}{z_3}\right) = \arg\left(\dfrac{z_2}{z_3}\right) \pm \pi$

$\Rightarrow \ \arg\left(\dfrac{z_2 - z_1}{z_3 - z_1}\right) = \arg\left(\dfrac{z_2 - 0}{z_3 - 0}\right) \pm \pi$

$\Rightarrow \ \arg\left(\dfrac{z_2 - z_1}{z_3 - z_1}\right) - \arg\left(\dfrac{z_2 - 0}{z_3 - 0}\right) = \pm \pi$

Sum of angles at A and origin is $\pm \pi$. Hence points O, B, A, C are concyclic.

6. If $|2z - 4 - 2i| = |z| \sin\left(\dfrac{\pi}{4} - \arg z\right)$, then locus of z is/an

(a) Ellipse (b) Circle (c) Parabola (d) Pair of straight line

Key. A

Sol. Let $z = x + iy = r(\cos\theta + i\sin\theta)$, then the equation is

$$|2(x-2)+2i(y-1)| = r\left(\frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta\right) = \frac{1}{\sqrt{2}}(r\cos\theta - r\sin\theta)$$

$$\Rightarrow \sqrt{(x-2)^2+(y-1)^2} = \frac{1}{\sqrt{2}}\left(\frac{x-y}{\sqrt{2}}\right)$$

It is an ellipse with focus at $(2,1)$ and directrix $x - y = 0$ and eccentricity $= \frac{1}{\sqrt{2}}$.

7. If $|z-3i| = 3$, (where i = $\sqrt{-1}$) and arg z ∈ (0, π/2), then cot (arg (z)) $-\frac{6}{z}$ is equal to

a) 0 b) –i c) i d) none of these

Key. C

Sol. Conceptual

8. If the imaginary part of the expression $\frac{z-1}{e^{i\theta}} + \frac{e^{i\theta}}{z-1}$ be zero, then the locus of z can be

(a) a straight line parallel to x-axis.
(b) a parabola
(c) a circle of radius 1
(d) none of these.

Key. C

Sol. Conceptual

9. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ then $\frac{\sin 3\alpha + \sin 3\beta + \sin 3\gamma}{\sin(\alpha+\beta+\gamma)}$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3

Key. D

Sol. Let $a = e^{i\alpha}, b = e^{i\beta}, c = e^{i\gamma}$ clearly $a+b+c = 0 \Rightarrow \frac{1}{a}+\frac{1}{b}+\frac{1}{c} = 0$

$a+b+c = 0 \Rightarrow a^3+b^3+c^3 = 3abc$

$\Rightarrow \sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$.

10. If z is a complex number satisfying $z^4 + z^3 + 2z^2 + z + 1 = 0$, then the set of possible values of $|z|$ is

(a) $\{1,2\}$ (b) $\{1\}$ (c) $\{1,2,3\}$ (d) $\{1,2,3,4\}$

Key. B

Sol. The given equation is $(z^2+z+1)(z^2+1) = 0$.

$z = \pm i, \omega, \omega^2$, ω being an imaginary cube root of unity. Thus $|z| = 1$.

11. Let A, B and C represents the complex numbers z_1, z_2 and z_3 in the Argand plane. If circumcentre of the triangle ABC is at the origin, then the complex number corresponding to orthocentre is

(a) $\frac{1}{4}(z_1+z_2+z_3)$ (b) $\frac{1}{3}(z_1+z_2+z_3)$ (c) $\frac{1}{2}(z_1+z_2+z_3)$ (d) $z_1+z_2+z_3$

Key. D

Sol. Centroid of $DABC$ is at $\frac{z_1 + z_2 + z_3}{3}$.

Orthocentre divides Centroid andcircumcentre in 2:3 externally.

12. If $z = x + iy$ then the equation $\left|\frac{2z - i}{z + 1}\right| = m$ does not represent a circle when

(a) $m = 1/2$ (b) m =1 (c) m = 2 (d) m = 3

Key. C

Sol. The given equation is $\left|\frac{z - \frac{i}{2}}{z + 1}\right| = \frac{m}{2}$, which does not represent a circle when $\frac{m}{2} = 1$.

13. α, β, γ are the roots of $x^3 - 3x^2 + 3x + 7 = 0$ (w is cube root of unity) then $\left(\frac{\alpha-1}{\beta-1}+\frac{\beta-1}{\gamma-1}+\frac{\gamma-1}{\alpha-1}\right)$ is

(A) $\frac{3}{\omega}$ (B) ω^2 (C) $2\omega^2$ (D) 3ω

Key. A

Sol. We have $x^3 - 3x^2 + 3x + 7 = 0$

$\Rightarrow$ $(x - 1)^3 + 8 = 0$

$\Rightarrow$ $\left(\frac{(x-1)}{-2}\right)^3 = 1$

$\Rightarrow$ $\left(\frac{x-1}{-2}\right) = 1, \omega, \omega^2$

$\Rightarrow$ $x = -1;\ 1 - 2\omega;\ 1 - 2\omega^2$

$\therefore$ $\alpha = -1, \beta = 1 - 2\omega; \gamma = 1 - 2\omega^2$

$\therefore$ required expression = $3\omega^2$.

14. The complex number $3 + 4i$ is rotated about origin by an angle of $p/4$ and then stretched 2- times. The complex number corresponding to new position is

(a) $\sqrt{2}(-3 + 4i)$ (b) $\sqrt{2}(-1 + 7i)$ (c) $\sqrt{2}(3 - 4i)$ (d) $\sqrt{2}(-1 - 7i)$

Key. B

Sol. The new complex number is $2(3 + 4i)e^{ip/4} = \sqrt{2}(-1 + 7i)$.

15. If $(a + ib)^5 = \alpha + i\beta$ then $(b + ia)^5$ is equal to

(A) $\beta - i\alpha$ (B) $\beta + i\alpha$ (C) $\alpha - \beta$ (D) $-\alpha - i\beta$

Key. B

Sol. $(a + ib)^5 = \alpha + i\beta$

Taking complex conjugate

$(a - ib)^5 = \alpha - i\beta$

$(-i^2 a - ib)^5 = \alpha - i\beta$

$(-i)^5 (b + ai)^5 = \alpha - i\beta$

$(b + ai)^5 = -\frac{\alpha}{i} + \beta$

$= \alpha i + \beta$

16. The complex number a + i, a – i, 1 + ai, 1 – ai where a $\in$ R taken in that order on the Argand plane represent the vertices of a parallelogram if
(A) a = 1 (B) a = – 1 (C) a = 0 (D) none of these

Key. B

Sol. Diagonals of Parallelogram intersects at midpoint

Solution : $\frac{a+1}{2} = \frac{a+1}{2}$

$\frac{-1-a}{2} = \frac{a+1}{2}$

2a = -2

a = -1

17. If (1 + i) (1 + 2i) (1 + 3i) .....(1 + ni) = α + iβ, then $2 \cdot 5 \cdot 10.....(1+n^2)$ is equal to $(where\, \alpha, \beta, n \in R)$
(A) α -iβ (B) $\alpha^2 - \beta^2$ (C) $\alpha^2 + \beta^2$ (D) none of these

Key. C

Sol. (1 + i) (1 + 2i) (1 + 3i) ………..(1 + ni) = α + iβ
α - iβ = (1 – i) (1 – 2i) (1 – 3i)…….. (1 – ni)
$\alpha^2 + \beta^2 = 2 \cdot 5 \cdot 10......(1+n^2)$

18. If $z = (1+\sqrt{3}i)^{10} + (1-\sqrt{3}i)^{10}$, then arg z is
(A) $\frac{\pi}{2}$ (B) π (C) $\frac{\pi}{4}$ (D) none of these

Key. B

Sol. $z = a + \bar{a}$
= always real
$\Rightarrow$arg z = 0 or π.

19. If α, β, γ are the cube roots of p(< 0), then $\frac{x\alpha + y\beta + z\gamma}{x\beta + y\gamma + z\alpha}$ for any x, y, z is equal to (where ω is complex cube root of unity)
(A) 1 (B) 0 (C) ω^2 (D) 3

Key. C

Sol. $x = -p$

$x^{1/3} = p^{1/3}(-1)^{1/3}$

$\alpha = -p^{1/3}$ $\beta = -p^{1/3}w$ $g = -p^{1/3}w^2$

$$= \frac{1}{w}\frac{xw + yw^2 + z}{xw + yw^2 + z} = w^2$$

20. If the complex numbers z_1, z_2, z_3 satisfying $3z_1 = 5z_2 - 2z_3$, then z_1, z_2 and z_3 lie in a

(A) circle (B) parabola (C) line (D) hyperbola

Key. C

Sol. $3z_1 = 5z_2 - 2z_3$

$z_1 = \frac{5z_2 - 2z_3}{5-2}$

$\Rightarrow$ z_1 divides line joining z_2 and z_3 externally in ration 5 : 2

$\Rightarrow$ z_1, z_2, z_3 are collinear.

21. If z_1 and z_2 are two complex numbers satisfying the equation $\left|\frac{z_1+z_2}{z_1-z_2}\right| = 1$, then $\frac{z_1}{z_2}$ is a number which is

(A) Positive real (B) Negative real

(C) Zero or purely imaginary (D) None of these

Key. C

Sol. $\left|\frac{z_1+z_2}{z_1-z_2}\right| = 1$

$\left|\frac{\frac{z_1}{z_2}+1}{\frac{z_1}{z_2}-1}\right| = 1$

$\left|\frac{z_1}{z_2}+1\right| = \left|\frac{z_1}{z_2}-1\right|$

$\left|\frac{z_1}{z_2}+1\right| = \left|\frac{z_1}{z_2}-1\right|$

$\Rightarrow$ $\frac{z_1}{z_2} = 0$ or purely imaginary

22. If $|z_1| = 2$, $|z_2| = 3$, $|z_3| = 4$ and $|z_1 + z_2 + z_3| = 5$ then $|4z_2z_3 + 9z_3z_1 + 16z_1z_2|$ =

a) 20 b) 24 c) 48 d) 120

Key. D

Sol. $|4z_2z_3 + 9z_3z_1 + 16z_1z_2|$

= $|z_1\overline{z_1}z_2z_3 + \overline{z_2}z_2z_3z_1 + \overline{z_3}z_3z_1z_2|$

= $||z_1||z_2||z_3||z_1 + z_2 + z_3| = 120$

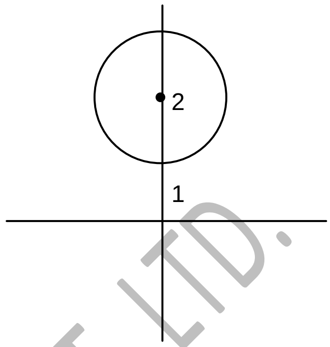
23. The value of $\sin\left[\log_e\left\{\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^z\right\}\right]$ is, where z satisfies the equation $|z - 2i| = 1$ and has least modulus

(a) 1 (b) 0 (c) -1 (d) $\frac{1}{2}$.

Key. C

Sol.

$$A = \sin\left[\log\left\{\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^z\right\}\right]$$



$$= \sin\left[\log e^{z\pi/2i}\right]$$

$$= \sin\left(\frac{z\pi}{2}i\right)$$

Again |z – 2i| = 1 is a circle centered at (0, 2) with radius 1.

Therefore a point on circle of least modulus is z = i.

$\therefore$ By equation θ

$$A = \sin\left(-\frac{\pi}{2}\right)$$

= - 1

24. If $c^2 + s^2 = 1$, then $\frac{1+c+is}{1+c-is}$ is equal to

(a) $c - is$ (b) $c + is$ (c) $s + ic$ (d) $s - ic$.

Key. B

Sol. $\frac{1+c+is}{1+c-is} = \frac{(1+c)+is}{(1+c)-is} \times \frac{(1+c)+is}{(1+c)+is}$

$$= \frac{\left((1+c)^2 - s^2\right) + i\left(s(1+c) + s(1+c)\right)}{(1+c)^2 + s^2}$$

$$= \frac{\left(1+c^2+2c-s^2\right) + i\left(2s(1+c)\right)}{(1+c)^2 + s^2}$$

$$= \frac{2c(c+1) + i2s(c+1)}{2+2c}$$

= c + is

25. If $\omega \neq 1$ be a cube root of unity and $(1+\omega)^7 = l + m\omega$, then the value of $l + m =$

(a) 0 (b) 1 (c) 2 (d) -1

Key. C

Sol. $\because \omega$ is cube root of unity

$\therefore\ 1+\omega+\omega^2=0$

$\Rightarrow\ 1+\omega=-\omega^2$

Now if $(1+\omega)^7=l+m\omega$

$\Rightarrow\ \left(-\omega^2\right)^7=l+m\omega$

$\Rightarrow\ -\omega^{14}=l+m\omega$

$\Rightarrow\ -\omega^{12}\cdot\omega^2=l+m\omega$

$\Rightarrow\ -\left(\omega^3\right)^4\omega^2=l+m\omega$

$\Rightarrow\ -\omega^2=l+m\omega$

$\Rightarrow\ 1+\omega=l+m\omega$

can comparison l = 1, m = 1

26. One vertex of an equilateral triangle is at the origin and the other two vertices are, roots of $2z^2+2z+k=0$, then k is

(A) 1 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{1}{2}$.

Key. C

Sol. $2z^2+2z+k=0$

$z=\frac{-2\pm\sqrt{4-8k}}{4}$

Since 'z' is a complex number

4 – 8k will be negative

$\Rightarrow$k> ½

(0, 0), $\left(\frac{-1}{2},\frac{\sqrt{2k-1}}{2}\right)\left(\frac{-1}{2},\frac{-1}{2}\sqrt{2k-1}\right)$

Since triangle is equilateral

$\therefore \frac{1}{4}(2k-1)+\frac{1}{4}=(2k-1)$

$\Rightarrow$ k = 2/3.

27. If $\log_{\tan 30^\circ}\left(\frac{2|z|^2+2|z|-3}{|z|+1}\right)<-2$ then

a) $|z|<\frac{3}{2}$ b) $|z|>\frac{3}{2}$ c) $|z|>2$ d) $|z|<2$

Key. C

Sol. $\log_{\tan 30^\circ}\left(\frac{2\left|z^2\right|+2|z|-3}{|z+1|}\right)<-2$

$\Rightarrow\frac{2\left|z^2\right|+2|z|-3}{|z|+1}>3$

$\Rightarrow\left((|z|-2)(2|z|)+3\right)>0$

$\Rightarrow$ |z| > 2

28. The number of common roots of the equations $x^3 + 2x^2 + 2x + 1 = 0$ and $x^{2012} + x^{2014} + 1 = 0$ is

(a) 1 (b) 2 (c) 3 (d) 4

Key. D

Sol. $x^3 + 2x^2 + 2x + 1 = 0 \Rightarrow x = -1, w, w^2$

But $x = w, w^2$ will only satisfy $x^3 + 2x^2 + 2x + 1 = 0$ and $x^{2012} + x^{2014} + 1 = 0$.

29. If $|z| = \min\{|z-1|, |z+1|\}$ then $|z + \bar{z}| =$

(a) 1 (b) 2 (c) 3 (d) 4

Key. A

Sol. If $|z| = |z-1|$

Then $|z|^2 = |z-1|^2$

$\Rightarrow z + \bar{z} = 1$

If $|z| = |z+1|$

Then $|z|^2 = |z+1|^2$

$\Rightarrow z + \bar{z} = 1$

$\Rightarrow |z + \bar{z}| = 1$

30. If the roots of $z^3 + iz^2 + 2i = 0$ represent the vertices of a ΔABC in the argand plane then the area of the triangle is (in square units)

A) 3 B) 1 C) 4 D) 2

Key. D

Sol. $(z-i)(z^2 + 2iz - 2) = 0 \Rightarrow z = i, 1-i, -1-i$

Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} -1 & 1 \\ 2 & 2 \end{vmatrix} = 2$ square units.

31. If $n \geq 3$ and $1, \alpha_1, \alpha_2, \alpha_3 ......\alpha_{n-1}$ are n roots of unity, then value of $\sum_{1 \leq i < j \leq n-1} \alpha_i \alpha_j$ is

(a) 0 (b) 1 (c) -1 (d) $(-1)^n$

Key. B

Sol. $x^n - 1 = (x-1)(x-\alpha_1)(x-\alpha_2)............(x-\alpha_{n-1})$

$= x^n - x^{n-1}(1 + \alpha_1 + ... + \alpha_{n-1}) + x^{n-2}\left(\sum_{i+j} \alpha i\, \alpha_j + \alpha_1 + \alpha_2 ..... + \alpha_{n-1}\right)$ +..........-1 = 0

$\Rightarrow \sum_{i+j} \alpha i\, \alpha_j + \alpha_1 + \alpha_2 ..... + \alpha_n = 0$

$\sum_{i+j} \alpha i\, \alpha_j = 1$

32. Let $z=\cos\theta+i\sin\theta$. Then, the value of $\sum_{m=1}^{15} lm\left(z^{2m-1}\right)$ at $\theta=2^0$ is

(A) $\frac{1}{2^0}$ (B) $\frac{1}{3\sin 2^0}$ (C) $\frac{1}{2\sin 2^0}$ (D) $\frac{1}{4\sin 2^0}$

Key. D

Sol. Given that $z=\cos\theta+isin\theta=e^{i\theta}$

$$\therefore \sum_{m=1}^{15}\left(z^{m-1}\right)=\sum_{m=1}^{15} lm\left(e^{i\theta}\right)^{2m-1}$$

$$=\sum_{m=1}^{15} lm\, e^{i(2m-1)\theta}$$

$$=\sin\theta+\sin 3\theta+\sin 5\theta+.......+\sin 29\theta$$

$$=\frac{\sin\left(\frac{\theta+29\theta}{2}\right)\sin\left(\frac{15\times 2\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)}$$

$$=\frac{\sin(15\theta)\sin(15\theta)}{\sin\theta}=\frac{1}{4\sin 2^0}$$

33. If z_1 is a root of the equation $a_0z^n+a_1z^{n-1}+.....+a_{n-1}z+a_n=3$, where $|a_i|<2$ for $i=0,1,.....n$. Then

(A) $|z_1|>\frac{1}{3}$ (B) $|z_1|<\frac{1}{4}$ (C) $|z_1|>\frac{1}{4}$ (D) $|z|<\frac{1}{3}$

Key. A

Sol. $a_0z^n+a_1z^{n-1}+a_2z^{n-2}+..............+a_{n-1}z+a_n=3$

$$|3|=\left|a_0z^n+a_1z^{n-1}+a_2z^{n-2}+..............+a_{n-1}z+a_n\right|$$

$$3\le|a_0||z|^n+|a_1||z|^{n-1}+.......+a_{n-1}|z|+|a_n|$$

$$3<2\left(|z|^n+|z|^{n-1}+.......+|z|+1\right)$$

$$\frac{3}{2}<1+|z|+|z|^2+......+|z|^n$$

$$\frac{1-|z|^{n+1}}{1-|z|}>\frac{3}{2}$$

$$2-2|z|^{n+1}<3|z|-1$$

$$3|z|-1>0$$

$$|z|>\frac{1}{3}$$

34. If x = a+ib is a complex number such that x^2 =3+4i and x^3=2+11i where i = $\sqrt{-1}$ then a+b =_____________________________

1. 1 2. 2 3. 3 4. 4

Key. 3

Sol. $x=\frac{x^3}{x^2}=\frac{2+11i}{3+4i}=\frac{(2+11i)(3-4i)}{25}$

$\therefore a+ib=\frac{6+44+25i}{25}=2+i$

$\Rightarrow a=2, b=1 \Rightarrow a+b=3$

35. If the complex number Z satisfying $Z+|Z|=2+8i$ then value of $|Z|$ =

1. 8 2.17 3. 15 4. 24

Key. 2

Sol. Let $z=a+ib$

$\Rightarrow a+ib+\sqrt{a^2+b^2}=2+8i$

$\Rightarrow b=8,\ a+\sqrt{a^2+64}=2$

$a^2+64=a^2-4a+4$

$\Rightarrow -4a=60 \Rightarrow a=-15$

$\therefore |z|=\sqrt{a^2+b^2}=\sqrt{225+64}=\sqrt{289}=17$

36. If $|Z+2-i|=5$ then maximum value of $|3Z+9-7i|=$ ______________

1. 20 2.15 3. 5 4. 16

Key. 1

Sol. $|3Z+9-7i|=|3Z+6-3i+3-4i|$

$\le |3(z+2-i)+|=|3-4i|$

$\le 3|z+2-i|+\sqrt{3^2+4^2}$

$3(5)+5=20$

37. If Z-lies on the circle $|z-2i|=2\sqrt{2}$ *then value of* $\arg\left(\frac{Z-2}{Z+2}\right)$ *is*

1. $\frac{\pi}{3}$ 2. $\frac{\pi}{4}$ 3. $\frac{\pi}{6}$ 4. $\frac{\pi}{2}$

Key. 2

Sol. Circle with centre (0,2) cuts X-axis at A(-2,0) and B(2,0). Now AB subtends an angle 90^0 at the center C.

AB subtends an angle $\frac{\pi}{4}$ at any point z on the major arc circle arg $\left(\frac{z-2}{z+2}\right)=\frac{\pi}{4}$

38. If $\lambda \in R$ and non real roots of $2Z^2 + 2Z + \lambda = 0$ and (0,0) forms vertices of an equilateral triangle then λ =

1. 1 2. $\frac{1}{2}$ 3. $\frac{1}{3}$ 4. $\frac{2}{3}$

Key. 4

Sol. Let $z_{1,} z_2$ be roots of $2z^2 + 2z + \lambda = 0$

$z_1 + z_2 = -1 \quad z_1 z_2 = \frac{\lambda}{2}$

When origin , $z_1 z_2$ forms equilateral Δ^{le}

We have $z_1^2 + z_2^2 = z_1 z_2$

$(z_1 + z_2)^2 = 3z_1 z_2$

$1 = \frac{3.\lambda}{2} \Rightarrow \lambda = \frac{2}{3}$

39. The greatest positive argument of z satisfying $|Z - 4| = \mathrm{Re}(Z)$

1. $\frac{\pi}{3}$ 2. $\frac{2\pi}{3}$ 3. $\frac{\pi}{2}$ 4. $\frac{\pi}{4}$

Key. 4

Sol. $|x + iy - 4| = x$

$(X - 4)^2 + y^2 + x^2$

$y^2 - 8x + 16 = 0$

z lies on the parabola with vertex (2,0) focus (4,0) and tangents from (0,0) ie a point on the directrix in x always include 90^0

$\therefore$ greatest arg(z) is $45^0 = \frac{\pi}{4}$

40. If Z and W are two complex numbers such that $\bar{z} + i\,\overline{w} = 0$ and arg (Zw)= π then arg (Z) = ________

1. $\frac{\pi}{4}$ 2. $\frac{\pi}{2}$ 3. $\frac{3\pi}{4}$ 4. $\frac{5\pi}{4}$

Key. 3

Sol. $\bar{z} + i\bar{w} \Rightarrow z - iw = 0 \Rightarrow z = iw$

Arg $(zw) = \pi \Rightarrow \arg(z) + \arg(w) = \pi$

$\arg(iw) + \arg w = \pi$

$\arg i + 2\arg w = \pi$

$\frac{\pi}{2}+2\arg w=\pi$

$2\arg w=\frac{\pi}{2}$

$\arg w=\frac{\pi}{4}\Rightarrow \arg(z)=\frac{3\pi}{4}$

41. If A (Z_1) B(Z_2) C(Z_3) are vertices of a triangle such that $Z_3=\left(\frac{Z_2-iZ_1}{1-i}\right)$ *and* $|Z_1|=3, |Z_2|=4$ *and* $|Z_2+iZ_1|=|Z_1|+|Z_2|$ then area of triangle ABC is

1. $\frac{5}{2}$ 2. 0 3. $\frac{25}{2}$ 4. $\frac{25}{4}$

Key. 4

Sol. $|z_2+iz_1|=|z_1|+|z_2|\Rightarrow z_2, iz_1, 0$ are collinear.

$\therefore \arg(iz_1=\arg z_2)$

$\Rightarrow \arg i+\arg z_1=\arg z_2$

$\Rightarrow \arg z_2-\arg z_1=\frac{\pi}{2}$

$z_3=\frac{z_2-iz_1}{l-i}$

$(l-i)z_3=z_2-iz_1$

$z_3-z_2=i(z_3-z_1)$

$\frac{z_3-z_2}{z_3-z_1}=i\Rightarrow \arg\left(\frac{z_3-z_2}{z_3-z_1}\right)=\frac{\pi}{2}$ and $|z_3-z_2|=|z_3-z_1|$

$\therefore$ AB=BC, $\therefore\ AB^2=AC^2+BC^2$

$25=2AC^2$

$\Rightarrow AC=\frac{5}{\sqrt{2}}$

Required area= $\frac{1}{2}\times\frac{5}{\sqrt{2}}\times\frac{5}{\sqrt{2}}=\frac{25}{4}$ sq. units

42. The radius of the circle given by arg $\left(\frac{Z-5+4i}{Z+3-2i}\right)=\frac{\pi}{4}$

1. $5\sqrt{2}$ 2. 5 3. $\frac{5}{\sqrt{2}}$ 4. $\sqrt{2}$

Key. 1

Sol. A(5,-4) B(-3,2) subtends an angle $\frac{\pi}{4}$ at C(z) on the circle hence $\frac{\pi}{2}$ at centre

$$M \to M.dAB \therefore AM = \frac{AB}{2}$$

$$= \frac{\sqrt{64+36}}{2}\frac{10}{2} = 5$$

Radius $= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$

43. f(x) = $2x^3 + 2x^2 - 7x + 72$ then $f\left(\frac{3-5i}{2}\right)$ = ___________

1. 1 2. 2 3. 3 4. 4

Key. 4

Sol. Let $x = \frac{3-5i}{2}$

$$2x = 3 - 5i$$

$$(2x-3)^2 = 5i$$

$$4x^2 - 12x + 9 = 25i^2$$

$$\Rightarrow 2x^2 - 12x + 34 = 0 \Rightarrow 2x^2 - 6x + 17 = 0$$

$$2x^2 - 6x + 17)2x^3 + 2x^2 = 7x + 72(x+4$$

$$2x^3 - 6x^2 + 17x$$

$$8x^2 - 24x + 72$$

$$8x^2 - 24x + 68$$

$$4$$

44. If $Cos\,\alpha + Cos\,\beta + Cos\gamma = 0 = Sin\,\alpha + Sin\,\beta + Sin\gamma$ then $Sin^2\,\alpha + Sin^2\,\beta + Sin^2\gamma$ = _______

1. $\frac{1}{2}$ 2. $\frac{3}{2}$ 3. 4 4. 1

Key. 2

Sol. Let $x = cis\alpha \quad y = cis\beta \quad z = cis\gamma$

Clearly $x+y+z=0, \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$

$$x^2+y^2+z^2=(x+y+z)^2-2xyz\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=0$$

$$=cis2\alpha+cis2\beta+cis2\gamma=0$$

$$\Rightarrow \cos 2\alpha+\cos 2\beta+\cos 2\gamma=0$$

$$1-2\sin^2\alpha+1-2\sin^2\beta+1-2\sin^2\gamma=0$$

$$\Rightarrow \sin^2\alpha+\sin^2\beta+\sin^2\gamma=\frac{3}{2}$$

45. If $Z_1\, and\, Z_2$ are two complex numbers such that $Z_1{}^2+Z_2{}^2\in R\ and\ Z_1(Z_1{}^2-3Z_2{}^2)=2$ $Z_2(3Z_1{}^2-Z_2{}^2)=11$ then $Z_1{}^2+Z_2{}^2=$

A) 5 2.125 3. 25 4. 15

Key. 1

Sol. $z_1\left(z_1^2-3z_2^2\right)=2$

$$z_1^2\left(z_1^4+9z_2^4-6z_1^2z_2^2\right)=4$$

$$\left(z_1^2\right)^3+9z_1^2z_2^4-6z_1^4z_2^2=4 \longrightarrow (1)$$

$$z_2^2\left(3z_1^2-z_2^2\right)^2=|121|$$

$$\Rightarrow\left(z_2^2\right)^3+9z_2^2z_1^4-6z_1^2z_2^4=121 \longrightarrow (2)$$

(1) + (2) $\Rightarrow\left(z_1^2+z_2^2\right)^3 125$

$$z_1^2+z_2^2=5$$

46. Let 'C' denote the set of complex numbers and define A & B by

$A=\{(z,w);z,w\in C \text{ and } |z|=|w|\}$

$B=\{(z,w);z,w\in C; \text{and } z^2=w^2\}$ then

A) $A=B$ B) $A\subset B$ C) $B\subset A$ D) none

Key: C

Hint: Conceptual

47. If $\left|z-|z+1|\right|=\left|z+|z-1|\right|$ where z is a complex number on the complex plane, then which of the following lies on the locus of z

A) line $y=0$ B) line $x=2$

C) circle $x^2+y^2=1$ D)line $x=0$ or on a line segment joining $(-1,0)$ to $(1,0)$

Key: D

Hint: $\left|z-|z+1|\right|^2=\left|z+|z-1|\right|^2$

$\Rightarrow (z+\bar{z})(|z+1|+|z-1|-2)=0$

$\Rightarrow$ z lies on y-axis or

Z lies on line segment joining the points $(-1,0)$ and $(1,0)$

48. If Z_1, Z_2 are two complex numbers such that $|Z_1|=1$, $|Z_2|=1$ then the maximum value of $|Z_1+Z_2|+|Z_1-Z_2|$ *is*

a) 2 b) $2\sqrt{2}$ c) 4 d) none of these

Key: B

$Z_1=\cos\alpha+i\sin\alpha,\quad Z_2=\cos\beta+i\sin\beta$

$|Z_1+Z_2|+|Z_1-Z_2|=\sqrt{2+2\cos(\alpha-\beta)}+\sqrt{2-2\cos(\alpha-\beta)}$

Hint: $let\,\alpha-\beta=\theta$

$2\cos\frac{\theta}{2}+2\sin\frac{\theta}{2}=2\sqrt{2}\sin\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$

49. If $|z-2|=\min\{|z-1|,|z-5|\}$, where Z is a complex number then

(A) $\mathrm{Re}(z)=\frac{3}{2}$ only (B) $\mathrm{Re}(z)=\frac{7}{2}$ only

(C) $\mathrm{Re}(z)\in\left\{\frac{3}{2},\frac{7}{2}\right\}$ (D) $\mathrm{Re}(z)\in\left\{\frac{1}{2},\frac{3}{2}\right\}$

Key: C

Hint: draw the locus Z in argand plane.

$\mathrm{Re}(z)\in\left\{\frac{3}{2},\frac{7}{2}\right\}$

50. If Z is a non-real complex number, then the minimum value of $\frac{\mathrm{Im}\,Z^5}{\mathrm{Im}^5 Z}$ is

(A) -1 (B) -2

(C) -4 (D) -5

Key: C

Hint: Let $Z=a+ib, b\neq 0$ where $\mathrm{Im}\,Z=b$

$Z^5=(a+ib)^5=a^5+{}^5C_1a^4bi+{}^5C_2a^3b^2i^2+{}^5C_3a^2b^3i^3+{}^5C_4i^4+i^5b^5$

$\mathrm{Im}\,Z^5=5a^4b-10a^2b^3+b^5$

$y=\frac{\mathrm{Im}\,Z^5}{\mathrm{Im}^5 z}=5\left(\frac{a}{b}\right)^4-10\left(\frac{1}{b}\right)^2+1$

Let $\left(\frac{a}{b}\right)^2 = x(\text{say}), x \in R^+$

$y = 5x^2 - 10x + 1 = 5\left[x^2 - 2x\right] + 1 = 5\left[(x-1)^2\right] - 4$

Hence $y_{min} = -4$.

51. Let $z_r\ (1 \le r \le 4)$ be complex numbers such that $|z_r| = \sqrt{r+1}$ and

$|30z_1 + 20z_2 + 15z_3 + 12z_4| = k|z_1z_2z_3 + z_2z_3z_4 + z_3z_4z_1 + z_4z_1z_2|$

Then the value of k equals

(A) $|z_1z_2z_3|$ (B) $|z_2z_3z_4|$

(C) $|z_4z_1z_2|$ (D) None of these

Key: C

Hint: We have $\left|\frac{z_1}{2} + \frac{z_2}{3} + \frac{z_1}{4} + \frac{z_4}{5}\right| = \frac{k}{60}|z_1z_2z_3z_4|\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \frac{1}{z_4}\right|$

Now, $z_1\overline{z}_1 = 2,\ z_2\overline{z}_2 = 3,\ z_3\overline{z}_3 = 4$ and $z_4\overline{z}_4 = 5$

So, $k = \frac{60}{|z_1z_2z_3z_4|} = \frac{60}{\sqrt{2}\sqrt{3}\sqrt{4}\sqrt{5}} = \sqrt{30} = |z_4z_1z_2|$

Note for objective take $z_1 = \sqrt{2}; z_2 = \sqrt{3}; z_3 = 2; z_4 = \sqrt{5}$ |

52. If P(z) and A(z_1) two be variable points such that $zz_1 = |z|^2$ and $|z - \overline{z}| + |z_1 + \overline{z}_1| = 10$ then area enclosed by the curve formed by them

(A) 25π (B) 20 π

(C) 50 (D) 100

Key: C

53. A particle starts to travel from a point P on the curve $C_1 : |z - 3 - 4i| = 5$, where $|z|$ is maximum. From P, the particle moves through an angle $\tan^{-1}\frac{3}{4}$ in anticlock wise direction on $|z - 3 - 4i| = 5$ and reaches at point Q. From Q, it comes down parallel to imaginary axis by 2 units and reaches at point R. Complex number corresponding to point R in the Argand plane is

(A) (3+5i) (B) (3+7i) (C) (3+8i) (D) (3+9i)

Key: B

Hint: $|z - 3 - 4i| = 5$

$\Rightarrow (x-3)^2 + (y-4)^2 = 25$

R is (3,7)

54. If $|z_1| = 2$, $|z_2| = 3$, $|z_3| = 4$ and $|2z_1+3z_2+4z_3|=4$, then absolute value of $8z_2z_3+27z_3z_1+64z_1z_2$ equals

(a) 24 (b) 48 (c) 72 (d) 96

Key: D

Hint: $|8z_2z_3+27z_3z_1+64z_1z_2| =$

$|z_1||z_2||z_3|\left|\frac{8}{z_1}+\frac{27}{z_2}+\frac{64}{z_3}\right|=(2)(3)(4)\left|\frac{8\bar{z}_1}{|z_1|^2}+\frac{27\bar{z}_2}{|z_2|^2}+\frac{64\bar{z}_3}{|z_3|^2}\right|$

$=24\left|2\bar{z}_1+3\bar{z}_2+4\bar{z}_3\right|=24\left|\overline{2z_1+3z_2+4z_3}\right|=24\left|2z_1+3z_2+4z_3\right|$

= (24) (4) = 96

55. If the ratio $\frac{1-z}{1+z}$ is purely imaginary, then

(a) $0 < |z| < 1$ (b) $|z|=1$

(c) $|z|>1$ (d) bounds for $|z|$ can not be decided

Key: b

Hint: $0=\frac{1-z}{1+z}+\frac{1-\bar{z}}{1+\bar{z}}=\frac{(1-z)(1+\bar{z})+(1-\bar{z})(1+z)}{(1+z)(1+\bar{z})}=\frac{2(1-|z|)^2}{|1+z|^2}\Rightarrow$ $|z| = 1$

56. If P and Q are represented by the numbers z_1 and z_2 such that $\left|\frac{1}{z_2}+\frac{1}{z_1}\right|=\left|\frac{1}{z_2}-\frac{1}{z_1}\right|$, then the circumcentre of ΔOPQ, (where O is the origin) is

(A) $\frac{z_1-z_2}{2}$ (B) $\frac{z_1+z_2}{2}$

(C) $\frac{z_1+z_2}{3}$ (D) $z_1 + z_2$

Key : B

Sol : $\left|\frac{1}{z_2}+\frac{1}{z_1}\right|=\left|\frac{1}{z_2}-\frac{1}{z_1}\right|$

$\Rightarrow$ $|z_1 + z_2| = |z_1 - z_2|$

$\Rightarrow$ $z_1\bar{z}_2+z_2\bar{z}_1=0$

$\Rightarrow$ $\frac{z_1}{z_2}$ is purely imaginary

$\Rightarrow$ $\arg\left(\frac{z_1}{z_2}\right)=\pm\frac{\pi}{2}$

$\Rightarrow$ $\angle POQ=\frac{\pi}{2}$

Circumcentre of ΔPOQ is the mid point of PQ i.e.

57. If α is non real root of $x^7 = 1$, then $1 + 3\alpha + 5\alpha^2 + 7\alpha^3 + ... + 13\alpha^6$ is equal to

(A) 0 (B) $\frac{14}{1-\alpha}$

(C) $\frac{14}{\alpha-1}$ (D) none of these

Key: C

Hint: Let A = $1 + 3\alpha + 5\alpha^2 + 7\alpha^3 + ... + 11\alpha^5 + 13\alpha^6$

$\alpha A = \alpha + 3\alpha^2 + 5\alpha^3 + 7\alpha^5 + ... + 11\alpha^7 + 13\alpha^7$

$(1 - \alpha) A = 1 + 2\alpha + 2\alpha^2 + 2\alpha^3 + ... + 2\alpha^6 - 13\alpha^7$

$= -12 + 2[\alpha + \alpha^2 + ... + \alpha^6) = -14$

$A = -\frac{14}{1-\alpha}$

58. If z_1, z_2 are two complex numbers satisfying the equation $\left|\frac{z_1+z_2}{z_1-z_2}\right|=1$, then $\frac{z_1}{z_2}$ is a number which is

(A) Positive real (B) Negative real

(C) Zero (D) Lying on imaginary axis

Key. D

Sol. $\left|\frac{z_1+z_2}{z_1-z_2}\right|=1 \Rightarrow \frac{z_1+z_2}{z_1-z_2}=\cos\alpha+i\sin\alpha$

where α is the argument of $\frac{(z_1+z_2)}{(z_1-z_2)}$. Applying componendo and dividendo, we get

$$\frac{z_1}{z_2}=\frac{1+\cos\alpha+i\sin\alpha}{-1+\cos\alpha+i\sin\alpha}$$

$$=\frac{2\cos\left(\frac{\alpha}{2}\right)\left[\cos\left(\frac{\alpha}{2}\right)+i\sin\left(\frac{\alpha}{2}\right)\right]}{2i\sin\left(\frac{\alpha}{2}\right)\left[\cos\left(\frac{\alpha}{2}\right)+i\sin\left(\frac{\alpha}{2}\right)\right]}=-i\cot\left(\frac{\alpha}{2}\right)$$

Purely imaginary in nature

59. If z_1, z_2 and z_3 are the vertices of $\Delta ABC,$ which is not right angled triangle taken in anti-clock wise direction and z_0 is the circumcentre, then $\left(\frac{z_0-z_1}{z_0-z_2}\right)\frac{\sin 2A}{\sin 2B}+\left(\frac{z_0-z_3}{z_0-z_2}\right)\frac{\sin 2C}{\sin 2B}$ is equal to

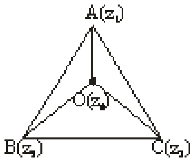
A) 0 B) 1 C) -1 D) 2

Key. C

Sol. Taking rotation at $'O'$

$$\frac{z_0-z_1}{z_0-z_2}=\cos 2C-i\sin 2C$$

$$\frac{z_0 - z_3}{z_0 - z_2} = \cos 2A + i \sin 2A$$



Now $\left(\frac{z_0 - z_1}{z_0 - z_2}\right)\frac{\sin 2A}{\sin 2B} + \left(\frac{z_0 - z_3}{z_0 - z_2}\right)\frac{\sin 2C}{\sin 2B}$

$$= \frac{\sin 2A\cos 2C - i\sin 2A\sin 2C + \cos 2A\sin 2C + i\sin 2A\sin 2C}{\sin 2B}$$

$$= \frac{\sin(2A + 2C)}{\sin 2B} = -1$$

60. If a complex number 'z' lies in the interior or on the boundary of a circle of radius 3 and centre at $(-4, 0)$, then the greatest and least values of $|z+1|$ are respectively

A) $5, 0$ B) $6, 1$ C) $6, 0$ D) $5, 1$

Key. C

Sol. It is given that $|z+4| \le 3$

Hence the greatest value of $|z+1| is 6$

Since the least value of the modulus of a complex number is zero, therefore

$|z+1| = 0 \Rightarrow z = -1 \quad \Rightarrow |z+4| = |-1+4| = 3$

$\Rightarrow |z+4| \le 3$ is satisfied by $z = -1$

Therefore the least value of $|z+1| is 0$

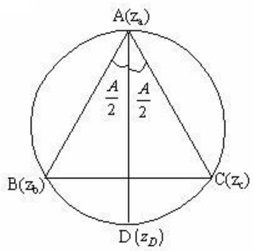
61. $A(z_a), B(z_b), C(z_c)$ be the vertices of ΔABC taken in anticlockwise direction whose circum–circle is $|z| = r$

If the internal angular bisector of angle A meets the circum circle again at $D(z_D)$ then

A) $z_D = z_a z_c$ B) $z_D^2 = z_b z_c$ C) $z_D = \frac{z_b z_c}{z_a}$ D) $z_D = \frac{-z_b z_c}{z_a}$

Key. B

Sol. Let D represents the complex number $z = (z_D)$



$\angle BAD = \angle CAD = A/2$

$$\frac{z-z_a}{|z-z_a|} = \frac{z_b-z_a}{|z_b-z_a|}e^{iA/2}$$

$$\frac{(z-z_a)^2}{|z-z_a|^2} = \frac{(z_b-z_a)^2}{|z_b-z_a|^2}e^{iA} \quad \text{----------- (1)}$$

Similarly $\frac{(z_c-z_a)^2}{|z_c-z_a|^2} = \frac{z-z_a}{|z-z_a|^2}e^{iA}$ ----------- (2)

From (1) & (2)

$$\frac{z-z_a}{\overline{z}-\overline{z}_a} = \frac{z_b-z_a}{\overline{z}_b-\overline{z}_a}e^{iA}, \quad \frac{z_c-z_a}{\overline{z}_c-\overline{z}_a} = \frac{z-z_a}{\overline{z}-\overline{z}_a}e^{iA} \qquad \Rightarrow \frac{z-z_a}{\overline{z}-\overline{z}_a}\times\frac{\overline{z}_c-\overline{z}_a}{z_c-z_a} = \frac{z_b-z_a}{\overline{z}_b-\overline{z}_a}\,\frac{\overline{z}-\overline{z}_a}{z-z_a}$$

$$\Rightarrow \left(\frac{z-z_a}{\overline{z}-\overline{z}_a}\right)^2 = \frac{z_b-z_a}{\overline{z}_b-\overline{z}_a}\cdot\frac{z_c-z_a}{\overline{z}_c-\overline{z}_a}$$

$$\Rightarrow \left(\frac{z-z_a}{\frac{r^2}{z}-\frac{r^2}{z_a}}\right)^2 = \left(\frac{z_b-z_a}{\frac{r^2}{z_b}-\frac{r^2}{z_a}}\right)\left(\frac{z_c-z_a}{\frac{r^2}{z_c}-\frac{r^2}{z_a}}\right)\left(\begin{array}{l}\because z_a, z_b, z_c \text{ and } z \text{ lie on } |z|=r\\ \therefore |z_a|=|z_b|=|z_c|=r\end{array}\right)$$

$$\Rightarrow (z\,z_a)^2 = (z_a z_b)(z_a z_c) \Rightarrow z^2 = z_b z_c \Rightarrow z_D^2 = z_b z_c$$

62. The least positive integer 'n' for which $\left(\frac{1+i}{1-i}\right)^n = \frac{2}{\pi}\sin^{-1}\left(\frac{1+x^2}{2x}\right)$, where $x>0$ and $i=\sqrt{-1}$ is

A) 2 B) 4 C) 8 D) 12

Key. B

Sol. $\because -1 \le \frac{1+x^2}{2x} \le 1$

$$\Rightarrow \left|\frac{1+x^2}{2x}\right| \le 1 \Rightarrow \frac{1+x^2}{2|x|} \le 1 \Rightarrow \frac{1+|x^2|}{2|x|} - 1 \le 0 \Rightarrow \frac{(|x|-1)^2}{|x|} \le 0$$

$$\therefore |x|>0, \therefore (|x|-1)^2 \le 0 \quad \Rightarrow (|x|-1)^2 = 0$$

$\therefore |x| = 1 \quad \Rightarrow x = \pm 1$

$\therefore x = 1 \; (\because x > 0)$

$\left(\frac{1+i}{1-i}\right)^n = \frac{2}{\pi}\cdot \sin^{-1}(1) = \frac{2}{\pi}\cdot\frac{\pi}{2} = 1 \quad \Rightarrow \left(\frac{(1+i)^2}{2}\right)^n = 1$

$(i)^n = 1$

$\Rightarrow (-1)^{n/2} = (-1)^2, (-1)^4, (-1)^6, ... \Rightarrow \frac{n}{2} = 2$

$\therefore n = 4$ (least positive value)

63. If $'a'$ is a complex number such that $|a| = 1$, then the values of $'a'$, so that equation $az^2 + z + 1 = 0$ has one purely imaginary root is

A) $a = \cos\theta + i\sin\theta; \theta = \cos^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$

B) $a = \sin\theta + i\cos\theta,\ \theta = \cos^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$

C) $a = \cos\theta + i\sin\theta; \theta = \cos^{-1}\left(\frac{\sqrt{5}-1}{4}\right)$

D) $a = \sin\theta + i\cos\theta,\ \theta = \cos^{-1}\left(\frac{\sqrt{5}-1}{4}\right)$

Key. A

Sol. $az^2 + z + 1 = 0....(i)$

Taking conjugate of both sides, $\overline{az^2 + z + 1} = \bar{0} \quad \Rightarrow \bar{a}(\bar{z})^2 + \bar{z} + \bar{1} = 0$

$\bar{a}z^2 - z + 1 = 0$ (since $\bar{z} = -z$ as $'z'$ is purely imaginary) ....(ii)

Eliminating $'z'$ from both the equations, we get $(\bar{a} - a)^2 + 2(a + \bar{a}) = 0$

Let $a = \cos\theta + i\sin\theta$ (since $|a| = 1$) so that $(-2i\sin\theta)^2 + 2(2\cos\theta) = 0$

$\Rightarrow \cos\theta = \frac{-1 \pm \sqrt{1+4}}{2}$

Only feasible value of $\cos\theta$ is $\frac{\sqrt{5}-1}{2}$

Hence $a = \cos\theta + i\sin\theta$, where $\theta = \cos^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$

64. The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} - i\cos\frac{2k\pi}{11}\right)$ is

A) -1 B) 0 C) $-i$ D) i

Key. D

Sol.
$$=-i\left(\cos\frac{2k\pi}{11}+i\sin\frac{2k\pi}{11}\right)=-i\left(e^{i\frac{2\pi}{11}}\right)^k$$

Let $e^{i\frac{2\pi}{11}}=z$

$$\therefore \sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11}-i\cos\frac{2k\pi}{11}\right)=-i\sum_{k=1}^{10}z^k$$

$$=-i\left[z+z^2+z^3+.......+z^{10}\right]$$

$$=-i\left[\frac{z\left(z^{10}-1\right)}{z-1}\right]=-i\left[\frac{z^{11}-z}{z-1}\right]=i$$

65. If $'z'$ lies on the circle $|z-2i|=2\sqrt{2}$, then the value of $\arg\left(\frac{z-2}{z+2}\right)$ is equal to

A) $\frac{\pi}{3}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{6}$ D) $\frac{\pi}{2}$

Key. B

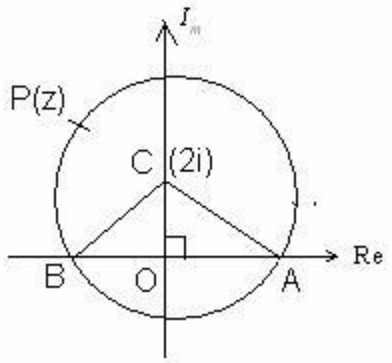
Sol. $CA=CB=2\sqrt{2}\cdot OC=2\Rightarrow OA=OB=2$

$\Rightarrow A=2+0i, B=-2+0i$

Clearly, $\angle BCA=\frac{\pi}{2}$

$\Rightarrow \angle BPA=\frac{\pi}{4}$

$\Rightarrow \arg\left(\frac{z-2}{z+2}\right)=\frac{\pi}{4}$



66. If $'z'$ is a complex number such that equation $\left|z-a^2\right|+\left|z-2a\right|=3$ always represents an

ellipse, then range of $a\left(\in R^{+}\right)$ is

A) $\left(1,\sqrt{2}\right)$ B) $\left[1,\sqrt{3}\right]$ C) $\left(-3,1\right)$ D) $\left(0,3\right)$

Key. D

Sol. $\left|a^2-2a\right|<3$

$\Rightarrow -3<a^2-2a<3 \Rightarrow -3+1<a^2-2a+1<3+1 \Rightarrow -2<\left(a-1\right)^2<4$

$\therefore 0\le\left(a-1\right)^2<4 \Rightarrow -2<a-1<2$ or $-1<a<3$

But $a\in R^{+}$

$\therefore\ 0<a<3 \Rightarrow a\in\left(0,3\right)$

67. ω is a non real complex cube root of unity and $a,b\in R$ .If ω,ω^2 are roots of $\frac{1}{a+x}+\frac{1}{b+x}=\frac{3}{x}$ then *a,b* are roots of

a) $3x^2-6x+2=0$
b) $6x^2-3x+2=0$
c) $2x^2-3x+6=0$
d) $6x^2-2x+3=0$

Key. B

Sol. The given equation simplifies $x^2+2x\left(a+b\right)+3ab=0$, whose roots are given table ω,ω^2

Hence $a+b=\frac{1}{2},ab=\frac{1}{3}$ .So a,b are roots of $x^2-x\left(\frac{1}{2}\right)+\frac{1}{3}=0$

68. If z is a complex number such that $\left|z-1\right|$ =1 then *arg* $\left(\frac{1}{z}-\frac{1}{2}\right)$ may be

a) $\frac{\pi}{6}$ b) $-\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $-\frac{\pi}{4}$

Key. B

Sol. Since $\left|z-1\right|=1\Rightarrow z-1=cis\theta\Rightarrow z=\left(1+\cos\theta\right)+i\sin\theta=2\cos\frac{\theta}{2}cis\frac{\theta}{2}$

$$\therefore\frac{1}{z}-\frac{1}{2}=\frac{cis\left(-\frac{\theta}{2}\right)}{2\cos\frac{\theta}{2}}-\frac{1}{2}=-\frac{i}{2}\tan\frac{\theta}{2}$$ which is purely imaginary

69. $\theta\in\left[0,2\pi\right]$ and z_1,z_2,z_3 are three complex numbers such that they are collinear and $\left(1+\left|\sin\theta\right|\right)z_1+\left(\left|\cos\theta\right|-1\right)z_2-\sqrt{2}\ z_3$ =0 .If at least one of the complex numbers z_1,z_2,z_3 is non- zero then number of possible values of θ is

a) Infinite b) 4 c) 2 d) 8

Key. B

Sol. If z_1, z_2, z_3 are collinear and $az_1 + bz_2 + cz_3 = 0$ then a+b+c =0 .Hence

$$1+|\sin\theta|+|\cos\theta|-1-\sqrt{2}=0 \Rightarrow |\sin\theta|+|\cos\theta|=\sqrt{2}$$

$$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

70. Let $A(z_1), B(z_2), C(z_3)$ be the vertices of a triangle oriented in anti clock wise direction. If $BC : CA : AB = 2 : \sqrt{2} : \sqrt{3}+1$, then the imaginary part of $\left(\frac{z_3 - z_1}{z_2 - z_1}\right)^4$ is

A) 0 B) $-7+2\sqrt{6}$

C) $7-2\sqrt{6}$ D) cannot be determined

Key. A

Sol. $\cos A = +\frac{1}{\sqrt{2}} \Rightarrow A = \frac{\pi}{4}$

$$\therefore \frac{z_3 - z_1}{z_2 - z_1} = \left|\frac{z_3 - z_1}{z_2 - z_1}\right| \text{cis } (\pi/4)$$

$$\Rightarrow \left(\frac{z_3 - z_1}{z_2 - z_1}\right)^4 = \left(\frac{\sqrt{2}}{\sqrt{3}+1}\right)^4 e^{i\pi} \Rightarrow \left(\frac{z_3 - z_1}{z_2 - z_1}\right)^4 = -\left(\frac{\sqrt{3}-1}{2}\right)^4$$

71. A,B,C are vertices of a triangle inscribed in the circle $|z|=1$. Altitude from A meets the circumcircle again at D. If D,B,C represents the complex number z_1, z_2, z_3 respectively then the complex number representing the reflection of D in the line BC, is

A) $\frac{z_1 z_2 + z_1 z_3 + z_2 z_3}{z_1}$ B) $\frac{z_1 z_2 + z_2 z_3 + z_1 z_3}{z_1 z_2 z_3}$

C) $\frac{z_1 z_2 + z_1 z_3 - z_2 z_3}{z_1}$ D) $\frac{z_1 z_2 + z_1 z_3 - z_2 z_3}{z_1 z_2 z_3}$

Key. C

Sol. image of D w.r.t sides of triangle is orthocenter

72. A point P representing the complex number z moves in the Argand plane so that it lies always in the region defined by $|z-1| \le |z-i|$ and $|z-2-2i| \le 1$. If P describes the boundary of this

region then the value of $|z|$ when the arg(z) has least value, is

A) $\sqrt{5}$ B) 7 C) $\sqrt{7}$ D) 5

Key. C

Sol. $|z| = OR = \sqrt{8-1} = \sqrt{7}$

73. Let P(z) be a variable point in the complex plane such that $|z| = \min\{|z-1|, |z+1|\}$ then the value of $\left(z+\bar{z}\right)$ is

A) 1 if Re(z)>0 B) 1 if Re z<0 C) 0 if Re z>0 D) 0 if Re z <0

Key. A

Sol. Let $|z| = |z-1|$ if Re $Z > 0$

$\Rightarrow$ lies line $z = \frac{1}{2}$

$\Rightarrow z + \bar{z} = \frac{1}{2} + \frac{1}{2} = 1$

74. If z is a complex number satisfying $|z|^2 + 2(z+\bar{z}) + 3i(z-\bar{z}) + 4 = 0$, $i = \sqrt{-1}$, then the complex number $z+3+2i$ will lie on a circle with

A) centre $1-5i$, radius 4 B) centre $1+5i$, radius 4

C) centre $1+5i$, radius 3 D) centre $1-5i$, radius 3

Key. C

Sol. Given $|z+(2-3i)| = 3$, Let $w = \left(z+3+2i\right) = z+2-3i+1+5i$

$\Rightarrow |w-(1+5i)| = |z+2-3i| = 3$.

75. The value of $i\log_e(x-i) + i^2\pi + i^3\log_e(x+i) + i^4\left(2\tan^{-1}x\right)$, $x > 0$, $i = \sqrt{-1}$ is

A) 0 B) 1 C) 2 D) 3

Key. A

Sol. Let $i\log\frac{x-i}{x+i} - \pi + 2\tan^{-1}x = k$

$\Rightarrow \log\left(\frac{x+i}{x-i}\right) = \left(k+\pi-2\tan^{-1}x\right)i = i\theta$

$\Rightarrow \frac{x+i}{x-i} = e^{i\theta} \Rightarrow x = \cot\frac{\theta}{2} \Rightarrow \theta = 2\cot^{-1}x$

$\therefore k+\pi-2\tan^{-1}x = 2\cot^{1}x \Rightarrow k = 0$

76. If $\left|\frac{z_1}{z_2}\right| = 1$ and arg $(z_1\ z_2) = 0$, then

A) $z_1 = z_2$ B) $|z_2|^2 = z_1 z_2$ C) $z_1 z_2 = 1$ D) $z_1 z_2 = 2$

Key. B

Sol. $\left|\frac{z_1}{z_2}\right| = 1 \Rightarrow |z_1| = |z_2| = r_1$ as $z_1 = r_1(\cos\theta_1 + \sin\theta)$

$$\arg(z_1 z_2) = 0 \quad \Rightarrow z_2 = r_1\left(\cos(-\theta_1) + i\sin(-\theta_2)\right)$$

$$\Rightarrow z_2 = \overline{z}_1 \quad \Rightarrow \overline{z}_2 = z_1 \Rightarrow |z_2|^2 = z_1.z_2$$

77. Let z be a complex number and $a_k, b_k (k = 1, 2, 3)$ are real numbers then the value of

$$\begin{vmatrix} a_1 z + b_1 \overline{z} & a_2 z + b_2 \overline{z} & a_3 z + b_3 \overline{z} \\ b_1 z + a_1 \overline{z} & b_2 z + a_2 \overline{z} & b_3 z + a_3 \overline{z} \\ b_1 z + a_1 & b_2 z + a_2 & b_3 z + a_3 \end{vmatrix} =$$

A) $(a_1 a_2 a_3 + b_1 b_2 b_3)|z|^2$ B) $(a_1 a_2 a_3 - b_1 b_2 b_3)|z|^2$

C) $a_1^{\ 2} - a_1^{\ 2}$ D) $|z|^2$

Key. C

Sol. $$\begin{vmatrix} z & \overline{z} & 1 \\ \overline{z} & z & 1 \\ 1 & z & \overline{z} \end{vmatrix} \begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix} = 0$$

78. If z_1, z_2, z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 1$ then area of region common to given triangle and another triangle having vertices $-z_1, -z_2, -z_3$, is

A) $\frac{\sqrt{3}}{2}$ B) $\frac{\sqrt{3}}{4}$ C) $\frac{7\sqrt{3}}{4}$ D) $\frac{5\sqrt{3}}{4}$

Key. A

Sol. Area of common region

= Area of ΔABC - 3 Area of $AB^{'}C^{'}$

$$= 3\frac{\sqrt{3}}{4} - 3.\frac{1}{2}.\frac{1}{2}\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{2}.$$

79. If $az^2 + bz + 1 = 0$, $a, b, z \in C$ and $|a| = \frac{1}{2}$, have a root α such that $|\alpha| = 1$ then $|a\overline{b} - b| =$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{5}{4}$ D) $\frac{3}{4}$

Key. D

Sol. $a\alpha^2 + b\alpha + 1 = 0$

$$\bar{a}\bar{\alpha}^2 + \bar{b}\bar{\alpha} + 1 = 0$$

$$\Rightarrow \alpha^2 + \bar{b}\alpha + \bar{a} = 0$$

$$\frac{\alpha^2}{\bar{a}b - \bar{b}} = \frac{\alpha}{1-|a|^2} = \frac{1}{a\bar{b} - b} \Rightarrow |a\bar{b} - b| = 1 - |a|^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

80. If $|z-1| \le 2$ & $|wz - 1 - w^2| = a$ (where 'w' is a cube root of unity) then complete set of values of a is

a) $0 \le a \le 2$ b) $\frac{1}{2} \le a \le \frac{\sqrt{3}}{2}$

c) $\frac{\sqrt{3}}{2} - \frac{1}{2} \le a \le \frac{1}{2} + \frac{\sqrt{3}}{2}$ d) $0 \le a \le 4$

Key. D

Sol. $|wt - 1 - w^2| = a$

$\Rightarrow |w||z+1| = a$

$\Rightarrow |z-1| + 2 \ge a$

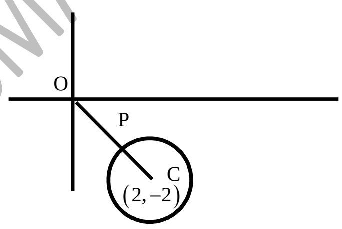
81. If z is a complex number having least absolute value and $|z - 2 + 2i| = 1$ then z =

a) $\left(2 - \frac{1}{\sqrt{2}}\right)(1-i)$ b) $\left(2 - \frac{1}{\sqrt{2}}\right)(1+i)$

c) $\left(2 + \frac{1}{\sqrt{2}}\right)(1-i)$ d) $\left(2 + \frac{1}{\sqrt{2}}\right)(1+i)$

Key. A

Sol. OP = OC - CP



$= 2\sqrt{2} = 1$

$\therefore (0,0)\ (2,-2)$

$2\sqrt{2} - 1 : 1$

$$\frac{2\left(2\sqrt{2}-1\right)}{2\sqrt{2}}, \frac{-2\left(2\sqrt{2}-1\right)}{2\sqrt{2}}$$

$$=\left(\left(2-\frac{1}{\sqrt{2}}\right), -\left(2-\frac{1}{\sqrt{2}}\right)\right)$$

82. Sum of common roots of the equation $z^3+2z^2+2z+1=0$ and $z^{1985}+z^{100}+1=0$ is

a) -1 b) 1 c) 0 d) 1

Key. A

Sol. $\left(z+1\right)\left(z^2+z+1\right)$

$\Rightarrow z=-1, w, w^2$

Let $f\left(z\right)=z^{1985}+z^{100}+1$

$f\left(-1\right)\neq 0, f\left(w\right)=f\left(w\right)=0$

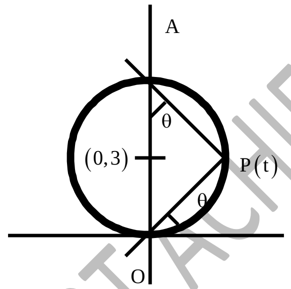
$\therefore w+w^2=-1$

83. Let z be a complex number having the argument $\theta,\ 0<\theta<\frac{\pi}{2}$ and satisfying the equation, $\left|z-3i\right|=3$. Then $\cot\theta-\frac{6}{z}=$

a) i b) – i c) 2i d) – 2i

Key. A

Sol. $r=0A\sin\theta=6\sin\theta$



$z=6\sin\theta\left(\cos\theta+i\sin\theta\right)$

$\Rightarrow \cot\theta-\frac{6}{z}=i$

84. If a^2+b^2, $ab+bc$ and b^2+c^2 are in G.P. then a, b, c are in

a) A.P b) G.P

c) H.P d) None

Key. B

Sol. $\left(ab+bc\right)^2=\left(a^2+b^2\right)\left(b^2+c^2\right)$

85. If $H_n=1+\frac{1}{2}+....+\frac{1}{n}$ then the value of $S_n=1+\frac{3}{2}+\frac{5}{3}+....+\frac{99}{50}$ is___

a) $H_{50}+50$ b) $100-H_{50}$

c) $49 + H_{50}$ d) $H_{50} + 100$

Key. B

Sol. $S_n = (2-1) + \left(2\frac{-1}{2}\right) + \left(2 - \frac{1}{3}\right) + \ldots + \left(2 - \frac{1}{50}\right)$

$= 100 - H_{50}$

86. Let z be a complex number satisfying $|z+16| = 4|z+1|$ then

a) $|z| = 4$ b) $|z| = 5$

c) $|z| = 6$ d) $3 < |z| < 6$

Key. A

Sol. $|z+16|^2 = 16|z+1|^2 \Rightarrow (z+16)(\bar{z}+16) = 16(z+1)(\bar{z}+1)$

$\Rightarrow z\bar{z} + 16z + 16\bar{z} + 256 = 16z\bar{z} + 16z + 16\bar{z} + 16$

$\Rightarrow z\bar{z} = 16 \Rightarrow |z|^2 = 16 \Rightarrow |z| = 4$

87. Let z_1 and z_2 be any two complex numbers then $\left|z_1 + \sqrt{z_1^2 - z_2^2}\right| + \left|z_1 - \sqrt{z_1^2 - z_2^2}\right|$ is equal to

a) $|z_1^2 - z_2^2| + |z_1^2 + z_2^2|$ b) $|z_1 - z_2| + |z_1^2 + z_2^2|$

c) $|z_1 + z_2| + |z_1^2 + z_2^2|$ d) $|z_1 + z_2| + |z_1 - z_2|$

Key. D

Sol. If $z_1 + \sqrt{z_1^2 - z_2^2} = u$ and $z_1 - \sqrt{z_1^2 - z_2^2} = v$,

We have

$|u|^2 + |v|^2 = \frac{1}{2}|u+v|^2 + \frac{1}{2}|u-v|^2$

$= 2|z_1|^2 + 2|z_1^2 - z_2^2|$

And so

$(|u| + |v|)^2 = 2\left\{|z_2|^2 + |z_z^1 - z_2^2| + |z_2|^2\right\}$

$= |z_1 z_2|^2 + |z_2 - z_2|^2 + 2|z_1^2 - z_2^2|$

$= (|z_1 + z_2| + |z_1 - z_2|)^2$

88. Both the roots of the equation $z^2 + az + b = 0$ are of unit modulus if

a) $|a| \le 2, |b| = 1, \arg b = 2\arg a$ b) $|a| \le 2, |b| = 1, \arg b = \arg a$

c) $|a| \ge 2, |b| = 2, \arg b = 2\arg a$ d) $|a| \ge 2, |b| = 2, \arg b = \arg a$

Key. A

Sol. Let $z_1 = \cos\phi_1 + i\sin\phi_1, \& z_2 = \cos\phi_2 + i\sin\phi_2$

Be the roots of $z^2 + az + b = 0$

$z_1 + z_2 = (-a)$ & $z_1 z_2 = b$

$$-2\cos\left(\frac{\phi_1 - \phi_2}{2}\right)\left[\cos\frac{\phi_1+\phi_2}{2} + i\sin\frac{\phi_1+\phi_2}{2}\right] = a$$

$$\Rightarrow \arg(a) = \frac{\phi_1+\phi_2}{2}$$

$\arg b = \phi_1 + \phi_2$

$\therefore \arg b = 2\arg a$

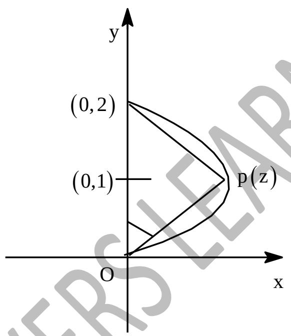
Also $|z_1 z_2| = |b| = 1$ and $|a| \le 2$

89. If $|z - i| = 1$ and $\arg(z) = \theta$ where $\theta \in \left(0, \frac{\pi}{2}\right)$, then $\cot\theta - \frac{2}{z}$ equals

a) $2i$ b) 3i c) i d) – i

Key. C

Sol. $< AOP = \frac{\pi}{2} - \theta$



$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta = \frac{|z|}{2}$$

Also $\frac{2i}{z} = \frac{OA}{OP}(\sin\theta + i\cos\theta) = \frac{2}{|z|}(\sin\theta + i\cos\theta)$

$= 1 + i\cot\theta$

$\frac{2}{z} = -i + \cot\theta$

$\Rightarrow \cot\theta - \frac{2}{z} = i$

90. Let $z = \frac{z_1 - z_2}{z_1\bar{z}_2 - 1}, z_1 \ne \frac{1}{\bar{z}_2}, 0 < |z_2| < 1$. If $|z| \le 1$ then

a) $|z_1| > 1$ b) $|z_1| \le 1$

c) $2 < |z_1| < 3$ d) $2 < |z_1| < 8$

Key. B

Sol. $z\bar{z} - 1 = \left(\frac{z_1 - z_2}{z_1\bar{z}_2 - 1}\right)\left(\frac{\bar{z}_1 - \bar{z}_2}{\bar{z}_1 z_2 - 1}\right) - 1$

$$=\frac{z_1\bar{z}_1+z_2\bar{z}_2-z_1z_2\bar{z}_1\bar{z}_2-1}{z_1z_2\bar{z}_1\bar{z}_2-z_1\bar{z}_2-\bar{z}_1z_2+1}$$

$$=\frac{\left(1-z_2\bar{z}_2\right)\left(z_1\bar{z}_1-1\right)}{\left(z_1\bar{z}_2-1\right)\left(\bar{z}_1z_2-1\right)}$$

$$\Rightarrow |z|^2-1=\frac{\left(1-|z_2|^2\right)\left(|z_1|^2-1\right)}{\left|z_1\bar{z}_2-1\right|^2}$$

$$\therefore |z_1|^2-1\le 0 \ \Rightarrow |z_1|\le 1$$

91. Let ‘z’ be a complex number satisfying $|z-2-i|\le 5$, Then $|z-14-6i|$ lies in

a) $[8,18]$ b) $(2,8)$

c) $[0,2]$ d) $[3,7]$

Key. A

Sol. $|z-14-6i|=|(z-2i)-(12+5i)|\le|z-2-i|+|12+5i|$

$\Rightarrow |z-14-6i|\le 5+13=18$

$\therefore$ Option a is correct

The complete solution can be obtained geometrically

92. If z_1, z_2 are complex numbers such that $z_1^3-3z_1z_2^2=2$ and $3z_1^2z_2-z_2^3=11$ then $\left|z_1^2+z_2^2\right|=$

A) 3 B) 4 C) 5 D) 6

Key. C

Sol. $z_1^3-3z_1z_2^2+3iz_1^2z_2-iz_2^3=2+11i \Rightarrow (z_1+iz_2)^3=2+11i$

Similarly, $(z_1-iz_2)^3=2-11i$

$\left|z_1^2+z_2^2\right|=\left|(z_1+iz_2)(z_1-iz_2)\right|=\left|(2+11i)^{1/3}(2-11i)^{1/3}\right|=5$

93. The circle $|z|=2$ intersects the curve whose equation is $z^2=\left(\bar{z}\right)^2+4i$ in the points $A, B, C, D.$ If z_1, z_2, z_3, z_4 represent the affixes of these points, then

A) $z_1z_2z_3z_4=1$

B) $\overline{z_1}+\overline{z_2}+\overline{z_3}+\overline{z_4}=0$

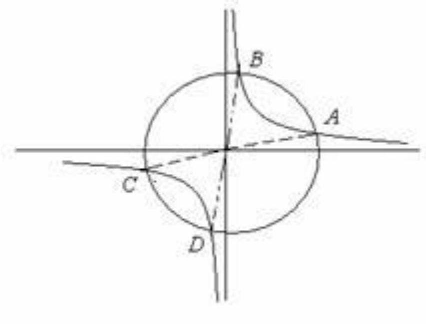
C) $z_1+z_2+z_3+z_4=2$

D) $\arg z_1 + \arg z_2 + \arg z_3 + \arg z_4 = 2k\pi,\ k = 0,1\ or\ -1$

Key. B

Sol. $z^2 = \left(\bar{z}\right)^2 + 4i \Rightarrow \left(\frac{z+\bar{z}}{2}\right)\left(\frac{z-\bar{z}}{2i}\right) = 1$ or $xy = 1$ (where $z = x + iy$)

The circle $x^2 + y^2 = 4$ intersects the rectangular hyperbola in four points, which are symmetrical about the origin in parts.



94. If $a_1, a_2, ..., a_n$ are real numbers with $a_n \neq 0$ and $\cos\alpha + i\sin\alpha$ is a root of $z^n + a_1 z^{n-1} + a_2 z^{n-2} + ... + a_{n-1} z + a_n = 0,$ then the sum $a_1\cos\alpha + a_2\cos 2\alpha + a_3\cos 3\alpha + ... + a_n \cos n\alpha$ is

A) 0 B) 1 C) -1 D) $\frac{1}{2}$

Key. C

Sol. $\cos\alpha + i\sin\alpha$ is a root of $a_n\left(\frac{1}{z}\right)^n + a_{n-1}\left(\frac{1}{z}\right)^{n-1} + ... + a_2\left(\frac{1}{z}\right)^2 + a_1\left(\frac{1}{z}\right) + 1 = 0.$

Equating real parts on both sides,

$a_n \cos n\alpha + a_{n-1}\cos(n-1)\alpha + ... + a_1\cos\alpha + 1 = 0$

95. .If $\left(\frac{3-z_1}{2-z_1}\right)\left(\frac{2-z_2}{3-z_2}\right) = k$, then points $A(z_1)$, $B(z_2)$, $C(3,0)$ and $D(2,0)$ (taken in clockwise sense) will

A) lie on a circle only for $k > 0$

B) lie on a circle only for $k < 0$

C) lie on a circle $\forall\ k \in R$

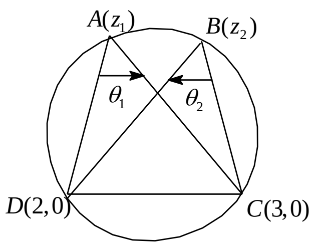
D) be vertices of a square $\forall\ k \in (0,1)$

Key: A

Sol : $\arg\left(\frac{3-z_1}{2-z_1}\right) + \arg\left(\frac{2-z_2}{3-z_2}\right)$

$= \arg\left(\frac{3-z_1}{2-z_1}\right)\left(\frac{2-z_2}{3-z_2}\right)$

If $k > 0$, its argument will be zero



So, θ_1 & θ_2 are equal in magnitude but opposite sign.

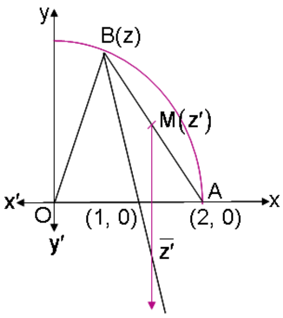
So DC subtends equal angle at A & B. So, points are concyclic if $k > 0$

96. If A(2,0) and B(z) are two points on the circle | z | = 2. $M(z')$ is the point on AB. If the point $\overline{z'}$ lies on the median of the triangle OAB where O is origin then $\arg(z')$ is

a) $\tan^{-1}\left(\frac{\sqrt{15}}{5}\right)$ b) $\tan^{-1}\left(\sqrt{15}\right)$ c) $\tan^{-1}\left(\frac{5}{\sqrt{15}}\right)$ d) $\frac{\pi}{2}$

Key: A

Sol: $M(z')$ is mid-point of AB, so $z' = \frac{z+2}{2}$



$$\Rightarrow \overline{z'} = \frac{\bar{z}+2}{2}$$

$\Rightarrow z, 1, \frac{\bar{z}}{2}+1$ are collinear

$$\Rightarrow \begin{vmatrix} z & \bar{z} & 1 \\ \frac{\bar{z}}{2}+1 & \frac{z}{2}+1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow z\left(\frac{z}{2}+1-1\right) - \bar{z}\left(\frac{\bar{z}}{2}+1-1\right) + 1\left(\frac{\bar{z}}{2}-\frac{z}{2}\right) = 0$$

$$\Rightarrow \quad \frac{z^2}{2} - \frac{\bar{z}^2}{2} + \frac{(\bar{z}-z)}{2} = 0$$

$$\Rightarrow \quad (z-\bar{z})(z+\bar{z}-1) = 0$$

$$\Rightarrow \quad z-\bar{z} = 0 \text{ or } (z+\bar{z}-1) = 0$$

$$\Rightarrow \quad z+\bar{z} = 1 \text{ or } \mathrm{Re}(z) = \frac{1}{2}$$

$$|z| = 2 \Rightarrow \frac{1}{4} + \mathrm{Im}(z)^2 = 4$$

$\Rightarrow \qquad \text{Im}(z) = \dfrac{\sqrt{15}}{2}$

1. $\quad z = \dfrac{1}{2} + \dfrac{i\sqrt{15}}{2}$

2. $\quad \arg(z') = \tan^{-1}\left(\dfrac{\sqrt{15}}{5}\right)$

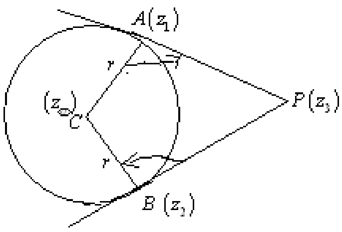
97. If the tangents at z_1, z_2 on the circle $|z - z_o| = r$ intersect at z_3, *then* $\dfrac{(z_3 - z_1)(z_o - z_2)}{(z_o - z_1)(z_3 - z_2)}$ equals

a) 1 b) -1 c) i d) – i

Key: B

Hint: $\dfrac{z_3 - z_1}{z_o - z_1} = \left(\dfrac{PA}{AC}\right) i \text{ and } \dfrac{z_o - z_2}{z_3 - z_2} = \left(\dfrac{BC}{BP}\right)(i)$

$$\frac{(z_3 - z_1)(z_o - z_2)}{(z_o - z_1)(z_3 - z_2)} = \left(\frac{PA}{AC} \times \frac{BC}{PB}\right)(-1) = -1$$



98. If Z is a complex number then the number of complex numbers satisfying the equation $Z^{2009} = \overline{Z}$ is

A) 3 B) 2009 C) 2010 D) 2011

Key: D

Sol. $Z^{2009} = \overline{Z} \Rightarrow |Z| = 0$ or $|Z| = 1$

99. If $a_1, a_2, ..., a_n$ are real numbers with $a_n \neq 0$ *and* $\cos\alpha + i\sin\alpha$ *is a root of* $z^n + a_1 z^{n-1} + a_2 z^{n-2} + ... + a_{n-1} z + a_n = 0,$ then the sum $a_1 \cos\alpha + a_2 \cos 2\alpha + a_3 \cos 3\alpha + ... + a_n \cos n\alpha$ is

a) O b) 1 c) – 1 d) 1/2

Key: C

Sol. $\cos\alpha + i\sin\alpha$ is a root of $a_n\left(\dfrac{1}{z}\right)^n + a_{n-1}\left(\dfrac{1}{z}\right)^{n-1} + ... + a_2\left(\dfrac{1}{z}\right)^2 + a_1\left(\dfrac{1}{z}\right) + 1 = 0.$ Equating real parts on both sides, $a_n \cos n\alpha + a_{n-1}\cos(n-1)\alpha + ... + a_1 \cos\alpha + 1 = 0$.

100. If is a cube root of unity, then $\omega+\omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}+......\infty\right)}=$

a) i b) i^2 c) 0 d) i^3

Key: B

Sol. $\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+..........=\frac{1}{2}\left(1+\frac{3}{4}+\frac{9}{16}+..........\infty\right)$

$=\frac{1}{2}\left[\frac{1}{1-\frac{3}{4}}\right]=\frac{1}{2}\times 4=2.$

101. If z_1, z_2, z_3 are any three complex numbers on Argand plane then $z_1\left(\text{Im}\left(\bar{z}_2 z_3\right)\right)+z_2\left(\text{Im}\left(\bar{z}_3 z_1\right)\right)+z_3\left(\text{Im}\left(\bar{z}_1 z_2\right)\right)$ is equal to

(A) 0 (B) $z_1+z_2+z_3$

(C) $z_1 z_2 z_3$ (D) $\left(\frac{z_1+z_2+z_3}{z_1 z_2 z_3}\right)$

Key: A

Hint: $z_1\left(\frac{\bar{z}_2 z_3-z_2\bar{z}_3}{2i}\right)+z_2\left(\frac{\bar{z}_3 z_1-z_3\bar{z}_1}{2i}\right)+z_3\left(\frac{\bar{z}_1 z_2-z_1\bar{z}_2}{2i}\right)=\frac{1}{2i}\times 0=0$

102. Let points P and Q correspond to the complex numbers α and β respectively in the complex plane. If $|\alpha|=4;$ and $4\alpha^2-2\alpha\beta+\beta^2=0,$ then the AREA OF THE $\Delta OPQ,$ O being the origin equals

A) $8\sqrt{3}$ B) $4\sqrt{3}$ C) $6\sqrt{3}$ D) $12\sqrt{3}$

Key: A

Hint: Conceptual

103. Suppose two complex numbers $z=a+ib; w=c+id$ satisfy the equation $\frac{z+w}{z}=\frac{w}{z+w}$ then

A) both a & c are zeros B) both b & d are zeros

C) both b & d must be non zeros D) at least one of b & d is non-zero

Key: D

Hint: $(z+w)^2=zw\Rightarrow z^2+zw+w^2=0$

Let $\frac{z}{w}=t\Rightarrow\frac{z}{w}=\frac{-1\pm\sqrt{3}i}{2}$

$\arg z-\arg w=\frac{2\pi}{3}$ or $\arg z-\arg w=-\frac{2\pi}{3}$

104. If x = a+ib is a complex number such that x^2 =3+4i and x^3=2+11i where i = $\sqrt{-1}$ then a+b =_____________________________

1. 1 2. 2 3. 3 4. 4

Key. 3

Sol. $x = \frac{x^3}{x^2} = \frac{2+11i}{3+4i} = \frac{(2+11i)(3-4i)}{25}$

$\therefore a+ib = \frac{6+44+25i}{25} = 2+i$

$\Rightarrow a = 2, b = 1 \Rightarrow a+b = 3$

105. If the complex number Z satisfying $Z + |Z| = 2+8i$ then value of $|Z|$ =

1. 8 2.17 3. 15 4. 24

Key. 2

Sol. Let $z = a+ib$

$\Rightarrow a+ib+\sqrt{a^2+b^2} = 2+8i$

$\Rightarrow b = 8,\ a+\sqrt{a^2+64} = 2$

$a^2+64 = a^2-4a+4$

$\Rightarrow -4a = 60 \Rightarrow a = -15$

$\therefore |z| = \sqrt{a^2+b^2} = \sqrt{225+64} = \sqrt{289} = 17$

106. If $|Z+2-i| = 5$ then maximum value of $|3Z+9-7i| =$ ______________

1. 20 2.15 3. 5 4. 16

Key. 1

Sol. $|3Z+9-7i| = |3Z+6-3i+3-4i|$

$\leq |3(z+2-i)+| = |3-4i|$

$\leq 3|z+2-i| + \sqrt{3^2+4^2}$

$3(5)+5 = 20$

107. If Z-lies on the circle $|z-2i| = 2\sqrt{2}$ *then value of* $\arg\left(\frac{Z-2}{Z+2}\right)$ *is*

1. $\frac{\pi}{3}$ 2. $\frac{\pi}{4}$ 3. $\frac{\pi}{6}$ 4. $\frac{\pi}{2}$

Key. 2

Sol. Circle with centre (0,2) cuts X-axis at A(-2,0) and B(2,0). Now AB subtends an angle 90^0 at the center C.

AB subtends an angle $\frac{\pi}{4}$ at any point z on the major arc circle arg $\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$

108. If $\lambda \in R$ and non real roots of $2Z^2+2Z+\lambda = 0$ and (0,0) forms vertices of an equilateral triangle then λ =

1. 1
2. $\frac{1}{2}$
3. $\frac{1}{3}$
4. $\frac{2}{3}$

Key. 4

Sol. Let $z_{1,}z_2$ be roots of $2z^2+2z+\lambda=0$

$z_1+z_2=-1 \quad z_1z_2=\frac{\lambda}{2}$

When origin , z_1z_2 forms equilateral Δ^{le}

We have $z_1^2+z_2^2=z_1z_2$

$(z_1+z_2)^2=3z_1z_2$

$1=\frac{3.\lambda}{2}\Rightarrow\lambda=\frac{2}{3}$

109. The greatest positive argument of z satisfying $|Z-4|=\text{Re}(Z)$

1. $\frac{\pi}{3}$
2. $\frac{2\pi}{3}$
3. $\frac{\pi}{2}$
4. $\frac{\pi}{4}$

Key. 4

Sol. $|x+iy-4|=x$

$(X-4)^2+y^2+x^2$

$y^2-8x+16=0$

z lies on the parabola with vertex (2,0) focus (4,0) and tangents from (0,0) ie a point on the directrix in x always include 90^0

$\therefore$ greatest arg(z) is $45^0=\frac{\pi}{4}$

110. If Z and W are two complex numbers such that $\bar{z}+i\bar{w}=0$ and arg (Zw)= π then arg (Z) = ________

1. $\frac{\pi}{4}$
2. $\frac{\pi}{2}$
3. $\frac{3\pi}{4}$
4. $\frac{5\pi}{4}$

Key. 3

Sol. $\bar{z}+i\bar{w}\Rightarrow z-iw=0\Rightarrow z=iw$

Arg $(zw)=\pi\Rightarrow\arg(z)+\arg(w)=\pi$

$\arg(iw)+\arg w=\pi$

$\arg i+2\arg w=\pi$

$$\frac{\pi}{2}+2\arg w=\pi$$

$$2\arg w=\frac{\pi}{2}$$

$$\arg w=\frac{\pi}{4}\Rightarrow \arg(z)=\frac{3\pi}{4}$$

111. If A (Z_1) B(Z_2) C(Z_3) are vertices of a triangle such that

$Z_3=\left(\frac{Z_2-iZ_1}{1-i}\right) and\ |Z_1|=3, |Z_2|=4\ and\ |Z_2+iZ_1|=|Z_1|+|Z_2|$ then area of triangle ABC is

1. $\frac{5}{2}$ 2. 0 3. $\frac{25}{2}$ 4. $\frac{25}{4}$

Key. 4

Sol. $|z_2+iz_1|=|z_1|+|z_2|\Rightarrow z_2, iz_1, 0$ are collinear.

$$\therefore \arg(iz_1=\arg z_2)$$

$$\Rightarrow \arg i+\arg z_1=\arg z_2$$

$$\Rightarrow \arg z_2-\arg z_1=\frac{\pi}{2}$$

$$z_3=\frac{z_2-iz_1}{l-i}$$

$$(l-i)z_3=z_2-iz_1$$

$$z_3-z_2=i(z_3-z_1)$$

$\frac{z_3-z_2}{z_3-z_1}=i\Rightarrow \arg\left(\frac{z_3-z_2}{z_3-z_1}\right)=\frac{\pi}{2}$ and $|z_3-z_2|=|z_3-z_1|$

$\therefore$ AB=BC, $\therefore\ AB^2=AC^2+BC^2$

$$25=2AC^2$$

$$\Rightarrow AC=\frac{5}{\sqrt{2}}$$

Required area= $\frac{1}{2}\times\frac{5}{\sqrt{2}}\times\frac{5}{\sqrt{2}}=\frac{25}{4}$ sq. units

112. The radius of the circle given by arg $\left(\frac{Z-5+4i}{Z+3-2i}\right)=\frac{\pi}{4}$

1. $5\sqrt{2}$ 2.5 3. $\frac{5}{\sqrt{2}}$ 4. $\sqrt{2}$

Key. 1

Sol. A(5,-4) B(-3,2) subtends an angle $\frac{\pi}{4}$ at C(z) on the circle hence $\frac{\pi}{2}$ at centre

$$M \to M.dAB \therefore AM = \frac{AB}{2}$$

$$= \frac{\sqrt{64+36}}{2}\frac{10}{2} = 5$$

Radius $= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$

113. f(x) = $2x^3 + 2x^2 - 7x + 72$ then $f\left(\frac{3-5i}{2}\right)$ = ____________

1. 1 2. 2 3.3 4.4

Key. 4

Sol. Let $x = \frac{3-5i}{2}$

$$2x = 3-5i$$

$$(2x-3)^2 = 5i$$

$$4x^2 - 12x + 9 = 25i^2$$

$$\Rightarrow 2x^2 - 12x + 34 = 0 \Rightarrow 2x^2 - 6x + 17 = 0$$

$$2x^2 - 6x + 17)2x^3 + 2x^2 = 7x + 72(x+4$$

$$2x^3 - 6x^2 + 17x$$

$$8x^2 - 24x + 72$$

$$8x^2 - 24x + 68$$

$$4$$

114. If $Cos\,\alpha + Cos\,\beta + Cos\gamma = 0 = Sin\,\alpha + Sin\,\beta + Sin\gamma$ then $Sin^2\,\alpha + Sin^2\,\beta + Sin^2\gamma$ = _______

1. $\frac{1}{2}$ 2. $\frac{3}{2}$ 3. 4 4. 1

Key. 2

Sol. Let $x = cis\alpha$ $y = cis\beta$ $z = cis\gamma$

Clearly $x + y + z = 0, \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

$$x^2 + y^2 + z^2 = (x+y+z)^2 - 2xyz\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 0$$

$$= cis2\alpha + cis2\beta + cis2\gamma = 0$$

$$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0$$

$1-2\sin^2\alpha+1-2\sin^2\beta+1-2\sin^2\gamma=0$

$\Rightarrow \sin^2\alpha+\sin^2\beta+\sin^2\gamma=\frac{3}{2}$

115. If $Z_1\, and\, Z_2$ are two complex numbers such that $Z_1{}^2+Z_2{}^2\in R\; and\; Z_1(Z_1{}^2-3Z_2{}^2)=2$ $Z_2(3Z_1{}^2-Z_2{}^2)=11$ then $Z_1{}^2+Z_2{}^2=$

A) 5 2.125 3. 25 4. 15

Key. 1

Sol. $z_1\left(z_1^2-3z_2^2\right)=2$

$z_1^2\left(z_1^4+9z_2^4-6z_1^2z_2^2\right)=4$

$\left(z_1^2\right)^3+9z_1^2z_2^4-6z_1^4z_2^2=4 \longrightarrow$ (1)

$z_2^2\left(3z_1^2-z_2^2\right)^2=|121|$

$\Rightarrow\left(z_2^2\right)^3+9z_2^2z_1^4-6z_1^2z_2^4=121 \longrightarrow$ (2)

(1) + (2) $\Rightarrow\left(z_1^2+z_2^2\right)^3 125$

$z_1^2+z_2^2=5$

116. Let $z=\cos\theta+i\sin\theta$. Then, the value of $\sum_{m=1}^{15} lm\left(z^{2m-1}\right)$ at $\theta=2^0$ is

(A) $\frac{1}{2^0}$ (B) $\frac{1}{3\sin 2^0}$ (C) $\frac{1}{2\sin 2^0}$ (D) $\frac{1}{4\sin 2^0}$

Key. D

Sol. Given that $z=\cos\theta+isin\theta=e^{i\theta}$

$\therefore \sum_{m=1}^{15}\left(z^{m-1}\right)=\sum_{m=1}^{15} lm\left(e^{i\theta}\right)^{2m-1}$

$=\sum_{m=1}^{15} lm\, e^{i(2m-1)\theta}$

$=\sin\theta+\sin 3\theta+\sin 5\theta+.......+\sin 29\theta$

$=\dfrac{\sin\left(\frac{\theta+29\theta}{2}\right)\sin\left(\frac{15\times 2\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)}$

$=\dfrac{\sin(15\theta)\sin(15\theta)}{\sin\theta}=\dfrac{1}{4\sin 2^0}$

117. If z_1 is a root of the equation $a_0z^n+a_1z^{n-1}+.....+a_{n-1}z+a_n=3$, where $|a_i|<2$ for $i=0,1,.....n$. Then

(A) $|z_1|>\frac{1}{3}$ (B) $|z_1|<\frac{1}{4}$ (C) $|z_1|>\frac{1}{4}$ (D) $|z|<\frac{1}{3}$

Key. A

Sol. $a_0z^n + a_1z^{n-1} + a_2z^{n-2} + ... + a_{n-1}z + a_n = 3$

$|3| = |a_0z^n + a_1z^{n-1} + a_2z^{n-2} + ... + a_{n-1}z + a_n|$

$3 \le |a_0||z|^n + |a_1||z|^{n-1} + ... + a_{n-1}|z| + |a_n|$

$3 < 2\left(|z|^n + |z|^{n-1} + ... + |z| + 1\right)$

$\frac{3}{2} < 1 + |z| + |z|^2 + ... + |z|^n$

$\frac{1-|z|^{n+1}}{1-|z|} > \frac{3}{2}$

$2 - 2|z|^{n+1} < 3|z| - 1$

$3|z| - 1 > 0$

$|z| > \frac{1}{3}$

118. If $n \ge 3$ and $1, \alpha_1, \alpha_2, \alpha_3......\alpha_{n-1}$ are n roots of unity, then value of $\sum\limits_{1\le i<j\le n-1} \alpha_i \alpha_j$ is

(a) 0 (b) 1 (c) -1 (d) $(-1)^n$

Key. B

Sol. $x^n - 1 = (x-1)(x-\alpha_1)(x-\alpha_2)............(x-\alpha_{n-1})$

$= x^n - x^{n-1}(1+\alpha_1 + ... + \alpha_{n-1}) + x^{n-2}\left(\sum\limits_{i+j} \alpha i\, \alpha_j + \alpha_1 + \alpha_2 ..... + \alpha_{n-1}\right)$ +..........-1 = 0

$\Rightarrow \sum\limits_{i+j} \alpha i\, \alpha_j + \alpha_1 + \alpha_2 ..... + \alpha_n = 0$

$\sum\limits_{i+j} \alpha i\, \alpha_j = 1$

119. If the equation $z^2 + z + \alpha = 0$ has a purely imaginary root and α lies on the circle $|z| = 1$ then the imaginary part of that root, is (are)

(A) $\pm\sqrt{2}$ (B) 0

(C) $\pm\sqrt{2-\sqrt{2}}$ (D) $\pm\sqrt{\frac{\sqrt{5}-1}{2}}$

Key. D

Sol. Let z = iβ (β∈R) be a root, then

$-\beta^2 + i\beta + \alpha = 0 \Rightarrow \alpha = \beta^2 - i\beta$

Now as $|\alpha| = 1$

$\Rightarrow \beta^4 + \beta^2 = 1 \Rightarrow \beta^2 = \frac{-1+\sqrt{5}}{2}$

120. Let $z(\alpha, \beta) = \cos\alpha + e^{i\beta}\sin\alpha$ ($\alpha, \beta \in R$, $i = \sqrt{-1}$) then the exaustive set of values of modulus of $z(\theta, 2\theta)$, as θ varies, is

(A) [0, 1] (B) $[0, \sqrt{2}\,]$

(C) [1, 2] (D) $[\sqrt{2}, 2]$

Key. B

Sol. $|z)\theta, 2\theta) = |\cos\theta + e^{i2\theta}\sin\theta|$

$= |\cos\theta + \sin\theta\cos 2\theta + i\sin\theta\sin 2\theta| = \sqrt{(\cos\theta+\sin\theta\cos 2\theta)^2+\sin^2\theta\cos^2\theta}$

$= \sqrt{1+\sin 4\theta} \in [0, \sqrt{2}]$

121. If |z| = 1 and z ≠ ±1 then one of the possible values of arg(z) – arg (z + 1) – arg (z – 1), is

(A) $-\pi/6$ (B) $\pi/3$

(C) $-\pi/2$ (D) $\pi/4$

Key. C

Sol. arg(z) – avg |z + 1) – arg(z–1) = arg $\left(\frac{z}{z^2-1}\right) = \arg\left(\frac{z}{z^2-z\bar{z}}\right)$

$= \arg\left(\frac{1}{z-\bar{z}}\right)$ = arg (purely imaginary no.)

122. If z_1, z_2, z_3 are three distinct complex numbers and a, b, c are three positive real numbers such that

$$\frac{a}{|z_2-z_3|}=\frac{b}{|z_3-z_1|}=\frac{c}{|z_1-z_2|} \text{ then } \frac{a^2}{z_2-z_3}+\frac{b^2}{z_3-z_1}+\frac{c^2}{z_1-z_2} \text{ is}$$

a) 3 abc b) $(abc)^3$ c) a + b + c d) 0

Key. D

Sol. $\frac{a}{|z_2-z_3|}=\lambda \Rightarrow \frac{a^2}{z_2-z_3}=\lambda^2\left(\bar{z}_2-\bar{z}_3\right)$ *etc*

123. If $|z_1|=2, |z_2|=3, |z_3|=4$ and $|2z_1+3z_2+4z_3|=4$ then the absolute value of $8z_2z_3 + 27z_3z_1 + 64z_1z_2$ equals

(A) 24 (B) 48 (C) 72 (D) 96

Key. D

SOL. $|8z_2z_3+27z_3z_1+64z_1z_2|$

$= |z_1z_2z_3|\left|\frac{8}{z_1}+\frac{27}{z_2}+\frac{64}{z_3}\right|$

$= 24\left|2\overline{z_1}+3\overline{z_2}+4\overline{z_3}\right|$

$= 24\times 4 = 96$

124. If ω is a cube root of unity, then $\omega+\omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}+....+\infty\right)}$ is

(A) i (B) i^2

(C) 0 (D) none of these

Key. B

Sol. $\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}+....+\infty = \frac{1}{2}\left(1+\frac{3}{4}+\frac{9}{16}+\frac{27}{64}+....+\infty\right)$

$$= \frac{1}{2}\left(\left(\frac{3}{4}\right)^0 + \left(\frac{3}{4}\right)^1 + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + ...\infty\right) = \frac{1}{2}\left(\frac{1}{1-\frac{3}{4}}\right) = \frac{1}{2}.\frac{1}{\frac{1}{4}} = 2$$

So expression = $\omega + \omega^2 = -1 = i^2$.

125. Let P be a point on the circumcircle of the triangle whose vertices A, B, C (P, A, B, C are in order) are represented by the complex numbers ω^2, $2i\omega$ and -4 (ω is a non real cube root of unity) respectively such that PA.BC = PC. AB then the complex number associated with the mid–point of PB is

(A) $\omega - 1$ (B) 0

(C) $-i$ (D) $\omega - \omega^2$

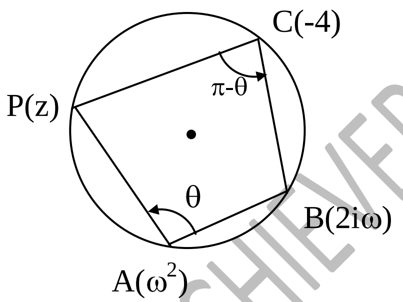
Key. B

Sol. Applying rotation formula at A and C,

$$\frac{z-\omega^2}{zi\omega-\omega^2} = \frac{PA}{AB}e^{i\theta}, \ \frac{2i\omega+4}{z+4} = \frac{BC}{PC}e^{i(\pi-\theta)}$$

Multiplying we get, , $\dfrac{z-\omega^2}{2i\omega-\omega^2} \times \dfrac{2i\omega+4}{z+4} = -1$

$\Rightarrow z = -2i\omega$



126. The complex numbers satisfying (3Z+1) (4Z+1) (6Z+1)(12Z+1) = 2 is

a) $\dfrac{\sqrt{33}-5}{4}$ b) $\dfrac{\sqrt{33}+5}{24}$ c) $\dfrac{-i\sqrt{23}-5}{24}$ d) $\dfrac{-i\sqrt{23}+5}{24}$

Key. C

Sol. Given equation can be written as $(144z^2 + 60z+4)(144z^2+60z+6) = 48 \Rightarrow t(t+2) = 48$

Where $t = 144z^2 + 60z+4$

$\therefore$ t = 6 or -8 hence $z = \dfrac{-5\pm\sqrt{33}}{24}, \dfrac{-5\pm i\sqrt{23}}{24}$

127. If |z| = 1 and $z' = \dfrac{1+z^2}{z}$, then

(A) z′ lie on a line not passing through origin

(B) $|z'| = \sqrt{2}$

(C) Re (z′) = 0

(D) Im (z′) = 0

Key. D

Sol. z′ = $\frac{1+z^2}{z}.\frac{\bar{z}}{\bar{z}}$ = $\frac{\bar{z}+z}{z\bar{z}} = z+\bar{z}$ which is purely real.

$\Rightarrow$ Im (z′) = 0.

128. For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of

$|z_1 - z_2|$ is

a) 0 b) 7 c) 2 d) 17

Key. C

Sol. Conceptual

129. If $y_1 = \max\left||z-w| - |z-w^2|\right|$, where $|z| = 2$ and

$y_2 = \max\left||z-w| - |z-w^2|\right|$, where $|z| = \frac{1}{2}$ and $w \text{ and } w^2$

are complex cube roots of unity, then

a) $y_1 = \sqrt{3}; y_2 = \sqrt{3}$ b) $y_1 < \sqrt{3}; y_2 = \sqrt{3}$

c) $y_1 = \sqrt{3}; y_2 < \sqrt{3}$ d) $y_1 > \sqrt{3}; y_2 < \sqrt{3}$

Key. C

Sol. We have $\left||z_1| - |z_2|\right| \le |z_1 - z_2|$ and equality holds only when $\arg z_1 = \arg z_2$.

$\Rightarrow \left||z-w| - |z-w^2|\right| \le |w^2 - w| \le \sqrt{3}$ and equality can hold only when $|z| = 2$ and not when $|z| = \frac{1}{2}$

130. Let $f(x)$ be the remainder obtained on dividing $x^{2007} - 1$ by $(x^2+1)(x^2+x+1)$, then $f(x)$ is a polynomial of degree

a) 0 b) 1 c) 2 d) 3

Key. D

Sol. Let $x^{2007} - 1 = (x^2+1)(x^2+x+1)p(x) + f(x)$

Put $x = \pm i, w, w^2$ for get $f(x)$

131. If $\alpha \ne 1$ is any of 7th roots of unity then real part of $\alpha^{2009} + 3\alpha^{2010} + 5\alpha^{2011} + ----+$ up to 7 terms is

a) 7 b) 14 c) -7 d) -14

Key. C

Sol. Let $\alpha = cis\frac{2K\pi}{7}(K = 0\,to\,6)$ $(\alpha^7 = 1)$ S $= \alpha^{2009} + 3\alpha^{2010} + 5\alpha^{2011} + ----- + 13\alpha^{2015}$

$$= 1+3\alpha+5\alpha^2+--------+13\alpha^6 (AGP)$$

$$= \frac{-14}{1-\alpha} = \frac{-14}{1-cis\frac{2K\pi}{7}}$$

$$= -7\left[1+i\cot\frac{K\pi}{7}\right]$$

132. All the complex numbers *z* that satisfy the equation $z^{10} = (1-z)^{10}$ lie on

a) $x = \frac{1}{2}$ b) $x = -\frac{1}{2}$ c) $y = \frac{1}{2}$ d) $y = -\frac{1}{2}$

Key. A

$$\frac{z^{10}}{(1-z)^{10}} = 1 \Rightarrow \frac{z}{1-z} = 1^{1/10} = cis\frac{2K\pi}{10}(K = 0\ to\ 9)$$

Sol.

$$\Rightarrow z = \frac{cis\frac{2K\pi}{10}}{1+cis\frac{2K\pi}{10}} = \frac{1}{2}+\frac{i}{2}\tan\frac{K\pi}{10}$$

133. If z is a complex number such that $|z-1|$ =1 then *arg* $\left(\frac{1}{z}-\frac{1}{2}\right)$ may be

a) $\frac{\pi}{6}$ b) $-\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $-\frac{\pi}{4}$

Key. B

Sol. Since $|z-1| = 1 \Rightarrow z-1 = cis\theta \Rightarrow z = (1+\cos\theta)+i\sin\theta = 2\cos\frac{\theta}{2}cis\frac{\theta}{2}$

$\therefore \frac{1}{z}-\frac{1}{2} = \frac{cis\left(-\frac{\theta}{2}\right)}{2\cos\frac{\theta}{2}}-\frac{1}{2} = -\frac{i}{2}\tan\frac{\theta}{2}$ which is purely imaginary

134. $\theta \in [0, 2\pi]$ and z_1, z_2, z_3 are three complex numbers such that they are collinear and $(1+|\sin\theta|)z_1+(|\cos\theta|-1)z_2-\sqrt{2}\ z_3$ =0 .If at least one of the complex numbers z_1, z_2, z_3 is non- zero then number of possible values of θ is

a) Infinite b) 4 c) 2 d) 8

Key. B

Sol. If z_1, z_2, z_3 are collinear and $az_1+bz_2+cz_3 = 0$ then a+b+c =0 .Hence

$$1+|\sin\theta|+|\cos\theta|-1-\sqrt{2} = 0 \Rightarrow |\sin\theta|+|\cos\theta| = \sqrt{2}$$

$$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

135. If $\alpha_0, \alpha_1, \alpha_2, ...\alpha_{n-1}$ be the n, n^{th} roots of unity, then value of $\sum_{i=0}^{n-1}\frac{\alpha_i}{(3-\alpha_i)}$ is equal to

A) $\frac{n}{3^n-1}$ B) $\frac{n-1}{3^n-1}$ C) $\frac{n+1}{3^n-1}$ D) $\frac{n+2}{3^n-1}$

Key. A

Sol. Let $P=\sum_{i=0}^{n-1}\frac{\alpha_i}{3-\alpha_i}=-\sum_{i=0}^{n-1}\frac{(3-\alpha_i)-3}{(3-\alpha_i)}=3\sum_{i=0}^{n-1}\frac{1}{3-\alpha_i}-\sum_{i=0}^{n-1}1$ --- (i)

$Z^n-1=\prod_{i=0}^{n-1}(Z-\alpha_i)\ \log\left(Z^n-1\right)=\sum_{i=0}^{n-1}\ln(Z-\alpha_i)$

Diff. both sides w.r.t Z

$\frac{nZ^{n-1}}{Z^n-1}=\sum_{i=0}^{n-1}\frac{1}{z-\alpha_i}$ Put $Z=3$

$\Rightarrow\frac{n3^{n-1}}{3^n-1}=\sum_{i=0}^{n-1}\frac{1}{3-\alpha_i}$

$P=\frac{3n3^{n-1}}{3^n-1}-n=\frac{n3^n}{3^n-1}-n=\frac{n}{3^n-1}$

136. Let $|Z_1-1|=1,|Z_2+4|=2$ then maximum value of $|Z_1-Z_2|$ is

A) 8 B) 5 C) 4 D) 2

Key. A

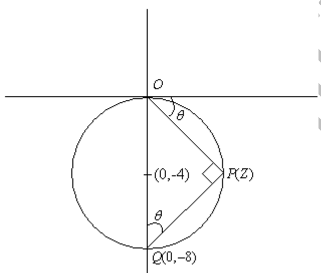
Sol. Max. distance between two curves lies along their common normal.

137. The complex number Z has argument θ, $-\frac{\pi}{2}<\theta<0$ and $|Z+4i|=4$, then $\cot\theta+\frac{8}{Z}=$

A) $1-i$ B) $1+i$ C) i D) $-i$

Key. C

Sol.



Applying rotation at P $\frac{Z+8i}{Z}=-i\cot\theta$

138. Let $|Z-(1+i)|<2$ then $|iZ+1+2i|$

A) < 7 B) < 9 C) < 5 D) < 10

Key. C

Sol. $|iZ+1+2i|=|i(z-(1+i)+3i|\le|Z-(1+i)|+3<2+3$

139. If $x^6=2x^3-1$ and *x* is not real then $\sum_{r=1}^{50}(x^r+x^{2r})^3=$

A) 0 B) 256 C) 76 D) 94

Key. D

Sol. $x = \omega, \omega^2$

$$\omega^r + \omega^{2r} = \begin{cases} 2 \text{ ,if } r \text{ is a multiple of 3} \\ -1 \text{ ,if } r \text{ is not of a multiple of 3} \end{cases}$$

140. If $A(z_1), B(z_2), C(z_3)$ be the vertices of triangle ABC in which $\underline{|ABC} = \frac{\pi}{4}$ and $\frac{AB}{BC} = \sqrt{2}$ then z_2 is equal to

(a) $z_3 + i(z_1 + z_3)$ (b) $z_3 - i(z_1 - z_3)$ (c) $z_3 + i(z_1 - z_3)$ (d) $z_3 - i(z_2 - z_1)$

Key. B

Sol. $\frac{z_1 - z_2}{z_3 - z_2} = \sqrt{2} e^{\frac{i\pi}{4}}$

141. If $|z_1| = 2$, $|z_2| = 3$, $|z_3| = 4$ and $|z_1 + z_2 + z_3| = 5$ then $|4z_2z_3 + 9z_3z_1 + 16z_1z_2|$ =

a) 20 b) 24 c) 48 d) 120

Key. D

Sol. $|4z_2z_3 + 9z_3z_1 + 16z_1z_2|$

= $|z_1\overline{z_1}z_2z_3 + \overline{z_2}z_2z_3z_1 + \overline{z_3}z_3z_1z_2|$

= $\|z_1\|z_2\|z_3\|\|z_1 + z_2 + z_3| = 120$

142. If $\log_{\tan 30^\circ}\left(\frac{2|z|^2 + 2|z| - 3}{|z| + 1}\right) < -2$ then

a) $|z| < \frac{3}{2}$ b) $|z| > \frac{3}{2}$ c) $|z| > 2$ d) $|z| < 2$

Key. C

Sol. $\log_{\tan 30^\circ}\left(\frac{2|z^2| + 2|z| - 3}{|z+1|}\right) < -2$

$\Rightarrow \frac{2|z^2| + 2|z| - 3}{|z| + 1} > 3$

$\Rightarrow \left((|z| - 2)(2|z|) + 3\right) > 0$

$\Rightarrow$ |z| > 2

143. z_1 and z_2 be two complex numbers with α and β as their principal arguments, such that $\alpha + \beta > \pi$, then principal Arg(z_1z_2) is

a) $\alpha + \beta + \pi$ b) $\alpha + \beta - \pi$ c) $\alpha + \beta - 2\pi$ d) $\alpha + \beta$

Key. C

Sol. Take z_1 = i, z_2 = ω Arg z_1 = $\frac{\pi}{2}$, Arg z_2 = $\frac{2\pi}{3}$

$Argz_1 + Argz_2 = \frac{7\pi}{6}$ should be equivalent to $\frac{7\pi}{6} - 2\pi$

144. If the square root of $\frac{x^2}{y^2} + \frac{y^2}{x^2} + \frac{1}{2i}\left(\frac{x}{y} + \frac{y}{x}\right) + \frac{31}{16}$ is $\pm\left(\frac{x}{y} + \frac{y}{x} - \frac{i}{m}\right)$ then m is

a) 2 b) 3 c) 4 d) 5

Key. C

Sol. $\left(\frac{x}{y} + \frac{y}{x} - \frac{i}{m}\right)^2 = \frac{x^2}{y^2} + \frac{y^2}{x^2} + \frac{1}{2i}\left(\frac{x}{y} + \frac{y}{x}\right) + \frac{31}{16}$

$L.H.S =$

$= \left(\frac{x}{y} + \frac{y}{x}\right)^2 - \frac{2i}{m}\left(\frac{x}{y} + \frac{y}{x}\right) - \frac{1}{m^2}$

$= \frac{x^2}{y^2} + \frac{y^2}{x^2} + 2 + \frac{4}{m}.\frac{1}{2i}\left(\frac{x}{y} + \frac{y}{x}\right) - \frac{1}{m^2}$

m = 4

145. If $\left|\frac{z_1 - 2z_2}{2 - z_1\overline{z_2}}\right|$ = 1 and $|z_2| \neq 1$, then value of $|z_1|$ =

a) 2 b) 1 c) 4 d) 5

Key. A

Sol. Conceptual

146. If $A_1(z_1)$, $A_2(\overline{z_1})$ are the adjacent vertices of a regular polygon. If $\frac{\text{Im}(\overline{z_1})}{\text{Re}(z_1)} = 1 - \sqrt{2}$ then number of sides of the polygon is equal to

a) 6 b) 8 c) 16 d)12

Key. B

Sol. Clearly origin is the centre of the polygon

Let $z_1 = re^{i\theta}$

$\overline{z_1} = re^{-i\theta}$

Re (z) = rcosθ

Im $(\overline{z_1})$ = – rsinθ

$\Rightarrow -\frac{\sin\theta}{\cos\theta}$ = 1 – $\sqrt{2}$ $\Rightarrow$ tan (θ) = $\sqrt{2} - 1$

$\Rightarrow \theta = \frac{\pi}{8}$ if 'n' be the no. of sides then θ = $\frac{\pi}{n}$

$\Rightarrow$ n = 8

147. If exactly one root of $z^2 + az + b = 0$ where a, b $\in C$ is purely imaginary, then

a) $\left(\bar{b} - b\right)^2 = -\left(a\bar{b} + \bar{a}b\right)\left(a + \bar{a}\right)$

b) $\left(\bar{b} - b\right)^2 = -\left(a\bar{b} + \bar{a}b\right)\left(a - \bar{a}\right)$

c) $\left(\bar{b} - b\right)^2 = -\left(a\bar{b} - \bar{a}b\right)\left(a + \bar{a}\right)$

d) $\left(\bar{b} - b\right)^2 = -\left(a\bar{b} - \bar{a}b\right)\left(a - \bar{a}\right)$

Key. A

Sol. $z^2 + az + b = 0$

Let $z_0$is the purely imaginary root of the equation

Then $z_0{}^2 + az_0 + b = 0$

$\Rightarrow z_0 + \overline{z_0}$ = 0

$\Rightarrow \overline{z_0} = -z_0$

We have $\overline{\overline{z_0}^2 + az_0 + b} = 0 \Rightarrow \overline{z_0}^2 + \overline{az_0} + \bar{b} = 0$

Now $\overline{z_0}^2 + az_0 + b$ and $z_0{}^2 - \bar{a}z_0 + \bar{b} = 0$

We should have a common root. Find common root.

148. z_1 and z_2 are the roots of $z^2 - az + b = 0$, where $|z_1| = |z_2| = 1$ and a, b are non-zero complex numbers, then

a) Arg(a) = 2 Arg(b)

b) 2 Arg(a) = Arg(b)

c) Arg(a) = Arg(b)

d) none of these

Key. B

Sol. $z_1 + z_2 = a \qquad z_1 z_2 = b$

Since $|z_1| = |z_2| = a$

$\Rightarrow$ Arg(a) = $\frac{1}{2}\left[Arg\left(z_1\right) + Arg\left(z_2\right)\right]$

Also Arg (b) = Arg $\left(z_1 z_2\right)$

$\therefore$ Arg(a) = $\frac{1}{2}\left(Arg\left(b\right)\right) \Rightarrow 2Arg\left(a\right) = Arg\left(b\right)$

149. If $|z - 2 + 2i|$ = 1, then the least value of $|z|$ is

a) $\sqrt{8} + 1$

b) $\sqrt{6} + 1$

c) $\sqrt{6} - 1$

d) $\sqrt{8} - 1$

Key. D

Sol. $|z - 2 + 2i| = 1 \Rightarrow$ z – 2 + 2i = cosθ + isinθ

z = (2 + cosθ) + i(sinθ – 2)

$|z| = \sqrt{4 + 4\cos\theta + \cos^2\theta + 4 - 4\sin\theta + \sin^2\theta}$

= $\sqrt{9 + 4\left(\cos\theta - \sin\theta\right)}$

= $\sqrt{9+4\sqrt{2}\cos\left(\theta+\frac{\pi}{4}\right)}$

|z| is least if cos $\left(\theta+\frac{\pi}{4}\right)$ = –1 = $\sqrt{9-4\sqrt{2}}$

= $\sqrt{9-2\sqrt{8}}$ = $\sqrt{8}-1$

150. If the imaginary part of $\frac{2z+1}{iz+1}$ is -4, then the locus of the point representing z in the complex plane is

1) straight line 2) a parabola 3) a circle 4) an ellipse

Key. 3

Sol. Let $z=x+iy$

$\frac{2z+1}{iz+1}=\frac{2(x+iy)+1}{i(x+iy)+1}$

$=\frac{(2x+1)+2iy}{(1-y)+ix}$

$=\frac{[(2x+1)+2iy][(1-y)-ix]}{(1-y)^2+x^2}$

since $\text{Im}\left(\frac{2z+1}{iz+1}\right)=-4$, we get $\frac{2y(1-y)-x(2x+1)}{x^2+(1-y)^2}=-4$

$\Rightarrow 2x^2+2y^2+x-6y+4=0$

Which represents a circle.

151. If $Z_K=Cos\frac{K\pi}{10}+iSin\frac{K\pi}{10}$ then $z_1z_2z_3z_4$ is equal to

1) -1 2) 1 3) -2 4) 2

Key. 1

Sol. Let $z_K=w^K$ where $w=\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}$

$\therefore z_1z_2z_3z_4=w.w^2.w^3.w^4$

$=w^{10}$

$=\cos\frac{10\pi}{10}+i\sin\frac{10\pi}{10}$ ($\because$ Demoviere's theorem)

$=\cos\pi+i\sin\pi$

$=-1$

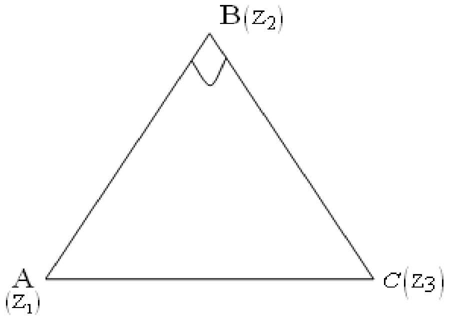
152. If z_1,z_2,z_3 are the vertices of an isosceles triangle, right angled at the vertex z_2, then value of $(z_1-z_2)^2+(z_2-z_3)^2$ is

1) -1 2) 0 3) $(z_1 - z_3)^2$ 4) None of these

Key. 2

Sol. Since $A(Z_1), B(Z_2), C(Z_3)$ is an Isosceless right angled triangle with right angle at B

$BA = BC$ and $\underline{|ABC} = 90^0$



$\Rightarrow |Z_1 - Z_2| = |Z_3 - Z_2|$ and $\arg\left(\dfrac{z_3 - z_2}{z_1 - z_2}\right) = \pi/2$

$$\therefore \frac{z_3 - z_2}{z_1 - z_2} = \frac{|z_3 - z_2|}{|z_1 - z_2|}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = i$$

$$(z_3 - z_2)^2 = -(z_1 - z_2)^2$$

$$\Rightarrow (z_1 - z_2)^2 + (z_2 - z_3)^2 = 0$$

153. If a, b, c, p, q, r are three non-zero complex numbers such that

$\dfrac{p}{a} + \dfrac{q}{b} + \dfrac{r}{c} = 1 + i$ and $\dfrac{a}{p} + \dfrac{b}{q} + \dfrac{c}{r} = 0$ then value of $\dfrac{p^2}{a^2} + \dfrac{q^2}{b^2} + \dfrac{r^2}{c^2}$ is

1) 0 2) -1 3) 2i 4) -2i

Key. 3

Sol. We have $(1+i)^2 = \left(\dfrac{p}{a} + \dfrac{q}{b} + \dfrac{r}{c}\right)^2$

$$1 - 1 + 2i = \frac{p^2}{a^2} + \frac{q^2}{b^2} + \frac{r^2}{c^2} + 2\left(\frac{qr}{bc} + \frac{rp}{ca} + \frac{pq}{ab}\right)$$

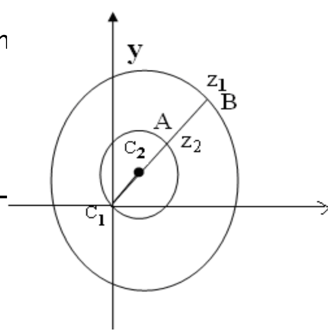
$$2i = \frac{p^2}{a^2} + \frac{q^2}{b^2} + \frac{r^2}{c^2} + \frac{2abc}{pqr}\left(\frac{a}{p} + \frac{q}{b} + \frac{r}{c}\right)$$

$$= \frac{p^2}{a^2} + \frac{q^2}{b^2} + \frac{r^2}{c^2} + \frac{2abc}{pqr}(0)$$

$$= \frac{p^2}{a^2} + \frac{q^2}{b^2} + \frac{r^2}{c^2}$$

154. For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minim

of $|z_1 - z_2|$ is



1) 0 2) 2 3) 7 4) 17

Key. 2

Sol. $|z_1| = 12 \Rightarrow z_1$ lies on circle with centre c_1 at origin and radius 12

$|z_2 - 3 - 4i| = 5 \Rightarrow z_2$ lies on the circle with centre $c_2\,(3+4i)$ and radius 5.

$\therefore |z_1 - z_2|$ will be minimum.

If z_1 and z_2 lies on the line joining c_1 and c_2 i.e on the line $z = 3+4i$

Minimum value of $|z_1 - z_2| = AB$

$= c_1B - c_1A$

$12 - 10 = 2$

155. If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$ then $|z_1 + z_2 + z_3|$ is

1) equal to 1 2) less than 1 3) greater than 3 4) equal to 3

Key. 1

Sol. $\because |z_1| = |z_2| = |z_3| = 1$ *we get* $z_1\bar{z}_1 = z_2\bar{z}_2 = z_3\bar{z}_3 = 1$

$\therefore 1 = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| \qquad \Rightarrow \bar{z}_1 = \frac{1}{z_1}, \bar{z}_2 = \frac{1}{z_2}, \bar{z}_3 = \frac{1}{z_3}$

$= |\bar{z}_1 + \bar{z}_2 + \bar{z}_3|$

$= |z_1 + z_2 + z_3|$

156. The complex numbers z_1, z_2 and z_3 satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$ are the vertices of a triangle which is

1) of area $\sqrt{3}$ 2) right angled and isosceles

3) equilateral 4) obtuse-angled and isosceles

Key. 3

Sol. $\left|\frac{z_1 - z_3}{z_2 - z_3}\right| = \left|\frac{1 - i\sqrt{3}}{2}\right| \qquad \Rightarrow \frac{|z_1 - z_3|}{|z_2 - z_3|} = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

$\Rightarrow |z_1 - z_3| = |z_2 - z_3|$

Again $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$

$\frac{z_1 - z_3}{z_2 - z_3} - 1 = \frac{1 - i\sqrt{3}}{2} - 1$

$$\frac{z_1 - z_2}{z_2 - z_3} = \frac{-1 - i\sqrt{3}}{2}$$

$$\left|\frac{z_1 - z_2}{z_2 - z_3}\right| = \left|\frac{-1}{2} \frac{-i\sqrt{3}}{2}\right|$$

$$\frac{|z_1 - z_2|}{|z_2 - z_3|} = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$$

$$\Rightarrow |z_1 - z_2| = |z_2 - z_3|$$

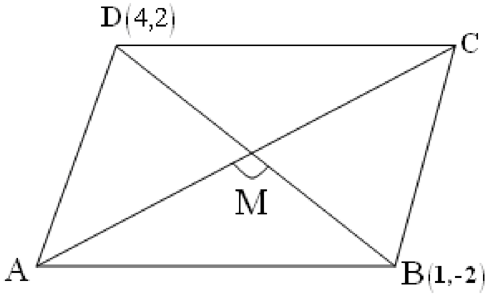
$\therefore$ z_1, z_2 and z_3 are the vertices of an equilateral triangle.

157. The vertices B and D of a parallelogram are $1-2i$ and $4+2i$ respectively. If the diagonals are at right angles and $|AC| = 2|BD|$, then the complex number representing A is

1) $\frac{3}{2}i + \frac{1}{2}$ 2) $3i - 4$ 3) $3i - \frac{3}{2}$ 4) $\frac{5}{2}$

Key. 3

Sol. Let affix of A be Z.
M = Mid point of BD



$$= \left(\frac{5}{2}, 0\right)$$

$$\underline{|AMB} = 90^0$$

$$\text{Arg}\left(\frac{1 - 2i - \frac{5}{2}}{z - \frac{5}{2}}\right) = \frac{\pi}{2}$$

$$\Rightarrow \frac{1 - 2i - \frac{5}{2}}{z - \frac{5}{2}} = \left|\frac{1 - 2i - \frac{5}{2}}{z - \frac{5}{2}}\right| cis\frac{\pi}{2}$$

$$= \frac{|\text{BM}|}{|\text{AM}|} i$$

$$= \frac{\left|\frac{BD}{2}\right|}{\left|\frac{Ac}{2}\right|} i$$

$$= \frac{|BD|}{|AC|} i$$

$$= \frac{1}{2} i \quad (\because |AC| = 2|BD|)$$

$$\therefore \left(\frac{-3}{2} \quad -2i\right)\frac{2}{i} = z - \frac{5}{2}$$

$\Rightarrow z = \frac{5}{2} - \frac{3}{i} - c_1$

$= \frac{-3}{2} + 3i$

Hence option(3)

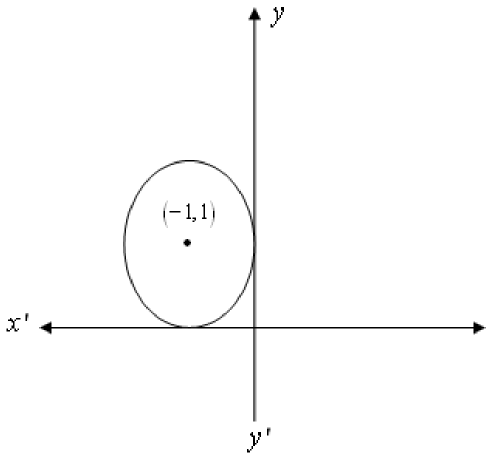
158. For all z satisfying $|z+1-i|=1$ we have

1) $\frac{\pi}{2} \le Argz \le \pi$ 2) $-\pi \le Argz \le \frac{-\pi}{2}$ 3) $-\pi < Arg\ z \le \frac{\pi}{2}$ 4) None of these

Key. 1

Sol. $|z+1-i|=1 \Rightarrow |z-(-1)+i|=1$

$\therefore$ Locus of z is a circle whose centre is $(-1,1)$ and radius=1



The circle touches both the axes in the second quadrant

All points on this circle lie in the region $+ve$ Y-axis corresponding to Arg $z = \frac{\pi}{2}$

and -ve X-axis corresponds to Argz $=\pi$

$\therefore \frac{\pi}{2} \le Arg\ Z \le \pi$

Hence option (1)

159. The equation $|z-4i|+|z+4i|=10$ represents

1) a circle 2) an ellipse 3) a line segment 4) None of these

Key. 2

Sol. $p(z)$, $A(4i), B(-4i)$ then

$|AB| = |-4i-4i|$

$= |-8i|$

$= 8$

Now $|z-4i|+|z+4i|=10$

$\Rightarrow |PA| + |PB| = 10$

$> |AB|$

$\therefore$ Locus of P is an ellipse

Hence option is (2).

160. If $z^5 = (z-1)^5$ then the roots are represented in the argand plane by the points that are

1) Collinear
2) Concylic
3) Vertices of a parallelogram
4) None of these

Key. 1

Sol. Let Z be a complex number satisfying

$Z^5 = (Z-1)^5$

$\Rightarrow |Z^5| = |(Z-1)^5|$

$\Rightarrow |Z|^5 = |Z-1|^5$

$\Rightarrow |Z| = |Z-1|$

Thus Z lies on the perpendicular bisector of the segment joining the Origin and $A(1+i\,0)$ i.e Z lies on $\text{Re } Z = \frac{1}{2}$

Hence option (1)

161. Let | z – 5 + 12 *i* | ≤ 1 and the least and greatest values of | z | are *m* and *n* and if *l* be the least positive value of $\frac{x^2+24x+1}{x}$ (x > 0), then *l* is

(A) $\frac{m+n}{2}$ (B) m + n (C) m (D) n

Key. 2

Sol. r^{th} term of given expression

$= r(r+1-w)(r+1-w^2)$

$= (r+1-1)(r+1-w)(r+1-w^2)$

$= (r+1)^3 - 1 \left[\begin{array}{l} \therefore Let\ r+1 = x \\ (x-1)(x-w)(x-w^2) = x^3 - 1 \end{array} \right]$

$\therefore$ Given expression value $= \sum_{r=1}^{n-1} r(r+1-w)(r+1-w^2)$

$= \sum_{r=1}^{n} (r+1)^3 - 1$

$= 2^3 + 3^3 + ...n^3 - (n-1)$

$= (1^3 + 2^3 + 3^3 + ... + n^3) - n$

$$=\frac{n^2(n+1)^2}{4}-n$$

Hence option (1).

162. If $x=2+5i$ then the value of $x^3-5x^2+33x-19$ is equal to

1) -5 2) -7 3) 7 4) 10

Key. 4

Sol. $x=2+5i \quad \Rightarrow x-2=5i$

$\Rightarrow (x-2)^2=(5i)^2$

$x^2-4x+4=-25$

$\Rightarrow x^2-4x+29=0 \to (i)$

Dividing $x^3-5x^2+33x-19 \; by \; x^2-4x+29$

$x^2-4x+29)x^3-5x^2+33x-19(x-1$

$\underline{x^3-4x^2+29x}$

$-x^2+4x-19$

$\underline{-x^2+4x-29}$

10

$\therefore x^3-5x^2+33x-19=(x-1)(x^2-4x+29)+10$

$=(x-1)0+10(\because from(1))$

$=10$

Hence option (4).

163. The complex number z_1, z_2, z_3 are the vertices of an equilateral triangle. If z_0 is the circumcentre of the triangle then $z_1{}^2+z_2{}^2+z_3{}^2=$

1) $z_0{}^2$ 2) $3z_0{}^2$ 3) $z_0{}^3$ 4) $3z_0{}^3$

Key. 2

Sol. Since the triangle with Z_1, Z_2, Z_3 as vertices is an equilateral triangle, its circumcentre and centroid will coincide

$$\therefore Z_0=\frac{Z_1+Z_2+Z_3}{3}$$

$$(3\,Z_0)^2=Z_1{}^2+Z_2{}^2+Z_3{}^2+2(Z_2Z_3+Z_3Z_1+Z_1Z_2)\to(1)$$

Since the triangle is equilateral we have

$$Z_1{}^2+Z_2{}^2+Z_3{}^2=Z_1Z_2+Z_2Z_3+Z_3Z_1\to(2)$$

From (1) and (2) we get

$$9\,Z_0{}^2=Z_1{}^2+Z_2{}^2+Z_3{}^2+2(Z_1{}^2+Z_2{}^2+Z_3{}^2)$$

$$3\,Z_0{}^2=Z_1{}^2+Z_2{}^2+Z_3{}^2$$

Hence option (2).

164. The complex numbers $\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate to each other for

1) $x = n\pi$ 2) $x = 0$

3) $x = \left(n + \frac{1}{2}\right)\pi$ 4) no value of x

Key. 4

Sol. Let $Z_1 = \sin x + i\cos 2x,\ Z_2 = \cos x - i\sin 2x$

$$\overline{Z_1} = Z_2$$

$$\sin x - i\cos 2x = \cos x - i\sin 2x$$

$$\Rightarrow \sin x = \cos x \ \ and \ \ \cos 2x = \sin 2x$$

$$\Rightarrow \tan x = 1 \ \ and \ \ \tan 2x = 1$$

$\Rightarrow x = \frac{\pi}{4}$ *and* $x = \frac{\pi}{8}$ which is not possible .Hence there is no value of x

Hence option (4).

165. Suppose z_1, z_2, z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$ then z_2 may be

1) $1 + i\sqrt{3}, 2$ 2) $3 - i\sqrt{2}, 4$ 3) $1 - i\sqrt{3}, -2$ 4) $2 - i\sqrt{3}, 1$

Key. 3

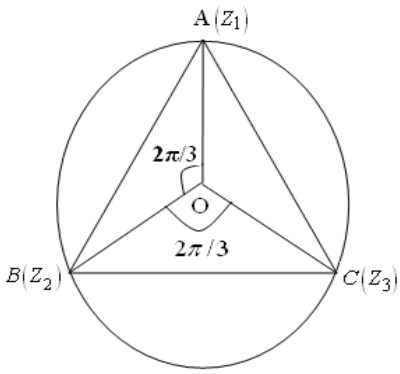
Sol. Let Z_1, Z_2, Z_3 are the vertices $\mathrm{A,B,C}$ of equilateral triangle ABC inscribed in a circle $|\mathrm{Z}| = 2$ with centre $(0,0)$ and radius 2.

Given $Z_1 = 1 + i\sqrt{3}$

Rotating OA about O by an angle $\frac{2\pi}{3}$ we have

$$\frac{Z-0}{1+i\sqrt{3}-0} = \frac{|Z-0|}{\left|+\left(\sqrt{3}-0\right)\right|} e^{\pm i\frac{2\pi}{3}}$$

$$Z = \left(1+i\sqrt{3}\right)\left(\cos\frac{2\pi}{3} \pm i\sin\frac{2\pi}{3}\right)$$



$$=\left(1+i\sqrt{3}\right)\left(\frac{-1}{2}\pm i\frac{\sqrt{3}}{2}\right)$$

$$=-\frac{\left(1+i\sqrt{3}\right)\left(1-i\sqrt{3}\right)}{2}\ (or)\ \frac{-\left(1+i\sqrt{3}\right)\left(1+i\sqrt{3}\right)}{2}$$

$$=\frac{-(1+3)}{2}\ or\ \frac{-\left(1-3+2i\sqrt{3}\right)}{2}$$

$$-2\ or\ 1-i\sqrt{3}$$

Hence option (3)

166. If $a=\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}$ then the quadratic equation whose roots are $\alpha=a+a^2+a^4$ and $\beta=a^3+a^5+a^6$ is

1) $x^2+x+2=0$
2) $x^2-5x+7=0$
3) $x^2-x+2=0$
4) $x^2+x-2=0$

Key. 1

Sol. $a=\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}$

$$a^7=\left(\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}\right)^7$$

$$=\cos 2\pi+i\sin 2\pi$$

$=1$ ....(1)

Sum of roots = $\alpha+\beta$

$$=a+a^2+a^4+a^3+a^5+a^6$$

$$=a+a^2+a^3+a^4+a^5+a^6$$

$$=\frac{a\left(1-a^6\right)}{1-a}$$

$$=\frac{a-a^7}{1-a}$$

$=\frac{a-1}{1-a}$

$=-1$....................(2)

Product of roots = $\alpha\beta$

$=\left(a+a^2+a^4\right)\left(a^3+a^5+a^6\right)$

$=a^4+a^5+a^7+a^6+a^7+a^9+a^7+a^8+a^{10}$

$=a^4+a^5+1+a^6+1+a^2+1+a+a^3 \quad (\because \textit{ from } (1))$

$=3+a+a^2+a^3+a^4+a^5+a^6$

$=3+(-1) \quad (\because \textit{ from } (2))$

$=3-1=2$

Required equation is $x^2-x(-1)+2=0$

$$x^2+x+2=0$$

Hence option (1)

167. Let z and w be two non zero complex numbers such that $|z|=|w|$ and $\arg z+\arg w=\pi$.

Then z

1) w 2) $\bar{w}$ 3) $-\bar{w}$ 4) $2w$

Key. 3

Sol. Let $\arg w=\theta$

$\therefore Arg\ z=\pi-\theta$

$w=|w|(\cos\theta+i\sin\theta)$ and $z=|z|\left[\cos(\pi-\theta)+i\sin(\pi-\theta)\right]$

$=|w|(-\cos\theta+i\sin\theta)$

$=-|w|(\cos\theta-i\sin\theta)$

$=-\bar{w}$

Hence option (3)

168. If $|z_1-1|\le 1, |z_2-2|\le 2, |z_3-3|\le 3$ then the greatest value of $|z_1+z_2+z_3|$ is

1) 6 2) 7 3) 9 4) 12

Key. 4

Sol. $|z_1+z_2+z_3| = |(z_1-1)+(z_2-2)+(z_3-3)+6|$

$\le |z_1-1|+|z_2-2|+|z_3-3|+6$

$\le 1+2+3+6$

≤ 12

Greatest value of $|z_1+z_2+z_3| = 12$

Hence option (4)

169. The greatest and least value of $|z_1+z_2|$ if $z_1 = 24+7i$ and $|z_2| = 6$

1) 31, 19 2) 25, 6 3) 31, 6 4) 19, 6

Key. 1

Sol. $|z_1+z_2| \le |z_1|+|z_2|$

$= |24+7i|+6$

$= \sqrt{(24)^2+7^2}+6 = 25+6 = 31$

Also $|z_1+z_2| = |z_1-(-z_2)|$

$\ge ||z_1|-|z_2|| = |25-6| = 19$

$\therefore$ Least value = 19, Greatest value = 31

Hence option (1)

170. If $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$ then Modulus and argument of $(1+\cos 2\alpha)+i\sin 2\alpha$ is

1) $-2\sin\alpha, \frac{\pi}{6}$ 2) $-2\cos\alpha, \alpha-\pi$ 3) $-2\sin\alpha, \alpha-\pi$ 4) None of these

Key. 2

Sol. Let $z = (1+\cos 2\alpha)+i\sin 2\alpha$

$= 2\cos^2\alpha + 2i\sin\alpha\cos\alpha$

$= 2\cos\alpha[\cos\alpha + i\sin\alpha]$

$$= -2\cos\alpha\left[-\cos\alpha - i\sin\alpha\right]$$

$$= -2\cos\alpha\left[\cos(\alpha-\pi) + i\sin(\alpha-\pi)\right] \qquad \left[\because \frac{\pi}{2} < \alpha < \frac{3\pi}{2}\right]$$

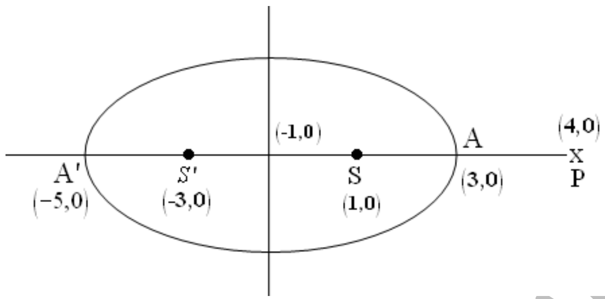
$$\therefore |z| = -2\cos\alpha \ \ \textit{and} \ \ \arg z = \alpha - \pi$$

Hence option (2)

171. If $|z-1| + |z+3| \le 8$ then the minimum and maximum values of $|z-4|$ respectively is

1) 1, 9 2) 10, 15 3) 16, 22 4) 13, 18

Key. 1



Sol.

Given $|z-1| + |z+3| \le 8$

$\therefore$ z lies inside or on the ellipse whose foci are (1, 0) and (-3, 0) and vertices are (-5,0) and (3,0). Clearly the minimum and maximum values of $|z-4|$ are 1 and 9 respectively representing the distances PA and PA'.

$$\therefore 1 \le |z-4| \le 9$$

Hence option (1)

172. If $|z-25i| \le 15$ then |maximum argz – minimum argz| equals

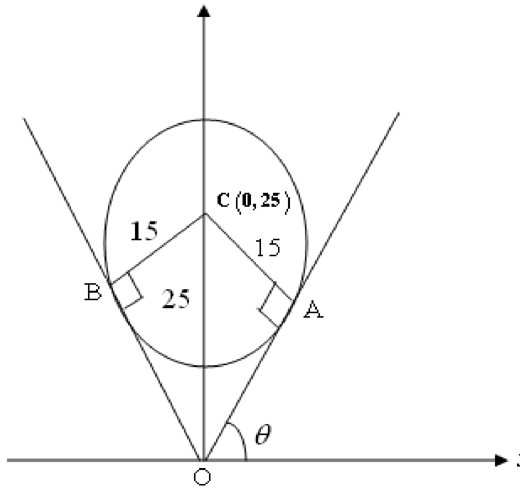
1) $2\cos^{-1}\frac{3}{5}$ 2) $2\cos^{-1}\frac{4}{5}$

3) $\frac{\pi}{2} + \cos^{-1}\frac{3}{5}$ 4) $\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{3}{5}$

Key. 2

Sol. If $|z-25i| \le 15$ then z lies either in the interior and or on the boundary of the circle with centre at $C(0,25)$ and radius equal to 15 .

The least argument is for point A and greatest argument is for point B from right



$$\Delta OAC, \cos\left(\frac{\pi}{2}-\theta\right)=\frac{OA}{OC}=\frac{20}{25}=\frac{4}{5}$$

$$\frac{\pi}{2}-\theta=\cos^{-1}\left(\frac{4}{5}\right)$$

Now for $|z-25i|\le 15$

|Maximum argz – Minimum argz| = |ArgB – ArgA|

$$=\underline{|BOA}$$

$$=\underline{|BOX}-\underline{|AOX}=\frac{\pi}{2}+\frac{\pi}{2}-\theta-\theta$$

$$=\pi-2\theta$$

$$=2\cos^{-1}\frac{4}{5}$$

Hence option (2)

173. Let z_1 and z_2 be two non-zero complex numbers such that $\frac{z_1}{z_2}+\frac{z_2}{z_1}=1$ then the origin and points represented by z_1 and z_2

1) lie on a straight line 2) form a right triangle

3) form an equilateral triangle 4) None of these

Key. 3

Sol. Let $\frac{z_1}{z_2}=z$ then $z+\frac{1}{z}=1$

$$\Rightarrow z^2-z+1=0$$

$$\Rightarrow z=\frac{1\pm i\sqrt{3}}{2}$$

$$\therefore \frac{z_1}{z_2}=\frac{1\pm i\sqrt{3}}{2}$$

If z_1 and z_2 are represented by A and B respectively and O be the origin, then

$$\frac{OA}{OB}=\frac{|z_1|}{|z_2|}=\left|\frac{1\pm\sqrt{3}i}{2}\right|=\sqrt{\frac{1}{4}+\frac{3}{4}}=1$$

$\Rightarrow OA = OB$

Also, $\frac{AB}{OB} = \frac{|z_2 - z_1|}{|z_2|} \qquad = \left|1 - \frac{z_1}{z_2}\right|$

$= \left|1 - \left(\frac{1}{2} \pm \frac{\sqrt{3}}{2} i\right)\right|$

$\left|\frac{1}{2} \mp \frac{\sqrt{3}}{2} i\right| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

$\Rightarrow AB = OB$

Thus $OA = OB = AB$

$\therefore \Delta AOB$ is an equilateral triangle.

Hence option (3)

174. If the equation, $z^4 + a_1 z^3 + a_2 z^2 + a_3 z + a_4 = 0$, where a_1, a_2, a_3, a_4 are real coefficients different from zero has a pure imaginary root then the expression $\frac{a_3}{a_1 a_2} + \frac{a_1 a_4}{a_2 a_3}$ has the value equal to:

(A) 0 (B) 1 (C) – 2 (D) 2

Key. 2

Sol. we know that

$$\frac{1}{x-1} + \frac{1}{x-w} + \frac{1}{x-w^2} + ... + \frac{1}{x-w^{n-1}} = \frac{n\left(x^{n-1}\right)}{2^n - 1}$$

Put $x = 2$ we get

$$\frac{1}{2-1} + \frac{1}{2-w} + \frac{1}{2-w^2} + ... + \frac{1}{2-w^{n-1}} = \frac{n\, 2^{n-1}}{2^n - 1}$$

$$\frac{1}{2-w} + \frac{1}{2-w^2} + ... + \frac{1}{2-w^{n-1}} = \frac{n.2^{n-1}}{2^n - 1} = \frac{n.2^{n-1} - 2^n + 1}{2^n - 1} = \frac{2^n (n-2) + 2}{2\left(2^n - 1\right)}$$

Hence option (4)

175. If α, β be the roots of the equation $u^2 - 2u + 2 = 0$ & if $\cot \theta = x + 1$, then $\frac{(x+\alpha)^n - (x+\beta)^n}{\alpha - \beta}$ is equal to

(A) $\frac{\sin n\theta}{\sin^n \theta}$ (B) $\frac{\cos n\theta}{\cos^n \theta}$ (C) $\frac{\sin n\theta}{\cos^n \theta}$ (D) $\frac{\cos n\theta}{\sin^n \theta}$

Key. 1

$$S = \quad 1+3\alpha+5\alpha^{2\alpha^n}+(2n-3)^{n-2}\,4(2n-1)\alpha^{n-1}$$
$$\alpha S = \quad \alpha+3\alpha^2+.................+(2n-3)\alpha^{n-1}+(2n-1)\alpha^n$$

Sol. Let $S(1-\alpha)=1+2\alpha+2\alpha^2+...........2\alpha^{n-1}-(2n-1)\alpha^n$

$$=1+2\alpha\left(1+\alpha+\alpha^2+..........\alpha^{n-2}\right)-(2n-1)\alpha^n$$

$$=1+\frac{2\alpha\left(1-\alpha^{n-1}\right)}{1-\alpha}-(2n-1)\alpha^n$$

$$=1+\frac{2\left(\alpha-\alpha^n\right)}{1-\alpha}-(2n-1)\alpha^n$$

$$=1+\frac{2(\alpha-1)}{1-\alpha}-(2n-1)\quad \left(\because \alpha^n=1\right)$$

$$=1-2-2n+1=-2n$$

$$S=\frac{-2n}{1-\alpha}=\frac{2n}{\alpha-1}$$

Hence option (4)

176. If $z=(\lambda+3)-i\sqrt{5-\lambda^2}$ then the locus of z is

1) ellipse 2) semi circle 3) parabola 4) straight line

Key. 2

Sol. Let $z=x+iy$ then $x=\lambda+3,\ \ y=-\sqrt{5-\lambda^2}$

$$\Rightarrow (x-3)^2=\lambda^2 \ \ and\ \ y^2=5-\lambda^2$$

(1) (2)

From (1) and (2) $(x-3)^2=5-y^2$

$$\Rightarrow (x-3)^2+y^2=5$$

Clearly it is a semi circle as $y<0$. Hence part of the circle lies below the x-axis.

Hence option (2)

177. If $x^2+x+1=0$ then the value of $\left(x+\frac{1}{x}\right)^2+\left(x^2+\frac{1}{x^2}\right)^2+.......\left(x^{27}+\frac{1}{x^{27}}\right)^2$ is

1) 27 2) 72 3) 45 4) 54

Key. 4

Sol. $x^2+x+1=0 \Rightarrow x=w \ or \ w^2$

Let $x=w$ then $x+\frac{1}{x}=w+\frac{1}{w}=w+w^2=-1$

$x^2+\frac{1}{x^2}=w^2+\frac{1}{w^2}=w^2+w=-1$

$x^3+\frac{1}{x^3}=w^3+\frac{1}{w^3}=1+1=2$

$x^4+\frac{1}{x^4}=w^4+\frac{1}{w^4}=w+\frac{1}{w}=-1$ etc.

$\therefore \left(x+\frac{1}{x}\right)^2+\left(x^2+\frac{1}{x^2}\right)^2+\left(x^3+\frac{1}{x^3}\right)^2+.......\left(x^{27}+\frac{1}{x^{27}}\right)^2$

$=18+9(2)^2=54$

Hence option (4)

178. If centre of a regular hexagon is at origin and one of the vertices on Argand diagram is $1+2i$, then its perimeter is

1) $2\sqrt{5}$ 2) $6\sqrt{2}$ 3) $4\sqrt{5}$ 4) $6\sqrt{5}$

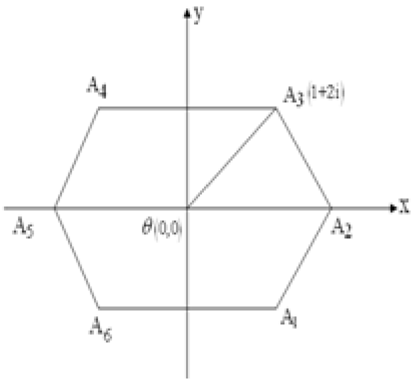
Key. 4

Sol. Let the vertices be $z_1, z_2, z_3, z_4, z_5, z_6$ w.r.t centre O at origin $|z_3|=\sqrt{5}$

Now $\Delta\, O\, A_2A_3$ is equilateral $\Rightarrow OA_2=OA_3=A_2A_3=\sqrt{5}$

Perimeter = $6\sqrt{5}$

Hence option (4)

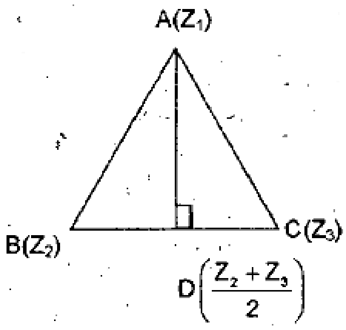


179. Let A(Z_1), B(Z_2), C(Z_3) be the vertices of an equilateral triangle ABC, then the value of $\arg\left(\dfrac{Z_2+Z_3-2Z_1}{Z_3-Z_2}\right)$ is equal to

a) $\dfrac{\pi}{3}$ b) $\dfrac{\pi}{4}$ c) $\dfrac{\pi}{2}$ d) $\dfrac{\pi}{6}$

Ans. c

$$\arg\left(\frac{Z_2+Z_3-2Z_1}{Z_3-Z_2}\right)=\arg 2\left\{\frac{\left(\frac{Z_2+Z_3}{2}-Z_1\right)}{Z_3-Z_2}\right\}$$



$$=\arg\left\{\frac{\left(\frac{Z_2+Z_3}{2}-Z_1\right)}{Z_3-Z_2}\right\}=\frac{\pi}{2}$$

Clearly $AD \perp BC$

180. If $|z-1-i|=1$, then the locus of a point represented by the complex number 5(z – i) – 6 is

a) circle with centre (1, 0) and radius 3 b) circle with centre (-1, 0) and radius 5
c) line passing through origin d) line passing through (-1, 0)

Ans. b

Let w = 5(z – i) – 6

$\Rightarrow |w+1| = 5|z-1-i| = 5$

181. Let z be a complex number satisfying $\left|z^2+2z\cos\alpha\right|\le 1,\ (a\in R)$ then maximum value $\cot|z|$ must be

a) $\sqrt{2}+1$ b) $\sqrt{3}-1$ c) $\sqrt{3}+1$ d) $\sqrt{6}$

Ans. c

$\left|z^2+2z\cos\alpha\right|\le 1 \Rightarrow |z||z+2\cos\alpha|\le 1$

$\Rightarrow |z+2\cos\alpha|\le |z|+|2\cos\alpha|$

$\Rightarrow |z|^2\left(|z|+|2\cos\alpha|\right)^2\le 1$

$\Rightarrow |z| \in \left[0, \sqrt{3}+1\right]$

182. Z_1 and Z_2 are the roots of $Z^2 - aZ + b = 0$ where $|Z_1| = |Z_2| = 1$ and $a, b \in C$, then

a) arg (a) = arg (b) b) arg(a) = 2 arg(b) c) 2arg(a) = arg(b) d) none of these

Ans. c

$Z_1 + Z_2 = a,\ Z_1 Z_2 = b\ and\ |Z_1| = |Z_2| = 1$

$\therefore\ \arg(a) = \frac{1}{2}\{\arg(Z_2) + \arg(Z_1)\}$ and $\arg(b) = \arg(Z_1 Z_2) = \arg(Z_1) + \arg(Z_2)$

$\therefore$ 2 arg(a) = arg (b)

183. If $|z-1| = 1$ and arg $z = \theta (z \neq 0)$ and $0 < \theta < \pi/2$, then $1 - \frac{2}{z}$ is equal to

a) $\tan\theta$ b) $i\ \tan\theta$ c) $\tan\frac{\theta}{2}$ d) $i\ \tan\frac{\theta}{2}$

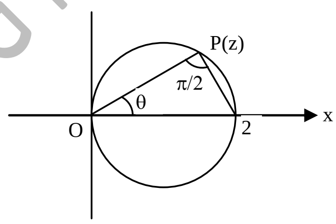
Ans. b

$\arg\left(\frac{2-z}{0-z}\right) = \frac{\pi}{2}$

$\Rightarrow \frac{z-2}{z} = \frac{AP}{OP} i$

$\tan\theta = \frac{AP}{QP}$

then $\frac{z-2}{z} = i\tan\theta$



184. If complex number z satisfies $|z - 6i| = lm(z)$, then range of $(\arg z - \arg \bar{z})$ will be

a) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ b) $\left[\frac{2\pi}{3}, \frac{4\pi}{2}\right]$ c) $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$ d) $\left[\frac{3\pi}{4}, \frac{5\pi}{3}\right]$

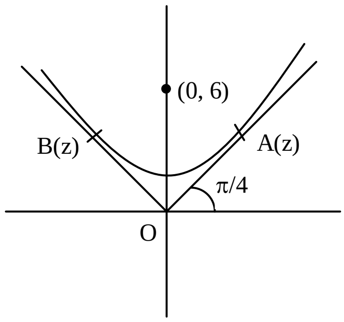
Ans. a

Clearly z lies on a parabola focus at (0, 6) and x-axis as directrix as $\arg \bar{z} = -\arg z$. Point of contact of the tangent drawn from origin to the parabola will corresponds to the maximum and minimum argument.

$(\arg z)_{\min} = \frac{\pi}{4}$

$(\arg z)_{\max} = \frac{3\pi}{4}$

$\frac{\pi}{2} \leq 2\arg \leq \frac{3\pi}{2}$



185. If z_1, z_2, z_3 are three distinct complex numbers and a, b, c are three positive real numbers such that $\frac{a}{|z_2 - z_3|} = \frac{b}{|z_3 - z_1|} = \frac{c}{|z_1 - z_2|}$ then the value of $\frac{a^2}{z_2 - z_3} + \frac{b^2}{z_3 - z_1} + \frac{c^2}{z_1 - z_2}$ is

a) 0 b) 1 c) 2 d) 3

Ans. a

$$\left(\frac{a^2}{z_2 - z_3}\right) + \left(\frac{b^2}{z_3 - z_1}\right) + \left(\frac{c^2}{z_1 - z_2}\right) = \lambda\left(\overline{z_2} - \overline{z_3} + \overline{z_5} - \overline{z_4} + \overline{z_1} - \overline{z_2}\right) = 0$$

186. If $|z-1| + |z+3| \le 8$ then the range of values of $|z-4|$ is

a) [0, 7] b) [1, 8] c) [1, 9] d) [2, 5]

Ans. c

z lies inside or on the ellipse with foci (1, 0) and (-3, 0). Hence minimum and maximum values of $|z-4|$ are 1 and 9.

187. If x_1, x_2, x_3 …. x_n are the roots of $x^n + ax + b = 0$, then the value of $(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)$ ….. $(x_1 - x_n)$ is

a) $nx_1^n + a$ b) $nx_1^{n-1} + a$ c) $nx_1 + a^{n-1}$ d) $nx_1 + a^n$

Ans. b

$$x^n + ax + b = 0 = (x - x_1)(x - x_2)(x - x_3)....(x - x_n)$$

$$\Rightarrow (x - x_2)(x - x_3)......(x - x_n) = \frac{x^n + ax + b}{x - x_1}$$

$$\therefore (x_1 - x_2)(x_1 - x_3)(x_1 - x_4)....(x_1 - x_n) = \lim_{x \to x_1}\left(\frac{x^n + ax + b}{x - x_1}\right) = nx_1^{n-1} + a$$

188. Triangle ABC, $A(z_1), B(z_2)$ and $C(z_3)$ is inscribed in the circle | z | = 5. If $H(z_H)$ be the orthocenre of triangle ABC, then Z_H is equal to

(A) $\frac{2}{3}(z_1 + z_2 + z_3)$ (B) $\frac{4}{3}(z_1 + z_2 + z_3)$

(C) $(z_1 + z_2 + z_3)$ (D) $3(z_1 + z_2 + z_3)$

Key. C

Sol. Circumcentre of triangle ABC is origin. Let $G(Z_G)$ be it's centriod, then

$Z_G = \frac{1}{3}(z_1 + z_2 + z_3)$ the points O (0), $G(z_G), H(z_H)$ are collinear and OG : GH = 1 : 2

$$Z_G = \frac{2 \times 0 + 1 \times Z_H}{3} = Z_H = 3Z_G = z_1 + z_2 + z_3$$

189. If tangents drawn to circle $|z| = 2$ at $A(z_1)$ and $B(z_2)$ meet at $P(z_P)$, then

(A) $Z_P = \left(\frac{z_1 + z_2}{2}\right)$ (B) $Z_P = \frac{2(z_1 + z_2)}{\sqrt{z_1 z_2}}$

(C) $Z_P = \frac{2z_1 z_2}{z_1 + z_2}$ (D) $Z_P^2 = z_2 z_1$

Key. C

Sol. Equation of tangentat $A(z_1)$ is

$$\frac{z}{z_1} + \frac{\bar{z}}{\bar{z}_1} = 2 \Rightarrow \frac{z}{z_1} + \frac{\bar{z}}{4} = 2$$

$\Rightarrow \ \frac{z}{z_1^2}+\frac{\bar{z}}{4}=\frac{2}{z_1}$

Equation of tangent at B (z_2) is

$\frac{z}{z_2^2}+\frac{\bar{z}}{4}=\frac{2}{z_2}$

$\Rightarrow \ z\left(\frac{1}{z_1^2}-\frac{1}{z_2^2}\right)=2\left(\frac{1}{z_1}-\frac{1}{z_2}\right)$

$\Rightarrow z=\frac{2z_1z_2}{z_1+z_2}$

190. If $t^2+t+1=0$, then the value of $\left(t+\frac{1}{t}\right)^2+\left(t^2+\frac{1}{t^2}\right)^2+.....+\left(t^{27}+\frac{1}{t^{27}}\right)^2$ is

(A) 27 (B) 72
(C) 45 (D) 54

Key. D

Sol. $s=\left(t+\frac{1}{t}\right)^2+\left(t^2+\frac{1}{t^2}\right)^2+.....+\left(t^{27}+\frac{1}{t^{27}}\right)^2$

Let $t=\omega$ then

$S=\left\{(-1)^2+(-1)^2+.....18\text{ terms}\right\}+\left\{(2)^2+.....9\text{ terms}\right\}$

$=18+9\times4=18+36=54$

191. Let n is of the form of 3P where P is an odd integer then,
${}^nc_0+{}^nc_3+{}^nc_6+{}^nc_9+.....+{}^nc_n$ equals

(A) $\frac{1}{3}\left(2^n-2\right)$ (B) $\frac{2}{3}\left(2^n-2\right)$

(C) $\frac{1}{3}\left(2^{n-1}-2\right)$ (D) $\frac{2}{3}(2^n+2)$

Key. A

Sol. $(1+x)^n=c_0+c_1x+c_2x^2+.....+c_nx^n$

$(1+\omega)^n=c_0+c_1\omega+c_2\omega^2+.....+c_n\omega^n$

$(1+\omega^2)^n=c_0+c_1\omega^2+c_2\omega^4+.....+c_n\omega^{2n}$

$2^n=c_0+c_1+c_2+.....+c_n$

$2^n+(-\omega)^n+\left(-\omega^2\right)^n=3c_0+3c_3+.....+3\,{}^nc_n$

$c_0+c_3+c_6+.....+c_n=\frac{1}{3}\left[2^n+(-1)^n\omega^n+(-1)^n\omega^{2n}\right]$

$=\frac{1}{3}\left[2^n+(-1)^{3P}\omega^{3P}+(-1)^{3P}\omega^{6P}\right]$

$=\frac{1}{3}\left[2^n-1-1\right]=\frac{1}{3}\left[2^n-2\right]$

192. Let z_1 and z_2 be two complex numbers with α and β as their principal arguments, such that $\alpha+\beta>\pi$, then principal arg(z_1z_2)is given by

(A) $\alpha+\beta+\pi$ (B) $\alpha+\beta-\pi$

(C) $\alpha+\beta-2\pi$ (D) $\alpha+\beta$

Key. C

Sol. arg (z_1z_2) = arg (z_1) + arg(z_2) + $2m\pi$, $m \in I$

$= \alpha + \beta - 2\pi$ which should be equivalent to negative angle $\frac{7\pi}{6} - 2\pi$

193. Let z and ω be two complex numbers, such that $|z|^2\omega - |\omega|^2 z = z-\omega$ and $z \neq \omega$, then

(A) $z=\bar{\omega}$ (B) $z\bar{\omega}=1$

(C) $\bar{z}\omega=2$ (D) $z\overline{\omega}=2$

Key. B

Sol. $|z|^2\omega - |\omega|^2 z = z-\omega$

$\Rightarrow \omega\left[1+|z|^2\right] = z\left[1+|\omega|^2\right]$

$\omega\bar{\omega} = \frac{\left[1+|z|^2\right]}{\left[1+|\omega|^2\right]} = z\bar{\omega} = |\omega|^2\left\{\frac{1+|z|^2}{1+|\omega|^2}\right\}$

$\Rightarrow z\bar{\omega}$ is real number and therefore

$z\bar{\omega} = \omega\bar{z}$ …… (1)

$|z|^2\omega - |\omega|^2 z = z-\omega$

$z\bar{z}\omega - \omega\bar{\omega}z - z + \omega = 0$

$z(\bar{z}\omega - 1) - \omega(\bar{\omega}z - 1) = 0$ ….. (2)

From (1) and (2)

$z(z\bar{\omega}-1) - \omega(z\bar{\omega}-1) = 0$

$(z\bar{\omega}-1)(z-\omega) = 0$

$z\bar{\omega} = 1 = \bar{z}\omega$ Since $z \neq \omega$

194. The point of intersection of the curves $\arg(z-3i) = \frac{3\pi}{4}$ and $\arg(2z+1-2i) = \frac{\pi}{4}$ is

(A) $\frac{3}{4}+i\frac{9}{4}$ (B) $1+3i$

(C) $1+i$ (D) no solution

Key. D

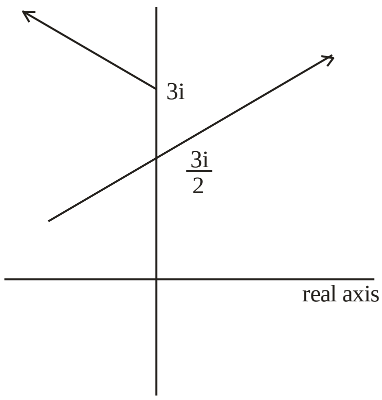
Sol.

$\arg(z-3i) = \frac{3\pi}{4}$ **…… (1)**

$\arg(2z+1-2i) = \frac{\pi}{4}$

$\Rightarrow \arg\left(z+\frac{1}{2}-i\right) + \arg(2) = \frac{\pi}{4}$

$\Rightarrow \arg\left[z - \left(-\frac{1}{2}\right) + i\right] = \frac{\pi}{4}$ **….. (2)**



No point of intersection of (1) and (2)

195. Let $\left|(z_1-2z_2)/\left(2-z_1\overline{z_2}\right)\right|=1$ and $|z_2|\neq 1$ where z_1 and z_2 are complex numbers. The value of $|z_1|$ is

(A) 1 (B) 2
(C) cannot be obtained (D) none of these

Key. B

Sol. $\left|z_1-2z_2\right|^2=\left|2-z_1\overline{z_2}\right|^2$ i.e.,

$$\left(z_1-2z_2\right)\left(\overline{z_1}-2\overline{z_2}\right)=\left(2-z_1\overline{z_2}\right)\left(2-\overline{z_1\bar{z}_2}\right)=(2-z_1\overline{z_2})(2-\bar{z}_1z_2)$$

$\Rightarrow\ z_1\overline{z_1}-2z_2\overline{z_1}-2z_1\overline{z_2}+4z_2\overline{z_2}=4-2z_1\overline{z_2}-2\overline{z_1}z_2+|z_1|^2|z_2|^2$

$\Rightarrow (Z1)^2-4)(|z_2|^2-1)=0$

Since $|z_2|\neq 1$,

$\Rightarrow |z_1|^2=4\Rightarrow |z_1|=2$

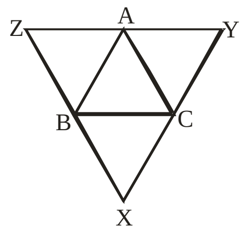
196. Points $A(z_1)$, $B(z_2)$ and $C(z_3)$ form a triangle with centroid z_0. If triangles XCB, CYA and BAZ similar to triangle ABC are out wordly drawn on the sides of ΔABC, then centroid of ΔXYZ is

(A) $3z_0$ (B) $-z_0$
(C) z_0 (D) $-2z_0$

Key. C

Sol.

The quadrilateral abxc is a parallelogram. if z is the affix of x,



$$\frac{1}{2}(z_1+z)=\frac{1}{2}(z_2+z_3)$$

$z=z_2+z_3-z_1$

similarly affix of y is $z_1+z_3-z_2$ and that of z is $z_1+z_2-z_3$

centroid of ΔXYZ is

$$\frac{1}{3}(z_2+z_3-z_4+z_1+z_3-z_2+z_1+z_3-z_3)$$

$$=\frac{1}{3}(z_1+z_2+z_3)=z_0$$

197. The roots of $1+z+z^3+z^4=0$ are represented by the vertices of

(A) a square (B) an equilateral triangle
(C) a rhombus (D) a rectangle

Key. B

SOL. The given equation is $(1+z)(1+z^3)=0$ the distinct roots being -1, $-\omega$, $-\omega^2$ which if be represented by points a, b and c in that order

$ab=|1-\omega|=|\omega|\,|\omega^2-1|=|\omega^2-1|$

$bc=|\omega-\omega^2|=|\omega^2|\,|\omega^2-1|=|\omega^2-1|$

$ca=|\omega^2-1|$

THE THREE POINTS REPRESENT THE VERTICES OF AN EQUILATERAL TRIANGLE.

198. If ω is any complex number such that zω = $|z|^2$ and $|z-\bar{z}|+|\omega+\bar{\omega}|=4,$ then as ω varies, then the area bounded by the locus of z is
(A) 4 sq. Units (B) 8 sq. Units
(C) 16 sq. Units (D) 12 sq. Units

Key. B

Sol. zω = $|z|^2 \Rightarrow \omega = \bar{z}$

$|z+\bar{z}|+|z-\bar{z}|=4$

$|x| + |y| = 2$

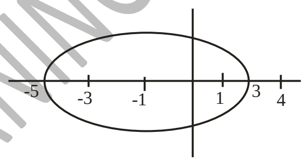
Which is a square ∴ Area = 8 sq. Units

199. If $|z - 1| + |z + 3| \le 8$, then the range of values of $|z - 4|$ is,
(A) (0, 8) (B) [1, 9]
(C) [0, 8] (D) [5, 9]

Key. B

Sol.

z lies inside or on the ellipse. Clearly the minimum distance of z from the given point 4 is 1 and maximum distance is 9



200. The reflection of the complex number $\frac{6+10i}{(1+i)^2}$ in the straight line $i\bar{z}=z,$ is
(A) – 3 + 5i (B) – 3 - 5i
(C) 3 – 5i (D) 3 + 5i

Key. A

Sol. $\frac{6+10i}{(1+i)^2}=\frac{6+10i}{2i}=5-3i$

Put z = x + iy

i(x – iy) = x + iy

ix + y = x + iy

⇒ (x – y) – i(x – y) = 0

⇒ x – y = 0

Reflection is (–3 + 5i)

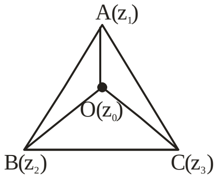
201. If z_1, z_2 and z_3 be the vertices of ΔABC, taken in anti-clock wise direction and z_0 be the circumcentre, then $\left(\frac{z_0-z_1}{z_0-z_2}\right)\frac{\sin 2A}{\sin 2B}+\left(\frac{z_0-z_3}{z_0-z_2}\right)\frac{\sin 2C}{\sin 2B}$ is equal to
(A) 0 (B) 1
(C) – 1 (D) 2

Key. C

Sol. Taking rotation at O

$$\frac{z_0-z_1}{z_0-z_2}=\cos 2C-i\sin 2C$$

$$\frac{z_0 - z_3}{z_0 - z_2} = \cos 2A + i\sin 2A$$



Now $\left(\frac{z_0 - z_1}{z_0 - z_2}\right)\frac{\sin 2A}{\sin 2B} + \left(\frac{z_0 - z_3}{z_0 - z_2}\right)\frac{\sin 2C}{\sin 2B}$

$$= \frac{\text{sin2A cos2C - isin2A sin2C + cos2Asin2C + isin2Asin2C}}{\sin 2B}$$

$$= \frac{\sin(2A + 2C)}{\sin 2B} = -1$$

202. If z = cosα + i sinα, 0 < α < π/6 then the argument of $\frac{z^4 - 1}{z^3 + 1}$ is

(A) $\frac{\pi}{2} + \frac{\alpha}{2}$ (B) $\frac{\pi}{2} - \frac{\alpha}{2}$

(C) $\frac{3\alpha}{2}$ (D) $2\alpha - \frac{\pi}{2}$

Key. A

Sol. $\arg\left(\frac{z^4 - 1}{z^3 + 1}\right) = arg(z^4 - 1) - arg(z^3 + 1) = \left(2\alpha - \frac{\pi}{2} + \pi\right) - \frac{3\alpha}{2} = \frac{\pi + \alpha}{2}$

203. If |z| = 1 and z′ = $\frac{1 + z^2}{z}$, then

(A) z′ lie on a line not passing through origin (B) |z′| = $\sqrt{2}$

(C) Re (z′) = 0 (D) Im (z′) = 0

Key. D

Sol. Conceptual

204. The number of complex numbers z satisfying $|z + \bar{z}| + |z - \bar{z}| = 4$ and |z + 2i| + |z – 2i| = 4 is/are

(A) 0 (B) 1

(C) 2 (D) 4

Key. C

SOL. ∴ |x| + |y| = 2 …..(i)

|z + 2i| + |z – 2i| = 4 …..(ii)

eq. (i) represent square & (ii) represent line segment solution are z = ± 2i.

205. If z_1, z_2 are complex numbers such that $z_1^3 - 3z_1z_2^2 = 2$ and $3z_1^2z_2 - z_2^3 = 11$ then $\left|z_1^2 + z_2^2\right|$ =

A) 3 B) 4 C) 5 D) 6

Key. C

Sol. $z_1^3 - 3z_1z_2^2 + 3iz_1^2z_2 - iz_2^3 = 2 + 11i \Rightarrow (z_1 + iz_2)^3 = 2 + 11i$

Similarly $(z_1 - iz_2)^3 = 2 - 11i$

$\left|z_1^2 + z_2^2\right| = \left|(z_1 + iz_2)(z_1 - iz_2)\right| = \left|(2 + 11i)^{1/3}(2 - 11i)^{1/3}\right| = 5$

Complex Numbers

Integer Answer Type

1. If $\frac{3iz_2}{5z_1}$ is purely real, then find $5\left|\frac{3z_1+7z_2}{3z_1-7z_2}\right|$.

Key. 5

Sol. Let $\frac{3iz_2}{5z_1}=K$ (real)

$$\frac{z_2}{z_1}=\frac{5K}{3i}$$

$$5\left|\frac{3+7\frac{z_2}{z_1}}{3-7\frac{z_2}{z_1}}\right|=5\left|\frac{3+7\frac{35K}{3i}}{3-\frac{35K}{3i}}\right|$$

$$5\left|\frac{35K+9i}{35K-9i}\right|=5$$

2. Let $1,w,w^2$ be the cube root of unity. The least possible degree of a polynomial with real coefficients having roots $2w,\left(2+3w\right),\left(2+3w^2\right),\left(2-w-w^2\right)$, is

Key. 5

Sol. Roots are $2w,\left(2+3w\right)\left(2+3w^2\right)\left(2-w-w^2\right)\left(2+3w\right)$ and $2+3w^2$ are conjugate each other 2w is complex root, then other root must be 2w^2 (as conjugate root occur in conjugate pair)

$2-w-w^2=2-\left(-1\right)=3$ which is real.

Hence least degree of the polynomial : 5.

3. If a complex number z satisfies $\left|z-8-4i\right|+\left|z-14-4i\right|=10$, then the maximum value of $\arg\left(z\right)=\tan^{-1}\frac{11}{3k}$, find k.

Key. 4

Sol. (k = 12) locus of z is an ellipse $\frac{\left(x-11\right)^2}{25}+\frac{\left(y-4\right)^2}{16}=1$

Equation of tangent is $y-4=m\left(x-11\right)+c\Rightarrow c=11m-4$

As $c^2=a^2m^2+b^2$ for standard ellipse

$$\Rightarrow\left(11m-4\right)^2=25m^2+16\Rightarrow m=0\ or\ m=\frac{11}{12}$$

$$\therefore\tan\theta=\frac{11}{12}\Rightarrow\theta=\tan^{-1}\frac{11}{12}$$

4. If ‘a’ and ‘b’ are complex numbers. One of the roots of the equation $x^2+ax+b=0$ is purely real and the other is purely imaginary then $a^2-\bar{a}^2 = kb$, find k

Key. 4

Sol. Let α and $i\beta$, $\alpha,\beta \in R$ are roots of

$x^2+ax+b=0 \Rightarrow \alpha+i\beta=-a, i\alpha\beta=b$

$\alpha-i\beta=-\bar{a}$

$\Rightarrow 2\alpha=-\left(a+\bar{a}\right)$ and $2i\beta$ =-$\left(a-\bar{a}\right)$

$\therefore 4i\alpha\beta=a^2-\bar{a}^2 \Rightarrow 4b=a^2-\bar{a}^2$

5. There are two complex numbers z such that $|z-2-i|=1$ and $\arg z=\frac{\pi}{4}$. The product of modulus of these two complex number is k. find k

Key. 8

Sol. (K=8) Two points B and D

$\therefore B=|z_1| \Rightarrow |z_1|=2\sqrt{2}$

$z_1=2+2i$

and $D(z_2), z_2=1+1i \Rightarrow |z_2|=\sqrt{2}$

$\therefore |z_1z_2|=2\sqrt{2}\times\sqrt{2}=8$

6. The sum of the real parts of the complex numbers satisfying the equations $\left|\frac{z-4i}{z-2i}\right|=1$ and $\left|\frac{z-8+3i}{z+3i}\right|=\frac{3}{5}$ is $\frac{k}{5}$, find k.

Key. 5

Sol. $\left|\frac{z-4i}{z-2i}\right|=1 \Rightarrow z=x+3i$ using this in $\left|\frac{z-8+3i}{z+3i}\right|=\frac{3}{5} \Rightarrow 5|x-8+6i|=3|x+6i|$

$\Rightarrow x=8,17$

Two complex numbers 8+3i, 17+3i

Sum of real part = 8+ 17 =25

7. If the equation $z^4+a_1z^3+a_2z^2+a_3z+a_4=0$ where a_1,a_2,a_3,a_4 are real coefficient different from zero has purely imaginary roots then find the value of the expression $\frac{a_3}{a_1a_2}+\frac{a_1a_4}{a_2a_3}$.

Key. 1

Sol. Let $z=iy$

$\Rightarrow y^4-a_1y^3i-a_3y^2+ia_3y+a_4=0$

$\Rightarrow y^4-a_2y^2+a_4=0 \quad -(1)$ and $-a_1y^3+a_3y=0$

$\Rightarrow y=0$ or $y^2=\frac{a_3}{a_1} \quad -(2)$

From (1) and (2)

$$\frac{a_3^2}{a_1^2}-a_2\frac{a_3}{a_1}+a_4=0$$

$$\Rightarrow \frac{a_3}{a_1a_2}+\frac{a_1a_4}{a_2a_3}=1$$

8. $\sum_{j=1}^{n-1}\frac{1}{1-e^{\frac{2\pi ij}{n}}}=\frac{n-1}{k}$, find k. $\left(i=\sqrt{-1}\right)$

Key. 2

Sol. k = 2,

Let $e^{i\frac{2\pi}{n}}=\alpha$ then $\sum_{j=1}^{n-12}\frac{1}{1-e^{j}}=\frac{1}{1-\alpha}+\frac{1}{1-\alpha^2}+.......+\frac{1}{1-\alpha^{n-1}}$

Where α is a nth root of unity $\left(\alpha,\alpha^2,\alpha^3,.......\alpha^{n-1}\right)$ are the roots of

$$\frac{x^n-1}{x-1}=(x-\alpha)\left(x-\alpha^2\right).......\left(x-\alpha^{n-1}\right)$$

Taking log on both side

$$\log\frac{x^n-1}{x-1}=\log(x-\alpha)+\log\left(x-\alpha^2\right)+.......+\log\left(x-\alpha^{n-1}\right)$$

Diff w.r.t. x and use $\lim\limits_{x\to1}$

$$\Rightarrow \frac{n-1}{2}=\frac{1}{1-\alpha}+\frac{1}{1-\alpha^2}+........+\frac{1}{1-\alpha^{n-1}}$$

9. Let A,B,C be equilateral triangle with $\frac{\sqrt{3}}{2}A=e^{i\pi/2}$, $\frac{\sqrt{3}}{2}B=e^{-i\pi/6}$, $\frac{\sqrt{3}}{2}C=e^{-i5\pi/6}$. Let P be any point on the incricle of ΔABC. Find the value of $PA^2+PB^2+PC^2$

Key. 5

Sol. Given triangle is an equilateral triangle

$\therefore$ incircle is $x^2+y^2=\frac{1}{3}$

Let point on the in circle is (x,y)

$$\therefore PA^2+PB^2+PC^2=x^2+\left(y-\frac{2}{\sqrt{3}}\right)^2+(x-1)^2+\left(y+\frac{1}{\sqrt{3}}\right)^2+(x+1)^2+\left(y+\frac{1}{\sqrt{3}}\right)^2$$

$$=3\left(x^2+y^2\right)+4$$

$$=1+4=5$$

10. Two lines $zi-\bar{z}i+2=0$ and $z(1+i)+\bar{z}(1-i)+2=0$ intersect at P. The complex numbers of points on the second line which are at a distance of 2 unit form the point P are $z=i\pm e^{i\frac{\pi}{k}}$, find k

Key. 4

Sol. k = 4

The lines are $zi - \bar{z}i + 2 = 0$ and $z(1+i) + \bar{z}(1-i) + 2 = 0$ at i.

Let point on the line be z then $|z - i| = 2$

$\Rightarrow 2e^{i\theta} + i$ putting this in second equation $\Rightarrow \theta = \pi/4$

$\therefore$ points are $z = i \pm e^{i\pi/4}$

11. If $z_1, z_2, z_3, ..., z_n$ are in G.P with first term as unity such that $z_1 + z_2 + z_3 + ... + z_n = 0$. Now if $z_1, z_2, z_3, ..., z_n$ represents the vertices of n-polygon, then the distance between incentre and circumcentre of the polygon is represented by 4k. Find k.

Key. 0

Sol. Let vertices be $1, \alpha, \alpha^2, ..., \alpha^{n-1}$.

Given $1 + \alpha + \alpha^2 + ... + \alpha^{n-1} = 0 \Rightarrow a^n - 1 = 0$

$\Rightarrow$ $z_1, z_2, z_3, ..., z_n$ are roots of $\alpha^n = 1$

Which form regular polygon. So distance is zero.

12. Let λ, z_0 be two complex numbers. $A(z_1)$, $B(z_2)$, $C(z_3)$ be the vertices of a triangle such that $z_1 = z_0 + \lambda$, $z_2 = z_0 + \lambda e^{i\pi/4}$, $z_3 = z_0 + \lambda e^{i7\pi/11}$ and $\angle ABC = \frac{3k\pi}{22}$ then the value of k is

Key. 5

Sol. $|z_1 - z_0| = |z_2 - z_0| = |z_3 - z_0| = |\lambda|$

$\frac{z_3 - z_0}{z_2 - z_0} = \frac{e^{i7\pi/11}}{e^{i\pi/4}} = e^{i17/44}$

$\Rightarrow \angle BSC = 17\frac{\pi}{44} \Rightarrow \angle BAC = 17\frac{\pi}{88}$

Similarly $\frac{z_2 - z_0}{z_1 - z_0} = e^{i\pi/4} \Rightarrow \angle ACB = \frac{\pi}{8}$

$\therefore \angle ABC = \pi - \frac{\pi}{8} - \frac{17\pi}{88} = \frac{15\pi}{22}$.

13. The roots of the equation $z^5 + z^6 + ...z^{10} = 0$ where $z \neq 0,1$ are represented by vertices of a pentagon having longest side length is equal d. Find d^2.

Key. 3

Sol. $d^2 = 3$. Equation reduces to $z^6 = 1$

$$\Rightarrow z = \cos 2k\frac{\pi}{6} + i\sin 2k\frac{\pi}{6},\ k = 1, 2, 3, 4, 5$$

The longest side = $\left|\cos\frac{\pi}{3} + i\sin\frac{\pi}{3} - \left(\cos 5\frac{\pi}{3} i\sin 5\frac{\pi}{3}\right)\right| = \sqrt{3}$.

14. The complex number z satisfying $|z+2+i|+|z-2+i|=4$, $0\le \arg(z+2+2i)\le \frac{\pi}{4}$ and $3\frac{\pi}{4}\le \arg(z-2+2i)\le \pi$ will lie on a line segment of the length k. Find k.

Key. 2

Sol. $(k=2)$ length $AB=2$

15. If the argument of $(z-a)(\bar{z}-b)$ is equal to that of $\frac{(\sqrt{3}+i)(1+\sqrt{3}i)}{1+i}$, where a,b are real numbers. If locus of z is a circle with centre $\frac{3+i}{2}$ then find $(a+b)$.

Key. 3

Sol. $\tan^{-1}\frac{(a-b)y}{x^2+y^2-(a+b)x+ab}=\frac{\pi}{4}$

$$\Rightarrow x^2+y^2-(a+b)x-(a-b)y+ab=0$$

Centre = $\frac{3+i}{2}\Rightarrow a+b=3$

16. If $Z=\frac{1}{2}\left(\sqrt{3}-i\right)$ then the least positive integral value of $'n'$ such that $\left(Z^{101}+i^{109}\right)^{106}=Z^n$ is $'k'$ then $\frac{2}{5}k=$

Key. 4

Sol. $Z=\frac{-1}{2}i\left(1+i\sqrt{3}\right)=i\omega^2$

$Z^{101}=i\omega$

$\left(Z^{101}+i^{109}\right)^{106}=-\left(i\omega^2\right)^{106}=-\omega^2$

$\therefore -\omega^2=\left(i\omega^2\right)^n=i^n\omega^{2n}$

$\omega^{2n-2}i^n=-1$

This is possible only when $N=4r+2$ and $2n-2$ is multiple of 3 i.e.,

$2(4r+2)-2$ is multiply of 3

i.e., $8r+2$ is multiple of $3\Rightarrow r=2$

$\therefore n=10 \quad \therefore \frac{2}{5}k=4$

17. If $\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5$ are the roots of the equation $x^5-1=0$, where $\alpha_k=\alpha^{k-1}$, $\alpha=e^{i2\pi/5}$ and $\lambda=\alpha_3^{1001},\mu=\alpha_4^{(669+1/3)}$, $\nu=\alpha_5^{(503+1/2)}$, then $[|\lambda^{2011}+\mu^{2011}+\nu^{2011}|]$ (where [.] denotes the greatest integer function) is

Key. 1

Sol. Clearly $\alpha_1 = 1$

$\alpha_2 = \alpha$

$\alpha_3 = \alpha^2$

$\alpha_4 = \alpha^3$

$\alpha_5 = \alpha^4$

where $\alpha = e^{i2\pi/5}$

$\therefore \ \lambda = \alpha_3^{1001} = (\alpha^2)^{1001} = \alpha^{2002} = \alpha^{5\times400+2} = \alpha^2$

$\mu = (\alpha_4)^{669+1/3} = (\alpha^3)^{(669+1/3)} = \alpha^{2008} = \alpha^3$

$\nu = (\alpha_5)^{503+1/2} = (\alpha^4)^{503+1/2}$

$= \alpha^{2014}. = \alpha^{5.402+4}$

$= \alpha^4$

Also sum of 2011[th] power of roots of unity is 0

So, $1 + \alpha^{2011} + \lambda^{2011} + \mu^{2011} + v^{2011} = 0$

$\lambda^{2011} + \mu^{2011} + v^{2011} = -(1 + \alpha^{2011})$

$\lambda^{2011} + \mu^{2011} + v^{2011} = -(1 + \alpha)$

$|\lambda^{2011} + \mu^{2011} + v^{2011}| = |-(1 + e^{i2\pi/5})|$

$= |1 + \cos 2\pi/5 + i \sin 2\pi/5| = |2\cos\pi/5(\cos\pi/5 + i \sin \pi/5)$

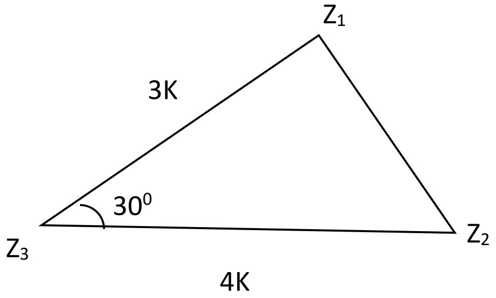
$= 2\dfrac{\sqrt{5}+1}{4} = \dfrac{\sqrt{5}+1}{2}$

$[|\lambda^{2011} + \mu^{2011} + v^{2011}|] = 1$

18. If $\left|Z_1 - Z_2\right| = \sqrt{25 - 12\sqrt{3}}$, and $\dfrac{Z_1 - Z_3}{Z_2 - Z_3} = \dfrac{3}{4} e^{i\frac{\pi}{6}}$, then area of triangle (in square units) whose vertices are represented by Z_1, Z_2, Z_3 is ……………….

Key: 3

Hint:



$$\frac{|Z_1 - Z_3|}{|Z_2 - Z_3|} = \frac{3}{4}$$

$$let\, |Z_1 - Z_3| = 3k, |Z_2 - Z_3| = 4k$$

$$angle\, at\, Z_3 = \frac{\pi}{6}$$

$$cos\, 30^0 = \frac{16k^2 + 9k^2 - 25 + 12\sqrt{3}}{2 \times 4k \times 3k} \Rightarrow k = 1$$

$$area = \frac{1}{2}.3.4 \sin 30^0 = 3$$

19. Two lines $zi - \overline{z}i + 2 = 0$ and $z(1+i) + \overline{z}(1-i) + 2 = 0$ intersect at a point P. There is a complex number $\alpha = x + iy$ at a distance of 2 units from the point P which lies on line $z(1+i) + \overline{z}(1-i) + 2 = 0$. Find $[|x|]$ (where [.] represents greatest integer function).

Key: 1

Hint: Solving the equation of the lines we get $z = -\overline{z} \Rightarrow z = i$

$|\alpha - 1| = 2; \alpha = 2e^{i\theta} + i$, put it in the equation of the second line, we get

$$\cos\theta - \sin\theta = 0$$

$$\alpha = i \pm 2e^{\frac{i\pi}{4}}$$

$$\therefore x = \pm\sqrt{2}$$

$$\Rightarrow [|x|] = 1$$

20. If $\alpha = e^{i2\pi/7}$ and $f(x) = A_0 + \sum_{k=1}^{20} A_k x^k$ and the value of $f(x) + f(\alpha x) + f(\alpha^2 x) + ... + f(\alpha^6 x)$ is $k(A_0 + A_7 x^7 + A_{14} x^{14})$ then find the value of k.

Key: 7

Hint: $f(x) + f(\alpha x) + f(\alpha^2 x) + ... + f(\alpha^6 x) = 7A_0 + \sum_{k=1}^{20} A_k x^k (1 + \alpha^k + ... + \alpha^{6k})$

but when $k \ne 7$ and $k \ne 14$, then $1 + \alpha^k + \alpha^{2k} + ... + \alpha^{6k} = 0$

Hence

$$f(x) + f(\alpha x) + ... + f(\alpha^6 x) = 7A_0 + 7A_7 x^7 + 7A_{14} x^{14} = 7(A_0 + A_7 x^7 + A_{14} x^{14})$$

k = 7

21. If $Z_n = \left(\cos\left(\frac{\pi}{n(n+1)(n+2)}\right) + i\sin\left(\frac{\pi}{n(n+1)(n+2)}\right)\right)$ for n = 1,2,3…. and the principle argument value of $z = \lim_{n\to\infty}(z_1 z_2 ...... z_n)$ is $\frac{k\pi}{24}$, then find the value of k

Key: 6

Hint: $z_n = \frac{i\pi}{e^{n(n+1)(n+2)}}$

$$\Rightarrow z = \lim_{n\to\infty} e^{\left(i\pi \sum_{n=1}^{n} \frac{1}{n(n+1)(n+2)}\right)}$$

$$\Rightarrow z = e^{\frac{i\pi}{4}} \Rightarrow \arg(z) = \frac{\pi}{4}$$

22. Suppose that w is the imaginary $(2009)^{th}$ roots of unity. If $\left(2^{2009}-1\right)\sum_{r=1}^{2008}\frac{1}{2-w^r} = (a)\left(2^b\right)+c$ where a, b, c, $\in$ N, and the least value of (a + b + c) is (2008)K. The numerical value of K is

Key: 2

Hint: Let x be the $(2009)^{th}$ root of unity $\neq 1$, then

$$x^{2009}-1 = (x-1)(x-w).....\left(x-w^{2008}\right)$$

Taking log on both sides, we get

$$\ln\left(x^{2009}-1\right) = \ln(x-1)+\ln(x-w)+\ln\left(x-w^2\right).....+\ln\left(x-w^{2008}\right)$$

$\therefore$ On differentiate both the side w.r.t. x, we get

$$\frac{(2009)x^{2008}}{x^{2009}-1} = \frac{1}{x-1}+\sum_{r=1}^{2008}\frac{1}{x-w^r} ....(2)$$

Putting x = 2 in equation (2) , we get

$$\Rightarrow 1+\sum_{r=1}^{2008}\frac{1}{2-w^r} = \frac{2009\left(2^{2008}\right)}{2^{2009}-1}$$

Multiplying both sides of above equation by $\left(2^{2009}-1\right)$, we get

$$\therefore \left(2^{2009}-1\right)\sum_{r=1}^{2008}\frac{1}{2-w^r} = 2009.2^{2008}-2^{2009}+1$$

$$= 2^{2008}(2009-2)+1 = 2^{2008}.2007+1 = \left[(a)\left(2^b\right)+c\right]$$

$$\therefore a = 2007, b = 2008, c = 1$$

Hence a + b + c = 4016

23. If $\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5$ are the roots of the equation $x^5-1=0$, where $\alpha_k = \alpha^{k-1}$, $\alpha = e^{i2\pi/5}$ and $\lambda = \alpha_3^{1001}, \mu = \alpha_4^{(669+1/3)}$, $\nu = \alpha_5^{(503+1/2)}$, then $[|\lambda^{2011}+\mu^{2011}+v^{2011}|]$ (where [.] denotes the greatest integer function) is

Key. 1

Sol. Clearly $\alpha_1 = 1$

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$= \alpha^{2014}. = \alpha^{5.402+4}$

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Also sum of 2011th power of roots of unity is 0

So, $1 + \alpha^{2011} + \lambda^{2011} + \mu^{2011} + v^{2011} = 0$

$\lambda^{2011} + \mu^{2011} + v^{2011} = -(1 + \alpha^{2011})$

$\lambda^{2011} + \mu^{2011} + v^{2011} = -(1 + \alpha)$

$|\lambda^{2011} + \mu^{2011} + v^{2011}| = |-(1 + e^{i2\pi/5})|$

= |1 + cos2π/5 + i sin 2π/5| = |2cosπ/5(cosπ/5 + i sin π/5)

$= 2\dfrac{\sqrt{5}+1}{4} = \dfrac{\sqrt{5}+1}{2}$

$[|\lambda^{2011} + \mu^{2011} + v^{2011}|] = 1$

24. Find the least +ve integral value of 'a' such that there is at least one complex number satisfying $\left|z+\sqrt{2}\right| < a^2 - 3a + 2$ and $\left|z + i\sqrt{2}\right| < a^2$

Key. 3

Sol. (a=3) Atleast one complex number z satisfy the required condition if the two circle intersect at two distinct points.

25. A triangle with vertices represented by z_1, z_2, z_3 has opposite sides of lengths in the ratio $2 : \sqrt{19} : 3$ respectively. Then the value of $4\left(z_1 - z_2\right)^2 + 6\left(z_1 - z_2\right)\left(z_3 - z_2\right) + 9\left(z_3 - z_2\right)^2$ is k. find k.

Key. 0

Sol. $\left(k=0\right) \cos B = -\dfrac{1}{2} \ B = \dfrac{2\pi}{3}$

By rotation $\dfrac{z_1 - z_2}{z_2 - z_2} = \left|\dfrac{z_1 - z_2}{z_3 - z_2}\right| e^{i\frac{2\pi}{3}}$

$$\Rightarrow 2\left(z_1 - z_2\right) + \frac{3}{2}\left(z_3 - z_2\right) = \left(z_3 - z_2\left(i\,\frac{3\sqrt{3}}{2}\right)\right)$$

Squaring to get the required result.

26. Let $1, w, w^2$ be the cube root of unity. The least possible degree of a polynomial with real coefficients having roots $2w, (2+3w), (2+3w^2), (2-w-w^2)$, is

Key. 5

Sol. Roots are $2w, (2+3w)(2+3w^2)(2-w-w^2)(2+3w)$ and $2+3w^2$ are conjugate each other 2w is complex root, then other root must be $2w^2$ (as conjugate root occur in conjugate pair)

$2-w-w^2 = 2-(-1) = 3$ which is real.

Hence least degree of the polynomial : 5.

27. If $2^7\cos^3\theta\cdot\sin^5\theta = a\sin 8\theta 0 b\sin 6\theta + c\sin 4\theta + d\sin 2\theta$ and θ is real then the value of a + b + c + d must be equal to

Ans. 7

Sol. Let $z = e^{i\theta}\cdot 2\cos\theta = \left(z+\frac{1}{z}\right)$ and $2i\sin\theta = \left(z-\frac{1}{z}\right)$

Now : $(2\cos\theta)^3(2i\sin\theta)^5 = \left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^5$

$= \left(z^8-\frac{1}{z^8}\right) - 2\left(z^6-\frac{1}{z^6}\right) - 2\left(z^4-\frac{1}{z^4}\right) + 6\left(z^2-\frac{1}{z^2}\right)$

Compare a = 1, b = 2, c = - 2, d = 6

a + b + c + d = 1 + 2 – 2 + 6 = 7

28. Let $A_1, A_2 \ldots A_n$ be vertices of an n sided regular polygon such that $\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$.

Then find values of n

Ans. 7

Sol. Let $A_1(Z_1), A_2(Z_2), A_3(Z_3), A_4(Z_4)$

$A_1A_2 = |Z_1|2\sin\frac{\pi}{n},\ A_1A_3 = |Z_1|2\sin\frac{2\pi}{n},\ A_1A_4 = |Z_1|2\sin\frac{3\pi}{n}$

$\Rightarrow \frac{1}{\sin\frac{\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}} + \frac{1}{\sin\frac{3\pi}{n}} \Rightarrow n = 7$

29. Let $A_1(Z_1), A_2(Z_2)$ be the adjacent vertices of a regular polygon. If $\frac{lm(\overline{Z}_1)}{\mathrm{Re}(Z_1)} = 1-\sqrt{2}$, then the number of sides of the polygon are ________

Ans. 8

Sol. Let $z_1 = re^{i\theta},\ \overline{Z_1}^{-i\theta} = re^{-i\theta},\ \mathrm{Re}(z_1) = r\cos\theta,\ lm(\overline{Z}_1) = -r\sin\theta$

$\Rightarrow -\frac{\sin\theta}{\cos\theta} = 1-\sqrt{2} \Rightarrow \tan\theta = \sqrt{2}-1 \Rightarrow \theta = \frac{\pi}{8}$

If the number of sides be n, then $\theta = \frac{\pi}{n} \Rightarrow n = 8$