Circles

Single Correct Answer Type

1. Let $C_1 : x^2 + y^2 = 1$; C_2: $(x - 10)^2 + y^2 = 1$ and C_3: $x^2 + y^2 - 10x - 42y + 457 = 0$ be three circles. A circle C has been drawn to touch circles C_1 and C_2 externally and C_3 internally. Now circles C_1, C_2 and C_3 start rolling on the circumference of circle C in anticlockwise direction with constant speed. The centroid of the triangle formed by joining the centres of rolling circles C_1, C_2 and C_3 lies on

(A) $x^2 + y^2 - 12x - 22y + 144 = 0$ (B) $x^2 + y^2 - 10x - 24y + 144 = 0$

(C) $x^2 + y^2 - 8x - 20y + 64 = 0$ (D) $x^2 + y^2 - 4x - 2y - 4 = 0$

Key. B

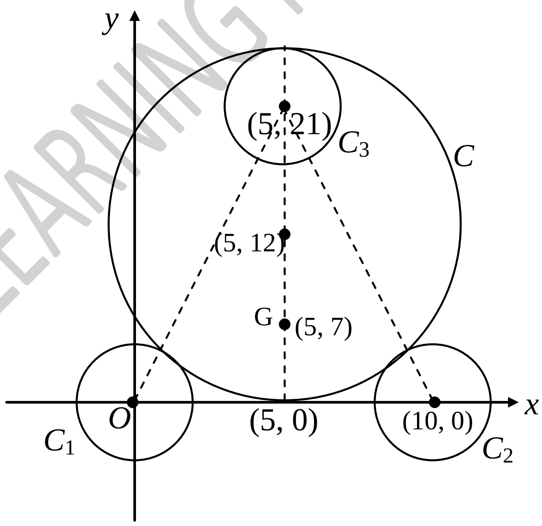
Sol.

The equation of circle C is

$$(x - 5)^2 + (y - 12)^2 = 12^2$$

This circle also touches x–axis at (5, 0).

From the geometry, centroid lies on the circle $(x - 5)^2 + (y - 12)^2 = 5^2$.



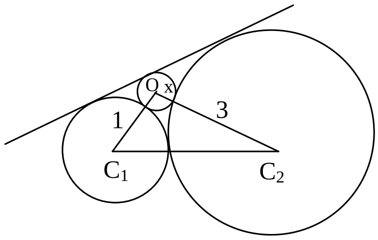
2. The circles $x^2 + y^2 - 6x + 6y + 17 = 0$ and $x^2 + y^2 - 6x - 2y + 1 = 0$, a common exterior tangent is drawn thus forming a curvilinear triangle. The radius of the circle inscribed in this triangle is

(A) $\frac{3}{2}\left(2+\sqrt{3}\right)$ (B) $\frac{1}{2}\left(2-\sqrt{3}\right)$ (C) $\frac{1}{2}\left(2+\sqrt{3}\right)$ (D) $\frac{3}{2}\left(2-\sqrt{3}\right)$

Key. D

Sol. The given circles are touching each other externally.

$$x = \frac{3}{(1+\sqrt{3})^2} = \frac{3}{2}\left(2-\sqrt{3}\right)$$



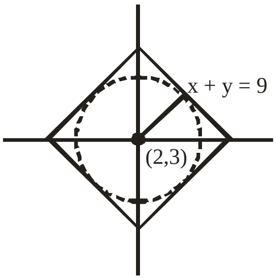
3. Equation of circle touching the line |x – 2| + |y – 3| = 4 will be

(A) $(x-2)^2 + (y-3)^2 = 12$ (B) $(x-2)^2 + (y-3)^2 = 4$

(C) $(x-2)^2 + (y-3)^2 = 10$ (D) $(x-2)^2 + (y-3)^2 = 8$

Key. D

Sol. PERPENDICULAR distance from centre to tangent = radius



$$r = \frac{|2+3-9|}{\sqrt{2}} = \frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

Equation of circle is $(x-2)^2 + (y-3)^2 = 8$

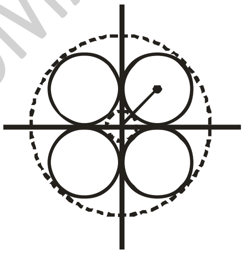
4. The equation of four circles are $(x \pm a)^2 + (y \pm a)^2 = a^2$. The radius of a circle touching all the four circles is

(A) $(\sqrt{2}-1)a$ or $(\sqrt{2}+1)a$ (B) $\sqrt{2}a$ or $2\sqrt{2}a$

(C) $(2-\sqrt{2})a$ or $(2+\sqrt{2})a$ (D) None of these

Key. A

Sol. Radius of smallest circle is



$$r + a = a\sqrt{2}$$
$$r = a\sqrt{2} - a$$

Another circle $\Rightarrow$ $r = a\sqrt{2} + a$

5. If two distinct chords, drawn from the point (p, q) on the circle $x^2+y^2=px+qy$ (where $(pq \neq 0)$ are bisected by the x-axis, then

A. $p^2=q^2$ B. $p^2=8q^2$ C. $p^2<8q^2$ D. $p^2>8q^2$

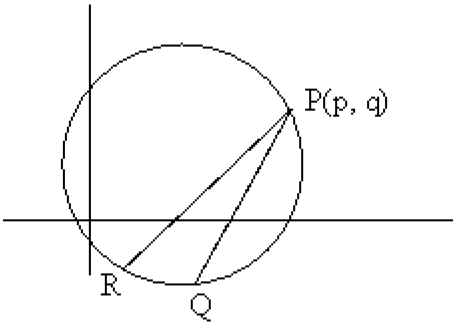
Key. D

Sol. Let PQ be a chord of the given circle passing through P(p, q) ad the coordinates of Q be (x, y). Since PQ is bisected by the x-axis, the mid-point of PQ lies on the x-axis which gives y = -q

Now Q lies on the circle $x^2+y^2px-qy=0$

So $x^2+q^2-px+q^2=0$

$\Rightarrow x^2-px+2q^2=0$



Which gives two values of x and hence the coordinates of two points Q and R (say), so that the chords PQ and PR are bisected by x-axis. If the chords PQ and PR are distinct, the roots of (i) are real distinct.

$\Rightarrow$ the discriminant $p^2-8q^2>0 \Rightarrow p^2>8q^2$

6. C_1 and C_2 are circles of unit radius with centres at (0, 0) and (1, 0) respectively. C_3 is a circle of unit radius, passes through the centres of the circles C_1 and C_2 and have its centre above x-axis. Equation of the common tangent to C_1 and C_3 which does not pass through C_2 is

A. $x-\sqrt{3}y+2=0$ B. $\sqrt{3}x-y+2=0$ C. $\sqrt{3}x-y-2=0$ D. $x+\sqrt{3}y+2=0$

Key. B

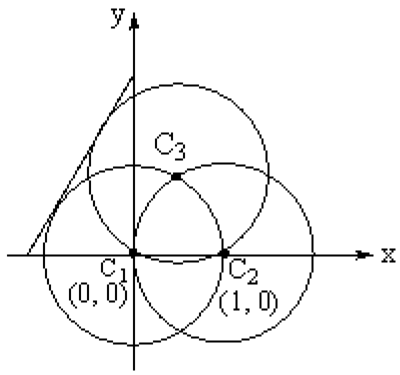
Sol. Equation of any circle through (0, 0) and (1, 0)

$$(x-0)(x-1)+(y-0)(y-0)+\lambda\begin{vmatrix} x & y & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix}=0$$

$\Rightarrow x^2+y^2-x+\lambda y=0$

If it represents C_3, its radius =1

$\Rightarrow 1=(1/4)+(\lambda^2/4) \Rightarrow \lambda=\pm\sqrt{3}$



As the centre of C_3, lies above the x-axis, we take $\lambda = -\sqrt{3}$ and thus an equation of C_3 is $x^2 + y^2 - x - \sqrt{3}y = 0$. Since C_1 and C_2 interest and are of unit radius, their common tangents are parallel to the joining their centres (0, 0) and $(1/2, \sqrt{3}/2)$.

So, let the equation of a common tangents be $\sqrt{3}x - y + 2 = 0$

It will touch C_1, if $\left|\frac{k}{\sqrt{3+1}}\right| = 1 \Rightarrow k = \pm 2$

From the figure, we observe that the required tangent makes positive intercept on the y-axis and negative on the x-axis and hence its equation to $\sqrt{3}x - y + 2 = 0$

7. The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of distinct non parallel lines. If constant c is changed as new constant k then new equation represents

A. Pair of lines B. Parabola C. Ellipse D. Hyperbola

Key. D

Sol. Since c is changed as k, $\Delta \neq 0$ and $h^2 > ab$

$\therefore$ new equation represents hyperbola

8. A circle of radius '5' touches the coordinate axes in the first quadrant. If the circle makes one complete roll on x-axis along the positive direction, then its equation in new position is

1) $x^2 + y^2 - 10(2\pi + 1)x - 10y + 100\pi^2 + 100\pi + 25 = 0$

2) $x^2 + y^2 + 10(2\pi + 1)x - 10y + 100\pi^2 + 100\pi + 25 = 0$

3) $x^2 + y^2 - 10(2\pi + 1)x + 10y + 100\pi^2 + 100\pi + 25 = 0$

4) $x^2 + y^2 + 10(2\pi + 1)x + 10y + 100\pi^2 + 100\pi + 25 = 0$

Key. 1

Sol. $c = (5,5)$ and $(5 + 10\pi, 5)$

$(x - 5 - 10\pi)^2 + (y - 5)^2 = 5^2$

9. From origin, chords are drawn to the circle $x^2+y^2-2y=0$. The locus of the middle points of these chords is

1) $x^2+y^2-y=0$
2) $x^2+y^2-x=0$
3) $x^2+y^2-2x=0$
4) $x^2+y^2-x-y=0$

Key. 1

Sol. $T=S_1$

i.e., $xx_1+yy_1-(y+y_1)=x_1^2+y_1^2-2y_1$

Passes through $(0,0)$

$\therefore x^2+y^2-y=0$

10. Circles are drawn through the point (2,0) to cut intercept of length '5' units on the x-axis. If their centres lie in the first quadrant then their equation is

1) $x^2+y^2-9x+2ky+14=0, k>0$
2) $3x^2+3y^2+27x-2ky+42=0, k>0$
3) $x^2+y^2-9x-2ky+14=0, k>0$
4) $x^2+y^2-2ky-9y+14=0, k>0$

Key. 3

Sol. $c=\left(\frac{9}{2},k\right)$

$\left(x-\frac{9}{2}\right)^2+(y-k)^2=\frac{25}{4}+k^2$

$(or)x^2+y^2-9x-2ky+14=0$

11. A line meets the coordinate axes in A and B. If a circle is circumscribed about the ΔAOB. If m, n are the distances of the tangent to the circle at the origin from the points A and B respectively, The diameter of the circle is

1) $m(m+n)$
2) $m+n$
3) $n(m+n)$
4) $2(m+n)$

Key. 2

Sol. $m=A(a,o)on(1)=\frac{a^2}{\sqrt{a^2+b^2}}$

$n=(o,b)\, on\ (1)=\frac{b^2}{\sqrt{a^2+b^2}}$

$d=\sqrt{a^2+b^2}=m+n$

12. The equation of the circle passing through the point $(2,-1)$ and having two diameters along the pair of lines $2x^2+6y^2-x+y-7xy-1=0$ is

1) $x^2+y^2+10x+6y-19=0$
2) $x^2+y^2+10x-6y+19=0$
3) $x^2+y^2+10x+6y+19=0$
4) $x^2+y^2-10x+6y+19=0$

Key. 1

Sol. $2x^2+6y^2-x+y-7xy-1=0$

$x--2y-1=0\ and\ \rightarrow(1)$

$2x-3y+1=0\rightarrow(2)$

$Cetre\ (-5,-3)$

$\therefore x^2+y^2+10x+6y-19=0$

13. If from any point on the circle $x^2+y^2=a^2$, tangents are drawn to the circle $x^2+y^2=b^2(a>b)$ then the angle between tangents is

1) $\sin^{-1}\left(b/a\right)$
2) $2\sin^{-1}\left(a/b\right)$
3) $2\sin^{-1}\left(b/a\right)$
4) $\sin^{-1}\left(a/b\right)$

Key. 3

Sol. $\sin\theta=\frac{b}{a}\Rightarrow\theta=\sin^{-1}\frac{b}{a}$

$Angle\ between\ the\ 2\theta=2\sin^{-1}\frac{b}{a}$

14. An equilateral triangle has two vertices $(-2,0)$ and $(2,0)$ and its third vertex lies below the x-axis, The equation of the circumcircle of the triangle is

1) $\sqrt{3}\left(x^2+y^2\right)-4y+4\sqrt{3}=0$
2) $\sqrt{3}\left(x^2+y^2\right)-4y-4\sqrt{3}=0$
3) $\sqrt{3}\left(x^2+y^2\right)+4y+4\sqrt{3}=0$
4) $\sqrt{3}\left(x^2+y^2\right)+4y-4\sqrt{3}=0$

Key. 4

Sol. $Vertex\ A\left(0,-\sqrt{12}\right)$

$Centroid\ G\left(0,\frac{-2}{\sqrt{3}}\right)$

$Circum\ \ radius=\sqrt{4+\frac{4}{3}}=\frac{4}{\sqrt{3}}$

$\therefore \sqrt{3}\left(x^2+y^2\right)+4y-4\sqrt{3=0}$

15. The coordinates of two points on the circle $x^2+y^2-12x-16y+75=0$, one nearest to the origin and the other farthest from it, are

1) $(3,4),(9,12)$
2) $(3,2),(9,12)$
3) $(-3,4),(9,12)$
4) $(3,4),(9,-12)$

Key. 1

Sol.

$c=(6,8),\ \ radius=5=AC$

$oc=\sqrt{36+64}=10$

$OA=5$

$Q\,OA:AC=5:5=1:1$

$A\ is\ midpo\text{int}\ 07\ OC$

$i.e.,\ (3:4)$

$Coordinate\ B\ be\ (h,k)$

$'c'\ is\ the\ midpo\text{int}\ 07\ AB$

$\therefore h=9,\ k=12$

$\therefore\ \ B(9,12)$

16. Two distinct chords drawn from the point (p,q) on the circle $x^2+y^2=px+qy$, where $pq\neq 0$, are bisected by the x-axis. Then

1) $|p|=|q|$
2) $p^2=8q^2$
3) $p^2<8q^2$
4) $p^2>8q^2$

Key. 4

Sol. $y=-q\ and\ \ x^2+y^2-px+qy=0$

Disc > 0

17. The centre of a circle of radius $4\sqrt{5}$ lies on the line $y = x$ and satisfies the inequality $3x + 6y > 10$. If the line $x + 2y = 3$ is a tangent to the circle, then the equation of the circle is

1) $\left(x-\frac{23}{3}\right)^2+\left(y-\frac{23}{3}\right)^2=80$ 2) $\left(x+\frac{17}{3}\right)^2+\left(y+\frac{17}{3}\right)^2=80$

3) $\left(x+\frac{23}{3}\right)^2+\left(y-\frac{23}{3}\right)^2=80$ 4) $\left(x-\frac{17}{3}\right)^2+\left(y-\frac{17}{3}\right)^2=80$

Key. 1

Sol. $c=(a,a)$

$radius = 4\sqrt{5} = length of the \perp\ from (a,a) to the line$

$i.e., \frac{|a+2(a)-3|}{\sqrt{4+1}} = \pm 4\sqrt{5} \Rightarrow a = \frac{23}{3}, \frac{-17}{3}$

$\therefore Centre \left(\frac{23}{3}, \frac{23}{5}\right) or \left(\frac{-17}{3}, \frac{-17}{3}\right)$

$3x+6y>10$

$C=\left(\frac{23}{3},\frac{23}{3}\right)$

$\therefore \left(x-\frac{23}{3}\right)^2+\left(y-\frac{23}{3}\right)^2 = 80$

18. The equation to the circle which is such that the lengths of the tangents to it from the points (1,0), (2,0) and (3,2) are $1, \sqrt{7}, \sqrt{2}$ respectively is

1) $2x^2+2y^2+6x+17y+6=0$ 2) $2x^2+2y^2+6x-17y-6=0$

3) $x^2+y^2+6x+15y+5=0$ 4) $x^2+y^2+6x-15y-5=0$

Key. 2

Sol. Let S=0 be the required circle

Apply $\sqrt{S_{11}}$

19. If the equations of four circles are $(x\pm 4)^2+(y\pm 4)^2=4^2$ then the radius of the smallest circle touching all the four circles is

1) $4\left(\sqrt{2}+1\right)$ 2) $4\left(\sqrt{2}-1\right)$ 3) $2\left(\sqrt{2}-1\right)$ 4) $\sqrt{2}-1$

Key. 2

Sol. $r=\sqrt{4^2+4^2}-4$ $i.e., 4\sqrt{2}-4$.

20. Let L_1 be a straight line passing through the origin and L_2 be the straight line x+y=1. If the intercepts made by the circles $x^2+y^2-x+3y=0$ on L_1 and L_2 are equal then which of the following equations can represent L_1?

1) $x+y=0$ 2) $x-y=0$

3) $7x-y=0$ 4) $x-7y=0$

Key. 2

Sol. $c\left(\frac{1}{2},\frac{-3}{2}\right)$

$$\frac{\left|\frac{1}{2}-\frac{3}{2}^{-1}\right|}{\sqrt{1+1}}=\frac{\left|\frac{m}{2}-\frac{3}{2}\right|}{\sqrt{1+m^2}}\Rightarrow m=1,\frac{-1}{7}$$

two chords are $y=x$ *and* $7x+y=0$

21. Let $A_0A_1A_2A_3A_4A_5$ be a regular hexogon inscribed in a circle of unit radius. Then the product of the lengths of the line segments A_0A_1, A_0A_2 and A_0A_4 is

1) $\frac{3}{4}$ 2) $3\sqrt{3}$ 3) 3 4) $\frac{3\sqrt{3}}{2}$

Key. 3

Sol. $A_0A_1=1$

$A_0A_1=\sqrt{3}$

Similarly, $A_0\ A_4=\sqrt{3}$

$\therefore (A_0A_1)\ (A_0A_2)\ (A_0A_4)=3$

22. The ΔPQR is inscribed in the circle $x^2+y^2=25$. If Q and R have coordinates (3,4) and (-4,3) respectively, then $\underline{|QPR}$ is equal to

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{6}$

Key. 3

Sol. $m_1=$ *slope of* $OP=\frac{4}{3}$ *and* m_2 *slope of* $OQ=\frac{-3}{4}$

$\Rightarrow m_1\ m_2=-1$

$\angle QOP=\Pi/2$

Thus $\angle QPR=\Pi/4$

23. A variable circle passes through the fixed point A(p,q) and touches x-axis. The locus of the other end of the diameter through A is

1) $(y-p)^2 = 4qx$ 2) $(x-q)^2 = 4py$

3) $(x-p)^2 = 4qy$ 4) $(y-q)^2 = 4px$

Key. 3

Sol. $(x-p)(x-\alpha)+(y-g)(y-\beta)=0 \quad (or)$

$x^2+y^2-(p+\alpha)x-(g+\beta)y+p\alpha+g\beta=0 \to (1)$

$Put\ y=0,\ we\ get\ x^2-(p+\alpha)x+p\alpha+g\beta=0 \to (2)$

$\therefore \Rightarrow Locus\ of\ B\ (\alpha,\beta)\ is\ (p-x)^2=4gy$

$(x-p)^2=4gy$

24. The locus of the mid point of the chord of the circle $x^2+y^2-2x-2y-2=0$ which makes an angle of 120^0 at the centre is

1) $x^2+y^2-2x-2y+1=0$ 2) $x^2+y^2+x+y+1=0$

3) $x^2+y^2-2x-2y-1=0$ 4) $x^2+y^2+x-y-1=0$

Key. 1

Sol. $Centre\ (1,1)\ and\ radius\ = 2 = OB$

$In\ \nabla OBP\ = 30^o$

$\therefore \sin 30^o\ \frac{op}{2}\ or\ op=1$

$\sin ce\ op=1$

$\Rightarrow x^2+y^2-2x-2y+1=0$

25. The chord of contact of tangents from a point 'P' to a circle passes through Q. If l_1 and l_2 are the lengths of the tangents from P and Q to the circle, then PQ is equal to

1) $\frac{l_1+l_2}{2}$ 2) $\frac{l_1-l_2}{2}$

3) $\sqrt{l_1^2+l_2^2}$ 4) $\sqrt{l_1^2-l_2^2}$

Key. 3

Sol. $P=(x_1,y_1)\ and\ Q=(x_2,y_2)$

$p=(x_1,y_1)\ to\ the\ given\ circle\ is\ xx_1+yy_1=a^2$

$Since\ it\ passes\ through\ Q\ (x_2,y_2)$

$\therefore xx_1+yy_1=a^2 \to (1)$

$Now,\ l_1=\sqrt{x_1^2+y_1^2-a^2},\ l_2=\sqrt{x_2^2+y_2^2-a^2}$

$and \;\; PQ = \sqrt{l_1^2 + l_2^2}$

26. If a chord of a the circle $x^2 + y^2 = 32$ makes equal intercepts of length l on the Co-ordinate axes, then

1) $|l| < 8$
2) $|l| < 16$
3) $|l| > 8$
4) $|l| > 16$

Key. 1

Sol. $Centre \; (0,0),$

$radius \left|\frac{l}{\sqrt{2}}\right| < \sqrt{32} \Rightarrow |l| < 8$

27. If the chord of contact of tangents from 3 points A,B,C to the circle $x^2 + y^2 = a^2$ are concurrent, then A,B,C will

1) be concyclic
2) Be collinear
3) Form the vertices of triangle
4) None of these

Key. 2

Sol. $xx_1 + yy_1 = a^2, \; xx_2 + yy_2 = a^2$

$and \;\; xx_3 + yy_3 = a^2$

$These \; lines \; will \; be \; conurrent$

$$\begin{vmatrix} x_1 & y_1 & -a^2 \\ x_2 & y_2 & -a^2 \\ x_3 & y_3 & -a^2 \end{vmatrix} = 0 \quad \begin{vmatrix} x_1 & y_1 & -1 \\ x_2 & y_2 & -1 \\ x_3 & y_3 & -1 \end{vmatrix} = 0$$

$Which \;\; is \; the \;\; condition \;\; to \; the \;\; collinearity \;\; of \;\; A, B, C.$

28. If the line passing through P=(8,3) meets the circle $S \equiv x^2 + y^2 - 8x - 10y + 26 = 0$ at A,B then PA.PB=

1) 5
2) 14
3) 4
4) 24

Key. 1

Sol. $PA.PB = | S_{11} |$

29. (a, b) is the mid point of the chord $\overline{AB}$ of the circle $x^2 + y^2 = r^2$. The tangent at A, B meet at C. then area of $\Delta ABC =$

1) $\frac{\left(a^2+b^2+r^2\right)^{\frac{3}{2}}}{\sqrt{a^2+b^2}}$

2) $\frac{\left(r^2-a^2-b^2\right)^{\frac{3}{2}}}{\sqrt{a^2+b^2}}$

3) $\frac{\left(a^2-b^2-r^2\right)^{\frac{3}{2}}}{\sqrt{a^2+b^2}}$

4) $\frac{\left(a^2-b^2+r^2\right)^{\frac{3}{2}}}{\sqrt{a^2+b^2}}$

Key. 2

Sol. Equation of the chord AB having (a,b)

$as\ M.P.\ S_1 = S_{11} \Rightarrow ax + by - (a^2 + b^2) = 0$

$chord\ \ length = 2\sqrt{r^2 - a^2 - b^2}$

$c = \left(\frac{-ar^2}{a^2+b^2}, \frac{br^2}{a^2+b^2}\right)$

$h = \frac{r^2 - a^2 - b^2}{\sqrt{a^2+b^2}}$

Area = ½ x b x h

30. The length and the midpoint of the chord $4x - 3y + 5 = 0$ w.r.t circle $x^2 + y^2 - 2x + 4y - 20 = 0$ is

1) $8, \left(-\frac{7}{5}, -\frac{1}{5}\right)$

2) $18, \left(\frac{7}{5}, \frac{1}{5}\right)$

3) $10, \left(-\frac{17}{5}, -\frac{11}{5}\right)$

4) $28, \left(-\frac{7}{5}, -\frac{8}{5}\right)$

Key. 1

Sol. $2\sqrt{r^2 - d^2} = 8$.

$M.P = \left(-7/5, -1/5\right)$

31. A variable circle passes through the fixed point (2, 0) and touches the y-axis then the locus of its centre is

1) a parabola

2) a circle

3) an ellipse

4) a hyperbola

Key. 1

Sol. $Circle\ (x-x_1)^2+(y-y_1)^2=x_1^2$

$y^2=4(x-1)\ Parabola$

32. If the lengths of the tangents from the point (1, 2) to the circles $x^2+y^2+x+y-4=0$ and $3x^2+3y^2-x-y-\lambda=0$ are in the ratio 4 : 3 then $\lambda=$

1) $\frac{23}{4}$ 2) $\frac{21}{4}$ 3) $\frac{17}{4}$ 4) $\frac{19}{4}$

Key. 2

Sol. $\frac{\sqrt{s_{11}}}{\sqrt{s_{11}^1}}=4/3$

33. If a tangent drawn from the point (4, 0) to the circle $x^2+y^2=8$ touches it at a point A in the first quadrant, then the coordinates of another point B on the circle such that AB = 4 are

1) $(2,-2)\ or\ (-2,2)$ 2) $(1,-2)\ or\ (-2,1)$

3) $(-1,1)\ or\ (1,-1)$ 4) $(3,-2)\ or\ (-3,2)$

Key. 1

Sol. equation of tangent through (4,0)

x+y-4=0

Point of contact = (2,2)

$AB=4\ \Rightarrow\ B=(2+4\cos\theta,\ 2+4\sin\theta)$

$\theta=\pi,\ \ \theta=\frac{3\pi}{2}$

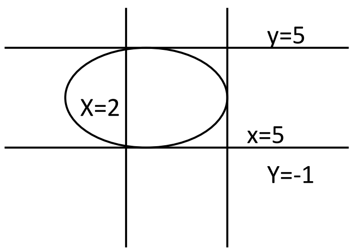
$(-2,2)\ \ (2,-2)$

34. The number of points common to the circle $x^2+y^2-4x-4y=1$ and to the sides of the rectangle formed by $x=2, x=5, y=-1,\ and\ y=5$ is

1) 5 2) 1 3) 2 4) 3

Key. 4

Sol. (3) points



35. A rectangle ABCD is inscribed in a circle with a diameter lying along the line $3y = x + 10$. If $A = (-6, 7),\ B = (4, 7)$ then the area of rectangle is

1) 80 2) 40 3) 160 4) 20

Key. 1

Sol. Area $= \pi r^2$

$r = \frac{\sqrt{17}}{4}$

36. Let ABCD be a quadrilateral with area 18, with side AB parallel to CD and AB = 2CD. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral ABCD touching all the sides, then its radius is

1) 3 2) 2 3) $\frac{3}{2}$ 4) 1

Key. 2

Sol. A(0,0); B(2a,o); C(a, 2r); D(0, 2r)
Equation of ABCD = ½ (2a+a) x 2r = 18
r=2

37. If OA and OB are two equal chords of the circle $x^2 + y^2 - 2x + 4y = 0$ perpendicular to each other and passing through the origin O, the slopes of OA and OB are the roots of the equation

1) $3m^2 + 8m - 3 = 0$ 2) $3m^2 - 8m - 3 = 0$

3) $8m^2 - 3m - 8 = 0$ 4) $8m^2 + 3m - 8 = 0$

Key. 2

Sol. equation of chords y-mx = 0
my + x = 0

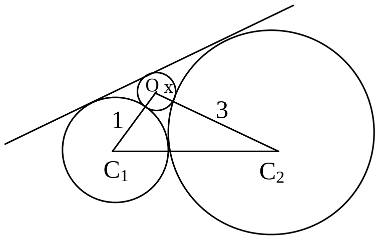
38. The circles $x^2 + y^2 - 6x + 6y + 17 = 0$ and $x^2 + y^2 - 6x - 2y + 1 = 0$, a common exterior tangent is drawn thus forming a curvilinear triangle. The radius of the circle inscribed in this triangle is

(A) $\frac{3}{2}\left(2+\sqrt{3}\right)$ (B) $\frac{1}{2}\left(2-\sqrt{3}\right)$ (C) $\frac{1}{2}\left(2+\sqrt{3}\right)$ (D) $\frac{3}{2}\left(2-\sqrt{3}\right)$

Key. D

Sol. The given circles are touching each other externally.

x = $\frac{3}{(1+\sqrt{3})^2} = \frac{3}{2}\left(2-\sqrt{3}\right)$



39.

40.

41. ABCD is a square of side 1 unit. A circle passes through vertices A,B of the square and the remaining two vertices of the square lie out side the circle. The length of the tangent drawn to the circle from vertex D is 2 units. The radius of the circle is

A) $\sqrt{5}$ B) $\frac{1}{2}\sqrt{10}$ C) $\frac{1}{3}\sqrt{12}$ D) $\sqrt{8}$

Key. B

Sol. Let $A=(0,1)$, $B=(0,0)$, $C=(1,0)$, $D=(1,1)$

Family of circles passing through A,B is $x^2+y^2-y+\lambda x=0$ $\sqrt{1+\lambda}=2 \Rightarrow \lambda=3$

42. The equation of circum-circle of a ΔABC is $x^2+y^2+3x+y-6=0$.

If $A=(1,-2)$, $B=(-3,2)$ and the vertex C varies then the locus of ortho-centre of ΔABC is a

A) Straight line B) Circle C) Parabola D) Ellipse

Key. B

Sol. Equation of circum-circle is $\left(x+\frac{3}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=\frac{17}{2}$

$C=\left(\frac{-3}{2}+\sqrt{\frac{17}{2}}\cos\theta, \frac{-1}{2}+\sqrt{\frac{17}{2}}\sin\theta\right)$

Circum centre of ΔABC is $\left(\frac{-3}{2},\frac{-1}{2}\right)$ Centroid can be obtained.

In a triangle centroid, circum centre and ortho centre are collinear.

43. The line $y = mx$ intersects the circle $x^2 + y^2 - 2x - 2y = 0$ and $x^2 + y^2 + 6x - 8y = 0$ at point A and B (points being other than origin). The range of $'m'$ such that origin divides AB internally is

A) $-1 < m < \frac{3}{4}$ B) $m > \frac{4}{3}$ or $m < -2$ C) $-2 < m < \frac{4}{3}$ D) $m > -1$

Key. A

Sol. The tangents at the origin to C_1 and C_2 are $x + y = 0$, $3x - 4y = 0$ respectively. Slope of the tangents are $-1, \frac{3}{4}$ respectively thus if $-1 < m < \frac{3}{4}$, then origin divides AB internally.

44. The equation of the smallest circle passing through the intersection of $x^2 + y^2 - 2x - 4y - 4 = 0$ and the line $x + y - 4 = 0$ is

(A) $x^2 + y^2 - 3x - 5y - 8 = 0$ (B) $x^2 + y^2 - x - 3y = 0$

(C) $x^2 + y^2 - 3x - 5y = 0$ (D) $x^2 + y^2 - x - 3y - 8 = 0$

Key. C

Sol. Conceptual

45. Three distinct points A, B and C are given in the two-dimensional coordinate plane such that the ratio of the distance of any one of them from (2,-1) to its distance from (-1, 5) is 1 : 2. Then the centre of the circle passing through A,B and C is

a) (1,1) b) (5, - 7)

c) (3,-3) d)(4,-8)

Key: C

Hint The circle ABC is the circle described on the join of (1,1) and (5, -7) as diameter.

46. Point A lies on y = x and point B on y = mx so that length AB = 4 units. Then value of 'm' for which locus of mid point of AB represents a circle is

a) m = 0 b) m = –1 c) m = 2 d) m = –2

Key: B

Hint Let co-ordinates of A(x_1, x_1) and B(x_2, mx_2).

Clearly $(x_1 - x_2)^2 + (x_1 - mx_2)^2 = 16$

Let mid point of P(h, k)

$\Rightarrow$ $x_1 + x_2 = 2h$ and $x_1 + mx_2 = 2k$

$\Rightarrow$ $(x_1 - x_2)^2 + 4x_1x_2 = 4h^2$ and

$$\left(x_1 - mx_2\right)^2 + 4mx_1x_2 = 4k^2$$

$$\left(x_1 - x_2\right)^2 + \left(x_1 - mx_2\right)^2 = 4h^2 + 4k^2 = 16$$

when m = –1

47. Equation of circle inscribed in |x – a| + |y – b| = 1 is

(A) $(x + a)^2 + (y + b)^2 = 2$ (B) $(x – a)^2 + (y – b)^2 = \frac{1}{2}$

(C) $(x – a)^2 + (y – b)^2 = \frac{1}{\sqrt{2}}$ (D) $(x – a)^2 + (y – b)^2 = 1$

KEY : B

HINT: Radius of the required circle is $\frac{1}{\sqrt{2}}$ and centre is (a, b)

Hence equation is $(x – a)^2 + (y – b)^2 = \frac{1}{2}$

48. Let L = 0 be a common normal to the circle $x^2 + y^2 - 2\alpha x - 36 = 0$ and the curve $S : (1+x)^y + e^{xy} = y$ drawn at a point x = 0 on S, then the radius of the circle is

A) 10 B) 5 C) 8 D) 12

Key: A

Hint: at x = 0 y = 2 $y'(0) = 4$

Equation of Normal is $x + 4y = 8$ $(\alpha,\ 0)$ lies on normal $\Rightarrow$ $\alpha = 8$

49. $x^2 + y^2 + 6x + 8y = 0$ and $x^2 + y^2 – 4x – 6y – 12 = 0$ are the equation of the two circles. Equation of one of their common tangent is

(A) $7x – 5y – 1 – 5\sqrt{74} = 0$ (B) $7x – 5y – 1 + 5\sqrt{74} = 0$

(C) $7x – 5y + 1 – 5\sqrt{74} = 0$ (D) $5x – 7y + 1 – 5\sqrt{74} = 0$

Key : C

Hint: Both the circles have radius = 5 and they intersect each other, therefore their common tangent is parallel to the line joining their centres.

Equation of the line joining their centre is 7x – 5y + 1 = 0.

$\therefore$ Equation of the common tangent is 7x – 5y = c

$\therefore \left|\frac{c+1}{\sqrt{74}}\right| = 5 \Rightarrow c = \pm 5\sqrt{74} - 1$

$\therefore$ Equation is $7x – 5y + 1 \pm 5\sqrt{74} = 0$.

Let each of the circles

$S_1 \equiv x^2 + y^2 + 4y – 1 = 0$

$S_2 \equiv x^2 + y^2 + 6x + y + 8 = 0$

$S_3 \equiv x^2 + y^2 – 4x – 4y – 37 = 0$

Touches the other two. Let P_1, P_2, P_3 be the point of contact of S_1 and S_2, S_2 and S_3, S_3 and S_1 respectively. Let T be the point of concurrence of the tangents at P_1, P_2, P_3 to the circles. C_1, C_2, C_3 are the centres of S_1, S_2, S_3 respectively.

50. P and Q are any two points on the circle $x^2 + y^2 = 4$ such that PQ is a diameter. If α and β are the lengths of perpendicular from P and Q on x + y = 1 then the maximum value of $\alpha\beta$ is

a) $\frac{1}{2}$ b) $\frac{7}{2}$ c) 1 d) 2

Key: B

$$P(2\cos\theta, 2\sin\theta), Q(-2\cos\theta, -2\sin\theta)$$

Hint: $$\alpha\beta = \frac{|2\cos\theta + 2\sin\theta - 1||-2\cos\theta - 2\sin\theta - 1|}{2}$$

$$= \frac{\left|4(\cos\theta + \sin\theta)^2 - 1\right|}{2} \le \frac{7}{2}$$

51. The equation of chord of the circle $x^2 + y^2 - 6x - 4y - 12 = 0$ which passes through the origin such that origin divides it in the ratio 3 : 2 is

(A) y + x = 0, 7y + 17x = 0 (B) y + 3x = 0, 7y + 3x = 0

(C) 4x + y = 0, 9y + 8x = 0 (D) y + 3x = 7, y + 3x = 0

Key: A

Hint:

Let AO = 2x, BO = 3x

Now, AO. BO = OE. OF

$X = \sqrt{2}$

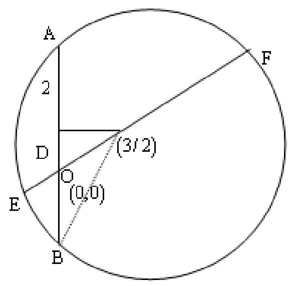
Now, D is mid point of chord AB

$AD = DB = \frac{5}{\sqrt{2}}$

Equation of AB is y = mx

$$\Rightarrow \frac{|3m - 2|}{\sqrt{1 + m^2}} = \frac{5}{\sqrt{2}} \Rightarrow m = -1, -17/7$$

Equation of AB is $y = -x$ and $y = -\frac{17}{7}x$



52. If two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then

(A) $2 < r < 8$ (B) $r < 2$

(C) $r = 2$ (D) $r > 2$

Key : A

Sol : Centres and radii of the given circles are C_1 (1,3), r_1 = r and $C_2 = (4, -1), r_2 = 3$ respectively since circles intersect in two distinct points, then

$\left|r_1 - r_2\right| < C_1C_2 < r_1 + r_2$

$\Rightarrow \left|r-3\right| < 5 < r+3$ ....(i)

from last two relations, $r > 2$

from firs two relations

$|r-3| < 5$

$\Rightarrow -5 < r-3 < 5$

$\Rightarrow -2 < r < 8$ ....(ii)

from eqs. (i) and (ii), we get $2 < r < 8$

53. (L-1)From a point P outside a circle with centre at C, tangents PA and PB are drawn such that $\frac{1}{(CA)^2} + \frac{1}{(PA)^2} = \frac{1}{16}$, then the length of chord AB is

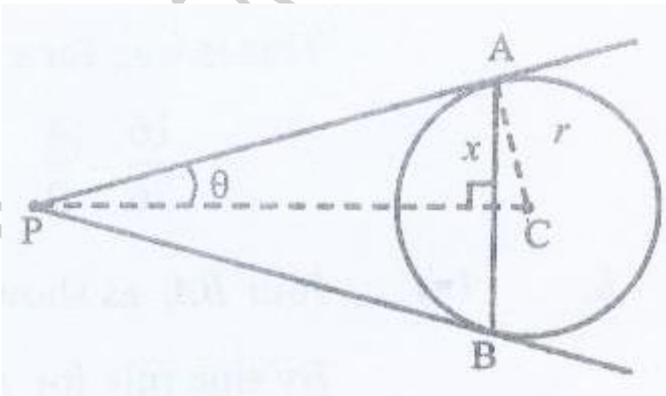
a) 8 b) 12

c) 16 d) none of these

Key : a

Sol : $\tan\theta = \frac{r}{PA}$

Given $\frac{1}{r^2} + \frac{1}{PA^2} = \frac{1}{16}$



$\Rightarrow \frac{\cot^2\theta + 1}{(PA)^2} = \frac{1}{16}$

$\Rightarrow PA\sin\theta = 4 = x \Rightarrow 2x = 8$

54. (L-II)Tangents PT_1 and PT_2 are drawn from a point P to the circle $x^2 + y^2 = a^2$. If the point P lies on the line px + qy + r = 0, then the locus of the centre of circumcircle of the triangle PT_1T_2 is

a) px + qy = r

b) $(x-p)^2+(y-q)^2=r^2$

c) $px+qy=\frac{r}{2}$

d) $2px+2py+r=0$

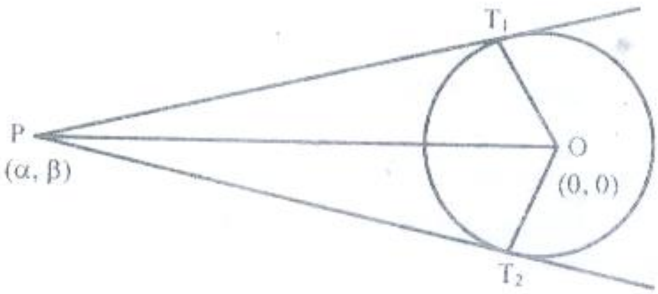
Key : d

Sol : P, T_2, O, T_1 are concylic points with PO as diameter

$\Rightarrow$ The circumcentre of ΔPT_1T_2 is $\left(\frac{\alpha}{2},\frac{\beta}{2}\right)$

New (α,β) lies on px + qy + r = 0

$\Rightarrow$ Locus of $\left(\frac{\alpha}{2},\frac{\beta}{2}\right)$ is 2px + 2qy + r = 0



55. (L-1)The circle $x^2+y^2=1$ cuts the x-axis at P and Q. Another circle with centre at Q and variable radius intersects to first circle at R above the X-axis and the line segment PQ at S. The maximum area of the triangle QSR is

a) $\frac{2}{9}$

b) $\frac{5\sqrt{2}}{7}$

c) $\frac{4\sqrt{3}}{9}$

d) $\frac{\sqrt{2}}{13}$

Key : c

Sol : Q is (-1, 0)

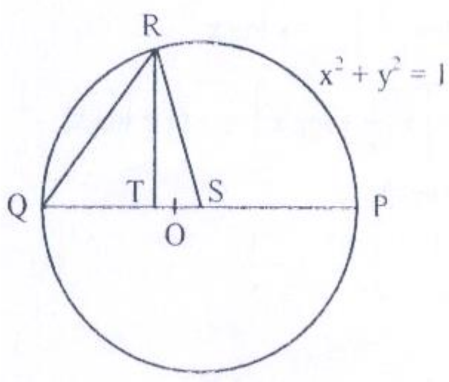
The circle with centre at Q and variable radius r has the equation

$(x+1)^2+y^2=r^2$

This circle meets the line segment QP at S where QS = r

It meets the circle $x^2+y^2=1$ at $R\left(\frac{r^2-2}{2},\frac{r}{2}\sqrt{4-r^2}\right)$ found by solving the equations of the two circles simultaneously.

A = area of the triangle QSR



$$= \frac{1}{2} QS \times RT$$

$$= \frac{1}{2} r\left(\frac{r}{2}\sqrt{4-r^2}\right)$$ since RT is the y coordinate of R

$$\frac{dA}{dr} = \frac{1}{4}\left\{2r\sqrt{4-r^2} + \frac{r^2(-r)}{\sqrt{4-r^2}}\right\} = \frac{\left\{2r\left(4-r^2\right)-4^3\right\}}{4\sqrt{4-r^2}} = \frac{8r-3r^3}{4\sqrt{4-r^2}}$$

$$\frac{dA}{dr} = 0 \text{ when } r\left(8-3r^2\right) = 0 \text{ giving } r = \sqrt{\frac{8}{3}}$$

$$\frac{d^2A}{dr^2} = \frac{4\sqrt{4-r^2}\left(8-9r^2\right) - \left(8r-3r^3\right)\frac{(-r)^4}{\sqrt{4-r^2}}}{16\left(4-r^2\right)}, \text{ where, } r = \sqrt{\frac{8}{3}}, \frac{d^2A}{dr^2} < 0$$

Hence A is maximum when r = $\sqrt{\frac{8}{3}}$ and the maximum area =

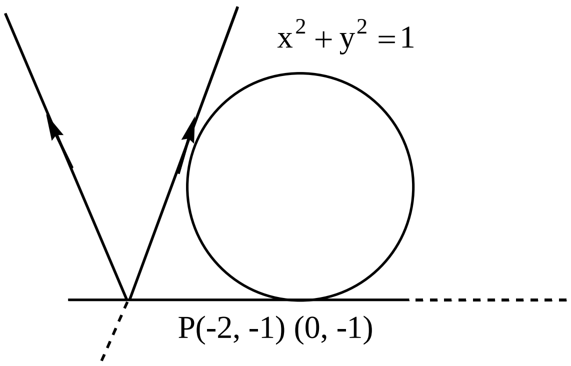
$$\frac{8}{4\times 3}\sqrt{4-\frac{8}{3}} = \frac{16}{12\sqrt{3}} = \frac{4}{3\sqrt{3}} = \frac{4\sqrt{3}}{9}$$

56. (L-I1)A ray of light incident at the point (-2, -1) gets reflected from the tangent at (0, -1) to the circle $x^2 + y^2 = 1$. The reflected ray touches the circle. The equation of the line along which the incident ray moved is

a) 4x – 3y + 11 = 0
b) 4x + 3y + 11 = 0
c) 3x + 4y + 11 = 0
d) 4x + 3y + 7 = 0

Key : b

Sol : Any line through $(-2,-1)$ is y + 1 = m(x + 2)



It touches the circle, if $\left|\frac{2m-1}{\sqrt{1+m^2}}\right| = 1$

$\Rightarrow m = 0, \frac{4}{3}$

$\therefore$ Equation of PB is $y+1 = \frac{4}{3}(x+2)$

$\Rightarrow 4x - 3y + 5 = 0$

A point of PB is (-5, -2)

Its image by the line y = -1 is (-5, -3)

Hence, equation of incident ray PP' is

$y - 3 = \frac{3+1}{-5+2}(x+5)$

$4x + 3y + 11 = 0$

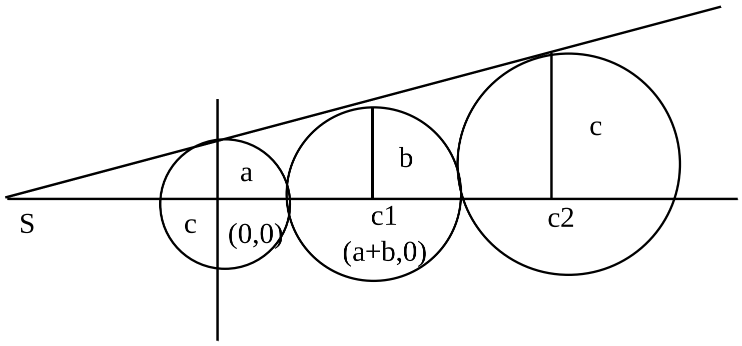
57. (L-I1)A circle C_1 of radius b touches the circle $x^2 + y^2 = a^2$ externally and has its centre on the positive x-axis; another circle C_2 of radius c touches the circle C_1 externally and has its centre on the positive x-axis. Given a < b < c, then the three circles have a common tangent if a, b, c are in

a) A.P. b) G.P.

c) H.P. d) none of these

Key : b

Sol : Similitude point wrt 0^4 s C and C_1 = Similitude point wrt – c_1c_2 the weget a, b, c and 14 G.



58. The locus of the centre of a circle which touches the circle $x^2+y^2-6x-6y+14=0$ externally and also the y-axis is given by

a) $x^2-6y-7y+14=0$ b) $x^2-10x-6y+14=0$

c) $y^2-6x-10y+14=0$ d) $y^2-10x-6y+14=0$

Key ; d

Sol : Let (x_1, y_1) be the centre. Since it touches y – axis its radius is $|x_1|$ Also it touches the given circle externally

$\therefore \sqrt{(x_1-3)^2+(y_1-3)^2} = |x_1|+2$ Squaring we get

$x_1^2+y_1^2-6x_1-6y_1+18 = x_1^2+4x_1+4$

$\Rightarrow y_1^2-10x_1-6y+14=0$

59. There is a system of circles, in which two pairs of circles have neither same nor parallel radical axis. If the number of radical axis of system is same as the number of radical centres, then number of circles in the system is

a) 4 b) 5 c) 6 d) 10

Key : b

Sol : Given $n_{c_2} = n_{c_3} \Rightarrow n = 3+2 = 5$

60. A line cuts the x-axis at A(4, 0) and the y-axis at B(0, 8). A variable line PQ is drawn perpendicular to AB cutting the x-axis in P and the y-axis in Q. If AQ and BP intersect at R, find the locus of R.

a) $x^2+y^2-2x-4y=0$ b) $x^2+y^2+2x+4y=0$

c) $x^2+y^2-2x+4y=0$ d) $x^2+y^2-4x-8y=0$

Key : d

Sol : Locus of R is a circle on AB as diameter ie $x^2+y^2-4x-8y=0$

61. Let P, $P \neq 0$ be any point inside a circle with centre at O. Draw a circle with diameter $\overline{OP}$. The point Q ($\neq p$) is any point on this circle. Extend $\overline{PQ}$ to meet the larger circle at A and B then which of the following statements is true

I) Q, P are points of trisection of AB   II) Q is mid point of AB

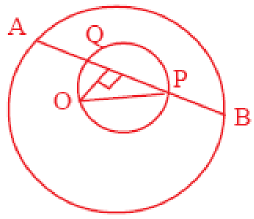
III) OA, OQ, OP, OB are in H.P

a) only I   b) only II

c) only II and II   d) all the three

Key : b

Sol : Angle in the semicircle is 90^0. Q is the midpoint of AB



62. From a point P outside a circle with centre at C, tangents PA and PB are drawn such that $\frac{1}{(CA)^2}+\frac{1}{(PA)^2}=\frac{1}{16}$, then the length of chord AB is

(A) 8   (B) 12

(C) 16   (D) none of these

Key : A

Hint : $\tan\theta = \frac{r}{PA}$

Given $\frac{1}{r^2}+\frac{1}{PA^2}=\frac{1}{16}$

$\Rightarrow \frac{\cot^2\theta+1}{(PA)^2}=\frac{1}{16}$

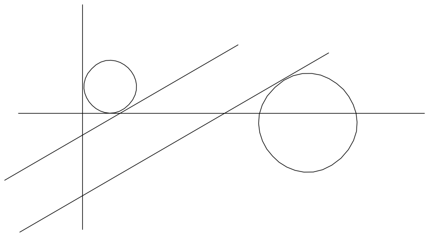
$\Rightarrow (PA)\sin\theta = 4 = x \Rightarrow 2x = 8$

63. If a variable line y = 2x + p lies between the circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 16x - 2y + 61 = 0$ without intersecting or touching either circles, then number of integral values of p is

a) 9   b) 8   c) 7   d) 6

Key : c

sol :



$\sin a - \sqrt{5} - 1 < P < 2\sqrt{5} - 15$

Integral value of P = 7

64. The locus of the centre of the circle which touches the y-axis and also touches the circle $(x+1)^2 + y^2 = 1$ externally is

A) $\{(x,y) \mid x^2 = 4y\} \cup \{(x,y) \mid y \le 0\}$

B) $\{(x,y) \mid y^2 = 4x\} \cup \{(x,y) \mid x \le 0\}$

C) $\{(x,y) \mid x^2 + 4y = 0\} \cup \{(x,y) \mid y \ge 0\}$

D) $\{(x,y) \mid y^2 + 4x = 0\} \cup \{(x,y) \mid x \ge 0\}$

Key. D

Sol. Let $P(x_1, y_1)$ be the centre of the touching $(x+1)^2 + y^2 = 1$ externally and touching y-axis

$\therefore 1 - x_1 = (x_1 + 1)^2 + y_1^2 \Rightarrow y_1^2 + 4x_1 = 0$

Also every circle with centre on positive x-axis and touching y-axis at origin satisfy the condition.

65. Three circles with centres at A, B, C intersect orthogonally. The point of intersection of the common chords is

A) Orthocentre of ΔABC

B) Circumcentre of ΔABC

C) Incentre of ΔABC

D) Centroid of ΔABC

Key. A

Sol. Common chord of two intersecting circles is $\perp^r$ to line of centres

66. The length of the common chord of the circles which are touching both the coordinate axes and passing through (2, 3) is

A) 3/2 B) 2/3 C) 2 D) $\sqrt{2}$

Key. D

Sol. y=x is the line joining the centres of the two circles.

67. A ray of light incident at the point (3, 1) gets reflected from the tangent at (0, 1) to the circle $x^2 + y^2 = 1$. The reflected ray touches the circle. The equation of the line containing the reflected ray is

A) $3x + 4y - 13 = 0$

B) $4x - 3y - 13 = 0$

c) $3x - 4y + 13 = 0$

D) $4x - 3y - 10 = 0$

Key. A

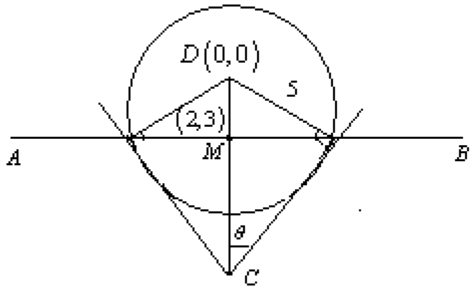
Sol. Angle of incidence is equal to angle of reflection.

68. AB is a chord of the circle $x^2+y^2=25$. The tangents to the circle at A and B intersect at C. If $(2,3)$ is the mid point of AB, then the area of quadrilateral OACB is

A) $\frac{50}{\sqrt{3}}$ B) $50\sqrt{\frac{3}{13}}$ C) $50\sqrt{3}$ D) $\frac{50}{\sqrt{13}}$

Key. B

Sol. From omB, $\cos(90-q)=\frac{\sqrt{13}}{5}$



$\Rightarrow \sin q=\sqrt{\frac{13}{5}}$

$\Rightarrow \cot q=\frac{2\sqrt{3}}{\sqrt{13}}$

Area of quad $OACB = 2\times\frac{1}{2}\times OB\times BC$

$= 5\times 5\cot q = 25\times\frac{2\sqrt{3}}{\sqrt{13}} = 50\sqrt{\frac{3}{13}}$

69. P(3,2) is a point on the circle $x^2+y^2=13$. Two points A, B are on the circle such that $PA=PB=\sqrt{5}$. The equation of chord AB is

A) $4x-6y+21=0$ B) $6x+4y-21=0$ C) $4x+6y-21=0$ D) $6x+4y+21=0$

Key. B

Sol. AB is common chord of $x^2+y^2=13$ and circle having centre at p and radius $\sqrt{5}$.

70. The point $([P+1],[P])$, (where [.] denotes the greatest integer function) lying inside the region bounded by the circle $x^2+y^2-2x-15=0$ and $x^2+y^2-2x-7=0$, then

a) $P\in[-1,0)\cup[0,1)\cup[1,2)$ b) $P\in[-1,2)-\{0,1\}$

c) $P\in(-1,2)$ d) $P\notin R$

Key. D

Sol. $x^2+y^2-2x-15=0 \Rightarrow [P]^2<8$

$x^2+y^2-2x-7=0 \Rightarrow 4<[P]^2$

71. The locus of centre of a circle which touches externally the circle $x^2+y^2-6x-6y+14=0$ and also touches the y – axis, is given by the equation

a) $x^2-6x-10y+14=0$ b) $x^2-10x-6y+14=0$

c) $y^2-6x-10y+14=0$ d) $y^2-10x-6y+14=0$

Key. D

Sol. Conceptual

72. The points A, B are the feet of O(0, 0) on $x-2y+1=0$, $2x-y-1=0$ respectively then the circum radius of the ΔOAB

a) 2 b) 1 c) $\sqrt{2}$ d) $1/\sqrt{2}$

Key. D

Sol. Point of meet $=(1,1)P$

$\therefore \angle OAP = 90^0 = \angle OBP$

$\therefore$ diameter = OP

73. A circle cuts x – axis at $A(a,0)$, $B(b,0)$ and y – axis at $C(0,c), D(0,2)$ then the orthocentre of the $\Delta ABC =$

a) $(2,0)$ b) $(-2,0)$

c) $(0,2)$ d) $(0,-2)$

Key. D

Sol. $O.C.(ABC) = \text{Image}$ of D, w.r.t $\overline{AB}$

74. The locus of the image of the point (2, 3) with respect to the line $(x-2y+3)+\lambda(2x-3y+4)=0 \ (\lambda \in R)$

a) $x^2+y^2-2x-4y+4=0$ b) $x^2+y^2+2x-4y+4=0$

c) $x^2+y^2-3x-4y-4=0$ d) $x^2+y^2-2x-4y+3=0$

Key. D

Sol. $(1,2)$ lie on both the lines and locus is $(h-1)^2+(k-2)^2=(2-1)^2+(3-2)^2$

75. ABCD is a rectangle . A circle passing through C touches AB,AD at M,N respectively . If the area of rectangle ABCD is K^2 units $(k>0)$ then $\perp^r$ distance from C to MN is

a) $2K$ b) K c) $\frac{K}{2}$ d) $4K$

Key. B

Sol. Taking AB,AD along axes and centre of the circle as E(h,h) we get M (h,0) N(0,h) and equation of MN as $x+y=h$. If $C=(\alpha,\beta)$ then given $\alpha\beta=k^2$ and also

$(\alpha-h)^2+(\beta-h)^2=h^2$, $\perp^r$ distance C to MN is $\frac{|\alpha+\beta-h|}{\sqrt{2}}=k$

$\therefore(\alpha+\beta-h)^2=\alpha^2+\beta^2-2h(\alpha+\beta)+2\alpha\beta=2k^2$

76. The number of integer values of λ for which the variable line $3x+4y=\lambda$ lies completely outside of circles $x^2+y^2-2x-2y+1=0$ and $x^2+y^2-18x-2y+78=0$ without meeting either circle, is

a) 8 b) 10 c) 12 d) 6

Key. A

Sol. The line given does not meet the circles if $(C_1=(1,1), C_2=(9,1)$

$$\frac{|3+4-\lambda|}{5}>1 \text{ and } \frac{|27+4-\lambda|}{5}>2$$

$$\Rightarrow |7-\lambda|>5 \& |31-\lambda|>10$$

$$\textit{But } 7-\lambda<0 \textit{ and } 31-\lambda>0.$$

Hence $\lambda>12 \& \lambda<21$

77. The curves $C_1: y=x^2-3$; $C_2: y=kx^2, k<1$ intersect each other at two different points. The tangent drawn to C_2, at one of the points of intersection $A=(a, y_1)(a>0)$ meets C_1 again at $B(1, y_2)$. $(y_1 \neq y_2)$. Then value of $a=$ ___?

a) 4 b) 3 c) 2 d) 1

Key. B

Sol. solving

$$C_1 \& C_2 \Rightarrow A\left(\sqrt{\frac{3}{1-k}}, \frac{3k}{1-k}\right)=(a, ka^2)\equiv(a, a^2-3).$$

$$\text{tangent } l \text{ to } C_2 \text{ at A is } y+a^2-3=2kx -----(1)$$

$$\Rightarrow B=(1,-2) \ (A\neq 1).$$

$$\text{from expression (1) } -2+a^2-3=2a\left(1-\tfrac{3}{a^2}\right).$$

$$\Rightarrow a=3, a=-2, a=1$$

$$\therefore a=3$$

78 Let A(1, 2), B(3, 4) be two points and C(x, y) be a point such that area of ΔABC is 3 sq.units and $(x-1)(x-3)+(y-2)(y-4)=0$. Then maximum number of positions of C, in the xy plane is

a) 2 b) 4 c)8 d) no such C exist

Key. D

Sol. (x,y) lies on the circle ,with AB as a diameter . Area

$$(\Delta ABC)=3$$

$$\Rightarrow \left(\tfrac{1}{2}\right)(AB)(\text{altitude})=3.$$

$$\Rightarrow \text{altitude } =\frac{3}{\sqrt{2}} \Rightarrow \text{no such "C" exists}$$

79. The equation of the smallest circle passing through the intersection of $x^2+y^2-2x-4y-4=0$ and the line $x+y-4=0$ is

(A) $x^2+y^2-3x-5y-8=0$ (B) $x^2+y^2-x-3y=0$

(C) $x^2+y^2-3x-5y=0$ (D) $x^2+y^2-x-3y-8=0$

Key. C

Sol. Family of circles passing through circle S = 0 and line L = 0 will be S + λ L = 0

$$x^2+y^2-2x-4y-4+\lambda(x+y-4)=0 \qquad \text{..... (1)}$$

For smallest circle line x + y – 4 = 0 will become the diameter for (1)

80. Equation of a straight line meeting the circle $x^2 + y^2 = 100$ in two points, each point is at a distance of 4 units from the point (8, 6) on the circle, is

(A) $4x+3y-50=0$ (B) $4x+3y-100=0$ (C) $4x+3y-46=0$ (D) $4x+3y-16=0$

Key. C

Sol.

$S_1 = x^2 + y^2 = 100$

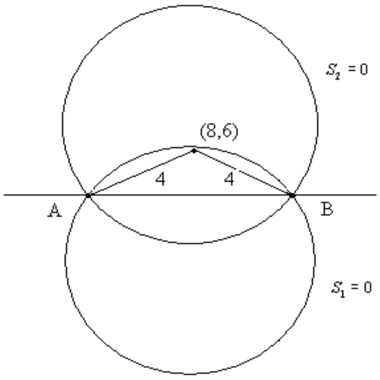
equation of circle centred at (8,6) & radius 4 units is

$(x-8)^2 + (y-6)^2 = 16$

required line AB is the common chord of $S_1 = 0$ & $S_2 = 0$, is

$S_1 - S_2 = 0$

$4x+3y-46=0$



81. The locus of the middle points of the chords of the circle of radius r which subtend an angle $\pi/4$ at any point on the circumference of the circle is a concentric circle with radius equal to

A. $r/2$ B. $2r/3$ C. $r/\sqrt{2}$ D. $r/\sqrt{3}$

Key. C

Sol. Equation of the circle be $x^2 + y^2 = r^2$. The chord which substends an angle $\pi/4$ at the circumference will subtend a right angle at the centre. Chord joining (r, 0) and (0, r) substends a right angle at the centre so(h, k) is $x^2 + y^2 = r^2/2$.

82. Two distinct chords drawn from the point (p,q) on the circle $x^2 + y^2 = px + qy$ where $pq \neq 0$, are bisected by the x-axis then

1) $|p| = |q|$ 2) $p^2 = 8q^2$ 3) $p^2 < 8q^2$ 4) $p^2 > 8q^2$

Key. 4

Sol. Let $A(p,q)$. Let $P(k,o)$ bisects the chord $\overline{AB}$

Then $B(2k-p,-q)$ lies on the circle

$\Rightarrow (2k-p)^2 + q^2 = p(2k-p) + q(-q)$

$\Rightarrow 4k^2 + p^2 - 4kp + q^2 = 2kp - p^2 - q^2$

$\Rightarrow 2k^2 - 3kp + (p^2 + q^2) = 0$

$b^2 - 4ac > 0 \Rightarrow 9p^2 - 8(p^2 + q^2) > 0$

$\Rightarrow p^2 > 8q^2$

83. The sum of the radii of inscribed and circumscribed circle of ‘n’ sided regular polygon of side ‘a’ is

1) $\frac{4}{a}\cot\left(\frac{\pi}{2n}\right)$ 2) $a\cot\left(\frac{\pi}{2n}\right)$ 3) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ 4) $2a\cot\left(\frac{\pi}{2n}\right)$

Key. 3

Sol. Circumradius, $R=\frac{a}{2}.\operatorname{cosec}\frac{\pi}{n}$

In radius, $r=\frac{a}{2}.\cot\frac{\pi}{n}$

Now, $R+r=\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$

84. B and C are points on the circle $x^2+y^2=a^2$. A point A(b,c) lies on the circle such that AB=AC=d. Then the equation of $\overline{BC}$ is

1) $bx+ay=a^2-d^2$ 2) $bx+ay=d^2-a^2$

3) $bx+cy=2a^2-d^2$ 4) $2(bx+cy)=2a^2-d^2$

Key. 4

Sol. Equation of the circle with centre at $A(b,c)$ and radius d is $(x-b)^2+(y-c)^2=d^2$

$\Rightarrow x^2+y^2-2bx-2cy+b^2+c^2-d^2=0$ --------- (1)

$\therefore$ $\overline{BC}$ is Radical axis of (1) and $x^2+y^2=a^2$

$\therefore$ $\overline{BC}$ is $2bx+2cy-b^2-c^2+d^2-a^2=0$

But, $b^2+c^2=a^2 \rightarrow 2bx+2cy=2a^2-d^2$

85. The locus of poles of the line $lx+my+n=0$ w.r.t the circle passing through $(-a,0),(a,0)$ is

1) $lx^2-mxy+nx+a^2l=0$ 2) $lx^2-mxy+ny+a^2l=0$

3) $ly^2-mxy+nx+a^2l=0$ 4) $ly^2-mxy+ny+a^2l=0$

Key. 3

Sol. Equation of the circle passing through $A(-a,0)$ $B(a,0)$ is $x^2+y^2-a^2+2\lambda(y)=0$

$\Rightarrow x^2+y^2+2\lambda y-a^2=0$

Polar of $P(x_1,y_1)$ is $xx_1+yy_1+\lambda(y+y_1)-a^2=0$ ------------- (1)

Given Polar, $lx+my+n=0$ ------------- (2)

Compare (1) & (2), eliminate λ, we get $ly^2-mxy+nx+a^2l=0$

86. If two circles which pass through the points $(0, a)$ and $(0, -a)$ cut each other orthogonally and touch the straight line $y = mx + c$, then

A) $c^2 = a^2(1 + m^2)$ B) $c^2 = a^2|1 - m^2|$

C) $c^2 = a^2(2 + m^2)$ D) $c^2 = 2a^2(1 + m^2)$

Key. C

Sol. Equation of a family of circles through $(0, a)$ and $(0, -a)$ is $x^2 + y^2 + 2\lambda ax - a^2 = 0.$ If two members are for $\lambda = \lambda_1$ and $\lambda = \lambda_2$ then since they intersect orthogonally

$$2\lambda_1\lambda_2 a^2 = -2a^2 \Rightarrow \lambda_1\lambda_2 = -1$$

Since the two circles touch the line $y = mx + c$

$$\left[\frac{-\lambda am + c}{\sqrt{1 + m^2}}\right]^2 = \lambda^2 a^2 + a^2$$

$$\Rightarrow a^2\lambda^2 + 2mca\lambda - c^2 + a^2(1 + m^2) = 0$$

$$\Rightarrow a^2(1 + m)^2 - c^2 = -a^2 \;\Rightarrow c^2 = (2 + m^2)a^2$$

87. Equation of a circle through the origin and belonging to the co-axial system, of which the limiting points are $(1, 2), (4, 3)$ is

A) $x^2 + y^2 - 2x + 4y = 0$ B) $x^2 + y^2 - 8x - 6y = 0$

C) $2x^2 + 2y^2 - x - 7y = 0$ D) $x^2 + y^2 - 6x - 10y = 0$

Key. C

Sol. Since the limiting points of a system of coaxial circles are the point circles (radius being zero), two members of the system are

$(x-1)^2 + (y-2)^2 = 0 \Rightarrow x^2 + y^2 - 2x - 4y + 5 = 0$ and

$(x-4)^2 + (y-3)^2 = 0 \Rightarrow x^2 + y^2 - 8x - 6y + 25 = 0$

The co-axial system of circles with these as members is

$$x^2 + y^2 - 2x - 4y + 5 + \lambda(x^2 + y^2 - 8x - 6y + 25) = 0$$

It passes through the origin if $5 + 25\lambda = 0$

or $\lambda = -(1/5)$

which gives the equation of the required circle as

$$5(x^2 + y^2 - 2x - 4y + 5) - (x^2 + y^2 - 8x - 6y + 25) = 0$$

$$\Rightarrow 4x^2 + 4y^2 - 2x - 14y = 0$$

$$\Rightarrow 2x^2 + 2y^2 - x - 7y = 0.$$

88. Circle are drawn to cut two circles $x^2 + y^2 + 6x + 5 = 0$ and $x^2 + y^2 - 6y + 5 = 0$ orthogonally. All such circles will pass through the fixed points.

A) $(1,-1)$ only B) $(2,-2)$ and $(0,0)$ C) $(-1,1)$ and $(-2,2)$ D) $(1,-1)$ and $(2,-2)$

Key. C

Sol. The radical axis of the given circles is $x+y=0.$ Let P $(\lambda,-\lambda)$ be any point on the above radical axis.

The length of the tangent drawn from P to any of the given circles is $l=\sqrt{\lambda^2+\lambda^2+6\lambda+5}$

A circle having centre at P and radius equal to l will be orthogonal to both the given circles.

Equation of such a circle, is $(x-\lambda)^2+(y+\lambda)^2=l^2=\lambda^2+\lambda^2+6\lambda+5$

i.e. $x^2+y^2+2\lambda^2-2\lambda x+2\lambda y=2\lambda^2+6\lambda+5$

i.e. $(x^2+y^2-5)-2\lambda(x-y+3)=0$

which represents a family of circles passing through the intersection points of $x^2+y^2-5=0....(i)$ and $x-y+3=0...(ii)$

Eliminating y we get

$x=-1,-2$ and the corresponding y=1, 2

Hence, the required points are $(-1,1)$ and $(-2,2)$.

89. If one circle of a co-axal system is $x^2+y^2+2gx+2fy+c=0$ and one limiting point is (a,b) then equation of the radical axis will be

A) $(g+a)x+(f+b)y+c-a^2-b^2=0$

B) $2(g+a)x+2(f+b)y+c-a^2-b^2=0$

C) $2gx+2fy+c-a^2-b^2=0$ D) None of these

Key. B

Sol. Given circle $S_1\equiv x^2+y^2+2gx+2fy+c=0...(i)$

$\therefore$ Equation of the second circle is $(x-a)^2+(y-b)^2=0$

$S_2\equiv x^2+y^2-2ax-2by+a^2+b^2=0...(ii)$

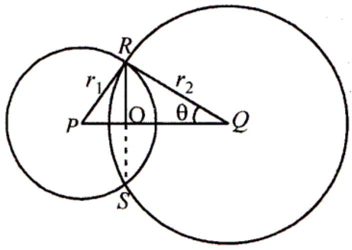
From (i) and (ii), equation radical axis is $S_1-S_2=0$

$\Rightarrow 2(g+a)x+2(f+b)y+c-a^2-b^2=0$

90. The circles having radii r_1 and r_2 intersect orthogonally. The length of their common chord is

A) $\frac{2r_1r_2}{\sqrt{r_1^2+r_2^2}}$ B) $\frac{2r_1^2+r_2^2}{\sqrt{r_1^2+r_2^2}}$ C) $\frac{r_1r_2}{\sqrt{r_1^2+r_2^2}}$ D) $\frac{2r_2^2+r_1^2}{\sqrt{r_1^2+r_2^2}}$

Key. A



Sol.

Let the centres of the circles be P and Q which intersect orthogonally at the point R, then $\angle PRQ = 90^0$

Let $\angle PQR = \theta$ then $\angle QPR = 90^0 - \theta$

$\therefore RO = r_2 \sin\left(90^0 - \theta\right) = r_1 \sin\theta$

$\Rightarrow \sin\theta = \frac{RO}{r_1}$ and $\cos\theta = \frac{RO}{r_2}$

$\Rightarrow RO^2\left(\frac{1}{r_1^2} + \frac{1}{r_2^2}\right) = 1 \Rightarrow RO = \frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$

$\therefore$ Length of common chord $RS = 2RO = \frac{2r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$

91. Radical centre of the three circles $x^2 + y^2 = 9,\ x^2 + y^2 - 2x - 2y = 5,\ x^2 + y^2 + 4x + 6y = 19$ lies on the line $y = mx$ if m is equal to

A) -1 B) $-2/3$ C) $-3/4$ D) 1

Key. D

Sol. The radical centre is the point of intersection of $2x + 2y = 4$ and $4x + 6y = 10$ i.e. $(1,1)$ which lies on $y = mx$ if $m = 1$.

92. If $\frac{x}{a} + \frac{y}{b} = 1$ touches the circle $x^2 + y^2 = r^2$ then $\left(\frac{1}{a}, \frac{1}{b}\right)$ lie on

(A) straight line (B) circle

(C) parabola (D) ellipse

Key. B

Sol. Let $\alpha = \frac{1}{a},\ \beta = \frac{1}{b}$

$x\alpha + y\beta - 1 = 0$ touches $x^2 + y^2 = r^2$

$\Rightarrow \left|\frac{-1}{\sqrt{\alpha^2 + \beta^2}}\right| = r$

$\Rightarrow \alpha^2 + \beta^2 = \frac{1}{r^2}$

$\Rightarrow \alpha, \beta$ lies on $x^2 + y^2 = \frac{1}{r^2}$

93. The value of 'c' for which the sets $\{(x, y) : x^2 + y^2 + 2x - 1 \le 0\} \cap \{(x, y) : x - y + c \ge 0\}$ contain only one point.

(A) – 1 only (B) 3 only

(C) both – 1 and 3 (D) 2

Key. A

Sol.

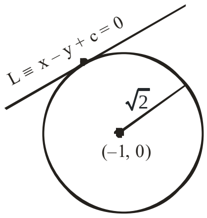
$$\frac{|-1+c|}{\sqrt{2}} = \sqrt{2}$$

c = 3, –1

L(–1, 0) > 0 when c = 3

< 0 when c = –1

$\Rightarrow$ c = –1



94. A circle of radius 'r' is inscribed in a square. The mid points of sides of square are joined to form a new square. The mid point of sides of resulting square are again joined so that a new square was obtained and so on. Then radius of circle inscribed in n^{th} square is

(a) $\left(2^{\frac{1-n}{2}}\right)r$ (b) $\left(2^{\frac{n-1}{2}}\right)r$

(c) $\left(2^{\frac{3-3n}{2}}\right)r$ (d) $\left(2^{-\frac{n}{2}}\right)r$

Key. A

SOL. CLEARLY RADIUS OF 2^{ND} CIRCLE $= \frac{\sqrt{r^2 + r^2}}{2} = \frac{r}{\sqrt{2}}$

AND THIRD CIRCLE $= \frac{r}{2}$

$\Rightarrow$ radius of *n*th circle $= \frac{r}{2^{\left(\frac{n-1}{2}\right)}}$

95. A variable circle touches the line y = x and passes through (0, 0). The common chord of the above circle and the circle $x^2 + y^2 + 6x + 8y - 7 = 0$ will pass through

(a) (0, 0) (b) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

(c) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, \frac{1}{2}\right)$

Key. D

SOL. EQUATION OF FAMILY OF CIRCLE TOUCHING

X = Y AT (0, 0) IS $X^2 + Y^2 + \lambda(X - Y) = 0$

REQUIRED COMMON CHORD $\equiv 6X + 8Y - 7 - \lambda(X - Y) = 0$

always passes through $\left(\frac{1}{2}, \frac{1}{2}\right)$

96. A circle passes through the points (2, 2) and (9, 9) and touches the *x*–axis. The *x*–cordinate of the point of contact is

(A) –2 or 2 (B) –4 or 4

(C) –6 or 6 (D) –9 or 9

Key. C

Sol. Any circle through (2, 2) and (9, 9) is

$$(x-2)(x-9)+(y-2)(y-9)+\lambda(y-x)=0 \qquad (1)$$

For the point of intersection with x–axis, we put y = 0 in (1), to get

$$(x-2)(x-9)+18-\lambda x=0$$

$D = 0 \Rightarrow (11+\lambda)^2 - 4\times 36 \Rightarrow \lambda = -23, 1$

97. From a fixed point on the circle $x^2 + y^2 = a^2$, two tangents are drawn to the circle $x^2 + y^2 = b^2$ (a > b). If the chord of contact touches a variable circle passing through origin, then the locus of the centre of the variable circle is

(A) a circle (B) a parabola

(C) an ellipse (D) a hyperbola

Key. B

Sol. The centre of the variable circle is always equidistant from the given chord of contact and the origin, its locus is a parabola.

98. If $9 + f''(x) + f'(x) = x^2 + f^2(x)$ be the differential equation of a curve and let P be the point of minima then the number of tangents which can be drawn from P to the circle $x^2 + y^2 = 9$ is

(A) 2 (B) 1

(C) 0 (D) either 1 or 2

Key. A

Sol. At the point of minima $f'(x) = 0$, $f''(x) > 0$

$\Rightarrow f''(x) = -9 + x^2 + f^2(x) > 0 \Rightarrow x^2 + y^2 - 9 > 0 \Rightarrow$ point P(x, f(x)) lies outside $x^2 + y^2 = 9$

$\Rightarrow$ two tangents are possible.

99. A point P lies inside the circles $x^2+y^2-4=0$ and $x^2+y^2-8x+7=0$. The point P starts moving under the conditions that its path encloses greatest possible area and it is at a fixed distance from any arbitrarily chosen fixed point in its region. The locus of P is

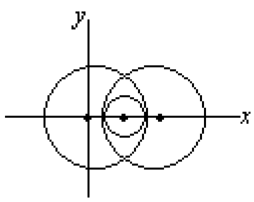
A) $4x^2+4y^2-12x+1=0$ B) $4x^2+4y^2+12x-1=0$

C) $x^2+y^2-3x-2=0$ D) $x^2+y^2-3x+2=0$

Key. D

Sol. For the point P to enclose greatest area, the arbitrarily chosen point should be $\left(\frac{3}{2},0\right)$ and P should move in a circle of radius ½ . Locus of P is a circle of radius ½ .

$$\left(x-\frac{3}{2}\right)^2+(y-0)^2=\frac{1}{4}\Rightarrow x^2+y^2-3x+2=0.$$



100. A circle of unit radius touches positive co-ordinate axes at A & B respectively. A variable line passing through origin intersects the circle in two points D and E. If the area of ΔDEB is maximum, then the reciprocal of the square of the slope of the line is

a) $\frac{1}{3}$ b) 3 c) $\frac{1}{2}$ d) 2

Key. B

Sol. Let 'm' the slope of the line (m > 0)

$$\Delta = \frac{\sqrt{2}\sqrt{m}}{m^2+1} \quad \Delta_{max} \Rightarrow m^2 = \frac{1}{3}$$

101. ABCD is a rectangle . A circle passing through C touches AB,AD at M,N respectively . If the area of rectangle ABCD is K^2 units $(k>0)$ then $\perp^r$ distance from C to MN is

a) $2K$ b) K c) $\frac{K}{2}$ d) $4K$

Key. B

Sol. Taking AB,AD along axes and centre of the circle as E(h,h) we get M (h,0) N(0,h) and equation of MN as $x+y=h.$ If $C=(\alpha,\beta)$ then given $\alpha\beta=k^2$ and also

$(\alpha-h)^2+(\beta-h)^2=h^2,$ $\perp^r$ distance C to MN is $\frac{|\alpha+\beta-h|}{\sqrt{2}}=k$

$\therefore (\alpha+\beta-h)^2=\alpha^2+\beta^2-2h(\alpha+\beta)+2\alpha\beta=2k^2$

102. The number of integer values of λ for which the variable line $3x+4y=\lambda$ lies completely outside of circles $x^2+y^2-2x-2y+1=0$ and $x^2+y^2-18x-2y+78=0$ without meeting either circle, is

a) 8 b) 10 c) 12 d) 6

Key. A

Sol. The line given does not meet the circles if $(C_1=(1,1),C_2=(9,1)$

$$\frac{|3+4-\lambda|}{5}>1 \text{ and } \frac{|27+4-\lambda|}{5}>2$$

$$\Rightarrow |7-\lambda|>5 \& |31-\lambda|>10$$

$$But\ 7-\lambda<0 \text{ and } 31-\lambda>0.$$

Hence $\lambda>12 \& \lambda<21$

103. All chords of the curve $x^2+y^2-10x-4y+4=0$, which make a right angle at (8,-2) pass through

a) (2,5) b) (-2,-5) c) (-5,-2) d) (5,2)

Key. D

Sol.(8,-2) lies on the circle $(x-5)^2+(y-2)^2=25$ and a chord making a right angle at (8,-2) must be a diameter of the circle .So they all pass through the centre (5,2).

104. The locus of the centre of the circle which touches the y-axis and also touches the circle $(x+1)^2+y^2=1$ externally is

A) $\{(x,y)\,|\,x^2=4y\}\cup\{(x,y)\,|\,y\le 0\}$ B) $\{(x,y)\,|\,y^2=4x\}\cup\{(x,y)\,|\,x\le 0\}$

C) $\{(x,y)\,|\,x^2+4y=0\}\cup\{(x,y)\,|\,y\ge 0\}$ D) $\{(x,y)\,|\,y^2+4x=0\}\cup\{(x,y)\,|\,x\ge 0\}$

Key. D

Sol. Let $P(x_1,y_1)$ be the centre of the touching $(x+1)^2+y^2=1$ externally and touching y-axis

$\backslash\ 1- x_1=(x_1+1)^2+y_1^2$ Þ $y_1^2+4x_1=0$

Also every circle with centre on positive x-axis and touching y-axis at origin satisfy the condition.

105. Three circles with centres at A, B, C intersect orthogonally. The point of intersection of the common chords is

A) Orthocentre of ΔABC B) Circumcentre of ΔABC

C) Incentre of ΔABC D) Centroid of ΔABC

Key. A

Sol. Common chord of two intersecting circles is ^ r to line of centres

106. The length of the common chord of the circles which are touching both the coordinate axes and passing through (2, 3) is

A) 3/2 B) 2/3 C) 2 D) $\sqrt{2}$

Key. D

Sol. y=x is the line joining the centres of the two circles.

107. A ray of light incident at the point (3, 1) gets reflected from the tangent at (0, 1) to the circle $x^2+y^2=1$. The reflected ray touches the circle. The equation of the line containing the reflected ray is

A) $3x+4y-13=0$ B) $4x-3y-13=0$

c) $3x-4y+13=0$ D) $4x-3y-10=0$

Key. A

Sol. Angle of incidence is equal to angle of reflection.

108. AB is a chord of the circle $x^2+y^2=25$. The tangents to the circle at A and B intersect at C. If $(2,3)$ is the mid point of AB, then the area of quadrilateral OACB is

A) $\frac{50}{\sqrt{3}}$ B) $50\sqrt{\frac{3}{13}}$ C) $50\sqrt{3}$ D) $\frac{50}{\sqrt{13}}$

Key. B

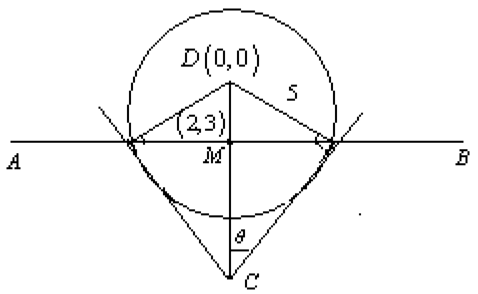
Sol. From omB, $\cos(90- q)=\frac{\sqrt{13}}{5}$

Þ $\sin q=\sqrt{\frac{13}{5}}$

Þ $\cot q=\frac{2\sqrt{3}}{\sqrt{13}}$

Area of quad $OACB = 2' \frac{1}{2}' OB' BC$

$= 5' 5\cot q = 25' \frac{2\sqrt{3}}{\sqrt{13}} = 50\sqrt{\frac{3}{13}}$



109. P(3,2) is a point on the circle $x^2 + y^2 = 13$. Two points A, B are on the circle such that $PA = PB = \sqrt{5}$. The equation of chord AB is

A) $4x - 6y + 21 = 0$ B) $6x + 4y - 21 = 0$

C) $4x + 6y - 21 = 0$ D) $6x + 4y + 21 = 0$

Key. B

Sol. AB is common chord of $x^2 + y^2 = 13$ and circle having centre at p and radius $\sqrt{5}$.

110. The range of a for which eight distinct points can be found on the curve $|x| + |y| = 1$ such that from each point two mutually perpendicular tangents can be drawn to the circle $x^2 + y^2 = a^2$ is

a) 1 < a < 2 b) $\frac{1}{2} < a < 1$ c) $\frac{1}{\sqrt{2}} < a < 1$ d) $\frac{1}{2} < a < \frac{1}{\sqrt{2}}$

Key. D

Sol. Director circle $x^2 + y^2 = 2a^2$ must cut square formed by $|x| + |y| = 1$ at 8 points

Min radius = OE

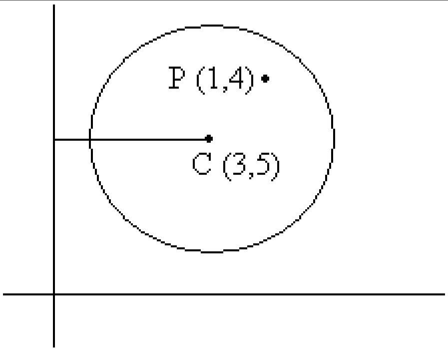
Max radius = OA

$\therefore \frac{1}{\sqrt{2}} < \sqrt{2}a < 1$

111. The point (1,4) lies inside the circle $x^2 + y^2 - 6x - 10y + p = 0$ which does not touch or intersects the coordinate axes then

a) 0 < p < 29 b) 25 < p < 29 c) 9 < p < 25 d) 9 < p < 29

Key. B



Sol.

CP < r < 3

$CP^2 < r < 9$

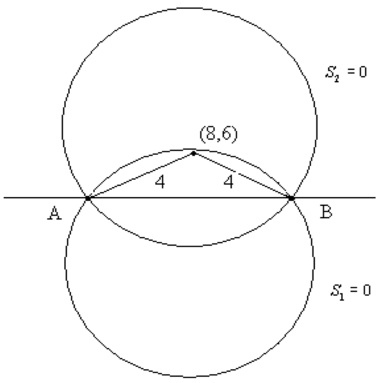
25 < P < 29

112. Equation of a straight line meeting the circle $x^2 + y^2 = 100$ in two points, each point at a distance of 4 from the point (8,6) on the circle, is

a) $4x+3y-50=0$ b) $4x+3y-100=0$ c) $4x+3y-46=0$ d) $4x+3y-16=0$

Key. C

Sol.



$S_1 = x^2 + y^2 = 100$

equation of circle centred at (8,6) & radius 4 units is $(x-8)^2 + (y-6)^2 = 16$

required line AB is the common chord of $S_1 = 0$ & $S_2 = 0$, is $S_1 - S_2 = 0$

$$4x+3y-46=0$$

113. Minimum radius of circle which in orthogonal with both the circles $x^2 + y^2 - 12x + 35 = 0$ and $x^2 + y^2 + 4x + 3 = 0$ is

a) 4 b) 3 c) $\sqrt{15}$ d) 1

Key. C

Sol. equation of the radical axis of two given circles is -16x + 32 = 0

$\Rightarrow$ x = 2

and it intersect the line joining the centres is y = 0 at the point (2,0)

$\therefore$ required radius is $\sqrt{4-24+35}=\sqrt{4+8+3}$

$$=\sqrt{15}$$

114. If $f(x+y)=f(x).f(y)$ for all x and y, $f(1)=2$ and $\alpha_n=f(n), n\in N$, then the equation of the circle having (α_1,α_2) and (α_3,α_4) as the ends of its one diameter is

A) $(x-2)(x-8)+(y-4)(y-16)=0$ B) $(x-4)(x-8)+(y-2)(y-16)=0$

C) $(x-2)(x-16)+(y-4)(y-8)=0$ D) $(x-6)(x-8)+(y-5)(y-6)=0$

Key. A

Sol. $f(x+y)=f(x).f(y)$, Q $f(1)=2$

Put $x=y=1\Rightarrow f(2)=2^2\Rightarrow f(n)=2^n$

Hence required circle is $(x-2)(x-8)+(y-4)(y-16)=0$

115. ABCD is a square of side 1 unit. A circle passes through vertices A, B of the square and the remaining two vertices of the square lie out side the circle. The length of the tangent drawn to the circle from vertex D is 2 units. The radius of the circle is

A) $\sqrt{5}$ B) $\frac{1}{2}\sqrt{10}$ C) $\frac{1}{3}\sqrt{12}$ D) $\sqrt{8}$

Key. B

Sol. Let $A=(0,1)$, $B=(0,0)$, $C=(1,0)$, $D=(1,1)$.

Family of circles passing through A, B is $x^2+y^2-y+\lambda x=0$.

$\sqrt{1+\lambda}=2\Rightarrow\lambda=3$

116. The point A lies on the circle $(x+3)^2+(y-4)^2=r^2$. Two chords of lengths 13 and 15 are drawn to the circle through A such that the distance between the mid points of these chords is 7. Then r =

A) $\frac{45}{4}$ B) $\frac{70}{9}$ C) $\frac{32}{3}$ D) $\frac{65}{8}$

Key. D

Sol. r is the circumradius of the triangle whose sides are a = 13, b = 15, c = 14 . $r=\frac{abc}{4\Delta}$

117. The equation of circumcircle of a Δ ABC is $x^2+y^2+3x+y-6=0$. If $A=(1,-2)$, $B=(-3,2)$ and the vertex C varies then the locus of orthocenter of Δ ABC is a

A) Straight line B) Circle C) Parabola D) Ellipse

Key. B

Sol. Equation of circumcircle is $\left(x+\frac{3}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=\frac{17}{2}$

$$C=\left(-\frac{3}{2}+\sqrt{\frac{17}{2}}\cos\theta,-\frac{1}{2}+\sqrt{\frac{17}{2}}\sin\theta\right)$$

Circum centre of ΔABC is $\left(-\frac{3}{2},-\frac{1}{2}\right)$

Centroid can be obtained.

In a triangle centroid, circum centre and ortho centre are collinear.

Circles

Integer Answer Type

1. Let $S_1 \equiv x^2 + y^2 - 4x - 8y + 4 = 0$ and S_2 its image in the line y = x. The radius of the circle touching y = x at (1, 1) and orthogonal to S_2 is $\frac{3}{\sqrt{\lambda}}$, then $\lambda^2 + 2 =$

Key. 6

Sol. Centre of circle $S_1 = (2, 4)$

Centre of circle $S_2 = (4, 2)$

Radius of circle S_1 = radius of circle S_2 = 4

$\therefore$ equation of circle S_2

$(x-4)^2 + (y-2)^2 = 16$

$\Rightarrow x^2 + y^2 - 8x - 4y + 4 = 0$ . . . (i)

Equation of circle touching y = x at (1, 1) can be taken as

$(x-1)^2 + (y-1)^2 + \lambda(x-y) = 0$

or, $x^2 + y^2 + x(\lambda - 2) + y(-\lambda - 2) + 2 = 0$ . . . (ii)

As this is orthogonal to S_2

$\Rightarrow 2\left(\frac{\lambda-2}{2}\right)\cdot(-4) + 2\left(\frac{-\lambda-2}{2}\right)\cdot(-2) = 4 + 2$

$\Rightarrow -4\lambda + 8 + 2\lambda + 4 = 6$

$\therefore$ required equation of circle is

$x^2 + y^2 + x - 5y + 2 = 0.$

Radius = $\sqrt{\frac{1}{4}+\frac{25}{4}-2} = \sqrt{\frac{26-8}{4}} = \sqrt{\frac{18}{4}} = \frac{3}{2}\sqrt{2}$.

2. The centre of each of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. The locus of any point in the set is a ring whose area is $\lambda\pi$, then $\frac{\lambda}{10} =$

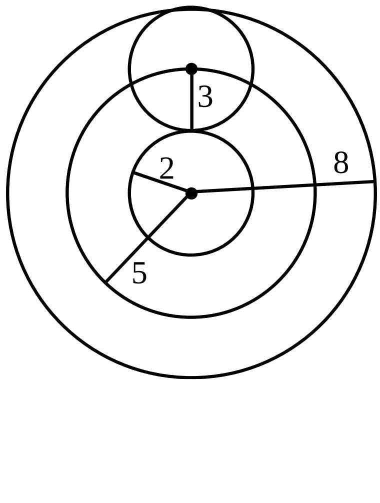
Key. 6

Sol.

From figure it is clear that point will lie between two concentric circles

$x^2 + y^2 = 4$ and $x^2 + y^2 = 64$

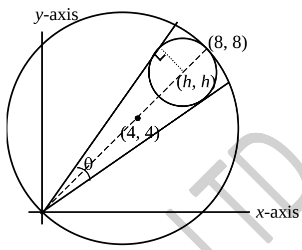
$\therefore$ Required locus $4 \le x^2 + y^2 \le 64$



3. If the radius of the circle touching the pair of lines $7x^2 - 18xy + 7y^2 = 0$ and the circle $x^2 + y^2 - 8x - 8y = 0$, and contained in the given circle is equal to k, then k^2 is equal to

Key. 8

Sol.



$\tan\theta = \left|\dfrac{2\sqrt{h^2 - ab}}{a+b}\right| = \dfrac{4\sqrt{2}}{7}$

$\tan\dfrac{\theta}{2} = \dfrac{1}{2\sqrt{2}}$ on solving

$\sin\dfrac{\theta}{2} = \dfrac{1}{3} = \dfrac{\sqrt{2}(8-h)}{\sqrt{2}\,h}$

Hence equation of circle is $(x-6)^2 + (y-6)^2 = 8$.

4. For the circle $x^2 + y^2 = r^2$ the value of r for which the area enclosed by the tangents from the point P(6, 8) to the circle and the chord of contact is maximum is ________

Key. 5

Sol. $f(r) = \Delta = \dfrac{r.s_{11}^{3/2}}{s_{11} + r^2} = \dfrac{r(100 - r^2)^{3/2}}{100}$

$f'(r) = 0 \Rightarrow -3r^2 + 100 - r^2 = 0 \Rightarrow r = 5$

5. Circles are drawn through (1, 1) touching the circle $x^2 + y^2 - 6x + 2y + 1 = 0$. If r_1 *and* r_2 are the radii of smallest and largest circles then the value of $(r_2 + r_1)^2 - (r_2 - r_1)^2$ equals ____

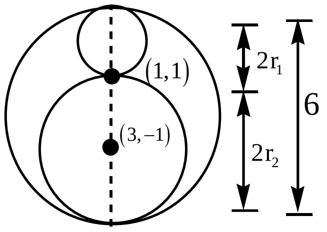
Key. 1

Sol. $2r_1 + 2r_2 = 6 \Rightarrow r_1 + r_2 = 3 \longrightarrow (1)$

Also $2r_2 - 2\sqrt{3} = 3 \Rightarrow r_2 = \dfrac{3 + 2\sqrt{2}}{2}$

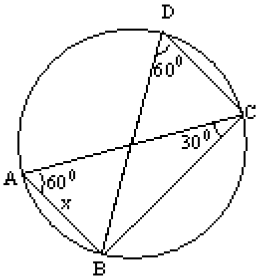
$\therefore\ r_1 = \dfrac{3 - 2\sqrt{2}}{2}$

$G.E = 4r_1r_2 = 4 \times \dfrac{1}{4} = 1$



6. Line segment AC and BD are diameters of circle of radius one. If $\angle BDC = 60^0$, the length of line segment AB is

Key. 1



Sol.

$\angle A = 60^0 = \angle D$

AC = 2(given)

$\angle ABC = 90^0$

$\Rightarrow x = 1$

7. If $m(x-2)+\sqrt{1-m^2}.y=3$, is tangent to a circle for all $m \in [-1,1]$ then the radius of the circle.

Key. 3

Sol. $(x-2)\cos\theta + y\sin\theta = 3$ is tangent to the circle $(x-2)^2 + y^2 = 3^2$

8. If the portion of the line $ax+by-1=0$ intercepted between the lines $ax+y+1=0$ and $x+by=0$ subtends a right angle at the origin, then the value of $4a+b^2+(b+1)^2$

Key. 1

Sol. Homogenise $(ax+by-1)(x+by)=0$ using $ax+y+1=0$

9. The tangents drawn from the origin to the circle $x^2+y^2-2rx-2hy+h^2=0$ are perpendicular then sum of all possible values of $\frac{h}{r}$ is ___

Key. 0

Sol. Combined equation of the tangents drawn from (0, 0)to the circle is

$\left(x^2+y^2-2rx-2hy+h^2\right)h^2=\left(-rx-hy+h^2\right)^2$ here coefficient of

$x^2+\text{coffecient of } y^2=0 \Rightarrow \left(h^2-r^2\right)+\left(h^2-r^2\right)=0$

$\Rightarrow \frac{h}{r}=\pm 1$

10. The number of points on $y=\tan^{-1}x,\ \forall\ x\in(0,\pi),$ whose image in $y=x$ is the centre of the circle with radius $\frac{\pi}{2\sqrt{2}}$ units and which is at a minimum distance of $\frac{\pi}{2\sqrt{2}}$ units from the circle.

Key. 8

Sol. Let (h,k) be the point on the curve $y=\tan^{-1}x$.

Image of (h,k) in $y=x$ is (k,h) which is the centre of a circle of radius $\frac{\pi}{2\sqrt{2}}$

Given $P.M=\frac{\pi}{2\sqrt{2}}$ (shortest distance)

And $C.M=\frac{\pi}{2\sqrt{2}}$ (radius of circle)

Now, $CP=\sqrt{(h-k)^2+(k-h)^2}=\frac{\pi}{2}$

$\Rightarrow \sqrt{2}\,|h-k|=\frac{\pi}{\sqrt{2}} \Rightarrow |h-k|=\frac{\pi}{2}$

$\Rightarrow h-k=\pm\frac{\pi}{2} \Rightarrow k=h\pm\frac{\pi}{2}$

Since, (h,k) lies on $y=\tan^{-1}x$

$\Rightarrow k=h-\frac{\pi}{2}$

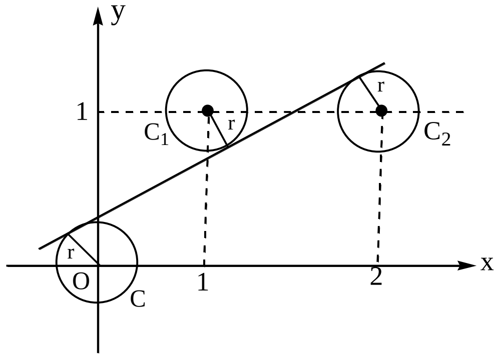
Now, $h=\frac{x}{2}=\tan^{-1}h$

Since, $0<h<\pi \Rightarrow \frac{-\pi}{2}<h-\frac{\pi}{2}<\frac{\pi}{2}$

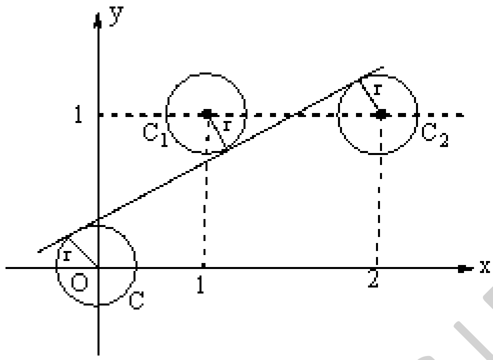
$h=\tan\left(h-\frac{\pi}{2}\right)=-\cot h \Rightarrow -h=\cot h$

11. As shown in figure three circles which have the same radius r, have centres at (0, 0), (1, 1), (2,

1). If they have a common tangent line, as shown, then the value of $10\sqrt{5}$ r is.



Key. 5



Sol.

Equation of line joining origin and centre of circle $C_2 \equiv (2,1)$ is, $y = \frac{x}{2}$

$\Rightarrow x - 2y = 0$

Let equation of common tangent is x – 2y + c = 0…………..(1)

$\therefore$ perpendicular distance from (0, 0) on this line

= perpendicular distance from (1, 1)

$$\Rightarrow \left|\frac{c}{\sqrt{5}}\right| = \left|\frac{c-1}{\sqrt{5}}\right|$$

$$\Rightarrow c = 1 - c \Rightarrow c = \frac{1}{2}$$

Equation of common tangent is

$x - 2y + \frac{1}{2} = 0$ or $2x - 4y + 1 = 0$ ………..(2)

Perpendicular from (2, 1) on the line (2)

$$r = \left|\frac{4-4+1}{\sqrt{20}}\right| = \frac{1}{2\sqrt{5}} = \frac{\sqrt{5}}{10}$$

12. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let 'P' be the midpoint of the line segment joining the centres of C_1 and C_2

and C be a circle touching circles C_1 and C_2 externally. If a common tangents to C_1 and C passing through $'P'$ is also a common tangent to C_2 and C. Then the radius of the circle C is

Key. 8

Sol. $(r+1)^2 = \alpha^2 + 9$

$r^2 + 8 = \alpha^2$

$\Rightarrow r^2 + 2r + 1 = r^2 8 + 9 \Rightarrow 2r = 16$

$\therefore r = 8$

13. The tangents drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ are perpendicular then sum of all possible values of $\frac{h}{r}$ is ___

Ans: 0

Hint. Combined equation of the tangents drawn from (0, 0)to the circle is

$\left(x^2 + y^2 - 2rx - 2hy + h^2\right)h^2 = \left(-rx - hy + h^2\right)^2$ here coefficient of

$x^2 + \text{coffecient of } y^2 = 0 \Rightarrow \left(h^2 - r^2\right) + \left(h^2 - r^2\right) = 0$

$\Rightarrow \frac{h}{r} = \pm 1$

14. If the curves $\frac{x^2}{4} + y^2 = 1$ and $\frac{x^2}{a^2} + y^2 = 1$ for suitable value of a cut on four concyclic points, then find the radius of the smallest circle passing through these 4 points

Key: 1

Hint: $\left(\frac{x^2}{4} + y^2 - 1\right) + \lambda\left(\frac{x^2}{a^2} + y^2 - 1\right) = 0$

$x^2\left(\frac{a^2 + 4\lambda}{4a^2(1+\lambda)}\right) + y^2 = 1$

Clearly radius is 1 unit

15. The neighbouring sides AB and BC of a square ABCD of side $\left(2+\sqrt{2}\right)$ units are tangents to a circle. The vertex D of the square lies on the circumference of the circle. The radius of the circle is ……….

Key : 4

Sol : Let y = mx +c be the tangnt to the given ellipse $y = 2x \pm \sqrt{a^2 + 4^e + l^2}$ Which passes the $(-2,0)$ $4a^2 + l^2 = 16$ let s = al $s^2 = a^2 l^2 = a^2\left(16 - 4a^2\right)$

$s_{\max} at - a\sqrt{l}\, then\, l = \sqrt{8}$

Max. value of al = 4

16. D, E, F are mid points of sides BC, CA, AB of ΔABC and the circum circles of ΔDEF, ΔABC touch each other then $\left[\sum \cos^2 A\right] =$ ________ (where [.] denotes N.G.I.F)

Key. 1

Sol. $(OS)^2 = (2SN)^2$

$= 4R^2\left[\sum \cos^2 A - \frac{3}{4}\right]$

17. The radius of the circles which pass through the point $(2,3)$ and cut off equal chords of length 6 units along the lines $y - x - 1 = 0$ and $y + x - 5 = 0$ is $'r'$ then $[r]$ is (where [.] denotes greatest integer function)

Key. 4

Sol. The given two lines pass through the point $(2,3)$ and are inclined at 45^0 and 135^0 to the x–axis the other ends of chords can easily be calculated as

$\left(2+3\sqrt{2}, 3+3\sqrt{2}\right)$ and $\left(2-3\sqrt{2}, 3-3\sqrt{2}\right)$

There is symmetry about the line $x = 2$ and therefore the centres of circles lie on $x = 2$

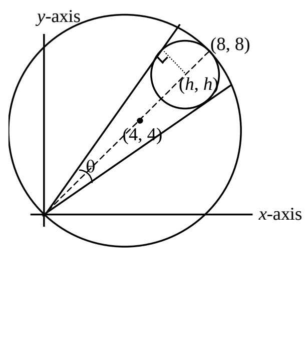
As the chords subtend right angles at the centre.

$\therefore\ 2r^2 = 6^2 \Rightarrow r = 3\sqrt{2}$

18. If the radius of the circle touching the pair of lines $7x^2 - 18xy + 7y^2 = 0$ and the circle $x^2 + y^2 - 8x - 8y = 0$, and contained in the given circle is equal to k, then k^2 is equal to

Key. 8

Sol.



tan θ = $\left|\frac{2\sqrt{h^2 - ab}}{a+b}\right| = \frac{4\sqrt{2}}{7}$

$\tan\frac{\theta}{2} = \frac{1}{2\sqrt{2}}$ on solving

$\sin\frac{\theta}{2} = \frac{1}{3} = \frac{\sqrt{2}(8-h)}{\sqrt{2}\,h}$

Hence equation of circle is $(x - 6)^2 + (y - 6)^2 = 8$.

19. The number of integral values of α for which the point (α – 1, α + 1) lies in the larger segment of the circle $x^2 + y^2 - x - y - 6 = 0$ made by the chord whose equation is $x + y - 2 = 0$ is

Key. 1

Sol. S(x, y) = $x^2 + y^2 - x - y - 6 = 0$ (1)

has centre at $C \equiv \left(\frac{1}{2}, \frac{1}{2}\right)$

According to the required conditions, the given point P(α - 1, α + 1) must lie inside the given circle.

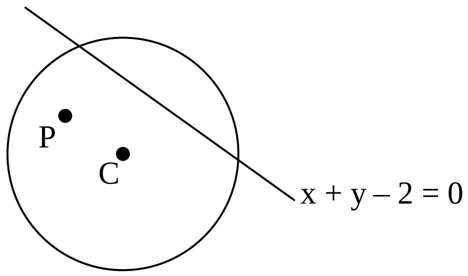
i.e. S(α - 1, α + 1) < 0

$\Rightarrow \quad (\alpha-1)^2 + (\alpha+1)^2 - (\alpha-1) - (\alpha+1) - 6 < 0$

$\Rightarrow \quad \alpha^2 - \alpha - 2 < 0$, i.e., $(\alpha-2)(\alpha+1) < 0$

$\Rightarrow \quad -1 < \alpha < 2$ (2)

Also P and C must lie on the same side of the line (see figure)



L (x, y) $\equiv x + y - 2 = 0$ (3)

i.e. L(1/2, ½) and $L(\alpha-1, \alpha+1)$ must have the same sign.

Since $L\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{1}{2} + \frac{1}{2} - 2 < 0$

$L(\alpha-1, \alpha+1) = (\alpha-1) + (\alpha+1) - 2 < 0$, i.e., $\alpha < 1$ (4)

Inequalities (2) and (4) together give the permissible values of α as $-1 < \alpha < 1$.

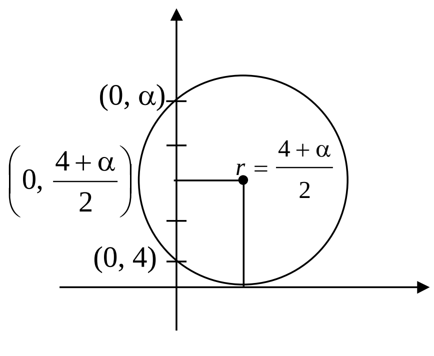
20. Radius of the smallest circle that can be drawn to pass through the point (0, 4) and touching the *x*–axis is

Key. 2

Sol.

$r = \frac{4+\alpha}{2}$, $\alpha \geq 0$

when α = 0, smallest radius = 2.



21. Let $M(-1, 2)$ and $N(1, 4)$ be two points in a plane rectangular coordinate system XOY. P is a moving point on the x-axis. When ∠MPN takes its maximum value, the x-coordinate of point P is

Key. 1

Sol. The centre of a circle passing through points M and N lies on the perpendicular bisector y = 3 – x of MN. Denote the centre by C(a, 3 – a), the equation of the circle is

$(x - a)^2 + (y - 3 + a)^2 = 2(1 + a^2)$

Since for a chord with a fixed length the angle at the circumference subtended by the corresponding arc will become larger as the radius of the circle becomes smaller. When $\angle MPN$ reaches its maximum value the circle through the three points M, N and P will be tangent to the x-axis at P, which means

$2(1 + a^2) = (a - 3)^2 \Rightarrow a = 1$ or $a = -7$

Thus the point of contact are P(1, 0) or $P'(-7, 0)$ respectively.

But the radius of circle through the points M, N and P' is larger than that of circle through points M, N and P.

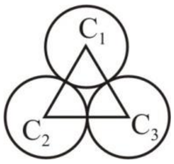
Therefore, $\angle MPN > \angle MP'N$. Thus P = (1, 0)

$\therefore$ x-coordinate of P = 1.

22. *r* be radius of incircle of triangle formed by joining centres of $(x - a)^2 + (y - b)^2 = 9$, $(x - a)^2 + (y - b - 7)^2 = 16$ and circle touching above two circles and having radius 5 units. Find r^2.

Key. 5

Sol. All three circles touch each other externally



$C_1C_2 = 7$

$C_2C_3 = 9$

$C_3C_1 = 8$

$$s = \frac{7 + 8 + 9}{2} = 12$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{12 \times 5 \times 3 \times 4}$$

$$r = \frac{\Delta}{s} = \sqrt{5}$$