Binomial Theorem

Single Correct Answer Type

1. If 7 divides $32^{32^{32}}$, the remainder is

A) 1 B) 0 C) 4 D) 6

Key. C

Sol. $32 = 2^5 \Rightarrow (32)^{32} = \left(2^5\right)^{32}$

$= 2^{160} = (3-1)^{160} = 3m+1, m \in N$

$\therefore (32)^{32^{32}} = (32)^{3m+1} = 2^{5(3m+1)}$

$2^{3(5m+1)}\, 2^2 = 4.8^{5m+1}$

$4(7+1)^{5m+1} = 4(7n+1), n \in N = 28n+4$

$\therefore$ When 7 divides $(32)^{32^{32}}$ remainder =4

2. If {x} represents the fractional part of x, then $\left\{\dfrac{5^{200}}{8}\right\}$ is

A) $\dfrac{1}{4}$ B) $\dfrac{1}{8}$ C) $\dfrac{3}{8}$ D) $\dfrac{5}{8}$

Key. B

Sol. $\dfrac{5^{200}}{8} = \dfrac{\left(5^2\right)^{100}}{8} = \dfrac{(1+24)^{100}}{8}$

$= \dfrac{1 + {}^{100}C_1.24 + {}^{100}C_2 (24)^2 + ... + {}^{100}C_{100} (24)^{100}}{8}$

$= \dfrac{1}{8} + \text{int}\,eger \Rightarrow \left\{\dfrac{5^{200}}{8}\right\} = \dfrac{1}{8}$

3. For n>3 ,1.2 ${}^nC_r - 2.3\ {}^nC_{r-1} + ... + (-1)^r (r+1)(r+2)$ is

A) ${}^{n-3}C_r$ B) 2. ${}^{n-3}C_r$ C) ${}^{n+3}C_{r+1}$ D) ${}^{n-2}C_r$

Key. B

Sol. We have $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + ...$

$+ {}^nC_{r-1}\ x^{r-1} + {}^nC_r\ x^r + ... + {}^nC_n x^n$ .....(1)

and $(1+x)^{-3} = 1 - {}^3C_1 x + {}^4C_2 x^2 ...... +$

$(-1)^{r-1}\ {}^{r+1}C_{r-1}\ x^{r-1} + (-1)^r$

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${}^{r+2}C_r\ x^r+........to\infty$ ....(2)

Multiply (1) and (2) , we get

$(1+x)^{n-3}=\left({}^nC_0+{}^nC_1x+{}^nC_2\ x^2+.......+{}^nC_n\ x^n\right)\left(1-{}^3C_1x+{}^4C_2\ x^2-.....\infty\right)$

Clearly , the coefficient of x^r from the product in R.H.S is

$1.{}^nC_r-{}^3C_1.{}^nC_{r-1}+{}^4C_2.{}^nC_{r-2}-......+(-1)^r\ {}^{r+2}C_r.{}^nC_0$

$={}^nC_r-3{}^nC_{r-1}+\frac{4.3}{2.1}\ {}^nC_{r-2}-\frac{5.4}{2.1}\ {}^nC_{r-3}$

$+......+(1)^r\frac{(r+2)(r+1)}{2.1}$

$=\frac{1}{2}\left[1.2\ {}^nC_r-2.3\,{}^nC_{r-1}+4.3\ {}^nC_{r-2}-......+(-1)r(r+2)(r+1)\right]$

$\therefore$ Required series = $2\times$ coefficient of x^r in $(1+x)^{n-3}=2.{}^{n-3}C_r$

4. The coefficient of x^{50} in the expansion of

$(1+x)^{1000}+2x(1+x)^{999}+3x^2(1+x)^{998}+.....+1001\ x^{1000}$

A) $1000_{C_{50}}$ B) $1001_{C_{50}}$ C) $1002_{C_{50}}$ D) 2^{1001}

Key. C

Sol. Let, $S=(1+x)^{1000}+2x(1+x)^{999}+2x(1+x)^{999}+3x^2(1+x)^{998}+.....+1001\ x^{1000}$

$\frac{x}{1+x}S=x(1+x)^{999}++2x^2(1+x)^{998}+......$

$+1000_x{}^{1000}+\frac{1001x^{1001}}{1+x}$

Subtract above equations,

$\left(1-\frac{x}{1+x}\right)S=(1+x)^{1000}+(1+x)^{999}+$

$x^2(1+x)^{998}+....+x^{1000}-\frac{1001x^{1001}}{1+x}$

$\Rightarrow S=(1+x)^{1001}+x(1+x)^{1000}+x^2(1+x)^{999}$

$+........+x^{1000}(1+x)-100]\ x^{1001}$

$=\frac{(1+x)^{1001}\left[\left(\frac{x}{1+x}\right)^{1001}-1\right]}{\frac{x}{1+x}-1}-1001\ x^{1001}$

[sum of G.P]

$=(1+x)^{1002}-x^{1002}-1002\ x^{1001}$

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$\therefore$ coefficient of x^{50} in S= coefficient of x^{50} in $\left[(1+x)^{1002}-x^{1002}-1002x^{1001}\right]={}^{1002}C_{50}$

5. The coefficient of the term independent of x in the expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$

A) 70 B) 112 C) 105 D) 210

Key. D

Sol. Given expression $=\frac{\left(x^{1/3}\right)^3+(1)^3}{x^{2/3}-x^{1/3}+1}-\frac{(x-1)}{x^{1/2}\left(x^{1/2}-1\right)}$

$=\frac{\left(x^{1/3}+1\right)\left(x^{2/3}-x^{1/3}+1\right)}{\left(x^{2/3}-x^{1/3}+1\right)}-\frac{\left(x^{1/2}+1\right)\left(x^{1/2}-1\right)}{x^{1/2}\left(x^{1/2}-1\right)}$

$=\left(x^{1/3}+1\right)-\left(1+x^{-1/2}\right)=x^{1/3}-x^{-1/2}$

$\Rightarrow\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$

$=\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$

$=\left(x^{1/3}-x^{-1/2}\right)^{10}$

T_{r+1} in $\left(x^{1/3}-x^{-1/2}\right)^{10}$ is

${}^{10}C_r\left(x^{1/3}\right)^{10-r}.(-1)^r.\left(x^{-1/2}\right)^r$

$=(-1)^r\ {}^{10}C_r\ x^{\left(\frac{10-r}{3}-\frac{r}{2}\right)}$ which is independent of x

If $\left(\frac{10-r}{3}-\frac{r}{2}\right)=0\Rightarrow r=4$

Hence required coefficient $={}^{10}C_4(-1)^4=210$

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6. Let n be an odd natural number greater than 1. Then the number of zeros at the end of the sum 99^n+1 is

(A) 3 (B) 4 (C) 2 (D) None of these

Key. C

Sol. $1+99^n=1+(100-1)^n=1+$

$\left\{{}^nC_0 100^n-{}^nC_1.100^{n-1}+.........-{}^nC_n\right\}$

Because n is odd $=100\left\{{}^nC_0.100^{n-1}-{}^nC_1.100^{n-2}+.........-\right.$

$\left.{}^nC_{n-2}.100+{}^nC_{n-1}\right\}$

= 100 $\times$ integer whose units place is different from 0

$\left[\because {}^nC_{n-1}=n, has\ odd\ digit\ at\ unit\ place\right]$

$\therefore$ There are two zeros at the end of the sum 99^n+1

7. If $C_0, C_1, C_2, ...C_n$ are the Binomial coefficients in the expansion $(1+x)^n$. 'n' being even, then $C_0+(C_0+C_1)+(C_0+C_1+C_2)+..........(C_0+C_1+C_2+....+C_{n-1})=$ is equal to

(A) $n2^n$ (B) $n.2^{n-1}$ (C) $n.2^{n-3}$ (D) $n.2^{n-2}$

Key. B

Sol. Sum $=\left\{C_0+\left(c_1+c_2+......+c_{n-1}\right)\right\}+=\left\{\left(c_0+c_1\right)+\left(c_0+c_1+.......+c_{n-2}\right)\right\}+$

$\left\{\left(c_0+c_1+c_2\right)+\left(c_0+c_1+......+c_{n-3}\right)\right\}+.......\ to\left(\frac{n}{2}\right)$

Terms = $=\left(c_0+c_1+......c_n\right)\times\frac{n}{2}=n.2^{n-1}$

8. If $\sum_{r=0}^{n}\left\{\frac{{}^nC_{r-1}}{{}^nC_r+{}^nC_{r-1}}\right\}^3=\frac{25}{24}$, then n is equal to

(A) 3 (B) 4 (C) 5 (D) 6

Key. C

Sol. Let $t_r=\frac{{}^nC_{r-1}}{{}^nC_r+{}^nC_{r-1}}=\frac{{}^nC_{r-1}}{{}^{n+1}C_r}=\frac{{}^nC_{r-1}}{\frac{n+1}{r}\ {}^nC_{r-1}}$

$\therefore l_r=\frac{r}{n+1}$

Now,

$S=\sum_{r=0}^{n}\{l_r\}^3\Rightarrow S=\sum_{r=0}^{n}\frac{r^3}{(n+1)^3}=\frac{1}{(n+1)^3}\sum_{r=0}^{n}r^3$

$\Rightarrow S=\frac{1}{(n+1)^3}\left\{\frac{n(n+1)}{2}\right\}^2\Rightarrow S=\frac{n^2}{4(n+1)}$

Now, $S=\frac{25}{24}$ (given) which is only possible for 5

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9. If m and n are any two odd positive integers with n < m, then the largest positive integer which divides all numbers of the form, $m^2 - n^2$ can be

(A) 4 (B) 6 (C) 8 (D) 9

Key. C

Sol. Let $m = 2k - 1$ *and* $n = 2p - 1, p < k$

Then $m^2 - n^2 = (m+n)(m-n)$

$= (2k + 2p - 2)(2k - 2p) = 4(k + p - 1)(k - p)$

Further if k and p both even, then k-p is even but k+p-1 is odd

If k and p both odd then k-p is even but k+p-1 is odd. If one is even and other odd then k-p is odd but k+p-1 is even. Thus in every case $(k-p)(k+p-1)$ even

$\therefore m^2 - n^2$ is divisible by $4 \times 2 = 8$. Hence, $m^2 - n^2$ is divisible by 8 or any multiple of 8. The largest integer among the given options is 8

10. The number of terms in $(a_1 + a_2 + a_3 + a_4)^3$ is

(A) 64 (B) 81 (C) 30 (D) 20

Key: D

Hint Any term of $(a_1 + \alpha_2 + a_3 + a_4)^3$ is of the form $a_1^{\alpha} . a_2^{\beta} . a_3^{\gamma} . a_4^{\delta}$.

where $\alpha + \beta + \gamma + \delta = 3$, $\alpha, \beta, \gamma, \delta \in \{0, 1, 2, 3\}$

Thus number of terms is 20.

11. The value of ${}^{12}C_2 + {}^{13}C_3 + {}^{14}C_4 + \ldots\ldots\ldots\ldots + {}^{999}C_{989}$ is

(A) ${}^{1000}C_{11} - 12$ (B) ${}^{1000}C_{11} + 12$ (C) ${}^{999}C_{11} - 12$ (D) ${}^{1000}C_{989}$

Key: A

Hint Since ${}^{10}C_0 + {}^{11}C_1 + {}^{12}C_2 + {}^{13}C_3 + \ldots\ldots\ldots + {}^{999}C_{989}$

$= {}^{1000}C_{989} = {}^{1000}C_{11}$

$\left(\text{Since}, {}^{10}C_0 = {}^{11}C_0 \text{ and } {}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r\right)$

So, ${}^{12}C_2 + {}^{13}C_3 + {}^{14}C_4 + \ldots\ldots + {}^{999}C_{989} = {}^{1000}C_{11} - 12$

12. The sum $S_n = \sum_{k=0}^{n} (-1)^K . {}^{3n}C_k$, where $n = 1, 2, \ldots\ldots\ldots\ldots$ is

(A) $(-1)^n . {}^{3n-1}C_{n-1}$ (B) $(-1)^n . {}^{3n-1}C_n$ (C) $(-1)^n . {}^{3n-1}C_{n+1}$ (D) None of these

Key: B

Hind: $S_n = {}^{3n}C_0 - {}^{3n}C_1 + {}^{3n}C_2 + \ldots\ldots\ldots + (-1)^n . {}^{3n}C_n$

But ${}^{3n}C_0 = {}^{3n-1}C_0$

$-{}^{3n}C_1 = -{}^{3n-1}C_0 - {}^{3n-1}C_1$

${}^{3n}C_2 = {}^{3n-1}C_1 + {}^{3n-1}C_2$

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$-{}^{3n}C_3 = -{}^{3n-1}C_2 - {}^{3n-1}C_3$

....

$(-1)^n . {}^{3n}C_n = (-1)^n . {}^{3n-1}C_{n-1} + (-1)^n . {}^{3n-1}C_n$

On adding we get $S_n = (-1)^n . {}^{3n-1}C_n$

13. The Coefficient of x^9 in $\left(x - {}^{21}C_0\right)\left(x - {}^{21}C_1\right)\left(x - {}^{21}C_2\right)........\left(x - {}^{21}C_{10}\right) is$

(A) $2^{40} - \frac{1}{2}\,{}^{42}C_{20}$ (B) $2^{39} - \frac{1}{2}\,{}^{42}C_{21}$ (C) $2^{40} - {}^{42}C_{20}$ (D) $2^{39} - \frac{1}{4}\,{}^{42}C_{21}$

Key: D

Hint: Coefficient x^{10} =sum of products of ${}^{20}C_0, {}^{20}C_1 .... {}^{20}C_{10}$. Taking '2' at a time.

14. $\sum_{K=1}^{10} \frac{(-1)^{K-1}}{K} . \left(10_{C_K}\right) =$

(A) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + --- + \frac{1}{11}$ (B) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + --- + \frac{1}{10}$

(C) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + --- + \frac{1}{9}$ (D) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + --- + \frac{1}{12}$

Key: B

Hint: Required value is $\frac{10_{C_1}}{1} - \frac{10_{C_2}}{2} + \frac{10_{C_3}}{3} ---- \frac{10_{C_{10}}}{10}$

To find which, consider $(1-x)^{10} = 10_{C_0} - 10_{C_1} x + 10_{C_2} x^2 ----- + 10_{C_{10}} x^{10}$

$$\Rightarrow \frac{(1-x)^{10} - 1}{x} = -\left[10_{C_1} - 10_{C_2} x + ------ + 10_{C_{10}} x^9\right]$$

$$\Rightarrow \int_0^1 \frac{1-(1-x)^{10}}{x} dx = \int_0^1 \left[10_{C_1} - 10_{C_2} x + --- + 10_{C_{10}} x^9\right] dx$$

$$= 10_{C_1} - \frac{10_{C_2}}{2} + \frac{10_{C_3}}{3} ----- \frac{10_{C_{10}}}{10}$$

To find LHS consider $I_n = \int_0^1 \frac{1-(1-x)^n}{x} dx \Rightarrow I_{n+1} - I_n = \int_0^1 (1-x)^n dx = \frac{1}{n+1}$

$$\therefore I_{n+1} = \frac{1}{n+1} + I_n$$

$$\therefore I_{10} = \int_0^1 \frac{1-(1-x)^{10}}{x} dx = \frac{1}{10} + I_9 = \frac{1}{10} + \frac{1}{9} + I_8 \approx ---$$

$$= \frac{1}{10} + \frac{1}{9} + \frac{1}{8} + ---- + 1$$

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15. The sum $\sum_{k=1}^{\infty}\frac{6^k}{\left(3^k-2^k\right)\left(3^{k+1}-2^{k+1}\right)}=$

(A) 1 (B) 2 (C) 3 (D) 4

Key: B

Hint: $\frac{6^k}{\left(3^k-2^k\right)\left(3^{k+1}-2^{k+1}\right)}=\frac{3^k-2^k}{\left(3^k-2^k\right)\left(3^{k+1}-2^{k+1}\right)}$

$$=\frac{3^k}{3^k-2^k}-\frac{3^{k+1}}{3^{k+1}-2^{k+1}}$$

$$\sum_{k=1}^{\infty}\frac{3^k}{3^k-2^k}-\frac{3^{k+1}}{3^{k+1}-2^{k+1}}=\frac{3}{3-2}-\underset{n\to\infty}{Lt}\frac{3^2}{3^n-2^n}$$

$$=3-1=2$$

16. The value of the expression ${}^{10}C_0 10^9-{}^{10}C_1 9^9+{}^{10}C_2 8^9..........-{}^{10}C_9$ *is*

A) $\lfloor\underline{9}$ B) $\lfloor\underline{10}$ C) $9\lfloor\underline{10}$ D) 0

Key: D

Hint: Given expression = No.of on to functions from a set of 9 elements to a set of 10 elements = 0

17. $\sum_{r=0}^{n}\frac{n-3r+1}{n-r+1}\frac{{}^nC_r}{2^r}$ is equal to

a) $\frac{1}{2^n}$ b) $\frac{1}{3^n}$ c) $\frac{1}{4^n}$ d) $\frac{1}{2^n}+1$

Key: A

Hint: $S=\sum_{r=0}^{n}\left(1-\frac{2r}{n-r+1}\right)\frac{{}^{n}C_{r}}{2^r}=\sum_{r=0}^{n}\frac{{}^{n}C_{r}}{2^r}-\sum_{r=0}^{n}\frac{{}^{n}C_{r-1}}{2^{r-1}}=\frac{1}{2^n}$

18. The sum of all the coefficient of those terms in the expansion of $(a + b + c + d)^8$ which contains b but not c

(A) 6305 (B) 6561 (C) 256 (D) 4^8

Key: A

Hint: Sum of the coefficients of the terms not containing c is 3^8 and of the term not containing b and c both is 2^8, so required sum = $3^8 - 2^8$.

19. If $\lfloor\underline{15}=2^{P_1}3^{P_2}5^{P_3}7^{P_4}11^{P_5}13^{P_6}$ then $\sum_{r=1}^{6}P_r$ is

(A) 24 (B) 23 (C) 22 (D)21

Key: A

Hint: $15!=2^{11}\times 3^6\times 5^3\times 7^2\times 11^1\times 13^1$

$$\therefore\sum_{r=1}^{6}P_r=11+6+3+2+1+1=24$$

20. The number of the positive integer pairs (x,y) such that $\frac{1}{x}+\frac{1}{y}=\frac{1}{2007}$ where $x<y$ is

(A) 5 (B) 6 (C) 7 (D) 8

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Key: C

Hint: $\frac{1}{x}+\frac{1}{y}=\frac{1}{2007}$

$\Rightarrow (x+y)2007 = xy$

$\Rightarrow xy-2007x-2007y=0$

$(x-2007)(y-2007)=2007^2=3^4\times 223^2$

The number of pairs is equal to the number of divisors of 2007^2 that is $(4+1)\times(2+1)=15$

Since $x<y$, so required number of pairs = 7

21. The greatest integer less than or equal to $\left(\sqrt{2}+1\right)^6$ is

(a) 194 (b) 195 (c) 196 (d) 197

Key: d

Hint: Let $\left(\sqrt{2}+1\right)^6$ = 1 + F, where I is an integer and 0 < F <1.

Let f = $\left(\sqrt{2}-1\right)^6$. We have

$\sqrt{2}-1=\frac{1}{\sqrt{2}+1} \Rightarrow 0<\sqrt{2}-1<1 \Rightarrow$ 0 < f < 1.

Also 1 + F + f = $\left(\sqrt{2}+1\right)^6+\left(\sqrt{2}-1\right)^6$

= 2[^6C_o. 2^3 + $^6C_2$. 2^2 + $^6C_4$ .2 + $^6C_6$]

= 2(8 + 60 + 30 + 1) = 198

Hence F + f = 198 – I is an integer. But 0 < F + f < 2.

Therefore F + f = 1, and thus, I = 197.

22. The coefficient of x^6 in the expansion of $\left(1+\frac{x}{\lfloor 1}+\frac{x^2}{\lfloor 2}+\frac{x^3}{\lfloor 3}+\frac{x^4}{\lfloor 4}+\frac{x^5}{\lfloor 5}\right)^2$ is

(A) $\frac{2}{15}$ (B) $\frac{4}{15}$ (C) $\frac{31}{360}$ (D) $\frac{2}{45}$

Key: C

Hint: Coefficient of x^6 is $\frac{1}{\lfloor 5}\frac{1}{\lfloor 1}+\frac{1}{\lfloor 4}\frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}\frac{1}{\lfloor 3}+\frac{1}{\lfloor 2}\frac{1}{\lfloor 4}+\frac{1}{\lfloor 1}\frac{1}{\lfloor 5}$

$=\frac{1}{\lfloor 6}\left[{}^6C_1+{}^6C_2+{}^6C_3+{}^6C_4+{}^6C_5\right]$

$=\frac{1}{\lfloor 6}\left(2^6-2\right)=\frac{31}{360}$

23. Coefficient of x^{2009} in $(1 + x + x^2 + x^3 + x^4)^{1001} (1 - x)^{1002}$ is

(A) 0 (B) 4. $^{1001}C_{501}$

(C) -2009 (D) none of these

Key: A

Hint: $(1 + x + x^2 + x^3 + x^4)^{1001} (1 - x)^{1002}$

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$= (1 - x)(1 - x^5)^{1001}$, so all the powers of x will be of the 5m or 5m + 1 ($m \in I$)

So coff. of x^{2009} will be 0

24. If $\left(x^2+2x+4\right)^n=\sum_{r=0}^{2n} a_r x^r$, then $\sum_{r=n}^{2r}\left(\frac{a_{2n-r}}{a_r}\right)$ is

a) $2^{2n+1}-1$ b) $\frac{4^{n+1}-1}{3}$

c) 4^{n-1} d) $\frac{4^{n-1}+1}{2}$

Key : b

Sol : Put x = 2x

$$\left(4x^2+4x+4\right)^n=\sum_{r=0}^{2n} a_r 2^r x^r$$

Put $x=\frac{1}{x}$

$$\left[1+2x+(2x)^2\right]^n=\sum_{r=0}^{2n} a_r\, x^{2n-r}$$

Put 2x = y

$$\left(1+y+y^2\right)^n=\sum_{r=0}^{2n} a_r \frac{y^{2n-r}}{2^{2n-r}}$$

Equating coefficient of $x^r \frac{1}{4^n} a_r 2^r = a_{2n-r} 2^{-r}$

$$\Rightarrow \frac{a_{2n-r}}{a_r}=\frac{1}{4n}4^r$$

$$\sum_{r=n}^{2n}\frac{a_{2n-r}}{a_n}=\sum_{r=n}^{2n}\frac{1}{4n}4r=\left(\frac{4^{n+1}-1}{3}\right)$$

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25. Coefficient of x^6 in $\left((1+x)\left(1+x^2\right)^2\left(1+x^3\right)^3.....\left(1+x^n\right)^n\right)$ is

a) 26 b) 28 c) 30 d) 35

Key : b

Sol : The coefficient of x^6 in the given expression = coefficient of x^6 in

$\left(1+{}^6C_1x^6\right)\left(1+{}^5C_1x^5\right)\left(1+{}^4C_1x^4\right)\left(1+{}^3C_1x^3+{}^3C_2x^6\right)\left(1+{}^2C_1x^2+{}^2C_2x^4\right)\left(1+x\right)$

= coefficient of x^6 in $\left(1+6x^6+5x^5+4x^4\right)\left(1+2x^2+3x^3+x^4+6x^5+3x^6\right)\left(1+x\right)$

= coefficient of x^6 in $\left(11x^5+17x^6\right)\ \left(1+x\right)$ = 28

26. The coefficient of x^2 in $\left(1+x\right)\left(1+2x\right)\left(1+4x\right).....\left(1+2^n x\right)$ is

a) $\dfrac{\left(2^n+1\right)\left(2^n+2\right)}{3}$ b) $\dfrac{\left(2^{n+1}-1\right)\left(2^{n+1}-2\right)}{3}$

c) $\dfrac{\left(2^{n+1}+1\right)\left(2^{n+1}+2\right)}{3}$ d) $\dfrac{\left(2^{n+1}+1\right)\left(2^{n+1}-2\right)}{5}$

Key : b

Sol : Coefficient of $x^2 = \dfrac{1}{2}\left[\left(1+2+2^2......+2^n\right)^2\right]-\left[1^2+2^2+....\left(2^n\right)^2\right]$

$$=\frac{1}{2}\left[\left(\frac{2^{n+1}-1}{1}\right)^2-\left(\frac{\left(2^2\right)^{n+1}-1}{3}\right)\right]$$

$$=\frac{1}{2}\left[\left(2^{n+1}-1\right)\left[\left(2^{n+1}-1\right)-\frac{2^{n+1}+1}{3}\right]\right]$$

$$=\frac{\left(2^{n+1}-1\right)\left(2^{n+1}-2\right)}{3}$$

27. Let $n\in N$ and $n<\left(5\sqrt{3}+8\right)^4$. Then the greatest value of n is

a) 77040 b) 77042

c) 77041 d) none of these

Key : c

Sol : Suppose $x=\left(5\sqrt{3}+8\right)^4$

= [x] + [x]

$0<5\sqrt{3}-8<1$

$0<\left(5\sqrt{3}-8\right)^4<1$

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Let $F=\left(5\sqrt{3}-8\right)^4$

$0 < F < 1$

$\left[x\right]+\left\{x\right\}+F=\left(5\sqrt{3}+8\right)^4+\left(5\sqrt{3}-8\right)^4$

$={}^4C_0\left(5\sqrt{3}\right)^4+{}^4C_1\left(5\sqrt{3}\right)^3(8)+....+\left({}^4C_0\left(5\sqrt{3}\right)^4-{}^4C_1\left(5\sqrt{3}\right)^3(8)+...\right)$

$=2.\left[C_0\left(5\sqrt{3}\right)^4+C_2(8)^2\left(5\sqrt{3}\right)^2+C_4(8)^4\right]$

$=2\left[625(9)+(6)64(25)(3)+(64)(64)\right]$

$=2\left[5625+28800+4096\right]=77042$

$\left\{x\right\}$ + F must be an integer

Also $0<\left\{x\right\}+F<2$

$\Rightarrow\left\{x\right\}+f=1$

$\Rightarrow\left[x\right]=77042-1=77041$

28. If $P_k=\dfrac{1-x^{k+1}}{1-x}$, the number of terms in the product $P_1.P_2....P_n$ is

a) $\dfrac{n(n+1)}{2}$

b) $\dfrac{n^2-n}{2}$

c) $\dfrac{n^2+n-2}{2}$

d) $\dfrac{n^2+n+2}{2}$

Key ; D

Sol : $P_k=\dfrac{1-x^{k+1}}{1-x}$

$$P_1P_2P_3.....P_n=\frac{\left(1-x^2\right)\left(1-x^3\right)\left(1-x^4\right).....\left(1-x^{n+1}\right)}{\left(1-x^n\right)}$$

No. of terms = 1 + max. power of x = $1+\dfrac{n(n+1)}{2}=\dfrac{n^2+n+2}{2}$

29. The value of $a_0+a_2+a_4+....$ is

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a) $\frac{2^n-1}{2}$ b) $\frac{2^n+1}{2}$

c) $\frac{(n-1)!}{2}$ d) $\frac{(n+1)!}{2}$

Key: d

Sol : $(1+x)(1+x+x^2)....(1+x+...+x^n) = a_0 + a_1x + a_2x^2 + a_3x^3 + ...$

Put, x = 1,

$2\times3\times4\times...(n-1) = a_0 + a_1 + a_2 + a_3 + ...$

$0 = a_0 - a_1 + a_2 - a_3 + ...$

Put x = -1,

$(n+1)! = 2[a_0 + a_2 + ...] \Rightarrow a_0 + a_2 + ... = \frac{(n+1)!}{2}$

30. The coefficient of abc^3de^2 in the expansion of $(a+b+c+d+e)^8$ is equal to

a) 3630 b) 3600

c) 3360 d) none of these

KEY : c

Sol : Coefficient of $abc^3de^2 is \frac{8!}{3!2!} = 3360$

31. If each coefficient in the expansion of the expression $x(1+x)^n (n \in N)$ in powers of 'x' is divided by the exponent of corresponding power, then the sum of the values thus obtained is equal to

A) $\frac{2^n}{n+1}$ B) $\frac{2^n-1}{n+1}$ C) $\frac{2^n+1}{n+1}$ D) $\frac{2^{n+1}-1}{n+1}$

Key. D

Sol. $\frac{c_0}{1} + \frac{c_1}{2} + \frac{c_2}{3} + ... + \frac{c_{n-1}}{n} + \frac{c_n}{n+1} = \frac{2^{n+1}-1}{n+1}$

32. If $x^{15} - x^{13} + x^{11} - x^9 + x^7 - x^5 + x^3 - x = 7$ then

(A) x^{16} is 15 (B) x^{16} is less than 15

(C) x^{16} greater than 15 (D) Nothing can be said about x^{16}

Key. C

Sol. $x^{15} - x^{13} + x^{11} - x^9 + x^7 - x^5 + x^3 - x$

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$= x(x^8+1)(x^4+1)(x^2-1)$

$x^{16}-1=(x^8+1)(x^4+1)(x^2+1)(x^2-1)$

$= \frac{7}{x}(x^2+1) = 7\left[\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2+2\right] > 14$

$\therefore x^{16} > 15$

33. The value of the expression $\sum_{0\le i}\sum_{<j\le n}(-1)^{i+j-1}\ {}^nC_i.{}^nC_j =$

1) ${}^{2n-1}C_n$ 2) ${}^{2n}C_n$ 3) ${}^{2n+1}C_n$ 4) none

Key: 1

Hint: Let the required value be S

$$\sum_{i=0}^{n}\sum_{j=0}^{n}(-1)^{i+j-1}\ {}^nC_i{}^nC_j = \sum_{i=0}^{n}(-1)^{2i-1}\left({}^nC_i\right)^2 + 2S$$

$$0 \quad = -\sum_{i=0}^{n}\left({}^nC_i\right)^2 + 2S$$

$$S = {}^{2n-1}C_{n-1} = {}^{2n-1}C_n$$

34. If $A_{(i,j)}$ be the co-efficient of $a^i b^j c^{2010-i-j}$ in the expansion of $(a+b-c)^{2010}$ then

(A) $A_{i,\,i}$ is defined for $i \le 1010$ (B) $A_{i,\,j} = A_{j,\,i}$

(C) $A_{2i,\,3i}$ is defined for $i \le 405$ (D) $A_{0,\,1} = 2000$

Key. B

Sol. Clearly, $A_{i,j} = \frac{2010!}{i!\ j!\,(2010-i-j)!}$

$A_{j,i} = \frac{2010!}{j!i!(2010-i-j)!}$

Hence, $A_{i,j} = A_{j,i}$

35. If n > 3 and $a, b \in R$, then the value of $ab - n(a-1)(b-1) + \frac{n(n-1)}{1.2}(a-2)$ $(b-2) - ..... + (-1)^n(a-n)(b-n)$ is equal to

(A) $a^n + b^n$ (B) $\frac{a^n - b^n}{a-b}$

(C) $(ab)^n$ (D) 0

Key. D

Sol. $T_{k+1} = (-1)^k\,.{}^nC_k(a-k)(b-k)$

$= (-1)^{k.n}C_k\left[ab - k(a+b) + k^2\right]$

Thus, the sum of series in (i)

$= \sum_{k=0}^{n}(-1)^k\,.{}^nC_k\left[ab - k(a+b) + k^2\right]$

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$= ab\sum_{k=0}^{n}(-1)^k.{}^nC_k - (a+b)\sum_{k=0}^{n}(-1)^k k.{}^nC_k + \sum_{k=0}^{n}(-1)^k k^2.{}^nC_k$

We know that

$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + ... + {}^nC_nx^n ...(ii)$

Differentiating both sides w.r.t. x, we get

$n(1+x)^{n-1} = 1.\,{}^nC_1 + 2.{}^nC_2x + 3.\,{}^nC_3x^2 + ... + n.{}^nC_nx^{n-1} ....(iii)$

Multiplying it by x we get

$n.x(1+x)^{n-1} = 1.{}^nC_1x + 2.\,{}^nC_2x^2 + 3.\,{}^nC_3x^3 + ... + n.{}^nC_nx_n$

Differentiating w.r.t. x, we get

$n(1+x)^{n-1} + n(n-1)x(1+x)^{n-2}$

$= 1^2.{}^nC_1 + 2^2.{}^nC_2x + ... + n^2.{}^nC_nx^{n-1}$ ...(iv)

Putting x = –1 in (ii), (iii) and (iv), we get

$\sum_{k=0}^{n}(-1)^k.{}^nC_k = 0$

$\sum_{k=0}^{n}(-1)^k k.{}^nC_k = 0$

$\sum_{k=0}^{n}(-1)^k k.^2\,{}^nC_k = 0$

Thus, the sum of the series is 0

36. The expression $\frac{1}{\sqrt{4x+1}}\left[\left[\frac{1+\sqrt{4x+1}}{2}\right]^7 - \left[\frac{1-\sqrt{4x+1}}{2}\right]^7\right]$ is a polynomial in x of degree

(A) 7 (B) 5 (C) 4 (D) 3

Key. D

Sol. put = y and expand

37. Co-efficient of α^t in the expansion of,

$(\alpha + p)^{m-1} + (\alpha + p)^{m-2}(\alpha + q) + (\alpha + p)^{m-3}(\alpha + q)^2 + ... (\alpha + q)^{m-1}$

where $\alpha \neq -q$ and $p \neq q$ is :

(A) $\frac{{}^mC_t\left(p^t - q^t\right)}{p-q}$ (B) $\frac{{}^mC_t\left(p^{m-t} - q^{m-t}\right)}{p-q}$

(C) $\frac{{}^mC_t\left(p^t + q^t\right)}{p-q}$ (D) $\frac{{}^mC_t\left(p^{m-t} + q^{m-t}\right)}{p-q}$

Key. B

Sol. E = $(\alpha + p)^{m-1}$

$\Rightarrow$ *co-efficient of* α^t = = =

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38. Let $\left(5+2\sqrt{6}\right)^n = p + f$ where $n \in N$ and $p \in N$ and $0 < f < 1$ then the value of, $f^2 - f + pf - p$ is :

(A) a natural number (B) a negative integer

(C) a prime number (D) are irrational number

Key. B

Sol. Ans is – 1

39. The coefficient of x^4 of in the expansion $(1+5x+9x^2+......\infty)(1+x^2)^{11}$ is

(A) ${}^{11}C_2 + 4\,{}^{11}C_1 + 3$ (B) ${}^{11}C_2 + 3\,{}^{11}C_1 + 4$

(C) $3\,{}^{11}C_2 + 4\,{}^{11}C_1 + 3$ (D) 171

Key. D

40. The number of distinct terms in the expansion of $\left(x+y^2\right)^{13} + \left(x^2+y\right)^{14}$ is

A) 27 B) 29 C) 28 D) 25

Key. C

Sol. To get common terms in both the expansions

$x^{r_1}\left(y^2\right)^{13-r_1} = \left(x^2\right)^{r_2}\left(y\right)^{14-r_2}$

$r_1 = 2r_2$ & $26 - 2r_1 = 14 - r_2$

$r_1 = 8;\ r_2 = 4$

$\therefore$ Only one term is common.

41. If $A_{(i,j)}$ be the co-efficient of $a^i b^j c^{2010-i-j}$ in the expansion of $(a+b-c)^{2010}$ then

(A) $A_{i,\,i}$ is defined for $i \le 1010$ (B) $A_{i,\,j} = A_{j,\,i}$

(C) $A_{2i,\,3i}$ is defined for $i \le 405$ (D) $A_{0,\,1} = 2000$

Key. A

Sol. Clearly, $A_{i,j} = \dfrac{2010!}{i!\,j!\,(2010-i-j)!}$

$A_{j,i} = \dfrac{2010!}{j!\,i!\,(2010-i-j)!}$

Hence, $A_{i,j} = A_{j,i}$

42. The value of $\dfrac{{}^{11}C_0}{1} + \dfrac{{}^{11}C_1}{2} + \dfrac{{}^{11}C_2}{3} + ... + \dfrac{{}^{11}C_{11}}{12}$ will be

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A) $\frac{1}{12}\left(2^{12}-1\right)$ B) $\frac{1}{12}\left(2^{11}-1\right)$ C) $\frac{1}{12}\left(2^{11}+1\right)$ D) None of these

Key. A

Sol. Using ${}^{n}C_{k}=\frac{n}{k}.{}^{n-1}C_{k-1}$

For $0\le k\le 11$

$\frac{{}^{11}C_{k}}{k+1}=\frac{{}^{12}C_{k+1}}{12}$

So, given expression is

$$\Rightarrow \frac{1}{12}\sum_{k=0}^{11}{}^{12}C_{k+1}\Rightarrow \frac{1}{12}\left[\sum_{k=0}^{11}{}^{12}C_{k}-{}^{12}C_{0}\right]\Rightarrow \frac{1}{12}\left(2^{12}-1\right)$$

43. $\sum_{m=1}^{n}\left(\sum_{k=1}^{m}\left(\sum_{p=k}^{m}{}^{n}C_{m}\cdot{}^{m}C_{p}\cdot{}^{p}C_{k}\right)\right)=$

a) $3^{n}-2^{n}$ b) $4^{n}-3^{n}$ c) $3^{n}+2^{n}$ d) $4^{n}-1$

Key: B

Hint: $\sum_{m=1}^{n}{}^{n}C_{m}\left(\sum_{k=1}^{m}\left(\sum_{p=k}^{m}\frac{m!}{p!(m-p)!}\cdot\frac{p!}{k!(p-k)!}\right)\right)$

$$=\sum_{m=1}^{n}{}^{n}C_{m}\left(\sum_{k=1}^{m}\left(\sum_{p=k}^{m}{}^{m-k}C_{p-k}\right)\frac{m!}{k!(m-k)!}\right)$$

$$=\sum_{m=1}^{n}{}^{n}C_{m}\left(\sum_{k=1}^{m}2^{m-k}\cdot{}^{m}C_{k}\right)$$

$$=\sum_{m=1}^{n}{}^{n}C_{m}\left((1+2)^{m}-2^{m}\right)=\sum_{m=1}^{n}\left({}^{n}C_{m}3^{m}-{}^{n}C_{m}2^{m}\right)$$

$$=(1+3)^{n}-1-(1+2)^{n}+1=4^{n}-3^{n}$$

44. The value of $2000_{C_2}+2000_{C_5}+2000_{C_8}+......+2000_{C_{2000}}=?$

a) $\frac{2^{1999}-1}{3}$ b) $\frac{2^{1999}+1}{3}$ c) $\frac{2^{2000}+1}{3}$ d) $\frac{2^{2000}-1}{3}$

Key: D

Hint $(1+x)^{n}=n_{C_0}+n_{C_1}x+n_{C_2}x^{2}+......+n_{C_n}x^{n}$

Put x = 1, w, w^{2} and add

$$\Rightarrow C_{2}+C_{5}+C_{8}+......=\frac{1}{3}\left\{2^{n}+(-1)^{n}\left(w^{2n+1}+w^{n+2}\right)\right\}$$

$$=\frac{1}{3}\left\{2^{2000}+(-1)^{2000}\left(w^{4001}+w^{2002}\right)\right\}$$

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$= \frac{2^{2000}-1}{3}$

45. $3^{10}\ {}^{20}C_0\ {}^{20}C_{10} - 3^9\ {}^{20}C_1\ {}^{19}C_9 + ... + {}^{20}C_{10}\ {}^{10}C_0$ equals (${}^nC_r$ denote coefficient of x^r in $(1 + x)^n$.)

(A) ${}^{10}C_{10}.3^{10}$ (B) ${}^{20}C_{10}.3^{10}$

(C) ${}^{20}C_{10}.2^{10}$ (D) ${}^{20}C_{10}\ {}^{10}C_8$

Key. C

Sol. Coefficient of x^{10} in $\left[{}^{20}C_0(3+x)^{20} - {}^{20}C_1(3+x)^{19} + ... + {}^{20}C_{10}(3+x)^{10}\right]$

$= x^{10}$ in $(3 + x - 1)^{20}$

$= x^{10}$ in $(2 + x)^{20}$

$= 2^{10}\ {}^{20}C_{10}$

46. Number of ways, 3 persons having 6 one rupee coins, 7 one rupee coins, 8 one rupee coins respectively donate 10 one rupee coin collectively is ……….

a) 29 b) 83 c) 44 d) 47

Key. D

Sol. Coeff of $x^{10}\left(1+x+x^2+-----+x^6\right)\left(1+x+--+x^7\right)\left(1+x+---+x^8\right)$ is

47. $\sum_{r=1}^{n}\sum_{p=0}^{r-1} {}^nC_r.\ {}^rC_p.2^p$ is equal to

a) $4^n - 3^n + 1$ b) $4^n - 3^n - 1$

c) $4^n - 3^n + 2$ d) $4^n - 3^n$

Key. D

Sol. $\sum_{r=1}^{n} nC_r\left((1+2)^r - 2^r\right) = \sum_{r=1}^{n} nC_r 2^r - \sum_{r=1}^{n} nC_r 2^r$

$=\left(4^n - 1\right) - \left(3^n - 1\right) = 4^n - 3^n$

48. The value of $\binom{50}{6} - \binom{5}{1}\binom{40}{6} + \binom{5}{2}\binom{30}{6} - \binom{5}{3}\binom{20}{6} + \binom{5}{4}\binom{10}{6}$ where $\binom{n}{r}$ denotes ${}^nC_r$, is

(A) 15625 (B) 0

(C) 1000000 (D) 2250000

Key. D

Sol. ${}^{50}C_6 - {}^5C_1\ {}^{40}C_6 + {}^5C_2\ {}^{30}C_6 - {}^5C_3\ {}^{20}C_6 + {}^5C_4\ {}^{10}C_6$ = coefficient of x^6 in $[{}^5C_0 (1 + x)^{50} - {}^5C_1 (1 + x)^{40} + {}^5C_2 (1 + x)^{30} - {}^5C_3 (1 + x)^{20} + {}^5C_4 (1 + x)^{10} - {}^5C_5 (1 + x)^0]$ = coefficient x^6 in $[(1 + x)^{10} - 1]^5$
= coefficient of x^6 in $({}^{10}C_1\ x + {}^{10}C_2\ x^2 + ...)^5 = {}^5C_1\ ({}^{10}C_2)\ ({}^{10}C_1)^4 = 2250000$.

49. The value of the expression ${}^{10}C_0 10^9 - {}^{10}C_1 9^9 + {}^{10}C_2 8^9 .... - {}^{10}C_9$ *is*

a) $\lfloor 9$ b) $\lfloor 10$ c) $9\lfloor 10$ d) 0

Key. D

Sol. Given expression = No.of on to functions from a set of 9 elements to a set of 10 elements = 0

50. The term independent of x and y in the expansion of

$[(\sqrt{x}+\frac{1}{\sqrt{x}})^2+(\sqrt{y}+\frac{1}{\sqrt{y}})^2+(\sqrt{xy}+\frac{1}{\sqrt{xy}})^2-4]^n$ is

a) $(\sum_{r=0}^{n} {}^nc_r)^2$ b) $\sum_{r=0}^{n} ({}^nc_r)^2$ c) $(\sum_{r=0}^{n} {}^nc_r)^3$ d) $\sum_{r=0}^{n} ({}^nc_r)^3$

Key. D

Sol. The given expression can be written as $(1+x)^n.(1+y)^n.(1+\frac{1}{xy})^n$. The constant term is clearly $C_0^3+C_1^3+-----+C_n^3$ where $c_r = {}^nc_r$.

51. The sum of the rational terms in the expansion of $\left(\sqrt{2}+\sqrt[5]{3}\right)^{10}$ is :

a) 41 b) 42 c) 39 d) 45

Key. A

Sol. $T_{r+1}=10C_r\,2^{\frac{10-r}{2}}.3^{\frac{r}{5}}$

This is rational , if $\frac{10-r}{2}$ and $\frac{r}{5}$ are integers.

$\therefore$ There are only two rational terms

Namely $10C_0\left(\sqrt{2}\right)^{10}\left(3^{\frac{1}{5}}\right)^{0}$ and $10C_{10}\left(\sqrt{2}\right)^{0}\left(3^{\frac{1}{5}}\right)^{10}$

$\therefore$ sum = 32 + 9 = 41

52. The values of ‘r’ such that $(100)C_r\left(\frac{1}{5^8}\right)^{100-r}\left(\frac{1}{2^6}\right)^{r}$ is rational is :

a) 84 b) 85 c) 86 d) 42

Key. A

Sol. Direct verification in sufficient.

53. If $\left(1+x\right)^n = a_0+a_1x+a_2x^2+.....+a_nx^n$ in which a_{n-3},a_{n-2},a_{n-1} are in AP, then

a) a_1,a_2,a_3 are in GP b) a_1,a_2,a_3 are in HP c) $n=7$ d) $n=14$

Key. C

Sol. $a_{n-3}=a_3, a_{n-2}=a_2, a_{n-1}=a_1\left(\because nC_r = nC_{n-r}\right)$

$\therefore$ (A) is correct.

a_1,a_2,a_3 are in AP $\Rightarrow n,\frac{n(n-1)}{2},\frac{n(n-1)(n-2)}{6}$ are in AP.

$$\frac{n+\frac{n(n-1)(n-2)}{6}}{2}=\frac{n(n-1)}{2}$$

$$6n+n^3-3n^2+2n=6n^2-6n$$

$n^3-9n^2+14n=0$, $(n-7)(n-2)=0$

$\therefore$ n = 7.

54. The coefficient of $a^8b^6c^4$ in the expansion of $\left(a+b+c\right)^{18}$ is :

a) $18C_4\times14C_6$ b) $18C_{10}\times10C_8$

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c) $18C_8 \times 12C_8$ d) $18C_{14} \times 14C_7$

Key. A

Sol. $(a+b+c)^{18} = \left[a+(b+c)\right]^{18}$

$= 18C_0 a^{18} + 18C_1 a^{17}(b+0^{71} + 18C_2 a^{16}(b+c)^2 + ... + 18C_{10} a^8 (b+c)^{10} + ...$

$\therefore$ coefficient of $a^8 b^6 c^4 = 18C_{10} \times 10C_6$

55. In the expansion of $(1 + x)(1 + x + x^2) ..... (1 + x + x^2 + ... + x^{2n})$, the sum of coefficients is
(A) 1 (B) (2n)!
(C) (2n)! + 1 (D) (2n + 1)!

Key. D

Sol. Let $(1 + x)(1 + x + x^2) ..... (1 + x + ... + x^{2n}) = a_0 + a_1 x + a_2 x^2 + ...$
Put x = 1, we get
$a_0 + a_1 + a_2 + ... = 2.3.4 ..... (2n + 1) = (2n + 1)!$

56. If x = $\left(\sqrt{3}+\sqrt{2}\right)^6$, then the largest integer not exceeding x is
(A) 729 (B) 971
(C) 969 (D) 970

Key. C

Sol. $x = (5 + 2\sqrt{6})^3$

Let x = k + f k$\in$N, 0 < f < 1

Consider: $5 - 2\sqrt{6} = \dfrac{1}{5+2\sqrt{6}} \Rightarrow 0 < 5 - 2\sqrt{6} < 1 \Rightarrow 0 < (5 - 2\sqrt{6})^n < 1$

Let $F = (5 - 2\sqrt{6})^n$, 0 < F < 1

$k + f + F = (5 + 2\sqrt{6})^3 + (5 - 2\sqrt{6})^3 = 2[5^3 + {}^3C_2\, 5.\, (2\sqrt{6})^2]$

= 2[125 + 360] = 970.

$\Rightarrow$ f + F is a integer, but 0 < f + F < 2

$\Rightarrow$ f + F = 1

$\Rightarrow$ k = 969.

57. The coefficient of x^{53} in the expansion of $\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} 2^m$ is

A) ${}^{100}C_{47}$ B) ${}^{100}C_{53}$ C) $-{}^{100}C_{53}$ D) $-{}^{100}C_{100}$

Key. C

Sol. $(x-3)^{100} + {}^{100}C_1 (x-3)^{99} 2^1 + ... + {}^{100}C_{100} 2^{100}$

$= \{(x-3)+2\}^{100} = (x-1)^{100} = (1-x)^{100} = {}^{100}C_{53}(-1)^{53} = -{}^{100}C_{53}$

58. If 7 divides $32^{32^{32}}$, the reminder is
A) 1 B) 0 C) 4 D) 81

Key. C

Sol. $32^{32^{32}} = 3m+1, m \in I^+$

$32^{32^{32}} = (2^5)^{3m+1} = 2^{15m+5} = 28n+5$

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59. The number of irrational terms in the expansion of $\left(5^{\frac{1}{3}}+2^{\frac{1}{4}}\right)^{100}$ is

A) 94 B) 92 C) 93 D) 91

Key. B

Sol. $\left(5^{\frac{1}{3}}+2^{\frac{1}{4}}\right)^{100}=\left(2^{\frac{1}{4}}+5^{\frac{1}{3}}\right)^{100}$

$T_{r+1}={}^{100}C_r\left(2^{\frac{1}{4}}\right)^{100-r}.\left(5^{\frac{1}{3}}\right)^{r}={}^{100}C_r\,2^{25-\frac{r}{4}}.5^{\frac{r}{3}}$

For rational terms, 'r' should be divisible by 12.

$\therefore$ No. of rational terms = 9

$\therefore$ No. of irrational terms = 101-9=92

60. The greatest integer less than or equal to $(\sqrt{3}+1)^6$ is

A) 416 B) 414 C) 417 D) 415

Key. D

Sol. $(\sqrt{3}+1)^6+(\sqrt{3}-1)^6=2\left[{}^6C_0(\sqrt{3})^6+{}^6C_2(\sqrt{3})^4+{}^6C_4(\sqrt{3})^2+{}^6C_6\right]=416$

Let $(\sqrt{3}+1)^6=I+F$ where I is integral part and F is fractional part

Let $(\sqrt{3}-1)^6=G$

$0<F<1; 0<G<1 \Rightarrow 0<F+G<2 \Rightarrow F+G=1$

$I+F+G=416 \Rightarrow I+1=416 \Rightarrow I=415$

61. $2\,{}^{10}C_0+\frac{2^2}{2}\,{}^{10}C_1+\frac{2^3}{3}\,{}^{10}C_2+......+\frac{2^{11}}{11}\,{}^{10}C_{10}=$

A) $2^{11}-1/11$ B) $3^{11}-1/11$ C) $2^{11}-2/11$ D) $4^{11}-1/11$

Key. B

Sol. Conceptual

62. If the fourth term in the expansion of $\left(2+\frac{3}{8}x\right)^{10}$ has the maximum numerical value then the range of x contains.

(a) $-\frac{64}{21}<x<-2$ (b) $1<x<\frac{64}{21}$ (c) $-2<x<\frac{64}{21}$ (d) $-\frac{64}{21}<x<2$

Key. A

Sol. Conceptual

63. If n is an odd natural number then $\sum_{r=0}^{n}\frac{(-1)^r}{n_{C_r}}$ is equal to

(a) 0 (b) 1 / n (c) $\frac{n}{2^n}$ (d) $n.2^n$

Key. A

Sol. Conceptual

64. If $(1+x)^n=\sum_{r=0}^{n}n_{c_r}x^r$ then $C_0^{\,2}+\frac{C_1^{\,2}}{2}+\frac{C_2^{\,2}}{3}+.......+\frac{C_n^{\,2}}{n+1}=$

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(a) $\frac{\lfloor 2n}{\left(\lfloor n\right)^2}$ (b) $\frac{\lfloor 2n+1}{\left(\lfloor n+1\right)^2}$ (c) $\frac{\lfloor 2n-1}{\left(\lfloor n-1\right)^2}$ (d) $\frac{\lfloor n}{\left(\lfloor n-1\right)^2}$

Key. B

Sol. Conceptual

65. In the expansion of $\left(\sqrt[3]{\frac{a}{b}}+\sqrt[3]{\frac{b}{\sqrt{a}}}\right)^{21}$ the term containing same powers of a and b is

(a) 11th (b) 13th (c) 12th (d) 6th

Key. B

Sol. $T_{r+1}=21_{C_r}\, a^{\frac{42-3r}{6}}\, b^{\frac{2r-21}{3}}$

$42-3r=4r-42\Rightarrow r=12$

66. The coefficient of x^4 of the expansion $(1+5x+9x^2+.......\infty)(1+x^2)^{11}$ is

(a) $11_{C_2}+4.11_{C_1}+3$ (b) $11_{C_2}+3.11_{C_1}+4$ (c) $3.11_{C_2}+4.11_{C_1}+3$ (d) 171

Key. D

Sol. Co-efficient of x^4

= $(1+5x+9x^2+13x^3+17x^4+......)(1+11x^2+11_{C_2}x^4+.....)$

$=11_{C_2}+99+17$

67. The coefficient of x^{n-2} in the polynomial $(x-1)(x-2)(x-3).....(x-n)$ is

(a) $\frac{n(n^2+2)(3n+1)}{24}$ (b) $\frac{n(n^2-1)(3n+2)}{24}$ (c) $\frac{n(n^2+1)(3n+4)}{24}$ (d) None

Key. B

Sol. $T_2=(x-\alpha_1)(x-\alpha_2)(x-\alpha_3)......(x-\alpha_n)$

$=x^n-\sum\alpha_i x^{n-1}+\alpha_1\alpha_2 x^{n-2}+...............$

68. If $1,\propto_1,\propto_2,\propto_3,\propto_4$ are the fifth roots of unity then $\sum_{i=1}^{4}\frac{1}{2-\propto_i}=$

(a) $\frac{51}{31}$ (b) $\frac{49}{31}$ (c) $\frac{25}{32}$ (d) $\frac{25}{16}$

Key. B

Sol. we know that $z^5-1=(z-1)(z-\alpha_1)(z-\alpha_2)(z-\alpha_3)(z-\alpha_4)$

Take log on both sides, diff.w.r.t. z and put z = 2.

69 If a_1 and a_2 be the coefficients of x^n in the expansion of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then

1. $a_2=2a_1$ **2.** $a_1=2a_2$ **3.** $a_1=a_2$ **4.** None of these

Key. **2**

Sol. Consider T_{r+1} in $(1+x)^{2n}$ $\therefore$ $T_{r+1}={}^{2n}C_r x^r$

$a_1=$ Coefficient of $x^n={}^{2n}C_n$

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$$=\frac{2n!}{n!n!}=\frac{2n(2n-1)!}{n(n-1)!(n)!}=\frac{2(2n-1)!}{(n-1)!(n!)}$$

Again coefficient of T_{r+1} in $(1+x)^{2n-1}$ is ${}^{2n-1}C_r$

$a^2=$ Coefficient of x^n in $(1+x)^{2n-1}$

$$={}^{2n-1}C_n=\frac{(2n-1)!}{n!(n-1)!}$$

$$a_2=\frac{1}{2}\frac{2(2n-1)!}{(n-1)!n!}=\frac{1}{2}a_1$$

$\therefore\ 2a_2=a_1$

70. If $C_0, C_1, C_2.....$ are binomial coefficients in the expansion $\sum_{r=0}^{n} C_r x^r$, then value of the expression (series)

$$\frac{2C_0}{1}+\frac{3C_1}{2}+\frac{4C_2}{3}+\frac{5C_3}{4}+......+\text{ is}$$

1. $\frac{2^n+1}{n+1}$ **2.** $\frac{2^n-1}{n+1}$ **3.** $\frac{2^n(n+3)-1}{n+1}$ **4.** $\frac{2^n(n+2)-1}{n+1}$

Key. **3**

Sol. Given

$$(1+x)^n=C_0+C_1x+c_2x^2+.....+C_nx^n$$

Integrating both sides with respect to x, we get

$$\frac{(1+x)^{n+1}}{n+1}=\frac{C_0x}{1}+\frac{C_1x^2}{2}+\frac{C_2x^3}{3}+.....+\frac{C_nx^{n+1}}{n+1}+k$$

Putting $x=0$,

We get $k=\frac{1}{n+1}$

$$\therefore\quad \frac{(1+x)^{n+1}-1}{n+1}$$

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$$= \frac{C_0 x}{1} + \frac{C_1 x^2}{2} + \frac{C_2 x^3}{3} + ... + \frac{C_n x^{n+1}}{n+1}$$

Multiplying with x both sides

$$\frac{x(1+x)^{n+1} - x}{n+1} = \frac{C_0 x^2}{1} + \frac{C_1 x^3}{2} + \frac{C_2 x^4}{3} + ... + \frac{C_n x^{n+1}}{n+1}$$

Differentiating with respect to x

$$\frac{(n+1)x(1+x)^n + (1+x)^{n+1} - 1}{n+1}$$

$$= \frac{2C_0 x}{1} + \frac{3C_1 x^2}{2} + \frac{4C_2 x^3}{3} + ... + \frac{(n+2)C_n x^{n+1}}{n+1}$$

Now putting $x = 1$ both sides, we get

$$\frac{2^{n+1} + (n+1)2^n - 1}{n+1}$$

$$= \frac{2C_0}{1} + \frac{3C_1}{2} + \frac{4C_2}{3} + ... + \frac{(n+2)C_n}{n+1}$$

$$\frac{2^n(n+3) - 1}{n+1} = \frac{2C_0}{1} + \frac{3C_1}{2} + \frac{4C_2}{3} + ... + \frac{(n+2)C_n}{n+1}$$

71. In the expansion of $(1+x)^{70}$,the sum of coefficients of odd powers of x is

1. 0 **2.** 2^{69} **3.** 2^{70} **4.** 2^{71}

Key. **2**

Sol. Fact. The sum of the coefficients of odd powers in the expansion of $(1+x)^n$ = sum of the coefficients of even powers in $(1+x)^n$

$= 2^{n-1}$

$2^{70-1} = 2^{69}$

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72. Number of irrational terms in the expansion of $\left(\sqrt[5]{2}+\sqrt[10]{3}\right)^{60}$ are

1. 54 **2.** 61 **3.** 30 **4.** 31

Key. **1**

Sol. Given $\left(\sqrt[5]{2}+\sqrt[10]{3}\right)^{60}=\left(2^{\frac{1}{5}}+3^{\frac{1}{10}}\right)^{60}$

Now L.C.M. of 5 and 10 is 10

$\therefore$ Number of rational terms let us writes $T_{r+1}={}^{60}C_r\left(2^{\frac{1}{5}}\right)^{60-r}\left(3^{\frac{1}{10}}\right)^{r}$

$={}^{60}C_r\,2^{12-\frac{r}{5}}3^{\frac{r}{10}}$

As $0\le r\le 60$

$\therefore\quad r=0,10,20,30,40,50,60$

$\therefore$ Number of rational terms is 7

$\therefore$ Number of irrational terms equals to

Total number of terms - Number of rational terms

$=61-7=54$

73. If $C_0,C_1,C_2,....,C_n$ are Binomial Coefficients, such that $S_n=\sum_{r=0}^{n}\frac{1}{{}^nC_r}$ and $t_n=\sum_{r=0}^{n}\frac{r}{{}^nC_r}$ then $\frac{t_n}{S_n}$ equals

1. $\frac{n}{2}$ **2.** $\frac{n(n+1)}{2}$ **3.** $\frac{n+1}{2}$ **4.** None of these

Key. **1**

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Sol. Given $t_n = \sum_{r=o}^{n} \frac{r}{C_r} = \sum_{r=0}^{n} \frac{n-(n-r)}{C_{n-r}} \quad \left(\because {}^n C_r = {}^n C_{n-r}\right)$

$$= \sum_{r=0}^{n} \frac{n}{C_{n-r}} - \sum_{r=0}^{n} \frac{n-r}{C_{n-r}}$$

$$= nS_n - \left[\frac{n}{C_n} + \frac{n-1}{C_{n-1}} + ... + \frac{1}{C_1} + 0\right]$$

$$t_n = nS_n - \sum_{r=0}^{n} \frac{r}{C_r}$$

$$t_n = nS_n - t_n$$

$$2t_n = nS_n$$

$$\therefore \ \frac{t_n}{S_n} = \frac{n}{2}$$

74. If $(1+x)^n = \sum_{r=0}^{n} C_r x^r$, then the value of $3C_1 + 7C_2 + \ 11C_3 + ... + (4n-1)C_n$ is

1. $(4n-1)2^n$ **2.** $(2n-1)2^n$ **3.** $(2n-1)2^n + 1$ **4.** $(4n-1)2^n - 1$

Key. **3**

Sol. Let $S = 3C_1 + 7C_2 + 11C_3 + ... + (4n-1)C_n$

Let us write

$$T_r = (4r-1)C_r$$

$$T_r = 4rC_r - C_r$$

$$= 4r\frac{n}{r}C_{r-1} - C_r \text{ using } {}^nC_r = \frac{n}{r}.{}^{n-1}C_{r-1} \backslash$$

$$\therefore \quad \sum_{r=1}^{n} T_r = 4n\sum_{r=1}^{n} C_{r-1} - \sum_{r=1}^{n} C_r$$

$$= 4 \ n \,.\, 2^{n-1}\left(2^n - 1\right)$$

$$= 2n.2^n - 2^n + 1$$

$S = 2^n(2n-1) + 1$ By using

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$(1+x)^{n-1} = C_0 + C_1x + C_2x^2 + ... + C_{n-1}x^{n-1}$

$2^{n-1} = C_0 + C_1 + C_2 + ... + C_{n-1}$

75. Middle term in the expansion of $\left(1-3x+3x^2-x^3\right)^{2n}$ is

1. $\frac{(6n)!x^n}{(3n)!(3n)!}$ **2.** $\frac{(6n)!.x^{3n}}{(3n)!}$ **3.** $\frac{(6n)!}{(3n)!(3n)!}(-x)^{3n}$ **4.** None of these

Key. 3

Sol. $\left(1-3x+3x^2-x^3\right)^{2n} = (1-x)^{6n}$

Concept: Index $= 6n$ which is even so most middle term

is $\left(\frac{6n}{2}+1\right)^{th}$ $i.e., (3n+1)^{th}$ term is middle term,

$T_{3n+1} = {}^{6n}C_{3n}(-x)^{3n} = \frac{6n!}{3n!3n!}(-x)^{3n}$

76. Let n be an odd natural number greater than 1. Then the number of zeros at the end of the sum $99^n + 1$ is

(A) 3 (B) 4 (C) 2 (D) None of these

Key. C

Sol. $1+99^n = 1+(100-1)^n = 1+$
$\left\{{}^nC_0 100^n - {}^nC_1.100^{n-1} + ... - {}^nC_n\right\}$
Because n is odd $= 100\left\{{}^nC_0.100^{n-1} - {}^nC_1.100^{n-2} + ... -\right.$
$\left.{}^nC_{n-2}.100 + {}^nC_{n-1}\right\}$
= 100 $\times$ integer whose units place is different from 0
$\left[\because {}^nC_{n-1} = n, \text{has odd digit at unit place}\right]$
$\therefore$ There are two zeros at the end of the sum $99^n + 1$

77. If $C_0, C_1, C_2, ...C_n$ are the Binomial coefficients in the expansion $(1+x)^n$. 'n' being even, then $C_0 + (C_0+C_1) + (C_0+C_1+C_2) + ...(C_0+C_1+C_2+....+C_{n-1}) =$ is equal to

(A) $n2^n$ (B) $n.2^{n-1}$ (C) $n.2^{n-3}$ (D) $n.2^{n-2}$

Key. B

Sol. Sum $= \left\{C_0 + \left(c_1 + c_2 + ...+c_{n-1}\right)\right\} + = \left\{\left(c_0+c_1\right) + \left(c_0+c_1+.......+c_{n-2}\right)\right\} +$

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$\{(c_0+c_1+c_2)+(c_0+c_1+......+c_{n-3})\}+.......\ to\ \left(\frac{n}{2}\right)$

Terms = $=(c_0+c_1+......c_n)\times\frac{n}{2}=n.2^{n-1}$

78. The positive integral values of n such that $1.2^1+2.2^2+3.2^3+4.2^4+5.2^5+....+n.2^n=2^{(n+10)}+2$ is

(A) 313 (B) 513 (C) 413 (D) 613

Key. B

Sol.

$2^1+2^2+2^3+...+2^n=2^{n+1}-2$

$2^2+2^3+...2^n=2^{n+1}-2^2$

$2^3+...+2^n=2^{n+1}-2^3$

... ...

$+2^n=2^{n+1}-2$

$=n(2^{n+1})-(2^{n+1}-2)$

$=2^{n+1}(n-1)+2$

Given that $2^{n+1}(n-1)+2=2^{2+10}+2$

$\Rightarrow(n-1)2^{n+1}=2^{n+10}$

$\Rightarrow n-1=2^9$

$\Rightarrow n=2^9+1=513$

79. If $\sum_{r=0}^{n}\left\{\frac{{}^nC_{r-1}}{{}^nC_r+{}^nC_{r-1}}\right\}^3=\frac{25}{24}$, then n is equal to

(A) 3 (B) 4 (C) 5 (D) 6

Key. C

Sol. Let $t_r=\frac{{}^nC_{r-1}}{{}^nC_r+{}^nC_{r-1}}=\frac{{}^nC_{r-1}}{{}^{n+1}C_r}=\frac{{}^nC_{r-1}}{\frac{n+1}{r}\ {}^nC_{r-1}}$

$\therefore l_r=\frac{r}{n+1}$

Now,

$S=\sum_{r=0}^{n}\{l_r\}^3\Rightarrow S=\sum_{r=0}^{n}\frac{r^3}{(n+1)^3}=\frac{1}{(n+1)^3}\sum_{r=0}^{n}r^3$

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$$\Rightarrow S=\frac{1}{(n+1)^3}\left\{\frac{n(n+1)}{2}\right\}^2 \Rightarrow S=\frac{n^2}{4(n+1)}$$

Now, $S=\frac{25}{24}$ (given) which is only possible for 5

80. If m and n are any two odd positive integers with n < m, then the largest positive integer which divides all numbers of the form, m^2-n^2 can be

(A) 4 (B) 6 (C) 8 (D) 9

Key. C

Sol. Let $m=2k-1$ *and* $n=2p-1, p<k$

Then $m^2-n^2=(m+n)(m-n)$

$=(2k+2p-2)(2k-2p)=4(k+p-1)(k-p)$

Further if k and p both even, then k-p is even but k+p-1 is odd

If k and p both odd then k-p is even but k+p-1 is odd. If one is even and other odd then k-p is odd but k+p-1 is even. Thus in every case $(k-p)(k+p-1)$ even

$\therefore m^2-n^2$ is divisible by $4\times2=8$. Hence , m^2-n^2 is divisible by 8 or any multiple of 8. The largest integer among the given options is 8

81. If $C_0, C_1, C_2, ..., C_n$ denote the binomial coefficients in the expansion of $(1+x)^n$, then $\sum_{r=0}^{n}(-1)^r\ {}^nC_r\frac{1+r\log_e 10}{(1+\log_e 10^n)^r}$ is equal to

A) 0 B) 1 C) 2 D) 3

Key. A

Sol. Let $\log_e 10=x$

$$=\sum_{r=0}^{n}(-1)^r\ {}^nC_r\frac{1+rx}{(1+nx)^r}$$

$$=\left(1-\frac{1}{1+nx}\right)^n-\frac{nx}{1+nx}\left(1-\frac{1}{1+nx}\right)^{n-1}=0$$

82. In the expansion of $\left(\sqrt[3]{\frac{a}{b}}+\sqrt[3]{\frac{b}{\sqrt{a}}}\right)^{21}$ the term containing same powers of a and b is

A) 11th B) 13th C) 12th D) 6th

Key. B

Sol. $t_{r+1}={}^{21}C_r\left(\frac{a}{b}\right)^{\frac{21-r}{3}}\frac{b^{r/3}}{a^{r/6}}={}^{21}C_r\ a^{\frac{42-3r}{6}}\ b^{\frac{2r-21}{3}}$

$\therefore$ 42 – 3r = 4r – 42 i.e. r = 12

$\therefore$ 13th term contains same powers of a and b

83. The coefficient of x^4 of in the expansion $\left(1+5x+9x^2+....\right)\left(1+x^2\right)^{11}$ is

A) ${}^{11}C_2+4\ {}^{11}C_1+3$ B) ${}^{11}C_2+3\ {}^{11}C_1+4$ C) $3\,{}^{11}C_2+4\ {}^{11}C_1+3$ D) 171

Key. D

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Sol. Coefficient of x^4 is $\left(1+5x+9x^2+...\right)\left(1+x^2\right)^{11}$

$=\left(1+5x+9x^2+...\right)\left(1+11x^2+{}^{11}C_2\left(x^2\right)^2+...\right)$

$=\left(1+5x+9x^2+13x^3+17x^4+...\right)\left(1+11x^2+{}^{11}C_2x^4+...\right)$

Coefficient of x^4 is ${}^{11}C_2+9+11+17=55+99+17=171$

84. If $\left(1+x+x^2\right)^{25}=a_0+a_1x+a_2x^2+...+a_{50}x^{50}$, then $a_0+a_2+a_4+...+a_{50}$ is

A) even
B) odd and of the form 3n
C) odd and of the form (3n – 1)
D) odd and of the form (3n + 1)

Key. A

Sol. putting x = 1 and – 1 and adding

$a_0+a_2+...+a_{50}=\dfrac{3^{25}+1}{2}=\dfrac{\left(1+2\right)^{25}+1}{2}$

$=\dfrac{{}^{25}C_0+{}^{25}C_1 2+{}^{25}C_2.2^2+...+{}^{25}C_{25}.2^{25}+1}{2}$

$=\dfrac{2\left[1+{}^{25}C_1+{}^{25}C_2.2+...+{}^{25}C_{25}.2^{24}\right]}{2}=2\left[13+{}^{25}C_2+...+{}^{25}C_{25}.2^{23}\right]$ is an even integer

85. The co-efficient of x^{n-2} in the polynomial (x – 1) (x – 2) (x – 3) … (x – n) is

A) $\dfrac{n\left(n^2+2\right)\left(3n+1\right)}{24}$
B) $\dfrac{n\left(n^2-1\right)\left(3n+2\right)}{24}$
C) $\dfrac{n\left(n^2+1\right)\left(3n+4\right)}{24}$
D) none of these

Key. B

Sol. $E=\left(x-\alpha_1\right)\left(x-\alpha_2\right)\left(x-\alpha_3\right)...\left(x-\alpha_n\right)$ wher $\alpha_1=1,\ \alpha_2=2$ etc

$=x^n-\left(\sum\alpha_1\right)x^{n-1}+\left(\sum\alpha_1\alpha_2\right)x^{n-2}+...$

Hence co-efficient of x^{n-2} = sum of all the products of the first 'n' natural numbers taken two at a time $=\dfrac{\left(1+2+3+...+n\right)^2-\left(1^2+2^2+...+n^2\right)}{2}=\dfrac{n\left(n^2-1\right)\left(3n+2\right)}{24}$

86. The remainder when 27^{40} is divided by 12 is

A) 3
B) 7
C) 9
D) 11

Key. C

Sol. $27^{40}=3^{120}$

$3^{119}=\left(4-1\right)^{119}={}^{119}C_0 4^{119}-{}^{119}C_1 4^{118}+{}^{119}C_2 4^{117}-{}^{119}C_3 4^{116}+...+{}^{119}C_{118}4-1$

$\therefore$ $3^{119}=4k-1$

$\therefore$ $3^{120}=12k-3=12\left(k-1\right)+9$

$\therefore$ The required remainder is 9

87. If $\sum_{r=0}^{2n}a_r\left(x-1\right)^r=\sum_{r=0}^{2n}b_r\left(x-2\right)^r$ and $b_r=\left(-1\right)^{r-n}$ for all $r\geq n$, then $a_n=$

A) ${}^{2n+1}C_{n-1}$
B) ${}^{3n}C_n$
C) ${}^{2n+1}C_n$
D) 0

Key. C

Sol. Let x – 1 = t, then

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$$\sum_{r=0}^{2n} a_r t^r = \sum_{r=0}^{2n} b_r (t-1)^r$$

$\therefore \quad a_n =$ coefficient of t^n in $\sum_{r=0}^{2n} b_r (t-1)^r$

$=$ coefficient of t^n b in $\left(b_0 + b_1(t-1) + ... + b_n(t-1)^n + b_{n+1}(t-1)^{n+1} + ... + b_{2n}(t-1)^{2n}\right)$

$= b_n\,{}^{n}C_0 + b_{n+1}\,{}^{n+1}C_1(-1)^1 + b_{n+2}\,{}^{n+2}C_2(-1)^2 + ... + b_{2n}\,{}^{2n}C_n(-1)^n$

$= (-1)^{n-n}.{}^{n}C_0 + (-1)^{n+1-n+1}, {}^{n+1}C_1 + ... + (-1)^{2n-n+n}\,{}^{2n}C_n$

$= {}^{n}C_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + ... + {}^{2n}C_2 + ... + {}^{2n}C_n = {}^{2n}C_n = {}^{2n+1}C_{n+1}$

$= {}^{2n+1}C_n$

88. In the expansion of $\left(7^{\frac{1}{3}} + 11^{\frac{1}{9}}\right)^{6561}$ total number of terms free from radical signs is

A) 729 B) 730 C) 731 D) none of these

Key. B

Sol. $T_{r+1} = {}^{6561}C_r\, 7^{\frac{6561-r}{3}}\, 11^{r/9}$

The term is free from radical sign, if r is multiple of 9 and 6561 – r is a multiply of 3

i.e. r = 0, 9, 18, 27, … 6561. These are 730 in number,

89. The last two digits of the number $(23)^{14}$ are

A) 01 B) 03 C) 09 D) None of these

Key. C

Sol. $(23)^{14} = (529)^7 = (530-1)^7$

$= {}^{7}C_0(530)^7 - {}^{7}C_1(530)^6 + ... - 7C_5(530)^2 + {}^{7}C_6 530 - 1$

$= {}^{7}C_0(530)^7 - {}^{7}C_1(530)^6 + ... + 3710 - 1$

= 100 m + 3709

$\therefore$ last two digits are 09

90. $\sum\sum_{0 \le i < j \le n} \left(C_i + C_j\right)^2 =$ ____

a) $(n-1).{}^{2n}C_n + 2^{2n}$ b) ${}^{2n}C_n + 2^{2n}$ c) ${}^{2n}C_n - (n+1)2^n$ d) None

Key. A

Sol. $\sum\sum\left(C_i + C_j\right)^2 = \sum\sum C_i^2 + C_j^2 + 2C_iC_j = n\left(C_0^2 + C_1^2 + ... + C_n^2\right) + 2\sum\sum C_iC_j$ is

$$= n.{}^{2n}C_n + 2\left(\frac{2^{2n} - {}^{2n}C_n}{2}\right)$$

Since $\left(C_0 + C_1 + ... + C_n\right)^2 = C_0^2 + C_1^2 + ... + C_n^2 + 2\sum\sum C_iC_j$

$2^{2n} = {}^{2n}C_n + 2\sum\sum C_iC_j$

91. The value of $\frac{1}{\lfloor 1 \lfloor 15} + \frac{1}{\lfloor 3 \lfloor 13} + \frac{1}{\lfloor 5 \lfloor 11} + \frac{1}{\lfloor 7 \lfloor 9}$ is

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a) $\frac{2^{14}}{\lfloor 15}$ b) $\frac{2^{15}}{\lfloor 16}$ c) $\frac{2^{10}}{\lfloor 15}$ d) $\frac{2^{13}}{\lfloor 15}$

Key. C

Sol. Multiply and divide by 16!

92. The term independent of x and y in the expansion of

$$\left[\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2+\left(\sqrt{y}+\frac{1}{\sqrt{y}}\right)^2+\left(\sqrt{xy}+\frac{1}{\sqrt{xy}}\right)^2-4\right]^n \text{ is}$$

a) $\left(\sum_{r=0}^{n}{}^nC_r\right)^2$ b) $\sum_{r=0}^{n}\left({}^nC_r\right)^2$ c) $\left(\sum_{r=0}^{n}{}^nC_r\right)^3$ d) $\sum_{r=0}^{n}\left({}^nC_r\right)^3$

Key. D

Sol. It can be simplified as $(1+x)^n(1+y)^n\left(1+\frac{1}{xy}\right)^n$

The constant term is $C_0^3+C_1^3+.....+C_n^3$

93. If $C_r={}^nC_r$, then $(C_0-C_2+C_4-C_6+...)^2+(C_1-C_3+C_5-C_7+...)^2$ is

a) 2^{2n} b) 2^n c) 2^{n^2} d) $2^{\frac{n+1}{2}}$

Key. B

Sol. Put $x=i$ in the expansion of $(1+x)^n$ we get

$(C_0-C_2+C_4-C_6.....)+i(C_1-C_3+C_5....)=2^{n/2}\left[\cos n\pi/4+i\sin n\pi/4\right]$ Take modulus both sides and square it

94. The coefficient of x^3 in $\left(2x-\frac{3}{x^2}\right)^9$ is

a) 41472 b) $2^8.3^5$ c) $2^8.3^4$ d) 44172

Key. A

Sol. r = 2 , coefficient $={}^9C_2\,(2)^7(-3)^2=(2)^9(3)^4$

95. If the coefficient of x in $\left(x^2+\frac{k}{x}\right)^5$ is 270, then the value of k is

a) 2 b) 3 c) 4 d) 5

Key. B

Sol. r = 3 , ${}^5C_3\,k^3=270$,k = 3

96. In the expansion of $\left(2+\frac{x}{3}\right)^n$, coefficient of x^7 and x^8 are equal. Then the value of n is

a) 49 b) 50 c) 55 d) 56

Key. C

Sol. ${}^nC_7\frac{2^{n-7}}{3^7}={}^nC_8\frac{2^{n-8}}{3^8}$, n = 55.

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97. Value of the numerically greatest term in the expansion of $\left(1+\frac{1}{\sqrt{3}}\right)^{20}$ is

a) ${}^{20}C_7\,\frac{1}{27}$ b) ${}^{20}C_8\,\frac{1}{27}$

c) ${}^{20}C_8\,\frac{1}{27\sqrt{3}}$ d) ${}^{20}C_7\,\frac{1}{27\sqrt{3}}$

Key. D

Sol. $\frac{21}{\sqrt{3}+1}=7g.....,T_{7+1}={}^{20}C_7\,\frac{1}{27\sqrt{3}}.$

98. The coefficients of 9th, 10th and 11th terms in the expansion of $(1+x)^n$ are in A.P. then the Value of n is

a) 23 b) 7 c) 4 d) 21

Key. A

Sol. r = 9, $n^2-(4\times9+1)n+4\times9^2-2=0$, n = 14 or 23.

99. The number of terms in the expansion of $(p+q+r+s)^n$ is

a) $\frac{n(n+1)}{2}$ b) $\frac{(n+1)(n+2)(n+3)}{6}$

c) $\frac{n(n+1)(2n+1)}{6}$ d) $\frac{(n+1)^2n^2}{4}$

Key. B

Sol. Standard formulae.

100. The number of irrational terms in the expansion of $\left(\sqrt{3}+\sqrt[4]{5}\right)^{124}$ is

a) 125 b) 32 c) 93 d) 34

Key. C

Sol. No of rational terms are $\left[\frac{124}{4}\right]+1$ = 32,

101. If a_1,a_2,a_3,a_4 are any 4 consecutive binomial coefficients of the expansion of $(1+x)^n$ Respectively, then $\frac{a_1}{a_1+a_2},\frac{a_2}{a_2+a_3},\frac{a_3}{a_3+a_4}$ are in

a) A.G.P b) G.P c) H.P d) A.P

Key. D

Sol. $\frac{1}{1+3},\frac{3}{3+3},\frac{3}{3+1}=\frac{1}{4},\frac{3}{6},\frac{3}{4}=\frac{3}{12},\frac{6}{12},\frac{9}{12}$, A.P.

102. If the sum of the coefficients in the expansion $(x - 2y+3z)^n$ is 128, then the greatest coefficient in $(1+x)^n$ is

a) 25 b) 35 c) 45 d) 31

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Key. B

Sol. Put $x = y = z = 1$ then $2^n = 128$, n=7, $7_{C_3} = \frac{7.6.5}{1.2.3} = 35$

103. If n = 2009, then N = $2009^n - 1982^n - 1972^n + 1945^n$ is divisible by

a) 658 b) 1977 c) 1988 d) 2009

Key. B

Sol. Since n is odd $x^n + y^n$ has divisor x+y.

104. If C_0, C_1 …C_{10} are the binomial coefficient in the expansion of $(1+x)^{10}$, then

$2C_0 + \frac{2^2}{2}C_1 + \frac{2^3}{3}.C_2 + \ldots + \frac{2^{11}}{11}.C_{10}$

a) $\frac{2^{11}}{11}$ b) $\frac{2^{11}-1}{11}$ c) $\frac{3^{11}}{11}$ d) $\frac{3^{11}-1}{11}$

Key. D

Sol. $\int_0^2 (1+x)^{10}dx = 10_{C_0}2 + \frac{2^2 10_{C_1}}{2} + \ldots\ldots\ldots \frac{2^{11}10_{C_{10}}}{11} = \frac{3^{11}-1}{11}$

105. If $(1+px+x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then $\sum_{r=0}^{2n}(2r+1)a_r =$

a) $(p+2)^n$ b) $(2p+1)(p+2)^2$

c) (2n+1) $(p+2)^n$ d) $(p+2)^{n+1}$

Key. C

Sol. $(2\sum_{r=0}^{2n} ra_r + \sum_{r=0}^{2n} a_r = 2n(2+p)^{n-1}(P+2) + (2+P)^n$

$= (2n+1)(P+2)^n$

106. Let $(x^3 + \alpha x^2 + 2x - 5)^{19}(x^2 + \beta x - 41)^8 (x^4 - x^3 + x - 7)^6 =$

$x^{97} + 391x^{96} + a_{95}x^{95} + a_{94}x^{94} + \ldots + a_1 x + a_0$ be an identity, where

$\alpha, \beta, a_{95}, a_{94}, \ldots, a_1, a_0$ are integers. If $\alpha + \beta < 10$, then the smallest possible value of α is

a) 7 b) 8 c) 31 d) 23

Key. C

Sol. It will be an identity even if we replace x by $\frac{1}{y}$ and considering numerator alone.

Differentiating on both sides with respect to y, at y =0 we get $19\alpha + 8\beta = 397$, $\alpha + \beta = 10 - k$ where k is positive integer. Put $\beta = 10 - \alpha - k$ in first equation we get $11\alpha - 8k = 317$

$\therefore \alpha = 31$

107. The number of different terms in the expansion of

$(1+x)^{2009} + (1+x^2)^{2008} + (1+x^3)^{2007}$ is

a) 3683 b) 4017 c) 4018 d) 4352

Key. B

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Sol. $(1+x)^{2009}$ has 2010 terms in total. $(1+x^2)^{2008}$ has a constant, even power of x starting from 2 to 4016 but already even powers of x from 2 to 2008 were enumerated in $(1+x)^{2009}$. The remaining terms containing even powers of x are from 2010 to 4016. They are 1004 in number. In $(1+x^3)^{2007}$ has a constant, multiples of 3 as powers of x. Even multiples of 3 from 6 to 4014 were already enumerated in above expansions. The remaining even multiples of 3 from 4020 to 6018 which are 334 in number. Odd multiples of 3 as powers of x from 3 to 2007 were enumerated in above expansions and the remaining from 2013 to 6021 are to be enumerated. They are 669 in number.

$\therefore$ the number of terms in the expansion = 2010 + 1004 + 669 + 334 = 4017.

108. The term independent of x and y in the expansion of

$$[(\sqrt{x}+\frac{1}{\sqrt{x}})^2+(\sqrt{y}+\frac{1}{\sqrt{y}})^2+(\sqrt{xy}+\frac{1}{\sqrt{xy}})^2-4]^n$$ is

A) $(\sum_{r=0}^{n} {}^nc_r)^2$ B) $\sum_{r=0}^{n} ({}^nc_r)^2$ C) $(\sum_{r=0}^{n} {}^nc_r)^3$ D) $\sum_{r=0}^{n} ({}^nc_r)^3$

Key. D

Sol. The given expression can be written as $(1+x)^n.(1+y)^n.(1+\frac{1}{xy})^n$. The constant term is clearly $C_0^3+C_1^3+----+C_n^3$ where $c_r={}^nc_r$.

109. The sum of the rational terms in the expansion of $\left(\sqrt{2}+\sqrt[5]{3}\right)^{10}$ is :

A) 41 B) 42 C) 39 D) 45

Key. A

Sol. $T_{r+1}=10C_r\,\frac{10-r}{2^2}.3^{\frac{r}{5}}$

This is rational , if $\frac{10-r}{2}$ and $\frac{r}{5}$ are integers.

$\therefore$ There are only two rational terms

Namely $10C_0\left(\sqrt{2}\right)^{10}\left(3^{\frac{1}{5}}\right)^0$ and $10C_{10}\left(\sqrt{2}\right)^0\left(3^{\frac{1}{5}}\right)^{10}$

$\therefore$ sum = 32 + 9 = 41

110. If $(1+x)^n=a_0+a_1x+a_2x^2+.....+a_nx^n$ in which a_{n-3},a_{n-2},a_{n-1} are in AP, then

A) a_1,a_2,a_3 are in GP B) a_1,a_2,a_3 are in HP

C) $n=7$ D) $n=14$

Key. C

Sol. $a_{n-3}=a_3,a_{n-2}=a_2,a_{n-1}=a_1\left(\because nC_r=nC_{n-r}\right)$

$\therefore$ (A) is correct.

$a_,,a_2,a_3$ are in AP $\Rightarrow n,\frac{n(n-1)}{2},\frac{n(n-1)(n-2)}{6}$ are in AP.

$$\frac{n+\frac{n(n-1)(n-2)}{6}}{2}=\frac{n(n-1)}{2}$$

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$$6n+n^3-3n^2+2n=6n^2-6n$$

$$n^3-9n^2+14n=0,\ (n-7)(n-2)=0$$

$\therefore$ n = 7.

111. If x = $\left(2+\sqrt{3}\right)^n$, then the value of $x-x^2+x[x]$ where [.] denotes the greatest integer function, is equal to

A) 1 B) 2 C) 2^{2n} D) 2^n

Key. A

Sol. $x-x^2+x[x]=x-x(x-[x])=x(1-\{x\})$

Now $x+x_1$ = even integer where $x_1=\left(2-\sqrt{3}\right)^n$ clearly $x_1 \in (0, 1)\ \forall\ n \in N$.

$\therefore \{x\}+x_1$= integer

$\Rightarrow \{x\}+x_1=1$

112. In the binomial expansion of $\left(\sqrt{x}+\dfrac{1}{2\sqrt[4]{x}}\right)^n$, $n \in N$ the coefficients of first, second and third terms form an A.P. The number of rational terms in the expansion is (Assume that x is a rational number and $\sqrt{x}$, $\sqrt[4]{x}$ are irrational)

A) 1 B) 2 C) 3 D) 4

Key. C

Sol. $$\left(\sqrt{x}+\frac{1}{2\sqrt[4]{x}}\right)^n=\sum_{r=0}^{n}{}^nC_r\left(\sqrt{x}\right)^{n-r}.\left(\frac{1}{2\sqrt[4]{x}}\right)^r=\sum_{r=0}^{n}{}^nC_r.\frac{1}{2^r}x^{\frac{2n-3r}{4}}$$

1, $\dfrac{n}{2}$, $\dfrac{n(n-1)}{8}$ are in A.P. $\Rightarrow n=8$

113. $\left(\lfloor\underline{1}+2\lfloor\underline{2}+3\lfloor\underline{3}+.......+2009\lfloor\underline{2009}\right)+1$

A) $2\lfloor\underline{2010}$ B) $\lfloor\underline{2010}$ C) $\lfloor\underline{2011}$ D) $2\lfloor\underline{2011}$

Key. B

Sol. $\lfloor\underline{1}+2\lfloor\underline{2}+3\lfloor\underline{3}+..........+n\lfloor\underline{n}=\lfloor\underline{n+1}-1$

114. The sum of coefficients of the terms of degree 'm' in the expansion of $(1+x)^n(1+y)^n(1+z)^n$ is

(A) $\left({}^nC_r\right)^3$ (B) $3\left({}^nC_r\right)$ (C) ${}^nC_{3r}$ (D) ${}^{3n}C_m$

Key. D

Sol. Putting y = z = x we get $(1+x)^{3n}$ coeff $x^m={}^{3n}C_m$

115. If C_r denotes ${}^nC_r$ then the value of $\dfrac{C_0}{2}-\dfrac{C_1}{3}+\dfrac{C_2}{4}-\dfrac{C_3}{5}+------(-1)^n\dfrac{C_n}{n+2}=$

(A) $\dfrac{1}{n+1}$ (B) $\dfrac{1}{n+2}$ (C) $\dfrac{1}{n(n+1)}$ (D) $\dfrac{1}{(n+1)(n+2)}$

Key. D

Sol. Req sum = $\displaystyle\int_0^1 x(1-x)^n\,dx=\frac{1}{(n+1)(n+2)}$

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116. If the 4th term in the expansion of $\left(2+\frac{3x}{8}\right)^{10}$ has the maximum numerical value, then 'x' lies in the interval

(A) $\left(\frac{-64}{21},-2\right)\cup\left(2,\frac{64}{4}\right)$ (B) $\left(\frac{-60}{23},-2\right)\cup\left(2,\frac{64}{23}\right)$

(C) $\left(\frac{-64}{21},2\right)$ (D) $\left(-2,\frac{-64}{21}\right)$

Key. A or B

Sol. $\left|\frac{t_3}{t_4}\right|<1 \;\; and \left|\frac{t_5}{t_4}\right|<1$

i.e., $\left|\frac{2}{x}\right|<1; \;\; \left|\frac{21}{64}x\right|<1$

$\therefore x\in\left(\frac{-64}{21},-2\right)\cup\left(2,\frac{64}{21}\right)$

117. The Value of $\sum_{r=0}^{15} {}^{15}C_r\left(r-\frac{15}{2}\right)^2$ is

A) $2^{10}.15$ B) $2^{12}.15$ C) $2^{13}.15$ D) $2^{15}.15$

Key. C

Sol. $\sum_{r=0}^{15} {}^{15}C_r.r^2 - 15\sum_{r=0}^{15} r.{}^{15}C_r + \frac{225}{4}\times 2^{15}$

$\sum_{r=0}^{15} r^2.{}^{15}C_r = 15\sum_{r=1}^{15}[r-1+1]\times {}^{14}C_{r-1} = 15.2^{14} + 15.14.2^{13} = 2^{15}.60$

$\therefore$ Required Sum $= \frac{225}{4}\times 2^{15} + 2^{15}.60 - 225.2^{14}$

118. The Value of $\frac{1}{\underline{|1}\underline{|15}}+\frac{1}{\underline{|3}\underline{|13}}+\frac{1}{\underline{|5}\underline{|11}}+\frac{1}{\underline{|7}\underline{|9}}$ is

A) $\frac{2^{14}}{\underline{|15}}$ B) $\frac{2^{15}}{\underline{|16}}$ C) $\frac{2^{10}}{\underline{|15}}$ D) $\frac{2^{13}}{\underline{|15}}$

Key. C

Sol. Let S be the required Sum. Then we have $2S\times\angle 16 = {}^{16}C_1 + {}^{16}C_3 + {}^{16}C_5 + {}^{16}C_7 + ... + {}^{16}C_{15}$

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Binomial Theorem

Integer Answer Type

1. The sum of last 3 digits of 3^{100} is

Key. 1

Sol. We have

$$3^{100}=\left(3^4\right)^{25}$$
$$=\left(81\right)^{25}$$
$$=\left(80+1\right)^{25}$$
$$={}^{25}C_0.\left(80\right)^{25}+{}^{25}C_1.\left(80\right)^{24}+....+{}^{25}C_{22}\left(80\right)^3+{}^{25}C_{23}\left(80\right)^2+{}^{25}C_{24}\left(80\right)+{}^{25}C_{25}$$
$$=10^3\left[{}^{25}C_0 8^{25}\times10^{22}+{}^{25}C_1\times8^{24}\times10^{21}+.....+{}^{25}C_{22}\times8^3\right]$$
$$+\frac{25\times24}{2}\times\left(80\right)^2+25\times80+1$$
$$=10^3 m+1920000+2000+1,\ where\ m\in N$$
$$=10^3\left(m+1920+2\right)+1$$
$$\Rightarrow 3^{100}-1=10^3\left(m+1922\right)$$
$$\Rightarrow 3^{100}-1 \text{ is divisible by 1000}$$

Thus, last three digits of 3^{100} are 001, last two digits of 3^{100} are 01 and the last digit of 3^{100} is 1

2. If $n\in N$ *and* $C_k={}^nC_k$, and $\sum_{k=1}^{n}k^3\left(\frac{{}^nC_k}{{}^nC_{k-1}}\right)^2=\frac{n\left(n+1\right)^2\left(n+2\right)}{3p}$ then p is

Key. 4

Sol. We have

$$\frac{{}^nC_k}{{}^nC_{k-1}}=\frac{n-k+1}{k}$$
$$\therefore \sum_{k=1}^{n}k^3\left(\frac{{}^nC_k}{{}^nC_{k-1}}\right)^2$$
$$=\sum_{k=1}^{n}k^3\left(\frac{n-k+1}{k}\right)^2$$
$$=\sum_{k=1}^{n}k\left\{\left(n+1\right)-k\right\}^2$$
$$=\sum_{k=1}^{n}k\left\{\left(n+1\right)^2-2\left(n+1\right)k+k^2\right\}$$
$$=\sum_{k=1}^{n}\left[k\left(n+1\right)^2-2\left(n+1\right)k^2+k^3\right]$$
$$=\left(n+1\right)^2\left(\sum_{k=1}^{n}k\right)-2\left(n+1\right)\left(\sum_{k=1}^{n}k^2\right)+\left(\sum_{k=1}^{n}k^3\right)$$

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$$=(n+1)^2\left\{\frac{n(n+1)}{2}\right\}-2(n+1)\frac{n(n+1)(2n+1)}{6}+\left\{\frac{n(n+1)}{2}\right\}^2$$

$$=\frac{n(n+1)^2}{2}\left\{(n+1)-\frac{2}{3}(2n+1)+\frac{n}{2}\right\}$$

..... (i) $=\frac{n(n+1)^2}{2}\times\frac{6(n+1)-4(2n+1)+3n}{6}$

$$=\frac{n(n+1)^2(n+2)}{12}$$

3. If $C_0, C_1, C_2....C_n$ denote binomial coefficient in the expansion of $(1+x)^n$ and given a=b=2 and n=10 , then
$aC_O+(a+b)C_1+(a+2b)C_2+.....+(a+nb)C_n=4K.2^9$ where k is ____

Key. 6

Sol. We have
$aC_0+(a+b)C_1+(a+2b)C_2+....+(a+nb)C_n$

$$\sum_{r=0}^{n}(a+rb)\,{}^nC_r$$

$$=a\left(\sum_{r=0}^{n}{}^nC_r\right)+b\left(\sum_{r=0}^{n}r\,{}^nC_r\right)$$

$$=a\left(\sum_{r=0}^{n}{}^nC_r\right)+b\left(\sum_{r=1}^{n}r\,\frac{n}{r}\,{}^{n-1}C_{r-1}\right)$$

$$=a\left(\sum_{r=0}^{n}{}^nC_r\right)+bn\left(\sum_{r=1}^{n}{}^{n-1}C_{r-1}\right)$$

$$=a,\ 2^n+bn\ 2^{n-1}\qquad\left[\because\sum_{r=0}^{n}{}^nC_r=2^n,\sum_{r=1}^{n}{}^{n-1}C_{r-1}=2^{n-1}\right]$$

$$=(2a+bn)2^{n-1}$$

4. Given ${}^8C_1x(1-x)^7+2.{}^8C_2x^2(1-x)^6+3.{}^8C_3x^3(1-x)^5+.....+8.x^8=ax+b, Find\ a+b$

Key. 8

Sol. ${}^8C_1x(1-x)^7+2.{}^8C_2\ x^2(1-x)^6$
$+3.{}^8C_3x^3(1-x)^5+.....+n.{}^8C_8\ x^8=8x$
Solution we have ,
${}^nC_1\ x(1-x)^{n-1}+2.\ {}^nC_2x^2(1-x)^{n-2}$
$+3.{}^nC_3x^3(1-x)^{n-3}+.....+n.\ {}^nC_nx^n$

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$$= \sum_{r=1}^{n} r.{}^{n}C_r x^r (1-x)^{n-r}$$

$$= \sum_{r=1}^{n} r. \frac{n}{r} n-1\ C_{r-1}\ x^r (1-x)^{n-r}$$

$$= n\sum_{r=1}^{n} {}^{n-1}C_{r-1}\ x.\ x^{r-1}(1-x)^{(n-1)-(r-1)}$$

$$= nx\sum_{r=1}^{n} {}^{n-1}C_{r-1}\ x^{r-1}(1-x)^{(n-1)-(r-1)}$$

$$= nx\left[x+(1-x)\right]^{n-1}$$

=nx .=8x(n=8)

a + b = 8

5. Let $\alpha_n = \sum_{r=0}^{n} \frac{1}{n_{C_r}}$, $Let\ \beta_n = \sum_{r=0}^{n} \frac{r}{n_{C_r}}$ for n = 10, find β_n / α_n

Key. 5

Sol. We have $\alpha_n = \sum_{r=0}^{n} \frac{1}{{}^{n}C_r}$: n=10

$$= \left\{ \sum_{r=0}^{(n/2)-1} \left(\frac{1}{{}^{n}C_r} \right) + \frac{1}{{}^{n}C_{n-r}} \right\} + \frac{1}{{}^{n}C/2}$$

$$= \left\{ \sum_{r=0}^{(n/2)-1} \frac{2}{{}^{n}C_r} \right\} + \frac{1}{{}^{n}C_{n/2}}$$

$$= 2\left\{ \sum_{r=0}^{(n/2)-1} \frac{2}{{}^{n}C_r} \right\} + \frac{1}{{}^{n}C_{n/2}} \quad \text{......(ii)}$$

$$\therefore \sum_{r=0}^{n} \frac{r}{{}^{n}C_r}$$

$$= \sum_{r=0}^{(n/2)-1} \left\{ \frac{r}{{}^{n}C_r} + \frac{n-r}{{}^{n}C_r} \right\} + \frac{n/2}{{}^{n}C_{n/2}}$$

$$= \left\{ \sum_{r=0}^{n/2-1} \frac{n}{{}^{n}C_r} \right\} + \frac{n}{2.{}^{n}C_{n/2}}$$

$$= \frac{n}{2}\left[\left\{ \sum_{r=0}^{n/2-1} \frac{1}{{}^{n}C_r} \right\} + \frac{1}{{}^{n}C_{n/2}} \right] = \frac{n}{2}\alpha_n \quad \text{[Using (ii)]}$$

Hence ,

$$\sum_{r=0}^{n} \frac{r}{{}^{n}C_r} = \frac{n}{2}\alpha_n\ for\ all\ n \in N$$

6. If $\sum_{r=0}^{n} (-1)^r \frac{{}^{n}C_r}{{}^{(r+2)}C_r} = \frac{k}{n+2}$, find k

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Key. 2

Sol. We have

$$\sum_{r=0}^{n}(-1)^{r}\frac{{}^{n}C_{r}}{{}^{n+2}C_{r}}$$

$$=\sum_{r=0}^{n}(-1)^{r}\frac{n!}{(n-r)!r!}\times\frac{2!r!}{(r+2)!}$$

$$=2\sum_{r=0}^{n}(-1)^{r}\frac{n!}{(n-r)!(r+2)!}$$

$$=\frac{2}{(n+1)(n+2)}\sum_{r=0}^{n}(-1)^{r}\frac{(n+2!)}{\{(n+2)-(r+2)\}!(r+2)!}$$

$$=\frac{2}{(n+1)(n+2)}\sum_{r=0}^{n}(-1)^{r+2}\ {}^{n+2}C_{r+2}$$

$$=\frac{2}{(n+1)(n+2)}\sum_{s=2}^{n+2}(-1)^{S}\ {}^{2}C_{s}$$

$$=\frac{2}{(n+1)(n+2)}\left[\left(\sum_{s=0}^{n+1}(-1)^{s}\ {}^{n+2}C_{s}\right)-\left({}^{n+2}C_{0}-{}^{n+2}C_{1}\right)\right]$$

$$=\frac{2}{(n+1)(n+2)}\left[0-\{1-(n+2)\}\right]$$

$$=\frac{2}{n+2}$$

7. $s=a+(a+d)+(a+2d)+......+(a+nd)$ and

$A=a+(a+d)^{n}C_{1}+(a+2d)n_{c_2}+.....+(a+nd)n_{c_n}$ then

$(n+1)A=k^{n}S$ *where* $k=$ __________

Key. 2

Sol. We have

$$s=a+(a+d)+(a\_2d)+......+(a+nd)$$

$$\Rightarrow s=\frac{n+1}{2}\left[2a+(n+1-1)d\right]$$

$$\Rightarrow s=\frac{n+1}{2}(2a+nd)$$

Now,

$$A=a+(a+d)\ {}^{n}C_{1}+(a+2d)\ {}^{n}C_{2}+.....+(a+nd)^{n}C_{n}$$

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$$=\sum_{r=0}^{n}(a+rd)\ {}^{n}C_r$$

$$=a\left(\sum_{r=0}^{n}{}^{n}C_r\right)+d\left(\sum_{r=0}^{n}r.\ {}^{n}C_r\right)$$

$$=a.2^n+d.n\ 2^{n-1}\qquad\left[\because\sum_{R=0}^{N}{}^{r}C_r=2^n,\sum_{r=0}^{n}r\ {}^{n}C_r=n\ 2^{n-1}\right]$$

$$=(2a+nd)2^{n-1}$$

$$=\left\{\frac{n+1}{2}(2a+nd)\right\}\left\{\frac{2}{n+1}\times2^{n-1}\right\}$$

$$=\frac{s.2^n}{n+1}$$

$$\therefore(n+1)A=2^nS$$

8. Consider two polynomials $f(x)$ and $g(x)$ as $g(x)=\sum_{r=0}^{200}\alpha_r x^r$ and $f(x)=\sum_{r=0}^{200}\beta_r x^r$. Given (i) $\beta_r=1\forall r\geq100$, (ii) $f(x+1)=g(x)$. Let $A=\sum_{r=100}^{200}\alpha_r$. Find the remainder when A is divided by 15.

Key: 1

Hint: $\sum_{r=0}^{200}\alpha_r x^r=\sum_{r=0}^{200}\beta_r(r+x)^r$

$\alpha_0+\alpha_1x+\alpha_2x^2+......+\alpha_{200}x^{200}$

$=\beta_0+\beta_1(1+x)+......+\beta_{200}(1+x)^{200}$

Equating coefficient of x^{100}, we get $\alpha_{100}={}^{100}C_{100}+{}^{101}C_{100}+.......{}^{200}C_{100}={}^{201}C_{101}$

Similarly we cant find $\alpha_{100}.....\alpha_{200}$

$\sum_{r=100}^{200}\alpha_r={}^{201}C_{101}+{}^{201}C_{102}+.....{}^{201}C_{201}$

$A=2^{200}$

When A is divided by 15 remainder is 1.

9. (L-1)Coefficient of x^6 in $\left((1+x)(1+x^2)^2(1+x^3)^3....(1+x^n)^n\right)$ is 4k. The numerical value of k is

Key : 7

Hint : The coefficient of x^6 in the given expression = coefficient of x^6 in

$(1+{}^6C_1x^6)(1+{}^5C_1x^5)(1+{}^4C_1x^4)(1+{}^3C_1x^3+{}^3C_2x^6)(1+{}^2C_1x^2+{}^2C_2x^4)(1+x)$

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= coefficient of x^6 in $\left(1+6x^6+5x^5+4x^4\right)\left(1+2x^2+3x^3+x^4+6x^5+3x^6\right)\left(1+x\right)$

= coefficient of x^6 in $\left(11x^5+17x^6\right)\left(1+x\right)$

= 28

10. $(1+x)(1+x+x^2)(1+x+x^2+x^3)\ldots\ldots(1+x+x^2+\ldots..+x^{100})$

When written in the ascending power of x then (the highest exponent of x) -5045 is

Key. 5

Sol. Highest exponent of $x = 1 + 2 + 3 + \ldots.. + 100 = \frac{100(101)}{2} : 5050$

11. Find the Coefficient of x^{103} in $(1 + x + x^2 + x^3 + x^4)^{199} (x - 1)^{201}$.

Key. 0

Sol. Coefficient of x^{103} in $(1 + x + x^2 + x^3 + x^4)^{199} (x - 1)^{201}$

Coefficient of x^{103} in $-(1 - x)^2 (1 - x^5)^{199}$

Coefficient of x^{103} in $-(1 - 2x + x^2) (1 - {}^{199}C_1 x^5 + {}^{199}C_2 x^{10} - {}^{199}C_3 x^{15} + \ldots..)$

Coefficient of $x^{103} = 0$

12. Given ${}^8C_1x(1-x)^7+2.{}^8C_2x^2(1-x)^6+3.{}^8C_3x^3(1-x)^5+\ldots..+8.x^8=ax+b, Find\ a+b$

Key. 8

Sol. ${}^8C_1x\left(1-x\right)^7+2.{}^8C_2\ x^2\left(1-x\right)^6$

$+3.{}^8C_3x^3\left(1-x\right)^5+\ldots..+n.{}^8C_8\ x^8=8x$

Solution we have ,

${}^nC_1\ x\left(1-x\right)^{n-1}+2.\ {}^nC_2x^2\left(1-x\right)^{n-2}$

$+3.{}^nC_3x^3\left(1-x\right)^{n-3}+\ldots..+n.\ {}^nC_nx^n$

$=\sum_{r=1}^{n} r.{}^nC_rx^r\left(1-x\right)^{n-r}$

$=\sum_{r=1}^{n} r.\ \frac{n}{r}n-1\ C_{r-1}\ x^r\left(1-x\right)^{n-r}$

$=n\sum_{r=1}^{n} {}^{n-1}C_{r-1}\ x.\ x^{r-1}\left(1-x\right)^{(n-1)-(r-1)}$

$=nx\sum_{r=1}^{n} {}^{n-1}C_{r-1}\ x^{r-1}\left(1-x\right)^{(n-1)-(r-1)}$

$=nx\left[x+\left(1-x\right)\right]^{n-1}$

=nx .=8x(n=8)

a + b = 8

13. Let $\alpha_n=\sum_{r=0}^{n}\frac{1}{n_{C_r}}, Let\ \beta_n=\sum_{r=0}^{n}\frac{r}{n_{C_r}}$ for n = 10, find β_n/α_n

Key. 5

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Sol. We have $\alpha_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$: n=10

$$= \left\{ \sum_{r=0}^{(n/2)-1} \left(\frac{1}{{}^nC_r} \right) + \frac{1}{{}^nC_{n-r}} \right\} + \frac{1}{{}^nC/2}$$

$$= \left\{ \sum_{r=0}^{(n/2)-1} \frac{2}{{}^nC_r} \right\} + \frac{1}{{}^nC_{n/2}}$$

$$= 2\left\{ \sum_{r=0}^{(n/2)-1} \frac{2}{{}^nC_r} \right\} + \frac{1}{{}^nC_{n/2}} \quad \text{......(ii)}$$

$$\therefore \sum_{r=0}^{n} \frac{r}{{}^nC_r}$$

$$= \sum_{r=0}^{(n/2)-1} \left\{ \frac{r}{{}^nC_r} + \frac{n-r}{{}^nC_r} \right\} + \frac{n/2}{{}^nC_{n/2}}$$

$$= \left\{ \sum_{r=0}^{n/2-1} \frac{n}{{}^nC_r} \right\} + \frac{n}{2.{}^nC_{n/2}}$$

$$= \frac{n}{2}\left[\left\{ \sum_{r=0}^{n/2-1} \frac{1}{{}^nC_r} \right\} + \frac{1}{{}^nC_{n/2}} \right] = \frac{n}{2}\alpha_n \quad \text{[Using (ii)]}$$

Hence ,

$$\sum_{r=0}^{n} \frac{r}{{}^nC_r} = \frac{n}{2}\alpha_n \ \text{for all } n \in N$$

14. If $\sum_{r=0}^{n} (-1)^r \frac{{}^nC_r}{{}^{(r+2)}C_r} = \frac{k}{n+2}$, find k

Key. 2

Sol. We have

$$\sum_{r=0}^{n} (-1)^r \frac{{}^nC_r}{{}^{n+2}C_r}$$

$$= \sum_{r=0}^{n} (-1)^r \frac{n!}{(n-r)!r!} \times \frac{2!r!}{(r+2)!}$$

$$= 2\sum_{r=0}^{n} (-1)^r \frac{n!}{(n-r)!(r+2)!}$$

$$= \frac{2}{(n+1)(n+2)} \sum_{r=0}^{n} (-1)^r \frac{(n+2!)}{\{(n+2)-(r+2)\}!(r+2)!}$$

$$= \frac{2}{(n+1)(n+2)} \sum_{r=0}^{n} (-1)^{r+2} \, {}^{n+2}C_{r+2}$$

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$$=\frac{2}{(n+1)(n+2)}\sum_{s=2}^{n+2}(-1)^{S}\ {}^{2}C_s$$

$$=\frac{2}{(n+1)(n+2)}\left[\left(\sum_{s=0}^{n+1}(-1)^{s}\ {}^{n+2}C_s\right)-\left({}^{n+2}C_0-{}^{n+2}C_1\right)\right]$$

$$=\frac{2}{(n+1)(n+2)}\left[0-\{1-(n+2)\}\right]$$

$$=\frac{2}{n+2}$$

15. $s=a+(a+d)+(a+2d)+......+(a+nd)$ and

$A=a+(a+d)^{n}C_1+(a+2d)n_{c_2}+.....+(a+nd)n_{c_n}$ then

$(n+1)A=k^{n}S\ where\ k=$__________

Key. 2

Sol. We have

$$s=a+(a+d)+(a\_2d)+......+(a+nd)$$

$$\Rightarrow s=\frac{n+1}{2}\left[2a+(n+1-1)d\right]$$

$$\Rightarrow s=\frac{n+1}{2}(2a+nd)$$

Now,

$$A=a+(a+d)\ {}^{n}C_1+(a+2d)\ {}^{n}C_2+.....+(a+nd)^{n}\ C_n$$

$$=\sum_{r=0}^{n}(a+rd)\ {}^{n}C_r$$

$$=a\left(\sum_{r=0}^{n}{}^{n}C_r\right)+d\left(\sum_{r=0}^{n}r.\ {}^{n}C_r\right)$$

$$=a.2^{n}+d.n\ 2^{n-1}\qquad\left[\because\sum_{R=0}^{N}\ {}^{r}C_r=2^{n},\sum_{r=0}^{n}r\ {}^{n}C_r=n\ 2^{n-1}\right]$$

$$=(2a+nd)2^{n-1}$$

$$=\left\{\frac{n+1}{2}(2a+nd)\right\}\left\{\frac{2}{n+1}\times 2^{n-1}\right\}$$

$$=\frac{s.2^{n}}{n+1}$$

$$\therefore(n+1)A=2^{n}S$$

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16. The sum of the series $3.{}^{2007}C_0 - 8.{}^{2007}C_1 + 13\,{}^{2007}C_2 - 18.{}^{2007}C_3 + ...$ upto 2008 terms is K, then K is

Key. 0

Sol. Series $= \sum_{r=0}^{2007} (-1)^r (5r+3)\ {}^{2007}C_r = 0 \Rightarrow K = 0$

17. If n is a +ve integer greater than 3 such that $(1+x^2)^2(1+x)^n = \sum_{K=0}^{n+4} a_K x^K$ and a_1, a_2, a_3 are in AP then maximum value of n is ________.

Key. 4

Sol. $(1+x^2)^2(1+x)^n = \sum_{K=0}^{n+4} a_K x^K$

$a_1 = nC_1,\ 2 + {}^nC_2 = a_2$

${}^nC_3 + 2.{}^nC_1 = a_3$

But a_1, a_2, a_3 are in AP

$(n-2)(n-3)(n-4) = 0 \Rightarrow n = 2, 3, 4$

Maximum value of n is 4.

18. If C_r is a binomial co-efficient in the expansion of $(1+x)^n$, find the value of $\sum_{i=1}^{n}\sum_{j=1}^{n} (i+j)C_iC_j$.

Sol. Note: $C_0 + C_1 + C_2 + ... + C_n = 2^n$ &$1.C_1 + 2.C_2 + ... + n.C_n = n\,2^{n-1}$

$\sum_{i=1}^{n}\sum_{j=1}^{n}(i+1)C_iC_j = \Sigma\Sigma i\ C_iC_j + \Sigma\Sigma j\ C_i C_j$

$= \sum_{i=1}^{n} i\,C_i.\left(\sum_{j=1}^{n} C_j\right) + \sum_{i=1}^{n} jC_j.\left(\sum_{j=1}^{n} C_i\right) = \left(2^n - 1\right)2\Sigma iC_i = \left(2^n-1\right)2n.2^{n-1} = n\,2^n\left(2^n - 1\right)$

19. Let $\left(1+x^3\right)^n = \sum_{r=0}^{n} a_r x^r (1-x)^{3n-2r}$, $n > 2$, then find the value of n so that a_1, a_2, a_3 are in A.G.P.

Sol. If $\left(1-x^3\right)^n = \sum_{r=0}^{n} a_r x^r (1-x)^{3n-2r}$

$\Rightarrow \quad \left[(1-x)\left(1+x+x^2\right)\right]^n = (1-x)^n \sum_{r=0}^{n} a_r x^r \left[(1-x)^2\right]^n = \sum_{r=0}^{n} a_r x^r \left[(1-x)^2\right]^{n-r}$

$\Rightarrow \quad \sum_{r=0}^{n} {}^nC_r (3x)^r \left[(1-x)^2\right]^{n-r} = \sum_{r=0}^{n} a_r x^r \left[(1-x)^2\right]^{n-r}$

Comparing the coefficients of like power of x on both sides, we get $a_r = {}^nC_r.\,3^r$.

$\therefore \quad a_1 = 3.\,{}^nC_1,\ a_2 = 9.\,{}^nC_2$ and $a_3 = 27.\,{}^nC_3$

$\therefore \quad a_1, a_2, a_3$ are in A.G.P. iff ${}^nC_1,\ {}^nC_2,\ {}^nC_3$ are in A.P.

$\therefore \quad$ n = 7 Ans.

20. Prove that ${}^nC_0\,{}^nC_1\,{}^nC_2 ... {}^nC_n < \dfrac{(n-1)!\left(2^{n+1}-n-2\right)^n}{n^{n-1}(n+1)^{n-1}}$.

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Sol. $$\left[\frac{1}{(n+1)!}\left({}^{n}C_0.{}^{n}C_1.\ {}^{n}C_2....{}^{n}C_n\right)\right]^{\frac{1}{n}}=\left[\frac{1}{2}\,{}^{n}C_1.\frac{1}{3}\,{}^{n}C_2\,\frac{1}{4}\,{}^{n}C_3...\frac{1}{n+1}\,{}^{n}C_n\right]^{\frac{1}{n}}$$

$$\le\frac{\frac{1}{2}\,{}^{n}C_1+\frac{1}{3}\,{}^{n}C_2+\frac{1}{4}\,{}^{n}C_3+...+\frac{1}{n+1}\,{}^{n}C_n}{n}=\frac{\sum_{r=1}^{n}{}^{n+1}C_{r+1}}{n\left(n=1\right)}=\frac{2^{n+1}-n-2}{n\left(n+1\right)}$$

$$\therefore\qquad {}^{n}C_0\,{}^{n}C_1\,{}^{n}C_2...\ {}^{n}C_n<\frac{(n+1)!\left(2^{n+1}-n-2\right)^n}{n^n\left(n+1\right)^n}=\frac{(n-1)!\left(2^{n+1}-n-2\right)^n}{n^{n-1}\left(n+1\right)^{n-1}}$$

21. If $\left(1+2x+2x^2\right)^n=a_0+a_1x+a_2x^2+...+a_{2n}x^{2n}$ where n is even, then find the value of $a_0a_{2n}-a_1a_{2n-1}+...+a_{2n}a_0$.

Sol. $\left(1+2x+2x^2\right)^n=a_0+a_1xa_2x^2+...+a_{2n}x^{2n}$ …(i)

Replace x by – x

$\left(1-2x+2x^2\right)^n=a_0-a_1x+a_2x^2-...+a_{2n}x^{2n}$ …(ii)

$\therefore$ $a_0a_{2n}-a_1a_{2n-1}...$ = coefficient of x^{2n} in the product of RHS of (i) and (ii)

= coefficient of x^{2n} in the product of LHS

= coefficient of x^{2n} in $\left[\left(1+2x^2\right)^2-\left(2x\right)^2\right]^n$

i.e. in $\left(1+4x^4\right)^n$

= coefficient of y^n in $\left(1+4y^2\right)^n$ where $x^2=y$

$={}^{n}C_{\frac{n}{2}}.2^n$ (n is even)

22. If the middle term of $\left(\frac{1}{x}+x\sin x\right)^{10}$ is equal to $7\frac{7}{8}$, then find the value of x.

Sol. There are 11 terms in the expansion $\therefore$ 6^{th} term is the middle term.

$T_6={}^{10}C_5\left(\frac{1}{x}\right)^5\left(x\sin x\right)^5$

${}^{10}C_5\left(\sin x\right)^5=\frac{63}{8}$

$\left(\sin x\right)^5=\frac{63}{8}\times\frac{1}{252}=\frac{1}{32}$

$\Rightarrow$ $\sin x=\frac{1}{2}$

$\therefore$ $x=n\pi+\left(-1\right)^n\frac{\pi}{6}$, $n\in I$ Ans.

23. Find the last three digits of 17^{256}

Sol. We have $17^2=289=290-1$

Now $17^{256}=\left(17^2\right)^{128}=\left(290-1\right)^{128}$

$$={}^{128}C_0\left(290\right)^{128}-{}^{128}C_1\left(290\right)^{127}+{}^{128}C_2\left(290\right)^{126}-.....$$

$$-{}^{128}C_{125}\left(290\right)^3+{}^{128}C_{126}\left(290\right)^2-{}^{128}C_{127}\left(290\right)+1$$

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$$= 1000\,m + \frac{(128)(127)}{2}(290)^2 - (128)(290) + 1$$

Where m is a positive integer.

= 1000 m + (128) (290) [(127) (145) – 1] + 1
= 1000 m + (128) (290) (18414) + 1
= 1000 m + 683527680 + 1
= 1000 [m + 683527] + 680 + 1
= 1000 [m + 683527] + 681

Thus, the last three digits of 17^{256} are 681. Ans.

24. Find sum of rational coefficients in expansion of $\left(\sqrt[3]{5}x + \sqrt{3}y + z\right)^6$

Sol. Any term of the expansion is of the form $\frac{6!}{a!b!c!}\left(5^{1/3}x\right)^a\left(3^{1/2}y\right)^b z^c$

z^c, a, b, c non-negative integers and a + b + c = 6. For rational coefficients 'a' must be multiple of 3 and b must be multiple of 2.
The following are the possibilities

Sum of coefficients $= \frac{6!}{6!} + \frac{6!}{2!4!}.3 + \frac{6!}{4!2!}3^2 + \frac{6!}{6!}3^3 + \frac{6!}{3!3!}5 + \frac{6!}{3!2!}.5.3 + \frac{6!}{6!}5^2 = 1233$ Ans.

25. Find the algebraically second largest term in the expansion of $(3-2x)^{15}$ at $x = \frac{4}{3}$.

Sol. $(3-2x)^{15}$ at $x = \frac{4}{3}$

$$r \le \frac{16|2x|}{3+|2x|} = \frac{16\times 2\times \frac{4}{3}}{3+\frac{8}{3}} = \frac{128}{17}$$

r = 7

$\therefore$ $t_{r+1} = t_8 = {}^{15}C_7 3^8 (-2x)^7 = -{}^{15}C_7 3^8 \left(\frac{8}{3}\right)^7$ is numerically greatest term.

$\therefore$ t_7 and t_9 are positive terms

$t_7 = {}^{13}C_6\, 3^9 (-2x)^6 = {}^{15}C_6\, 3^9 (8/3)^6 = {}^{15}C_6\, 8^6.3^3$

and $t_9 = {}^{15}C_8\, 3^7 (-2x)^8 = {}^{15}C_8.\, 3^7 (8/3)^8 = {}^{15}C_8\, 8^8.3^{-1}$

$\therefore$ $t_9 t_7 = \frac{15!}{8!7!}\frac{8^8}{3} - \frac{15!}{8!6!}8^6.3^3 = \frac{15!}{8!6!}\frac{8^6}{3}\left[\frac{64}{7} - \frac{3^4}{9}\right] > 0$

$\therefore$ $t_7 < t_9$

$t_{11} = {}^{15}C_{10}\frac{8^{10}}{3^5}$

$\therefore$ $t_7 - t_{11} = 10935 - 4096 > 0$

$\therefore$ $t_7 > t_{11}$

Thus $t_9 > t_7 > t_{11}$

Hence t_7 is the second largest term.

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26. Coefficient of x^{203} in $(x-1)(x^2-2)(x^3-3)......(x^{20}-20)$ is k then sum of the digits of k is

Key. 4

Sol. (x–1) (x^2–2) (x^3–3) .. (x^n–n)

Highest power of x = 1+2+3+.....+20 = 210 we require, coefficient of x^{210-7}

$\Rightarrow$ either, we should leave

X^7–7, (x–1) (x^6–6), (x^2–2) (x^5–5), (x^3–3) (x^4–4), (x–1) (x^2–2) (x^{4-}–4)
$\Rightarrow$ required coefficient is 12 + 10+ 6 – 7 – 8 = 13

27. If the coefficient of x^7 in $\left(ax^2+\frac{1}{bx}\right)^{11}$ and the coefficient of x^{-7} in $\left(ax-\frac{1}{bx^2}\right)^{11}$ are equal if b = $\frac{1}{9}$ then the value of a is

Key. 9

Sol. ab = 1

28. Let f(n) = $\left(\sqrt{2}+1\right)^n$, 'n' being an odd positive integer [.] the greatest integer function then the value of 'n' for which $\left[f(x)\right]=82$ is.

Key. 5

Sol. $\left(\sqrt{2}+1\right)^n-\left(\sqrt{2}-1\right)^n=2\left\{{}^nC_1\left(\sqrt{2}\right)^{n-1}+{}^nC_3\left(\sqrt{2}\right)^{n-3}+...+{}^nC_n\right\}$

= integer

$\Rightarrow \left(\sqrt{2}-1\right)^n$ is the fractional part of $\left(\sqrt{2}-1\right)^n$

$\therefore$ [$f_{(n)}$] = $\left[\left(\sqrt{2}+1\right)^n\right]=\left(\sqrt{2}+1\right)^n-\left(\sqrt{2}-1\right)^n$ is satisfied for n = 5

29. If α and β are the roots of equation x^2 + 4x + p = 0 where p = $\sum_{r=0}^{n} n_{C_r}\frac{1+rx}{(1+nx)^r}(-1)^r$ then the value of | α-β | is ……

Key. 4

Sol. p = $\sum_{r=0}^{n} n_{C_r}\cdot\left(\frac{-1}{1+nx}\right)^r+\sum_{r=0}^{n} n_{C_r}\frac{(-1)^r rx}{(1+nx)^r}$

= $\left(1-\frac{1}{1+nx}\right)^n+x\sum_{r=1}^{n}{}^{n-1}C_{r-1}\frac{(-1)^r x}{(1+nx)^r}$

= $\left(\frac{nx}{1+nx}\right)^n-\left(\frac{nx}{1+nx}\right)\left(1-\frac{1}{1+nx}\right)^{n-1}$

= 0

$\therefore |\alpha-\beta|=4$

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30. ${}^{n}C_{o}\,{}^{4n}C_{n} - {}^{n}C_{1}\,{}^{(4n-3)}C_{n} + {}^{n}C_{2}\,{}^{4n-6}C_{n} - {}^{n}C_{3}\,{}^{4n-9}C_{n} + \ldots +$ to n terms is equal to $(1+k)^{n}$ then the value of k is

Key. 2

Sol. Coefficient x^n in $\left\{{}^{n}C_{o}(1+x)^{4n} - {}^{n}C_{1}(1+x)^{4n-3} + {}^{n}C_{2}(1+x)^{4n-6} \ldots + (-1)^{n}\,{}^{n}C_{n}(1+x)^{n}\right\}$

$\Rightarrow$ Coefficient x^n in $(1+x)^{n}\left\{(1+x)^{3}-1\right\}^{n} \Rightarrow 3^{n}$

31. If the sum of the coefficient in the expansion of $\left(\alpha x^{2}-2x+1\right)^{35}$ is equal to the sum of the coefficients in the expansion of $(x-\alpha y)^{35}$,then the value of α is.

Key. 1

Sol. Conceptual

32. The value of $99^{50} - 99.98^{50} + \frac{99.98}{1.2}(97)^{50} - \ldots\ldots + 99$ is

Key. 0

Sol. ${}^{99}C_{0}99^{50} - {}^{99}C_{1}(99-1)^{50} + {}^{99}C_{2}(99-2)^{50} - \ldots\ldots + {}^{99}C_{98}(99-98)^{50} - {}^{99}C_{99}(99-99)^{50}$

$= 99^{50}\left[{}^{99}C_{0} - {}^{99}C_{1} + {}^{99}C_{2} - \ldots. + {}^{99}C_{98} - {}^{99}C_{99}\right] + {}^{50}C_{1}99^{49}\left[{}^{99}C_{1} - 2.\,{}^{99}C_{2} + 3.\,{}^{99}C_{3} - \ldots\right] + \ldots. = 0$

33. At which x > 0 the 6th term in the expansion of the binomial $\left(\sqrt{2^{\log\left(10-3^{x}\right)}} + \sqrt[5]{2^{(x-2)\log 3}}\right)^{m}$ is equal to 21, if it is known that the binomial coefficient of the 2nd, 3rd and 4th term in the expansion represent respectively the 1st, 3rd and 5th terms of an A.P (the symbol log stands for logarithm to the base 10).

Key. 2

Sol. $2.{}^{m}C_{2} = {}^{m}C_{1} + {}^{m}C_{3}$

$\Rightarrow m = 7$

$\Rightarrow 21 = {}^{7}C_{5}\left[\sqrt{2^{\log\left(10-3^{x}\right)}}\right]^{7-5} \times \left[\sqrt[5]{2^{(x-2)\log 3}}\right]^{5}$

$\Rightarrow x = 2$

34. When 32^{33} is divided by 34, it leaves the remainder 3k + 5 then the value of k is _____.

Key. 9

Sol. $32^{33} = 2^{165} = 2 \times 16^{41} = 2 \times (17-1)^{41} = 2 \times (17k-1) = 34k - 34 + 32$

So the remainder is 32.

35. If the sum of all the coefficients of the terms in the expansion of $(x + y + z + w)^{6}$ which contain x but not y, is 95t then the value of t is _____.

Key. 7

Sol. The sum of coefficients of terms not containing y = 3^{6}

The sum of coefficients of terms not containing both x & y = 2^{6}

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So the required number = $3^6 - 2^6 = 665$.

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