3D-Geometry

Single Correct Answer Type

1. In a three dimensional co - ordinate system P, Q and R are images of a point A(a, b, c) in the x - y the y - z and the z - x planes respectively. If G is the centroid of triangle PQR then area of triangle AOG is (O is the origin)

a) 0 b) $a^2 + b^2 + c^2$ c) $\frac{2}{3}\left(a^2 + b^2 + c^2\right)$ d) none of these

Key. A

Sol. Point A is (a, b, c)

$\Rightarrow$Points P, Q, R are (a, b, –c), (-a, b, c) and (a, –b, c) respectively.

$\Rightarrow$centroid of triangle PQR is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

$\Rightarrow G \equiv \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

$\Rightarrow$A, O, G are collinear $\Rightarrow$ area of triangle AOG is zero.

2. The four lines drawing from the vertices of any tetrahedron to the centroid of the opposite faces meet in a point whose distance from each vertex is 'k' times the distance from each vertex to the opposite face, where k is

a) $\frac{1}{3}$ b) $\frac{1}{2}$ c) $\frac{3}{4}$ d) $\frac{5}{4}$

Key. C

Sol. Let A (x_1, y_1, z_1) B (x_2, y_2, z_2) C (x_3, y_3, z_3) D (x_4, y_4, z_4) be the vertices of tetrahydron. If E is the centroid of face BCD and G is the centroid of A B C D the AG= $3/4(AE) \therefore K = 3/4$

3. The coordinates of the circumcentre of the triangle formed by the points (3, 2, -5), (-3, 8, -5) (-3, 2, 1) are

a) (-1, 4, -3) b) (1, 4, -3) c) (-1, 4, 3) d) (-1, -4, -3)

Key. A

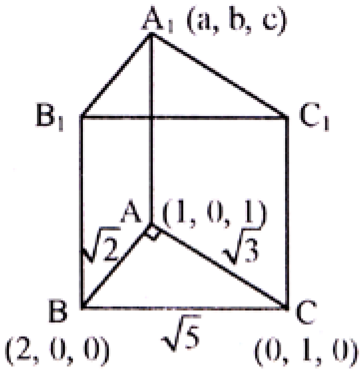
Sol. Triangle formed is an equilateral $\Rightarrow$ Circum centre = centroid = (-1, 4, -3)

4. The volume of a right triangular prism $ABCA_1B_1C_1$ is equal to 3. Than the co-ordinates of the vertex A_1, if the co-ordinates of the base vertices of the prism are A(1, 0, 1), B(2, 0, 0) and C(0, 1, 0)

a) (–2, 2, 2) or (0, –2, 1) b) (2, 2, 2) or (0, –2, 0)
c) (0, 2, 0) or (1, –2, 0) d) (3, –2, 0) or (1, –2, 0)

Key. B

Sol. Volume = Area of base $\times$ height



$3=\frac{1}{2}{'}\ \sqrt{2}{'}\ \sqrt{3}{'}\ h$

h = $\sqrt{6}$

$(A_1A)^2 = h^2 = 6$

$\overrightarrow{A_1A}.\overrightarrow{AB} = 0$

$\overrightarrow{A_1A}.\overrightarrow{AC} = 0$

$\overrightarrow{AA_1}.\overrightarrow{BC} = 0$

solving we get position vector of A_1 are (0, –2, 0) or (2, 2, 2)

5. If $\vec{a},\vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a}+\vec{b}+\vec{c}$ is also a unit vector and θ_1,θ_2 *and* θ_3 are angles between the vectors $\vec{a},\vec{b};\vec{b},\vec{c}$ *and* $\vec{c},\vec{a}$, respectively, then among $\theta_1,\ \theta_2$ *and* θ_3.

a) all are acute angles b) all are right angles

c) at least one is obtuse angle d) None of these

Key. C

Sol. Since $\left|\vec{a}+\vec{b}+\vec{c}\right|=1 \Rightarrow \left(\vec{a}+\vec{b}+\vec{c}\right).\left(\vec{a}+\vec{b}+\vec{c}\right)=1 \Rightarrow \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=-1$

$\Rightarrow \cos\theta_1+\cos\theta_2+\cos\theta_3=-1$

So, at least one of $\cos\theta_1,\cos\theta_2$ *and* $\cos\theta_3$ must be negative

6. Given that the points $A(3,2,-4),B(5,4,-6)$ *and* $C(9,8,-10)$ are collinear, the ratio in which B divides $\overline{AC}$ is :

1)1 : 2 2)2 : 1 3)3 : 2 4)2 : 3

Key. 1

Sol. $\left(\frac{9m+3n}{m+n},\frac{8m+2n}{m+n},\frac{-10m-4n}{m+n}\right)=(5,4,-6)$

$$\frac{m}{n}=\frac{1}{2}$$

7. If $A(0,1,2), B(2,-1,3) and\ C(1,-3,1)$ are the vertices of a triangle, then its circumcentre and orthocenter are situated at a distance of

1)3 units 2)2 units 3)3/2 units 4) $3/\sqrt{2}$ units

Key. 4

Sol. ortho center- $(2,-1,3)$

Circum center- $\left(\frac{1}{2},-1,\frac{3}{2}\right)$

8. Equation of the plane passing through the origin and perpendicular to the planes $x+2y+z=1, 3x-4y+z=5$ is

1) $x+2y-5z=0$ 2) $x-2y-3z=0$ 3) $x-2y+5z=0$ 4) $3x+y-5z=0$

Key. 4

Sol. $\begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 3 & -4 & 1 \end{vmatrix}=0$

$A=3i+j-5k$

$\Rightarrow 3x+y-5z=0$

9. If θ is the angle between $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}z+4=0$ and is such that $\sin\theta$ =1/3, the value of λ =

1) $-\frac{4}{3}$ 2) $\frac{4}{3}$ 3) $-\frac{3}{5}$ 4) $\frac{5}{3}$

Key. 4

Sol. $Sin\theta=\left|\frac{2-2+2\sqrt{\lambda}}{3\sqrt{5+\lambda}}\right|=\frac{1}{3}$

$\lambda=\frac{5}{3}$

10. The image of the point $(-1,3,4)$ in the plane x-2y=0 is

1) $(15,11,4)$ 2) $\left(-\frac{17}{3},-\frac{19}{3},1\right)$ 3) $\left(\frac{9}{5},-\frac{13}{5},4\right)$ 4) $\left(-\frac{17}{3},-\frac{19}{3},4\right)$

Key. 3

Sol. $\frac{h+1}{1}=\frac{k-3}{-2}=\frac{p-4}{0}=-2\left(\frac{-1-6}{5}\right)$

$(h,k,p)=\left(\frac{9}{5},\frac{-13}{5},4\right)$

11. The plane passing through the points $(-2,-2,2)$ and containing the line joining the points $(1,1,1)$ and $(1,-1,2)$ makes intercepts on the coordinates axes, the sum of whose lengths is

1. 3 2. 4 3. 6 4. 12

Key. 4

Sol. Equation of the plane be $a(x+2)+b(y+2)+c(z-2)=0.$ As it passes through $(1,1,1)$ and $(1,-1,2), \frac{a}{1}=\frac{b}{-3}=\frac{c}{-6}$. Equation of the plane is $\frac{x}{-8}+\frac{y}{8/3}+\frac{z}{8/6}=1$ and the required sum $=12.$

12. An equation of the plane containing the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and the point $(0,7,-7)$ is

1. $x+y+z=0$
2. $x+2y-3z=35$
3. $3x-2y+3z+35=0$
4. $3x-2y-z=21$

Key. 1

Sol. Equation of the plane is $A(x+1)+B(y-3)+C(z+2)=0$ where $3A+2B+1=0$ and $A+B(7-3)+C(-7+2)=0$

13. The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the coordinate axes at A, B, C respectively. D and E are the mid-points of AB and AC respectively. Coordinates of the mid-point of DE are

1. $(a, b/4, c/4)$ 2. $(a/4, b, c/4)$ 3. $(a/4, b/4, c)$ 4. $(a/2, b/4, c/4)$

Key. 4

Sol. $A(a,0,0), B(0,b,0), C(0,0,c), D(a/2,b/2,0), E(a/2,0,c/2)$ so midpoint of DE is $(a/2, b/4, c/4).$

14. The coordinates of a point on the line $x=4y+5, z=3y-6$ at a distance $3\sqrt{26}$ from the point $(5,0,-6)$ are

1. $(17,3,3)$ 2. $(-7,3,-15)$ 3. $(-17,-3,-3)$ 4. $(7,-3,15)$

Key. 1

Sol. Line is $\frac{x-5}{4/\sqrt{26}}=\frac{y}{1/\sqrt{26}}=\frac{z+6}{3/\sqrt{26}}$. A point on this line at a distance $3\sqrt{26}$ from $(5,0,-6)$ is $(5\pm(3\times4), \pm3, -6\pm9)=(17,3,3)$ or $(-7,-3,-15).$

15. The points $(0,7,10), (-1,6,6)$ and $(-4,9,6)$ are the vertices of

1. A right angled isosceles triangle
2. Equilateral triangle
3. An isosceles triangle
4. An obtuse angled triangle

Key. 1

Sol. Length of the sides are 18, 18 and 36.

16. Equation of a plane bisecting the angle between the planes $2x-y+2z+3=0$ and $3x-2y+6z+8=0$ is

1. $5x-y-4z-45=0$
2. $5x-y-4z-3=0$
3. $23x+13y+32z-45=0$
4. $23x-13y+32z+5=0$

Key. 2

Sol. Equations of the planes bisecting the angle between the given planes are

$$\frac{2x-y+2z+3}{\sqrt{2^2+(-1)^2+2^2}}=\pm\frac{3x-2y+6z+8}{\sqrt{3^2+(-2)^2+6^2}}$$

$\Rightarrow\ 7(2x-y+2z+3)=\pm 3(3x-2y+6z+8)$

$\Rightarrow\ 5x-y-4z-3=0$ taking the $+ve\ sign$, and $23x-13y+32z+45=0$ taking the – ve sign.

17. If the perpendicular distance of a point P other than the origin from the plane $x+y+z=p$ is equal to the distance of the plane from the origin, then the coordinates of P are

1. $(p,2p,0)$
2. $(0,2p,-p)$
3. $(2p,p,-p)$
4. $(2p,-p,2p)$

Key. 3

Sol. The perpendicular distance of the origin $(0,0,0)$ from the plane $x+y+z=p$ is

$$\left|\frac{-p}{\sqrt{1+1+1}}\right|=\frac{|p|}{\sqrt{3}}.$$

If the coordinates of P are (x,y,z), then we must have

$$\left|\frac{x+y+z-p}{\sqrt{3}}\right|=\frac{|p|}{\sqrt{3}}$$

$\Rightarrow\ |x+y+z-p|=|p|$

Which is satisfied by (c)

18. If p_1,p_2,p_3 denote the distances of the plane $2x-3y+4z+2=0$ from the planes $2x-3y+4z+6=0, 4x-6y+8z+3=0$ and $2x-3y+4z-6=0$ respectively, then

1. $p_1+8p_2-p_3=0$
2. $p_3^2=16p_2^2$

3. $8p_2^2 = p_1^2$ 4. $p_1 + 2p_2 + 3p_3 = \sqrt{29}$

Key. 1 or 4

Sol. Since the planes are all parallel planes, $p_1 = \frac{|2-6|}{\sqrt{2^2+3^2+4^2}} = \frac{4}{\sqrt{4+9+16}} = \frac{4}{\sqrt{29}}$

Equation of the plane $4x - 6y + 8z + 3 = 0$ can be written as $2x - 3y + 4z + 3/2 = 0$

So $p_2 = \frac{|2-3/2|}{\sqrt{2^2+3^2+4^2}} = \frac{1}{2\sqrt{29}}$ and $p_3 = \frac{|2+6|}{\sqrt{2^2+3^2+4^2}} = \frac{8}{\sqrt{29}}$

$\Rightarrow \quad p_1 + 8p_2 - p_3 = 0$

19. The radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is cut by the plane $x + 2y + 2z + 7 = 0$ is

1. 2 2. 3 3. 4 4. 1

Key. 2

Sol. Centre of the sphere is $(-1,1,2)$ and its radius is $\sqrt{1+1+4+19} = 5.$

Length of the perpendicular from the centre on the plane is $\left|\frac{-1+2+4+7}{\sqrt{1+4+4}}\right| = 4$

Radius of the required circle is $\sqrt{5^2 - 4^2} = 3.$

20. The shortest distance from the plane $12x + 4y + 3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is

1. $11\frac{3}{4}$ 2. 13 3. 39 4. 26

Key. 2

Sol. The centre of the sphere is $(-2,1,3)$ and its radius is $\sqrt{4+1+9+155} = 13$

Length of the perpendicular from the centre of the sphere on the plane is

$$\left|\frac{-24+4+9-327}{\sqrt{144+16+9}}\right| = \frac{338}{13} = 26$$

So the plane is outside the sphere and the required distance is equal to $26 - 13 = 13.$

21. An equation of the plane passing through the line of intersection of the planes $x+y+z=6$ and $2x+3y+4z+5=0$ and the point $(1,1,1)$ is

1. $2x+3y+4z=9$ 2. $x+y+z=3$ 3. $x+2y+3z=6$ 4. $20x+23y+26z=69$

Key. 4

Sol. Equation of any plane through the line of intersection of the given planes is $2x+3y+4z+5+\lambda(x+y+z-6)=0$
It passes through $(1,1,1)$ if $(2+3+4+5)+\lambda(1+1+1-6)=0 \Rightarrow \lambda=14/3$ and the required equation is therefore, $20x+23y+26z=69.$

22. The volume of the tetrahedron included between the plane $3x+4y-5z-60=0$ and the coordinate planes is

1. 60 2. 600 3. 720 4. None of these

Key. 2

Sol. Equation of the given plane can be written as $\frac{x}{20}+\frac{y}{15}+\frac{z}{-12}=1$

Which meets the coordinates axes in points $A(20,0,0), B(0,15,0)$ and $C(0,0,-12)$ and the coordinates of the origin are $(0,0,0).$

$\therefore$ the volume of the tetrahedron $OABC$ is

$$\frac{1}{6}\begin{vmatrix} 0 & 0 & 0 & 1 \\ 20 & 0 & 0 & 1 \\ 0 & 15 & 0 & 1 \\ 0 & 0 & -12 & 1 \end{vmatrix} = \left|\frac{1}{6}\times 20\times 15\times(-12)\right| = 600.$$

23. Two lines $x=ay+b, z=cy+d$ and $x=a^1y+b^1, z=c^1y+d^1$ will be perpendicular, if and only if

1. $aa^1+bb^1+cc^1=0$ 2. $(a+a^1)(b+b^1)(c+c^1)=0$
3. $aa^1+cc^1+1=0$ 4. $aa^1+bb^1+cc^1+1=0$

Key. 3

Sol. Lines can be written as $\frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c}$ and $\frac{x-b^1}{a^1}=\frac{y}{1}=\frac{z-d^1}{c^1}$ which will be perpendicular if and only if $aa^1+1+cc^1=0$

24. A tetrahedron has vertices at $O(0,0,0), A(1,2,1), B(2,1,3)$ and $C(-1,1,2)$. Then the angle between the faces OAB and ABC will be

1. $\cos^{-1}(17/31)$ 2. 30^0 3. 90^0 4. $\cos^{-1}(19/35)$

Key. 4

Sol. Let the equation of the face OAB be $ax+by+cz=0$ where

$a+2b+c=0$ and $2a+b+3c=0 \Rightarrow \dfrac{a}{5}=\dfrac{b}{-1}=\dfrac{c}{-3}$

25. If the angle θ between the lines $\dfrac{x+1}{1}=\dfrac{y-1}{2}=\dfrac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}z+4=0$ is such that $\sin\theta=1/3$, then the value of λ is

1. $3/4$ 2. $-4/3$ 3. $5/3$ 4. $-3/5$

Key. 3

Sol. Since the line makes an angle θ with the plane in makes an angle $\pi/2-\theta$ with normal to the plane

$$\therefore \quad \cos(\frac{\pi}{2}-\theta)=\frac{2(1)+(-1)(2)+(\sqrt{\lambda})(2)}{\sqrt{1+4+4}\times\sqrt{4+1+\lambda}}$$

$$\Rightarrow \quad \frac{1}{3}=\frac{2\sqrt{\lambda}}{3\sqrt{\lambda+5}} \Rightarrow \lambda+5=4\lambda$$

$$\Rightarrow \quad \lambda=5/3$$

26. The ratio in which the yz plane divides the segment joining the points $(-2,4,7)$ and $(3,-5,8)$ is

1. $2:3$ 2. $3:2$ 3. $4:5$ 4. $-7:8$

Key. 1

Sol. Let yz plane divide the segment joining $(-2,4,7)$ and $(3,-5,8)$ in the ration $\lambda:1$. Then $\Rightarrow \dfrac{3\lambda-2}{\lambda+1}=0 \Rightarrow \lambda=\dfrac{2}{3}$ and the required ratio is $2:3$.

27. The coordinates of the point equidistant from the points $(a,0,0),(0,a,0),(0,0,a)$ and $(0,0,0)$ are

1. $(a/3,a/3,a/3)$ 2. $(a/2,a/2,a/2)$ 3. (a,a,a) 4. $(2a,2a,2a)$

Key. 2

Sol. Let the coordinates of the required point be (x,y,z) then

$x^2+y^2+z^2=(x-a)^2+y^2+z^2=x^2+(y-a)^2+z^2=x^2+y^2+(z-a)^2$

$\Rightarrow x=a/2=y=z.$ Hence the required point is $(a/2,a/2,a/2)$.

28. Algebraic sum of the intercepts made by the plane $x+3y-4z+6=0$ on the axes is

1. $-13/2$ 2. $19/2$ 3. $-22/3$ 4. $26/3$

Key. 1

Sol. Equation of the plane can be written as $\frac{x}{-6}+\frac{y}{-2}+\frac{z}{3/2}=1$

So the intercepts on the coordinates axes are $-6,-2,3/2$ and the required sum is

$-6-2+3/2=-13/2.$

29. If a plane meets the co-ordinate axes in A,B,C such that the centroid of the triangle ABC is the point $(1,r,r^2)$, then equation of the plane is

1. $x+ry+r^2z=3r^2$ 2. $r^2x+ry+z=3r^2$ 3. $x+ry+r^2z=3$ 4. $r^2x+ry+z=3$

Key. 2

Sol. Let an equation of the required plane be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

This meets the coordinates axes in $A(a,0,0),B(0,b,0)$ and $C(0,0,c)$.

So that the coordinates of the centroid of the triangle ABC are

$(a/3,b/3,c/3)=(1,r,r^2)\,(given)\Rightarrow a=3,b=3r,3r^2$ and the required equation of the plane is

$\frac{x}{3}+\frac{y}{3r}+\frac{z}{3r^2}=1$ or $r^2x+ry+z=3r^2$.

30. An equation of the plane passing through the point $(1,-1,2)$ and parallel to the plane $3x+4y-5z=0$ is

1. $3x+4y-5z+11=0$ 2. $3x+4y-5z=11$ 3. $6x+8y-10z=1$ 4. $3x+4y-5z=2$

Key. 1

Sol. Equation of any plane parallel to the plane $3x+4y-5z=0$ is $3x+4y-5z=K$

If it passes through $(1,-1,2)$, then $3-4-5(2)=K\Rightarrow K=-11$

So the required equation is 3x+4y-5z+11=0.

31. Equations of a line passing through $(2,-1,1)$ and parallel to the line whose equations are $\frac{x-3}{2}=\frac{y+1}{7}=\frac{z-2}{-3}$,is

1. $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-1}{2}$
2. $\frac{x-2}{2}=\frac{y+1}{7}=\frac{z-1}{-3}$
3. $\frac{x-2}{2}=\frac{y-7}{-1}=\frac{z+3}{1}$
4. $\frac{x-3}{2}=\frac{y+1}{-1}=\frac{z-2}{1}$

Key. 2

Sol. The required line passes through $(2,-1,1)$ and its direction cosines are proportional to $2,7,-3$ so its equation is $\frac{x-2}{2}=\frac{y+1}{7}=\frac{z-1}{-3}$

32. The ratio in which the plane $2x-1=0$ divides the line joining $(-2,4,7)$ and $(3,-5,8)$ is

1. 2:3
2. 4:5
3. 7:8
4. 1:1

Key. 4

Sol. Let the required ratio be $k:1$, then the coordinates of the point which divides the join of the points $(-2,4,7)$ and $(3,-5,8)$ in this ratio are given by $\left(\frac{3k-2}{k+1},\frac{-5k+4}{k+1},\frac{8k+7}{k+1}\right)$

As this point lies on the plane $2x-1=0$.

$\Rightarrow \frac{3k-2}{k+1}=\frac{1}{2} \Rightarrow k=1$ and thus the required ratio as $1:1$.

33. If l_1, m_1, n_1 and l_2, m_2, n_2, are d.c.'s of $\vec{OA}$, $\vec{OB}$ such that $\angle AOB=\theta$ where 'O' is the origin, then the d.c.'s of the internal bisector of the angle $\angle AOB$ are

(A) $\frac{l_1+l_2}{2\sin\theta/2},\frac{m_1+m_2}{2\sin\theta/2},\frac{n_1+n_2}{2\sin\theta/2}$

(B) $\frac{l_1+l_2}{2\cos\theta/2},\frac{m_1+m_2}{2\cos\theta/2},\frac{n_1+n_2}{2\cos\theta/2}$

(C) $\frac{l_1-l_2}{2\sin\theta/2},\frac{m_1-m_2}{2\sin\theta/2},\frac{n_1-n_2}{2\sin\theta/2}$

(D) $\frac{l_1-l_2}{2\cos\theta/2},\frac{m_1-m_2}{2\cos\theta/2},\frac{n_1-n_2}{2\cos\theta/2}$

Key. B

Sol. Let OA and OB be two lines with d.c's l_1, m_1, n_1 and l_2, m_2, n_2. Let OA = OB = 1. Then, the coordinates of A and B are (l_1, m_1, n_1) and (l_2, m_2, n_2), respectively. Let OC be the bisector of $\angle AOB$. Then, C is the mid point of AB and so its coordinates are $\left(\frac{l_1+l_2}{2},\frac{m_1+m_2}{2},\frac{n_1+n_2}{2}\right)$.

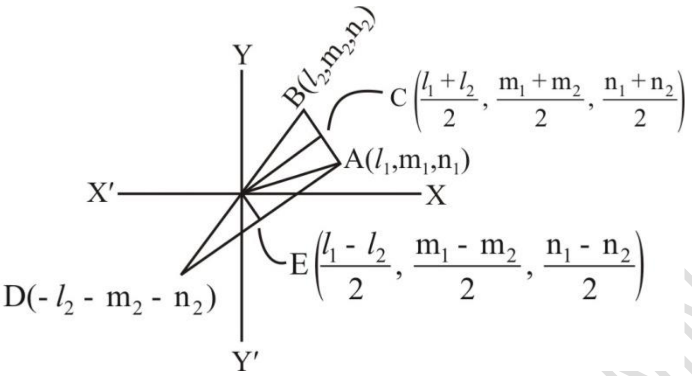
$\therefore$ d.r's of OC are $\frac{l_1+l_2}{2},\frac{m_1+m_2}{2},\frac{n_1+n_2}{2}$

We have, $OC=\sqrt{\left(\frac{l_1+l_2}{2}\right)^2+\left(\frac{m_1+m_2}{2}\right)^2+\left(\frac{n_1+n_2}{2}\right)^2}$

$=\frac{1}{2}\sqrt{\left(l_1^2+m_1^2+n_1^2\right)+\left(l_2^2+m_2^2+n_2^2\right)+2\left(l_1l_2+m_1m_2+n_1n_2\right)}$

$=\frac{1}{2}\sqrt{2+2\cos\theta}$ $\quad [\because \cos\theta=l_1l_2+m_1m_2+n_1n_2]$

$=\frac{1}{2}\sqrt{2(1+\cos\theta)}=\cos\left(\frac{\theta}{2}\right)$



$\therefore$ d.c's of OC are $\frac{l_1+l_2}{2(OC)},\frac{m_1+m_2}{2(OC)},\frac{n_1+n_2}{2(OC)}$

34. A line is drawn from the point $P(1,1,1)$ and perpendicular to a line with direction ratios $(1,1,1)$ to intersect the plane $x+2y+3z=4$ at Q. The locus of point Q is

A) $\frac{x}{1}=\frac{y-5}{-2}=\frac{z+2}{1}$ B) $\frac{x}{-2}=\frac{y-5}{1}=\frac{z+2}{1}$

C) $x=y=z$ D) $\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$

Key. A

Sol. Locus of 'Q' is the line of intersection of the plane $x+2y+3z=4$ and $1(x-1)+1(y-1)+1(z-1)=0 \Rightarrow$ then the line is $\frac{x}{1}=\frac{y-5}{-2}=\frac{z+2}{1}$

35. A line is drawn from the point P(1, 1, 1) and perpendicular to a line with direction ratios $(1,1,1)$ to intersect the plane $x+2y+3z=4$ at Q. The locus of point Q is

A) $\frac{x}{1}=\frac{y-5}{-2}=\frac{z+2}{1}$ B) $\frac{x}{-2}=\frac{y-5}{1}=\frac{z+2}{1}$ C) $x=y=z$ D) $\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$

Key: A

Hint: Locus of Q is the line of intersection of the plane $x+2y+3z=4$ and $1(x-1)+1(y-1)+1(z-1)=0\Rightarrow$ then line is $\frac{x}{1}=\frac{y-5}{-2}=\frac{z+2}{1}$

36. If a line with direction ratios 2 : 2: 1 intersects the line $\frac{x-7}{3}=\frac{y-5}{2}=\frac{z-3}{1}$ and $\frac{x-1}{2}=\frac{y+1}{4}=\frac{z+1}{3}$ at A and B then AB=.

a) $\sqrt{2}$ b) 2 c) $\sqrt{3}$ d) 3

Key:

Hint $A(7+3\alpha,5+2\alpha,3+\alpha),B(1+2\beta,-1+4\beta,-1+3\beta)$

Dr's of AB are 2:2:1

$\frac{6+3\alpha-2\beta}{2}=\frac{3+\alpha-2\beta}{1}=\frac{4+\alpha-3\beta}{1}$

$\alpha=-2,\beta=1$

A(1,1,1)B(3,3,2)

AB = 3

37. A, B, C are the points on x, y and z axes respectively in a three dimensional co-ordinate system with O as origin. Suppose the area of triangles OAB, OBC and OCA are 4, 12 and 6 respectively, then the area of the triangle ABC equals

(A) 16 (B) 14 (C) 28 (D) 32

Key: B

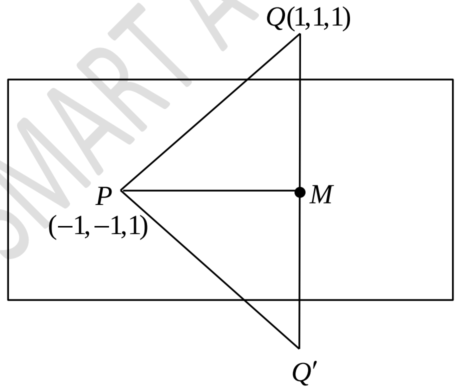
Hint $[ABC]=\sqrt{[OAB]^2+[OBC]^2+[OCA]^2}$

where $[ABC]$ = area of triangle ABC

38. The area of the figure formed by the points $(-1,-1,1);(1,1,1)$ and their mirror images on the plane $3x+2y+4z+1=0$ is

(a) $\frac{5\sqrt{33}}{29}$ (b) $\frac{4\sqrt{33}}{29}$ (c) $\frac{20\sqrt{33}}{27}$ (d) $\frac{20\sqrt{33}}{29}$

Key. D



Sol.

Req. area = ΔPQQ^1

$=2\Delta PQM$

$=2\cdot\frac{1}{2}\cdot QM\cdot PM$

39. If a plane passes through the point $(1,1,1)$ and is perpendicular to the line $\frac{x-1}{3}=\frac{y-1}{0}=\frac{z-1}{4}$ then its perpendicular distance from the origin is

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{7}{5}$ (D) 1

Key: C

Hint: The d.r of the normal to the plane is 3, 0, 4 . The equation of the plane is $3x+0y+4z+d=0$ since it passes through $(1,1,1)$ so; $d=-7$

Now distance of the plane $3x+4z-7=0$ from $(0,0,0)$ is $\frac{7}{\sqrt{3^2+4^2}}=\frac{7}{5}$ unit

40. Three straight lines mutually perpendicular to each other meet in a point P and one of them intersects the x-axis and another intersects the y-axis, while the third line passes through a fixed point (0, 0, c) on the z-axis. Then the locus of P is

A) $x^2+y^2+z^2-2cx=0$ B) $x^2+y^2+z^2-2cy=0$

C) $x^2+y^2+z^2-2cz=0$ D) $x^2+y^2+z^2-2c(x+y+z)=0$

Key: C

Hint: Let L_1, L_2, L_3 be the mutually perpendicular lines and P(x_0, y_0, z_0) be their point of concurrence. If L_1 cuts the x-axis at A(a, 0, 0), L_2 meets the y-axis at B(0, b, 0) and C(0, 0, c) $\in L_3$, then $L_1 11(x_0-a,\ y_0,\ z_0)$, $L_2 11(x_0,\ y_0-b,\ z_0)$ and $L_3 11(x_0,\ y_0,\ z_0-c)$. Hence

$$\left.\begin{array}{l} x_0(x_0-a)+y_0(y_0-b)+z_0^2=0 \\ x_0^2+(y_0-b)y_0+z_0(z_0-c)=0 \end{array}\right\}$$

$$x_0(x_0-a)+y_0^2+z_0(z_0-c)=0$$

Eliminating a and b from the equations, we get

$$x_0^2+y_0^2+z_0^2-2cz_0=0$$

41. The centroid of the triangle formed by (0, 0, 0) and the point of intersection of $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-1}{1}$ with $x=0$ *and* $y=0$ is

(a) $(1,1,1)$ (b) $\left(\frac{1}{6},\frac{-1}{3},\frac{1}{6}\right)$ (c) $\left(\frac{-1}{6},\frac{1}{3},\frac{-1}{6}\right)$ (d) $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

Key. B

Sol. Any point on the given line $(K+1,2K+1,K+1)$

but $x=0 \quad \Rightarrow A(0,-1,0)$

$y=0 \quad \Rightarrow B(\frac{1}{2},0,\frac{1}{2});0(0,0,0)$

42. The plane $x-y-z=4$ is rotated through 90^o about its line of intersection with the plane $x+y+2z=4$ and equation in new position is $Ax+By+Cz+D=0$ where A,B,C are least positive integers and $D<0$ then

(a) $D=-10$ (b) $ABC=-20$

(c) $A+B+C+D=0$ (d) $A+B+C=10$

Key. D

Sol. Given planes are $x-y-z=4$ ---------- (1) and $x+y+2z=4$ ---------- (2)

Since required plane passes through the line of intersection (1) & (2)

$\Rightarrow$ Its equation is $(x-y-z-4)+\alpha(x+y+2z-4)=0$

$\Rightarrow(1+\alpha)x+(\alpha-1)y+(2\alpha-1)z-(4\alpha+4)=0$ ---------- (3)

Since (1) & (3) are perpendicular

$\Rightarrow 1(1+\alpha)-1(\alpha-1)-1(2\alpha-1)=0$

$1+\alpha-\alpha+1-2\alpha+1=0 \qquad \Rightarrow \alpha=3/2$

$\Rightarrow$ Its equations is $(x-y-z-4)+\frac{3}{2}(x+y+2z-4)=0$

$5x+y+4z-20=0$

43. Three lines $y-z-1=0=x;\ z+x+1=0=y;\ x-z-1=0=y$ intersect the xy plane at A, B, C then orthocenter of triangle ABC is

(a) $(0,1,0)$ (b) $(-1,0,0)$ (c) $(0,0,0)$ (d) $(1,1,1)$

Key. A

Sol. Intersection of $y-z-1=0=x$ with xy plane gives $A(0,1,0)$ similarly $B(-1,0,0)$, $C(1,0,0)$

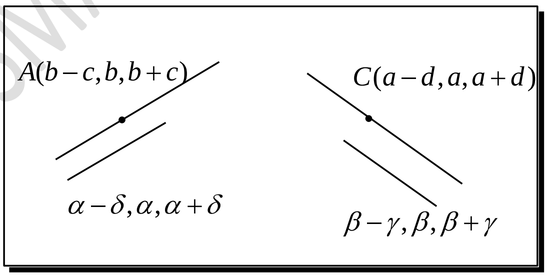
$\therefore$ orthocentre is $(0,1,0)$

44. The lines $\frac{x-a+d}{\alpha-\delta}=\frac{y-a}{\alpha}=\frac{z-a-d}{\alpha+\delta};\frac{x-b+c}{\beta-r}=\frac{y-b}{\beta}=\frac{z-b-c}{\beta+r}$ are coplanar and the equation of the plane in which they lie is

(a) $x+y+z=0$ (b) $x-y+z=0$ (c) $x-2y+z=0$ (d) $x+y-2z=0$

Key. C

Sol.



45. The reflection of the point P(1, 0, 0) in the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ is

(a) (3, –4, –2) (b) (5, –8, –4) (c) (1, –1, –10) (d) (2, –3, 8)

Key: b

Hint: Coordinates of any point Q on the given line are

(2r + 1, –3r – 1, 8r – 10) for some $r \in R$

So the direction ratios of PQ are 2r, –3r – 1, 8r – 10

Now PQ is perpendicular to the given line

if 2(2r) – 3 (–3r – 1) + 8(8r – 10) = 0

$\Rightarrow$ 77r – 77 = 0 $\Rightarrow$ r = 1

and the coordinates of Q, the foot of the perpendicular from P on the line are (3, –4, –2).

Let R(a, b, c) be the reflection of P in the given lines when Q is the mid-point of PR

$\Rightarrow \frac{a+1}{2}=3, \frac{b}{2}=-4, \frac{c}{2}=-2$

$\Rightarrow$ a = 5, b = –8, c = –4

and the coordinates of the required point are $(5,-8,-4)$.

46. Reflection of plane 2x + 3y + 4z + 1 = 0 in plane x + 2y + 3z – 2 = 0 is

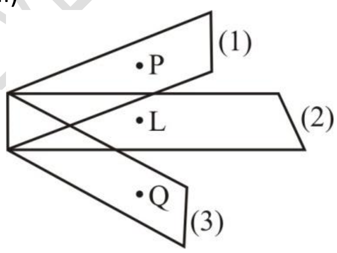
(A) 6x – 19y + 32z = 47 (B) 6x + 19y + 32z = 47

(C) 6x + 19y + 16z = 47 (D) 3x + 19y + 16z = 47

Key. B

Sol. 2x + 3y + 4z + 1 = 0 .....(i)

x + 2y + 3z – 2 = 0 .....(ii)



(iii) is reflection of plane

reflection of ax + by + cz + d = 0 in $a'x+b'y+c'z+d'=0$

$=(aa'+bb'+cc')(a'x+b'y+c'z+d')$

$=(a'^2+b'^2+c'^2)(ax+by+cz+d)$

$2(2+6+12)(x+2y+3z-2)=(1^2+2^2+3^2)(2x+3y+4z+1)$

$4(x+2y+32-2)=14(2x+3y+4z+1)$

$12x+38y+64z=94$

$\Rightarrow 6x+19y+32z=47$

47. The reciprocal of the distance between two points, one on each of the lines $\frac{x-2}{3}=\frac{y-4}{2}=\frac{z-5}{5}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$

(A) cannot be less than 9 (B) having minimum value $5\sqrt{3}$

(C) cannot be greater than 78 (D) cannot be $2\sqrt{19}$

Key. D

Sol. The shortest distance (SD) = $\frac{\left|\begin{vmatrix} 2-1 & 4-2 & 5-3 \\ 2 & 3 & 4 \\ 3 & 2 & 5 \end{vmatrix}\right|}{\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 2 & 5 \end{vmatrix}\right|}=\frac{1}{\sqrt{78}}$

So, $\frac{1}{SD}=\sqrt{78}$

48. Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is

(A) $x + 2y - 2z = 0$ (B) $3x + 2y - 2z = 0$

(C) $x - 2y + z = 0$ (D) $5x + 2y - 4z = 0$

Key. C

Sol. Vector along the required plane is $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix}$ = $8\hat{i}-\hat{j}-10\hat{k}$

So, normal vector $(\vec{n})$ to the plane is $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & -1 & -10 \\ 2 & 3 & 4 \end{vmatrix}=26\hat{i}-52\hat{j}+26\hat{k}$.

So, equation of the plane is $\vec{r}.\vec{n}=0 \Rightarrow x - 2y + z = 0$.

49. The distance between the plane $x - 2y + z - 6 = 0$ and the plane containing the sets of points $(1 + 2\lambda, 2 + 3\lambda, 3 + 4\lambda)$ and $(2 + 3\mu, 3 + 4\mu, 4 + 5\mu)$, where λ, μ are parameters, is

(A) $\sqrt{3/2}$ (B) $\sqrt{6}$

(C) $\sqrt{12}$ (D) $2\sqrt{6}$

Key. B

Sol. Normal vector : $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$ = $-\hat{i}+2\hat{j}-\hat{k}$

equation of plane: $-1(x - 1) + 2(y - 2) - 1(z - 3) = 0$

$\Rightarrow x - 2y + z = 0$

So, required distance = $\frac{|6|}{\sqrt{1+4+1}}$ = $\sqrt{6}$

50. If the point (0, λ, 1) lies within the triangular prism formed by the planes x = 0, 2y – z + 2 = 0 and 2y + 3z – 6 = 0 then the set of values of λ is

(A) (–2, 2)

(B) $\left(-\frac{1}{2}, \frac{3}{2}\right)$

(C) $\left(-4, -\frac{4}{3}\right)$

(D) (0, 4)

Key. B

Sol. The planes are 2y + z = 0, 5x – 12y = 13 and 3x + 4z = 10

Solving we get z = $\frac{11}{2}$

3D-Geometry

Integer Answer Type

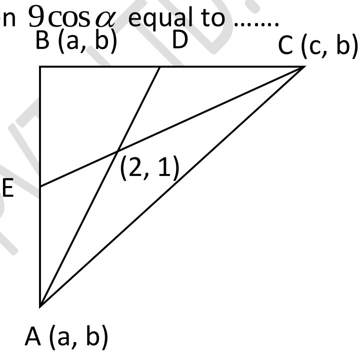
1. The foot of the perpendicular from (1,2,3) to the join of (6,7,7), (9,9,5) is (3,5, λ) then λ =

Key. 9

Sol. Any point of the line joing the given points can be taken as $\left(6+3t,7+2t,7-2t\right)$ if it is the required foot of the $\perp$ of (1,2,3) we get 3 (5+3t)+2 (5+2t)-2(4-2t) =0 $\Rightarrow t=-1$

2. The plane $2x-2y+z=3$ is rotated about the line where it cuts the xy plane by an acute angle α . If the new position of plane contains the point (3, 1, 1) then $9\cos\alpha$ equal to …….

Key: 7



Hint: Let equation of new plane $2x-2y+z-3+\lambda z=0$

Point (3, 1, 1) lie on $it \Rightarrow \lambda=-2$

Hence equation of new plane 2x – 2y –z = 3

$$\cos\alpha=\frac{4+4-1}{3.3}=\frac{7}{9}$$

3. Shortest distance between the z-axis and the line $\mathrm{x}+\mathrm{y}+2\mathrm{z}-3=0=2\mathrm{x}+3\mathrm{y}+4\mathrm{z}-4$ is ___

Ans: 2.

Hint : Equation of any plane ; continuing the general plane is

$$\mathrm{x}+\mathrm{y}+2\mathrm{z}-3+\lambda\left(2\mathrm{x}+3\mathrm{y}+4\mathrm{z}-4\right)=0---(1)$$

$$\text{if plane (1) is parallel to z-axis } \Rightarrow \lambda=-\frac{1}{2}$$

Therefore plane, parallel to z-axis is y+2=0-----(2)

Now, shortest distance between any point on z-axis (0, 0, 0) (say) from plane (2) is 2

4. The point P (1,2,3) is reflected in the xy – plane, then its image Q is rotated by 180°about the x – axis to produce R , and finally R is translated in the direction of the positive y – axis through a distance d to produce S (1,3,3). The value of d is

ANS : 3

Hint Reflecting the point (1,2,3) in the xy – plane produces (1,2,-3) . A half turn about the x – axis yields (1,-2,3). Finally translation 5 units will produce (1,3,3)

5. Let A,B,C be three non-collinear points. Then n be the no. of lines lying in plane containing the points A, B, C which are equidistant from all three points then $n+5=$

Key: 8

6. The equation of the plane passing through the intersection of the planes $2\mathrm{x}-5\mathrm{y}+\mathrm{z}=3$ and $\mathrm{x}+\mathrm{y}+4\mathrm{z}=5$ and parallel to the plane $\mathrm{x}+3\mathrm{y}+6\mathrm{z}=1$ is x + 3y + 6z = k, where k is

Key : 7

Sol : Equation of plane passing through the intersection of the planes $2x-5y+z=3$ and $x+y+4z=5$ is

$$(2x-5y+z-3)+\lambda(x+y+4z-5)=0$$

$$\Rightarrow(2+\lambda)x+(-5+\lambda)y+(1+4\lambda)-3-5\lambda=0 \qquad \text{....(i)}$$

which is parallel to the plane x + 3y + 6z = 1.

Then $\frac{2+\lambda}{1}=\frac{-5+\lambda}{3}=\frac{1+4\lambda}{6}$

Then, $\frac{2+\lambda}{1}=\frac{-5+\lambda}{3}=\frac{1+4\lambda}{6}$

$\therefore \lambda=\frac{-11}{2}$

from eq. (i),

$$-\frac{7}{2}x-\frac{21}{2}y-21z+\frac{49}{2}=0$$

$\therefore\ x+3y+6z=7$

Hence, k = 7

7. If the distance of a point lying on the plane 2x +3y + 6z = p from the point (3, 0, 1) is unity then the total number of possible values of p, where p is a prime number, is

Key. 6

Sol. $\frac{|2(3)+3+6(1)-p|}{\sqrt{2^2+3^2+6^2}}\le 1$

$\Rightarrow |12 - p| \le 7 \Rightarrow -7 \le p - 12 \le 7$

$\Rightarrow 5 \le p \le 19 \Rightarrow 5, 7, 11, 13, 17, 19$

i.e. six possible values of p.

8. A line from the origin meets the lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ and $\frac{x-8/3}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ at P and Q respectively. If the distance PQ = l then the value of [l] (where [.] represents the greatest integer function), is

Key. 2

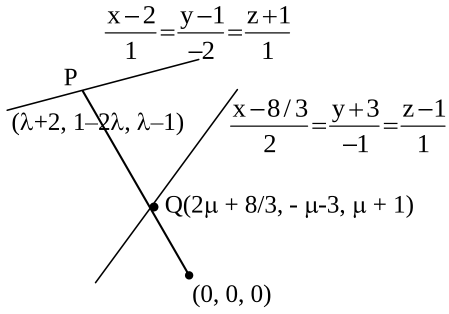
Sol. From the given conditions, we have,

$$\frac{2\mu+8/3}{\lambda+2}=\frac{\mu+3}{2\lambda-1}=\frac{\mu+1}{\lambda-1}$$

$\Rightarrow \lambda = 3,\ \mu = \frac{1}{3}$

$\Rightarrow P \equiv (5, -5, 2)\ Q \equiv \left(\frac{10}{3},\frac{-10}{3},\frac{4}{3}\right)$

$\Rightarrow l = PQ = \sqrt{6} \Rightarrow [l] = 2$



9. The shortest distance between the z-axis and the line, x + y + 2z – 3 = 0, 2x + 3y + 4z – 4 = 0 is :

Key. 2

Sol. The equation of any plane containing the given line is

$(x+y+2z-3)+\lambda(2x+3y+4z-4)=0$

$\Rightarrow (1+2\lambda)x+(1+3\lambda)y+(2+4\lambda)z-(3+4\lambda)=0 \quad ....(1)$

If the plane is parallel to z-axis whose direction cosines are 0, 0, 1 ; then the normal to the plane will be perpendicular to z-axis

$\therefore \quad (1+2\lambda)(0)+(1+3\lambda)(0)+(2+4\lambda)(1)=0$

$\Rightarrow \quad \lambda=-\frac{1}{2}$

Put in eq. (1), the required plane is

$(x+y+2z-3)-\frac{1}{2}(2x+3y+4z-4)=0 \Rightarrow y+2=0...(2)$

$\therefore$ S.D. = distance of any point say (0, 0, 0) on z-axis from plane (2)

$=\frac{2}{\sqrt{(1)^2}}=2$

10. If equation of the plane through the straight line $\frac{x-1}{2}=\frac{y+2}{-3}=\frac{z}{5}$ and perpendicular to the plane x – y + z + 2 = 0 is ax – by + cz + 4 = 0, then find the value of 10^3a+10^2b+10c

Ans. 1710

Sol. Let equation of a plane containing the line be $l(x-1)+m(y+2)+nz=0$

then $2l-3m+5n=0$ and $l-m+n=0$

$\therefore \quad \frac{l}{2}=\frac{m}{3}=\frac{n}{1}$

$\therefore$ the plane is 2(x – 1) + 3(y + 2) + z = 0

i.e. 2x + 3y + z + 6 = 0

$\therefore$ a = 2, b = - 3, c = 1

$\therefore \quad 10^3a+10^2b+10c=2000-300+10=1710$ Ans.

11. Find the equation to the line which intersects the lines

x + y + z = 1, 2x – y – z = 2

x + y – z = 3, 2x + 4y – z = 4

and passes through the point (1, 1, 1)

Ans. 19

Sol. The line intersecting the given lines is

$$\left.\begin{aligned}(x + y + z - 1) + \lambda\,(2x - y - z - 2) = 0\\(x - y - z - 3) + \mu\,(2x + 4y - z - 4) = 0\end{aligned}\right\} \qquad ...(i)$$

If it passes through (1, 1, 1), then we get from (1)

$\lambda = 1$ and $\mu = 4$

Hence the required equations to the intersecting line are x – 1 = 0 = 9x + 15y – 5z + 19. Ans

12. Find the shortest distance and the vector equation of the line of shortest distance between the lines given by $\vec{r} = 3\vec{i} + 8\vec{j} + 3\vec{k} + \lambda\left(3\vec{i} - \vec{j} + \vec{k}\right)$ and $\vec{r} = -3\vec{i} - 7\vec{j} + 6\vec{k} + \mu\left(-3\vec{i} + 2\vec{j} + 4\vec{k}\right)$.

Ans. $\vec{r} = 3\vec{i} + 8\vec{j} + 3\vec{k} + \lambda\left(-6\vec{i} - 15\vec{j} + 3\vec{k}\right)$

Sol. $\vec{r} = 3\vec{i} + 8\vec{j} + 3\vec{k} + \lambda\left(3\vec{i} - \vec{j} + \vec{k}\right)$ …(i)

$\vec{r} = -3\vec{i} - 7\vec{j} + 6\vec{k} + \mu\left(-3\vec{i} + 2\vec{j} + 4\vec{k}\right)$ …(ii)

Let L and M be points on the line (i) and (ii) respectively

So that LM is perpendicular to both the lines

Let position vector of L be $3\vec{i} + 8\vec{j} + 3\vec{k} + \lambda_0\left(3\vec{i} - \vec{j} + \vec{k}\right)$

and the position vector of M be $-3\vec{i} - 7\vec{j} + 6\vec{k} + \mu_0\left(-3\vec{i} + 2\vec{j} + 4\vec{k}\right)$

then $\overrightarrow{LM} = -6\vec{i} - 15\vec{j} + 3\vec{k} - \lambda_0\left(3\vec{i} - \vec{j} + \vec{k}\right) + \mu_0\left(-3\vec{i} + 2\vec{j} + 4\vec{k}\right)$

since $\overrightarrow{LM}$ is perpendicular to both the lines (i) and (ii)

$\therefore$ $\overrightarrow{LM}.\left(3\vec{i} - \vec{j} + \vec{k}\right) = 0$ and $\overrightarrow{LM}.\left(-3\vec{i} + 2\vec{j} + 4\vec{k}\right) = 0$

Thus – 18 + 15 + 3 - λ_0 (9 + 1 + 1) + μ_0 (- 9 – 2 + 4) = 0

i.e. $-11\lambda_0 - 7\mu_0 = 0$ …(iii)

and 18 – 30 + 12 $-\lambda_0$ (- 9 – 2 + 4) $+\mu_0$ (9 + 4 + 16) = 0

i.e. $7\lambda_0 + 29\mu_0 = 0$ …(iv)

from (iii) and (iv) we get

$\lambda_0 = \mu_0 = 0$

$\therefore$ $\overrightarrow{LM} = -6\vec{i} - 15\vec{j} + 3\vec{k}$

$\therefore$ $\left|\overrightarrow{LM}\right| = \sqrt{36 + 225 + 9} = \sqrt{270} = 3\sqrt{30}$

Position vector of L is $3\vec{i} + 8\vec{j} + 3\vec{k}$

$\therefore$ equation of the line of shortest distance (LM) is

$\vec{r} = 3\vec{i} + 8\vec{j} + 3\vec{k} + \lambda\left(-6\vec{i} - 15\vec{j} + 3\vec{k}\right)$

$\vec{r} = 3\vec{i} + 8\vec{j} + 3\vec{k} + \lambda\left(-6\vec{i} - 15\vec{j} + 3\vec{k}\right)$

13. If the lengths of external and internal common tangents to two circles

$x^2 + y^2 + 14x - 4y + 28 = 0$ and $x^2 + y^2 - 14x + 4y - 28 = 0$ are λ and μ. Then the value of $\left[\frac{\lambda+\mu}{4}\right]$ is equal to (where [.] denotes greatest integer function)

Ans. 4

Sol. $c_1c_2 > r_1 + r_2$

External = $\sqrt{d^2 - (r_2 - r_1)^2} = 14 = \lambda$

Internal = $\sqrt{d^2 - (r_1 + r_2)} = 4 = \mu$

$\lambda + \mu = 18 \qquad \left[\frac{\lambda+\mu}{4}\right] = 4$

14. Consider two concentric circle $C_1 : x^2 + y^2 = 1$ and $C_2 : x^2 + y^2 - 4 = 0$. A parabola is drawn through the points where C_1 meet the x-axis and having arbitrary tangent of C_2 as its directrix. Then locus of focus of drawn parabola is $\frac{3}{4}x^2 + y^2 = k$, then value of k is

Ans. 3

Sol. $(h-1)^2 + k^2 = (\cos\theta - 2)^2$ - (1)

$(h+1)^2 + k^2 = (\cos\theta + 2)^2$ - (2)

(2) – (1) gives us $\cos\theta = \frac{h}{2}$

(2) + (1)

$2(h^2 + k^2 + 1) = 2(\cos^2\theta + 4)$

$\frac{3}{4}x^2 + y^2 = 3$

15. All chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ that subtend a right angle at the origin, pass through a fixed point (h, k) then h – k is equal to

Ans. 3

Sol. Let the equation of the chord to y = mx + c

Combined equation of the line joining the point of intersection with origin is

$3x^2 - y^2 - 2(x - 2y)\left(\frac{y - mx}{c}\right) = 0$

$\Rightarrow x^2(3c + 2m) - y^2(c - 4) - 2xy(1 + 2m) = 0$

From the condition of perpendicularity, we get 3c + 2m – c + 4 = 0

$\Rightarrow m + c = -2$

i.e the line y = mx + c, passes through (1, -2)